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Division for Astronomy and Space Physics

Bachelor Thesis
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A differential spectroscopic study of a faint solar twin in the open cluster M67

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Abstract

A solar twin is a star with characteristics very similar to those of the Sun. This is, a Sun-like star. Earlier studies show that M67-1194 is one of the best solar twins found to date. But this star is not exactly like the Sun. There exist differences between them, in terms of chemical composition, that were defined as well. The goal of this study is to apply a new method to study the similarities of the mentioned star with the Sun. Hopefully a method that provides more precise results. The method uses difference spectra, result of subtracting the spectrum of the star from the solar spectrum, instead of the regular spectra. Then the question is: can this method cast more precise results than those obtained by Önehag et al. in 2011? The answer is that it does. The earlier study gave an uncertainty of $\pm 0.02\,dex$ and ours gave uncertainties between $\pm 0.01\,dex$ and $\pm 0.02\,dex$. Nevertheless, this conclusion must be taken with caution as the study is not as conclusive as the one out carried by Önehag et al..

Sammanfattning

En soltvilling är en stjärna som liknar solen väldigt mycket. Tidigare studier visar att M67-1194 är en av de bästa upptäckta soltvillingarna hittills. Men stjärnan är inte exakt som solen. Det finns skillnader mellan dem angående deras kemiska sammansättningar vilket har också visats i tidigare studier. Den här studiens syfte är att tillämpa en ny metod för att granska likheterna mellan den nämnda stjärnan och solen. En metod som förhoppningsvis ger noggrannare resultat. Metoden använder differensspektra som erhålls genom att subtrahera stjärnans spektrum från solens spektrum, i stället för att använda de vanliga spektra. Frågan är om den här nya metoden kan ge noggrannare resultat än dem som Önehag et al. fick 2011. Svaret är ja. Den tidigare studien visade en osäkerhet på $\pm 0.02\,dex$ medan vår gav osäkerheter mellan $\pm 0.01\,dex$ och $\pm 0.02\,dex$. Icke desto mindre måste våra slutsatser tas försiktigt ty vår studie är inte lika säker som Önehags et al..

Contents

1 Introduction

Solar twins are stars with characteristics very similar to those of the Sun. Of special interest as defining quantities are the effective temperature, the surface gravity and the chemical abundances.

The effective temperature of a star, T_{eff}, is defined to be the temperature of a black-body radiator with the same radius and total energy output as the star. It is quite close to the temperature of the visible surface of the star [9].

The surface gravity, $log\, g$, is the gravitational attraction experienced by an object placed on the surface of the star.

And the chemical abundance, $[A/B]$, which is the amount of element A, compared to the amount of element B, present in the composition of the star, is defined as

$$[A/B] = log_{10}(\frac{N_A}{N_B})_{Star} - log_{10}(\frac{N_A}{N_B})_{Sun}, \tag{1}$$

where N_A is the number of atoms of the element A per unit volume andN_B is the number of atoms of the element B per unit volume present in the star [4].

The problem that concerns us is that of the chemical abundance of the solar twins. One specially important characteristic is the abundance of metals (understanding metals as any element different from hydrogen or helium) in comparison to hydrogen. In the case of sun-like stars, $[Fe/H]$ is commonly very well determined spectroscopically [4]. We call this quantity *metallicity*. The metallicity is defined by equation 1 as

$$[Fe/H] = log_{10}(\frac{N_{Fe}}{N_H})_{Star} - log_{10}(\frac{N_{Fe}}{N_H})_{Sun}. \tag{2}$$

This study will focus on the precision when calculating the difference in the chemical abundance between the Sun and M67-1194 of 7 elements: Al, Cu, Mg, Mn, Na, O and Sc. Henceforth I will designate this difference as $\Delta[X/Fe]$ which is equivalent to $[X/H]$.

To be able to determine how similar a star is to the Sun, we need to compare, in the most precise way possible, the spectra of the studied star and the Sun .

I pursue to develop an improved method, based on the differential spectroscopic approach, this is line-by-line, which delivers more precise results than the method used nowadays.

To test the new method a star in M67 is chosen. M67 is an open cluster of stars. This means that it is a loosely clustered group of stars. Open clusters are

often confined to the galactic plane and normally found within spiral arms. The open clusters are usually young objects and contain up to a few hundred stars within a region of about 30 light-years across [25]. M67 is in the constellation of Cancer and it is around 4 billion years old.

