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Abstract

This study investigates the general equilibrium effects of a fertility shock under different intergenerational transfer schemes. The effects on lifetime income and utility for different generations, as well as the effects on factor prices, are analyzed in a three-period overlapping generations model where the workers provide for the young and the retired under different tax schemes. The economic effects of a fertility shock vary substantially with different intergenerational transfer schemes. How wages, interest rate and savings will evolve differs not only quantitatively but also qualitatively. To minimize the effects from a fertility shock it is vital that the effects on human capital are minimized. For a baby boom shock this implies that a higher fraction of output must be devoted to human capital accumulation, during the educational years of the baby boom generation. With respect to transfers to the old, the tax rate should not be fixed.

Key Words: Intergenerational transfers, demography, social security, education.

JEL classification: J13, H55, H52

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1 Introduction

In most OECD countries the old age dependency ratio is projected to increase dramatically during the first half of this century. This has lead to considerable anxiety regarding the financial viability of the social security programs. Less attention has been devoted to the fact that fertility has been the principal source of the changing demographic structure;[1] implying that changes to the old age dependency ratio are predated by changes in the young age dependency ratio. Just as the old age dependency is crucial for the social security financing, is the young age dependency crucial for the education financing. When analyzing the distributional effects between generations it is necessary to account for both the young age dependency and the old age dependency. One must also consider which type of intergenerational transfers that are in place, since it is well known from the pension literature that different schemes have very different distributional properties.

The pension literature identifies the main distinctions between different intergenerational transfer schemes, and investigates if one type of system dominates the other. The schemes respond differently to demographic and productivity disturbances.[2] Unfortunately, the pension literature does not investigate the general equilibrium effects from a demographic change. Factor prices are treated exogenous and changes in the young age dependency is seldom accounted for. The exclusion of the young age dependency may be misleading and the *ceteris paribus* assumption about factor prices is also most likely incorrect, unless the Feldstein-Horioka puzzle disappears altogether. To account for these factors, it seems necessary to use a general equilibrium framework.

Typically, simulation methods are required to analyze demographic effects in a general equilibrium framework. Pioneering in this direction was the work by Auerbach and Kotlikoff (1987), which created a tool for investigating the macroeconomic effects of demographic changes. Most of these studies, however, do not investigate different types of intergenerational transfer systems. At most, some studies investigate different types of pension systems, but with respect to the intergenerational transfers to the young they do not employ the same systematic treatment.[3]

[1] See Organization for Economic Cooperation and Development (1988) on the relative importance between fertility, mortality, and migration.

[2] See for instance Hassler and Lindbeck (1997), Thøgersen (1998), Lindbeck (2000) and Wagener (2003). They discuss different types of pay-as-you-go (PAYG) pension systems, but these are by definition intergenerational transfers.

[3] See for instance Auerbach and Kotlikoff (1985) and Blomquist and Wijkander (1994) for models with no human capital. While some studies that include human capital are

This paper investigates how the effects from a fertility shock, illustrated by a baby boom, vary with different types of intergenerational transfer systems. This is done from a theoretical perspective by the use of a three period overlapping generations (OLG) model. I focus on the intergenerational transfer schemes, as in the pension literature, while incorporating both the young age dependency and the effect on factor prices, as in the computable general equilibrium literature. The novelty of this paper consists of explicitly treating the young age dependency as a intergenerational transfer system that can respond to a demographic change in different ways.

A demographic shock which creates a gain in the financing of young age dependency, will create a burden in the old age dependency system (though not in the same period). Moreover, in both cases it is a distributional matter between the same two generations; the parent generation and the shock generation. Which generation that will be burdened and which will receive the gain depends on the type of transfer systems. There will also be an affect on the capital intensity. For a baby boom shock there will be two negative effects, the education burden and the capital dilution. There will also be a positive effect in the form of a gain in the pension system.

The relative outcome for different generations is highly dependent on the transfer schemes, and so are the transition paths for factor prices. Not only do the paths differ quantitatively, they can also differ qualitatively.

Using an *ax ante* approach to compare the different schemes, based on an utilitarian social welfare function, I find that the transfers that flow to the old should not be adjusted to achieve a fixed tax rate. This is counter to what Thøgersen (1998) finds, but supports what Wagener (2003) finds with an *ex post* approach. Regarding the transfers to the young, every child should be guarantied a certain fraction of output, that they can devote to human capital accumulation. In this way the effects on human capital after a demographic disturbance are minimized.

The remainder of this paper is organized as follows. In section 2, the different schemes for intergenerational transfers are presented. I section 3 a partial analysis on the implications from a demographic shock on the intergenerational transfers is conducted. Section 4 presents the general equilibrium model. In section 5, the model is calibrated and the steady state results are presented. Section 6 presents the results while section 7 concludes.

Docquier and Michel (1999), Fougère and Mérette (1999), Pecchenino and Utendorf (1999), and Pecchenino and Pollard (2002).

2 Intergenerational transfers

The aim of intergenerational transfers is to provide support during life-cycle periods with no labor activity. There are both formal and informal channels through which these intergenerational transfers flow from the active population to the inactive population. For the elderly the formal channels are dominant in the developed world, so it is quite natural to view this as a separate system since it is more or less an explicit intergenerational contract between the active population and the elderly. The transfers to the children have strong formal channels such as mandatory education, but even the informal channels can be viewed as an implicit intergenerational transfer due to custom and tradition. It seems reasonable to view these transfers as two separate systems. Similar to other applications which suffer from time inconsistency problem it is desirable that these systems or institutions are governed by laws which seldom change; in the spirit of Kotlikoff et al. (1988). This paper investigates different types of laws that can govern these transfers.

Assume that there is a system that handles all intergenerational transfers that flow from the working population to the elderly. A large portion of these transfers consist of PAYG pension, and for this reason I denote this system the **pension system**. Here the pension system incorporates medicare expenses and the like, but excludes non-intergenerational retirement solutions such as funded pensions.

For the young age support, let's assume a separate system. Since a large portion of the intergenerational transfers to the young consists of education, it is natural to refer to this system as the **education system**. The education system refers to the overall intergenerational transfers between the active population and the children.

What is characteristic of the pension system is that when entering the system (i.e. when entering the active labor age) individuals start by paying to the system and then later, when retired, they will receive. The education system is the opposite from the pension system in the sense that when entering the education system (i.e. at birth) individuals start by receiving and then later when joining the work force they will pay to the system. This, seemingly trivial, difference is crucial for the analysis of a demographic shock.

2.1 Modelling the transfers

The simplest way to capture both the education and the pension system is to use a three period OLG model. The OLG model consists of one period when young, one period when working, and one period when retired. The young receive contributions from the working population via the education system, and the retired receive contributions from the working population via the pension system. For the systems to be pure intergenerational transfers it is necessary that the budgets are balanced in each period. Thus assuming a period-by-period balanced budget for each system separately, makes it possible to state the transfers in period t as:[4]

$$b_{E,t}N_t = d_{E,t}N_{t-1}, \tag{1}$$

$$b_{P,t}N_{t-2} = d_{P,t}N_{t-1}, \tag{2}$$

where $b_{E,t}$ denotes the per child benefit from the education system, $d_{E,t}$ is the contribution per worker to the education system, $b_{P,t}$ is the benefit per retired from the pension system, and $d_{P,t}$ denotes the contribution per worker to the pension system. These are indexed with subscript t to denote that the transfer occurs in period t. The size of each generation is denoted by N, where the subscript t indicates in which period the generation is born. In period t the number of children is N_t, while the number of workers is N_{t-1}, and the number of retirees is N_{t-2}.

Suppose that each worker in period t has n_t children. Then the young age dependency ratio in period t, N_t/N_{t-1}, is denoted n_t and hence the old age dependency ratio, N_{t-2}/N_{t-1}, equals n_{t-1}^{-1}. From the balanced budget restrictions in equations (1) and (2) one can immediately see the impact of changes in the dependency ratios. Demographic changes will either change the received benefits or the contributions, or both.[5]

Above the contributions and benefits where not related to the level of income in the society. In a world with growing income over time it would not make sense to have fixed benefits/contributions over time. It is reasonable to relate the benefits/contributions to the income, where income refers to the mean income of the working generation.

Let $\tilde{w}_t$ denote the mean labor income of the workers in period t, and let $\tau_{E,t}$ and $\tau_{P,t}$ denote the *contribution rate* devoted for financing the education and the pension system, respectively. The contribution from the workers, $d_{i,t}$, where $i = E, P$, can then be stated as:

$$d_{i,t} = \tilde{w}_t\tau_{i,t}. \tag{3}$$

[4]The assumption regarding two separate systems is mainly based on the fact the existing social security programs have a very weak connection with the education system, if any. If the period-by-period balanced budget assumption was loosened, then there would be other financing opportunities for an open economy.

[5]Which it changes depends on the transfer schemes, this is however explained in subsection 2.2.

The received benefits, $b_{i,t}$, can also be related to the income level of the working population according to:

$$b_{i,t} = \tilde{w}_t \gamma_{i,t}, \tag{4}$$

where $\gamma_{i,t}$ are the *benefit rates* in the transfer systems. The benefit rates are the fraction of active workers income that each child/retired receives.[6]

The period-by-period balanced budget constraints for the two transfer systems can then be rewritten as:

$$\gamma_{E,t} = \tau_{E,t}/n_t, \tag{5}$$

$$\gamma_{P,t} = \tau_{P,t} n_{t-1}. \tag{6}$$

Changes in the dependency ratios will either affect the contribution rate or the benefit rate. By inserting equations (5) and (6) into equations (3) and (4) it is clear that the benefits/contributions will not only depend on demographic changes but also on how income changes.

2.2 Different schemes

The various intergenerational transfer schemes differ in how the benefits and the contributions respond to changes in demography and income. The difference between the schemes can be understood from the balanced budget restrictions.

From equations (5) and (6) two simple schemes emerge. Either the benefit rate is fixed, $\gamma_{i,t} = \gamma_i$, or the contribution rate is fixed, $\tau_{i,t} = \tau_i$. These schemes will simply be referred to as *fixed benefit rate, FB,* and *fixed contribution rate, FC.*[7] It is, however, possible to have a fixed benefit rate in the education system, while having a fixed contribution rate in the pension system since the systems operate independent of each other.

For the pension system one more scheme will be considered. This scheme will be labelled *fixed replacement rate, FR.* In this case the benefits received in the pension system are related to *previous* income instead of current income, i.e. the income from one's own active life. For the education system the

[6]The term benefit rate is, to my knowledge, not used in the literature. This is not to be confused with the term *replacement rate* which is used in the pension literature, and which will be described further on. The benefit rate is an theoretical abstraction and is also used in Lindbeck (2000), though not using the same term.

