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Endogenous Separations, Wage Rigidities and Employment Volatility

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Abstract

We show that in micro data, as well as in a search and matching model with endogenous separations and rigid wages, separations and hence employment volatility are non-neutral to wage rigidities of incumbent workers. In contrast to when all wages are flexible, the standard deviation of unemployment in a model with rigid wages for incumbent workers (only) matches the standard deviation in the data. Thus, the degree of wage rigidity for newly hired workers is not a sufficient statistic for determining the effect of wage rigidities on macroeconomic outcomes in this class of models.

Keywords: Search and matching, Unemployment volatility puzzle, Wage rigidities, Job Destruction

JEL classification: J63, J64, E30.
1 Introduction

In a recent very influential paper, Pissarides (2009) showed that in the baseline search and matching model job creation, and hence employment volatility, is only affected by wage setting in new matches. This is important, since it points to the degree of wage rigidity of new hires as the key statistic determining labor-market dynamics as opposed to wage rigidities in general. Naturally, this insight spurred a growing empirical literature studying wage setting for new hires (see e.g. Carneiro, Guimarães and Portugal, 2012, Martins, Thomas and Solon, 2012, Gertler, Trigari and Huckfeldt, 2014, Haefke, Sonntag and van Rens, 2013).

Pissarides (2009) analyzes the case with exogenous separations, a route supported by the influential finding of Shimer (2007, 2012) that separations contribute very little in (un)employment fluctuations. However, recent work by Barnichon (2012) show that this result hinges crucially on the assumption that the job finding and the job separation rates are two independent determinants of unemployment. Relaxing this assumption increases the role of separations in (un)employment volatility dramatically to about 40 percent of unemployment’s variance; see also Fujita and Ramey (2009). In light of this finding we take a step back in this paper and study this neutrality proposition in a search and matching model with endogenous separations as in e.g. Pissarides (1994), where wages of new hires are fully flexible. If all wages are fully flexible, unemployment volatility is substantially lower than in the data. Importantly, however, in a simple partial equilibrium setup, we show that wage rigidities of incumbent workers are important for separations and hence employment volatility. A shock to productivity increases all wages that can be adjusted, but with wage frictions some wages in existing matches are unchanged leading to low separations. Then, since the incumbent wage affects job separations, employment is affected. Thus, to only focus on wage setting for new hires is not enough in this framework to fully capture the link between wage-setting rigidities and employment volatility.

To provide evidence on the link between separations and incumbent wages, we rely on linked Swedish employer-employee micro data. We show that when incumbent workers’ wages are flexible there should be no relationship between the firm wage and separations when controlling for the marginal revenue product and outside options. In contrast, the data give stark evidence for a strong positive relationship as expected when incumbent worker wages are rigid. This finding is thus in line with the literature studying the cyclicality of wages documenting wage rigidities in incumbents’ wages; see Pissarides (2009) for an overview of this large literature.

Moreover, since general equilibrium feedback effects may overturn partial equilibrium intuition,
we proceed by introducing endogenous separations in combination with rigid wages for incumbent workers in a DSGE model. In this setup, we find that wage rigidities for incumbent workers have large quantitative effects on employment volatility even when wages for new hires are fully flexible. In contrast to a model where all wages are flexible, the standard deviation of unemployment matches the standard deviation in the data. Also, the improved fit in unemployment volatility does not come from an overall worsening in fit in other variables. Thus, all in all, it seems that the degree of wage rigidity for newly hired workers is not a sufficient statistic for determining the effect of wage rigidities on macroeconomic outcomes. Instead, wage frictions for incumbent workers turn out to have large effects on employment volatility, despite wages for new hires being flexible, in the proposed class of models.

Three related papers are Bils, Chang, and Kim (2014), Schoefer (2015) and Fujita and Ramey (2012). Bils, Chang, and Kim (2014) argues that endogenous effort can break the neutrality result of wages for existing workers. Even though wages for new hires are flexible, future effort choices are affected by wage frictions, in turn affecting job creation and employment. However, in the baseline model, where equilibrium effort depends on the individual worker’s wage, the difference compared to a model with fully flexible wages is small. In Schoefer (2015), there is a financial friction in the form of a requirement on firms to use internal funds when hiring workers. Wage rigidities then make firm internal funds vary substantially with shocks, in turn leading to a large volatility in hiring and employment. In both papers, any effects of wage frictions on employment volatility work through the hiring margin, though. Fujita and Ramey (2012) analyses a model with endogenous separations and on-the-job search with flexible wages. In their calibrated model, they find that unemployment volatility is more in line with the data than in the classical search and matching model, albeit still on the low side. However, we show that the wage elasticity with respect to productivity is too high under flexible wages. Specifically, in a model with flexible wages, endogenous separations and on-the-job search, we find that the wage elasticity is substantially larger than in the data, while it is close to the data under wage frictions.

This paper is outlined as follows. In Section 2 we present the basic mechanism we have in mind, in Section 3 we present micro-data evidence supporting that incumbent wages affect separations, in Section 4 we outline the framework for the quantitative evaluation and in Section 5 we present the calibration and the quantitative results. Finally, Section 6 concludes.

\footnote{Barnichon (2012) also points out the importance of separations.}
2 The Mechanism

To set ideas, it is helpful to first focus on a stylized model of the labor market featuring the mechanism we have in mind. Let firms and workers determine wages \( w_{jt} \) by the Nash-Bargaining solution. We assume that there are search and matching frictions captured by a constant returns matching function, giving rise to a surplus to be bargained over. Moreover, separations are endogenous along the lines of Pissarides (1994). Thus, an idiosyncratic productivity shock \( a_{jt} \) is drawn in each firm, following the cdf \( G \). The firm decides on a cutoff level of idiosyncratic productivity, denoted \( R_{jt} \), where the firm is indifferent between terminating the match and keeping the worker. Firm marginal revenue product, given the idiosyncratic productivity shock, is

\[
p_{jt} z_t a_{jt}
\]

where \( p_{jt} \) is the price of the firm and \( z_t \) an aggregate productivity shock.

We denote the surplus of the firm (worker) when wages change by \( J_{jt} (H_{jt}) \). Letting \( w_{jt} (a_{jt}) \) denote the rebargained wage, the expected firm value for the firm is,

\[
J_{jt} (a_{jt}) = p_{jt} z_t a_{jt} - w_{jt} (a_{jt}) + \alpha \beta \rho \int_0^1 \max\{J_{jt+1} (r), 0\} dG (r)
\]

\[
+ (1 - \alpha) \beta \rho \int_0^1 \max\{\tilde{J}_{jt+1} (r, w_{jt} (a_{jt})), 0\} dG (r),
\]

where \( \beta \) is the discount factor, \( \alpha \) the probability that wages are adjusted, \( \tilde{J}_{jt+1} (r, \tilde{w}_{jt}) \) the surplus of the firm when wages are fixed at \( \tilde{w}_{jt} \) and \( \rho \) is the fixed probability that the match survives into the next period, capturing an exogenous component of separations (i.e. voluntary quits). Also, for firms where the wage is fixed at \( \tilde{w}_{jt} \), noting that \( \tilde{w}_{jt} \) is a state variable,

\[
\tilde{J}_{jt} (a_{jt}, \tilde{w}_{jt}) = p_{jt} z_t a_{jt} - \tilde{w}_{jt} + \alpha \beta \rho \int_0^1 \max\{J_{jt+1} (r), 0\} dG (r)
\]

\[
+ (1 - \alpha) \beta \rho \int_0^1 \max\{\tilde{J}_{jt+1} (r, \tilde{w}_{jt}), 0\} dG (r).
\]

