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Students' depictions of quantum mechanics: a contemporary review and some implications for research and teaching

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Abstract

This thesis presents a comprehensive review of research into students' depictions of quantum mechanics. A taxonomy to describe and compare quantum mechanics education research is presented, and this taxonomy is used to highlight the foci of prior research. A brief history of quantum mechanics education research is also presented.

Research implications of the review are discussed, and several areas for future research are proposed. In particular, this thesis highlights the need for investigations into what interpretations of quantum mechanics are employed in teaching, and that classical physics – in particular the classical particle model – appears to be a common theme in students' inappropriate depictions of quantum mechanics.

Two future research projects are presented in detail: one concerning interpretations of quantum mechanics, the other concerning students' depictions of the quantum mechanical wave function.

This thesis also discusses teaching implications of the review. This is done both through a discussion on how Paper 1 can be used as a resource for lecturers and through a number of teaching suggestions based on a merging of the contents of the review and personal teaching experience.

List of papers and conference presentations

Falk, J & Linder, C. (2005). Towards a concept inventory in quantum mechanics. Presentation at the Physics Education Research Conference, Salt Lake City, Utah, August 2005.

Falk, J., Linder, C., & Lippmann Kung, R. (in review, 2007). Review of empirical research into students' depictions of quantum mechanics. Manuscript submitted to Educational Research Review.

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Paper 1

Appendix 1

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1. Introduction

1.1. Introduction to the problem

Possibly the greatest joy of conducting education research in quantum mechanics is the feeling of making a substantial contribution to a research field that is very important. Although I am aware that each researcher probably considers her or his own research field as particularly important, I cannot help feeling that I am particularly favoured in my choice of research field, since the necessity of quantum mechanics education research is dramatically obvious.

Why is this? In short, it is because quantum mechanics is an extremely important and influential physics theory, and because teaching and learning quantum mechanics is a challenging task for both lecturers and students. In this section I will elaborate on this, in order to make the importance of quantum mechanics education research clear.

1.1.1. The importance of quantum mechanics

I believe that most physicists if asked by a non-physicist whether quantum mechanics is important, would start to smile. Some would do this out of sheer politeness. A few would possibly do it while thinking "how can anyone ask such a stupid question?" But most, I believe, would do it because they feel that they finally have a physics question that they can answer in a way that laypeople would understand.

To a physicist, the importance of quantum mechanics is self evident. She, or he, knows that quantum mechanics is *the* theory to use when it comes to microscopic phenomena: no other theory has been able to describe and predict, for example, atomic behaviour nearly as accurately as quantum mechanics has been able to. A physicist would also know that microscopic phenomena are extremely important when it comes to understanding matter at a larger scale. For example, by understanding the atomic structure you can explain why leaves are green, why certain plastics bend when others break, why metals conduct electricity, why it takes so much energy to heat up water in relation to equal amounts of many other substances, and why chemical reactions take place the way they do.

A physicist might also tell you that quantum mechanics is also a remarkable predictor. In fact, quantum mechanics has produced one of the most detailed predictions verified so far, with the so-called fine structure constant, describing the strength of electromagnetic interaction in the cosmos. The fine structure constant has so far been measured to a precision of twelve digits (Gabrielse et al. 2006). For comparison, a twelve digit precision can be exemplified by measuring the circumference of Earth with a precision of a hair width (or a twentieth of a millimetre).

A physicist will probably also start talking about string theory, physics' best attempt so far for a *grand unified theory* – a theory of everything. It so happens that string theory is anchored in quantum mechanics, along with the general theory of relativity. It is likely that some physicists would also go on to talk about quarks, radiation, anti-particles and other aspects of quantum mechanics, but at this point the listener is likely to get tired of examples from physics: OK, I get the point – quantum mechanics is an important theory in physics.

Influence of quantum mechanics outside physics

However, I am not completely satisfied with the conclusion that quantum mechanics is important in physics. Yes, quantum mechanics *is* an extremely important theory in physics, but the importance of quantum mechanics goes well beyond physics theories. To make this clear, we will also ask a few other imaginary representatives from other professions.

An engineer familiar with quantum mechanics would tell us that if it was not for quantum mechanics, we would not be able to make semiconductors in the way we do today. This means, for example, that we would not have cell phones, LCD displays, computers, light emitting diodes, and basically all other electronic equipment. A medical doctor would add that we, among other things, would not have magnetic resonance imaging; a powerful tool used for imaging the inside of our body. Also, the medical doctor would agree with the molecular biologist that quantum mechanics has made it possible to simulate how medical substances interact with the proteins of our body – an efficient and safe first step in testing new medical substances.

If we would go on to ask a science fiction writer about quantum mechanics, she or he would probably get excited. The writer would talk about quantum computers that are immensely more powerful than our ordinary computers; about quantum teleportation, creating an exact replica of whatever is teleported and at the same time destroying the original; or about quantum cryptography, a way of transmitting information without even a theoretical possibility of eavesdropping. This may seem a bit far-fetched, and indeed, quantum computers and quantum teleportation still have a long way to go before they can leave the laboratory environment. But quantum cryptography is actually commercially available, even if the range of communication is limited[1].

Finally, we turn to a philosopher, to ask about the significance of quantum mechanics. Assuming that the philosopher knows quantum mechanics well – and there are definitely some who do – she or he would tell us that quantum mechanics has had a profound impact on what we mean by space and time, and possibly more importantly, cause and effect. Quantum mechanics has also shown that our world cannot be described by a so-called *local realistic theory*, which is basically that every part of the world is in itself a determined reality that can be observed. Instead, the world must either be described by a non-realistic theory – that the world is not determined before we observe it; or a non-local realistic theory – basically saying that what we perceive as two different places in space are in some aspects actually the same place (or are in direct contact).

1.1.2. The importance of quantum mechanics education research

Given the vast impact quantum mechanics has made on modern society, it is quite surprising that so few know about quantum mechanics. If quantum mechanics is that important, why are we not taught at least the impact of quantum mechanics in school?

This question is of course very difficult to answer, but it is reasonable to look for an answer in terms of quantum mechanics being *difficult* – in one way or another. Indeed, it is easy to find examples of people describing quantum mechanics as difficult: in popular science and the media, quantum mechanics often appears with epithets such as "weird" or "strange", but perhaps more surprisingly, these descriptions also appear in physics articles (for example, Hilgartner & DiRienzi, 1995) and physics education articles (for example, Aravind, 2001; Müller & Wiesner, 2002), and one particularly popular quote is Richard Feynman's statement "no one understands quantum mechanics" (see, for example, Singh, Belloni, & Christian, 2006).

If quantum mechanics is considered difficult, a question that immediately arises is *why* it would be difficult, and possibly if it really is difficult or merely described as such. But before continuing this line of thought, I wish to make the point that it actually is a problem that quantum mechanics is not well understood, or even known, by the public. It seems fair that a typical citizen in a modern society should not have to know technical details of quantum mechanics any more than she or he should have to know the name of the 20 amino acids building all proteins, or be able to explain how a nuclear power plant works. But surely we expect a typical citizen to know something about DNA and the theory of evolution, and some reliable knowl-

[1] See, for example, http://idquantique.com/, http://www.smartquantum.com/ or http://magiqtech.com/ for examples of quantum cryptography products.

edge about supplying our society with the power it needs. Why should we not be aware about a theory as influential as quantum mechanics?

Seen in this light, asking why quantum mechanics is difficult is not a trivial question, but a step towards solving a very real problem. If we know what makes quantum mechanics difficult, we may be able to start to change the experience of learning it.

But there is also another, more alarming problem: it appears that not only laypeople, but also quantum mechanics students find quantum mechanics extremely challenging to understand. Research shows that many students experience grave problems in the learning of quantum mechanics. These studies have shown that students have problems in virtually all aspects of quantum mechanics – be it problem-solving, describing concepts and phenomena, or relating it to the world and society we inhabit. This problem threatens not only to make quantum mechanics become even more "alien", but also to slow down research in the development of applications or – perhaps more likely – to move research and application development to countries where quantum mechanics education has been more effective. When bringing in the question on how governmental money for education is used, or misused, it becomes obvious that the teaching and learning of quantum mechanics needs to be considered in terms of the economy of a country.

On top of this rather large-scale perspective, there is of course also the individual aspect of physics students missing out parts of their education that could, and should, be rewarding and stimulating.

In all, quantum mechanics education is an important educational and economic issue, and it should be investigated with precision and expertise. And, as the reader may have concluded, this is exactly what quantum mechanics education research is about: exploring teaching and learning of quantum mechanics, with the immediate or ultimate goal of improving the conditions for learning.

1.2. Significance of this thesis

The previous section provides a number of reasons as to why education research in quantum mechanics is important. If we accept this, then the subsequent question naturally becomes: what should the focus of this research be? In what way can the conditions for learning quantum mechanics – and the experience of this – be improved? What research is likely to produce results useful for improving quantum mechanics teaching, or understanding the process of learning quantum mechanics? And, of course, what research has already been done into the teaching and learning of quantum mechanics?

This latter question – what research has been done – is the obvious way of trying to start answering the other questions, as results from previous research informs future research questions. Unfortunately, the question of what research has been done into teaching and learning quantum mechanics is a far from a trivial one, for several reasons.

The first reason is that there, until recently, has been no broadly comprehensive summary of quantum mechanics education research. (Some limitations of this claim are discussed in Paper 1.) Thus, until now, education researchers have had to search across many different and diffuse sources to obtain an overview of even parts of the research field.

In other words (and as Paper 1 shows) a fair proportion of the research has been reported only at conferences, or in unpublished theses. This means that it may be difficult not only to obtain copies of research reports, but also to even become aware of their existence. Finding publications in quantum mechanics education research is also made difficult by the fact that publications are often spread across many different journals, as the field is still too young to have its own channel of publication[2].

Finally, the question about what education research has been done into teaching and learning quantum mechanics becomes difficult due to lack of tools for comparing research and research results. For example, research may focus on quantum mechanics at different levels, may involve students with different backgrounds, and different research projects may focus on different aspects of the same quantum mechanics topic.

The main contribution of this thesis is to summarise one branch of the research done into quantum mechanics education, namely that of *students' depictions of quantum mechanics*. This is complemented by discussion of the themes in research and research results, and recommendations for research questions for future work. In this way the thesis also provides tools to aid contrasting and comparing quantum mechanics education research.

Focusing on students' depictions of quantum mechanics, which is about student learning, is deliberately chosen before looking at teaching aspects of quantum mechanics education. This is because research into quantum mechanics education still has a long way to go, and I would argue that research into student learning informs teaching practice, rather than vice versa[3]. Thus, I consider a review of learning aspects of quantum mechanics to be a way of optimising review efforts, at this point in time.

As an example of how research into student learning may inform teaching practice, this thesis presents a number of suggestions for teaching that draw on the review presented in Paper 1. However, it should be noted that this

[2] There are of course also arguments *for* publishing in physics journals, physics education journals, and general learning research journals, as quantum mechanics education research must relate and communicate with physics lecturers and other education researchers. But nevertheless, this diversity makes it more difficult to find published research.

[3] One possible example of teaching research informing research into learning would be evaluation of different teaching materials: if one textbook would prove much better than another, the textbook contents could be analysed to inform research into student learning. However, I find the opposite relation more plausible – that research into learning informs the writing of textbooks.

thesis mainly focuses on facilitating future education research in quantum mechanics, rather than on directly impacting on quantum mechanics teaching practice.

1.3. Introduction to physics education research

Broadly, the goal of physics education research (PER) is to better understand how students and people in general learn physics, and how this experience may be improved. More specifically, physics education research can, for example, involve studies of students' attitudes towards learning physics, the social learning environment, how students perceive and describe physics concepts, and also developing and evaluating curricula and approaches to teaching.

This introduction to physics education research will focus on conceptual understanding, complemented by research into teaching methods, and some conceptual and theoretical frameworks for analysing the experience of learning physics. While some aspects of physics education research such as the studies of physics as a social and cultural environment are omitted, the focus of the introduction is chosen on the basis of, what I argue, is relevant to education research in quantum mechanics.

For a more extensive description of physics education research, I recommend two annotated research summaries. The first is a resource letter written by McDermott and Redish (1999). The research presented in this resource letter is primarily conducted in the US, and the intended audience is both educators and education researchers.

The second, a summary of physics education research and its impact, has been written by Thacker (2003). It reviews research conducted since 1990 from an international perspective. It concludes that PER has affected not only teaching, but also course content, curriculum design and textbooks:

> If one examines physics curricula and teaching methods around the world, what stands out, is not the differences, but the similarity of goals, assessment and methods. In particular, if one examines the PER done around the world and the changes in teaching methods in the last 10 [now 15] years, one notices that students' across the world have the same conceptual difficulties and that very similar changes are being made in curriculum design and classroom instruction. [...] Traditional curricula and methods of instruction are being questioned and re-evaluated, based on the results of PER. (Thacker, 2003, p. 1848, references removed)

1.3.1. Conceptual understanding and "misconceptions"

In many aspects, physics education research is a result of difficulties inherent in the teaching and learning of physics. One of the problems that have attracted most attention is that physics students exhibit what has been characterised as inadequate and inappropriate *conceptual understanding*. This means that many students – even students skilled at problem-solving and passing examinations – have problems recognising and relating to physics concepts in a way that is consistent with accepted physics models (see, for example, Aalst, 2000).

In the early 1980s, investigations into students' conceptual understanding led to what was then widely known as *misconceptions research* – a highly influential branch of physics education research. For the purposes of this thesis, "misconceptions" can be described as common and inappropriate formation of physics understanding; for example, that heavy objects fall faster than light objects, that a motion is always paired with an acting force in the same direction as the motion, or that metal objects generally have a lower temperature than, for example, things made out of wood.

Some different terms have been used for "misconceptions", usually with slightly different meanings or theoretical and epistemological implications[4]. A few examples are "preconceptions" (which students are assumed to have prior to instruction) or "alternative conceptions" (which acknowledges that the "incorrect" conceptions may also be useful). Some more examples of terms are provided by Smith, diSessa and Roschelle (1993).

Smith, diSessa and Roschelle (1993) have described a number of assumptions made in misconceptions research, summarised as:

- students have misconceptions;
- misconceptions originate in prior learning (in classroom or some other part of the world);
- misconceptions can be stable and widespread among students, and misconceptions can be strongly held and resistant to change;
- misconceptions interfere with learning;
- misconceptions must be replaced;
- instruction should confront misconceptions;
- research should identify misconceptions.