The star chosen is M67-1194 a faint star of apparent luminosity $V \approx 15$. We find this star around 2700 ly away from the Sun. It is a G-type main-sequence star [30].

The chemical abundance of the star M67-1194 have been already studied by Önehag et al. (2011) and the results obtained state that this solar twin has "stellar parameters indistinguishable from the solar values, with the exception of the overall metallicity" [32]. The metallicity they obtained was $[Fe/H] = 0.023 \pm 0.015\, dex$.

The method used by Önehag et al. analyses the spectrum of the star through fitting theoretical spectra derived from models of the stellar and solar spectra. Among other interesting results, they got an uncertainty in $\Delta[X/Fe]$ of 0.02 dex (dex: decimal exponents).

Our method proposes the spectra of the Sun and the studied star to be subtracted from each other obtaining a difference spectrum which will be analysed in search for a theoretical function that will cast the difference in chemical abundance, possibly with more precision. This is, an uncertainty less than 0.02 dex.

2 Background

2.1 Previous study on M67-1194

The first study where the star M67-1194 is mentioned to be a solar analogue is that by Pasquini et al. (2008) [24]. In that work the strength of the H_α-line was used to determine which stars in the cluster M67 had effective temperatures similar to that of the Sun.

Their results showed that M67-1194 and more than nine other stars in M67 had effective temperatures within $100\,K$ of the solar value.

Then, in 2011 a paper by Önehag et al. was published showing that, through differential analysis of the spectrum of M67-1194, the stellar parameters of this star, including the metallicity were similar enough to those of the Sun to label the star as a solar twin. In fact, M67-1194 is considered to be "the best solar twin found to date" [30].

In 2014, the solar twin M67-1194 has been used, as a model of solar twin, by Önehag et al. (2014), in a study of the abundance of elements in other stars of the cluster M67 [31].

More recently Liu et al. (2016) studied the chemical abundances of some solar twins in M67 including M67-1194, obtaining an uncertainty less than 0.02 dex [19].

2.2 Other related works

In 2009 Meléndez et al. conducted an element abundance analysis of the Sun in comparison with 11 solar twins [21]. This study was used by Önehag et al. (2011) to compare M67-1194 to other solar twins and in this way they determined the peculiar similarity of this star with the Sun.

3 Analysing the data

3.1 Software

3.1.1 SIU

SIU (Spectral Investigation Utility) is a software by Johannes Reetz (University of Munich Observatory) provided with indispensable tools for the analysis of spectra like computing of equivalent width, continuum rectification, calculation of difference spectra, and analysis of spectral lines.

3.1.2 MatLab

MatLab (MATrix LABoratory) is numerical computing environment by MathWorks. I used this tool to execute all the numerical treatment of the data. Of special utility were the capacity of the software to manage data as matrices, some built-in functions as the ones used to calculate mean value, standard deviation, correlation between set of data, etc.

Another useful feature in MatLab was its Curve Fitting Toolbox and of course the possibility of writing and running scripts with part of the calculations so they were performed faster.

3.2 Data

The data for M67-1194 utilized for this project were obtained from the spectrometer FLAMES-UVES at ESO-VLT in the spring of 2009. The spectrometer was set to yield a resolving power of $R = 47000$ and a wavelength range from 4800 Å to 6700 Å. In the case of the Sun, the spectrum was a day-time spectrum of the sky taken by the same instrument as above in 2004. These were the same data used by Önehag et al. (2011).

3.3 Normalization of spectra

Raw data obtained from the spectrograph attached to a telescope is reduced to obtain the spectrum of the observed object. This spectrum is nothing but a tabulation of the flux of the object against wavelength. In the case of stars, the obtained spectrum is mostly an absorption spectrum. These absorption spectra are characterized by a continuum and the spectral lines eating flux from the continuum.

In order to compare one spectrum to another, it is recommendable to normalize both spectra. This is, calibrate the data so that the continuum gets the value 1. I did this in SIU applying the Continuum Rectification tool.

In figures 1 and 2, we can appreciate how a spectrum is modified by normalization.