With right to manage, firms choose separations (i.e. the cutoff productivities \( R_{jt} \) and \( \tilde{R}_{jt} (\tilde{w}_{jt}) \)) so that

\[
J_{jt} (R_{jt}) = 0 \text{ and } \tilde{J}_{jt} \left( \tilde{R}_{jt} (\tilde{w}_{jt}), \tilde{w}_{jt} \right) = 0.
\]
Similarly, the surplus for the worker when wages change are

\[
H_{jt}(a_{jt}) = w_{jt}(a_{jt}) - b + \alpha \beta \left[ \rho \int_{r \geq R_{jt+1}}^1 H_{jt+1}(r) dG(r) - s_t H^e_{t+1} \right] + (1 - \alpha) \beta \left[ \rho \int_{r \geq \tilde{R}_{jt+1}(w_{jt}(a_{jt}))}^1 \tilde{H}_{jt+1}(r, w_{jt}(a_{jt})) dG(r) - s_t H^e_{t+1} \right],
\]

(5)

where \( b \) is (real) income received when unemployed, \( s \) the probability of finding a job and \( H^e_t \) the average value of being employed across all firms in the economy. When wages are not rebargained, \( \tilde{H}_{jt}(a_{jt}, \tilde{w}_{jt}) \) is defined along the lines of (5) with \( \tilde{w}_{jt} \) replacing \( w_{jt}(a_{jt}) \). Wages are determined in bargaining and are given by the Nash Bargaining Solution (NBS);

\[
\max_{w_{jt}(a_{jt})} (H_{jt}(a_{jt}))^\varphi (J_{jt}(a_{jt}))^{1-\varphi}.
\]

(6)

Note that, when all wages are fully flexible and separations are bargained over (which is not the case above), the separation cutoff is determined so that the total surplus \( S_{jt} = J_{jt} + H_{jt} \) is zero, i.e. \( S_{jt}(R_{jt}) = 0 \). Since \( J_{jt} = (1 - \varphi) S_{jt} \), the solution is the same as under right to manage; see also Fujita and Ramey (2012). Moreover, wages do not affect separations in equilibrium, since wages just redistribute surplus between the firm and the worker. If wages are sticky, they will have allocative effects through separations, though. For workers that don’t renege their wage, the separation cutoff \( \tilde{R}_{jt} \) is determined so that \( J_{jt} \left( \tilde{R}_{jt}(\tilde{w}_{jt}), \tilde{w}_{jt} \right) = 0 \), which implies that

\[
\tilde{R}_{jt}(\tilde{w}_{jt}) = \tilde{w}_{jt} - \alpha \beta \rho \int_0^1 \max\{J_{jt+1}(r), 0\} dG(r) - (1 - \alpha) \beta \rho \int_0^1 \max\{\tilde{J}_{jt+1}(r, \tilde{w}_{jt}), 0\} dG(r).
\]

(7)

Assuming that \( \frac{dJ_{jt+1}(r,\tilde{w}_{jt})}{d\tilde{w}_{jt}} < 0 \), we get \( d\tilde{R}_{jt}/d\tilde{w}_{jt} > 0 \) and thus separations increase in the wage. When wages are fully flexible, separations are determined so that \( S_{jt}(R_{jt}) = 0 \), implying that

\[
R_{jt} = \frac{b - \left[ \beta \rho \int_0^1 \max\{S_{jt+1}(r), 0\} dG(r) - \beta s_t \varphi S^e_{t+1} \right]}{p_{jt} z_t},
\]

(8)

which does not depend on wages, but only on current and future (through \( S_{jt+1} \) and \( S^e_{t+1} \)) outside options and marginal products.

Thus, regressing actual separations on wages and real marginal revenue product, while controlling for the outside option, is informative in the present framework for guiding the choice of wage-setting framework.
3 Are Separations Driven by Incumbent Wages?

To build an empirical test of which model of wage setting that is best aligned with micro data, we rely on equations (7) and (8). Moreover, we impose a log-linear technology (Cobb-Douglas) to account for that most firms have many employees and assume that the variation in the outside option is common across workers in the same sector and can be captured by the interaction of time and sector dummies, denoted \( \lambda_{it} \).\(^3\)\(^4\) Using the IV strategy outlined below, we then expect \( \beta_w > 0 \) when controlling for marginal revenue product and outside options if wages are rigid for incumbent workers. We then run the following regression

\[
\ln sep_{jt} = \alpha_j + \beta_{mrp} \ln mrp_{jt} + \beta_w \ln w_{jt} + \lambda_{jt} + \epsilon_{jt},
\]

where, letting \( y_{jt} \) denote real value added and \( l_{jt} \) employment, \( sep_{jt} \), \( mrp_{jt} (= p_{jt}y_{jt}/l_{jt}) \) and \( w_{jt} (= wage\text{bill}_{jt}/l_{jt}) \) denote the number of separations, the nominal marginal revenue product of labor and the nominal wage at firm \( j \) at time \( t \), respectively, and \( \alpha_j \) captures all firm-level constants.\(^5\) Essentially, the time by sector dummies pin down the curve of interest in the system determining endogenous variables in the model. To handle simultaneity (and potential measurement errors) we need instruments correlated with \( \ln mrp_{jt} \) and \( \ln w_{jt} \), providing independent variation vis-a-vis the outside option of the workers, but uncorrelated with any idiosyncratic shocks simultaneously driving \( \ln sep_{jt} \).

Naturally, these restrictions leaves a very small set of potential instruments. However, as shown by Fujita and Ramey (2012), matching a model with endogenous separations and on-the-job search to the data requires idiosyncratic shocks that are, for all practical purposes, to be regarded as i.i.d. on the annual frequency (see section 5 below for a discussion). This results opens up for plausibly using \( \ln mrp_{jt-1} \) and \( \ln w_{jt-1} \) as instruments. Importantly, even with non-persistent idiosyncratic shocks, wage stickiness, as included in the model, still gives rise to a positive correlation between \( \ln w_{jt} \) and \( \ln w_{jt-1} \), through unchanged incumbent worker wages, which we can use for identification. Similarly, although not explicitly modeled, the presence of price stickiness in the data (as reported by Carlsson and Nordström Skans, 2012) generates a positive correlation between \( \ln mrp_{jt} \) and \( \ln mrp_{jt-1} \), which also can be used for identification. In Appendix A we present results from the first-stage regressions that are in line with these predictions. Moreover, from the model, idiosyncratic technology shocks that drives up separations would also drive down the wage. Thus, autocorrelation in the idiosyncratic shocks

\[^3\]We will also experiment with the triple interaction of time, sector and county below, taking into account the possibility that the reservation wage varies across regions.

\[^4\]See Carlsson, Messina and Nordström Skans, 2014, for direct evidence on the importance of sectoral variation in outside options for wage setting.

\[^5\]Future values affect today’s separations and we have common autocorrelated shocks in the model outlined below. Note, however, that this common variation in future values will be captured by the time dummies.
technology shocks would bias us towards finding a negative sign on the key parameter of interest in this exercise, $\beta_w$. Thus in this sense the positive estimate of $\beta_w$ presented below is to be regarded as a lower bound. Note also that relying on lagged wage as an instrument for the wage effectively implies that the parameter on the log wage, $\beta_w$, is identified by variation in the incumbent workers’ wages, which is exactly the variation we want to use. Finally, note that the time (by sector) dummies will handle that the model above is stated in real terms, whereas the explanatory variables in the regression (9) are stated in nominal terms.