The most extensive research into "misconceptions" has been conducted in the area of Newtonian mechanics. This has, among other things, led to the development of a multiple-choice concept inventory for investigating conceptual understanding of force and motion (Hestenes, Wells, & Swackhamer, 1992). McDermott and Redish describes the questionnaire as follows: "The most widely used and thoroughly tested assessment instrument is the Force Concept Inventory (FCI). Each test item requires that students distinguish between correct Newtonian answers and erroneous 'common-sense'

[4] That I have chosen to use the term "misconception" should not be interpreted as entering the debate over which term is most appropriate. It is merely used for historical reasons.

beliefs. Widespread administration of the FCI has raised the awareness of faculty to the failure of most lectures to promote conceptual development." (McDermott & Redish, 1999, p. 760)

Even if classical mechanics[5] comprises the largest fraction of research into misconceptions, there is also research into conceptual understanding of electromagnetism, optics, thermodynamics, fluid mechanics, waves and sound, and more (see the two annotated research summaries listed earlier for further examples). There are also some more multiple-choice instruments for investigating conceptual understanding, for example, the Conceptual Survey of Electricity and Magnetism (Maloney, O'Kuma, Hieggelke, & Heuvelen, 2001), or the Test of Understanding Graphs in Kinematics (Beichner, 1994).

1.3.2. Research into physics teaching

One important aspect of physics education is the teaching. Thus, an important aspect of physics education research is studying teaching, and investigations and development of approaches to teaching.

One prominent trend found in the research discussions of studies into physics teaching is about what is characterised as *traditional teaching methods* – "relying primarily on passive-student lectures, recipe labs, and algorithmic-problem exams" (Hake, 1998, p. 65). Such methods have been shown ineffective as far as building conceptual understanding is concerned. Indeed, a large investigation involving more than 6000 physics students shows that conceptual learning in mechanics is more or less independent of the lecturer, when the teaching relies on such traditional approaches to teaching (Hake, 1998).

Instead, physics education research points towards using approaches to teaching that engage students, relating physics to real-world situations, and generally focussing more on students' learning than experts' experiences of physics in curriculum development (for example Fensham, 1984). Research-based curricula may, for example, take specific student difficulties into account (McDermott, 1991). It has reliably been shown that teaching where students are engaged interactively results in significantly better conceptual understanding of physics. It has also been shown that students with better conceptual understanding acquire better skills in standard problem-solving (as opposed to the sometimes less mathematical conceptual problems) (Hake, 1998).

Another example of the benefits of student engagement is that students who merely observe a teaching demonstration construct no better understanding of the underlying concepts than do students who have not seen the demonstration at all. However, students who engage with the demonstration,

[5] In this thesis, "Newtonian mechanics" and "classical mechanics" are used as synonyms.

for example, by predicting its outcome, display significantly better understanding (Crouch et al. 2004).

Of course, interactive-engaging teaching styles become more feasible in small or very small classes. However, some teaching approaches for increasing the rate of interaction in large lecture classes have also been developed and tested, with encouraging results (for example, Meltzer & Manivannan, 2002). A drawback appears to be that it is not possible to cover as much material as in traditional courses, and although most students favour the modified lecturing style after an initial period, a small fraction (less than 10 %) of the students seem highly uncomfortable with the teaching approach even at the end of the course (for example, Meltzer & Manivannan, 2002).

Recently, an approach to interactive teaching known as *variation theory* has received increasing attention. In short, variation theory focuses on teaching where "a critical feature of what we would like to teach is purposely varied while other aspects are purposely kept invariant." (Linder, Fraser, & Pang, 2006, p. 589) In-practice examples and recommended further reading can be found in (Linder, Fraser, & Pang, 2006).

Epistemology and learning physics

In researching conceptual learning among students, not only have approaches to teaching been investigated, but also attitudes and metacognition among students (Gunstone, 1991). Linder (1992) has argued that the epistemology reflected in physics teaching – whether physics is truths and facts that should be learnt, or tools and approximate models – may affect students understanding physics conceptually. Research has also shown connections between students' epistemological beliefs and their conceptual understanding of and general achievement level in physics (Lising & Elby, 2005).

1.3.3. Models for describing physics learning

Physics education research has clearly shown that the view of students as "blank slates", ready to be imprinted with facts and knowledge about physics, is not suitable for understanding student learning (for example, Driver & Erickson, 1983). Instead, new models of learning are being developed and refined, some borrowing from psychology and cognitive science, but also with elements or entire models specific for physics education research (see, for example, Aalst, 2000; diSessa, 1993; Redish, 2003). In this section, a few models are described briefly.

In the early period of "misconception" research, it was suggested that physics students as well as laypersons spontaneously combined everyday experiences with taught physics. In particular, it was noticed how many "misconceptions" in kinematics resembled Aristotelian physics (which, for example, treats impetus in a completely different way than Newtonian physics) (see, for example, McCloskey, 1983). However, later research has led to the argument that students often apply inconsistent "frameworks[6]" when describing physical systems, and in particular, few students consistently apply Aristotelian physics when describing physical systems (see, for example, Finegold & Gorsky, 1991).

Most of the later models for describing physics learning rely on a constructivistic framework, where the learner actively constructs knowledge through different forms of "cognitive resources". In such constructivist models, the prior knowledge of the learner becomes an essential aspect of learning. Constructivist models of learning also readily reflect the fact that different people being exposed to seemingly the same teaching, may have completely different learning outcomes.

One influential model of learning has been proposed by diSessa (1993), where physics knowledge is assembled in terms of what diSessa calls *phenomenological primitives*. These primitives are simple ideas, which could, for example, be proportionality (for example, that more force results in higher speed) or attenuation (for example, that the heat of a fire declines with distance from its centre). According to diSessa, simple learning experiences in specific contexts are generalised into patterns and laws. In this framework, "misconceptions" as pre-built blocks are replaced by, for example, overgeneralisations of the patterns primitives show.

Inspired by the theory of diSesssa, and a few similar theories, another model for understanding physics learning has been proposed by Hammer et al. (2005). This model focuses in particular on how students describe physics differently in different situations. The core of the model involves *activation of resources,* and how the cognitive resources that are activated may shift between contexts. For example, a student discussing the tossing of a ball may do this in terms of a force acting upwards while the ball is rising; but at the apex, the student may instead describe the upwards and downwards forces as balanced. This second description suddenly includes a downwards force, and also an upwards force even though the ball is not moving upwards, thereby logically contradicting the prior description. In the model of Hammer et al. (2005), this may be explained by triggering of different resources, where, for example, a turning point may activate cognitive resources such as forces "being balanced" (while a ball moving upwards triggers other resources).

1.3.4. Physics education research in relation to quantum mechanics educations research

When comparing education research in quantum mechanics to general physics education research, it can be seen that quantum mechanics education

[6] Here, "framework" means a set of principles or rules a student employs to make sense of physics experiences. Frameworks do not have to be internally consistent.

research makes up just a very small part of the total work done to-date. (see, for example, Duit, 2006; McDermott & Redish, 1999).

Most effort in quantum mechanics education research has been put into investigating so called “misconceptions”, and considerable effort has also been put into developing and evaluating teaching approaches that are informed by this knowledge.

Only very little work has been published on models for quantum mechanics learning (Hadzidaki, Kalkanis, & Stavrou, 2000; Kalkanis, Hadzidaki, & Stavrou, 2003). There have also been some efforts to relate quantum mechanics learning to the learning model of diSessa (Oliver & Bao, 2002).

Correlations between students’ epistemological beliefs of quantum mechanics and their understanding of the subject have been found, however, the changes are not clear-cut (see, for example, Ireson, 1999; Mashhadi, 1996).

There has been some discussion on whether or not the historical development of quantum mechanics should and could inform teaching (for example, Strnad, 1981).

More extensive references are provided in Paper 1, and Appendices 1 and 2.

1.4. Introduction to quantum mechanics

This section briefly introduces one way of looking at quantum mechanics. The description follows the so-called modal interpretation of quantum mechanics, although this will not be particularly important at this introductory level.

It is, however, important to note that this is *one* way of looking at quantum mechanics. I have chosen this view because I believe it to be relevant for a non-physicist reader. It introduces quantum mechanical concepts in a way that (hopefully) can be understood without prior knowledge of quantum mechanics or sophisticated mathematics.

Some limitations of this particular view are discussed in the end of this section.

1.4.1. The context of quantum mechanics

Before introducing any technical aspects of quantum mechanics, it is important to know the contexts where quantum mechanics is used.

Although quantum mechanics can be used to describe the motion of billiard balls and pressure of gases, the main realm of quantum mechanics is the atomic level. At sizes visible to our eyes, the differences between quantum mechanics and classical mechanics is so small that, in almost all cases, quantum mechanics provides but an insignificant correction to classical physics – just as the theory of relativity is irrelevant when dealing with velocities much smaller than the speed of light.

However, quantum mechanics should not be mistaken as a theory that only applies to nano-scale environments, which becomes "correct" only when sizes are sufficiently small. You could rather say that in nano-scale environments, classical mechanics is no longer a useful approximation of quantum mechanics. Still, there are examples of quantum mechanical phenomena visible in our everyday environment, such as diffraction patterns of light.

1.4.2. States, eigenstates and probabilities

One absolutely crucial aspect of quantum mechanics is that particles may have undetermined values for properties such as energy, velocity and angular momentum (rotation). This means that, for example, an electron may have *several* values of its potential energy. However, in the event of measuring the electrons potential energy, the potential energy will change into *one* of the possible values. This is what is often referred to as the "collapse of the wave function", and is an important example, that, according to quantum mechanics, measurement is not something passive, but an active process that may affect a particle.

An electron which only has one possible value for potential energy – such as an electron whose potential energy has just been measured – is said to be in an *eigenstate* of potential energy. An electron that is not in an eigenstate of potential energy can always be written as a *sum of eigenstates*, for example $\Psi = 0.1\ E_1 + 0.5\ E_2 + 0.2\ E_3 + \ldots$ (where Ψ represents the non-eigenstate of the electron, and E_i represents the different eigenstates of potential energy).

The coefficients in front of the different eigenstates may be used for calculating the probabilities for the specific potential energy associated to that eigenstate. This is done by squaring the coefficient (or actually by taking the absolute value of the square, since the coefficients may be complex-valued). Thus, if we were to measure the potential energy of the electron above, we have a 0.01 probability of obtaining the potential energy associated to the eigenstate E_1 and a 0.25 chance of obtaining the E_2 potential energy.

In the example above we are using potential energy, but the same reasoning holds for any measurable property of a particle. Thus, the state of an electron may be described as a sum of eigenstates of potential energy, but also another set of eigenstates of velocity, angular momentum, and so on.

1.4.3. Spatial distribution

In particular, the reasoning in the section above may be applied to the property position, which is a measurable property of a particle.

This means that a particle *does not*, in general, have a definite position in space. By measuring the position, however, the particle is forced into assuming one of the possible positions.

That a particle does not have a well-defined position contradicts how we normally conceptualize particles – as a point in space, with mass but no spatial size. It is tempting to visualize the quantum mechanical particle as a localised point-particle, that we do not know where it is, but that still has a pre-determined position even before we measure it. However, it turns out that this view is counterproductive when trying to understand quantum mechanics. *Properties are not determined until they are measured*[7], and there have actually been quite a number of experiments proving this[8].

As with other properties, the state of a particle may be described as a sum of *eigenstates of position*. Since position most often is a continuous variable, the coefficients are not described through discrete sets of numbers, but through a function describing a distribution. For every possible position, the function has a particular value. Taking the absolute value squared will yield the probability density of detecting the particle at that particular point in space.

This function, describing a distribution in space, is usually called the *wave function* of a particle, and is commonly denoted $\mathbf{\Psi}(\mathbf{x}, t)$, where $\mathbf{x}$ denotes a position (in one or more dimensions), and t denotes time. For example, $\mathbf{\Psi}(\mathbf{0}, t_0)$ would give us information of the probability of detecting a particle at the origin, at a certain time t_0.

The wave function is a central part of quantum mechanics. Often, especially in practical situations, the probability distribution (being the absolute value of the wave function squared) is more useful than the wave function itself, not least since the probability distribution may be empirically tested through repeated measurements. Still, it should be noted that the spatial expansion of a particle's state is only one of several possible ways of describing the state.

1.4.4. The Schrödinger equation

One part of quantum mechanics formalism, often seen as perhaps the most important one (see, for example, Singh et al., 2006), is the Schrödinger equa-

[7] This statement needs minor clarification, since there are consistent interpretations of quantum mechanics where measurement outcomes may be pre-determined. However, these will need to include more dimensions than our usual three, and a useful way of understanding the interpretations is to first understand why the properties cannot be pre-determined in ordinary three-dimensional space. Thus, viewing properties as unknown but pre-determined is, for the moment, unproductive when trying to understand quantum mechanics.

[8] See, for example, Aspect (1999) for a brief discussion on experiments confirming non-locality of quantum mechanics.

tion. The Schrödinger equation describes how a quantum mechanical system changes with time:

$$\frac{\partial \Psi}{\partial t} = \frac{-i}{\hbar} \hat{H} \Psi$$

Figure 1. The Schrödinger equation. ∂ represents a derivative and Ψ represents the wave function. i is the imaginary unit (where $i^2 = -1$) and h is Planck's constant (where the bar denotes that it is divided by 2π). The H with hat is an operator, acting on the wave function.

The left side of this equation is the *time derivative* of the wave function, describing the rate of change in the wave function as time changes. The right side of the equation consists of a complex factor, and then an operator corresponding to the *total energy* of the particle. The operator, called the Hamilton operator, involves kinetic energy, potential energy, and sometimes more terms relating, for example, to attraction or repulsion between particles. Thus, the Hamilton operator depends on the environment of a particle, and so does the time development of the particle.

A full explanation of the Schrödinger equation is beyond the scope of this brief introduction, but it should still be noted that in the case of a particle in an eigenstate of total energy, the time evolution of the system becomes a trivial problem. Because of this, it is often very informative to analyze the eigenstates of the total energy. Indeed, many very important quantum mechanical applications deal solely with finding or approximating these eigenstates.

1.4.5. Some comments

This introduction provides, as previously noted, only one possible way of interpreting and presenting quantum mechanics.

One limitation of this presentation is how the term *particle* is used. In quantum mechanics, the notion of particles becomes somewhat problematic when a system involves several *identical particles* – particles that by any measurements are indistinguishable, such as two electrons. In these cases, a measurement may perhaps be better described not as corresponding to a property of a physical particle, but as an event of interaction between two fields.

The point of this section is, however, primarily to give the non-physicist reader an idea of the principles constituting quantum mechanics, and not to provide an in-depth analysis of it.

2. Introduction to Paper 1

2.1. The purpose of Paper 1

As described in the introduction to physics education research (Section 1.3), one important aspect of this kind of research has been to identify and explore so called "misconceptions". However, I have decided to use the term "alternative conceptions" because the construct explicitly acknowledges that "incorrect" conceptions may also be useful. This construct is also anchored in the theoretical framing known as constructivism, which highlights the pedagogical principle that people make sense of the world in terms of what they already know. (cf. Driver & Erickson, 1983). Research into alternative conceptions has led to creating new textbooks and curricula, and has also provided more direct tools for teaching, such as demonstrations, tutorials, and instruments, to find out what alternative conceptions are common in any particular class.

Looking at education research in quantum mechanics, however, it is very difficult to draw conclusions on what alternative conceptions exist, why they appear, and how common they are. This is partly because there is comparatively little education research carried out in quantum mechanics, but also because prior to my literature review (Paper 1), there has been no comprehensive summary of quantum mechanics education research. Thus, one of the main purposes of Paper 1 has been to facilitate future research into students' conceptions of quantum mechanics.

