Consistency and Quality Control

Differences and similarities regarding exam task construction and syllabus content between IB Maths SL and Maths 1-4 at the Swedish Natural Sciences Programme

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The recent trend in didactical research regarding mathematics in Sweden has pointed towards the need to distinguish between rote learning of algorithms and an actual understanding of mathematics, often in connection with a variety of “competencies” displayed by the student through different types of examinations. Despite the didactical shift this has caused in the Swedish mathematics curriculum, however, the declining results of Swedish students in international surveys such as PISA and TIMSS continue. Since improved results have yet to be seen, this essay investigates the differences and similarities regarding syllabus content and exam task construction between the current maths courses of the Swedish Natural Sciences (NV) programme and the closest corresponding maths course in the International Baccalaureate (IB) Diploma Programme, Maths SL. The aim is to distinguish what sets the Maths SL course apart regarding consistency and quality control, and why the Swedish national tests as of yet fail to provide the nation-wide consistency at a high quality level that is sought after.
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INTRODUCTION

Swedish students’ declining results in international surveys such as TIMSS and PISA have led to a decade long reform process of the Swedish school system. As a backlash to what is commonly referred to as “flumskolan”¹, the Swedish education system has been pushed in the direction of more standardised tests, earlier grades, increased discipline, and teacher in-job training to secure higher quality teaching². The most recent national high school curriculum, Gy11, is a standards-based, goal-oriented curriculum featuring more details and a higher-level mathematical content than the preceding Lp94, as well as an outright demand for teaching within Swedish secondary education to be “based on scientific principles and well-tried experience”³. Despite an ongoing expansion within Swedish research on the didactics of mathematics and the implementation of the curriculum, however, the desired improvements in students’ mathematical achievement following the implementation are yet to be seen.

As a contrast, the International Baccalaureate Diploma Programme (IB DP), which is also offered by a number of Swedish high schools, has seen no significant decline in student results over the years⁴. The IB, which was founded in 1968 and is now offered across all continents, features extensive goal-oriented syllabi within each subject, all encompassed within the “IB Learner Profile” that promotes critical thinking, transdisciplinarity, compassion and knowledgeability. Originally based on the didactical work of for example Dewey, Neill, Piaget, Bruner and Peterson⁵, there is today an entire field of research concerning international education in which the IB often features, and where IB curricula or didactical models are often the objects of investigation⁶. Due to the large attention that Swedish students’ declining results in general, but especially those in mathematics, have received, the aim of this paper is therefore identify the similarities and differences between the Swedish high school mathematics syllabus and that of the IB Maths Standard Level (SL), as well as how the respective syllabus content is assessed and what implications such assessment brings for students’ learning. Based on the findings, suggestions are then made as to what kind of changes, if any, could be beneficial for Swedish high school mathematics education.

Students at the Natural Sciences Programme in Sweden are required to take a total of four courses in mathematics, each finalized with a standardised test, during their three years of high school. In comparison, the IB Maths SL exam consists of two exam papers, taken one or a few days apart, that cover the entire syllabus all at once. During the two-year SL course, students are also required to submit an internal assessment called “Mathematical Exploration”. Due to these structural differences, Section I of this paper provides an overview

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¹ “Flumskolan” is a commonly used invective that refers to an educational system where pupil democracy and inclusion are considered more important than the gaining of traditional knowledge, and where homework and tests are considered unnecessary (Bråting & Österman, 2015, 3). There are very few certainties regarding this concept since it was conceived as a political battering ram, but according to Borzoo Kavakoli (DN, 2014-06-04) “flumskolan” grew from the 1960’s and -70’s curriculum reforms. Its didactical principles are most clearly distinguishable in the 1994 Swedish national curriculum (Lgr94).

² Bråting & Österman, 2015, p.3

³ Skolverket, 2011, p. 5. Author’s translation

⁴ IBO, 2014a, p.7

⁵ IBO, 2015

⁶ See for example The International Education Research Database (IERD) and The Journal of Research in International Education (JRIE).
of each education system, as well as in-depth descriptions of the respective syllabbi. Section II compares the content and structure of Maths SL exam questions with exam paper questions from all four levels of the Swedish national tests in mathematics, focusing on what cognitive demands they place on the learner. Based on this analysis, an attempt to draw conclusions about how well the exams relate to or fail to relate to the respective syllabbi, as well as what implications such an alignment or misalignment brings for students, is made.

The term syllabus is used in this essay when referring to the documents that outline the aim and scope of specific courses, course content and assessment criteria. The more general term curriculum is used when referring to the over-arching goals and the aggregate of courses of an education system. Translations into English of the Swedish syllabus have been made by the author.

WHAT DOES IT MEAN TO “KNOW” MATHEMATICS?

Many teachers, philosophers and didactical researchers have tried to answer the questions “Why should students study mathematics?”; “What is the best way to learn mathematics?”; and “What does it mean to know mathematics?”. Today’s Swedish high school mathematics syllabus mainly stresses the practical and social aspects of mathematics as the answer to the first question\(^7\), while the IB mathematics syllabus outlines mathematics as “a well-defined body of knowledge, […] an abstract system of ideas [and] a useful tool” that is not only of practical use but also “an important key to understanding the world in which we live”\(^8\). The answers to the these three questions are thus intimately linked, in that defining what one means by knowing mathematics and what the uses of such knowledge is, also in many ways decides what the best ways of learning mathematics are. In order to define what one means by knowing mathematics, however, one must first define what one means by knowledge in general, and learning in general.

Most of today’s research within the field of education traces its roots to the works of Jean Piaget, Lev Vygotsky and John Dewey, who in the 20\(^{th}\) century made great contributions to, and set the foundation for, the modern Philosophy of Education. The, for this paper, most noteworthy feature of the Piagetian school is its focus on the mind’s cognitive functions and the integration of new information into a constructed but ever-changing personal world view by means of interaction with objects or subjects\(^9\). Vygotsky presented the idea of a Zone of Proximal Development (ZPD), which led to an increased focus on the teacher as the main actor when it comes to drawing out students’ potential. He also stressed, as did Dewey, that the learner is not a single entity separated from the society around them, and thus learns by imitating and, at higher levels, through transmission by language\(^10\). The ideas of John Dewey

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\(^7\) The only non-practical reasons given for the study of mathematics are phrased as such: “[mathematics] is developed from both practical needs and human curiosity and desire to explore mathematics as such” and “[u]ltimately, mathematics is about discovering patterns and formulating general linkages”. The syllabus also states that the mathematics education should provide students with “experience of the logic of mathematics, its generalizability, creative qualities and multi-faceted character” (Skolverket, Gy11, Kursplan i matematik).

\(^8\) IBO, 2012, p. 4

\(^9\) Phillips and Soltis, 2009, p. 49.

\(^10\) Phillips and Soltis, 2009, p. 57-59
have arguably had the greatest impact on current didactical research and we will come back to his concepts of the formal (logical) aspects and the psychological aspects of knowledge. One more researcher who is interesting to note in this context, however, is Jerome Bruner. His idea of education as training in the now to tackle problems in the future, i.e. to create a transfer of learning, is evident in both the Swedish and the IB curriculum and provides what is today one of the most common answers to the first question asked in this section. The analysis in Section III aims to decide whether the respective mathematics syllabi and assessment methods can be said to fulfil such a requirement and, if so, which transferrable skills they encourage.

Modern Philosophy of Education sprung from the works of the above and is now a vast field of research with input from Psychology, Philosophy and Biology, to mention but a few. The ideas of knowledge as personally assimilated but socially constructed and transferred with the use of language in a social setting, have had a great impact on how knowledge of mathematics is defined. Traditionally, knowing mathematics was intimately linked to showing skills in arithmetics and logical deduction. With the widening and deepening of mathematics as a field of study, however, these became but two parts of what is now referred to as mathematical competence. Two large contributions to this modern definition of knowledge of mathematics come from the 2001 American study Adding it Up by Kilpatrick et. al. and the 2002 Danish Kompetencer og Matematiklæring (KOM) project led by Mogens Niss. Both studies present sets of abilities\(^\text{11}\), see the table below, that they consider to constitute mathematical competence. Note that although there are strong similarities, there are, since they have been developed within slightly different discourses, no direct parallels between the two sets.

<table>
<thead>
<tr>
<th>KOM-project</th>
<th>Adding it Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Thinking mathematically (mastering mathematical modes of thought)</td>
<td>Conceptual understanding (comprehension of mathematical concepts, operations and relations)</td>
</tr>
<tr>
<td>2 Posing and solving mathematical problems</td>
<td>Procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately)</td>
</tr>
<tr>
<td>3 Modelling mathematically (analysing and building models)</td>
<td>Strategic competence (ability to formulate, represent and solve mathematical problems)</td>
</tr>
<tr>
<td>4 Reasoning mathematically</td>
<td>Adaptive reasoning (capacity for logical thought, reflection, explanation, and justification)</td>
</tr>
<tr>
<td>5 Representing mathematical entities (objects and situations)</td>
<td>Productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy)</td>
</tr>
<tr>
<td>6 Handling mathematical symbols and formalisms</td>
<td></td>
</tr>
<tr>
<td>7 Communicating in, with, and about mathematics</td>
<td></td>
</tr>
<tr>
<td>8 Making use of aids and tools</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1: Overview of the components of mathematical competence as defined by the KOM project and Kilpatrick et. al. in Adding it Up.