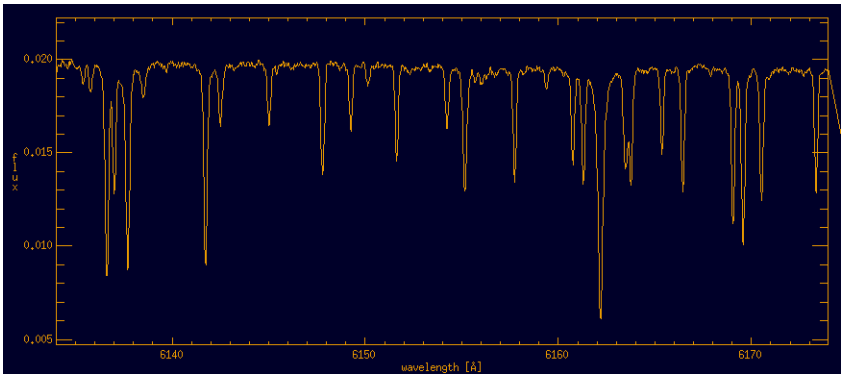

Figure 1: Spectrum before normalization. The continuum lies around 0.020 in this case.

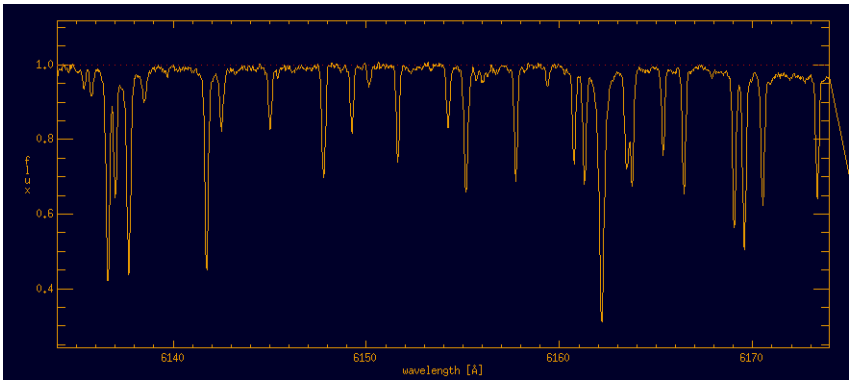

Figure 2: Spectrum after normalization. The continuum lies at 1.000.

3.4 Difference spectrum

After both the spectrum of the Sun and that of M67-1194 have been normalized by the method described in the previous section, I proceed to obtain the difference spectrum which is simply a subtraction of one spectrum from the other. I did this using the Difference Spectrum tool built into in SIU.

As it can be observed in figure 3, for the presented line, there seems to be

almost no difference between the solar spectrum (red dots) and the spectrum of the studied star (black dots). However an almost unnoticeable perturbation in the difference spectrum (green dots) appears just under the spectral line. This tiny bump is my object of study in this work.

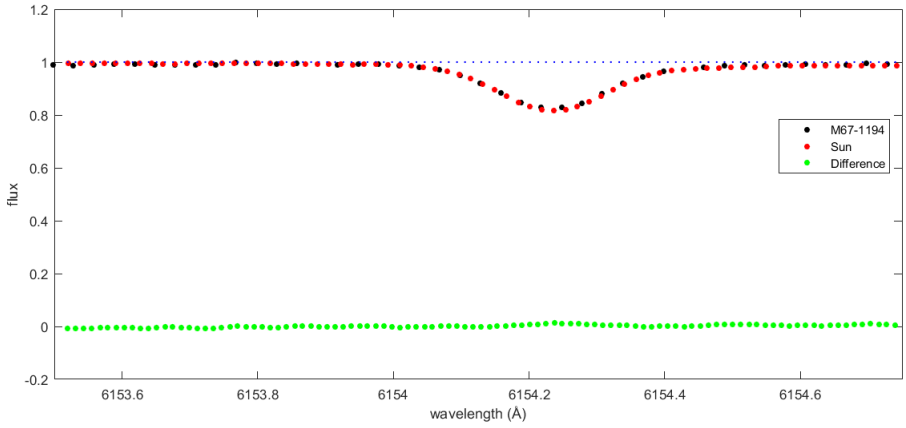

Figure 3: Spectrum around the spectral line Na I (6154.234 Å).

In figure 4 we can see a zoom into the difference spectrum comprehending a wide range of wavelength on both sides of the spectral line. The regions outside the very line will be of importance in our study too.