However, the review in Paper 1 does not explicitly look into students' conceptions of quantum mechanics. It reviews research into student *depictions* of quantum mechanics. Why is this?

2.1.1. Student depictions?

There are three reasons for reviewing the literature in terms of depictions instead of conceptions. The first is that the term conception is often used when applying a cognitive approach to learning. Since Paper 1 has no ambition of promoting any particular theoretical framework for describing learning, the term conception was considered not to be ideal. The second reason is that alternative conceptions – as the term often used in science education research – implies that a student is fairly consistent in the way she or he describes a certain phenomenon for a specific context: if, for example, a student provides a new explanation to why cold air is heavier every second time she or he is asked the question, the explanations would probably not be called alternative conceptions. Since many of the articles reviewed in Paper 1 did not review the consistency of student descriptions across contexts, this makes the use of the term concepts even less ideal.

Finally, a few of the learning problems reviewed in Paper 1 are arguably not of a conceptual nature. For example, students' inability to calculate time-dependence might be better described as a difficulty in problem-solving. Thus, what is reviewed is something wider than "students' conceptions". I chose to use the term *depictions*, because Paper 1 is a review of students' depictions of quantum mechanics through verbal descriptions, drawing, writing, and the language of mathematics. The intention being to choose an inclusive term, that would allow many types of student descriptions, and also to create a neutral theoretical-perspective stance for these descriptions.

3. Review method

3.1. Why a review method?

Doing scientific research involves using a systematic method for gathering and evaluating data. Reporting on scientific inquiries involves not only reporting on the results of a study, but also reporting on the methods used and the framework supporting these methods. In qualitative research, where methods and results are not completely separable, the need for reporting on method is even greater: not reporting on methods used implies leaving out important parts of how the research results should or should not be interpreted.

Considering this literature review a scientific inquiry, I thus wish to report on the methods used. The methods were not chosen beforehand, but rather evolved as a result of structured work, careful planning and continuous reflection. In large, the review of the literature may be divided into five phases, described below.

3.2. Phase A: starting point

The purpose of doing this literature review was to get to know the field in which I intend to conduct my own research. When starting the literature review, I already had some ideas regarding the focus of my research. Thus, articles that focused on these planned research topics were read with greater interest and detail.

The research topics of particular interest were the following:

- students using classical depictions to describe quantum systems;
- surveys for investigating students' depictions of quantum mechanics;
- inappropriate depictions linked to pictures and diagrams in quantum mechanics teaching and textbooks.

3.3. Phase B: pre-study

The first part of the literature study concerned research in students' depictions of orbitals and atoms. This topic was covered in quite some detail, and during this phase the method of working was changed and refined. The results of the research were summarised in a separate review, which I intended to distribute to quantum mechanics lecturers. This was not done to the extent that was initially planned, but the goal of presenting the results still guided the work of gathering quantum mechanics education research and summarising it in an accessible and accurate way. The result is presented in Appendix 2.

Although this was not the intention, the orbitals and atoms literature review served as a kind of pre-study. When the review was finished it resulted in an evaluation of routines of working ending up with a more structured approach for the remaining literature research. This refined method is presented under the next phase.

The pre-study also resulted in an extended and refined list of keywords for my classification and sorting of articles.

3.4. Phase C: main literature study

In the main part of the literature study, considerable effort was put into gathering articles and references concerning quantum mechanics education. When any article was read particular attention was put on the reference lists, and all articles that seemed of any relevance were located and gathered as far as possible. Even articles that could not be accessed were entered in the reference manager, along with any useful notes.

The main method of locating articles was using internet and library searches. The main article retrieval method was electronic library resources, complemented by physical library resources and e-mailing authors. All available literature was gathered; not only published articles, but also conference presentations, proceedings, theses, and unpublished monographs. The gathered material was stored and catalogued electronically, with very few exceptions.

The reading of articles was somewhat themed, either focusing on a particular author or on a particular sub-topic. However, the order in which articles were read was not particularly strict, and generally focused on articles that seemed the most interesting.

The way articles were covered and processed for the literature review could be summarised in the following four steps:

- crude categorisations in the reference manager, as articles were gathered. The categorisation followed the keywords from the pre-

study, and the list of keywords was only slightly extended during this phase;
- reading of an article, accompanied by notes made in the reference manager. All articles were scanned for any further references that had not already been recorded;
- when an article was read, very brief summaries were written in a separate document. In contrast to the notes in the reference manager, comments in this document were sorted by topics (which were basically the same as the list of keywords);
- every week, or every second week, comments from the key-topics document were included in a fully readable review. This review was thus re-read and rewritten several times during this phase.

As the work went on, the initial torrent of new references eventually turned into a trickle.

3.5. Phase D: finalising the review

The literature study then moved on to the next phase. At that time, 80 out of nearly 200 articles related to quantum mechanics education research had been covered. A large proportion of the unread articles did not concern empirical education research, but rather opinions and general teaching suggestions. It was also decided that empirical research into teaching methods and aids should be excluded from the review, in order to enable an in-depth review of student depictions of quantum mechanics within the time constraints.

In this phase of the literature study, the review manuscript was revised. This included discerning of main research results and reworking the internal order of the review.

The previous crude categorisation of articles made it possible to be sure that most relevant articles had been covered, and thus that no further drastic changes of the review manuscript would be necessary.

4. Significance of Paper 1

In this section, the significance of Paper 1 concerning quantum mechanics education research in general, and quantum mechanics teaching in particular, is described and discussed. I will also present a number of suggestions for quantum mechanics teaching, based on my experiences as an education researcher, lecturer and student.

4.1. What does this literature review mean for researchers?

The literature review presented in Paper 1 contributes significantly to quantum mechanics education research. In this section, I will elaborate on these contributions.

4.1.1. Introduction to research and resource list

The most obvious contribution of the literature review is that it summarises research made into students' depictions of quantum mechanics. Even though the research field is now 25 years old, no comprehensive summary of research has so far been accessible in English. What has been done so far is described below.

- Fletcher (1997) provides a fairly comprehensive resource list of education research in quantum mechanics, and also some related education research in chemistry. Only a few studies are described in detail.
- Müller (2003) has published a summary of education research in quantum mechanics. This review covers the plentiful early German research in great detail, and also a large proportion of more recent and international research. However, it is written in German.
- Fletcher (2004) presents an updated resource list of education research in quantum mechanics education in his doctoral thesis. Unfortunately, the period 1997–2004 is not covered as extensively as the period prior to 1997. Again, some selected studies are presented in detail.

The literature review presented in Paper 1 will provide future researchers and lecturers with a contemporary resource that introduces and summarises research into students' conceptions of quantum mechanics.

Not only will the literature review serve as an introduction to the research field – it is also a resource, with a wide-ranging list of references for further reading.

4.1.2. Taxonomy

The second contribution of the literature review in Paper 1 is a broad and pragmatic taxonomy for categorising quantum mechanics education research. Through the filter of this taxonomy, it becomes clear that nearly all research has been into quantum mechanics at what is called a pre-quantum or introductory quantum level. It also becomes obvious that there are virtually no quantum education studies that have involved 500 participants or more.

Apart from categorising previous research, the presented taxonomy is also useful when presenting new research. Following the taxonomy, it is possible to, for example, describe a quantum mechanics course as "an introductory quantum mechanics course, also involving Dirac formalism and some aspects of identical particles", which is a compact way of describing course content, facilitating comparison with other studies.

The taxonomy also stresses the importance of reporting on course level and content, in what year the course is offered, the location of the study, and number of participants.

4.1.3. Suggesting research questions

In summarising research into quantum mechanics education, it also becomes possible to draw conclusions on a larger scale. Paper 1 presents a number of important future research topics.

These questions involve not only extending research to include large-scale studies and more advanced quantum mechanics topics, but also investigating what interpretations are employed in quantum mechanics teaching. It is also suggested that investigations into the relation between classical mechanics and quantum mechanics in students' depictions is an important key for understanding the learning of quantum mechanics.

4.1.4. Historical overview

This literature review also provides an historical outline of quantum mechanics education research. In this outline, not only *when* is presented, but also *where* and to some extent *what* education research has been conducted.

4.2. What does this literature review mean for lecturers?

Though Paper 1 focuses on students' depictions of quantum mechanics, excluding research into teaching methods and aids, the review is still a useful resource for quantum mechanics lecturers. In this section, I will elaborate on how Paper 1 can contribute to the teaching aspect of quantum mechanics teaching and learning.

4.2.1. Introduction to education research

Starting from a somewhat pessimistic view, it is likely that many quantum mechanics lecturers do not know what education research has to say about the teaching and learning of quantum mechanics, or are even aware of the existence of such research. To such lecturers, Paper 1 can serve as an introduction to quantum mechanics education research, or to complement other new examples of quantum mechanics education research. Since Paper 1 is structured according to quantum mechanics topics, it may also be used as a read-only-your-own-topic resource, while its extensiveness still shows the span of quantum mechanics topics investigated, as well as providing exemplars of physics education research.

Since the literature review also includes accepted depictions of quantum mechanics, it is not necessary that the reader be familiar with quantum mechanics on an everyday basis (though she or he will probably need to have studied quantum mechanics to appreciate the content). For someone starting to teach quantum mechanics, Paper 1 may be particularly useful, since it summarises both students' inappropriate depictions and the accepted depictions.

4.2.2. Awareness that students have problems in quantum mechanics

A second somewhat pessimistic view is that some quantum mechanics lecturers may not be aware of the possible difference between what is taught and what is learnt in the classroom. I am convinced that all lecturers who have assessed exams or home assignments in a quantum mechanics course are aware that students sometimes "don't get it", but there is a pedagogically important difference between "not getting it" and "getting it wrong".

Paper 1 leaves no doubt that there is more to the problems in quantum mechanics teaching and learning than students "not getting it". The consistency of at least some inappropriate depictions shows that students at different ages and in different countries share some of the same difficulties: for some reason, students are "getting it wrong" in the same ways. It is reasonable to believe that quantum mechanics teaching can be improved and informed by investigating these inappropriate depictions.

Thus, this literature review may serve as an introduction as to why it is useful not only to study how students understand quantum mechanics, but also how they *mis*understand it.

4.2.3. Awareness of inappropriate depictions

The main contribution of Paper 1, from a teaching perspective, is the awareness of different inappropriate depictions students use in quantum mechanics.

A lecturer aware of common inappropriate depictions has a chance of adjusting her or his teaching to deal with these depictions, even if only mentioning them and pointing out their inappropriateness. Research included in the review (Paper 1) has shown that students are apt to retain at least some of their inappropriate depictions, and thus counteracting them should not be considered an easy task. Still, a lecturer aware of some common inappropriate depictions stands a much better chance of dealing with them, be it through examples the lecturer uses, questions that students ask, particularly useful assignments, or similar.

Another benefit of being aware of inappropriate depictions is that some teaching approaches may actually facilitate inappropriate depictions: as a lecturer (at least for me), it easily happens that I draw wave function and potential in the same diagram, or talk about electrons moving about or being located in certain orbitals. As a lecturer, it may be obvious that certain ways of illustrating or talking about quantum mechanics phenomena have constraints and should not be taken literally. But for a student, knowing which aspects of a metaphor to hold on to or disregard may be very difficult. A lecturer aware of common inappropriate depictions stand a much better chance of avoiding possibly misleading phrases or illustrations – or at least to comment afterwards that, for example, a diagram in the textbook should more accurately be separated into two diagrams.

4.3. Suggestions for teaching

In this section, I make a number of suggestions for the teaching of quantum mechanics. These suggestions are mainly based on the research presented in Paper 1, but also draw on my experiences as both lecturer and student in quantum mechanics courses.

In writing this section, I wish to acknowledge that teaching is a skill and an art, that each lecturer must craft for her- or himself. The points presented here are, thus, merely suggestions. They should be treated as a source of inspiration and new input, not as any proven effective method of teaching quantum mechanics.

I doubt that there are any universally effective methods of teaching, even if I am convinced that some methods are more effective than others, and I argue that one of the most important qualities of teaching is the joy the subject brings and the ability to communicate this fascination. Consequently, I argue that every lecturer must find and develop her or his own way of teaching. However, I also strongly argue for the usefulness of lecturers sharing their experiences and ideas in order to facilitate each others' crafting of teaching, and this is what I intend to do in this section.

Please note that a list of research into quantum mechanics teaching methods and aids can be found in Appendix 1.

4.3.1. Recognise the influence of the examinations

The first suggestion for teaching quantum mechanics is a general suggestion, probably applicable to all physics education. The suggestion is to recognise the influence of the examinations. My experiences as a student, and also observations of students as a lecturer and researcher, strongly tells me that one factor heavily influencing student learning is what they expect from the examination. Thus, the form and content of the examination can be a very powerful tool in guiding student learning.

Designing a good examination requires much thought about the course, and the purpose of the course. What are students expected to understand after finishing the course? What skills are they supposed to have acquired? For what parts of the course is not in-depth understanding fundamental?

All these questions are related to the purpose of the course. Is the course supposed to prepare students for tasks they encounter as professionals? Or is it a preparatory course for future courses? What aspects are important in that case, and what is the aim of the more advanced course?

How could an examination be designed in order to probe the crucial aspects of the course? I argue that designing the perfect examination is an impossible task, and certainly beyond what should be expected from lecturers. Nevertheless, considering what an examination actually probes is an important aspect of improving teaching.

I strongly argue that examinations in many quantum mechanics courses would benefit from including essays or oral presentations, and not only standard problem-solving.

4.3.2. Make use of "easy" questions

Outcomes of research into quantum mechanics education presented in Paper 1 clearly show that students have problems with many, even basic, aspects of quantum mechanics. In teaching, this means that not only are the complex questions needed, but the power of the seemingly "easy" questions should also be recognised.

Making use of the "easy" questions has several advantages. One is that students are guided into considering basic concepts of quantum mechanics and not only "drilled" in solving calculation problems where the underlying concepts sometimes are hidden. For example, consider the following "easy" questions. *What are the possible energy measurement outcomes for this (simple) state? What is the distribution between the measurements, on average?*

A lecturer employing seemingly easy questions in teaching also acquires a powerful tool in discerning problems that students have: a lecturer using difficult questions has a tool for identifying students who don't have problems with quantum mechanics, while a lecturer using easy questions has a tool for discerning the difficulties students struggle with.

Relating to the previous section, I argue that quantum mechanics examinations in general would benefit from a section with "easy" questions. A list of non-complex questions that all students should be able to answer after completing the course would be a reason for students to look into even the basic questions of quantum mechanics, not only focusing on skills in calculation and problem-solving. Also, someone failing "easy" questions is a clear indication of a student not reaching the appropriate goals for the course. As an example, *what is the difference between expectation value and measurement outcome?* This should hardly be a difficult question for someone familiar with quantum mechanics, but incorrect answers would provide information on problems students have with the learning of quantum mechanics. I argue that an effective strategy for designing such questions is not to look for questions that students might fail, but rather to look for questions in which all students are expected to succeed.