\(^{11}\) The term used in the KOM project report are competencies (Niss, 2002). In order to avoid confusion, however, in this paper the components of mathematical competence will be referred to as abilities.
According to some researchers, a reflective ability is now “considered to be the major source of knowledge on all levels of mathematics”\textsuperscript{12}, which would certainly signify a large shift in the way we view mathematics as a school subject. A more balanced view is upheld by many, however, arguing that although problem solving, reflection and mathematical communication are important, one cannot go further than one’s knowledge of concepts and mathematical procedures allow\textsuperscript{13}. The components of mathematical competence outlined above reflect a similar, balanced view of mathematics but contain, apart from the inclusion in both sets of conceptual knowledge as well as problem solving and reasoning skills, some notable differences. Most interesting to note, Kilpatrick et al have chosen to add productive disposition as one strand of mathematical proficiency, while the KOM abilities include communication and tool usage abilities. A similar difference between the IB Maths SL assessment tools and the components of the Swedish National Test will be expounded upon in Section II.

The inclusion of communication as a key skill coincides with a similar shift in Swedish mathematics education, where over the past thirty years more and more demands have been made for Swedish students to express themselves verbally, making it more difficult for students with language difficulties to aspire for higher grades in mathematics\textsuperscript{14}. Bråting & Österman’s (2015) analysis of mathematics syllabi from the 1960’s up to today’s syllabus shows that focus has shifted from mainly securing that students can show efficacy in arithmetic calculations and evidence of logical thought, to them having to not only perform the calculations but also to discuss and evaluate the methods they used in their calculations\textsuperscript{15}. The adoption of such a metaperspective, while relying on vast amounts of data claiming that only rote learning of mathematical concepts and procedures does not stimulate mathematical understanding\textsuperscript{16}, has, Bråting & Österman claim, unfortunately been pushed too far in some areas, leading to an actual decrease, rather than increase, in students’ mathematical skills due to the lessened focus on pure mathematical procedures\textsuperscript{17}. Whether or not such a marked shift can be seen in today’s Swedish exams will be investigated and determined in relation to the IB Maths SL exam in the analysis of exam task structure in Section II.

As can be seen in both the KOM project abilities and from the five abilities listed in \textit{Adding it Up}, problem solving is considered an important mathematical skill for students to acquire. Much research has been made regarding the processes of problem-solving\textsuperscript{18} and also its link to our minds’ long term memory and working memory. Szabo (2013) combines the long term memory with the processes of the working memory into what he calls our mathematical memory. This is crucial for problem-solving because it relies on procedural knowledge stored in the long term memory to be called upon by the working memory, and also on generalisations of past problems that the student has encountered\textsuperscript{19}. As also Ahlberg (1992) and Fey (1994) have stressed, “pupils’ understanding of a problem depends on their prior

\textsuperscript{12} Scholz, in Biehler et al, 1994, p. 229
\textsuperscript{13} See for example Fischbein and Lompscher, in Biehler et al, 1994
\textsuperscript{14} This view is also supported by Dahland who states that “Many students’ difficulties with mathematics originate in the poor handling of language, as regards both Swedish and the language of mathematics” (Dahland, in Strässer, 2003, p. 74)
\textsuperscript{15} Bråting & Österman, 2015, p.6
\textsuperscript{16} See for example Jonsson et al, 2014, Bergqvist, 2000, etc.
\textsuperscript{17} Bråting & Österman, 2015, p. 15
\textsuperscript{18} See for example Silver, 1987, and Szabo, 2013.
\textsuperscript{19} Szabo, 2013
experiences” and they therefore need “a repertoire of significant conceptual and procedural knowledge and the ability to transfer that knowledge”\textsuperscript{21}. Lack of procedural knowledge, therefore, causes stress and uncertainty and hinders the problem solving process\textsuperscript{22}, while having encountered many types of mathematical problems earlier significantly facilitates problem solving.

We might conclude, then, that in order for students to learn mathematics, they first need to acquire solid conceptual and procedural knowledge. It is the task of the teacher, however, to ensure that such knowledge is put to use in such a way that the students can gain and store problem solving strategies for later use. So far, this is in line with both the KOM project abilities and those described by Adding it Up. Both sets of abilities also stress students’ reasoning abilities, which can be verbal or expressed in mathematical notation. One notable difference, however, is that Kilpatrick et. al. have chosen to add productive disposition as one strand of mathematical proficiency, while the KOM abilities include communication and tool usage abilities. These slightly differing views on mathematical competence reflect a wider disagreement of what it actually means to know mathematics, and are reflected in differing curricula, as we will see in Section I. It becomes even clearer, however, when one looks at assessment related to said curricula, which will be the topic of Section II.

SECTION I: Educational Structure, Syllabus and Assessment

GENERAL EDUCATIONAL STRUCTURE AND ASSESSMENT PRACTICES

The topic of assessment will be dealt with more thoroughly in Section II, but the main traits of the IB DP and the Swedish high school system are outlined here.

Swedish high school education normally spans three years for students aged 16-19, and students are divided into so called programmes. Students choose programmes depending on their general interests and each programme specializes in certain areas of knowledge. Some programmes are vocational (focusing on carpentry, nursing, etc, while providing basic academic knowledge) while others are almost exclusively academic and prepare students for further studies within the humanities, social sciences, engineering or natural sciences. On top of the common subjects, such as mathematics, Swedish and English, each programme has certain programme-specific subjects and courses that all students must take. In general, students study 10-14 subjects, each divided into courses. Students are given a choice in years 2 and 3 as to what higher-level courses they wish to take in order to complete their full course of studies. In order to graduate, students must also complete a gymnasiarbeten, a long-term task in which they alone or within a group delve deeper into an area of interest, and plan, execute, write a report on and orally present a project related to their studies\textsuperscript{23}. At the end of

\textsuperscript{20} Ahlberg, 1992, in Strässer, 2005, p. 53
\textsuperscript{21} Fey, in Biehler et al, 1994, p. 20
\textsuperscript{22} Szabo, 2013
\textsuperscript{23} http://www.gymnasium.se/om-gymnasiet/om-program-gymnasiet-5143, accessed 2016-02-05
each subject course or assignment, a grade is set by the teacher based on the student’s performance.

In Sweden, the International Baccalaureate Diploma Programme (IB DP) is one of the programmes that students can choose within the national high school education system. Its assessment and organisation, however, rests with the International Baccalaureate Organisation (IBO) and not under the Swedish National Agency for Education (Skolverket). The IB DP is in itself a two-year programme, but for Swedish students it has been extended by a Pre-Diploma (Pre-DP) year in order to conform to Swedish standards. While the Pre-DP year is structured similarly to the first year of a national academic high school programme, students that continue on to the IB DP narrow their studies to only six subjects. Each is chosen from one of six groups of subjects: Language and Literature (first language), Language Acquisition (second language), Individuals and Societies (social sciences), Experimental Sciences (natural sciences), Mathematics, and the Arts (this group includes Drama, Music and Fine Arts but the subject can also be chosen freely from groups 1-5). The curriculum is designed to encourage the study of a broad range of academic areas, while at the same time providing in-depth knowledge in each subject. Apart from the six subjects, students must complete a course in Theory of Knowledge (TOK), write an Extended Essay (a 4000 word academic essay concerning an optional topic), and so called CAS (Creativity, Action, Service) assignments. Assessment within the six subjects is mainly made by external examiners following a final exam at the end of year two. Internal assessments, corrected by the subject teachers and moderated by external examiners, account for twenty percent of the final grade in each subject.

THE SWEDISH MATHEMATICAL SYLLABUS

The recent developments within Swedish didactical research traces the international widening of the field of mathematical philosophy and has to a large extent been influenced by the KOM project definition of mathematical competence as “the ability to understand, judge, do, and use mathematics across a variety of mathematical situation”. The current mathematics syllabus centres around seven abilities that closely resemble those presented in the KOM report. Therefore, starting from the assumption that the Swedish high school mathematics syllabus aims to promote students’ mathematical competence as defined by Niss, the following overview aims to identify what type of formal, conceptual, procedural and topic-specific mathematical knowledge it actually requires from the student. It also outlines the syllabus structure and potential didactical implications.