[7]It is possible to let both the contribution rate and the benefit rate vary, this is a convex combination of these two extreme cases that will not be explored in this paper. Wagener (2004) analyzes convex combinations between the FC and FR scheme for the PAYG pension system.

benefits will always be related to current income, since when entering the education system the individuals have no previous income.

The motivation for investigating the FC and FR scheme is that existing PAYG pension systems often belong to one of these schemes. The motivation for the FB scheme is that this scheme from a theoretical point is the opposite of the FC scheme, according to equations (5) and (6). Also, when investigating the education system this is the only natural alternative to the FC scheme.

Below the different schemes are presented and distinguished according to their benefit formula.[8]

2.2.1 Fixed benefit rate, FB

A fixed benefit rate in either the education or the pension system, i.e. $\gamma_{i,t} = \gamma_i$, gives the following benefit formula in period t:

$$b_{i,t}(\tilde{w}_t) = \gamma_i \tilde{w}_t. \tag{7}$$

In this case the benefit in period t only depends on the current income. How the dependency ratio evolves over time does not directly matter for the benefit. With respect to demographic changes it is the workers contribution that is altered to fulfill the budget restriction. The retired and/or the children are always promised a certain amount of the current income independent on how many they are in relation to the working population.

2.2.2 Fixed contribution rate, FC

In this case the workers are promised to pay a certain fraction, τ_i, of their income to the young and/or the pension system. This will result in the following benefits in the education and pension systems:

$$b_{E,t}(\tilde{w}_t, n_t) = \tau_E \tilde{w}_t / n_t, \tag{8}$$

$$b_{P,t}(\tilde{w}_t, n_{t-1}) = \tau_P \tilde{w}_t n_{t-1}. \tag{9}$$

Benefits will in this case not only fluctuate with income, but also with demographic fluctuations. On the other hand, the contributions from workers will only fluctuate with current income.

[8] Alternatively, the contribution formula could be used. Which is used does not matter, if the benefit formula is known then the contribution formula is given via the balanced budget restrictions. Here the benefit formula is used since the common approach in the pension literature is to identify the pension formula.

2.2.3 Fixed replacement rate, FR

Benefits are a fraction of the retired person's own income while working and this fraction is referred to as the *replacement rate*.[9] Let $\tilde{\gamma}$ denote the replacement rate, which implies that the benefit rate can be stated as, $\gamma_{P,t} = \tilde{\gamma}/\theta_t$ where $\theta_t = \tilde{w}_t/\tilde{w}_{t-1}$. In this case, the benefit formula can be stated as:

$$b_{P,t}(\tilde{w}_{t-1}) = \tilde{\gamma}\tilde{w}_{t-1}. \tag{10}$$

With respect to demographic shocks, this benefit formula is similar to the benefit formula in the FB scheme. In both cases the benefit is independent of the dependency ratios. The difference is that past income instead of current income determines the benefit. After income realization the workers know what their future retirement benefit will be, irrespective of future wages and demographic structure.[10] In the previous schemes the contributors and the beneficiaries have shared the income uncertainty, in this case the workers bear the full cost of both demography and income uncertainty.

For the transfers to the children, i.e. the education system, it is not reasonable to assume such a benefit formula since they have no past earnings.

2.3 Generation t from crib to grave

Here generation t is followed over the life-cycle to illustrate how the benefit rates, total tax rate, and the implicit interest rates will differ under the transfer schemes.

2.3.1 Benefit rates and the total tax rate

Table 1 shows the benefit rates for generation t under the different schemes. The fraction of income that the generation pays to the systems can be summarized as a total tax rate. Let τ_t be the total tax rate in period t, which can be expressed as:

$$\tau_t = \tau_{P,t} + \tau_{E,t} = \gamma_{P,t}/n_{t-1} + \gamma_{E,t}n_t. \tag{11}$$

The total tax rate in period $t+1$, which generation t pays, can thus be stated as a function of the benefit rates in the education and the pension systems. For each of the six possible scheme combinations there is a corresponding total tax rate, which is presented in table 2.

[9] In the literature it sometimes occurs that the replacement rate refers to the fraction of current income (what is referred to as the benefit rate in this paper). This is, however, conceptually obscure since the benefits of the present pensioners does not replace the wages

Table 1: The benefit rates for generation t.

	Education $\gamma_{E,t}$	Pension $\gamma_{P,t+2}$
FR	-	$\tilde{\gamma}/\theta_{t+2}$
FB	γ_E	γ_P
FC	τ_E/n_t	$\tau_P n_{t+1}$

Table 2: Total tax rate for generation t, τ_{t+1}.

Pension	Education	
	FB	FC
FR	$\tilde{\gamma}/\theta_{t+1}n_t + \gamma_E n_{t+1}$	$\tilde{\gamma}/\theta_{t+1}n_t + \tau_E$
FB	$\gamma_P/n_t + \gamma_E n_{t+1}$	$\gamma_P/n_t + \tau_E$
FC	$\tau_P + \gamma_E n_{t+1}$	$\tau_P + \tau_E$

2.3.2 Implicit interest rate

In the education system the generations start by receiving benefits which implicitly will be repaid in the next period when working. The implicit gross interest rate on intergenerational loans between period t and $t+1$ will be denoted $R_{E,t+1}$ where the subscript E indicates the education system. It is thus generation t that has to pay the implicit interest rate $R_{E,t+1}$. Since each generation implicitly borrows in the education system it want this interest rate to be as low as possible.

In the pension system the generations start by making contributions when working and then when retired they receive benefits. Thus, there is an implicit rate of return on the contributions made. The interest rate received by generation t is denoted $R_{P,t+2}$, where the subscript indicates that it is an implicit rate of return on investment made between period $t+1$ and $t+2$. Since it is an implicit investment, each generation wants the interest rate in the pension system to be as high as possible. By definition the implicit interest rates in the education system and the pension system, for generation t, can be stated as:

$$R_{E,t+1} = d_{E,t+1}/b_{E,t}, \tag{12}$$

of present workers. Augustinovics (1999), among others, has also pointed at this misuse i the literature.

[10]This holds under the assumption that the feasibility constraint is not violated.

$$R_{P,t+2} = b_{P,t+2}/d_{P,t+1}. \quad (13)$$

The implicit interest rate for generation t under the different transfers schemes is presented in table 3. From table 3 it is clear that if there where no changes

Table 3: Implicit interest rate for generation t.

	Education $R_{E,t+1}$	Pension $R_{P,t+2}$
FR		$\theta_{t+1}n_t$
FB	$\theta_{t+1}n_{t+1}$	$\theta_{t+2}n_t$
FC	$\theta_{t+1}n_t$	$\theta_{t+2}n_{t+1}$

to income development nor population growth, the schemes would be identical. A increase in the population growth (or productivity growth) will imply a burden in the education system due to higher interest rate on "loans"; while it will imply a gain in the pension system, due to higher interest rate on "investments".

What also emerges is that the education and the pension systems respond in the opposite way after a demographic shock. In the education system a FB scheme implies that the implicit interest rate for generation t is determined by the population growth between generation t and its children. If the education system is a FC scheme then the implicit interest rate for generation t is determined by the population growth between generation t and its parents.

The interest rate in the pension system of FR or FB type is determined by the population growth between generation t and its parents. While if it is a FC type then the interest rate is given by the population growth between generations t and its children.

Regarding the income growth, the interest rate in the education system is always determined by the income growth between generation t and its parents. The interest rate in the pension system is dependent on the income growth between the generation t and its children. The exception is the pension system of FR type where the interest rate depends on the growth between generation t and its parents.

3 Demographic shock: partial analysis

In this section a partial analysis of the effects from a demographic shock on the intergenerational transfers is conducted. The interaction between demography and income is ignored. Since fertility has been the principal source of the changing demographic structure it is natural to consider a baby boom shock as the demographic shock. A baby boom shock can be expressed as $n_{t+j} = n \; \forall j \neq 0$ and $n_t > n$, i.e. a baby boom in period t. To simplify the analysis, the steady state population growth is normalized to zero, i.e. the gross birth rate is $n = 1$. These assumptions imply that all generations prior to the baby boom generation are of equal size smaller than the baby boom generation; while all generations after the baby boom generation are of the same size as the baby boom generation.

What will be considered is how the different generations are affected during their life-cycle, in terms of deviation from steady state. Lets first investigate how the generations are affected in the education system. This is presented in table 4. If the education system operates under a FB scheme it

Table 4: The effect on the education system, from $n_t > 1$.

gen.	FB	FC
$t-2$	0	0
$t-1$	$(1-n_t)\,\gamma_E \tilde{w}_t$	0
t	0	$(1/n_t - 1)\,\tau_E \tilde{w}_t$
$t+1$	0	0

Note: Deviation from steady state outcome over the life-cycle for the different generations (gen.).

will be generation $t-1$, i.e. the parent generation, that pays a higher tax rate (see table 2). Under the FC scheme it is the benefit rate in period t that is lower, this accrues to generation t, i.e. the boom generation (see table 1). The allocation of the cost in the education system, implied by the higher n_t, will thus be a distributional matter between the boom generation and its parent generation.

Table 5 presents how the generations are affected in the pension system. If the pension system operates under a FB or FR scheme then an increase in birth rates, $n_t > 1$, will lead to a lower tax rate in period $t+1$, which is paid by generation t, i.e. the boom generation. The other generations will have the same tax rate as in the steady state. If the pension system operates under a FC scheme then the gain of a higher benefit rate will accrue to generation $t-1$, the parents of the baby boom generation. Thus the gain in the pension system can either accrue to the boom generation or the parent generation.

Table 5: The effect on the pension system, from $n_t > 1$.

gen.	FB	FC	FR
$t-2$	0	0	0
$t-1$	0	$(n_t - 1)\,\tau_P \tilde{w}_{t+1}$	0
t	$(1 - 1/n_t)\,\gamma_P \tilde{w}_{t+1}$	0	$(1 - 1/n_t)\,\tilde{\gamma}\tilde{w}_t$
$t+1$	0	0	0

Note: Deviation from steady state outcome over the life-cycle for the different generations (gen.).

3.1 Partial analysis conclusion

The baby boom shock implies a burden in the education system and a gain in the pension system. In both systems it will be a distributional matter between the baby boom generation and its parent generation. Whether the burden/gain accrues to the boom generation or its parent generation depends on the transfer schemes.

If the education and the pension system operate under opposite schemes both the burden and the gain will accrue to the same generation. This happens either if the education system is of FB type and the pension system is of FC type, or if the education system is of FC type when the pension system is of FB or FR type. With respect to demographic uncertainty opposite schemes seem to have a desirable risk sharing feature if the factor prices are unaffected.