Another possible benefit of making use of "easy" questions is that quantum mechanics may seem less difficult to students. Of course, making use of the easy questions must not turn a quantum mechanics course into a dull repetition of basic statements, or compromise the contents of the course.

4.3.3. Emphasise difference with classical mechanics

Since one of the main themes in the literature review is students applying classical depictions to quantum mechanical systems, it is natural to suggest that teaching should try to increase the distinction between classical physics and quantum mechanics.

Looking at the research into students' depictions, it seems that the difference that needs to be emphasised is between how classical physics and quantum mechanics model particles: many students seem to depict particles as always being localised, as in classical physics.

I argue that the difference between classical physics and quantum mechanics can be approached through the quantum wave function, which in some aspects replaces the classical particle. Focusing extensively on the wave function – its role in quantum mechanics, what relation it has to a particle, and maybe also making use of essay assignments on the topic – may form a robust platform for learning quantum mechanics.

4.3.4. Mention interpretations of quantum mechanics

When discussing the difference between classical physics and quantum mechanics, interpretations of quantum mechanics become essential. This is also true when discussing the quantum wave function.

Introducing details of different interpretations of quantum mechanics is arguably beyond the scope of any undergraduate course, and I find it plausible that introducing multiple interpretations at an introductory level will cause confusion rather than clarity. Still, emphasising that what is employed is one particular interpretation, and that other interpretations exist, could facilitate students viewing quantum mechanics as a theory, and not a truth about nature that could and should not be questioned.

4.3.5. Concerning atoms and orbitals

The main problem in teaching and learning the quantum mechanical atomic model seems to be that students retain a classical orbit depiction of the atom. There are several methods of introducing the quantum mechanical orbital model, and some of them have shown encouraging results.

A list of such articles can be found in Appendix 1, and I will not discuss them further here. However, I would like to express my opinion on teaching the Bohr atomic model, which I believe is counterproductive, and is taught more because of tradition than because of its usefulness.

4.3.6. Concerning the wave-nature of matter

In summarising the problems students have in depicting quantum mechanics, I argue that the largest and most important hurdle for their learning is that particles are represented differently in quantum mechanics and classical physics (as already has been stated).

This problem is unlikely to be solved easily, in particular when considering that a classical particle model is not only supported by classical physics, but also to a large extent by everyday experiences – where objects appear as solid, localised entities with definite trajectories and other properties typical of the classical particle model.

To help students form an accepted depiction of the wave-nature of matter, I argue that depicting quantum particles as alternating between waves and classical particles should be avoided in teaching. I also argue that it is worth stressing that quantum mechanics poses a non-contradictory model for describing particles.

Some education researchers have suggested totally new words for quantum particles, for example *quanticles* (Taber, 2002). Since classical and quantum mechanical particles are described quite differently, I believe that it may be useful to introduce quantum mechanical particles using new terms. However, I also argue that physics students studying quantum mechanics should be used to the term *particle*, in order to communicate well with physicists.

4.3.7. Concerning one-dimensional square potentials

Square potential barriers, being a central part of introductory quantum mechanics, have received surprisingly little attention when it comes to empirical research into teaching methods. (See Appendix 1 for a list of articles on the topic.) The comparatively extensive research in students' depictions of square potentials shows that there are substantial problems to be dealt with.

Considering the problems, I argue that it is important to take care in explaining the basic situation when introducing this topic. McKagan and Wieman (2006) exemplify one-dimensional square potentials through electrons in thin wires, and then different potentials may be represented by different materials and even gaps in the wire. This kind of example would probably help some students appreciate the one-dimensionality of the potential system.

In the case of a particle with a definite total energy, I argue that it is beneficial to complement a potential energy diagram with a diagram of (local) kinetic energy. This diagram could help make a clear distinction between total energy, potential energy and kinetic energy, hopefully making it easier for some students not to confuse the different energies.

Furthermore, a diagram of (local) kinetic energy for a particle may be very useful when approaching the wave function for the particle. When combined with the time-independent Schrödinger equation – involving a second derivative of the wave function representing kinetic energy – it becomes obvious that the wave function will look fundamentally different in sections with negative kinetic energy compared to positive cases. Following this, it is also natural to point out that the curvature or wavelength is related to (local) kinetic energy, while the amplitude of the wave function is unrelated to energy.

Finally, I argue that continuously returning to a concrete example – as an electron in a thin wire – may help students focus not only on calculations, but also on what the calculations represent: what do these calculations tell us about the electron?

In particular, I maintain that it is unwise to show diagrams where potential energy and wave function or probability distributions are plotted together. This also includes the case of the infinite square well potential, where I argue that it may be counterproductive to draw wave functions centred on the different energy levels. “Particle in a box” is an expression that may lead students to depict the system as two-dimensional. For lack of a better phrase, I instead suggest “electron in a wire” (or possibly “nano-wire”).

4.3.8. Concerning time-dependence

Time-dependence appears to be a problematic topic in quantum mechanics, and virtually no research has been done on its teaching.

Some researchers have suggested that the problems with time-dependence are partly explained by the fact that large parts of introductory quantum mechanics only deal with time-independent situations (for example, Singh, 2006). Introducing time-dependence earlier is probably not a good solution, since this would mean introducing more complex aspects of quantum mechanics in topics that already are problematic.

Perhaps a more feasible suggestion is to merely *mention* the time-dependence at the end of time-independent problems, and also to continually stress that a stationary wave function (energy eigenfunction) is actually only the time-independent factor of a complete wave function.

I would also like to support a suggestion made by Styer (1996): to use the term “energy eigenequation” instead of “time-independent Schrödinger equation”. While the latter term certainly is correct, it is also easily and inaccurately shortened to simply the Schrödinger equation, and even when using the full term it may be difficult for students to appreciate the importance of the “time-independent” prefix. Reserving the term Schrödinger equation for the time-dependent version may help ensure that time is included as a parameter when dealing with the Schrödinger equation.

4.3.9. Concerning Dirac notation

Research into teaching approaches that use the Dirac notation, and students’ depictions of Dirac notation is, respectively, non-existent and close to non-existent. However, the findings – that students are uncomfortable with the Dirac notation – is in line with my own and many of my fellow students’ experiences during an introductory quantum mechanics course. The following suggestions are based on these experiences, and also experiences from later discussions with students and quantum mechanics lecturers.

The first thing to note when discussing the Dirac notation is that it is useful in quantum mechanics. The notation system simplifies calculations, enables compact expressions, and also helps focusing on quantum states rather than analytical expressions. Thus, it is sensible to include the Dirac notation in the curriculum, and students' problems with the Dirac notation should not be resolved by simply removing the topic.

Secondly, the main problem my fellow students and I experienced when being introduced to the Dirac notation was that we could not relate it to the spatial wave function, which we had been using throughout almost the entire course. The analytically expressed wave function was our platform for describing quantum systems, and we saw no point in introducing the Dirac notation for doing a task for which we more or less already had the tools required. Indeed, it took some time before we learned that there was information about a quantum particle – for example spin – that could be fitted into the Dirac notation, but not the analytical wave function.

Thirdly, after becoming more acquainted with quantum mechanics, I now view the Dirac notation as a simple way of writing down quantum systems, while the analytically expressed wave functions are messy and often the source of calculation errors and other mistakes.

Taking all this into account, I would like to suggest that the Dirac notation is used from the start in introductory quantum mechanics courses. Though the Dirac notation is not very useful when discussing one-dimensional square potentials, it is perfectly suitable for introducing and discussing measurements, expectation values and spin.

Then, once the Dirac notation has become familiar, the analytical wave functions may be introduced. Since the analytical wave function is merely a special case in the Dirac notation – namely a quantum state expanded in position eigenstates – this way of introducing the analytical wave function will hopefully be less problematic than using the analytical wave function for introducing the Dirac notation.

4.3.10. Use quantum mechanics as a theme in physics education

There are several education researchers who argue that quantum mechanics should be introduced at an earlier stage in school (for example, Redish, Bao, & Jolly, 1997), on the basis that quantum mechanics is important and also that it takes time to get familiar with the concepts.

Considering the many prerequisites for learning quantum mechanics in the in-depth aspects required for physicists, it might seem difficult to study quantum mechanics prior to the second or third year at university. However, I argue that the prerequisites could also work towards getting the students accustomed to some of the more critical fundamentals of quantum mechanics: the many connections between other parts of physics, and not least mathematics, could be used to introduce quantum mechanics over a longer period of time. For example, one- and multidimensional calculus, linear algebra, differential equations, Fourier analysis and transform theory, wave physics and electromagnetism – just to mention some courses – could all be directly related to examples in quantum mechanics.

Being introduced to quantum mechanics over a longer period of time would arguably increase the possibilities for effective learning. Another possible strength is that quantum mechanics is an essential part of modern physics, and in being so it is also one of the more exotic and exciting aspects of physics. Thus, a quantum mechanics physics programme has a chance of appealing to students. Also, having an explicit goal with many of the introductory mathematics and physics courses could help to make them seem less tiresome and more meaningful.

Finally, the vast number of applications of quantum mechanics would make it easy to justify an undergraduate physics programme centred on quantum mechanics that emphasises possible work opportunities.

5. Description of research plans

This section contains descriptions of two research projects I plan to carry out in my continued PhD studies.

5.1. Study 1: What standard interpretation?

Investigating how students depict quantum mechanics, and in particular how students *inappropriately* depict quantum mechanics, is complicated by the fact that there is no consensus on how to interpret quantum theory: While the mathematical formalism is clear-cut, there is no consensus on what the mathematical entities *mean*. Even in the Copenhagen interpretation, claimed to be the most widely used (Stapp, 1972), there exists several different interpretations of important concepts (Stapp, 1972).

The quantum wave function is a good example of differences between interpretations. In the modal interpretation of quantum mechanics – very similar to the Copenhagen interpretation – the amplitude squared of a wave function gives the probability density of detecting (measuring) a particle at a certain location. In the Bohmian interpretation on the other hand, the wave function is described as a field "guiding" the particle in a way similar to force field (see, for example, Albert, 1994 for an introduction). In contrast to the Copenhagen interpretation, this particle is localised at all times, and can have a definite trajectory in space. Thus, not only is the wave function interpreted differently in the two interpretations, but also particles are depicted in radically different ways.

Still, when investigating students' inappropriate depictions of quantum mechanics, it is vital to know what interpretations the students rely on.

It may be argued that research into students' depictions of quantum mechanics may be carried out without considering what is "correct" or "incorrect". This is certainly true, and determining the accepted versus the inappropriate depictions does not have to be the only goal of investigating students' depictions. However, in order to interpret students' depictions in a meaningful way, it is crucial to have a good grasp of quantum mechanics. If a researcher is relying on a different interpretation than the students are, then the students' depictions are likely to seem strange, and then the researcher's interpretations risk being shallow and miss important details. Indeed, this is the very point in having a good understanding of quantum mechanics when investigating students' depictions of the theory.

As Paper 1 has shown, very little research has been done into what interpretations are employed for quantum mechanics teaching. All current literature points towards the Copenhagen interpretation being dominant. However, it is not obvious what the Copenhagen interpretation actually means. Also, at least here at Uppsala University, I have experienced different attitudes towards hidden variables[9] in different quantum mechanics courses.

The question about which interpretation is employed for teaching is not only of interest for quantum education research, but arguably also for quantum physicists: there is no consensus on what the so-called "standard" interpretation of quantum mechanics actually is. Investigations on what interpretation is employed for teaching does not only indicate what interpretation future quantum physicists will have adopted, but also what interpretation today's lecturers are using.

5.1.1. Research questions

The proposed research questions are divided into one overall and several detailed questions. The overall research question is:

- What interpretations of quantum mechanics are used in the teaching of quantum mechanics at Swedish universities?

In particular, this will be investigated through the following questions:

- Do the interpretations used match any (historically) established interpretation? In particular, do the interpretations agree with the Copenhagen interpretation(s)?
- To what extent do Swedish quantum mechanics courses present different interpretations of quantum mechanics?
- Are there differences in the interpretations that are employed within or between universities? If so, do these differences relate to how advanced the courses are?
- Are there differences between universities in the interpretations that are employed?

[9] Hidden variables are unknown but determined variables influencing the outcome of measurements. The presence or absence of hidden variables affect, for example, if quantum mechanics could be considered deterministic or not.

5.1.2. Research method

The scope of this study is to investigate quantum mechanics courses taught across Sweden. To make this feasible, I propose that the data collection be done through the use of questionnaires. To improve distribution and collecting of data, the questionnaires will be web-based and online.

In order to avoid asking lecturers what interpretation they believe that they are using, the questionnaire will not rely on questions of the type "do you use the many-worlds interpretation of quantum mechanics?" Instead, it will use a number of statements or examples, where the answers may be used to judge what interpretations are used. An example of such a statement is "hidden variables determine measurement outcomes", while another one may be "it is possible to calculate probabilities for measurement outcomes *for single particles*". Lecturers will be asked to respond to these statements by choosing one response in the following (preliminary) list:

- Yes, this is the main interpretation used in my teaching.
- Yes, this interpretation is mentioned in my teaching, but it is not the main interpretation used.
- No, this interpretation is treated as incorrect in my teaching.
- This interpretation is not mentioned in my teaching.
- This interpretation is not relevant for the topics included in my courses.
- I don't understand this statement/question. / I do not wish to answer.

Participants will have the opportunity to comment on any of the statements.

The questionnaire will also contain background information on the participants. One example of a background question could be, "which university are you teaching at?"

5.1.3. Reporting

An important aspect of carrying out scholarly research is to report on the findings. This study will result in an article, submitted to a journal with preferably both physics lecturers and other physicists as readers.

Another way of reporting of this study will be to offer all participants a summary of the results.

5.1.4. Possible ways of expanding the project

The scope of this project is to map what interpretations of quantum mechanics are employed at Swedish universities. If this study brings clear results, one possible way of expanding it would be to extend the study to outside of Sweden.

It would probably not be possible to investigate all quantum mechanics courses in, for example, Germany. Instead, a reasonable goal would be to cover some larger and smaller universities across a number of countries in- and outside the E.U. The results from these could then be compared to the detailed picture of Swedish quantum courses, to see if there is reason to believe that quantum mechanics courses, for example, in the USA there may be different interpretations than in Sweden. To provide a fully reliable picture of similarities and differences, more detailed investigations would of course be necessary.

5.2. Study 2: Students' depictions of the quantum wave function

A common thread in the literature summarised for Paper 1 is that many students tend to draw on classical reasoning when describing quantum systems or quantum phenomena. For example, the classical orbits depiction is used by 20–60 % of students at pre-quantum level. The research also shows, so far, that 20–50 % of introductory quantum students depict transmission and reflection in terms of classical energy limits, and that 30–50 % depict a probability peak as something similar to a classical particle. It also seems that classical reasoning is present in a number of other thematic difficulties, such as energy loss due to tunnelling; depicting the wave-particle duality as classical particles moving in wave-shaped trajectories; depicting quantum uncertainty as measurement error; lower potential implying higher probability for finding a particle; and, to some extent in inappropriate depictions of quantum measurements and spin.