MATEMATIK 1C, 2C, 3C and 4

As mentioned above, Swedish high school, or gymnasium, is divided into programmes based on students’ general areas of interest (natural sciences, humanities, arts, etc). Since 2011, mathematics education has been conducted in three “strands” of courses called the “a”, “b”,

24 IBO, Mathematics SL guide, 2012
25 Niss, in Blum et al, 2007
and “c” strands, where the choice of programme decides which strand students will take. Pupils at vocational programmes study the “a” strand and are obligated to complete one course, Ma1a, while further courses are optional. Students preparing for further studies within the humanities or social studies are obligated to take at least two “b” strand courses, Ma1b and Ma2b, which require a higher level of abstraction than the corresponding “a” strand courses. On an even higher level of abstraction, and with more focus on mathematical rigour and formalism, is the “c” strand, which is mandatory for students at the natural sciences (NV) programme and the technical (TE) programme. NV and TE students are required to complete a minimum of three courses, Ma1c, Ma2c and Ma3c, but a majority of students opt to continue on to Ma4, from which some continue even further on to Ma5. Since this essay aims to compare the syllabi of two different types of curriculum, it is important for the general level of mathematics to be compatible. I have therefore opted to compare the IB Maths SL syllabus with a combination of Ma1c, 2c, 3c and 4 because the mathematical content is similar, albeit not identical; they provide similar qualifications when applying to university; and both aim to prepare students for further studies within a multitude of areas.

Since the Swedish courses are all examined and graded separately, some topics reoccur in the respective syllabi, but the level of skills demanded increases as the courses progress. According to the stated intent of the syllabus, the general aim of mathematics education in Swedish high schools is to

“allow students to develop the ability to work mathematically […], develop an understanding of mathematical concepts and methods and develop different strategies for solving mathematical problems and using mathematics in social and vocational situations.”

The syllabus also stresses students’ ability to “place mathematics in different contexts and see its importance to individuals and society at large.” No concrete examples of such contexts are given; rather, a high level of abstraction is maintained in the syllabus that leaves room for teachers’ own interpretation. However, the syllabus lists and repeatedly comes back to the seven skills that students are expected to acquire. These permeate and span across all content topics so that students are expected to, when having finished the courses, be able to:

1. Use and describe the meaning of mathematical concepts and relations between concepts.
2. Manage operations and solve standard tasks with and without tools.
3. Formulate, analyse and solve mathematical problems and assess chosen strategies, methods and results.
4. Interpret realistic situations and form mathematical models. Also to use models and evaluate their properties and limitations.
5. Follow, form and evaluate mathematical lines of reasoning.
6. Communicate mathematical thoughts orally, in writing, and through actions.

26 Skolverket, 2011a, p.1
27 Ibid., p.1
28 The actual wording is: "Undervisningen i ämnet matematik ska ge eleverna förutsättningar att utveckla förmågan att […]" (Skolverket, 2011a, p.1). This implies that the main responsibility for students’ learning lies with the teacher, who should give them “the pre-requisites for developing their skills”. The role of the teacher is a topic of hot debate in Sweden at the time of writing, but since it is outside the scope of this essay to continue that discussion, I have chosen to interpret the statement as a general demand towards both students and teachers. After all, students who fail to fulfil the criteria, no matter who is to blame, fail the courses.
7. Relate mathematics as a science to its importance and usage within other sciences, in a vocational, social and historical context.\(^{29}\)

These seven skills form the basis of the assessment criteria, which are outlined in each course syllabus. All assessment criteria rely on descriptive distinctions between levels of knowledge, where a passing level (the grade E) is commonly defined with words such as “synoptic”, “simple” and “with some certainty”. A pass with merit grade, C, requires “elaborate” descriptions or definitions, “some certainty”, and that the student chooses between procedures and also comments on why a specific method was chosen. To pass with distinction, i.e. to acquire an A grade, the student must show confidence and effectiveness, perform mathematical operations “with certainty”, and form and evaluate “sound and nuanced” mathematical arguments.\(^{30}\) Since the criteria differ very slightly between the courses, figure 1 shows the full assessment criteria (translated from Swedish) for the course Ma1c, with the only additional demands from the courses 3c and 4 added in brackets.

Even though the general skills required, and therefore the assessment criteria, do not change significantly over the courses, the mathematical content,\(^{31}\) and thereby the technical level,\(^{32}\) does. Figure 2 shows this progression from Ma1c to Ma4. Problem solving skills are required at all levels, but the content changes from a more diverse coverage in courses 1c and 2c, including arithmetics, basic algebra, classic geometry, vectors, probability and statistics, to purely algebraic and analytic content in courses 3 and 4. It is worth noting that this algebraic and analytic content in some aspects exceeds that of the IB Maths SL syllabus, described below, both when it comes to number theory (complex numbers are studied in the more comprehensive Maths Higher Level, HL, but not in Maths SL) and analytic concepts and procedures (differential equations are, again, included in Maths HL).

\(^{29}\) Skolverket, 2011a, p.1
\(^{30}\) Ibid.
\(^{31}\) The term used in the syllabus is ”Centralt Innehåll” which can be translated in a number of ways: as ”content,” ”central content,” or “fundamental content”. The term leaves it open for teachers to add any content they deem suitable, but such content cannot be used for assessment.
\(^{32}\) The technical level is one of three dimensions of mathematical competence described by the KOM project and refers to what level of conceptual or technical difficulty each ability can be activated at. The other two dimensions are the coverage grade (whether students display only receptive or also productive abilities regarding a chain of reasoning) and action radius, the number and variety of contexts in which a student can apply a certain skill (Helenius, 2006, p.12-13)
<table>
<thead>
<tr>
<th>E (Pass)</th>
<th>C (Pass with Merit)</th>
<th>A (Pass with Distinction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The student is able to give a synoptic explanation of mathematical concepts with the help of some representations, and can synoptically explain how different concepts are related. (The student also alternates between these representations with some certainty) The student can, with some certainty, use these related concepts in order to solve familiar mathematical problems and problems within programme specific setting. The student uses a few simple procedures and solves standard tasks with some certainty, with and without digital tools.</td>
<td>The student is able to give an elaborate explanation of mathematical concepts with the help of some representations, and can elaborately explain how different concepts are related. The student also alternates between these representations with some certainty. The student can, with some certainty, use these related concepts in order to solve mathematical problems and problems within programme specific setting. The student uses several procedures (including advanced arithmetic and algebraic expressions) and solves standard tasks with certainty, with and without digital tools.</td>
<td>The student is able to give an elaborate explanation of mathematical concepts with the help of several representations, and can elaborately explain how different concepts are related. The student can, with certainty, use these related concepts in order to solve mathematical problems and problems within programme specific setting. The student uses several procedures (including advanced arithmetic and algebraic expressions) and solves standard tasks effectively and with certainty, with and without digital tools.</td>
</tr>
<tr>
<td>The student is able to formulate, analyse and solve simple mathematical problems. These include a few concepts and demand simple interpretations. The student transforms realistic problem situations into mathematical terms by applying given mathematical models. The student can, with simple judgements, evaluate the reasonableness of their results and the models, strategies and methods employed to reach it.</td>
<td>The student is able to formulate, analyse and solve mathematical problems. These include several concepts and demand advanced interpretations. The student transforms realistic problem situations into mathematical terms by choosing and applying mathematical models. The student can, with simple judgements, evaluate the reasonableness of their results and the models, strategies and methods employed to reach it, as well as suggest alternative solutions.</td>
<td>The student is able to formulate, analyse and solve complex mathematical problems. These include several concepts and demand advanced interpretations. In problem solving, the student discovers general correlations and expresses them with symbolical algebra. The student transforms realistic problem situations into mathematical terms by choosing, applying and adapting mathematical models. The student can, with nuanced judgements, evaluate the reasonableness of their results and the models, strategies and methods employed to reach it, as well as suggest alternative solutions.</td>
</tr>
<tr>
<td>The student is able to conduct simple mathematical lines of reasoning and can, with simple arguments, evaluate their own and others’ thoughts and actions and differentiate between guesses and sound statements. The student can, with some certainty, express themselves orally, in writing and with actions that incorporate simple mathematical symbols and other representations.</td>
<td>The student is able to conduct sound mathematical lines of reasoning and can, with nuanced arguments, evaluate their own and others’ thoughts and actions and differentiate between guesses and sound statements. The student can, with some certainty, express themselves orally, in writing and with actions, and uses mathematical symbols and other representations with some adjustments to different purposes and situations.</td>
<td>The student is able to conduct sound and nuanced mathematical lines of reasoning and can, with nuanced arguments, evaluate and advance their own and others’ thoughts and actions and differentiate between guesses and sound statements. (The student can also conduct mathematical proofs). The student can, with certainty, express themselves orally, in writing and with actions, and uses mathematical symbols and other representations with adjustments to different purposes and situations.</td>
</tr>
<tr>
<td>By giving examples, the student relates some course content to its importance within other subjects, vocational and social situations, and mathematical history. The student can also participate in a simple discussion regarding the relevance of these examples.</td>
<td>By giving examples, the student relates some topic content to its importance within other subjects, vocational and social situations, and mathematical history. The student can also put forward sound and nuanced arguments regarding the relevance of these examples.</td>
<td>By giving examples, the student relates some topic content to its importance within other subjects, vocational and social situations, and mathematical history. The student can also put forward sound and nuanced arguments regarding the relevance of these examples.</td>
</tr>
</tbody>
</table>