This partial analysis did not consider any interaction between demographic changes and income development. The analysis did, however, identify how the education and the pension system differ with respect to the income growth. If the pension system is of FR type then the return in the pension system depends on the income growth between generation t and its parent generation; otherwise it depends on the income growth between generation t and its child generation. The latter gives incentives, at least on the aggregate level, to care about the ability of the future workforce. To include such consideration in the analysis makes it necessary to account for the income development, which was not done in this partial analysis.

In the general equilibrium income growth will depend on demographic changes, both via factor price movements and via human capital accumulation. Also, the general equilibrium analysis will give a quantitative estimate of the gain in the pension system, the burden in the education system, and the effects on factor price changes from changes in the capital labor ratio.

Note that in this partial analysis the effects from the baby boom only lasted for two periods, in period t and period $t+1$. This will not be the case in the general equilibrium, and thus it will be necessary to use simulation methods in the analysis.

4 The model

The general equilibrium model adds a production function and capital accumulation to the three period OLG model. The model consists of three components: individuals that maximize their lifetime utility, firms that maximize their profit, and the intergenerational transfer systems. The transfer systems are exogenous and permanent and they can operate according to the schemes above. Agents know under which scheme the systems operate and they have perfect foresight. Except for the exogenous intergenerational contract (i.e. the transfer systems) there is no altruism between generations. The model is a simpler version of Pecchenino and Pollard (2002) who include altruism and uncertainty about time of death.[11]

4.1 Individuals

Individuals live for three periods. During young age, children invest all their time (one unit) in human capital accumulation, from which they all receive the same utility. Children's time input is combined with education benefits, provided by the workers, to develop their human capital which will be used when working. Any difference in the per child education benefit will thus not affect the utility in the first period of life, but will instead alter the human capital. In the next period, when working, all supply inelastically their effective labor, the product of their one unit of time and their human capital, to firms and receive wage income. A fraction of this wage income will finance the education and pension systems; the remaining part will be divided between savings and consumption. In the third and final period, individuals are retired and consume their own savings and income from the pension system.

Since all generations gain the same utility when young this period is suppressed. The lifetime utility of an individual, belonging to generation $t-1$, is assumed to be additively separable according to:

$$U_{t-1} = \ln c_{w,t} + \beta \ln c_{r,t+1}, \tag{14}$$

[11] Pecchenino and Utendorf (1999) show that using intergenerational loans for education financing instead of including altruism does not alter their results in a significant way.

where β is the subjective discount factor and thus a measure of the individual's impatience to consume. Consumption per worker in period t is denoted with $c_{w,t}$, while consumption per retired in period t is denoted by $c_{r,t}$.

Denote by h_t the human capital for generation $t-1$ while at work. This is a product of the benefits from the education system in period $t-1$, i.e.:

$$h_t = b_{E,t-1}^{\sigma}, \tag{15}$$

where $\sigma \in (0,1]$ measures the elasticity of scale in the production of human capital. The human capital determines the effective labor supply for each individual in period t. The individuals take their human capital, wages, the interest rate, the tax rate, and the benefits in the pension system, as given. Their only decision variable is savings, which they choose as to maximize the lifetime utility, according to equation (14), subject to the following budget constraints:

$$c_{w,t} = (1-\tau_t)\, w_t h_t - s_t, \tag{16}$$

$$c_{r,t+1} = R_{t+1} s_t + \gamma_{P,t+1} \tilde{w}_{t+1}, \tag{17}$$

where s_t denotes the per worker savings in period t, w_t is the wage for one unit of effective labor, and R_{t+1} denotes the gross interest rate on savings between period t and $t+1$. As before, τ_t denotes the total tax rate used in the financing of the education and the pension systems, $\gamma_{P,t+1}$ is the benefit rate received when retired, and $\tilde{w}_t = w_t h_t$.

Maximizing the objective function (14) under the constraints (16) and (17) yields the familiar intertemporal Euler equation:

$$c_{r,t+1} = \beta R_{t+1} c_{w,t}. \tag{18}$$

4.2 Production

The aggregate production function in the economy is assumed to be of Cobb-Douglas type and homogeneous of degree 1. Production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where L_t is the aggregate effective labor, i.e. $L_t = h_t N_{t-1}$, K_t is the aggregate capital stock in period t, and A is a scaling parameter. The capital stock K_t depreciates fully during the production process. Defining production in terms of output per worker yields:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha}, \tag{19}$$

where $y_t = Y_t / N_{t-1}$, and $k_t = K_t / N_{t-1}$.

The prices of the factor inputs are obtained from the firms maximization problem, and since perfect competitive factor markets are assumed these prices equal their marginal product, that is:

$$R_t = A\alpha k_t^{\alpha-1} h_t^{1-\alpha}, \tag{20}$$

$$w_t = A(1-\alpha) k_t^{\alpha} h_t^{-\alpha}, \tag{21}$$

where R_t is the price on physical capital, and w_t is the price per unit of human capital, both in period t.

4.3 Market clearing

All markets are assumed to be perfectly competitive and for the goods market to clear the following condition must be satisfied:

$$y_t = s_t + c_{w,t} + c_{r,t}/n_{t-1} + b_{E,t}n_t,$$

which states that supply of goods must equal demand, which comprises consumption, savings, and education expenditures.

Using firm's and individual's first-order conditions together with the balanced budget restriction for the transfer systems this condition can be reduced to:

$$k_{t+1} = s_t/n_t. \tag{22}$$

Next period's capital labor ratio is determined by current savings and the workforce growth. If n_t increases without an equivalent increase in savings there will be capital dilution in the next period.

4.4 Equilibrium

Given the initial capital stock, $k_0 > 0$, the initial human capital stock, $h_0 > 0$, and the population growth, $\{n_t\}_{t=0}^{\infty}$, a competitive equilibrium for this economy is a sequence of: prices $\{w_t, R_t\}_{t=0}^{\infty}$, allocations $\{c_{w,t}, c_{r,t}, s_t\}_{t=0}^{\infty}$, human and physical capital stocks $\{k_t, h_t\}_{t=0}^{\infty}$, and benefit rates and tax rates $\{\gamma_{E,t}, \gamma_{P,t}, \tau_{E,t}, \tau_{P,t}\}_{t=0}^{\infty}$, such that the individuals maximize their utility, firms maximize their profits, markets clear, and that the budgets of the transfer systems are balanced.

Individual saving decisions fully characterize the equilibrium, since it defines the equilibrium trajectory for $\{k_t\}_{t=0}^{\infty}$ via eq. (22). Eqs. (16)-(18) and (20)-(22) yield the following saving function in equilibrium:

$$s_t = \frac{\beta\alpha(1-\tau_t) w_t h_t}{\lambda_t}, \tag{23}$$

where $\lambda_t = \alpha\,(1+\beta)+(1-\alpha)\,\gamma_{P,t+1}/n_t$. Saving is a fraction of the disposable income and independent of the interest rate in the economy; this is a result from the utility function which has an intertemporal elasticity of substitution equal to unity.

The savings respond, as expected, negatively to an increase in the future pension benefit rate, since these two are substitutes.

4.4.1 The steady state

There are two different types of steady state equilibria depending on whether the production function for human capital exhibits diminishing returns or not. If there are diminishing returns, i.e. $\sigma < 1$, then there is a stationary equilibrium with no growth in the per capita variables. If there is constant returns, i.e. $\sigma = 1$, then there is a balanced growth equilibrium such that the per capita variables $\{y_t, k_t, h_t\}$ grow at a constant gross rate equal to:

$$\theta = A\,(1-\alpha)\,\gamma_E^{(1-\alpha)}\left[\frac{\beta\alpha\,(1-\tau)}{\lambda n}\right]^{\alpha}, \tag{24}$$

where $\theta_t = y_t/y_{t-1}$ and in steady state $\theta_t = \theta\ \forall\ t$.

4.5 The intergenerational welfare function

To obtain a compact measure of how all generations are affected by a fertility shock a welfare function is defined according to:

$$W = \sum_{t=1}^{T} \phi_t (U_t - U_{t,ss}), \tag{25}$$

where $U_{t,ss}$ is the lifetime utility in steady state prior to the shock of an individual born in period t. This is a pure utilitarian welfare function, implying neutrality towards the inequality in the distribution of utility.[12] The separability assumption made above is standard, but a comment on the weighting factor, ϕ, is in order.

There are different views on how the per capita lifetime utility of generation t should be weighted. The question is if the utility should be weighted by the generation size, and/or by a social discount factor. Not to dwell to much on this issue it seems more or less necessary to account for the generation size, otherwise there would be an unequal treatment of individuals belonging to generations of different size. A social discount rate will also be

[12]Choosing a general utilitarian welfare function with aversion towards inequality between generations utility would strengthen the results obtained later in the paper.

included which allows for sensitivity analysis when varying this parameter. The weighting factor used will be the following:

$$\phi_t/\phi_{t-1} = \beta_s n_t, \tag{26}$$

where β_s is the social discount rate. In the benchmark simulation the social discount rate will be set equal to the individuals discount factor, i.e. $\beta_s = \beta$. The formulation allows for varying the social discounting as long as $\beta_s \in (0, 1/n]$. If there is population growth then the discount rate should not exceed the inverse of the population growth; if it does, then the future generations would get an ever increasing impact on the welfare function, due to their larger number.[13]

5 Calibration

5.1 Demographic shock

The baby boom shock under consideration can be stated as $n_{t+j} = n \; \forall j \neq 0$ and $n_t > n$. To get an estimate of the shock the U.S. experience will be used. In figure 1 the birth rates per 1000 inhabitants for the U.S. between the period 1910 to 2001 are presented.

A shock is by definition a large and sudden deviation from expectations. To estimate the size of the shock one needs to know what the expectations were, and obviously the outcome. To avoid historic researching of what the expectations actually were, figure 1 can be used to assess what the expectations might have been. Moreover, knowing that the official years of the U.S. baby boom generation are 1946 to 1964 makes it possible to at least view this period as a shock period.