3.1 Micro Data

The firm-level micro data we use to estimate equation (9) are drawn from two sources. First, we use the database Företagens Ekonomi (FEK) maintained by Statistics Sweden, which contains information on annual frequency on value added, labor costs, the number of employees (in terms of full-time equivalents) and a five-digit (NACE) sector code for all Swedish firms in the private sector from 1997 to 2008. We then compute $mrp_{jt}$ as nominal value added ($p_{jt}y_{jt}$) divided by the number of full-time equivalent workers ($l_{jt}$). To obtain a measure of the firm wage we divide total firm-level nominal labor costs ($wagebill_{jt}$) by the same measure of firm-level employment as for the marginal revenue product of labor. This gives a proper measure of the average firm-level, full-time wage since the labor input measure accounts for both the extensive and intensive labor-input margin.

Second, to compute separations we use the Register Based Labor Market Statistics (RAMS) database. This database is also maintained by Statistics Sweden and contains information about labor earnings for all employment spells in the Swedish private sector. Importantly, the RAMS data contains a plant and a firm identifier, which allows us to match the individual employment spells to the employing firm in the FEK database. The raw data is collected from employers by the Swedish Tax Authority in order to calculate taxes. Data include information on annual earnings, as well as the first and last remunerated month in the year. Using this information, we can construct a firm measure of separations. Here, separations are defined in the same way as they enter into the flow equation of employment, i.e. $Employment_{jt} = Employment_{jt-1} + Hires_{jt} - sep_{jt}$. Since the firm identity also changes with a new owner, mergers, etc., we do not include observations for the entry year of a firm.

The baseline definition of separations we employ is based on the primary employment of full-time

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*Unfortunately, the sectoral coding system was subject to a substantial change (from the SNI2002 system to the SNI2007 system) with no unique mapping between the two systems (2008 being the last year with the SNI2002 system) so here we limit the end year of the study to 2008.

*Swedish collective agreements, covering about 90 percent of the work force, are bargained at the sectoral level, but a substantial part of wage bargaining is done by local parties at the firm level, see Nordström Skans, Edin, and Holmlund (2009).

*For the entry year the separations would be zero by construction, which for the reasons stated above is very likely to be wrong. Naturally, the exit year cannot be included since we do not observe the other variables included in the regression (9) in that year.
workers. The data lacks information on actual hours, so to restrict attention to workers that are reasonably close to full time workers we only consider a person to be a full-time employee if the (monthly) wage exceeds 75 percent of the mean (monthly) wage of janitors employed by municipalities. Also, since we are aiming to identify full-time workers we only count an individual as employed by at most one firm each year by only keeping the employment with the highest wage in November (which is the reference month used by Statistics Sweden). Thus, in other words, with this definition we focus on individual’s primary employment.\(^9\) Self-employed workers are not counted as employed in any of the definitions of separations used in the paper.

Finally, note also that the RAMS database contains geographical information on the plant where the worker is employed. Using this information we can also experiment by including a control for regional variation in the workers’ outside option. Thus, we can include the triple interaction of time, two-digit (NACE) sector and county (NUTS3) as a control for outside options.\(^10\) Matching the RAMS and the FEK data, we end up with a sample of 189,199 firms and 1,014,960 firm/year observations where we can compute all the information we need for estimation.

Two additional complications arise from the fact that many firms are small and there are many instances of zero separations (about 55 percent of the data). To handle very large swings in separations we first require that the firms have at least five full-time employees (according to the strict definition used to compute separations in the RAMS data), leaving a sample of 79,651 firms and 395,912 firm/year observations. Secondly, to conserve on the data and not throw out all zero observations on separations we use the approximation \(\text{sep}_{jt}/\text{sep}_j\), where \(\text{sep}_j\) denotes the firm average of separations, instead of \(\ln \text{sep}_{jt}\), as the dependent variable in the regression (9).\(^{11}\) However, even with this approximation we need to drop firms with a zero firm average of separations. Thus, all in all, the baseline estimation sample amounts to 69,471 firms and 378,395 firm/year observations. For this sample, we control for outside options by using 528 sector by time dummies (when using sector by time by county dummies to control for outside options, we use 8,934 dummies).\(^{12}\)

In Appendix A we present robustness exercises to address the definition of separations, the dropping small firms and to only looking at a subsample consisting of the manufacturing sector.

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\(^9\) Nordström Skans, Edin, and Holmlund (2009) and Carlsson, Messina, and Nordström Skans (2014) use a similar approach to identify full-time workers’ primary employment.

\(^{10}\) Note, though, that in this latter experiment we need to drop firms with employees in more than one county.

\(^{11}\) Note that in (9), \(\beta_w = \frac{d \ln \text{sep}_{jt}}{d \ln w_{jt}} = \frac{(d \text{sep}_{jt}/\text{sep}_{jt})}{(d \ln w_{jt})} \simeq \frac{(d \text{sep}_{jt}/\text{sep}_{jt})}{(d \ln w_{jt})},\) where the later expression is estimated when the approximation is used.

\(^{12}\) Estimation is performed using the felsdvreg routine for Stata; see Cornelissen (2008).
3.2 Empirical Results

As can be seen in the first column of Table 1, IV estimation, using lags as instruments, yields a statistically significant estimate of $\beta_w = 2.709$ (clustered s.e. 0.150), thus rejecting the null of flexible incumbent wages with a sign consistent with the presence of wage frictions in the data. Moreover, the coefficient sign on the marginal revenue product $\beta_{mrp}$ of $-1.830$ (clustered s.e. 0.083) is also in line with what is expected from the model. As reported in Appendix A, the instruments are highly relevant with F-statistics of 491 and 196, in respective first-stage regression.

From column (2) we see that dropping the crisis year of 2008 does not affect the results. Moreover, in column (3) we learn that also allowing for geographical variation, over and above sector time variation, in reservation wages does not change the results. Specifically, including the triple interaction of sector (NACE two-digit) by time by county yields very similar results as to relying on only the interaction of sector (NACE two-digit) by time. Note that the number of observations changes slightly when also allowing for geographical variation in the reservation wage since we lose firms with multi-county production in this version of the regression.

In Appendix A we also show that the results are robust to: (i) employing a loose measure of firm-level separations in the regression relying on all employment spells of all workers regardless of their degree of firm attachment, (ii) including very small firms in the sample and (iii) focusing on the manufacturing sector only. Thus, all in all, we conclude from this exercise that the micro data is strongly and robustly in favor of the incumbent wages being allocative.

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<tr>
<th>Table 1: Are Wages Allocative?</th>
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<td>Observations</td>
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* (***) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes. All regressions include firm-level fixed effects.
4 A Model for Quantitative Evaluation

The next step in our analysis attempts to realistically evaluate the macroeconomic quantitative importance of the mechanism outlined above by embedding it in a standard Pissarides (1994) model. The basic framework for the quantitative evaluation shares many elements of standard real models.

In contrast to many previous papers studying the unemployment volatility puzzle (e.g., Shimer (2005), Hall (2005)), our model features endogenous separations and on-the-job search. On-the-job search is important in order to generate a Beveridge curve with a negative slope, but not for the overall findings of this paper. Firms use labor to produce output and post vacancies on a search and matching labor market. Wages are bargained between workers and firms in a setting with stochastic impediments to rebargaining, akin to Calvo (1983). New hires, however, always bargain their wage. Separations and on-the-job search are endogenous along the lines of Pissarides (1994). Unemployed workers receive unemployment benefits paid by the government that are financed via lump-sum taxes.