Figure 2: Assessment criteria for the Swedish NV and TE mathematics syllabus
<table>
<thead>
<tr>
<th>Topic</th>
<th>Ma1c</th>
<th>Ma2c</th>
<th>Ma3c</th>
<th>Ma4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number theory, arithmetic and algebra</td>
<td>Properties of integers, prime numbers and divisibility</td>
<td>Logarithms, proof and use of the laws of logarithms</td>
<td>Absolute value</td>
<td>Methods for calculations with complex numbers in rectangular and polar form</td>
</tr>
<tr>
<td></td>
<td>Arithmetic procedures, including exponents, and strategies for the use of digital tools</td>
<td>Proof and use of algebraic identities</td>
<td>Extension of basic arithmetic to deal with polynomials and rational expressions</td>
<td>The complex plane</td>
</tr>
<tr>
<td></td>
<td>Manipulating algebraic expressions</td>
<td>Algebraic and graphical methods for solving exponential, quadratic and power equations, as well as linear equations with two or three unknowns</td>
<td>Properties of the equation of a circle</td>
<td>The absolute value and conjugate of a complex number</td>
</tr>
<tr>
<td></td>
<td>Linear inequalities</td>
<td>Complex numbers</td>
<td>Use of the unit circle to define trigonometric concepts</td>
<td>Use of de Moivre’s formula</td>
</tr>
<tr>
<td></td>
<td>Algebraic and graphical methods for solving power equations and linear equations and inequalities</td>
<td></td>
<td>Proof and use of the cosine, sine and area formula</td>
<td>Algebraic and graphical methods of solving simple polynomial equations with complex roots</td>
</tr>
<tr>
<td></td>
<td>Definitions of sine, cosine and tangent in regards to right-angled triangles</td>
<td>Definition of the curve concept, equations for straight lines and parables, and how analytic geometry combines geometric and algebraic concepts</td>
<td></td>
<td>Proof and use of trigonometric formulae</td>
</tr>
<tr>
<td></td>
<td>Vectors and their representations</td>
<td>Use of classic theorems regarding uniformity, congruence, and angles</td>
<td></td>
<td>Algebraic and graphical methods of solving trigonometric equations</td>
</tr>
<tr>
<td></td>
<td>Adding, subtracting and scaling vectors</td>
<td>Illustration of the concepts definition, theorem and proof</td>
<td></td>
<td>Mathematical proofs; arithmetic, algebraic and geometrical</td>
</tr>
</tbody>
</table>

Fig. 3: Syllabus content for Ma1c, Ma2c, Ma3c and Ma4
<table>
<thead>
<tr>
<th>Correlations and Transformations</th>
<th>Probability and Statistics</th>
<th>Problem Solving</th>
</tr>
</thead>
</table>
| *Parts per thousand, ppm and percentage units*  
*Compounding percentages, indices, interest rates and mortgages*  
*Domain, range and properties of linear, power and exponential functions*  
*Representations of functions: words, function expressions, tables and graphs*  
*Differentiating equations, inequalities, algebraic expressions and functions* | *Analysis of how statistical methods and results are used in social and scientific settings*  
*Dependent and independent events, probability calculations in several steps* | *Strategies for problem solving, including the use of digital media and tools*  
*Mathematical problems relating to personal finances, social life and applications in other subjects*  
*Mathematical problems relating to mathematical history* |
| *Properties of quadratic equations*  
*Construction of graphs for functions and calculating values and roots of functions, with or without digital tools* | *Statistical methods for reporting observations and measurements, including regression analysis*  
*Methods for calculating different measures of central tendency and variability, including standard deviation*  
*Properties of normally distributed data* | *Strategies for problem solving, including the use of digital media and tools*  
*Mathematical problems relating to personal finances, social life and applications in other subjects*  
*Mathematical problems relating to mathematical history* |
| *Continuous and discrete functions*  
*The concept of a limit*  
*Properties of higher degree polynomial functions*  
*Secant, tangent, rate of change and derivative of a function*  
*Deduction of and use of differentiation rules for power and exponential functions and sums of functions*  
*Properties of the number $e$*  
*Algebraic and graphical methods of finding the value of a function's derivative*  
*Algebraic and graphical methods of solving for maximum and minimum values, including sign changes and the second derivative*  
*Correlation between the graph of a function and the first and second derivative*  
*Primitive functions and definite integrals*  
*The relation between integrals and derivatives*  
*Calculation of simple integrals* | *Sketching of graphs, including asymptotes*  
*Proof and use of differentiation rules for trigonometric, logarithmic, exponential and composite functions, as well as the product and quotient rule*  
*Algebraic and graphical methods for finding integrals, with and without digital tools, including calculating physical quantities and probability distributions*  
*Properties of differential equations and applications in programme specific subjects* | *Strategies for problem solving, including the use of digital media and tools*  
*Mathematical problems relating to personal finances, social life and applications in other subjects*  
*Mathematical problems relating to mathematical history* |

Fig. 3: Syllabus content for Ma1c, Ma2c, Ma3c and Ma4
THE IB CURRICULUM: MATHEMATICS

The first IB syllabi were designed by an international team of educators in 1964 and aimed to include beneficial curricular and didactical practices from many different nations\textsuperscript{33}. Since then they have been the subject of international comparisons and research, and each subject syllabus is regularly updated every five years\textsuperscript{34}. All students taking the IB Diploma Programme are required to study mathematics as one of their six subjects, but can choose to do so at different levels. In terms of qualification for Swedish higher education, the Mathematics Studies course corresponds to the amount and level of mathematics studied at the Swedish Social Studies and Humanities programmes (the b-strand courses). The Standard Level course is considered equal to the mathematics studied at the Natural Sciences and Technical programmes (the c-strand courses) up to and including Ma4, but as mentioned above some elements differ between the Swedish syllabus and the Maths SL syllabus. IB students can also opt for the Mathematics Higher Level course, which corresponds more or less to the c-strand courses up to and including Ma5.

The IB Maths syllabi are more detailed than the Swedish counterparts in that they provide teachers with examples of what is and what is not part of the required content, a section on how the mathematics courses relate to the rest of the IB curriculum, expected prior knowledge, approaches to and aims of the teaching and learning of mathematics SL, as well as a list of command terms and notation\textsuperscript{35}. The presentation below is therefore an overview of the overall aims of IB mathematics education, a summary of the required content and the internal and external assessments of the Standard Level course, and some comments on what skills students are expected to acquire.

IB MATHS SL

The mathematical content of the IB syllabus is assessed as a whole during the final exam and the internal assessment, and is therefore divided by topic only\textsuperscript{36}. Most of the content in Ma1c (apart from functions, see table 2) is included in what is expected to be prior learning\textsuperscript{37} and is therefore, in most schools, covered during the Pre-DP year. Any gaps left behind are expected to be addressed during the instruction of new content covered in the SL syllabus. The six topics covered in the SL course are, in brief:

\textsuperscript{33} Ateskan et al, 2014, p.15
\textsuperscript{34} As Faas & Friesenhahn (2014) write, the IB has a rich source of national curricula to compare and align its subject content and cognitive demands with. The mathematics syllabi were last updated in 2012, to be examined starting May 2014.
\textsuperscript{35} IBO, 2012
\textsuperscript{36} As opposed to the Swedish syllabus, where content is divided both by topic and by course.
\textsuperscript{37} IBO, 2012, p.15
1. **Algebra**  
Sequences and series, exponents and logarithms, binomial expansion.

2. **Functions and equations**  
The concept of a function, the properties of linear, quadratic, exponential, logarithmic and reciprocal functions and their graphs, and graphical and analytical equation solving that includes real-life situations.

3. **Circular functions and trigonometry**  
Radian measures, properties of the circle, trigonometric concepts defined in terms of the unit circle, trigonometric identities and rules, trigonometric functions and some applications, solving trigonometric equations.

4. **Vectors**  
Different representations of vectors and vector operations, scalar product, vector equations, the angle between and point of intersection between two lines.

5. **Statistics and Probability**  
Statistical concepts, presentation of discrete, continuous and grouped data, statistical measures and their interpretations, dispersion, cumulative frequency, linear correlation of bivariate data, scatter diagrams and lines of best fit, use of the regression line of y on x for prediction purposes, concepts of probability, Venn diagrams, conditional probability, applications of discrete random variable probability distributions and expected value for discrete data, binomial distribution, normal distribution, normal distributions and curves, properties of the normal distribution.

6. **Calculus**  
Limit and convergence, derivative from first principles, interpretation and applications of the derivative of \( x^n (n \in \mathbb{Q}) \), \( \sin x \), \( \cos x \), \( \tan x \), \( e^x \) and \( \ln x \), differentiation of sums and real multiples of these functions, chain rule, product and quotient rules, equations of tangents and normals, second and higher derivatives, maximum and minimum points, points of inflexion, relationship between the graphs of \( f, f' \) and \( f'' \), optimization, indefinite and definite integrals of \( x^n (n \in \mathbb{Q}) \), \( \sin x \), \( \cos x \), \( \frac{1}{x} \), \( e^x \) and composites of these with the function \( ax + b \), integration by inspection or substitution of the form \( \int f(g(x))g'(x)dx \), areas under and between curves, volumes of revolution about the x-axis, kinematic problems.