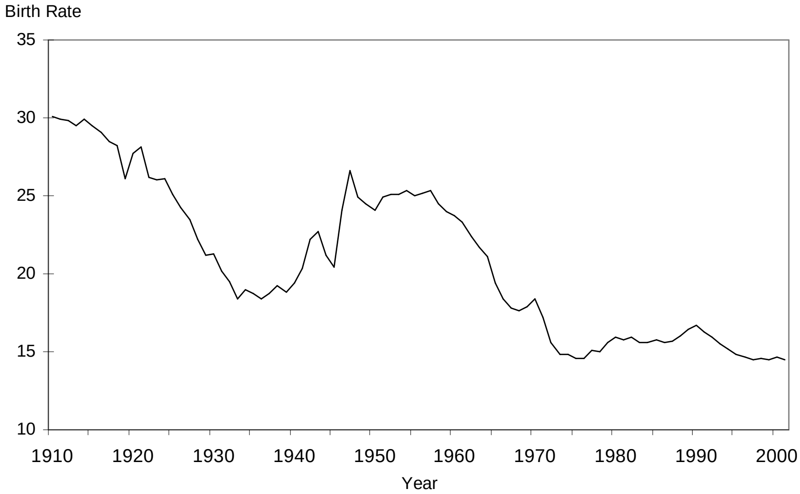
In the model every period represents roughly 27 years. Using a 27 years period length while trying to assess the magnitude of the shock with different specifications for the expectations, does not seem to yield estimates lower than 20 percent, according to figure 1.[14] For this reason the magnitude of the shock used will be 20 percent, i.e. $n_t = 1.2n$. Regarding the steady state gross population growth n this will be set to 1.3, based on the annual average for the U.S. between 1910-2001.[15]

[13]See for instance Blanchet and Kessler (1991) and Boadway et al. (1991) for a short comment concerning the weighting problem.

[14]If instead a period length of 19 years is used, to fit the official years of the baby boom, the estimates of the shock are around 30 percent depending on the specification of the expectations.

[15]The annual average is approximately 1.01, which implies that per period $n = 1.01^{27}$.

Figure 1: Birth Rate per 1000 inhabitants for the U.S. between 1910 to 2001.



Source: National Vital Statistics Reports 51, no. 2 (2002).

The demographic structure used in the simulation can be stated as $n_{t+j} = 1.3 \; \forall j \neq 0$ and $n_t = 1.56$.

5.2 Preferences

Regarding preferences, β is the standard measure of the individual's impatience to consume. Using the one year estimate from Auerbach and Kotlikoff (1987) of 0.98 translates to $\beta = 0.6$, since every period represents about 27 years.

5.3 Production

There are two parameters in the production function that need to be calibrated, α and A. The share of capital income in the national product, α, is calibrated to one third. The scale parameter A can in the benchmark simulation be chosen freely since it will not alter the relative outcome in any significant way. However, in the sensitivity analysis when allowing for endogenous growth, i.e. $\sigma = 1$, the growth rate of the economy will depend on A. Since A can be chosen freely when $\sigma < 1$, but not when $\sigma = 1$, I will let the latter decide the value for A; which will be chosen to yield an annual

Normalizing $n = 1$, as in the partial analysis, would not alter the results in any significant way.

growth rate per worker of 2.5%, which corresponds to U.S. historical rates, in the balanced growth case.[16]

5.4 Human capital

The production function of human capital has only one exogenous parameter, σ, but this parameter is the most difficult to calibrate. Based on estimates from the education literature an attempt is made to find a reasonable range for σ. Since it is not straight forward to assess σ from these estimates the argument for how this is done is left to the appendix.

Previous studies by Chakrabarti et al. (1993) and Pecchenino and Utendorf (1999) that use the same production function for human capital have calibrated σ to 0.6 and 1, respectively. Here a compromise between these two values will be used, such that $\sigma = 0.8$.

The benchmark calibration is thus $\sigma = 0.8$, while the sensitivity analysis in the appendix will show how the results change with σ.

5.5 Intergenerational transfers

Calibrating the education and the pension system amounts to calibrating the benefit rates in steady state, $\gamma_{i,ss}$. For the pension system it is possible to use the existing PAYG pension systems as a guideline. According to the Social Security Office of the Chief Actuary the current benefit ratio, i.e. benefit to the average wage ratio in the same period, is 0.42. In reality, however, the ratio between working years and years of retirement is almost 2, while in this three period model it is 1. For this reason the benefit rate in the pension system is chosen such that $\gamma_{P,ss} = 0.2$ .This together with the assumption that $n = 1.3$ in steady state implies that $\tau_{P,ss} = 0.15$.[17]

When the pension system operates under the FR scheme it is the replacement rate, $\tilde{\gamma}$, that is fixed. The replacement rate is calibrated such that the

[16]There are many empirical studies that try to estimate this growth rate. A short review is given in Pecchenino and Utendorf (1999), which find 2.5% to be the best compromise between the different estimates.

[17]This contribution rate might seem high compared to the social security tax rate, but bear in mind that the contribution rate also incorporates medicare expenses and the like. Also, compared to other countries, e.g. Sweden, this is not an especially high social security tax rate. Moreover, due to the models time assumption the benefit rate and the contribution rate cannot be in line with reality at the same time. This is in line with previous three period OLG models, see e.g. Blomquist and Wijkander (1994). A compromise is made such that the benefit rate is lower than in reality while the contribution rate is higher. Note, however, that it is assumed that the high tax rate will not affect the labor participation rate.

same benefit rate is obtained in steady state, i.e. $\tilde{\gamma} = 0.2\theta_{ss}$.

To obtain the benefit rate in the education system it is possible to use estimates of the GDP share devoted to education. For the U.S. the GDP share for primary and secondary school spending has approximately been 4 percent during the last three decades and the GDP share for higher education is close to 3 percent.[18] The total share of GDP spent on formal education thus amounts to 7 percent.

Besides the formal education and the children's own time input there is a large amount of leisure time, mainly parental time, invested in educating children. Leibowitz (1974) estimates that 131.6 minutes per day of an average couple's non-sleeping time is spent on educational care. This would imply that 6.9 percent of an individuals non-sleeping time is spent on educational care of children.

Letting expenditures on education be a composite of formal education and home education implies that $\tau_E = 0.12$. As for the pension system the model's time ratio between education and working time is not realistic. If one were to use the real contribution rate then the benefit rate would be underestimated severely. Once again a compromise is made such that the contribution rate is higher and the benefit rate is lower than in real life. The following contribution rate will be used $\tau_{E,ss} = 0.16$, which implies that the benefit rate is $\gamma_{E,ss} = 0.12$.

Table 6: Calibrated values for the exogenous parameters.

Parameter		Value
Time preference	β	0.6
Share of capital income	α	1/3
Efficiency in human capital production	σ	0.80
Steady state benefit rate in the pension system	γ_P	0.20
Steady state benefit rate in the education system	γ_E	0.12
Population gross growth rate	n	1.3
Baby Boom shock	n_t/n	1.2
Steady state gross growth rate	θ	1
Total factor productivity	A	21.6

5.6 Steady state

Before the model is used to study the effects of demographic changes, it is useful to report the steady state values for some key variables, according to

[18]See Rangazas (2002) p. 947.

the calibration in table 6. In steady state all the cases are identical and it is possible to obtain analytical results. To obtain the numerical results, presented in table 7, it is enough to plug in the parameter values from table 6 into the analytical solution.

Table 7: Steady state values according to calibration in table 6.

Output per worker	y	1821
Cons. per worker	c_w	571
Cons. per retired	c_r	1037
Wage rate per effective unit of labor	w	22
Gross interest rate for capital	R	3.02
Saving rate	S/Y	3.4%
Capital output ratio	k/y	0.11

To see how the model fits stylized facts the last three variables from table 7 are of most interest. The magnitude of the interest rate is quite realistic when adjusting for the time length in the model.[19] The saving ratio in life-cycle models with no bequests has notorious difficulties to fit empirical facts.[20] As for other similar models the saving rate is considerably below the comparable U.S. rate, which is around 6.7 percent. This should not cause large problems as long as the capital output ratio is within reasonable range. From table 7 the capital output ratio is 0.11, which on annual basis becomes 3. This is slightly higher than the comparable U.S. ratio, which is about 2.5, but still within reason.

6 Results

To identify the winners and losers it is natural to present the results regarding the lifetime utilities. Since the utility is based on consumption the results for the discounted lifetime consumption will also be presented, which will be denoted by C_{t-1}.[21] This will capture how different generations are affected from the demographic shock in terms of net discounted lifetime income.

[19] The reported interest rate is the compounded interest rate over 27 years, which on annual basis becomes 4.2%.

[20] See Kotlikoff and Summers (1981).

[21] The discounted lifetime consumption for the working generation in period t, C_{t-1}, is calculated according to: $C_{t-1} = c_{w,t} + c_{r,t+1}/R_{t+1}$. The subscript indicates that is the discounted lifetime consumption for generation $t-1$. Note that C_{t-1} also must equal the discounted net lifetime income for generation $t-1$.

To compare the different transfer schemes on a more general basis, than how it affects a particular generation, it is necessary to investigate the social welfare results. The results from the baby boom shock are presented, but social welfare considerations need to be based on an *ex ante* approach. For this reason the expected social welfare, given that there is equal probability of positive and negative birth rate shock, is analyzed.

Other variables that will be presented are the aggregate savings by the workers, $S_{w,t}$, and the factor prices. The savings is presented to obtain a measure of how the capital labor ratio differs between the cases, while the factor prices is what ultimately affects the generations.[22]

6.1 Savings

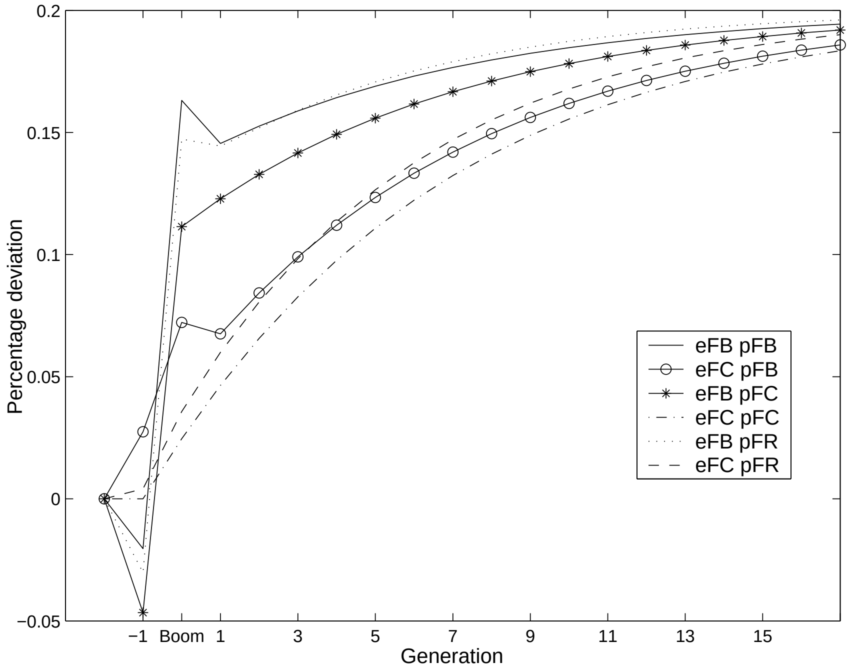
Before analyzing the effects on factor prices it is useful to consider the effects on savings. Aggregate savings by the workers will determine the capital labor ratio in the next period, which in turn will determine the factor prices. To offset the capital dilution of the 20 percent larger workforce in period $t+1$, and onwards, the aggregate savings by the workers need to increase accordingly. Clearly, the change in aggregate savings in period t is not even close to what would be needed to compensate for the coming workforce increase; on the contrary, in half of the cases the change in savings will aggravate the capital dilution.