4.1 Firms

Firms each employ one worker to produce a homogenous good with a constant returns technology that is sold at price $p_t$ to retailers. Firm revenue is $p_t z_t a_{jt}$, where $z_t$ is an aggregate productivity shock, $w_{jt}$ the wage for a firm and $a_{jt}$ and idiosyncratic productivity shock. We normalize $p_t$ to unity. The idiosyncratic shock is assumed to follow the cdf $G$ with upper and lower bounds, $a_{ub}$ and $a_{lb}$, respectively. As in the example in section 2, if idiosyncratic productivity is sufficiently low, the firm will cease operations and lay off the worker.

4.2 Search and Matching and the Hiring Decision

Letting $u_t$ denote unemployment, $\nu_t$ vacancies and $\phi_t$ the number of matched workers searching, the total number of searching workers is $u_t + \phi_t$. Match formation is governed by the Cobb-Douglas matching function

$$m(u_t + \phi_t, \nu_t) = \sigma \nu_t \phi_t^{\sigma - 1}. \quad (10)$$

Labor-market tightness is given by

$$\theta_t = \frac{\nu_t}{u_t + \phi_t}. \quad (11)$$

Vacancies are determined as usual by the equalization of the vacancy cost, denoted $c$, of an employee and the expected value of the worker to the firm. As in Pissarides (1994), when workers enter a firm, they enter at the highest idiosyncratic productivity $a_{ub}$. Job creation is then given by

$$c = \beta E_t q(\theta_t) J_{t+1} (a_{ub}), \quad (12)$$
where $\beta$ is the discount factor, $q(\theta_t)$ the probability of filling a vacancy and $J_t$ the value of a firm. A detailed description of employment flows, which are somewhat involved, can be found in section B.1 in the appendix.

### 4.3 Value Functions

Let $H^s$ and $H^{ns}$ denote worker surplus when the worker searches and does not search on the job, respectively. We assume that workers face a cost $\sigma$ of searching on the job. With probability $\lambda$, workers’ idiosyncratic productivity changes and is again drawn from the distribution $G$ and with probability $(1 - \lambda)$ that the probability is unchanged. Note that the wage will depend on idiosyncratic productivity $a_t$. Let $w^s(a_t)$ ($w^{ns}(a_t)$) denote the worker wages when searching (not searching). The expected net surplus for an employed worker in a firm that resets the wage this period is, where $I_t$ is an indicator function that is equal to one of the worker searches on the job and zero otherwise, again suppressing the aggregate state variable $z_t$,

$$H_t^i(a_t) = w_t^i(a_t) - b - \mathbb{I}_t \sigma + \beta E_t \alpha \rho^i \left( \lambda \int_0^1 H_{t+1}^i(r) dG(r) + (1 - \lambda) H_{t+1}^s(a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{H}_{t+1}^i(r, w_t^{ns}(a_t)) dG(r) + (1 - \lambda) \hat{H}_{t+1}^s(a_t, w_t^{ns}(a_t)) \right)$$

$$+ \beta E_t (g^i - f(\theta_t)) H_{t+1}(a_{\omega t}),$$

where $b$ is the replacement rate, $g^{ns} = 0$ and $g^s = f(\theta_t)$, $\rho^{ns} = (1 - s)$ and $\rho^s = (1 - f(\theta_t)) (1 - s)$,

$$H_t(a_t) = \begin{cases} \max\{H_t^{ns}(a_t), H_t^s(a_t)\} & \text{if } a_t > \hat{R}_t^{ns} \text{ and } a_t > \hat{R}_t^s \\ H_t^{ns}(a_t) & \text{if } a_t > \hat{R}_t^{ns} \text{ and } a_t \leq \hat{R}_t^s \\ H_t^s(a_t) & \text{if } a_t \leq \hat{R}_t^{ns} \text{ and } a_t > \hat{R}_t^s \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\hat{H}_t(a_t, \hat{w}_{jt}) = \begin{cases} \max\{\hat{H}_t^{ns}(a_t, \hat{w}_{jt}), \hat{H}_t^s(a_t, \hat{w}_{jt})\} & \text{if } a_t > \hat{R}_t^{ns} \text{ and } a_t > \hat{R}_t^s (\hat{w}_{jt}) \\ \hat{H}_t^{ns}(a_t, \hat{w}_{jt}) & \text{if } a_t > \hat{R}_t^{ns} \text{ and } a_t \leq \hat{R}_t^s (\hat{w}_{jt}) \\ \hat{H}_t^s(a_t, \hat{w}_{jt}) & \text{if } a_t \leq \hat{R}_t^{ns} \text{ and } a_t > \hat{R}_t^s (\hat{w}_{jt}) \\ 0 & \text{otherwise.} \end{cases}$$
In case wages are not reset but remain at the level \( \hat{w}_{jt} \) from the previous period, the wage \( \hat{w}_{jt} \) is a state variable and the surplus is

\[
\hat{H}_t^{i} (a_t, \hat{w}_{jt}) = \hat{w}_{jt} - b - \Pi_t \sigma + \beta E_t \alpha \rho^i \left( \lambda \int_0^1 H_{t+1} (r) dG (r) + (1 - \lambda) H_{t+1} (a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{H}_{t+1} (r, \hat{w}_{jt}) dG (r) + (1 - \lambda) \hat{H}_{t+1} (a_t, \hat{w}_{jt}) \right) + E_t (g^i - f (\theta_t)) H_{t+1} (a_{ub}).
\]

(16)

For firms that change wages, the surplus is, when there is no on-the-job search

\[
J_t^{i} (a_t) = z_t a_t - w_t^{i} (a_t) + \beta E_t \rho^i \alpha \left( \lambda \int_0^1 J_{t+1} (r) dG (r) + (1 - \lambda) J_{t+1} (a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{J}_{t+1} (r, w_t^{i} (a_t)) dG (r) + (1 - \lambda) \hat{J}_{t+1} (a_t, w_t^{i} (a_t)) \right),
\]

where

\[
J_t (a_t) = \begin{cases} 
\max (J_t^{ns} (a_t), 0) & \text{if } a_t > R_t^S \\
\max (J_t^s (a_t), 0) & \text{if } a_t \leq R_t^S
\end{cases}
\]

(18)

and

\[
\hat{J}_t (a_t, \hat{w}_{jt}) = \begin{cases} 
\max (\hat{J}_t^{ns} (a_t, \hat{w}_{jt}), 0) & \text{if } a_t > \hat{R}_t^S (\hat{w}_{jt}) \\
\max (\hat{J}_t^s (a_t, \hat{w}_{jt}), 0) & \text{if } a_t \leq \hat{R}_t^S (\hat{w}_{jt})
\end{cases}
\]

(19)

In case wages are not reset but remain at the level \( \hat{w}_{jt} \) from the previous period, the values are

\[
\hat{J}_t^{i} (a_t, \hat{w}_{jt}) = z_t a_t - \hat{w}_{jt} + \beta E_t \rho^i \alpha \left( \lambda \int_0^1 J_{t+1} (r) dG (r) + (1 - \lambda) J_{t+1} (a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{J}_{t+1} (r, \hat{w}_{jt}) dG (r) + (1 - \lambda) \hat{J}_{t+1} (a_t, \hat{w}_{jt}) \right).
\]

(20)