The full syllabus includes more details and further guidance as to what kind of operations and knowledge the students are expected to acquire within the subtopics. It also outlines how the teacher may relate the content to mathematical history from an international perspective, and to applications within other subjects such as Physics, Theory of Knowledge, Biology, Economics, etc.\(^{38}\)

In comparison to the Swedish mathematics curriculum described above, the SL syllabus notably does not include differential equations or complex numbers. It does, however, cover probability, statistics and vectors in more depth by adding the binomial distribution, discrete random variable probability distributions, Venn diagrams, grouped data statistics, cumulative frequency, the scalar product and vector equations.

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\(^{38}\) IBO, 2012
The IB recognizes the importance of both formative and summative assessment “as an integral part of teaching and learning” and encourages teachers to use formative assessment in the process leading up to the final exam, which is a summative assessment, and during the process of creating the internally assessed Mathematical Exploration (see section II for a more detailed description). The skills that students are expected to demonstrate within their Internal Assessment and the final exam are:

1. **Knowledge and understanding**: recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts.

2. **Problem-solving**: recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems.

3. **Communication and interpretation**: transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation.

4. **Technology**: use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems.

5. **Reasoning**: construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.

6. **Inquiry approaches**: investigate unfamiliar situations, both abstract and real-world, involving organizing and analysing information, making conjectures, drawing conclusions and testing their validity.

There are many similarities between the seven expected skills of the Swedish syllabus and the above, but the composition differs in several notable ways that bear resemblance to the differences between the KOM project and the *Adding it Up* definitions of the components of mathematical competence. Firstly, “knowledge and understanding” within the IB syllabus combines the *knowledge of mathematical facts and concepts with techniques* in a variety of familiar and unfamiliar contexts. The Swedish demands for knowledge of concepts and procedures, or techniques, are split apart and presented in a disconnected manner from unfamiliar tasks, focusing instead on “standard tasks” and “interpret[ing] realistic situations”.

The IB syllabus also demands that students apply their problem-solving skills not only to real but also to abstract contexts, something which is not mentioned outright in the Swedish syllabus. Interesting to note is that while the Swedish syllabus is more vague on what is considered a mathematical argument, the construction of such arguments is instead included in the list of content for the course Ma1c. The SL assessment objectives outline mathematical arguments as the “use of precise statements, logical deduction and inference” by using and manipulating mathematical expressions, but it is not included in the syllabus content itself. Rather, it is assumed that such a skill will follow from the process of acquiring other mathematical knowledge. Conversely, it is spelt out in the IB syllabus content, rather

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39 IBO, 2012, p.37
40 Skolverket, 2011
41 This could be because of a marked shift in the focus of Swedish didactics of mathematics, from the logical aspects of mathematics to its psychological aspects. See Bråting and Österman, 2015, for a comment on this.
than as an assessment criterion, how the mathematical content should be related to questions regarding Theory of Knowledge, links to mathematical history and applications within other sciences, while criterion 7 but not the content of the Swedish syllabus covers this. The sixth requirement relates mainly to the skills assessed in the Mathematical Exploration and places higher demands of active mathematical investigation than the corresponding Swedish criterion of relating mathematics to its usages within other sciences and contexts. It is mentioned in the section “Course Aims” of the Swedish syllabus that “[t]he teaching of mathematics shall include varied work models and operation methods, including investigative activities”42, but there is no mandatory assessment that includes such activities.

SECTION II: Exam Task Structure and Assessment

STRUCTURE OF EXAMINATIONS

SWEDISH ASSESSMENT AND NATIONAL TESTS

According to Swedish law, education at all levels should be equal no matter which school a student attends43. However, in a bid to maintain Swedish teachers’ professional integrity (or “space,” see below) each teacher is responsible for assessing and grading their own students’ work. Such grading can be both continuous, for formative purposes, but also, and of more interest to this essay, summative, resulting in a final grade at the end of each course. According to the guidelines provided by Skolverket, ”the teacher shall use all available information about the student’s knowledge, set in relation to the national assessment criteria, when awarding grades”44. This leaves full responsibility with teachers and allows them to include both students’ classroom performance, in-course assignment results, and results on the national test for the course into the final grade. It has, however, led to a discrepancy between the grades awarded on the national test compared to students’ final grades, most often in favour of a higher final grade than test result.45 This is notably very common in mathematics, where the largest difference is between those who fail the national test but still receive a passing grade. According to surveys made by Skolinspektionen (the Swedish School Inspectorate), this discrepancy indicates that students are given a chance to make up for their failed test by some other means. In terms of equality, this leaves much to be desired, since a student’s fate is then determined by who their teacher is and how much support said teacher provides46.

In order to minimise such regional and local differences, the execution and grading of the national tests has been scrutinized and Skolverket now provides extensive support material for teachers on these matters. From a political point of view, thus, schools are encouraged to strive towards alignment between results from the national tests and the final grades. Any infringement on teacher sovereignty regarding grading and assessment is yet to be seen.

42 Skolverket, 2011a, p.1, author’s translation
43 Skolverket, 2011b, p.6
44 Skolverket, 2012, p.10, author’s translation
45 Skolverket, 2014, p.11-12
46 Skolverket, 2009, p.6
however, as the Swedish tradition has long been to trust the teacher as a professional interpreter and execution of curricula, a process deemed to include assessment\textsuperscript{47}. Therefore, teachers can base their assessment on any in-course examination tasks they wish (although these are commonly standard tests\textsuperscript{48}), but all students must take the national tests and the results must be taken into account when setting the final grade.

NATIONAL TESTS: MATHEMATICS

The national tests in mathematics, at all levels, consist of one oral and three written parts. The oral examination is conducted by the teacher over a period of five weeks and often focuses on content areas such as statistics and probability in Ma1c, functions in Ma2c, derivatives and integrals in Ma3c, and complex numbers and trigonometric and differential equations in Ma4c. Students take the oral test in groups of 3-4 students (although exceptions are allowed for students with for example social anxiety) and the abilities that are to be assessed are procedural knowledge, problem solving, reasoning and communication.

The first written part, hereafter called Part 1 (IB exam papers will be referred to as Paper 1 and Paper 2 for easy distinction between the two systems) is performed without any digital tools and mainly assesses students’ conceptual and procedural knowledge, with some elements of problem solving. Once a student has handed in the first part, he or she can bring out their calculators and move on to Part 2. This paper mainly checks for deeper understanding of concepts and more advanced procedures. The third paper consists of more modelling and problem solving tasks and allow students to show reasoning and communication skills at higher levels. All in all, students taking Ma1c have 90 minutes to finish Parts 1 and 2 and 120 minutes to finish Part 3, while higher courses allow 120-150 minutes for Part 1 and 2 and 120 minutes for Part 3.

IB MATHS SL EXAMINATIONS

The Maths SL exam consists of two papers, Paper 1 and Paper 2. Graphic Display Calculators (GDCs) are allowed only for Paper 2 and the total time allowed is 90 minutes per exam paper. The two papers are most often written on separate but consecutive days during the worldwide exam period, which takes place over three weeks in May and in November each year.

Section A of Paper 1 consists of short-response questions and aims to test “students’ knowledge and understanding across the breadth of the syllabus”, while Section B consists of extended-response questions that test “students’ knowledge and understanding of the syllabus in depth”\textsuperscript{49}. Since no GDC is allowed, the questions mainly involve analytic approaches to solutions and marks may be awarded for method, accuracy, answers and reasoning. Paper 2 similarly consists of one short-response and one extended-response section but assesses

\textsuperscript{47} Swedish teachers’ professional space has traditionally included not only teaching but also grading and assessment. The term is translated from the Swedish word “frirum”, used by Skolverket to describe the professional liberties of the teacher to translate curriculum content into actual teaching. The term indicates a willingness of the government to guide but not control teachers’ professional choices (Molin, 2006, p.15).

\textsuperscript{48} Helena Korp refers to “konventionella prov”, i.e. written tests that consist of more or less standardised tasks that become progressively more difficult from the first task to the last (Jönsson, 2013, p.22)

\textsuperscript{49} IBO, 2012, p.40-41, author’s italics
students’ knowledge of the topics in greater detail. Efficiency when using the GDC and conveying the results from the process is necessary to have time to answer all questions, but it is not assessed separately.

The final exam accounts for 80% of the students’ final grade in each subject. The remaining 20% are, in mathematics, decided by what is known as the Mathematical Exploration, a compulsory internal assessment with which students can investigate an area of mathematics that interests them. The final report should be 6-12 pages long and show academic honesty, a clear understanding expressed with good mathematical writing and mathematical communication by use of formulae, diagrams, graphs, etc, and thoughtful reflection. The IB encourages teachers to incorporate work on the exploration into the course so that they can give students proper guidance and opportunities to discuss potential topics or methods of investigation during class hours. Otherwise the teacher should act as a supervisor and guide up until the time that a final draft is handed in. Teachers correct the final version using the assessment criteria provided by the IBO, and a random sample of three explorations per class is sent to external examiners for moderation.