3.5 Synthetic or theoretical spectra

As seen in figure 4, the difference spectrum of two stellar spectra seems to follow no pattern. However, there is a pattern! The largest peak in the difference spectrum coincides with the presence of spectral lines meanwhile the smaller ones seem to correspond to noise in both original spectra or errors in the normalization process.

The almost imperceptible bump that we saw in figure 3 becomes now a large peak in figure 4 which makes it easier to study. Nevertheless the peak has still a quite complex shape difficult to quantify. What I did was comparing the peak in the difference spectrum with the one obtained from a synthetic or theoretical spectrum. This is, a spectrum created by a mathematical model.

The first step in shaping the theoretical difference spectrum is getting a regular theoretical spectrum of the Sun and another of the studied star. In order

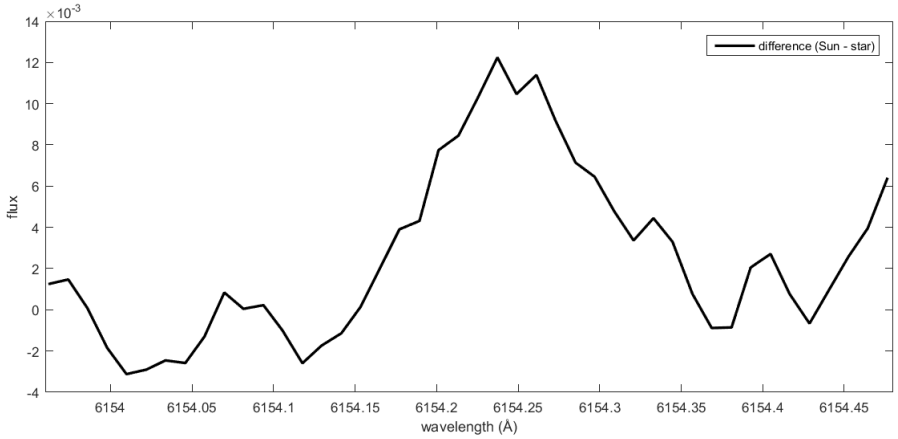

Figure 4: Difference spectrum around the spectral line Na I (6154.234 Å).

to do this I used the Line Formation tool built into SIU.

In figure 5, the utility panel of the Line Formation tool is shown. The red frame shows the stellar parameters of the Sun:

- $Teff = 5780\,\mathrm{K}$, the effective temperature of the Sun.
- $log(g) = 4.44\,\mathrm{cm/s^2}$, the gravitational acceleration on the surface of the Sun.
- $[Fe/H] = 0.00$ the metallicity of the Sun.
- $Xi = 1.00\,\mathrm{km/s}$, the microturbulence in the Sun.

In the bottom line of the panel we find Element Abundance Deviations, whose value OFF corresponds with the solar values but it can be changed specifying both the element and the quantity. I used the value OFF to obtain theoretical spectra of the Sun and changed the value to get theoretical spectra of M67-1194.

I created theoretical spectra for values from -0.09 to 0.09 with a difference of 0.01 between them. The result can be seen in figure 6.

Once I had theoretical spectra for the Sun and for different values of the Element Abundance Deviations I obtained difference spectra using again the Difference Spectrum tool built into SIU. A display of the resulting spectra can be seen in figure 7.