4.4 Endogenous Separations and On the Job Search

As in Mortensen and Pissarides (1994) and Den Haan, Ramey, and Watson (2000), firms have the right to manage. A firm lays off workers if idiosyncratic productivity is at most equal to a cutoff level \( R^i \) and \( \hat{R}^i \) for \( i \in \{ns, s\} \). The separation cutoffs are

\[
J_t^{i} (R^i) = 0 \text{ and } \hat{J}_t^{i} \left( \hat{R}^i (\hat{w}_{jt}) \right) = 0 \text{ for } i \in \{ns, s\}.
\]

(21)

Similarly, the decisions for workers when to search on the job depend on the level of the idiosyncratic productivity shock. A worker searches on the job when idiosyncratic productivity is at most equal to
a cutoff level $R^S$ and $\hat{R}^S$. The on-the-job search cutoffs are

$$H_t^S (R^S_t) = H_t^{ns} (R^S_t) \quad \text{and} \quad \hat{H}_t^S (\hat{w}_jt), \hat{w}_jt) = \hat{H}_t^{ns} (\hat{R}^S_t (\hat{w}_jt), \hat{w}_jt). \quad (22)$$

### 4.5 Wage Bargaining

The nominal wage, when wages are rebargained, is chosen such that it solves the Nash product

$$\max_{\tilde{w}_i(a_{jt})} \left( H_i^t (a_{jt}) \right) ^{\varphi} \left( J_i^t (a_{jt}) \right) ^{1-\varphi}, \quad (23)$$

where $i \in \{ns, s\}$ and $\varphi$ denote the bargaining power of the family.

### 4.6 The Resource and Government Budget Constraints

Let $n_t(a)$ denote employment in firms with idiosyncratic productivity $a$. The aggregate resource constraint can be written as

$$c_t + \nu_t = \int_0^1 n_t(a) z_t ada. \quad (24)$$

The government uses lump-sum taxes to finance unemployment benefits. Thus, $\tau_t = (1 - n_t) b_r$.

### 5 Quantitative Evaluation

#### 5.1 Calibration

The baseline calibration of the structural parameters is presented in Table 2 and is based on standard values for a monthly parametrization. We set $\beta$ to 0.9966, which generates a real interest rate of

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$  Time preference</td>
<td>0.9966</td>
</tr>
<tr>
<td>$s$ Exogenous separation rate</td>
<td>0.0215</td>
</tr>
<tr>
<td>$\varphi$ Family bargaining power</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_a$ Matching function</td>
<td>0.5</td>
</tr>
<tr>
<td>$\lambda$ Prob. of new idiosyncratic draw</td>
<td>0.299</td>
</tr>
<tr>
<td>$\alpha$ Calvo prob. of wage adjustment</td>
<td>0.138</td>
</tr>
<tr>
<td>$b$ Payoff when unemployed</td>
<td>0.7</td>
</tr>
<tr>
<td>$\sigma_G$ Idiosyncratic productivity distr. variance</td>
<td>0.081 0.271</td>
</tr>
<tr>
<td>$\sigma_\mu$ Matching function productivity</td>
<td>0.531 0.531</td>
</tr>
<tr>
<td>$c$ Vacancy cost</td>
<td>0.309 0.283</td>
</tr>
<tr>
<td>$\sigma$ Search cost</td>
<td>0.045 0.036</td>
</tr>
</tbody>
</table>

The wage frictions and flexible wages

around 4 percent. We set the worker outside option $b$ to 0.7, in line with Pissarides (2009). For
job separations, we set total quarterly separations to 0.1, implying a monthly rate of 0.0345. We set $s = 0.0215$ to match the share of non-layoff separations in JOLTS, which is 0.624 for the period 2001:Q1-2007:Q1. We set the bargaining power $\varphi = 0.5$, implying symmetrical bargaining in the baseline calibration. We choose $\sigma_a$ to yield a matching function elasticity of 0.5 to ensure that the (basic) Hosios condition is satisfied, following Pissarides (2009). As is commonly assumed, see e.g. Den Haan, Ramey, and Watson (2000) and Christiano, Trabandt, and Walentin (2010), we assume that the idiosyncratic productivity shock follows a log-normal distribution with mean zero and standard deviation parameter $\sigma_G$. We approximate the idiosyncratic distribution by a grid with 60 gridpoints with lower (upper) bound of 0.6 (1.0). The parameter $\lambda$ that determines the degree of persistence in the idiosyncratic productivity process is set to 0.299, as in Fujita and Ramey (2012). This implies a very low yearly probability of remaining in the same productivity state of 0.01. The Calvo parameter is set to 0.138, following Christiano, Eichenbaum, and Evans (2005). We follow Hagedorn and Manovskii (2008) closely when calibrating the productivity process. We approximate, through a 5-state Markov chain, the continuous-valued $AR(1)$ process $\log z_t = \rho_z \log z_{t-1} + \varepsilon_t^z$ where $\rho_z \in (0, 1)$ and $\varepsilon_t^z \sim N(0, \sigma^2_z)$. In the BLS data (see below) we find an autocorrelation of 0.765 and an unconditional standard deviation of 0.013 for the HP-filtered productivity process with a smoothing parameter of 1,600. At a monthly frequency, this requires setting $\rho_z = 0.960$ and $\sigma^2_z = 0.0085$. We then choose the parameters $\sigma_{\mu}$, $c$, $\sigma_G$ and $\sigma$ so that the model has a labor market tightness of 0.72 as in Pissarides (2009), a total separation rate of 0.0345 which is based on a quarterly separation rate of 10 percent, a job-to-job finding rate of 0.032 as in Moscarini and Thomsson (2007) and a job finding probability of 0.450 based on a weekly rate of 0.139. The results are presented in Table 2.  

5.2 Solution algorithm

We use nonlinear solution techniques along the lines of Hagedorn and Manovskii (2008) to solve the model. As with a standard search and matching model, the system can be solved recursively, i.e. first solving for labor market tightness, wages and values and then for employment flows. Since the system (12), (13), (16), (17), (20) and (23) above do not depend directly on unemployment, we can solve

---

14 This is also roughly in line with Shimer (2012), where endogenous separations are around 0.025.
15 Their baseline quarterly estimate is 0.36, implying a duration of wage contracts of slightly below three quarters.
16 Separations and job finding probabilities computed using the data compiled by Robert Shimer between 1948:Q1 and 2007:Q1. See footnote above.
17 The number of months in the simulation is set so that it corresponds to the number of quarters in the quarterly data used below (where the period is 1951:Q1-2007:Q1). When choosing the parameters $\sigma_{\mu}$, $c$, $\sigma_G$ and $\sigma$, we simulate 100 data sets, aggregate to quarterly data and then compute the average labor market tightness, the separation rate, the job-to-job finding rate and the job finding probability.
without computing employment and unemployment. Wages that change today depend on the state variables, \( z_t \) and \( a_{jt} \). Similarly, a wage that is reset at some point \( t - k \) in the past depend on the state variables, \( z_{t-k} \) and \( a_{jt-k} \). The state for worker and firm surpluses when wages are rigid then depends on the current states, \( z_t \) and \( a_{jt} \), and the states when the wage was last reset. Letting \( Z \) denote the state space for \( z_t \) and \( A \) the state space for \( a_{jt} \), the state space for \( J^i \) and \( H^i \) is \( Z \times A \) and the state space for \( \hat{J}^i \) and \( \hat{H}^i \) is \( Z \times A \times Z \times A \).

Given the redefinition of the state space above, we guess a solution for firm and worker surpluses, wages and labor market tightness and compute new revised values using value function iteration until convergence. The model is then simulated to generate the synthetic variables required to compute the moments that we match in the calibration.