EXAM TASK STRUCTURE

In order to make comparisons between the two systems, each exam will be decomposed using qualitative content analysis. Specifically, three questions are to be answered:

1. How high is the prevalence of exam tasks that contain non-mathematical text?

As mentioned above, concerns have been raised that more and more tasks within the Swedish education system rely heavily on students’ reading abilities. In this essay a distinction between “text-based” tasks (which are stated using text that contains non-mathematical terminology) and “non-text-based” tasks (which rely solely on mathematical terminology or other modes of mathematical representation) is made based on whether or not all the information necessary to solve the task is given using mathematical symbols, formulae, graphs or terminology specific to mathematics.

Fig 4: Example of non-text based question in the Swedish National Test

Fig 5: Example of text based question in the Swedish National Test

IBO, 2012, p.43-46
In figure 4, for example, knowledge of the term *bestäm*, which is context specific to mathematics, together with the number axis and the symbolical expression $|x_1 - x_2|$, provide enough information for a student who speaks very little Swedish to solve the task. The question in figure 5, however, is entirely text based and requires language knowledge outside the area of mathematics for solving.

2. How many and what kind of tasks are assessed as problem solving tasks? Are they more often than not related to information given in text, or vice versa?

Assessment criterion number 4 within the Swedish syllabus stresses that students should interpret *realistic* situations in order to form mathematical models, while the demands within the IB concerning problem solving apply to both *real and abstract* contexts. Is there thus any difference in how problem solving tasks are presented?

3. How many tasks, if any, encourage students to create a, for them, novel reasoning sequence? Do the tasks build up towards such a reasoning or is the student left on his or her own to create the necessary steps?

For example Johan Lithner (Umeå University) has pointed towards a difference between imitative, or rote, learning and therefore reasoning, and creative reasoning. Lithner differentiates between Memorised Reasoning (MR) based on the recollection of complete answers, Algorithmic Reasoning (AR) based on the recollection of suitable algorithms that render problems *trivial*, and Creative Mathematically founded Reasoning (CMR) that involves students creating a, to them, new reasoning sequence or re-creating a forgotten one; supporting their strategy choices and implementations with reference to truth or plausibility; and anchoring their arguments in “intrinsic mathematical properties of the components involved in the reasoning.”

Differentiating between standard tasks, problem solving tasks that students have encountered before, and tasks that are novel to students, is difficult, not to mention impossible, since all students have different mathematical backgrounds. However, a for this essay appropriate view of creativity in mathematics is suggested by Silver (1997). He argues that although some insights have indeed occurred in a rapid and exceptional manner, creativity can more often be said to involve “thinking processes [that] are related to deep, flexible knowledge in content domains and associated with long periods of work and reflection”.

As such, both syllabi aim for this kind of knowledge but the modes of assessment differ. Below there will be an analysis as to how and with what kind of tasks such an aim is translated into practical use.

In order to stay within the scope of this essay, one exam from each system has been selected. Thus, the Swedish National Tests are represented by the 2012 exam while the IB Maths SL

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51 Skolverket, 2012, Np Ma3c
52 Ibid.
53 Lithner, 2007, p.259-66
54 Interpretation by Lithner, 2007, p.267-8
exam is represented by the 2015 May exam\textsuperscript{55}. For a more accurate result, however, a more in-depth study of more exam papers would be preferable.

NATIONAL TESTS:

EXAM TASKS IN MA1C, MA2C, MA3C AND MA4

In the table below, the number of questions in each part of each exam of the Swedish courses is listed, together with the number and percentage of text-based questions. As we can see, the percentage of text-based questions decreases markedly as the courses progress. At the level of Ma3 and Ma4, most of the text within questions only involves specifically mathematical terminology and requires less general reading skills than in Ma1c and Ma2c. Part 3 of each exam generally involves more applied problem solving than the other parts (with the exception of Part 2 in Ma1c which consists of one continuous question with practical applications) and thus has a higher percentage of text-based questions. On average, over the full course of study, text-based questions account for 40.5\% of tasks within the Swedish National Tests.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
 & Ma1c & Text & Text (\%) & Ma2c & Text & Text (\%) & Ma3c & Text & Text (\%) \\
\hline
Part 1 & 14 & 5 & 36 & 19 & 5 & 26 & 14 & 1 & 7 \\
Part 2 & 4 & 4 & 100 & 7 & 2 & 29 & 8 & 0 & 0 \\
Part 3 & 15 & 12 & 80 & 14 & 12 & 86 & 14 & 7 & 50 \\
Oral exam & 2-4 (8)* & 8 & 100 & 1-2 (6)* & 1 & 17 & 1-2 (6)* & 2 & 33 \\
Total % & 71 & 39 & 24 & 28 & & & & & \\
\end{tabular}
\caption{Amount of questions and percentage text-based questions in the Swedish National Tests. *Total number of questions, out of which each student is required to answer but a few.}
\end{table}

Each test comes with an assessment matrix that shows which skills are required for each task, and at what level\textsuperscript{56}. The table in figure 7 is based on this data and shows how many problem solving tasks (hereafter called PL tasks from the Swedish word problemlösning) are found in each part, and how many of those are text-based\textsuperscript{57}. Interestingly, the lowest percentage of text-based problem solving tasks was found in Ma3c rather than Ma4, but there is still a marked difference between the first two courses, where 93\% and 88\% of problem-solving questions are text-based, and the following two courses, where the majority of problem solving questions do not rely on superfluous text. This is in line with the results above and the

\textsuperscript{55} The choice of exams was made based on availability and closeness to the implementation of syllabus changes (2011 for the Swedish syllabus, to be first examined in 2012, and 2012, to be first examined in 2014, for the IB syllabus).

\textsuperscript{56} See appendix 1

\textsuperscript{57} Checking for how many percent of the entire exams that consist of problem solving, we find that 34\% of tasks in Ma1c, 35\% of tasks in Ma2c, 29\% of tasks in Ma3c and 28\% of tasks in Ma4 concern problem solving. That results in an average of 31.5\% PL tasks.
increased abstraction as the courses progress. On average, approximately 65% of problem solving questions are text-based over the full course of study.

<table>
<thead>
<tr>
<th>Number of questions</th>
<th>Ma1c</th>
<th>PL</th>
<th>PL text</th>
<th>Ma2c</th>
<th>PL</th>
<th>PL text</th>
<th>Ma3c</th>
<th>PL</th>
<th>PL text</th>
<th>Ma4</th>
<th>PL</th>
<th>PL text</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 1</td>
<td>14</td>
<td>2</td>
<td>2</td>
<td>19</td>
<td>4</td>
<td>4</td>
<td>14</td>
<td>2</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Part 2</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>8</td>
<td>3</td>
<td>0</td>
<td>14</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Part 3</td>
<td>15</td>
<td>6</td>
<td>5</td>
<td>14</td>
<td>11</td>
<td>10</td>
<td>14</td>
<td>7</td>
<td>4</td>
<td>11</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Oral exam</td>
<td>2-4</td>
<td>2</td>
<td>2</td>
<td>1-2</td>
<td>0</td>
<td>0</td>
<td>1-2</td>
<td>0</td>
<td>0</td>
<td>1-2</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>% text-based PL</td>
<td>93</td>
<td></td>
<td>88</td>
<td>33</td>
<td></td>
<td>45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 7: Amount of problem solving (PL) questions and percentage text-based problem solving (PL text) questions in the Swedish National Tests. *Total number of questions, out of which each student is required to answer but a few.

It is impossible to answer for each student whether a task sets in motion a novel reasoning sequence, or whether he or she has forgotten one but re-creates it. However, in order to check for tasks that require novel reasoning, those tasks that were assessed for reasoning skills were reviewed and sorted into Non-Novel (NN) if similar tasks were to be found in the corresponding text book\(^{58}\), Lead-Up (LU) if the questions required novel thinking but offered clues along the way, and Non-Lead-Up (NLU) if no clues were offered. The results are displayed in figure 8.

<table>
<thead>
<tr>
<th>Number of R tasks</th>
<th>Ma1c</th>
<th>NN</th>
<th>NLU</th>
<th>LU</th>
<th>Ma2c</th>
<th>NN</th>
<th>NLU</th>
<th>LU</th>
<th>Ma3c</th>
<th>NN</th>
<th>NLU</th>
<th>LU</th>
<th>Ma4</th>
<th>NN</th>
<th>NLU</th>
<th>LU</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parts 1,2,3</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Oral exam</td>
<td>7</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>6</td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Text based</td>
<td>15</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>% Text based</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td>42</td>
<td></td>
<td></td>
<td></td>
<td>17</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% NLU or LU</td>
<td>40</td>
<td>36</td>
<td>33</td>
<td>17</td>
<td>36</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td>17</td>
<td></td>
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</tr>
</tbody>
</table>

Fig. 8: Amount of tasks in the Swedish National Tests that are graded for Reasoning skills, R, and what type of task (Non-Novel, Non-Lead-Up or Lead-Up, as well as text based) they can be categorized as.