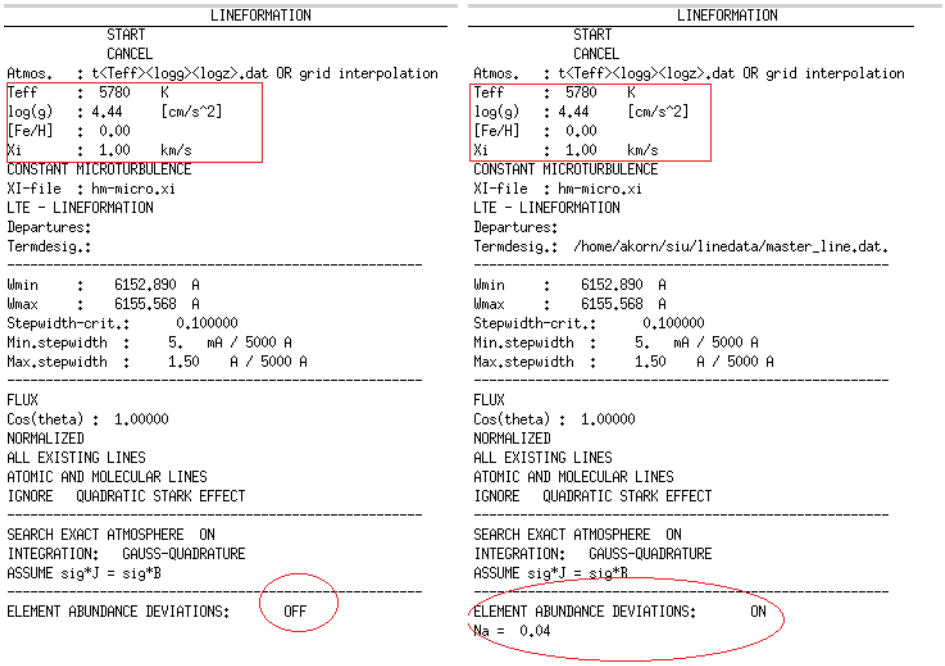

Figure 5: Line Formation Tool in SIU.

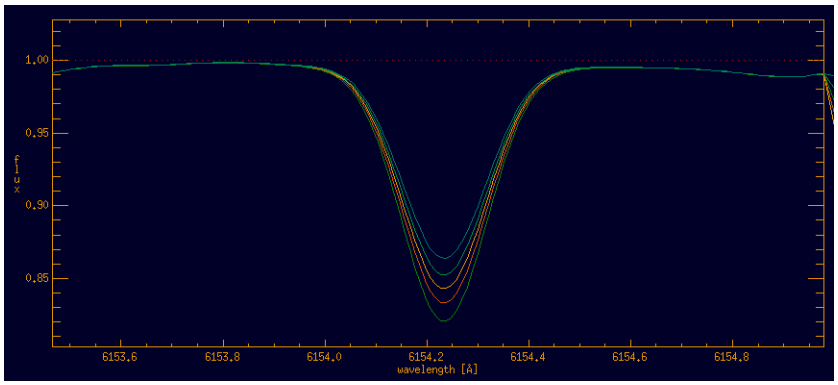

Figure 6: Theoretical spectra of the line Na6154. In descending order: -0.09, -0.04, 0.00 (Sun), 0.04 and 0.09.

3.6 Treatment of data and best fitting estimation

The search for a more precise estimation of the element abundance in M67-1194, which is my final goal, lead me to compare the actual observed difference spec-

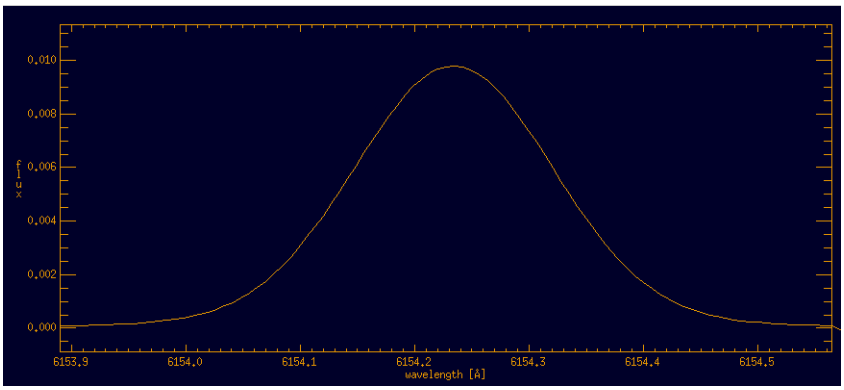

Figure 7: Theoretical difference spectrum for the spectral line Na6154 corresponding to an abundance deviation of 0.04.

trum with the synthetic spectra obtained as explained above.

In order to do this properly, the data required to receive some treatment which would help to eliminate some sources of error and make possible the direct comparison between pixels shaping the spectra. This treatment was done using MatLab.

What I needed to analyse was the difference spectra in the zone corresponding to the spectral line and not the continuum so I extracted, using MatLab, a new set of data containing exclusively the pixels shaping the zone of the spectral line. Using the MatLab function *interp1* upon all the spectra (observed spectrum and the theoretical ones for the different values of abundance deviation) I got an interpolation of the data so that a comparison wavelength-by-wavelength could be performed. See figure 9.