Note that in period $t+1$, when the boom generation saves, the aggregate savings are between 2 and 16 percent above the steady state value, which is not enough to restore the capital labor ratio for coming generations. How fast the capital dilution is offset depends on how hard the boom generation is burdened; the less they are burdened the faster will the capital labor ratio return to it's steady state value.

When the education system is of FB type the parent generation will be burdened with higher education tax, leaving them with less disposable income which reduces their savings. How much they reduce their savings depends on the design of the pension system. If the pension system is of FB type the parent generation will increase their savings to compensate for the future reduction in pension benefits, arising from the baby boom generations lower wage. Since the benefit rate is fixed, the wage decrease in the next period will punish their benefits without any cushioning. Thus they will increase their savings to compensate for future lower benefits, which will mitigate the boom generations capital dilution.

[22]For most variables graphs are presented as deviation from steady state, the corresponding tables for these graphs are presented in Appendix A.

Figure 2: Aggregate savings during working period as deviation from s.s. outcome.



If the pension system is of FC type then the demographic benefit from the pension system, i.e. the higher benefit rate due to increased worker/retiree ratio, will compensate the parent generation from the lower wage and higher interest rate in the next period. In this case the parent generation will not alter their savings due to future events. Thus, if the pension system is of FC type the parent generation will not increase their savings to mitigate the future effects on factor prices.

Under the FR pension scheme the parent generation will not receive the demographic benefit in the pension system, as they did in the FC scheme. They will, however, receive a higher benefit rate due to the reduction in wage growth. This increase in the benefit rate is not as high as in the FC scheme, but higher than the fixed benefit rate under the FB scheme. In this case the pension system yields incentives to increase savings but not as much as in the FB scheme.

The boom generation's capital dilution is thus mitigated most under case 2 (eFC pFB), when the education system is of FC type and the pension system is of FB type. However, even under this case is the increase in savings only 3 percent, far from the needed 20 percent. The factor price movements are aggravated most under case 3 (eFB pFC), when the parent generation has to pay for the burden in the education system, and when they have no incitament from the pension system to increase their savings.

For the parent generation to increase their savings it is a necessary condition that the pension system is either of FB or of FR type, this is an analytical result. The numerical results show that a second condition is that the education system is of FC type.

It is important to remember that these results are obtained under the assumption that the intertemporal elasticity of substitution is equal to unity. Savings are in this case not dependent on the expected interest rate. Most would argue that this assumption is false and that the intertemporal elasticity of substitution is less than unity; implying that an increase in the expected interest rate should result in an increase in current savings. Since the interest rate is higher when the baby boom generation works, i.e. in period t+1, the parents savings are probably somewhat underestimated due to the assumption. However, it is most unlikely that the change in savings arising from the interest rate is of such magnitude as to change the results.[23]

6.2 Factor price movements

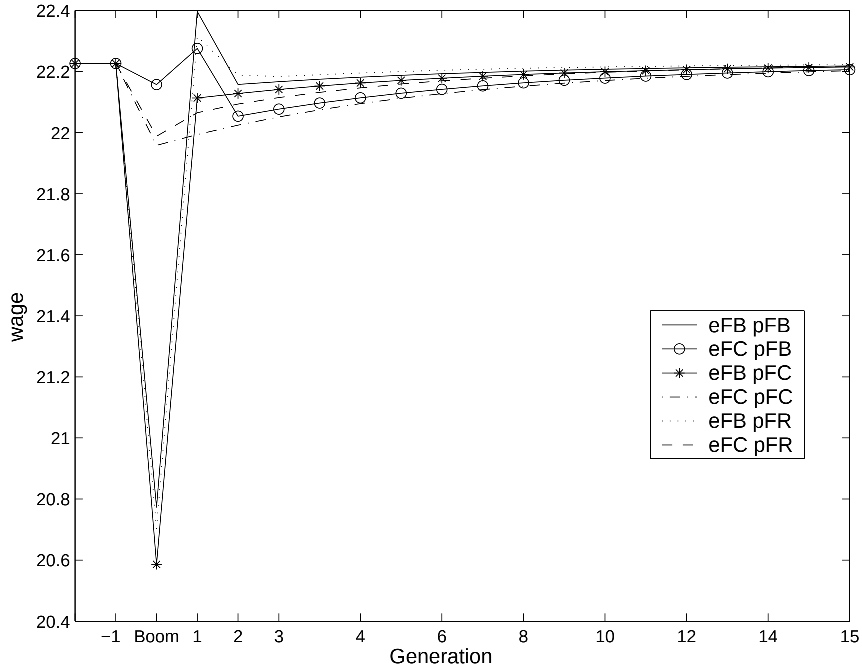
Since the wage and the interest move in the opposite direction, the discussion will focus on the outcome for the wage, which is presented in figure 3. There is especially one interesting fact that emerges, besides that the impact differs quite substantially between the cases. Note that the child generation have increased wage for some cases even though their capital labor ratio is below the steady state value. If human capital was excluded from the analysis the wage would be below the steady state value for all cases.

When trying to evaluate the effect on the factor prices from demographic changes it thus seems to be important to account for the intergenerational transfer systems and the human capital formation. The different cases yield once again very different predictions of how, and when, the factor prices are expected to adjust.

Of interest is also that the cases which have the smallest impact on factor prices, i.e. when the education system is of FC type, are also the cases that lead to the highest net lifetime income loss for the baby boom generation. The human capital effect is thus of greater importance than the effect on factor prices. For the baby boom, and their progeny, it is thus more important that the education system preserves their human capital, than to avoid

[23]The needed increase in savings to prevent capital dilution is 20 percent which implies that at least additional 16 percentage point increase is needed. Moreover, studies that use an elasticity of substitution lower than 1 are not able to prevent capital dilution either (see for instance Blomquist and Wijkander (1994)).

Figure 3: Wage per unit of human capital for the generations.



aggravating the capital labor dilution. Unfortunately, both effects are not attainable simultaneously, the parent generation must decrease their savings if the boom generation's human capital is to remain intact.[24]

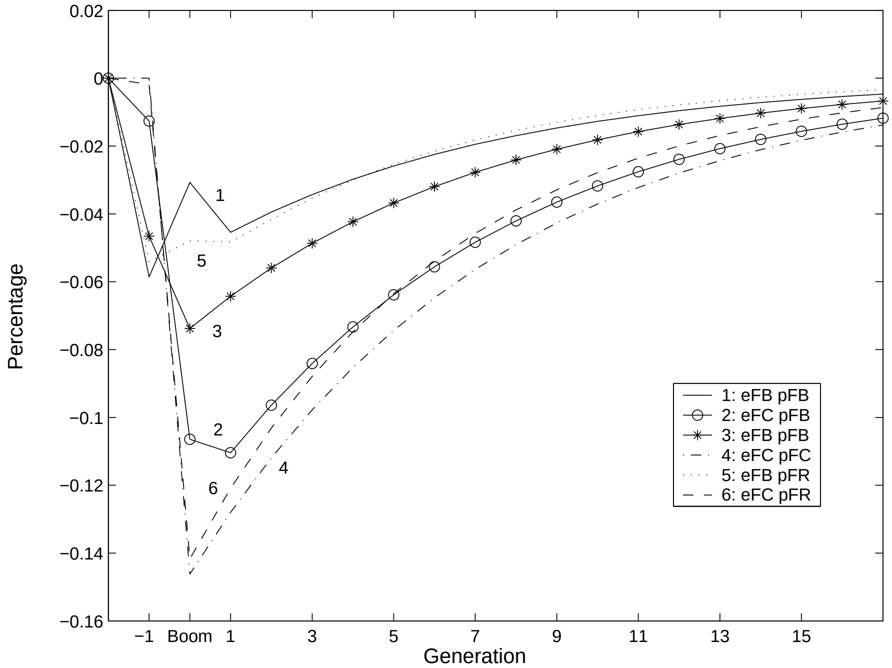
6.3 Lifetime consumption

In figure 4, one thing stands out. When there is a baby boom shock, there are no winners in terms of discounted lifetime consumption. At most the parent generation, i.e. generation $t-1$, is unaffected. This occurs during case 4 (eFC pFC), when both the education and the pension system operate under a FC scheme. In this case the parent generation obtains the gain from the pension system while it is not burdened from the education system. The gain from the pension system will perfectly cancel out the negative effect from the income loss of the boom generation, and the increase in the interest rate, leaving the parent generation unaffected.

What also emerges is that the magnitude of the burden that is put on different generations varies substantially between the different cases. For the parent generation the consumption loss differs between 0 and 6 percent

[24]One can, however, not exclude that this result is sensitive to the assumption about the intertemporal elasticity of substitution.

Figure 4: Discounted lifetime consumption as percentage deviation from s.s. outcome.



compared to steady state. For the baby boom generation the loss will vary between 3 and 15 percent; while the loss for the child generation, i.e. generation $t+1$, varies between 5 and 13 percent compared to steady state.

Previous studies have found that irrespective of the pension system a baby boom generation can never be fully compensated for the unfavorable factor price movements.[25] This result is supported here, however, what is of interest is that the baby boom generation is not necessarily worse off in terms of discounted net income, compared to surrounding generations. It may be the case that, relative to both its parent generation and its child generation, they have higher income.

The empirical literature concerning relative cohort size and inequality yields ambiguous results.[26] The simple stylized model presented here can to some extent reproduce these opposite findings. It can support both large and small relative effects, and moreover, it can both support negative and positive effects from cohort size on net lifetime income.