One fundamental difference between quantum mechanics and classical mechanics is the physical models used for describing particles. It appears that all the thematic difficulties listed above relate in one way or another to classical particles (in other words, particles that are always localised). Thus, investigating students' depictions of particles in quantum mechanics is likely to be an important research area when trying to understand the learning of quantum mechanics.

The proposed research project described below is directed at aspects of students' depictions of particles in quantum mechanics. The proposed research project will inform my future investigations of students' depictions of particles in quantum mechanics.

5.2.1. Research questions

Again, proposed research questions are divided into one overall and several detailed research questions. The overall research question is:

- How do quantum mechanics students depict the quantum wave function?

In particular, this will be investigated through the following questions:

- How do students who have successfully passed quantum mechanics courses depict the quantum wave function?
- How do students who struggle with introductory quantum courses depict the quantum wave function?
- How do students depict the relation between the quantum wave function and its associated particle?
- Do students use a consistent depiction of the quantum wave function when presented with different physical situations?

5.2.2. Research method

The main method of collecting data will be through interviews with quantum mechanics students. To avoid interviewing students' I am currently teaching or have previously taught, no interview participants will be collected at Uppsala University. This also eliminates having to interview friends or fellow PhD students.

Initially, up to 40 interview-participants will be sought from several levels of university physics studies – from pre- or introductory quantum courses to advanced quantum courses (and possibly beyond). Each participant will be interviewed once using a broad interview protocol. After analysing the initial interview sessions, about ten students will be chosen for further in-depth interviews. The choice of students will be based on getting as large a variety as possible with respect to depictions of the wave functions. The choice will also depend on getting participants who are willing to engage in discussion around the envisaged content of the interview. All interviews will be audio-recorded and analysed shortly after the interview session. Participants will be compensated, preliminary by coupons for a local lunch restaurant. The aim will be to keep the interview sessions to no longer than one hour.

The initial interview protocol will have a broad questioning approach to explore students' depictions of the quantum wave function, briefly discussing it in several aspects and contexts. In the in-depth interviews, each session will be focused on one or two aspects of the wave function. Examples of such aspects could be orbitals, localised wave functions, and how wave functions are affected by measurements.

Since this part of proposed future work is much more flexible than what I called Study 1 earlier, the method of inquiry will, to a larger extent, be shaped by the early findings in the proposed work. Thus, it is difficult to give a more detailed description of the method at this stage.

5.2.3. Theoretical framework for analysis

The analysis of the interview data will be carried out using the phenomenomgraphic approach (cf Marton & Booth, 1997) since the goal will be to find categories of qualitatively different ways of depicting the quantum wave function, and how this variation of depiction is related to the depictions of particles. In doing a phenomenographic analysis, it will be particularly useful to have a wide spectrum of student depictions.

This study will not look into how common any of the categories of depictions are amongst quantum mechanics students, but rather will look for the variation in the ways of depicting the constructs.

5.2.4. Possible ways of expanding the project

There are many ways of building on this study for future research, and indeed, Study 2 is intended as a study that could inform even larger postgraduate studies.

One natural way of building on Study 2 is to design a larger study using only one or a few of the topics chosen for focused interviews. This is, of course, a suitable project if one or a few of the topics seems particularly promising for understanding students' depictions of quantum mechanics – for example by shedding light on depictions of several quantum mechanics topics.

If my proposed next phase of studies is successful in describing students' depictions of the quantum wave function in great detail, one possible way of building on this could be to embark on quantitative investigations into students' depictions of the wave function. This could, for example, be made through the development of questionnaires to be used on larger populations, allowing a statistical analysis of students' depictions.

One further way of building on what I have proposed, as Study 2 is to relate students' depictions of the quantum wave function to inappropriate depictions of quantum mechanics. For example, the research question could take on the following form, "Do students with inappropriate depictions of the wave function also give inappropriate depictions of other topics in quantum mechanics? If so, which?"

Another way that seems particularly interesting to me at this early point, is to undertake closer investigations on how students' depictions of the quantum wave function relate to students' classical depictions of quantum phenomena. If clear relations between depictions of particles and wave functions can be found, depictions of wave functions may be a key to understanding inappropriate depiction in many quantum topics – since many depictions appear to be related to the classical particle model.

If my proposed future studies do *not* reveal any relation between students' depictions of the quantum wave function and their depictions of particles, one conclusion to draw may be that the quantum wave function is not an appropriate conceptual framing to use for investigating students' depictions of particles. In that case, one will need to choose between pursuing the question of students' depictions of particles, and students' depictions of the quantum wave function.

Finally, one possible way of building on my proposed future studies is to compare and contrast students' depictions of the quantum wave function, and from that develop course material to contribute towards efforts aimed at improving students' understanding of the wave function.

6. Sammanfattning på svenska

Denna licentiatavhandling handlar om inlärningsforskning i kvantmekanik, och mer specifikt tittar den på studenters beskrivningar av kvantmekanik. Det huvudsakliga bidraget denna avhandling ger till fysikens didaktik är en sammanställning över forskning som gjorts kring just studenters beskrivningar av kvantmekanik – något som hittills saknats[10], trots att sådan forskning funnits i 25 år.

Litteratursammanställningen, som presenteras i Paper 1, bidrar på flera sätt till framtida inlärningsforskning i kvantmekanik. Som alla litteratursammanställningar ger den en överblick över vilken forskning som gjorts, och vilka resultat som hittills rapporterats – vilket givetvis hjälper till att hitta intressanta forskningsfrågor och också ökar möjligheterna att jämföra nya forskningsresultat med gamla studier. Men litteratursammanställningen bidrar också med en taxonomi för att grovt kategorisera inlärningsforskning i kvantmekanik, vilket ytterligare bidrar till att uppmärksamma områden som varit underprioriterade.

Huvudresultat i litteratursammanställningen är följande:

- Studenter har i regel stora problem med att lära sig kvantmekanik. I princip alla områden som har undersökts visar att studenter har betydande problem, och det finns också anledning att tro att många studenter har svårt att relatera kvantmekanik till den dagliga tillvaron. De områden som undersökts kvantitativt i mer än en studie visar typiskt att 20–50 % av studenterna beskriver de valda kvantmekaniska begreppen på ett olämpligt sätt.
- Många av de olämpliga beskrivningar som studenter ger av kvantmekanik hänger samman med en klassisk partikelbild (istället för en kvantmekanisk partikelbild).
- Den mesta inlärningsforskningen i kvantmekanik har skett på kvantmekaniska begrepp som tas upp i introduktionskurser i kvantmekanik. Framförallt dominerar studier kring studenters beskrivningar av atomer, följt av beskrivningar av endimensionella potentialsystem (så som tunnlingsfenomenet i en dimension). Inlär-

[10] En omfattande litteratursammanställning över framförallt tysk forskning finns utgiven på tyska, emedan engelska översikter i princip har varit begränsade till listor över referenser. Detta presenteras närmare i artikel 1.

ningsforskning på mer avancerade kvantmekaniska begrepp är sällsynta eller extremt sällsynta.
- Storskaliga studier, som omfattar fler än 500 studenter, är extremt sällsynta.

Litteratursammanställningen uppmärksammar också att studier i kvantmekanikinlärning sällan presenterar vilken tolkning av kvantmekanik som använts i undervisningen. Då det finns flera olika tolkningar av den matematik som beskriver kvantmekaniken – och dessa kan skilja sig ganska kraftigt åt gällande exempelvis partikelbegreppet – torde även vilken tolkning studenter förväntas använda vara en viktig del av bakgrundsdata för studier.

Vidare presenterar avhandlingen också ett antal tips och förslag till kvantmekaniklärare, grundade i tidigare forskning kombinerat med erfarenheter som både lärare och student i kvantmekanikkurser.

Slutligen presenteras också ett antal förslag för framtida forskning, med utgångspunkt i litteratursammanställningen. Bland annat innehåller avhandlingen två detaljerade beskrivningar av forskningsprojekt för mina fortsatta doktorandstudier: en enkätstudie för att kartlägga vilka tolkningar av kvantmekanik som används i svensk undervisning, och en förberedande intervjustudie för att undersöka studenters beskrivningar av den kvantmekaniska vågfunktionen och hur dessa beskrivningar är relaterade till en klassisk bild av partiklar.

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8. References

Aalst, J. v. (2000). An Introduction to Physics Education Research. *Canadian Journal of Physics, 78*, 57-71.

Albert, D. Z. (1994). Bohm's alternative to quantum mechanics. *Scientific American, 270*(5), 58-67.

Aravind, P. K. (2001). The magic tesseracts and Bell's theorem. *American Journal of Physics, 69*(2001), 348-353.

Aspect, A. (1999). Bell's inequality test: more ideal than ever. *Nature, 398*(6724), 189-190.

Beichner, R. J. (1994). Testing student interpretation of kinematics graphs. *American Journal of Physics, 62*(8), 750-762.

Crouch, C. H., Fagen, A. P., Callan, J. P., & Mazur, E. (2004). Classroom demonstrations: Learning tools or entertainment? *American Journal of Physics, 72*(6), 835-838.

diSessa, A. A. (1993). Toward an Epistemology of Physics. *Cognition and Instruction, 10*(2-3), 105-226.

Driver, R., & Erickson, G. (1983). Theories-in-Action: Some theoretical and empirical issues in the study of students' conceptual frameworks in science. *Studies in Science Education, 10*, 37-60.

Duit, R. (2006). Students' and Teachers' Conceptions and Scince Education: University of Kiel.

Fensham, P. J. (1984). Misconceptions, Preconceptions and Alternative Frameworks. *Chemical Society Reviews, 13*(2), 199-217.

Finegold, M., & Gorsky, P. (1991). Students' concepts of force as applied to related physical systems: A search for consistency. *International Journal of Science Education, 13*(1), 97-113.

Fletcher, P. R. (1997). *How Students Learn Quantum Mechanics.* Unpublished master thesis, University of Sydney, Sydney.

Fletcher, P. R. (2004). *How Tertiary Level Physics Students Learn and Conceptualise Quantum Mechanics.* Unpublished doctoral thesis, University of Sydney, Sydney.

Gabrielse, G., Hanneke, D., Kinoshita, T., & Odom, B. (2006). New Determination of the Fine Structure Constant from the Electron g Value and QED. *Physical Review Letters, 97*(030802), 1-4.

Gunstone, R. F. (1991). *Constructivism and metacognition: Theoretical issues and classroom studies.* Paper presented at the Research in physics learning: Theoretical issues and empirical studies, University of Kiel, Germany.

Hadzidaki, P., Kalkanis, G., & Stavrou, D. (2000). Quantum mechanics: a systemic component of the modern phyics paradigm. *Physics Education, 35*(6), 386-392.

Hake, R. R. (1998). Interactive-engagement versus traditional methods: a six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*(1), 64-74.

Hammer, D., Elby, A., Scherr, R. E., & Redish, E. F. (2005). Resources, framing, and transfer. In J. Mestre (Ed.), *Transfer of Learning: Research and Perspectives* (pp. 89-120). Greenwich, UK: Information Age Publishing.

Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force Concept Inventory. *The Physics Teacher, 30*(3), 141-158.

Hilgartner, C., & DiRienzi, J. (1995). A Non-Aristotelian View of Quantum Theory. *Physics Essays, 8*(4), 472-505.

Ireson, G. (1999). A multivariate analysis of undergraduate physics students' conceptions of quantum phenomena. *European Journal of Physics, 20*(3).

Kalkanis, G., Hadzidaki, P., & Stavrou, D. (2003). An instructional model for a radical conceptual change towards quantum mechanics concepts. *Science Education, 87*(2), 257-280.

Linder, C. (1992). Is teacher-reflected epistemology a source of conceptual difficlty in physics? *International Journal of Science Education, 14*(1), 111-121.

Linder, C., Fraser, D., & Pang, M. F. (2006). Using a Variation Approach to Enhance Physics Learning in a College Classroom. *The Physics Teacher, 44,* 589-592.

Lising, L., & Elby, A. (2005). The impact of epistemology on learning: A case study from introductory physics. *American Journal of Physics, 73*(4), 372-382.

Maloney, D. P., O'Kuma, T. L., Hieggelke, C. J., & Heuvelen, A. v. (2001). Surveying students' conceptual knowledge of electricity and magnetism. *American Journal of Physics, 69*(7), S12-S23.

Marton, F., & Booth, S. (1997). *Learning and Awareness*: Lawrence Erlbaum Associates.

Mashhadi, A. (1996). *What is the nature of the understanding of the concept of "wave particle duality" among Advanced level Physics student?* Unpublished doctoral thesis, Oxford University, Oxford, United Kingdom.

McCloskey, M. (1983). Naive Theories of Motion. In D. Gentner & A. L. Stevens (Eds.), *Mental Models* (pp. 299-324): Lawrence Erlbaum Associates, Inc.

McDermott, L. C. (1991). Millikan Lecture 1990: What we teach and what is learned -- Closing the gap. *American Journal of Physics, 59*(4), 301-315.

McDermott, L. C., & Redish, E. F. (1999). Resource Letter: PER-1: Physics Education Research. *American Journal of Physics, 67*(9), 755-767.

McKagan, S. B., & Wieman, C. E. (2006, February 14 2006). *Exploring Student Understanding of Energy through the Quantum Mechanics Conceptual Survey.* Paper presented at the Physics Education Research Conference.

Meltzer, D. E., & Manivannan, K. (2002). Transforming the lecture-hall environment: The fully interactive physics lecture. *American Journal of Physics, 70*(6), 639-654.

Müller, R. (2003). *Quantenphysik in der Schule* (Vol. 26): Logos.

Müller, R., & Wiesner, H. (2002). Teaching quantum mechanics on an introductory level. *American Journal of Physics, 70*(3), 200-209.

Oliver, K. W., & Bao, L. (2002). Student Resources in quantum mechanics, or why students need meta-resources, *Physics Education Research Conference*: ARXIV.

Redish, E. F. (2003, July 15-25, 2003). *A Theoretical Framework for Physics Education Research: Modeling Student Thinking.* Paper presented at the International school of physics, "Enrico Fermi" Course CLVI, Varenna, Italy.

Redish, E. F., Bao, L., & Jolly, P. (1997). Student Difficulties with Energy in Quantum Mechanics, *AAPT winter meeting*. Phoenix, Arizona.

Singh, C. (2006). Student difficulties with quantum mechanics formalism, *2006 Physics Education Research Conference*.

Singh, C., Belloni, M., & Christian, W. (2006). Improving students' understanding of quantum mechanics. *Physics Today*, 43-49.

Smith, J. P. I., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: a constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences, 3*(2), 115-163.

Stapp, H. P. (1972). The Copenhagen Interpretation. *American Journal of Physics, 40*(8), 1098-1116.

Strnad, J. (1981). Pitfalls in the teaching of introductory quantum physics. *European Journal of Physics, 2*(4), 250-254.

Styer, D. F. (1996). Common misconceptions regarding quantum mechanics. *American Journal of Physics, 64*(1), 31-34.