The next step was to compare the theoretical spectra with the observed one to decide which synthetic model fitted best to the actual data. This was done following the weighted least squares method (it can be found as χ^2 in some literature). This method is based on the minimization of the quantity

$$Q = \sum_{i=1}^{n} \left(\frac{O_i - E_i}{\sigma_i}\right)^2 \tag{3}$$

where O_i is the observed data, E_i is the expected data (in our case, the theoretical data) and σ_i the error inherent to the observational data for the i^{th} pixel.

The implementation of the least squares method was done by running two simple MatLab programs of my own making combined, *MKA.m* and *MK.m*, which I attach in the appendix A and B respectively.

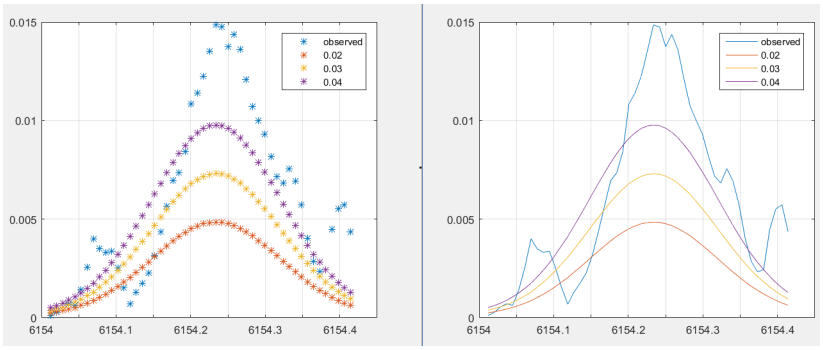

Figure 8: Left: Pixels in the different spectra. Each pixel in the observed spectra has a wavelengthwise matching pixel in each synthetic spectrum. Right: All the spectra as continuous lines.

The estimator Q is calculated for each value of $\Delta[X/Fe]$ and the one getting the least value of Q is the best fitting curve to the actual observed difference spectrum.

3.7 Uncertainties

In this work, the uncertainties or errors are the main goal of study. Therefore I explain in detail how these quantities were calculated.

The programs *MKA.m* and *MK.m* were also used to calculate, by propagation, the signal-to-noise ratio of the difference spectrum

$$SN_{diff} = \sqrt{(SN_{star})^2 + (SN_{sun})^2}, \tag{4}$$

where SN_{star} and SN_{sun} are the signal-to-noise ratio of the star and the Sun respectively, calculated as

$$\frac{1}{\sqrt{\sigma^2}} \tag{5}$$

where σ^2 is the covariance of the data shaping the continuum in the original spectra.

From the expression for the signal-to noise ratio, shown in equation 5, we obtain

$$\sigma_{diff} = \frac{1}{SN_{diff}}. \tag{6}$$

What the implementation of the combination of the programs *MKA.m* and *MK.m* does is to calculate Q for each theoretical spectrum, then moves the observed spectrum by the factor σ_{diff} and calculate Q again. The difference in abundance between the first and the second best fitting is the estimated error in the abundance deviation.

Let us consider a real example! For the spectral line Cu 5220 I obtained 0.01 as the first best fitting value and -0.01 as the best fitting value with the offset continuum, then the error is $S =\mid 0.01 - (-0.01) \mid = 0.02$. When the first and the second best fitting is the same theoretical curve $S = 0.00$, and then I conservatively adopt $S = 0.02$ as Önehag et al. did.

Once I got the total error for each line, neglecting errors due to stellar parameters which are small in solar twin studies, I needed to get both the mean value of $\Delta[X/Fe]$ and its uncertainty for the element. I calculated the mean value of chemical abundance deviation using weighted arithmetic mean

$$\Delta[X/Fe]_{Tot} = \frac{\sum_{i=1}^{n} \Delta[X/Fe]_i S_i^{-2}}{\sum_{i=1}^{n} S_i^{-2}}. \tag{7}$$

The propagated uncertainty of the weighted arithmetic mean is

$$u = \sqrt{\frac{1}{\sum_{i=1}^{n} S_i^{-2}}}. \tag{8}$$

The result of my study is given as $\Delta[X/Fe]_{tot} \pm u$.