### 5.3 Quantitative Results

The quarterly data set we use to calculate moments to be compared to the model moments cover the sample period 1951:Q1-2007:Q1 and are constructed as follows. Unemployment is taken from the BLS and is constructed from the Current Population Survey and is seasonally adjusted. The job finding and separation rates are constructed as quarterly averages of the monthly series provided by Robert Shimer.\(^{18}\) Vacancies are measured by quarterly average of the "Help-Wanted Index" described in Barnichon (2010). Labor productivity is taken from the BLS and is defined as real output per person in the nonfarm business sector. Following Hagedorn and Manovskii (2008), wages are computed as labor share times labor productivity, where the labor share is also taken from the BLS and is calculated for the nonfarm business sector. All variables are logged and HP-filtered with a penalty parameter equal to 1,600.

The simulated standard deviation for the unemployment rate, the job finding rate, total separation rate vacancies and the vacancy unemployment ratio are illustrated in Table 3. First, we see that wage rigidities for incumbent workers (only) generate more than double the standard deviation in the unemployment rate relative to a model with fully flexible wages for all workers (s.d. 0.13 vs. 0.06) and take the standard deviation of unemployment in the model up to the level of the observed standard deviation in the data (0.12). Also, wage rigidities move the correlation between the unemployment rate and labor productivity closer to what is observed in the data (from −0.96 to −0.72 as compared to −0.27 in the data).\(^{19}\) This in line with Fujita and Ramey (2012), which finds that a model with endogenous separations and fully flexible wages for all workers takes the volatility about a half-way towards realistic values (c.f. the results in Shimer, 2005). The results presented here show that a

\(^{18}\)Available at https://sites.google.com/site/robertshimer/research/flows.

\(^{19}\)It is worth remembering here that there is only an aggregate technology shock in the model, whereas the data can be driven by many types of shocks.
model with flexible wages for new hires, but with realistic amount of wage rigidities for incumbent workers, can actually generate unemployment volatility in the search and matching framework in line with the data. Thus, this extension of the model with a realistic standard value of the worker outside option of 0.7 is an alternative to the Hagedorn and Manovskii (2008) approach, where the value of unemployment is calibrated very closely to firm productivity and worker bargaining power is set close to zero, which achieves the same goal.

In terms of the job finding and separation rates, the model with wage frictions for incumbents is fairly similar to the model with fully flexible wages. Both have a job finding rate that is less volatile than in the data and a separation rate that has about the same volatility as the data. Regarding vacancies \( v_t/u_t \), the model with incumbent wage frictions has a volatility that is twice as high (50 percent) as in the data, while the model with fully flexible wages has a volatility of about one-fifth (one-third) as compared with the data, i.e. substantially smaller than in the data.

The correlation between unemployment and vacancies is \(-0.450\), indicating that the model generates a negatively sloped Beveridge curve. However, with fully flexible wages, the correlation is \(-0.833\), which is closer to the correlation \(-0.917\) observed in the data.

Moreover, the elasticity of wages with respect to productivity is substantially closer to the data when incumbent wages are rigid. The elasticity in the data is 0.463 while the elasticity in the model with wage frictions for incumbents is 0.517, whereas in the model with flexible wages it is 0.944. Note also that, in the model with wage frictions for incumbents, the elasticity of wages for new hires is substantially larger than the elasticity for aggregate wages, 0.825 versus 0.517.

Thus, the improved fit in unemployment volatility documented here does not come from an overall worsening in fit in other variables, and some dimensions become substantially better.

### Table 3: Comparison of Moments

<table>
<thead>
<tr>
<th></th>
<th>( u_t )</th>
<th>job find. rate</th>
<th>sep. rate</th>
<th>( v_t )</th>
<th>( v_t/u_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, std</td>
<td>0.124</td>
<td>0.039</td>
<td>0.052</td>
<td>0.139</td>
<td>0.257</td>
</tr>
<tr>
<td>Data, corr with labor p.</td>
<td>(-0.265)</td>
<td>0.272</td>
<td>(-0.533)</td>
<td>0.415</td>
<td>0.351</td>
</tr>
<tr>
<td>Model, std</td>
<td>0.130</td>
<td>0.026</td>
<td>0.047</td>
<td>0.256</td>
<td>0.345</td>
</tr>
<tr>
<td>Model, corr with labor p.</td>
<td>(-0.725)</td>
<td>0.960</td>
<td>(-0.349)</td>
<td>0.405</td>
<td>0.586</td>
</tr>
<tr>
<td>Flex wage model, std</td>
<td>0.063</td>
<td>0.026</td>
<td>0.046</td>
<td>0.028</td>
<td>0.088</td>
</tr>
<tr>
<td>Flex wage model, corr with labor p.</td>
<td>(-0.957)</td>
<td>0.957</td>
<td>(-0.982)</td>
<td>0.934</td>
<td>0.986</td>
</tr>
</tbody>
</table>

Note: All variables are logged and HP-filtered with a penalty parameter equal to 1600. The sample period for data is 1951:Q1-2007:Q1. Unemployment from the BLS is constructed from the CPS and seasonally adjusted. The job finding and separation rates are constructed as quarterly averages of the monthly series provided by Robert Shimer. Vacancies are measured by the quarterly average of the "Help-Wanted Index" provided by Regis Barnichon.
6 Concluding Discussion

In this paper we return to the question of whether or not wage rigidities for incumbent workers affect macroeconomic outcomes. By extending Pissarides (2009) and allowing for endogenous separations in line with Pissarides (1994), wage rigidities in existing matches are no longer neutral with respect to employment volatility. To provide evidence on how incumbent worker wage setting should be modeled, we rely on linked Swedish employer-employee micro data. In a simple model, we show that, when incumbent workers’ wages are flexible, there should be no relationship between the firm wage and separations, when controlling for the marginal revenue product and outside options. In contrast, the data give stark evidence of a strong positive relationship as expected under incumbent worker wage frictions. Finally, we show that the intuition, based on a simple partial equilibrium setup, also holds in a DSGE model allowing for general equilibrium feedback effects. Overall, we find that wage rigidities for incumbent workers have large quantitative effects on employment volatility even when wages for new hires are fully flexible. In contrast to a model where all wages are flexible, the standard deviation of unemployment matches the standard deviation in the data. Also, the improved fit in unemployment volatility does not come from an overall worsening in fit in other variables. Thus, all in all, it seems that the degree of wage rigidity for newly hired workers is not a sufficient statistic for determining the effect of wage rigidities on macroeconomic outcomes. Instead, wage frictions for incumbent workers turn out to have large effects on employment volatility in the proposed class of models.
References


A Appendix: Robustness of Empirical Results

This appendix addresses the robustness of the micro-econometric evidence supporting the right-to-manage bargaining framework presented in Section 3. In Table 4 we present the first-stage results corresponding to the baseline results presented in column (2) of Table 1. As can be seen in both columns, the instruments are strongly relevant with F statistics of 491 and 196, respectively. Also, a formal under-identification test confirms that the baseline IV specification is well identified (Kleibergen and Paap (2006) rk LM statistic: $\chi^2(1) = 375.32$, p-val = 0.000). Moreover, as expected under wage and price stickiness there is a strong relationship for each respective "own lag".

In Table 5 we perform various robustness exercises on our baseline results replicated in column (1) for convenience. In column (2) of Table 5 we first focus solely on the manufacturing sector (i.e. NACE codes 15-37). As can be seen in the table, this does not change the results. In column (3) we drop the restriction that the firm should have at least five full-time employees (according to the strict definition used to compute separations in the RAMS data) to be included in the sample. As can be seen in the table, the results are stronger relative to the baseline results in column (2) when loosening this restriction. This is not surprising since the small firms have larger percentage swings in separations. In the final column of Table 5, we use a much looser definition of employment, using all employment spells of all workers regardless of their degree of firm attachment. This means that a worker is counted as employed regardless of the (monthly) wage or the timing or length of the spell within a year. This gives smaller effects, but the results are qualitatively unchanged.