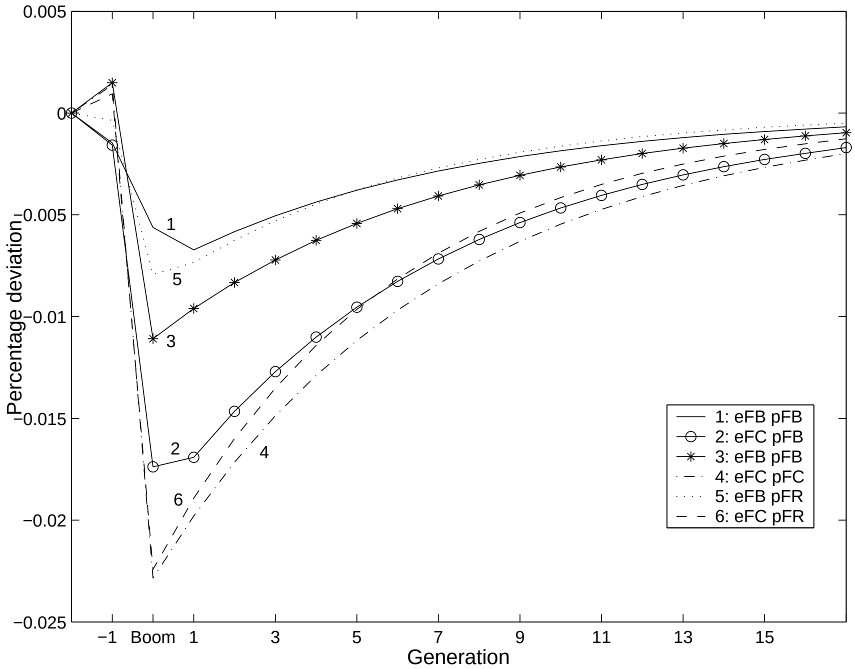
[25] E.g. Bohn (2001) and Blomquist and Wijkander (1994).

[26] E.g. Welch (1979), Berger (1985), Easterlin (1987), Macunovich (1998), and Dahlberg and Nahum (2003).

6.4 Lifetime utility

From the presentation above it was possible to see how the different generations lifetime income varied for the different cases. For the parent generation case 4 (eFC pFC) yielded the highest income while case 1 (eFB pFB) yielded the lowest discounted lifetime consumption. However, based on this it is not possible to conclude that case 4 is preferred by the parent generation, since the net discounted lifetime income is not equivalent to the generations utility.

Figure 5: Lifetime utility as deviation from s.s. outcome.



In figure 5 the lifetime utilities. It is clear that ranking the cases based on the discounted lifetime consumption is not the same as ranking the cases based on lifetime utility. This is not strange and should not come as a surprise, however, it is interesting that the parent generation can be better off in terms of utility compared to the steady state outcome. This happens in three out of six cases. Remarkable is that case 3 (eFB pFC) yields the best outcome for the parent generation, since in this case they have to pay the cost in the education system. Moreover, the parent generation would prefer that the education system is of FB type as long as the pension system is not of FR type.[27] If the pension system is of FR type every incentive to increase

[27]This result is however sensitive to the efficiency in the education system, i.e. calibration of σ. If the efficiency is low then the future benefit of education is small. The

future generations productivity is removed, and thus the parent generation would always prefer the education system of FC type.

For the boom generation, and their progeny, the outcome is always worse than the steady state outcome. They all would prefer case 1 (eFB pFB) since this leads to least utility loss. This case also implies that the children generation is worse off than the baby boom generation. The children generation is also worse off than the baby boom generation for case 2 (eFC pFB). The reason for this is that the baby boom generation receives the benefit from the pension system, whereas the children do not.

The boom generation and the child generation rank the cases in the same way, but this is not true for all their progeny. Case 5 implies faster convergence to the steady state outcome, leading to case 5 (eFB pFR) being the most preferred case for the generation born in period 8, and thereafter.

It is not strange that the generations make different ranking of the cases.[28] Notable, however, is that the parent generation prefer to pay for the burden in the education system if they later are going to receive the benefit in the pension system.

6.5 Intergenerational welfare

Since the generations rank the cases differently, it is necessary to adopt a social welfare function to try to obtain an "objective" ranking of the different systems. The social welfare function adopted is a pure utilitarian where the generations lifetime utility deviation from steady state enters additively, according to equation (25). Three social discount rates are applied: one, benchmark calibration where $\beta_s = \beta$, second, a high value as possible where $\beta_s = 1/n$, and third, a low value such that $\beta_s = 0.5\beta$. Besides the social discount rate the lifetime utilities are also weighted with generation size. Table 8 presents the social welfare measures under the different cases. The number of periods included in the calculation is set to 100, i.e. $T = 100$ in equation (25).[29]

The top three ranked cases all have an education system of fixed benefit type. It is vital that the human capital of the baby boom is preserved to

benchmark $\sigma = 0.8$ yields a marginal benefit for educating the baby boom generation. For σ smaller than benchmark the cost would be greater than the benefit. The sensitivity results are presented in Appendix C.

[28] From a democracy perspective it could be problematic that the generations have different preferences. Changes in old age dependency ratios would imply changes in voting power between workers and retirees (children have no voting power). A large cohort could in principle change intergenerational transfer schemes as to maximize it's own utility.

[29] $T = 100$ is more than enough since the ranking of cases does not change after $T = 23$.

Table 8: Social welfare for different social discount factors.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\beta_s = \beta$	-14.07	-35.99	-19.66	-41.58	-15.16	-38.93
Rank	1	4	3	6	2	5
$\beta_s = 1/n$	-55.21	-140.35	-78.97	-164.12	-53.91	-138.51
Rank	2	5	3	6	1	4
$\beta_s = 0.5\beta$	-1.89	-4.57	-2.02	-4.70	-1.93	-4.68
Rank	1	4	3	6	2	5

Note: Population scaling is included. Rank indicates the ranking of the cases, where 1 is the best and 5 the worse.

mitigate the capital dilution, and thus upholding the ability to finance the human capital of future generations. The other cases where the education system is of FC type yield far greater loss.

If holding the education system fixed, it is clear that the pension system should not have a fixed contribution rate. The FB and FR scheme do not differ much, while the FC scheme yields a notable higher loss.

6.6 Expected Intergenerational welfare

The baby boom was used to illustrate the importance of accounting for different types of intergenerational transfers. It is, however, problematic to use only the baby boom shock when trying to rank the cases. For a baby bust shock there will be an opposite reaction, i.e. a benefit in the education system, capital labor deepening, and a burden in the pension system. This would result in an opposite ranking of the cases.

If trying to choose between the cases one would want to adopt an *ex ante* approach, not an *ex post*. Assume that there is a fifty fifty probability of a positive and negative fertility shock, such that $E(n_t = 1.2n) = 0.5$ and $E(n_t = 0.8n) = 0.5$. Which case yields the highest expected welfare? This is answered in table 9.

For all three discount rates the highest expected welfare is obtained for case 5 (eFB pFR), the second for case 1 (eFB pFB), and the lowest for case 4 (eFC pFC). For the baby boom shock it was obvious why the cases ranged as they did. The reason why the same result is obtained from equal probability of positive and negative shock depends on that the utilities do not respond symmetrically; since the marginal benefit from consumption is decreasing.

From an *ex ante* perspective it seems as the education system of FB type is preferred, while the pension system should be of FB or FR type.

The result regarding the pension system, is the opposite of what Thøgersen (1998) found but supports the findings of Wagener (2003). Both however only

Table 9: Expected social welfare, $E[W]$, for different social discount factors.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\beta_s = \beta$	-2.55	-3.71	-2.85	-4.01	-2.48	-3.74
Rank	2	4	3	6	1	5
$\beta_s = 1/n$	-9.36	-15.09	-9.60	-15.33	-7.96	-13.10
Rank	2	5	3	6	1	4
$\beta_s = 0.5\beta$	-1.27	-1.85	-1.42	-2.00	-1.24	-1.87
Rank	2	4	3	6	1	5

Note: Population scaling is included. Rank indicates the ranking of the cases, where 1 is the best and 5 the worse.

investigate the FC contra FR scheme under income uncertainty, while not analyzing the FB scheme. Wagener (2003) finds the FR scheme is preferred over the FC scheme under an *ex post* comparison, while none dominated the other from an *ex ante* perspective. While Thøgersen (1998) finds that the FC scheme is strictly preferred from an *ex ante* perspective. The result in this paper indicates that the pension system should not be of FC type. Note that whatever risk-sharing feature the FC scheme could have with respect to wage uncertainty, the FB scheme analyzed here has the same feature.

7 Conclusion

The simulations show that it matters a great deal which intergenerational transfer schemes that are in place. It matters for how savings and factor prices will respond to a fertility shock, and for outcomes in terms of utility and lifetime consumption for different generations. Since the response of important economic variables, such as wages, could change both qualitatively and quantitatively it is of great importance to consider which type of intergenerational transfers that are in place when evaluating past events or when forming predictions.

The relative outcome between different generations is highly dependent on the transfer schemes. The baby boom generation can be better off than the surrounding generations in terms of life time consumption, while under other schemes the baby boom generation is worse off.

In the partial analysis it was shown that a baby boom creates a burden in the education system, and a gain in the pension system. The allocation of these is a distributional matter between the parent generation and the boom generation. When including the effect on factor prices I find a strong case that the parent generation should bear the burden in the education system while the boom generation should obtain the gain in the pension system. This since the boom generations suffers from the capital dilution cost.

Even if the parent generation would not prefer to bear the burden in the education system, they should still do so if we want to maximize social welfare, which is measured according to a pure utilitarian function. Moreover, neither the education system nor the pension system should be of fixed contribution rate type. This holds even from an *ex ante* perspective, based on expected social welfare when there is equal probability of positive and negative fertility shock. The education system should be of fixed benefit rate type since this will minimize the effect on human capital. The pension system should be of fixed benefit rate or fixed replacement rate type, to compensate for the capital labor ratio effect.

Though this paper does not analyze how society makes decision regarding the transfer systems, it can be mentioned that the parent generation would never prefer to finance the burden in the education if the pension system is of fixed replacement rate type. All incentives for increasing future generations productivity are removed in this case. Otherwise the parent generation could prefer to pay for the burden in the education system if the efficiency in the education system is high enough.

The results indicate that the education system should be of FB type while the pension system should be of FB or FR type (preferable FB with respect to incentives for the education financing). Many countries, including Sweden, have reformed there PAYG pension systems from a FR scheme to a FC scheme. One argument for the transformation was based on the risk-sharing properties regarding income uncertainty. However, the FB scheme within this paper responds in the same way as the FC scheme to income disturbances. Another argument for the FC transition is that it will not lead to higher payroll tax when the old dependency ratio increases. This could have beneficiary effects for the labor supply which are not accounted for in this analysis.