4 Results

The method explained above gave me the following results:

Table 1: Results obtained for the spectral lines considered during the analysis.

Element, λ (Å)	$\Delta[X/Fe]_{Sun-star}$ (dex)	Error (dex)
Al I		
6696.023	0.02	0.01
6698.681	0.05	0.02
Cu I		
5218.207	-0.03	0.03
5220.091	0.01	0.02
Mg I		
5711.092	0.09	0.02
6318.711	-0.02	0.01
6319.249	-0.09	0.02
Mn I		
6013.495	0.07	0.02
6016.647	0.07	0.01
6021.802	0.01	0.02
Na I		
6154.234	0.03	0.01
6160.758	0.01	0.01
O I		
6158.190	-0.09	0.02
Sc II		
5657.880	-0.04	0.01
5684.195	0.03	0.02
6245.626	0.02	0.02

Table 2: Results obtained for the chemical elements considered during the analysis.

Element	$\Delta[X/Fe]_{Sun-star}$ (dex)	Error (dex)
Al I	0.03	0.009
Cu I	0.02	0.017
Mg I	-0.01	0.008
Mn I	0.07	0.009
Na I	0.02	0.007
O I	-0.09	0.020
Sc II	0.02	0.008

5 Discussion

Table 2 shows the uncertainties related to the difference between the Sun and M67-1194 in the abundance of the studied elements and there it can be seen that I achieved errors between ≤ 0.02 dex which is a better result to that of ± 0.02 dex obtained by Önehag et al. (2011). In the case of the oxygen only one spectral line was studied and in such case we adopt the uncertainty to be 0.02 dex just as the it was done by Önehag et al. (2011).

During the study I tried another formulation to find the uncertainties using correlation between pixels. However this approach was not successful. The uncertainties obtained that way laid not far away from these obtained in earlier studies but on the other hand the values of $\Delta[X/Fe]$ were not reliable. Thus I made the decision of discarding it.

The results I got for $\Delta[X/Fe]$ differ from those obtained by Önehag et al, (2011) from 0.00 dex for some spectral lines to 0.09 dex for another spectral lines. This can be the consequence of at least two factors.

One reason could be the error introduced in the process of normalization of the spectra. The spectrum of the Sun and the one of M67-1194 were normalized separately and when combined it becomes appreciable some difference in the level of the continuum between the spectra.

The other reason could have to do with the fact that the two spectral lines presenting larger discrepancy, this is 0.09 dex, are Mg I 5711 and Mn I 6016 which are both of them stronger spectral lines. Meaning this, that they go below 0.8 in flux. The fitting of theoretical spectra to stronger spectral lines behaves different from that to weaker lines. For weaker lines the strength of the line is proportional to the chemical abundance but this proportionality deviates when studying stronger lines.

All in all it is worthy to mention that the study carried out by Önehag et al. (2011) encompasses a much larger amount of spectral lines including those of almost 20 different elements besides more than 30 lines of iron which are also important when studying the composition of Sun-like stars. So to achieved a more conclusive result a more extensive study, this is with more spectral lines, is required.

6 Conclusions

The method to determine the chemical abundance of the solar twin M67-1194 using difference spectra and a line-by-line (differential) approach seems to cast quite precise results ($error \leq 0.02$ dex). With even higher precision than other previous studies.

In despite of the small amount of lines studied, the apparatus used to quantify the uncertainties appears to be consistent. Even the mean element abundance deviation, $\Delta[X/Fe]$ for the most of the 7 considered elements, shown in table 2, confirms the Sun-like nature of M67-1194. This cast reliability upon the method.

Again, I insist a more extensive study embracing many more spectral lines should be carried out in order to test the method on the whole chemical composition of the star.

7 Further Research

As mentioned above more extensive study is required to determine if this difference spectrum method is more profitable than other methods. Such a study would include more spectral lines, especially those of iron.

Beyond all that, it could be possible to perform a more mathematically profound analysis of the difference spectra to establish some sort of mathematical tool, useful for this particular kind of spectra, similar to the equivalent width used on regular spectra.

Also, to minimize the error introduced by the normalization process one could get the difference spectrum directly from the unnormalized spectra and afterwards normalized the resulting difference spectra to zero.