All in all, 37% of questions in Ma1c, 24% in Ma2c, 29% in Ma3c and 30% in Ma4 include some part that is assessed for mathematical reasoning (an average of 30%). These tasks are exclusively text-based in Ma1c while text-based tasks account for 17-42% in the other three courses. Non-Novel tasks are the most frequent (total of 34 tasks), followed by Non-Lead-Up and Lead-Up (total of 8 tasks each). On average, 32% of R tasks are therefore NLU or LU questions. While a larger study covering more years than one would give a more accurate result, the conclusion can be made that the National Tests studied here mainly consists of

\(^{58}\) Although not all schools use the same textbook, most Swedish schools changed to the Matematik5000 series after the implementation of Gy11. This series has therefore been used for reference.
tasks that resemble those encountered in textbooks, but efforts are also made to create tasks that require a novel kind of (creative) reasoning.

ASSESSMENT

During the courses or in the final exam, students have to show proof of all seven skills at a certain level in order to receive the corresponding final grade. The National Tests, however, despite its intricate division of points into “Eₚ”, “Cₚ”, “Aₚ” or the like, ultimately distributes the grades according to a traditional point system with the added demands of level-specific points (see an example from Ma1c in the table below).

<table>
<thead>
<tr>
<th>Points</th>
<th>E</th>
<th>D</th>
<th>C</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
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<tr>
<td>Level-specific</td>
<td></td>
<td></td>
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<td>demands</td>
<td>At least 20</td>
<td>At least 32</td>
<td>At least 44</td>
<td>At least 54</td>
<td>At least 64</td>
</tr>
<tr>
<td></td>
<td>At least 11 points at C-level or above</td>
<td>At least 20 points at C-level or above</td>
<td>At least 7 points at A-level</td>
<td>At least 12 points at A-level</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9: Grade requirements, including level-specific demands, in the Swedish National Tests.

To aid assessment, teachers are given mark schemes and corrected student solutions for the written parts, as well as example answers to read up on before, and assessment matrices to fill in during, the oral examination. Matrices describing the requirements for each level of each skill are also provided.

IB MATHS SL:

PAPER 1, PAPER 2 AND THE MATHEMATICAL EXPLORATION

EXAM TASK STRUCTURE

The written Maths SL exam aims to assess students’ procedural and conceptual knowledge as well as problem-solving skills. Since it is an international exam that is corrected by professional examiners stationed around the world, the mark scheme focuses on how to eliminate discrepancies in the correction of tasks. Points are awarded for correct answers (A) that mostly require preceding marks for a valid method (M) given for shown working, and/or clear reasoning (R). Correct answers with no working shown can sometimes be awarded (N) marks. Since no specific points are awarded for problem solving, classification of tasks into such a category has been done by the author.
Paper 1 contains fewer text-based questions in general, as does Part A compared to Part B of both papers. The dominance of text-based questions in Part B of this May 2014 Paper 2 is largely due to two continuous questions with practical applications, each containing 4 and 5 tasks respectively. Combining the two papers, the average percentage for text-based questions becomes 34%.

As can be seen in figure 11, 40% of problem solving questions in the exam are text-based. Only 10 out of 61 (≈16%) of all tasks included problem solving, which seems to indicate that for the two exam papers, more emphasis is placed on assessing whether students have acquired and can apply the required procedural and conceptual knowledge, than on problem solving skills in particular. It is worth noting, however, that as a general rule, one point in the IB exam corresponds to approximately one minute of work. Translated into points rather than amount of tasks, problem solving instead constitutes $\frac{24}{90} = 26\%$ of Paper 1 and $\frac{26}{90} = 29\%$ of Paper 2, a distribution which more fairly conveys the time spent on such tasks.

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59 Each paper accounts for a maximum of 90 points and the time limit is 90 minutes.
Out of the total of six tasks (approximately 10% of all tasks) that were assessed for reasoning skills, only two were Non-Novel (33%). This seems to be in line with the IBO’s aim to include “a significant requirement within each examination paper for candidates to tackle questions and/or tasks that are in some way different from what they have done before.”

The third part of the IB Maths SL examination, the Mathematical Exploration (hereafter referred to as ME), cannot be analysed in the same manner as the exam papers since it is a long-term project in which each student decides, under teacher supervision, on a mathematical topic commensurate with the Maths SL syllabus to delve deeper into. The ME is graded according to the criteria Communication (0-4 points), Mathematical Presentation (0-3 points), Personal Engagement (0-4 points), Reflection (0-3 points) and Use of Mathematics (0-6 points). The teacher acts as a guide and supervisor throughout the project and corrects students’ first draft as formative assessment before they hand in the final version, making the ME a process-oriented as well as result-oriented assessment.

The ME is the result of a “[deliberate attempt] to give significant attention to the so-called “higher-order” cognitive skills” of analysis, synthesis and evaluation” and to assess “those skills and areas of understanding that are less appropriately addressed through external examination papers”.

Therefore, no matter what they choose to investigate, students’ reports should consist of an introduction, a rationale, a body, and a conclusion. Students are expected to use appropriate mathematical language, including notation, symbols and terminology, to define key terms where appropriate, and to use multiple forms of mathematical representations (formulae, diagrams, models, etc) where appropriate. Independent and/or creative thinking is encouraged and students must show some analysis and evaluation of the exploration.

The assessment criteria are expected to be available to students at all times to enable them to actively engage in the learning process and to take responsibility for their own learning.

**ASSESSMENT**

The IB awards grades ranging from 1-7 (where 4 is the lowest passing grade for Higher Level courses and 3 the lowest passing grade for Standard Level courses) and the grade boundaries for each subject vary from year to year based on statistical background data regarding previous year’s results, the distribution of predicted grades, etc. The variation is slight, however, and the May 2015 boundaries are yet to be publicly disclosed at the time of writing, so for this paper the boundaries for 2014 are used instead. The results on each part are weighted so that the results from the ME account for 20% of the grade, and Paper 1 and Paper 2 account for 40% respectively.

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60 In this case, calculating the amount of points rewarded yields very similar results, so no distinction will be made.
61 IBO, 2010, p.28
62 Ibid.
63 Ibid, p.54
64 IBO, 2012.
65 IBO, 2010, p.44.
### Table

<table>
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<th>Paper 1</th>
<th>Paper 2</th>
<th>Total</th>
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<td>Points</td>
<td>Grade</td>
<td>Points</td>
</tr>
<tr>
<td>1</td>
<td>0-2</td>
<td>1</td>
<td>0-17</td>
</tr>
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<td>2</td>
<td>3-5</td>
<td>2</td>
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<td>3</td>
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<td>9-11</td>
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<td>5</td>
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<td>6</td>
<td>15-17</td>
<td>6</td>
<td>66-75</td>
</tr>
<tr>
<td>7</td>
<td>18-20</td>
<td>7</td>
<td>76-90</td>
</tr>
</tbody>
</table>

Fig. 13: Grade requirements for the IB Maths SL examinations.

The IBO recognizes that “assessment of the DP is high-stakes, criterion-related performance assessment” and the written exam is therefore corrected and assessed by trained external examiners in order to achieve a sufficiently high level of reliability\(^67\). As mentioned above, with examiners located all over the world, the mark schemes aim to eliminate differences in the awarding of marks. This is done by detailed descriptions of when each type of mark should be awarded; differences between how the mark scheme communicates different methods and answers and the different acceptable modes of response; where clarity is necessary and when it is not; how accurate sketches should be; etc\(^68\). Possible solutions for each question are divided into steps and the mark scheme details what kind of mark can be awarded for each step in the working. The grades are then set exclusively based on the number of points that the student has accumulated, see figure 13.

For the internal assessment, teachers are provided with a guide that outlines the five criteria, what is assessed by each criterion, and what each level requires of the student. The levels are also linked to example MEs that have achieved it for that criterion. Teachers are instructed to consider one criterion at a time and read the description of each achievement level, starting from zero, until a level is found that has not been reached. The level of achievement is then considered to be the preceding level\(^69\). A random selection of each school’s papers are sent in for external moderation that can result in a general increase or decrease of the grades each teacher has awarded, or a confirmation of the teacher’s assessment. The maximum mark is 20 and all students must hand in an ME in order to receive their final grade.

\(^{67}\) IBO, 2010, p.12  
\(^{68}\) IB Maths SL exam, May 2015, Paper 1, TZ1  
\(^{69}\) IBO, 2012
SECTION III: Conclusions

CONCLUSIONS AND DISCUSSION

<table>
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<tr>
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<th>National Tests</th>
<th>IB exam</th>
<th>Points of interest</th>
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<td>Text based</td>
<td>40.5%</td>
<td>34%</td>
<td>ME not included in IB</td>
</tr>
<tr>
<td>Problem solving (PL)</td>
<td>31.5%</td>
<td>28%</td>
<td>IB percentage weighted using points</td>
</tr>
<tr>
<td>PL text based</td>
<td>65%</td>
<td>40%</td>
<td></td>
</tr>
<tr>
<td>Reasoning (R)</td>
<td>30%</td>
<td>10%</td>
<td>52% and 50% text based, respectively</td>
</tr>
<tr>
<td>Non-Lead-Up Reasoning</td>
<td>31%</td>
<td>33%</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 14: Comparison between the Swedish National Tests and the IB exam regarding the prevalence of text-based questions in general and problem solving and reasoning tasks in particular.