<table>
<thead>
<tr>
<th>Table 4: First Stage Results for Baseline Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) OLS</td>
</tr>
<tr>
<td><strong>Dependent Variable</strong></td>
</tr>
<tr>
<td>$\ln mrp_{jt-1}$</td>
</tr>
<tr>
<td><em>(0.007)</em>*</td>
</tr>
<tr>
<td>$\ln w_{jt-1}$</td>
</tr>
<tr>
<td><em>(0.009)</em>*</td>
</tr>
<tr>
<td><strong>Dummies:</strong></td>
</tr>
<tr>
<td>Firm</td>
</tr>
<tr>
<td>Sector by Time</td>
</tr>
<tr>
<td><strong>F Stat($\ln mrp_{jt-1} = \ln w_{jt-1} = 0$)</strong></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Firms</td>
</tr>
</tbody>
</table>

* (**) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes.
Table 5: Robustness

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>(\beta_{m_{\text{mrp}}})</td>
<td>-1.830</td>
<td>-1.645</td>
<td>-2.878</td>
<td>-1.177</td>
</tr>
<tr>
<td></td>
<td>(0.083)**</td>
<td>(0.150)**</td>
<td>(0.104)**</td>
<td>(0.060)**</td>
</tr>
<tr>
<td>(\beta_{w})</td>
<td>2.709</td>
<td>2.220</td>
<td>3.884</td>
<td>1.645</td>
</tr>
<tr>
<td></td>
<td>(0.150)**</td>
<td>(0.349)**</td>
<td>(0.133)**</td>
<td>(0.106)**</td>
</tr>
</tbody>
</table>

Dummies:
- Firm: YES YES YES YES
- Sector by Time: YES YES YES YES
- Manufacturing Only: NO YES NO NO
- \(\geq 5\) Full Time Employees: YES YES NO YES
- Separations Definition: BASELINE BASELINE BASELINE LOOSE

Observations: 378,395; 85,518; 799,739; 390,504
Firms: 69,471; 14,589; 131,862; 76,005

* (***) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes.

In Table 6 we evaluate the approximation \(sep_{jt}/\bar{sep}_j\), where \(\bar{sep}_j\) denotes the firm average of separations, instead of \(\ln sep_{jt}\) in the regressions above. To this end we estimate the baseline specification on a sample where all zero separation observations have been removed. As can be seen in the table, the approximation works very well with almost identical results. Interestingly, compared with the baseline results in column (2) of Table 1, we get a feel for the size of the bias towards zero from removing the zero separation observations.

B Appendix: Derivations

B.1 Employment flow

Let \(W^\text{grid}\) denote the wage grid. Let \(e_{t-1}(a, \hat{w})\) denote employment for workers with idiosyncratic productivity at most \(a\) with wage \(\hat{w}\) in period \(t - 1\). Total employment for workers with idiosyncratic productivity at most \(a\) in period \(t - 1\) is then

\[
e_{t-1}^{agg}(a) = e_{t-1}^{c}(a) + \sum_{\hat{w} \in W^\text{grid}} e_{t-1}^{nc}(a, \hat{w}).
\]
Table 6: Comparisson between Normalized and Log Separations

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>β_{m_r_p}</td>
<td>−1.202</td>
<td>−1.196</td>
</tr>
<tr>
<td></td>
<td>(0.065)**</td>
<td>(0.065)**</td>
</tr>
<tr>
<td>β_{w}</td>
<td>1.516</td>
<td>1.461</td>
</tr>
<tr>
<td></td>
<td>(0.117)**</td>
<td>(0.119)**</td>
</tr>
</tbody>
</table>

Dummies:
Firm: YES YES
Sector by Time: YES YES
Dependent Variable: Normalized Log
Firms: 291,570 291,570
Firms: 69,471 69,471

* (**) Denotes significance on the 5 (1) percent level from zero. Standard errors clustered on the firm level reported inside parenthesis. Sector denotes two-digit NACE codes.

Employment evolution for workers with idiosyncratic productivity at most a that change wages is, when a ∈ [R_{t-1}, R_{t-1}^S]

\[ e_t^c (a) = \alpha \rho \left[ \lambda (G (a) - G (R_t)) (e_{t-1}^{agg} (a_{ab}) - e_{t-1}^{agg} (R_{t-1}^S)) + (1 - f (\theta t)) e_{t-1}^{agg} (R_{t-1}) \right] + (1 - \lambda) (1 - f (\theta t)) (e_{t-1}^{agg} (a) - e_{t-1}^{agg} (R_t)) \]  \hspace{1cm} (B.2)

when a ∈ [R_{t-1}, a_{ab}]

\[ e_t^c (a) = \alpha \rho \left[ \lambda (G (a) - G (R_t)) (e_{t-1}^{agg} (a_{ab}) - e_{t-1}^{agg} (R_{t-1}^S)) + (1 - f (\theta t)) e_{t-1}^{agg} (R_{t-1}) \right] + (1 - \lambda) (e_{t-1}^{agg} (a) - e_{t-1}^{agg} (R_{t-1}^S)) + (1 - f (\theta t)) (e_{t-1}^{agg} (R_{t-1}^S) - e_{t-1}^{agg} (R_{t-1})) \]  \hspace{1cm} (B.3)

and when a = a_{ab}

\[ e_t^c (a) = \alpha \rho \left[ \lambda (G (a) - G (R_t)) (e_{t-1}^{agg} (a_{ab}) - e_{t-1}^{agg} (R_{t-1}^S)) + (1 - f (\theta t)) e_{t-1}^{agg} (R_{t-1}) \right] + (1 - \lambda) (e_{t-1}^{agg} (a) - e_{t-1}^{agg} (R_{t-1}^S)) + (1 - f (\theta t)) (e_{t-1}^{agg} (R_{t-1}^S) - e_{t-1}^{agg} (R_{t-1})) \]  \hspace{1cm} (B.4)

\[ + f (\theta t) (u_{t-1} + \phi_{t-1}) \]

where \phi_{t-1} are workers searching on the job. When R_{t-1} > R_{t-1}^S we have, for a ∈ [R_{t-1}, a_{ab}]

\[ e_t^c (a) = \alpha \rho \left[ \lambda (G (a) - G (R_t)) e_{t-1}^{agg} (a_{ab}) + (1 - \lambda) (e_{t-1}^{agg} (a) - e_{t-1}^{agg} (R_t)) \right] \]  \hspace{1cm} (B.5)
and for $a = a_{ubb}$

$$
\epsilon_t^c (a) = \alpha \rho \left[ \lambda (G (a) - G (R_t)) \epsilon_{t-1}^{agg} (a_{ubb}) + (1 - \lambda) (\epsilon_{t-1}^{agg} (a) - \epsilon_{t-1}^{agg} (R_t)) \right] + f (\theta_t) \left( u_{t-1} + \phi_{t-1} \right).
$$

Then we have

$$
n_t (a) = \epsilon_t^c (a) - \epsilon_t^c (a - 1) + \sum_{\hat{w} \in W_{grid}} [\epsilon_t^{nc} (a, \hat{w}) - \epsilon_t^{nc} (a - 1, \hat{w})].
$$