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Appendix A: Tables

Table A.1: Percentage deviation from steady state outcome, for the variables C_{t-1} and $S_{w,t}$, t periods after the shock.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-5.86	-2.04	-1.26	2.75	-4.66	-4.66	0.00	0.00	-5.41	-3.02	-0.19	0.41
1	-3.07	16.31	-10.65	7.22	-7.38	11.14	-14.62	2.46	-4.80	14.73	-14.17	3.57
2	-4.54	14.55	-11.04	6.75	-6.43	12.29	-12.80	4.64	-4.83	14.44	-12.09	6.00
3	-3.95	15.26	-9.64	8.43	-5.60	13.28	-11.19	6.57	-4.16	15.19	-10.31	8.06
15	-0.72	19.14	-1.80	17.84	-1.03	18.77	-2.11	17.47	-0.56	19.35	-1.43	18.35
25	-0.17	19.79	-0.43	19.48	-0.25	19.70	-0.51	19.39	-0.10	19.88	-0.27	19.69
50	0.00	19.99	-0.01	19.99	-0.01	19.99	-0.01	19.98	0.00	20.00	0.00	20.00

Table A.2: Percentage deviation from steady state outcome, for the variables R_t and w_t, t periods after the shock.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	14.48	-6.54	0.63	-0.31	16.57	-7.38	2.46	-1.21	15.25	-6.85	2.18	-1.07
2	-1.51	0.77	-0.44	0.22	1.03	-0.51	2.13	-1.05	-0.79	0.40	1.47	-0.73
3	0.62	-0.31	1.57	-0.78	0.89	-0.44	1.84	-0.91	0.34	-0.17	1.21	-0.60
15	0.11	-0.06	0.28	-0.14	0.16	-0.08	0.33	-0.16	0.06	-0.03	0.16	-0.08
25	0.03	-0.01	0.07	-0.03	0.04	-0.02	0.08	-0.04	0.01	-0.01	0.03	-0.01
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.3: Percentage deviation from steady state outcome, for the variable U_{t-1}, t periods after the shock.

t	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	-0.15	-0.16	0.15	0.14	-0.04	0.09
1	-0.56	-1.74	-1.11	-2.28	-0.79	-2.24
2	-0.67	-1.69	-0.96	-1.98	-0.73	-1.89
3	-0.58	-1.47	-0.83	-1.72	-0.63	-1.60
15	-0.10	-0.26	-0.15	-0.31	-0.08	-0.21
25	-0.02	-0.06	-0.04	-0.07	-0.02	-0.04
50	0.00	0.00	0.00	0.00	0.00	0.00

Appendix B: Human capital calibration

Here an attempt is made to find a reasonable range for σ. To insure that the model has stable properties it is necessary that $\sigma \leq 1$. Is it reasonable that one is the upper bound? One motivation for that it is a reasonable upper bound, is that explosive growth should have been experienced otherwise. Since the upper bound seems reasonable the task becomes to find a reasonable lower bound. Even though this model becomes stationary with $\sigma < 1$, the experienced growth does not motivate $\sigma = 1$. This since there is no guide whether growth in the model should be incorporated via h or A.

Before proceeding it could be useful to see how other simulation studies have calibrated the human capital formation. As mentioned in section 5.4, studies that use the same production function for human capital as in this paper, namely Chakrabarti et al. (1993) and Pecchenino and Utendorf (1999), have calibrated σ to 0.6 and 1, respectively. Related literature with production functions that have more than one single input often depict the production function of human capital as homogenous of degree one.[30] The most common use is that the human capital formation exhibits constant returns to scale. The motivation for the constant returns to scale assumption seems to be that growth is an empirical fact, and thus the model should allow for balanced growth. Such motivation is however doubtful since the growth could as well come from an exogenous source.

Since σ measures the elasticity of scale in the production of human capital, it seems naturale to search within the education literature. For calibration it is important to bear in mind how this stylized model differs from reality. The inputs to the production of human capital are in reality versatile. The main inputs, besides children's own time, consist of formal schooling, parental educational care, and there is also a possibility of externalities à la Lucas (1988). All these different types of inputs are in this stylized model collected in one single input, b_E. Hence, it is difficult to calibrate the elasticity of scale for this composite input, based on estimates from the different inputs. A plausible (conservative) lower bound for σ is, however, possible to obtain.

Expenditures per child for human capital accumulation, b_E, can be divided into two main inputs, formal education, and parental education. Both these inputs have a quantity dimension and a quality dimension. From the education literature there is more or less a consensus that the quantity dimension is productive, i.e. longer education at the same quality will increase human capital; however, with decreasing rate at the macro level.[31]

[30] See for instance Lord and Rangazas (1991), Rangazas (2002), and Pecchenino and Pollard (2002).

[31] See Psacharopoulos (1994) for a global comparison for the rate of return at different

The quality dimension can be measured as the pupil teacher ratio in formal education or as the number of children for family education. For formal education there has been a controversy if more spending per pupil at a given grade will increase productivity.[32] More recent studies, however, by Card and Krueger (1992) find that school quality has an effect on productivity.

If σ only where to capture the effect from the the formal education along the quality dimension the estimate from Card and Krueger (1992) would imply $\sigma = 0.17$. Is this the lower bound? Probably not, since this estimate only accounts for the quality effect from formal schooling, while σ should also incorporate the effects from the quantity dimension and the education at home. Since this model is not suited for the quantity dimension, due to fixed time periods, lets ignore the effects from the quantity dimension. This exclusion will most likely yield a conservative lower bound.

The effects from home education can, however, be incorporated. Moreover, by making three assumptions, the Card and Krueger (1992) estimate can be used to assess the σ for the single input b_E. Home education is included both because it is straightforward to include it in the model, and due to the large amount of time it comprises (see section 5.5).

The first assumption, is that the human capital formation is conducted according to a Cobb-Douglas production form, with home and formal education spending as inputs.[33] The second assumption, is that home education and formal education are of approximately the same magnitude, which is reasonable according to the estimates presented in section 5.5. If one adds the third assumption, that home education has the same elasticity of scale as the formal education it is possible to obtain a lower bound for $\sigma = 0.34$, based on the Card and Krueger (1992) estimate. Since the elasticity of scale probably is not constant and the class size is vastly greater than the number of children in the family, even if the third assumption is wrong this would imply that the lower bound reported is conservative.

Thus the reasonable range seems to be $\sigma \in [0.34, 1]$, nonetheless the sensitivity analysis will test for $\sigma \in [0.17, 1]$.

educational levels.

[32]The influential work by Hanushek (1986) established the previous convention that there is no systematic relation between school spending and achievement.

[33]This is how most of related literature define the human capital production function.

Appendix C: Sensitivity analysis

Here follows a report of how the results change when varying σ. Four different values for σ are considered (besides the benchmark), $\sigma = 0.17,\ 0.34,\ 0.6$, and 1. The lowest value is solely based on the estimates from Card and Krueger (1992) and should only capture the quality effect from formal schooling. The second value is obtained if one adds the effect from home education, and is according to the reasoning in Appendix B viewed as a reasonable lower bound. The last two values for σ are chosen based on related literature, i.e. Chakrabarti et al. (1993) and Pecchenino and Utendorf (1999).

C.1 Steady state

The steady state results relative to output do not change for $\sigma < 1$, except for the interest rate and the wage which have the same level. For $\sigma = 1$ the level for the interest rate goes up to 5.9 while the wage goes down to 15.9. What also happens when endogenous growth is introduced is that the capital output ratio goes down to 0.06 and the saving ratio goes up to 0.09. The reduction in the capital output ratio leads to the increase in the interest rate. The saving output ratio increases due to the increase in interest rate, which reduces the dissaving of the retirees. That these changes occur is not strange since the model exhibits two different steady states, one for $\sigma < 1$ and one for $\sigma = 1$.

What is important is how the results after a baby boom shock will change. For this reason the tables in Appendix A and the tables in the text concerning the social welfare and the expected social welfare, i.e. table 8 and 9 respectively, are reproduced for different σ.

C.2 Savings

The discussion will focus on how the main results are affected when varying σ. Before moving on to that discussion it is worth nothing that under case 4 (eFC pFC) and case 6 (eFC pFR) there is no effect on the aggregate saving by the workers when $\sigma = 1$, according to table C.1. This will imply that there is no effect on factor prices, presented in table C.2. That this is the case can be shown analytically.[34] What happens is that the physical capital to human capital ratio remains intact if the education system is of fixed contribution rate, when $\sigma = 1$. This means that the per worker income that the boom generation receives will vary inversely with the demographic

[34] These proofs could be sent on request.

shock, and thus not affecting the aggregate savings by the boom generation. Further, if the pension system then is of FC or FR type, $\sigma = 1$ will imply that the denominator, i.e. λ_t, in the savings function (eq. (23)) will not change.

C.3 Lifetime consumption

The interesting results about the generations lifetime consumption was that the boom generation could be better of than the parent generation. From table C.1 it emerges that this result still holds even when varying σ. The fact that a baby boom shock will lead to that there are no winners in terms of net discounted lifetime income is not sensitive to the calibration of σ. At most the parent generation can stay unaffected, this case, however, implies a large burden on the future generations.

C.4 Factor prices

One results was that the transition path of central variables, such as factor prices, was dependent on the intergenerational transfer schemes. The difference between the cases decreases with σ. The quantitative difference still remains non-negligible, but the qualitative difference is not present for the two lowest values tested for σ. Also, one notable effect about the factor prices was that the relative outcome between the child generation and their progeny could differ qualitatively. This result will not hold for the lowest and highest value of σ. The fact still remains that if trying to explain the evolution of factor prices, or trying to project the factor prices, it does not seem advisable to ignore how intergenerational transfers respond to demographic changes.

C.5 Social welfare

From table C.4 and C.5 it emerges that the results about social welfare and expected social welfare vary as expected with σ. A low value for σ implies that the education system is not efficient. Keeping the education tax rate fixed will imply that the cost increases proportionally with the demographic change, while the benefit from such action will decline with σ. Confronted with a baby boom shock the education system of FB type will not longer be the preferred one for $\sigma = 0.17$. If the welfare from future generations is discounted hard, i.e. $\beta_s = 0.5\beta$, then education system of FC type would be preferred for $\sigma = 0.34$ also. This is not surprising since the benefit for keeping the tax rate fixed arises in the future, which is discounted severely.

More interesting is how the *ex ante* welfare will change with σ. For low efficiency in the education system, i.e. the two lowest values for σ, the education system of FC type seems to be preferred. The result that the pension system should no be of FC type seems to hold regardless of σ.