Taber, K. S. (2002). Conceptualizing quanta: Illuminating the ground state of student understanding of atomic orbitals. *Chemistry Education: Research and Practice in Europe, 3*(2), 145-158.

Thacker, B. A. (2003). Recent advances in classroom physics. *Reports on Progress in Physics, 66*, 1833-1864.

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– SUBMITTED MANUSCRIPT – REVIEW OF EMPIRICAL STUDIES INTO STUDENTS' DEPICTIONS OF QUANTUM MECHANICS

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This review summarises and compares empirical studies on students' depictions of quantum mechanics. A taxonomy for classifying quantum mechanics education studies is presented, in order to facilitate comparisons between studies. Results are discussed, and suggestions for future research are presented.

Keywords: quantum mechanics education research; inappropriate depictions; student conceptions; alternative conceptions; misconceptions.

1 BACKGROUND AND MOTIVATION FOR THIS REVIEW

1.1 Why quantum mechanics education research?

Research has shown that quantum mechanics education is struggling with many severe problems. These problems range from topics in the introduction of modern physics, such as the wave-nature of matter, to introductory topics such as tunnelling, and more advanced topics such as time-development of quantum systems. Today, physics students typically study quantum mechanics in their second or third year of university studies, and in for example Germany, quantum physics is introduced already at pre-university level.

Despite the importance of quantum mechanics from a scientific point of view, and also the important role it plays in physics education, research in quantum mechanics education has received little attention compared to many other branches of physics. However, the rapidly growing number of studies in quantum education indicates that quantum mechanics education will play a more important role in physics education research in the future.

For a non-physicist, it may be difficult to fully appreciate the role quantum mechanics plays in our society, both in everyday life and scientific research. To help the non-physicist reader, a few examples are presented to give a picture of the importance of quantum mechanics.

Quantum mechanics is still the only theory that has been able to describe and predict nature at the atomic level without contradicting experimental results. In fact, considering numerical precision, quantum mechanics has so far produced the most accurate predictions any scientific theory has made. Quantum mechanics has, since its development during the beginning of the 1900's, not only turned into *the* fundamental theory for small-scale physics, which also plays a crucial role in chemistry, molecular biology and medicine.

Without quantum mechanics, things such as computers, light emitting diodes, magnetic resonance imaging, lasers, compact discs, and many more would not have been developed. The theory is also the basis of nanotechnology, a technology playing an increasingly important role in our society. Apart from practical applications, quantum mechanics has also altered the map of how we perceive nature, challenging what we call space, time, and causality. Quantum mechanics is also the foundation of string theory, physics' best attempt so far for attaining a theory of everything.

In short, quantum mechanics is an important scientific theory of today.

Despite its importance, it is uncommon for a non-physicist to even be aware of the impact quantum mechanics has on our society, let alone any technical details of the theory. Unfortunately, education research shows that this is also the case for many physics students.

The aim of education research in quantum mechanics is to understand and resolve the difficulties associated with teaching and learning quantum mechanics. We hope, with this review, to make a contribution to this.

1.2 Existing reviews

Education research in quantum mechanics has expanded rapidly over the last ten years, with an increased number of research groups investigating an increased span of research questions. The growing interest for education research in quantum mechanics and the associated increasing number of publications in the field creates a demand for a comprehensive summary of research results. Despite this, there are only a few reviews available.

The most comprehensive reviews are, so far:

- Fletcher (1997) provides a quite comprehensive resource list of education research in quantum mechanics, and also some related education research in chemistry. A few representative studies are also described in more detail;
- Müller (2003) has published a summary of education research in quantum mechanics. This review covers the plentiful early German research in great detail, but also a large proportion of later and international research. It is written in German;
- Fletcher (2004) also presents an updated resource list of education research in mechanics education in his doctoral thesis. Unfortunately, the period 1997–2004 is not covered as extensively as the research prior to 1997. Again, some selected studies are presented in detail.

This review contributes to the field in several ways. Firstly, it includes contemporary research, and also contains a number of older resources not included in previous reviews.

Secondly, this review does not only list references, but also presents a summary of research results relevant for the focus of this review. Thirdly, this review presents research in themes, allowing comparison between different studies. It also presents a taxonomy to categorise studies, further facilitating comparing and contrasting.

1.2.1 Scope of this review

This article reviews studies on *students' depictions of quantum mechanics*, since we believe that this is the most useful review focus for education researchers and also highly useful for quantum mechanics teachers.

By choosing this focus, education research into developing teaching methods and aids is left out. We feel that education research in quantum mechanics is still young, and our current focus would better suit the needs of the research field. Readers interested in teaching methods and aids are recommended to read Müller (2003), and also to investigate the reference lists presented by Fletcher (2004).

This review also only covers empirical studies, leaving out a large number of articles where authors share experiences and suggestions for teaching and curriculum design. Nor are historical descriptions of quantum mechanics covered. We feel that though such articles are important for the community of quantum mechanics teachers, education research must build on an empirical basis. A list of articles with teaching suggestions and presenting the historical development of quantum mechanics can be found in Fletcher (2004).

Due to language barriers, parts of the research published in German have not been included in this review. An extensive overview here is given by Müller (2003).

In doing this review, we have striven to collect research from many different sources. The main methods for finding articles have been examining reference lists, complemented by examining online publication lists of researchers. The main method of tracking down articles has been Google Scholar™, complemented by other library search resources and by contacting authors.

In order to make the review as comprehensive as possible, not only published articles are included, but also theses, conference presentations, proceedings, and some unpublished monographs.

1.3 Terms used in this review

When presenting research, we use a number of terms to describe the studies and the environment in which they were made. The purpose of using special terms is to make it easier to compare and contrast different studies.

We describe, as far as possible, the sizes of the studies, the locations of the studies, what year of university studies the participants come from, and how advanced the taught quantum mechanics was. In some cases, the population data has had to be deduced from the context. For example, in the cases where it is unclear at which university the study has been conducted, we use the location of the study by the country where the researcher is active.

1.3.1 Student depictions

The aim of this review is to summarise various problems in how students describe or apply quantum mechanics. In particular, the aim is to describe students' inappropriate depictions, in order to find suitable areas for future research in quantum mechanics education.

Throughout this review, we will consistently use the term student *depictions* when referring to how students describe quantum mechanics phenomena. The reason for choosing this word is *not* to employ any particular framework, and *not* to make any claims on how students' learning could or should be described. What is presented is how students depict different aspects of quantum mechanics, not how they think or learn.

We will use the term *inaccurate depiction* when a depiction is inconsistent with quantum mechanics or, where appropriate, the Copenhagen interpretation of quantum mechanics. In contrast, we will use the term *accepted depiction* when referring to a depiction that is consistent with quantum mechanics and, where appropriate, the Copenhagen interpretation of it.

1.3.2 Sizes of studies

This review uses three different categories to decribe the relative sizes of the studies: *small-scale studies* (30–99 participants), *mid-scale studies* (100–499 participants), and *large-scale studies* (500 participants or more).

Small-scale studies are reported in absolute numbers, whereas results from mid and large-scale studies usually are reported in percentage to facilitate comparison and contrast. Some small-scale studies are originally reported in percentage – in these cases absolute numbers have been calculated. Studies with fewer than 30 participants are only reported with single or multiple examples without any further quantitative reporting.

Only one large-scale study was encountered when doing this literature review. The reason, however, for still defining this category is to help stress the fact that large-scale studies are extremely rare.

1.3.3 Year of university studies

The participants in the reviewed studies are described as pre-university students, through their year of university studies, or as post-graduates. Students who are at the end of a given year of study are, where appropriate, categorised in the next level.

1.3.4 The quantum mechanics level

This review uses the collective categories *pre-quantum*, *introductory quantum*, *intermediate quantum*, and *advanced quantum* education. These categories serve as a tool to distinguish different levels of quantum mechanics education, for example, quantum mechanics as emission spectrum of hydrogen – which is fairly basic – and quantum mechanics as calculating the time-dependence of wave function – which is fairly advanced. The contents of the categories draw somewhat on the research done on quantum mechanics curricula (for example Cataloglu, 2002; Fletcher, 1997), but are largely put together by logical and experimental convenience. The categories are characterised as below.

- **Pre-quantum education**. Here, the education does not deal with quantum mechanics in an in-depth mathematical way. This category does not, for example, include solving the Schrödinger equation. Pre-quantum education could, for example, contain the wave nature of matter, basic properties of hydrogen atoms, atom emission and absorption, introduction of uncertainty principle, de Broglie wavelength, and some historical development of quantum mechanics.

- **Introductory quantum education**. Here, the education deals with basic quantum mechanics theory, also from a technical point of view. Introductory quantum education includes fundamentals such as the introduction of wave functions, the Schrödinger equation, time-independent solutions to some basic potentials in one dimension, tunnelling, derivation of energy quantisation, measurements, and time-evolution of simple systems. Angular momentum, orbitals and formal description of hydrogenic atoms are also included in this category.
- **Intermediate quantum education**. Here, the education has a focus that relies on both introductory quantum topics and the use of some notably advanced mathematics. Intermediate quantum education typically include spin, matrix representation, Dirac formalism, introduction to identical particles, creation and annihilation operators, the harmonic oscillator potential, energy eigenstates for some multi-dimensional potentials other than a Coulomb potential.
- **Advanced quantum education**. Here, the education deals with elaborations on topics from the intermediate level. Advanced quantum education could include quantum entanglement, time-dependent Hamilton operators, the Einstein-Podolsky-Rosen paradox, density operators, mixed states, scattering theory in multiple dimensions, and time-dependent perturbation theory.

Naturally, courses could span over more than one of these categories, especially in the case of introductory and intermediate quantum education. As already pointed out, this list is a pragmatic tool for categorising research, and should be treated as such. In particular, this is a not way of making a case for in which order quantum concepts should or could be introduced.

The labels pre to advanced quantum education can be seen as chosen more or less arbitrarily. We do not wish to define what appropriate quantum mechanics courses should include, nor to define how advanced or basic certain topics are. Certainly, beyond the advanced level there are a number of quantum mechanics topics, such as relativistic quantum mechanics, quantum field theory, quantum information theory, quantum electrodynamics, and chomodynamics.

1.4 Concerning quantum mechanics interpretations

The fact that there is not one, but several different interpretations of quantum mechanics, can make it difficult to evaluate students' depictions of quantum mechanics. This is particularly problematic when discussing students' descriptions of particles, which are delocalised in some interpretations, but always localised in others.

This review follows the Copenhagen interpretation of quantum mechanics, unless otherwise stated. This choice is not unproblematic, since there are no studies that convincingly show that the Copenhagen interpretation is the most prominently used in teaching.

The reason for choosing the Copenhagen interpretation in this review is that all researchers explicitly mentioning interpretation claim that the Copenhagen interpretation is employed in education (for example Singh, 2001). This is also backed up by some investigations on what interpretations are used in quantum mechanics textbooks (Mashhadi, 1994) and reports elsewhere (Stapp, 1972). In the research reviewed, there is no evidence that any other interpretations than the Copenhagen has been used in education, other in the intention of providing alternative teaching methods (for example Budde, Niedderer, Scott, & Leach, 2002).

1.5 Order of presentation

This introduction is followed by a presentation on general results from the review, along with an historical outline of quantum mechanics education research. Part 3 is the main part of this review, presenting research into students' depictions of quantum mechanics. Finally, we discuss the results of this review and make suggestions for future research.

We have chosen to structure this review by ordering research in terms of quantum mechanics topics. As a consequence, some articles are referred to in several places.

2 GENERAL RESULTS AND HISTORICAL BACKGROUND

2.1 General results

The most general and probably most obvious result in quantum mechanics education research is that many students struggle with the subject. This observation is underpinned by the following factors:

- qualitative studies show that many students have considerable problems to depicting a vast array of quantum mechanical topics in an accepted way. Indeed, according to the education research that has taken place so far, there is little reason to believe that there are *any* quantum mechanical topics that do not present considerable learning problems to students – with the possible exception of energy levels;
- quantitative studies of inappropriate depictions generally show that 20–50 % of the students have problems on the topics that have been chosen for investigation;
- students display so-called fragmented depictions of quantum mechanics – meaning an inability to use consistent depictions and a trend to change depictions depending on context rather than the physics involved. One example of this is students claiming that wave function amplitudes are related to energy in one context, but to probability in another (Bao, 1999);
- students seem to have problems relating quantum mechanics to the real world, or recognising the significance of quantum mechanics.[1]

2.1.1 Students' attitudes towards quantum mechanics

Considering students' learning difficulties, it is not surprising that quantum education researchers portray students as experiencing quantum mechanics as particularly difficult. However, the literature research uncovered only a few examples of student attitudes towards quantum mechanics, and only one study actually dealt explicitly with this

[1] One vivid example of this is a multiple-group study at the University of Sydney, where 48 students were asked to "name three things which quantum mechanics has given us" (Fletcher, 2004, p. 122). Although the students also were encouraged to name more than three items, the students managed on average 2.2 items. Even post-graduate students managed only 2.5 items on average. The students were themselves surprised and upset that they were unable to mention more items (Fletcher, 2004).

question. 236 pre-university, pre-quantum mechanics students all over Norway participated in this study, and the results showed that the attitude towards quantum physics is more or less the same as towards other topics in physics – with a tendency to rate quantum physics as more difficult, boring and unnecessary. There were only small and non-significant differences between male and female students (Olsen, 1999).

Apart from the plentiful research showing students' problem in understanding, there are also studies showing that student understanding in quantum mechanics *does* increase with the number of courses taken (Cataloglu & Robinett, 2002) and that how students interpret phenomena related to quantum mechanics changes as they study quantum mechanics (Ireson, 2000; Mashhadi, 1996).

2.2 Historical background

Empirical education research concerning quantum mechanics appears to have begun in the early eighties, where some research was carried out in Italy (Cervellati & Perugini, 1981) and the United Kingdom (Lawless, 1982). Full-blown research projects in quantum education started to emerge in the mid eighties at Frankfurt University, Germany (for example Göritz & Wiesner, 1984, in Müller, 2003). It was not long before such efforts spread to the University of Bremen and the University of Berlin (for example Bethge, 1988; Fischler & Lichtfeldt, 1991). This early research mostly focused on students' depictions of atoms.

In the 1990's, quantum education research was taken up in Spain (Gil & Solbes, 1993), more so in the United Kingdom (for example Mashhadi, 1994), the United States (for example Redish, Bao, & Jolly, 1997; Steinberg, Oberem, & McDermott, 1996), Australia (Fletcher, 1997), and in Norway (Olsen, 1999). In this period, studies on students' depictions of the wave-particle duality took on noteworthy focus.

More recently, quantum education research has been conducted in Turkey (Nakiboglu & Benlikaya, 2001), Finland (Mannila, Koponen, & Niskanen, 2002), Greece (Kalkanis, Hadzidaki, & Stavrou, 2003), Sweden (Falk, 2004), Taiwan (Ke, Monk, & Duschl, 2005), the Netherlands (Koopman & Ellermeijer, 2005), and Thailand (Wuttiprom, Chitaree, Soankwan, Sharma, & Johnston, 2006). Since the previous century, student depictions of more mathematical aspects of quantum mechanics, such as time development and tunnelling, have also been investigated by education researchers.