Employment for workers who do not change wages can be computed as follows. First, suppose $\hat{R}_{t-1}^S (\hat{w}) > \hat{R}_{t-1} (\hat{w})$. For wage state $\hat{w}$, when OJS is chosen, i.e., for $a \in [\hat{R}_{t-1} (\hat{w}), \hat{R}_{t-1} (\hat{w})]$, employment evolves according to

$$
e_{t}^{nc} (a, \hat{w}) = \left( 1 - \alpha \right) \rho \left[ \lambda \left( G (a) - G \left( \hat{R}_{t} (\hat{w}) \right) \right) \right] \left[ e_{t-1}^{nc} (a_{ubb}, \hat{w}) - e_{t-1}^{nc} \left( \hat{R}_{t-1}^S (\hat{w}), \hat{w} \right) \right] + (1 - f (\theta_t)) \epsilon_{t-1}^{nc} \left( \hat{R}_{t-1}^S (\hat{w}), \hat{w} \right) + (1 - \lambda) \left( 1 - f (\theta_t) \right) \left( e_{t-1}^{nc} (a, \hat{w}) - e_{t-1}^{nc} \left( \hat{R}_{t} (\hat{w}), \hat{w} \right) \right).
$$

and, when OJS is not chosen, i.e., for $a \in [\hat{R}_{t-1} (\hat{w}), a_{ubb}]$.

$$
e_{t}^{nc} (a, \hat{w}) = \left( 1 - \alpha \right) \rho \left[ \lambda \left( G (a) - G \left( \hat{R}_{t} (\hat{w}) \right) \right) \right] \left[ e_{t-1}^{nc} (a_{ubb}, \hat{w}) - e_{t-1}^{nc} \left( \hat{R}_{t-1}^S (\hat{w}), \hat{w} \right) \right] + (1 - f (\theta_t)) e_{t-1}^{nc} \left( \hat{R}_{t-1}^S (\hat{w}), \hat{w} \right) + (1 - \lambda) \left( 1 - f (\theta_t) \right) \left( e_{t-1}^{nc} (a, \hat{w}) - e_{t-1}^{nc} \left( \hat{R}_{t} (\hat{w}), \hat{w} \right) \right).
$$

Now, suppose $\hat{R}_{t-1}^S (\hat{w}) \leq \hat{R}_{t-1} (\hat{w})$. Then, for $a \in [\hat{R}_{t-1} (\hat{w}), \hat{R}_{t-1} (\hat{w})]$ we have $e_{t}^{nc} (a, \hat{w}) = 0$ and for $a \in [\hat{R}_{t} (\hat{w}), a_{ubb}]$ we have, modifying the expression above:

$$
e_{t}^{nc} (a, \hat{w}) = \left( 1 - \alpha \right) \rho \left[ \lambda \left( G (a) - G \left( \hat{R}_{t} (\hat{w}) \right) \right) \epsilon_{t-1}^{nc} (a_{ubb}, \hat{w}) \right] + (1 - \lambda) \left( 1 - f (\theta_t) \right) \left( e_{t-1}^{nc} (a, \hat{w}) - e_{t-1}^{nc} \left( \hat{R}_{t} (\hat{w}), \hat{w} \right) \right).
$$

Finally, the unemployment to employment transitions are

$$
UE_t = A \theta_{t-1}^{1-\alpha} u_{t-1}.
$$

---

20Note that workers with idiosyncratic productivity realization at or below $\hat{R}_{t-1}^S (\hat{w})$ search on the job and lose their job only in the current period.
and separations evolve according to, letting \( I_t = 1 \) if \( \hat{R}_{t-1}^S (\hat{w}) > \hat{R}_{t-1} (\hat{w}) \) and \( I_t = 0 \) otherwise,

\[
EU_t = (1 - \rho) \left( c_{t-1}^e (a_{ub}) + \sum_{\hat{w} \in W_{grid}} c_{t-1} (a_{ub}, \hat{w}) \right) + \alpha \rho \left[ \lambda G (R_t) (e_{t-1}^{agg} (a_{ub}) - e_{t-1}^{agg} (R_{t-1}) + (1 - f (\theta_t)) e_{t-1}^{agg} (R_{t-1})) \right] + (1 - \lambda) (1 - f (\theta_t)) e_{t-1}^{agg} (R_{t-1}) + (1 - \alpha) \sum_{\hat{w} \in W_{grid}} \rho \left[ \lambda G \left( \hat{R}_t (\hat{w}) \right) (e_t (a_{ub}, \hat{w}) - I_t f (\theta_t) e_t (\hat{R}_{t-1}^S (\hat{w}), \hat{w})) \right] + (1 - \lambda) (I_t (1 - f (\theta_t)) + (1 - I_t)) a_{ub} (\hat{R}_t (\hat{w}), \hat{w}) \].

(B.11)

### B.2 The algorithm

Since the system (12), (13), (16), (17), (20) and (23) above do not depend directly on unemployment, we can solve without using unemployment as a state variable. Now, for clarity, we do not suppress the dependence of wages, surpluses and labor market tightness on aggregate productivity. Then, since the values of newly created firms and newly hired workers depend on current and future productivities only (through future surpluses, tightness and \( H^c \)), the current wage depends only on the current productivities and tightness depends only on aggregate productivity. Hence, \( w_t^i \) is a function of \( z_t \) and \( a_t \) only. Then, for firm-worker pairs that did not reset their wage today, the wage depends on the productivity when the wage was last reset, say \( \hat{z} \) and \( \hat{a} \). We then write \( \hat{w} (\hat{z}, \hat{a}) \). Then worker surpluses are

\[
H^i (z_t, a_t) = w^i (z_t, a_t) - b - I_t \sigma + \beta E_t \alpha \rho^i \left( \lambda \int_0^1 H (z_{t+1}, r) dG (r) + (1 - \lambda) H (z_{t+1}, a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{H} (z_{t+1}, r, w_{t+1}^i (z_t, a_t)) dG (r) \right) + (1 - \lambda) \hat{H} (z_{t+1}, a_t, w_{t+1}^i (z_t, a_t)) + \beta E_t \left( g^i - f (\theta (z_t)) \right) H (z_{t+1}, a_{ub}) \]  

(B.12)

In case wages are not reset but remain at the level \( \hat{w}_{jt} \) from the previous period, the wage \( \hat{w}_{jt} \) is a state variable and the values are

\[
\hat{H}^i (z_t, a_t, \hat{w} (\hat{z}, \hat{a})) = \hat{w} (\hat{z}, \hat{a}) - b - I_t \sigma + \beta E_t \alpha \rho^i \left( \lambda \int_0^1 H (z_{t+1}, r) dG (r) + (1 - \lambda) H (z_{t+1}, a_t) \right) + \beta E_t (1 - \alpha) \rho^i \left( \lambda \int_0^1 \hat{H} (z_{t+1}, r, \hat{w} (\hat{z}, \hat{a})) dG (r) \right) + (1 - \lambda) \hat{H} (z_{t+1}, a_t, \hat{w} (\hat{z}, \hat{a})) + E_t \left( g^i - f (\theta (z_t)) \right) H (z_{t+1}, a_{ub}) \].

(B.13)
We can proceed similarly for the remaining value equations so that surpluses when wages are re-set depend on current productivity only and surpluses when wages are not rebargained depend on productivity at the last rebargain together with the current productivity.

We solve by fixing a solution for the wage, surpluses and tightness and then use value function iteration to find revised surpluses, wages and tightness. Given convergence of the value function iteration, we can then proceed to compute employment, unemployment, vacancies and separations.
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