Table C.1: Percentage deviation from steady state outcome, for the variables C_{t-1} and $S_{w,t}$, t periods after the shock, when varying σ.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$	C_{t-1}	$S_{w,t}$
$\sigma = 0.17$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-5.86	-2.04	-1.26	2.75	-4.66	-4.66	0.00	0.00	-5.41	-3.02	-0.76	1.64
1	-3.07	16.31	-3.54	15.76	-7.38	11.14	-7.82	10.61	-5.01	15.29	-5.60	14.65
2	-1.79	17.85	-2.00	17.60	-3.37	15.96	-3.57	15.71	-1.18	19.03	-1.38	18.84
3	-0.80	19.04	-0.90	18.92	-1.52	18.18	-1.61	18.07	-0.18	19.90	-0.24	19.85
15	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
25	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 0.34$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-5.86	-2.04	-1.26	2.75	-4.66	-4.66	0.00	0.00	-5.41	-3.02	-0.61	1.32
1	-3.07	16.31	-5.51	13.39	-7.38	11.14	-9.71	8.35	-4.95	15.14	-8.02	11.69
2	-2.54	16.95	-3.92	15.30	-4.20	14.96	-5.56	13.33	-2.19	17.83	-3.61	16.35
3	-1.43	18.28	-2.21	17.34	-2.38	17.15	-3.15	16.22	-0.96	19.05	-1.59	18.41
15	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
25	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 0.6$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-5.86	-2.04	-1.26	2.75	-4.66	-4.66	0.00	0.00	-5.41	-3.02	-0.37	0.81
1	-3.07	16.31	-8.45	9.86	-7.38	11.14	-12.51	4.98	-4.87	14.91	-11.57	7.12
2	-3.68	15.59	-7.62	10.85	-5.47	13.44	-9.34	8.79	-3.70	15.94	-8.01	11.12
3	-2.71	16.75	-5.65	13.22	-4.04	15.15	-6.94	11.67	-2.58	17.15	-5.53	13.88
15	-0.07	19.92	-0.14	19.83	-0.10	19.88	-0.17	19.79	-0.03	19.97	-0.06	19.94
25	0.00	20.00	-0.01	19.99	0.00	19.99	-0.01	19.99	0.00	20.00	0.00	20.00
50	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00	0.00	20.00
$\sigma = 1$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	-5.86	-2.04	-1.26	2.75	-4.66	-4.66	0.00	0.00	-5.41	-3.02	0.00	0.00
1	-3.07	16.31	-12.79	4.65	-7.38	11.14	-16.67	0.00	-4.73	14.55	-16.67	0.00
2	-5.40	13.52	-14.88	2.14	-7.38	11.14	-16.67	0.00	-5.92	12.91	-16.67	0.00
3	-5.40	13.52	-14.88	2.14	-7.38	11.14	-16.67	0.00	-6.02	12.78	-16.67	0.00
15	-5.40	13.52	-14.88	2.14	-7.38	11.14	-16.67	0.00	-6.02	12.77	-16.67	0.00
25	-5.40	13.52	-14.88	2.14	-7.38	11.14	-16.67	0.00	-6.02	12.77	-16.67	0.00
50	-5.40	13.52	-14.88	2.14	-7.38	11.14	-16.67	0.00	-6.02	12.77	-16.67	0.00

Table C.2: Percentage deviation from steady state outcome, for the variables R_t and w_t, t periods after the shock, when varying σ.

	Case 1 (eFB pFB)		Case 2 (eFC pFB)		Case 3 (eFB pFC)		Case 4 (eFC pFC)		Case 5 (eFB pFR)		Case 6 (eFC pFR)	
t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t	R_t	w_t
$\sigma = 0.17$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	14.48	-6.54	8.63	-4.06	16.57	-7.38	10.61	-4.92	15.25	-6.85	9.42	-4.40
2	1.32	-0.66	1.59	-0.79	4.33	-2.10	4.61	-2.23	1.88	-0.93	2.20	-1.08
3	1.01	-0.50	1.12	-0.56	1.91	-0.94	2.03	-1.00	0.30	-0.15	0.38	-0.19
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.34$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	14.48	-6.54	6.41	-3.06	16.57	-7.38	8.35	-3.93	15.25	-6.85	7.41	-3.51
2	0.55	-0.27	1.68	-0.83	3.43	-1.67	4.59	-2.22	1.15	-0.57	2.60	-1.28
3	1.14	-0.56	1.77	-0.88	1.91	-0.94	2.55	-1.25	0.54	-0.27	1.02	-0.51
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.6$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	14.48	-6.54	3.10	-1.52	16.57	-7.38	4.98	-2.40	15.25	-6.85	4.42	-2.14
2	-0.62	0.31	0.90	-0.45	2.07	-1.02	3.63	-1.77	0.05	-0.03	2.35	-1.16
3	1.00	-0.50	2.14	-1.05	1.51	-0.75	2.65	-1.30	0.58	-0.29	1.53	-0.75
15	0.02	-0.01	0.05	-0.03	0.04	-0.02	0.06	-0.03	0.01	0.00	0.01	-0.01
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 1$												
-1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1	14.48	-6.54	-1.79	0.91	16.57	-7.38	0.00	0.00	15.25	-6.85	0.00	0.00
2	-2.40	1.22	-2.40	1.22	0.00	0.00	0.00	0.00	-1.62	0.82	0.00	0.00
3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.13	0.06	0.00	0.00
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table C.3: Percentage deviation from steady state outcome, for the variable U_{t-1}, t periods after the shock, when varying σ.

t	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.17$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	-0.35	0.66	0.35	1.36	-0.09	0.94
1	-0.95	-1.08	-2.19	-2.32	-1.60	-1.78
2	-0.52	-0.58	-0.98	-1.04	-0.39	-0.45
3	-0.23	-0.26	-0.44	-0.46	-0.07	-0.08
15	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.34$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	-0.31	0.34	0.32	0.97	-0.08	0.67
1	-0.95	-1.63	-2.07	-2.76	-1.51	-2.40
2	-0.70	-1.08	-1.16	-1.55	-0.65	-1.07
3	-0.39	-0.61	-0.65	-0.87	-0.28	-0.47
15	0.00	0.00	0.00	0.00	0.00	0.00
25	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 0.6$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	-0.24	-0.03	0.24	0.45	-0.06	0.31
1	-0.82	-2.09	-1.69	-2.96	-1.22	-2.80
2	-0.83	-1.75	-1.24	-2.17	-0.87	-1.91
3	-0.61	-1.29	-0.91	-1.59	-0.60	-1.30
15	-0.01	-0.03	-0.02	-0.04	-0.01	-0.01
25	0.00	0.00	0.00	0.00	0.00	0.00
50	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = 1$						
-1	0.00	0.00	0.00	0.00	0.00	0.00
0	-0.14	-0.27	0.14	0.00	-0.03	0.00
1	-0.52	-1.87	-0.98	-2.33	-0.70	-2.33
2	-0.65	-1.90	-0.90	-2.15	-0.73	-2.15
3	-0.61	-1.76	-0.84	-1.99	-0.68	-1.99
15	-0.32	-0.94	-0.45	-1.06	-0.36	-1.06
25	-0.23	-0.68	-0.32	-0.77	-0.26	-0.77
50	-0.14	-0.40	-0.19	-0.45	-0.15	-0.45

Table C.4: Social welfare, with different social discount factors when varying σ.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.17$						
$\beta_s = \beta$	-4.87	-2.71	-7.52	-5.35	-5.11	-2.97
Rank	3	1	6	5	4	2
$\beta_s = 1/n$	-8.84	-6.42	-14.92	-12.49	-8.64	-6.22
Rank	4	2	6	5	3	1
$\beta_s = 0.5\beta$	-1.21	0.03	-1.19	0.05	-1.22	0.02
Rank	5	2	4	1	6	3
$\sigma = 0.34$						
$\beta_s = \beta$	-6.29	-7.77	-9.39	-10.87	-6.60	-8.26
Rank	1	3	5	6	2	4
$\beta_s = 1/n$	-12.63	-17.24	-20.15	-24.76	-12.34	-16.92
Rank	2	4	5	6	1	3
$\beta_s = 0.5\beta$	-1.37	-1.01	-1.38	-1.03	-1.38	-1.03
Rank	4	1	6	2	5	3
$\sigma = 0.6$						
$\beta_s = \beta$	-9.60	-19.73	-13.76	-23.89	-10.18	-21.01
Rank	1	4	3	6	2	5
$\beta_s = 1/n$	-24.66	-51.88	-36.78	-64.00	-24.09	-51.15
Rank	2	5	3	6	1	4
$\beta_s = 0.5\beta$	-1.64	-2.87	-1.72	-2.94	-1.67	-2.92
Rank	1	4	3	6	2	5
$\sigma = 1$						
$\beta_s = \beta$	-22.89	-68.25	-31.29	-76.65	-25.62	-76.65
Rank	1	4	3	6	2	5
$\beta_s = 1/n$	-797.20	-2317.98	-1101.56	-2622.34	-892.83	-2622.34
Rank	1	4	3	6	2	5
$\beta_s = 0.5\beta$	-2.19	-6.59	-2.38	-6.78	-2.25	-6.78
Rank	1	4	3	6	2	5

Table C.5: Expected social welfare, $E[W]$, with different social discount factors, when varying σ.

	Case 1 (eFB pFB)	Case 2 (eFC pFB)	Case 3 (eFB pFC)	Case 4 (eFC pFC)	Case 5 (eFB pFR)	Case 6 (eFC pFR)
$\sigma = 0.17$						
$\beta_s = \beta$	-1.19	-0.92	-1.50	-1.23	-1.19	-0.92
Rank	4	2	6	5	3	1
$\beta_s = 1/n$	-2.15	-1.74	-2.51	-2.09	-1.94	-1.52
Rank	5	2	6	4	3	1
$\beta_s = 0.5\beta$	-0.60	-0.46	-0.75	-0.62	-0.60	-0.46
Rank	4	2	6	5	3	1
$\sigma = 0.34$						
$\beta_s = \beta$	-1.40	-1.28	-1.71	-1.59	-1.38	-1.27
Rank	4	2	6	5	3	1
$\beta_s = 1/n$	-2.74	-2.68	-3.09	-3.03	-2.43	-2.34
Rank	4	3	6	5	2	1
$\beta_s = 0.5\beta$	-0.70	-0.64	-0.86	-0.79	-0.69	-0.64
Rank	4	2	6	5	3	1
$\sigma = 0.6$						
$\beta_s = \beta$	-1.89	-2.25	-2.19	-2.56	-1.84	-2.25
Rank	2	5	3	6	1	4
$\beta_s = 1/n$	-4.61	-6.03	-4.93	-6.34	-3.99	-5.24
Rank	2	5	3	6	1	4
$\beta_s = 0.5\beta$	-0.94	-1.13	-1.10	-1.28	-0.92	-1.12
Rank	2	5	3	6	1	4
$\sigma = 1$						
$\beta_s = \beta$	-3.85	-6.77	-4.13	-7.05	-3.82	-7.05
Rank	2	4	3	6	1	5
$\beta_s = 1/n$	-124.37	-242.96	-122.75	-241.34	-119.64	-241.34
Rank	3	6	2	5	1	4
$\beta_s = 0.5\beta$	-1.92	-3.38	-2.07	-3.53	-1.91	-3.53
Rank	2	4	3	6	1	5

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