Today, most of the education research in quantum mechanics is being carried out in the United States (for example Singh, Wittmann, McKagan and Ambrose) and Germany (for example Niedderer and Fischler), but also in Australia (for example Johnston).

3 STUDENTS' DEPICTIONS OF QUANTUM MECHANICS

In presenting students' depictions of quantum mechanics, I follow the categories of pre-quantum to advanced quantum education, as described earlier. Each category is divided into topics, and some topics are also divided into sub-topics.

Every topic starts off with a brief summary of an accepted depiction of the relevant aspects of the topic. In most cases, this depiction relies on the Copenhagen interpretation, but, where possible and appropriate, the accepted depiction is independent of interpretation. After the accepted depiction, students' depictions of the topic are presented.

Since the aim of this review is to summarise and compare research into students' depictions of quantum mechanics, the presentation is structured around these depictions. In three topics – atoms, square potentials, and time-dependence – research has allowed using inappropriate depictions as sub-headers of the topics. In some other topics, the presentation of depictions is ordered in sub-topics. In topics where there is still little research on students' depictions, the research findings are not divided into sub-headers at all.

The outline of the presentation of students' depictions may be summarised as follows:

1) level of quantum mechanics (for example introductory quantum education);
    a. a topic (for example atoms or quantum measurements);
        i. an accepted depiction of the topic;
        ii. inappropriate depictions (for example classical orbits), or depictions ordered in sub-topics of the topic (for example measurements' effect on the wave function).

We have chosen to introduce many of the topics with a quotation, either from researchers or from students that they have interviewed. The intention is to, when possible, provide a vivid insight to researchers' conclusions and students' main difficulties in the particular topic.

Part 3 ends with a presentation of research made into developing instruments for investigating students' conceptions of quantum mechanics, to be further discussed in part 4.

Pre-quantum level

Research carried out in the pre-quantum level mainly focuses on the topics atoms and orbitals, the wave-nature of matter and the uncertainty relation. Students' depictions of atoms and orbitals are the dominating topic.

3.1 Atoms and orbitals

> *"Karl: I used to think of stability in such a way that Coulomb force and centrifugal force are exactly equal and balance each other. ... Of course, that doesn't fit at all to quantum mechanics.*
>
> *Interviewer: Do you see any connection between your ideas about probabilities we just discussed, and the stability of atoms?*
>
> *Karl: No, I don't. For me there is, on the one hand, this idea of stability and, on the other hand, the matter of probability. I completely keep them apart. For me these are two different things. ... I have two completely different ideas. With respect to stability, I should say, it moves on an orbit. In the other case I would speak of probability of finding."*
>
> *(Bethge & Niedderer, 1996, p. 13)*
>
> *[Quotation illustrating a student using an inappropriate classical orbit depiction when discussing atomic stability.]*

3.1.1 An accepted orbital depiction

In this literature review, orbitals are considered to be the mathematical solutions to the Schrödinger equation that are associated with an atom or, in some cases, with a complex of atoms. The orbitals, or linear combinations of orbitals, are used for describing the probability distribution for an electron's location.

In the simplest case, orbitals are the solutions to the Schrödinger equation for a single-electron atom where the nucleus is approximated with a point charge. These solutions are then separated according to different angular momenta, and the projection of the angular momenta onto the z-axis.

In more complex and realistic descriptions, orbitals can be adjusted according to one or several of the following factors:

- repulsive electric forces between electrons bound to the same nucleus;
- a non point-like nucleus;
- electric forces from other nuclei in a molecule, sometimes resulting in molecular orbitals;
- external potential fields, for example due to other molecules or magnetic fields.

It is worth pointing out that in this depiction, orbitals are not the same thing as electrons bound to nuclei. Orbitals are a way of mathematically expanding the wave functions associated with these electrons. In particular, one difference is that in a multi-electron system, an electron must be in an symmetric anti-symmetric superposition of several orbitals.

In the Copenhagen interpretation of quantum mechanics, it makes no sense to say that an electron is located anywhere before a position measurement is carried out. Nor is it sensible to talk about a trajectory of an electron, as a particle does not have an absolute position and momentum at the same time. The wave functions associated with electrons merely give, when their amplitudes are squared, the distribution of outcomes of position measurements on a large number[2] of identically prepared electrons.

3.1.2 Inappropriate depiction: the classical orbit depiction

> *"It is remarkable that even if quantum mechanical ideas are mentioned, Bohr's model is almost always used as the starting point of the discussion. Because most of the students interviewed for this study had quantum physics courses in high school as well as in the university, it is legitimate to say that the Bohr model is a very dominant and stable conception."*
>
> *(Rainer Müller & Wiesner, 2002, p. 201)*

Student understanding of atoms, and attempts to improve this understanding, is the most prominent topic reported in quantum education research. Within this topic, the most salient finding is that many students use a classical atomic model, where the fictitious centrifugal forces and electrostatic attractive forces are balanced. These depictions are often very similar to the Bohr model of the atom, but do not necessarily stem directly from the Bohr model (see Mashhadi, 1994 for a brief discussion on this). In some studies,

[2] A "large number" should be interpreted as a number tending to infinity.

students explicitly compare atoms to a planetary system, but it is uncertain whether this is a trait of student depictions or if it is a consequence of the way questions are asked and the research reported. We will collectively call the depiction of electrons as classical particles orbiting the nucleus the *classical orbits* depiction.

A number of mid or small-size investigations of students' depictions of atoms have been carried out, allowing an estimation of how common the classical orbits depiction of the atom is amongst students: approximately 50 % of 142 German pre-university students (Bethge, 1992; Bethge & Niedderer, 1996); 20 % of 167 chemistry teacher students at Balekesir University in Turkey (Nakiboglu, 2003); 16 % of 199 French second-year students (Cros & Chastrette, 1988); 28 % of 302 first-year, probably Canadian students (MacKinnon, 1999), approximately 25 % of 57 British pre-university students (Mashhadi, 1994, 1995); six of 37 German pre-university students (Rainer Müller & Wiesner, 1999); and four to ten of 30 pre-university Canadian students (Griffiths & Preston, 1992) use the classical orbit depiction. Also, approximately 23 % of 200 Italian pre-university students describe a trajectory or path when asked to define what an orbital is (Cervellati & Perugini, 1981). In the studies mentioned, all students had been taught the orbital models of atoms to at least pre-quantum level.

In a German pre and post study, 63 % of 270 pre-university students used the classical orbits depiction prior to a course dealing with quantum mechanical orbital theory. In the group given a modified course, designed to deal with the classical orbit depiction, 22 % of the students still used the classical orbits depiction five weeks after the pre-quantum course. The corresponding ratio among the 120 students given the non-modified course was 60 % (Fischler & Lichtfeldt, 1992).

In a Norwegian study of 236 pre-university pre-quantum mechanics students, 19 % chose multiple-choice answers close to the classical orbit depiction. In another question in the same survey study, students were asked to comment on the statement "the structure of an atom is similar to the solar system" (translated from Norwegian). In the answers, 20 % mentioned fundamental differences; such as electrons do not have any defined orbits, and only probabilities for different locations. Another 29 % made an analogy accompanied by some simple differences; such as planets have elliptical orbits, name of forces are different, and electrons can 'jump' between orbits. A further 41 % of the students made a simple analogy, without mentioning differences (Olsen, 1999, 2001).

There are also a number of smaller descriptive studies concerning the classical orbit depiction of the atom (Euler, Hanselmann, & Müller, 1999; Taber, 2005), and more than one study reports that students' classical orbit depiction appears to be particularly robust to change (Bethge & Niedderer, 1996; Rainer Müller & Wiesner, 1999, 2002; Petri & Niedderer, 1998).

3.1.3 Inappropriate depiction: only certain orbits allowed

One semi-classical view of atoms held by some pre-university and university students is that only certain electron orbits are allowed (Bethge, 1992; Bethge & Niedderer, 1996; Rainer Müller & Wiesner, 1999; Taber, 2002b). This could be interpreted as an extension of the classical orbit depiction, combined with the notion of energy levels, but it is also similar to some pre-university chemistry students' depictions of atoms where electrons are fixed or may move around freely only on particular surfaces (Taber, 2002a, 2002b). There are also multiple examples of pre-university and university chemistry students using the terms orbitals and (electron) shells interchangeably (MacKinnon, 1999; Taber, 2002b).

A related inappropriate student depiction is the "smeared orbit", where the classical orbits are seen as the most probable path for electrons. Some findings suggest that the smeared orbit depiction is an intermediate step for students when moving towards the quantum mechanical depiction of the atom (Petri & Niedderer, 1998).

3.1.4 Inappropriate depiction: the bounded-volume orbital

Another cluster of inappropriate depictions seems to be related to students confusing orbitals with the probability envelope sometimes used for illustrating orbitals, most often a sphere or hourglass shape. Some pre-university students in the United Kingdom depict the electron as located inside this finite, bounded-volume orbital (Taber, 2002b). A mid-sized study of fourth-year chemistry students at University of Ioannina, Greece, showed that 19 % of the students inappropriately depict the orbital as a sub-part of space where there is typically a 90 % probability of finding an electron (Tsaparlis, 1997), resulting in the shapes described above. The study built on 506 written exams, but only included the 212 with a passing grade.

The depiction of an orbital as a finite and bounded volume is inconsistent with the Copenhagen interpretation in at least two important aspects: firstly, orbitals are not shapes describing a finite, bounded volume, but a complex-valued field stretching through entire space; secondly, electrons are not "located inside" the orbitals – the orbitals give a distribution of position measurement outcomes.

3.1.5 Inappropriate depiction: movement of electrons

Some research also reports on pre-university students inappropriately depicting electrons as moving about inside an atom (Budde et al., 2002). It appears that even students who accept and use a probabilistic description of the location of electrons may have difficulty not thinking in terms of trajectories (Bethge, 1992).

This depiction is inappropriate in the aspect that electrons are not localised, do not have trajectories, and do not move in the macroscopic sense of the word, according to the Copenhagen interpretation.

3.1.6 Inappropriate depiction: the atomic cell

There are some examples of pre-university students claiming that atoms are alive and even capable of growing and dividing (Griffiths & Preston, 1992; Harrison & Treagust, 1996). This inappropriate depiction has not been encountered in research on students in quantum courses, and is not comment further on in this review.

3.2 The wave-nature of matter

> *"The preliminary results of the study indicate that students have incorporated the 'new' quantum phenomena into the 'older' mechanistic conceptions."*
>
> *(Mashhadi, 1994, p. 324)*

> *"What is most striking is that the 'wave vs particle' view is persistent over time, despite the increasing exposure to quantum mechanics and self-selection toward becoming physics majors. There is no indication from the data that this way of thinking about quantum entities is a stepping stone, or progression towards another more sophisticated viewpoint. In other words, half the students retain the view that they first had in second year. When presented with a problem they choose, as a starting point, to think about either a simple wave or a simple particle."*
>
> *(Fletcher, 2004, p. 112)*

3.2.1 An accepted depiction of the wave-particle duality

It could be argued that the wave-particle duality is not an actual part of quantum mechanics, since both waves and classical particles are replaced by the quantum wave function. Following this line of reasoning, the wave-particle duality is a paradox emanating from classical physics, where, for example, electrons and photons sometimes acts as if they were classical particles – for example, being localised and having mass – and sometimes acts as waves – for example, being delocalised and causing interference patterns.

In the Copenhagen interpretation of quantum mechanics, the wave-particle duality may be described through a discontinuous change of the wave function caused by a measurement of position: the measurement changes the wave function into a highly localised distribution function, making the particle temporarily exhibit particle-like properties.

One may also argue that one important aspect of the wave-particle duality is that the physical models of waves and classical particles are no longer appropriate to describe quantum phenomena, and that quantum mechanics presents a new physical model.

In particular the following holds for the Copenhagen interpretation: quantum mechanics poses a unified, non-contradicting description of how particles sometimes exhibit wave properties, sometimes particle properties; particles do not have a particle part and a wave part; and the wave-nature of matter is not a matter of classical particles oscillating.

In the sections below, the term "particle" has, when appropriate, been changed to "classical particle", in order to distinguish from particles as described by quantum mechanics. Similarly, the term "quantum particle" is used to avoid confusion.

3.2.2 Sub-topic: depictions of waves and particles

A few studies have investigated how students describe characteristics of waves and classical particles. An Australian study on 33 third-year introductory quantum mechanics students showed that one common and inappropriate depiction was that one property telling particles from waves was that particles have momentum – while this property also may be exhibited by waves. The study also showed that students tended to use mathematical descriptions to characterise waves, such as Fourier expansions (Fletcher, 1997; Johnston, Crawford, & Fletcher, 1998). In a Norwegian study on 236 pre-university pre-quantum mechanics students, participants often appropriately described classical particles as being massive when asked to describe differences between waves and particles (Olsen, 1999).

In the Norwegian study, many students also depicted photons as a double-nature phenomenon, being both wave and classical particle, while electrons were depicted as classical particles only (Olsen, 2002). This was despite the Norwegian national curriculum's intention to use the similar behaviour of electrons and particles as a way of introducing wave-particle duality and modern physics. The finding is similar to a British study on 57 pre-university pre-quantum mechanics students. Here, most students described electrons as a "tiny ball", while photons also were described in terms of "energy packets", for example (Mashhadi, 1994; Mashhadi & Woolnough, 1999). Yet another study, on US pre-university pre-quantum mechanics students, showed that electrons were typically depicted as classical particles (Thacker, 2003). In contrast, a Finnish study found that a classical depiction of both electrons and photons was dominant in a group of physics students and teacher students (Mannila et al., 2002).

The depiction of particles as either classical particles or waves is inconsistent with the Copenhagen interpretation on the basis of neglecting the wave function as a consistent model for describing quantum particle behaviour – for photons as well as electrons.

3.2.3 Sub-topic: depictions of wavelength of electrons

In the Norwegian study described in 3.2.2, students displayed great difficulty when asked to explain what is meant by the wavelength of an electron (Olsen, 1999). In quantum mechanics, the wavelength of an electron could for example be related to interference phenomena.

An investigation in the US that involved over 450 students from first to third year at university, and from pre-quantum to introductory quantum level, showed that many students did not relate the de Broglie wavelength of an electron to its speed (Vokos, Shaffer, Ambrose, & McDermott, 2000). This is inconsistent with quantum mechanics, since the de Broglie wavelength of an object is inversely proportional to its momentum and hence with its speed, if mass is constant.

3.2.4 Sub-topic: depictions of wave-nature of matter

Students' depictions of the wave-nature of matter appear to be different from the accepted interpretation in many cases. In the Norwegian study described in 3.2.2 (Olsen, 1999), some students held that electrons and photons have one wave part and one classical-particle part, while other held that the experiment carried out determines whether an electron or photon is a particle or a wave. Both these depictions are inconsistent with the Copenhagen interpretation by neglecting the wave function as a coherent description for quantum particles. The study also showed very few students discussing the concept of physics models when commenting on the wave-nature of matter.