*In the IB exam, problem solving was evident in 16% of tasks but 28% of the total points awarded. For just comparison, both are displayed here.

Bråting & Österman (2015) have called attention to the increased focus on communicating mathematics and showing reasoning skills, rather than simply performing calculations, in Swedish mathematics education. Comparing the prevalence of Reasoning tasks between the Swedish National Tests and the IB Maths SL exams, there seems to be cause to believe that such a shift has taken place. Tasks that are designed to provoke novel or non-novel mathematical reasoning sequences account for 30% of tasks within the National Tests, but only 10% in the IB exam. Such tasks are generally considered to be of a higher order cognitive level since they demand in-depth knowledge of mathematical concepts and their properties, and are important tools for testing students’ mathematical understanding. However, the IB also, while focusing mainly on conceptual and procedural knowledge and problem solving in the written exams, assesses such skills and considers them essential. Instead of including such an assessment mainly in the final exam, however, the IB aims to assess the higher-level cognitive skills through the more in-depth and investigative Mathematical Exploration. The main difference between the two systems, then, appears not to lie in their requirement of student to reason mathematically, but in the means by which these skills are assessed. The Swedish system today uses scattered tasks in written or oral exams, where students are required to answer under the immediate pressure of an exam situation, while the IB has chosen to employ a long-term investigative project.

The development of problem solving skills is considered to be one of the main goals of mathematics education, in accordance with both the KOM report and Adding It Up. This shows in both systems’ syllabi and also in the relatively high prevalence of problem solving tasks (31.5% and 28%) in the exams. Approximately 65% of problem solving questions in the Swedish National Tests and 40% of problem solving questions in the IB Maths SL exam.

70 Jonsson et. al., 2014
71 *16%
are text-based. It is worth noting, that the percentage decreases markedly from the Swedish Ma1c (93%) and Ma2c (88%) to the following courses (33% and 45%, respectively), paralleling an increase in abstraction level. It thus appears as if the trend towards more realistic, and thus text based, tasks, that Bråting & Österman point out as worrisome, is apparent at the lower levels of the most difficult strand of mathematics in Swedish high school education. Should a similar pattern of a high prevalence of text-based tasks be seen in the a- and b-strand courses as well, where students rarely progress further than Ma1a/b or Ma2a/b, it could certainly exacerbate students’ mathematical difficulties if tasks are included that not only require mathematical knowledge but also reading skills. It is outside the scope of this paper to conduct an investigation into this matter, but since Skolverket has identified the largest decrease in reading comprehension amongst the weakest quartile of students, rather than the strongest quartile, such an investigation would be justified. A higher prevalence of text based questions increases demands for students’ reading and language skills and thus makes it more difficult for non-native Swedish speakers and generally weak readers to perform well on the tests; two social groups that are more prevalent in programmes outside of the Natural Sciences Programme or the Technical Programme. The IB exam, which is taken world-wide by a majority of non-native English speakers, has long aimed to decrease language-created bias by keeping questions short and paying attention to their wording. Given the increased heterogeneity of the Swedish society, there might be benefits to be had should Skolverket follow the IBO’s lead and investigate the prevalence of superfluous text for all courses in all strands of high school mathematics.

Webb (1997) points out the importance of alignment in education systems that rely on criterion-based achievement goals:

As more and more attention is paid to the accountability of education systems, alignment between assessments and expectations for learning becomes not only critical, but also essential.

Such alignment, Webb argues, can be assessed using 4 criteria: Content focus, articulation across grades and ages (i.e. students’ knowledge increases and develops over time), equity and fairness (giving students reasonable opportunity to show their level of knowledge and skills) and pedagogical implications (i.e. classroom practices). This paper does not cover classroom practices, but according to the criteria of content focus and articulation both the Swedish National Tests and the IB assessments appear to be in line with the demands of the respective syllabi. The content of the syllabi is thoroughly assessed and students are given opportunity to show their acquired skills, albeit in different manners in the respective systems.

In terms of equity and fairness, however, the fairness of both the Swedish National Tests and the IB exams is contestable since the two systems rely on differing views of what fairness entails. Since all Swedish teachers grade their own students’ work they can, despite the best of intentions, fall prey to bias or simply fail to interpret the assessment criteria in the way that was intended. It can be argued, however, that individual teachers follow their students throughout their course of study and can therefore rely on more solid material for assessment

73 IBO, 2004
74 Webb, 1997
75 Ibid.
than can external examiners, who are only presented with a snapshot of a student’s ability at the end of the course. Also, since a significantly higher prevalence of text-based questions has been identified in the first two Swedish courses, which benefits native speakers with high reading abilities, teachers can also allow some leeway for students who they know have language difficulties, or who freeze under the pressure of an exam situation. Such leeway is not possible during the IB examination; students are under tremendous pressure to perform at their best during the exam period, no matter any differences in language ability or mental fortitude in exam situations. According to some, however, this can be considered more fair, since all students experience the exact same conditions no matter who their teacher is, or what their relationship with said teacher has been.

Both the Swedish mathematics syllabus and the IB syllabus place high demands on students to acquire a large amount of conceptual and procedural knowledge, as well as problem solving and other higher-order cognitive skills. As has been shown by the content analysis of the respective final exams of both systems, these demands are translated into tasks that allow students to show such knowledge and skills. However, two main differences have been identified:

1. **The mode of assessment**: internal (teacher only) correction or external correction and/or moderation.
2. **The structure of assessment tasks**: 4 courses tested at the end of each course with exams consisting of three written papers and one oral part; or one course which includes the submission of a longer piece of academic text and ends with a final, two paper exam.

Studies have found that some students benefit from oral exams. However, in order to allow students time for creative reasoning, for communicating mathematical ideas and relating them to other contexts and for finding relations between concepts, the IB Mathematical Exploration or some similar longer term project appear more suitable than the oral component of the National Test, where students are asked to draw conclusions on a short notice and convey them to the teacher in a coherent manner. A project such as the ME also develops analytic skills and deepens students’ understanding of particular mathematical topics. With the current Swedish system, however, such a task would need to be coordinated with four different courses that all cover different topics. Suggestions have been made, by many different parties, to re-instate *subject grades* instead of *course grades* in Swedish high schools, i.e. to revert back to a system with only one course per subject in a manner similar to the IB model, albeit not necessarily with the same methods of examination. Proponents argue that a more coherent course would allow students more time to process and internalise mathematical knowledge and that it would decrease the inflation of grades that has been a long-term issue in Swedish education. Such a measure is supported by both of the teachers’ unions in Sweden and would allow other beneficial measures, such as instating a mandatory longer-term investigative project, to take place with positive effects.

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76 See for example Huynh et. al., 2004
77 Larsson, 2015; Mattsson, 2015; Bergman Alme et al, 2015
78 Directly translated from *betygsinflation*, a term borrowed from the field of Economics to express an increase in the general grade level without the corresponding increase in knowledge.
The current Minister of Education Aida Hadzialic has also suggested that Sweden introduces external marking of the National Tests, based on the IB model\textsuperscript{79}. The advantages and disadvantages in terms of fairness of such assessment have already been expanded upon above; one possible solution could be to combine external marking of the national tests with teacher assessment of students’ performance throughout the course, in order to achieve maximum fairness. In order for such an external marking process to be possible, however, regarding time and energy spent by both students, teachers and examiners, it would, again, have to be combined with a reversal back to subject grades to achieve a realistic number of examinations. As can be seen in the data, such a decrease from four to one examinations (with the addition of a longer-term project, or assignments set by individual teachers throughout the course) does not result in a decreased possibility for students to show the breadth and depth of their mathematical knowledge and skills. Also, the two measures combined could possibly decrease the continuous and incessant pressure on students to perform, as well as the pressure placed on the teacher to be both guide and examiner all at once and all the time\textsuperscript{80}. In a time and age when Swedish teachers and students alike report ever higher levels of stress, and students’ overall results show no signs of improving, that would be a welcome change.

\textsuperscript{79}\textsuperscript{79} Färlin, M., 2016

\textsuperscript{80}\textsuperscript{80} The increased pressure from both parent and students regarding grades and assessment has been linked to increased levels of stress, mental fatigue and sick leaves amongst Swedish teachers in recent years. See for example Dahlgren, P., 2016, and Lärarförbundet, 2015.
References


**Material provided by the IBO**


**IB Maths SL Exam Papers:**

M15/5/MATME/SP1/ENG/TZ1/XX/M

M15/5/MATME/SP2/ENG/TZ1/XX

**Material provided by the National Education Agency**


**Other online sources**

http://www.gymnasium.se/om-gymnasiet/om-program-gymnasiet-5143, accessed 2016-02-25


Appendix 1

Example of assessment matrix for the Ma2c National Test. The matrix shows what skills are assessed in each task and provides an overview of the skills that are prominent in each part of the exam.