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A *MKA.m*

```
% MKA assigns a Least Square value (applicating
% the program MK) to each theoretical curve,
% gives which curve is the best fitted. Then
% executes the offset of the continuun and gives the
% new best fitting.



ob=fline(:,2);

teo=ftd0(:,2);MK;Q0=Q;
teo=ftd1(:,2);MK;Q1=Q;
teo=ftd2(:,2);MK;Q2=Q;
teo=ftd3(:,2);MK;Q3=Q;
teo=ftd4(:,2);MK;Q4=Q;
teo=ftd5(:,2);MK;Q5=Q;
teo=ftd6(:,2);MK;Q6=Q;
teo=ftd7(:,2);MK;Q7=Q;
teo=ftd8(:,2);MK;Q8=Q;
teo=ftd9(:,2);MK;Q9=Q;
teo=ftdm1(:,2);MK;Qm1=Q;
teo=ftdm2(:,2);MK;Qm2=Q;
teo=ftdm3(:,2);MK;Qm3=Q;
teo=ftdm4(:,2);MK;Qm4=Q;
teo=ftdm5(:,2);MK;Qm5=Q;
teo=ftdm6(:,2);MK;Qm6=Q;
teo=ftdm7(:,2);MK;Qm7=Q;
teo=ftdm8(:,2);MK;Qm8=Q;
teo=ftdm9(:,2);MK;Qm9=Q;

Qg1=[Q0, Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Qm1, Qm2,
    Qm3, Qm4, Qm5, Qm6, Qm7, Qm8, Qm9]';
[B1,L1]=min(Qg1);
%-------------------------------

%-----------------OFFSET----------
sigg=1/SNgdiff; ffline=fline(:,2)-sigg;
hold on
plot(fline(:,1),ffline);
%-------------------------------

%-------------------------------

```

```
ob=ffline;
teo=ftd0(:,2);MK;Q0=Q;
teo=ftd1(:,2);MK;Q1=Q;
teo=ftd2(:,2);MK;Q2=Q;
teo=ftd3(:,2);MK;Q3=Q;
teo=ftd4(:,2);MK;Q4=Q;
teo=ftd5(:,2);MK;Q5=Q;
teo=ftd6(:,2);MK;Q6=Q;
teo=ftd7(:,2);MK;Q7=Q;
teo=ftd8(:,2);MK;Q8=Q;
teo=ftd9(:,2);MK;Q9=Q;
teo=ftdm1(:,2);MK;Qm1=Q;
teo=ftdm2(:,2);MK;Qm2=Q;
teo=ftdm3(:,2);MK;Qm3=Q;
teo=ftdm4(:,2);MK;Qm4=Q;
teo=ftdm5(:,2);MK;Qm5=Q;
teo=ftdm6(:,2);MK;Qm6=Q;
teo=ftdm7(:,2);MK;Qm7=Q;
teo=ftdm8(:,2);MK;Qm8=Q;
teo=ftdm9(:,2);MK;Qm9=Q;

Qg2=[Q0, Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Qm1, Qm2,
    Qm3, Qm4, Qm5, Qm6, Qm7, Qm8, Qm9]';
[B2,L2]=min(Qg2);
%-------------------------------

%-------------------------------------
Ref=[1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19;0.00
     0.01 0.02 .03 .04 .05 .06 .07 .08 .09 -.01 -.02 -.03
    -.04 -.05 -.06 -.07 -.08 -.09]'

Eg=[L1;L2]
%-------------------------------------

%-------------------END-------------

% By Carlos Fernandez Ortega
```

B *MK.m*

```
% MK calculates the signal-to-noise ratio,
% the error in the observated data and the
% Least Square Estimator.


%----SNR-----------------------------------
```

```
x = Csun(:,2);
SNgsun = 1/sqrt(cov(x));
x = Cstar(:,2);
SNgstar = 1/sqrt(cov(x));
SNgdiff = sqrt(SNgstar^2 + SNgsun^2);
%-------------------------------------------------

%-----------Residuals and errors-------
Residuals = ob - teo;
errors = ob/SNgdiff;
%-----------------------------------

%-----Least Square---------------------
n=length(ob);
C=zeros(n,1);
for  i=1:n
     C(i)=(Residuals(i)/errors(1))^2;
end

Q=sum(C);
%-------------------------------------

%---------------END-----------------------

% By Carlos Fernandez Ortega
% Original idea by Thomas Nordlander
```