In an Australian interview study of 50 students, 27 students held that the nature of the object depends on the situation – for example, what measurements are carried out. The interviewed students were from second-year level up to post-graduate level, and this depiction was represented in all groups (Fletcher, 2004).

One noteworthy misinterpretation of the wave-particle duality is that classical particles move in wave-shaped trajectories (Ambrose, Shaffer, Steinberg, & McDermott, 1999; Falk, 2004; Fletcher & Johnston, 1999; Koopman & Ellermeijer, 2005; Rebello & Zollman, 1999). Two investigations of 236 Norwegian pre-university, pre-quantum mechanics students (Olsen, 1999) and 189 probably second-year students at the University of Sydney (Fletcher, 1997) both showed that six to eight percent of students depict the wave-nature of matter in this way. In the only large-scale study covered by this literature review, 17 % of 550 German pre-university pre-quantum mechanics students depicted the photon as moving in a wave-shaped trajectory (Rainer Müller & Wiesner, 2002).

Mid-size studies on British pre-university, first-year, and second-year students showed that students' depictions of the wave-particle duality may be related to their view of physics knowledge. For example, how students depict the wave-particle duality may be related to if they depict electrons as something that actually exists, or merely a physical model for describing certain phenomena (Ireson, 1999a, 1999b; Mashhadi, 1996; Mashhadi & Woolnough, 1997).

3.3 Uncertainty and indeterminacy

3.3.1 An accepted model of uncertainty

In the Copenhagen interpretation of quantum mechanics, when considering two non-commutable measurable quantities, uncertainty is a fundamental property of any ensemble of particles. For example, an ensemble of identically prepared electrons will have certain distributions of momentum and position measurement outcomes. These distributions may be calculated from the wave function of the electrons, and the standard deviations of momentum and position will obey the Heisenberg uncertainty relation.

The uncertainty can, in some aspects, be better described as indeterminacy in the Copenhagen interpretation: even if all information about a particle would be known, the outcome of a measurement may still be unknown.

In particular, the Copenhagen interpretation does not contain any "hidden variables" that tells the "actual" values of for example position or momentum before them being measured; the uncertainty gives standard deviations, and not any maximum deviation; and the uncertainty applies to all objects, regardless of size.

3.3.2 Depictions of uncertainty

The Australian and Norwegian studies described in 3.2.2 indicate that a noteworthy portion of students depict uncertainty as uncertainty in classical measurements: that is, an error or deviation from a "correct" value (Fletcher, 1997; Johnston et al., 1998; Olsen, 1999). Similar results were also found in a study on third-year introductory quantum mechanics students at Ohio State University (Sadaghiani, 2005), a German pre-university, pre-quantum study (Petri & Niedderer, 1998), and also a German study on 37 pre-university mostly pre-quantum physics teacher students (Rainer Müller & Wiesner, 2002). The latter study revealed some different ways the students described uncertainty: classical measurement error, a span in which the "correct" measurement value is located, and also the somewhat more accepted depiction that one measurement disturbs other properties. Five students also adequately described uncertainty as the statistical distribution of measurement outcomes on identical systems (Müller & Wiesner, 1999).

An investigation on 137 examination scripts from Australian first- and second-year probably pre-quantum mechanics students revealed that 20 % of the students inappropriately depict uncertainty as not applicable at a macro level (Fletcher, 2004).

Introductory quantum level

Education research in introductory quantum education has so far focused on the topics one-dimensional potentials, quantum measurements, energy quantisation, and time-development of quantum mechanical systems. Tunnelling – and other aspects of one-dimensional potentials – is the dominating topic.

3.4 One-dimensional square potentials

> *"[students'] explanation of tunnelling [...] is patchy and expressed in terms of a particle model rather than a wave function or probability model. Their proficiency with the tools [in problem-solving] hides their lack of understanding of the physical situation."*
>
> *(Fletcher, 2004, p. 73)*

Student depictions of one-dimensional square potentials are the second most investigated topic in quantum mechanics education research, surpassed only by atoms. In this review, one-dimensional square potential barriers comprise quantum mechanical systems where the potential is piecewise constant. This includes potential wells, potential steps, and potential barriers. In this review, this topic also contains some research into students' depictions of one-dimensional potentials with constant slope – even though this is not a square potential system[3].

The most research presented in this section refers to student depictions of tunnelling.

3.4.1 An accepted depiction of one-dimensional square potentials

Since one-dimensional square potential is a quite diverse topic, it is difficult to give a depiction summarising all possible aspects. Instead, this section focuses the aspects relevant to the presented research.

A potential energy diagram describes the local potential energy for a particle, describing what potential energy a particle localised at a specified point would have. Based on the potential energy diagram, it is possible to solve the time-independent Schrödinger equation, resulting in a wave function piecewise built up by sums of exponentials (with complex arguments in the general case). One consequence is a wave function that may be non-zero in regions where the particles' total energy is less than the local potential energy – "classically forbidden regions" – and also "on the transmission side" of such regions, which results in a phenomenon called tunnelling.

One common special case in quantum mechanics is particles with a definite total energy. This results in wave functions with amplitudes constant in time, which implies that the distribution of measurement outcomes for large ensembles of identically prepared particles is also constant in time. In particular: such a particle has a definite total energy; a particle may theoretically have a definite energy without contradicting Heisenberg's uncertainty principle; a particle with less energy than a potential energy barrier may tunnel through it; and tunnelling does not cause a particle to lose energy.

Another relevant case is the rather localised wave function, where a probability peak describes the most frequent position measurement outcome. Such particles do not have a definite total energy, but their wave functions may be decomposed into a combination of wave functions with definite energies. The localised wave function – sometimes called a wave-packet – behaves as the sum of its monoenergetic components. The position of the probability peak will in general not be constant in time. In particular, a localised wave

[3] These potentials are included in the introductory quantum category, since they are less complex than, for example, the one-dimensional harmonic oscillator potential.

function colliding with a potential energy barrier will, in the general case, split into one transmitted and one reflected probability peak.

The distribution of position measurement outcomes for a large ensemble of identically prepared systems is, as pointed out earlier, described by the square of the amplitude of their wave functions. Thus, it is not obvious that the most particles will be detected at the point where the local potential energy is lowest – in fact, the opposite will be true in the semi-classical limit. It may also be noted that the local wavelengths of a wave function are inversely proportional to the local kinetic energy[4], while the amplitude of the wave function is related to distribution of position measurement outcomes.

3.4.2 Inappropriate depiction: energy loss

Of the sub-topics to student depictions of one-dimensional square potentials, depictions of tunnelling have been the subject of the most investigations.

The most commonly reported inappropriate depiction within tunnelling is what has been called the *energy loss* depiction: that a particle loses energy due to tunnelling (while quantum mechanics predicts a constant total energy). The energy loss depiction has been explored in a number of studies (Ambrose, 1999; Bao, 1999; McKagan & Wieman, 2006; Morgan, Wittmann, & Thompson, 2003; Wittmann, 2003; Wittmann, Morgan, & Bao, 2005), and appears to be supported by two inappropriate arguments: either a classical argument that "the particles expend kinetic energy 'breaking [through]' the barrier…" (Ambrose, 1999, p. 129, student quote); or a confusion between energy and wave functions or probability densities (Muller & Sharma, 2006). The latter may be caused by diagrams in textbooks and teaching where potential energy and wave function or probability density are plotted on the same axis (Domert, 2005; Domert, Linder, & Ingerman, 2005; Morgan et al., 2003).

The energy loss depiction has been the subject of a few small and mid-sized studies, exploring how common the depiction is among introductory quantum mechanics students: 40 % of 105 second-year students at Uppsala University, Sweden (Falk, 2004); 17 out of 80 first-year students at Lund University, Sweden (Falk, 2004); 39 of 64 second-year students at the University of Sydney (Muller & Sharma, 2006); and 58 % of 132 students at University of Colorado (McKagan & Wieman, 2006). The energy loss depiction appears to be quite resistant to change (Wittmann et al., 2005).

3.4.3 Inappropriate depiction: axis shift

Wittman and colleagues also report that the energy loss depiction is highly related to a particular and inappropriate way of drawing diagrams of the tunnelling process (Morgan et al., 2003; Redish, Wittmann, & Steinberg, 2000; Wittmann, 2003; Wittmann et al., 2005). This has been labelled *axis shift*. A typical example of an axis shift diagram is shown in the figure below: wave function and potential energy are plotted in the same diagram, and the axis around which the wave function oscillates is shifted through the potential energy barrier. This shift is typically depicted by students as the energy loss of the particle.

[4] With this somewhat unorthodox term, I refer to the difference between total energy and potential energy at any given point.

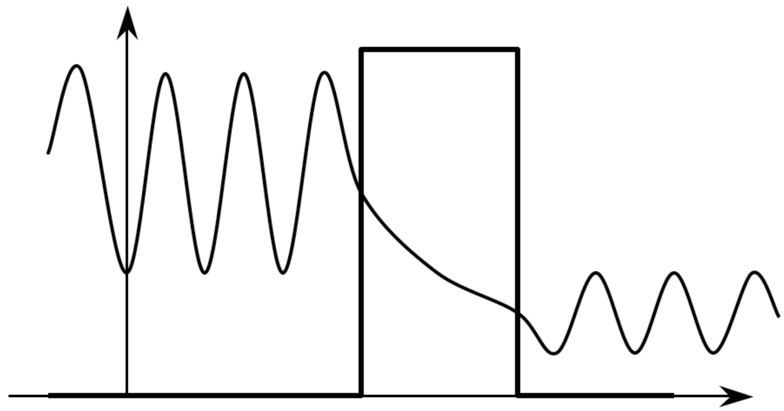

Figure 1. Sketch of tunnelling diagram with the typical axis shift properties: the axis around which the wavefunction oscillates is shifted when passing through the potential energy barrier. This shift is typically, and inappropriately, depicted as the energy loss of the particle.

The axis shift also appears in illustrations, but is not commented on, in a thesis presented in 1999 (Bao, 1999).

The axis shift depiction is not directly inconsistent with quantum mechanics, since it concerns presentation of quantum mechanics rather than quantum mechanics itself. Still, it could be argued to be an inappropriate depiction, since it mixes wave function and potential energy on the same axis, and also depicts the wave function as oscillating around a shifting and non-zero axis.

3.4.4 Inappropriate depiction: low potential energy means high probability

Some studies have reported on students inappropriately reasoning that a particle will most likely be found at the place where the potential energy is lowest (Ambrose, 1999). This is typically motivated through "if the potential is higher, the particle has a harder time being in that spot" (Ambrose, 1999, p. 140). This inappropriate depiction appears both when discussing sloping potentials and square potentials with one higher and one lower potential energy section.

This depiction is inconsistent with quantum mechanics, since at least in the semi-classical limit, the probability of detecting a particle is greater where the potential energy is higher, but still below the classical limit.

3.4.5 Inappropriate depiction: range of energies

> *"S: What I meant by 'the energy […]', was that each electron has slightly different energies. Therefore the energy of an electron could be represented by an energy distribution similar to – or identically – psi. […] The proportion that did make it [across the boundary] would be equal to the probability of an electron to have that high of energy."*
>
> *(Ambrose, 1999, p. 147, student quote)*
>
> *[Quotation illustrating a student using an inappropriate range-of-energy depiction when discussing transmission and reflection against a potential energy barrier.]*

Some students inappropriately depict transmission and reflection at one-dimensional barriers or potential energy steps as being caused by particles having a range of energies, so that only the particles with sufficient energy will be transmitted (Ambrose, 1999; Domert, 2006; Domert et al., 2005). This appears to be promoted by inappropriately using Heisenberg's uncertainty principle to motivate that all sets of particles have a range of energies, while quantum mechanics theoretically allows definite energies. There is also an example of students misinterpreting diagrams to suggest that particles have a range of energies (McKagan & Wieman, 2006).

Studies showed that this depiction was held by "nearly half" of 53 second-year students at University of Washington (Ambrose, 1999), 15–19 % of 185 first to second-year university students at two Swedish universities (Falk, 2004), and 27–30 % of 132 second-year students at the University of Colorado (McKagan & Wieman, 2006). Interviews show that students may retain this depiction, even when faced with contradictions (McKagan & Wieman, 2006). All the above studies were made at the introductory level of quantum mechanics.

A somewhat related depiction is reported by Falk (2004) and Domert et al. (2005): that the part of a particle with "enough energy" will be transmitted, the remaining will be reflected. The difference from the depiction above is the range of energies explicitly refers to one single particle, not only an ensemble of particles.

Both depictions are inconsistent with quantum mechanics, for example in the aspect that they predict that the width of a potential energy barrier does not affect the transmission probability.

3.4.6 Inappropriate depiction: probability peak as an entity

> *"If it turns into two wave packets […] I would interpret it as getting two particles."*
>
> *"Either it gets through or it doesn't, only with different probabilities. It is still a particle, described by a bunch of wave functions."*
>
> *(Falk, 2004, p. 45, two separate student quotes)*
>
> *[Quotation illustrating two students using an inappropriate probability-peak-as-an-entity depiction when discussing reflection and transmission against a potential energy barrier.]*

Another inappropriate depiction is that some students depict a probability peak as being transmitted or reflected as an entity upon encountering a potential energy barrier, while quantum mechanics in the general case predicts one reflected and one transmitted peak.

Studies at introductory quantum level show that this depiction is used by 29–33 % of 185 first- to second-year students at two Swedish universities (Falk, 2004), and "almost half" of 53 second-year and post-graduate students at the University of Washington (Ambrose, 1999). The latter study did not focus explicitly on this depiction, but reports on students who "did not include a reflected wave packet in their graphs" (Ambrose, 1999, p. 118). This depiction has also been explored qualitatively by Domert et al. (2005).

3.4.7 General problems with potential energy diagrams

Several investigations have shown that some students have problems interpreting diagrams of potential energy (Ambrose, 1999; McKagan & Wieman, 2006). One particular problem is depicting the total energy of the particle as an entity that must be positive (Redish et al., 1997), while this can be shifted arbitrarily – and often is chosen negative for bound particles. Some evidence suggest that students are more comfortable with kinetic energy, and indeed interpret E as kinetic energy, which may shed light on some inappropriate depictions (Bao, 1999; Muller & Sharma, 2006).

Focusing on kinetic energy is not incorrect in quantum mechanics from a formal point of view, but may cause problems since much of the formalism is centred on total energy. Also, in many cases the kinetic energy, in contrast to total energy, is local rather than a global property of a particle.

Two other inappropriate depictions are students not appreciating the one-dimensionality of the potential systems (Bao, 1999; Bao & Redish, 2002; Falk, 2004; Rebello & Zollman, 1999) and also depicting potential energy barriers as obstacles dissipating a particles energy in a way similar to friction (Brookes & Etkina, 2005; Koopman & Ellermeijer, 2005; Morgan et al., 2003).

Another inappropriate depiction associated with potential energy barriers is not linking a wave function's local wavelength to the local kinetic energy. In drawings, some students inappropriately use a constant ratio wavelength/amplitude, instead of relating amplitude to probability and wavelength to local kinetic energy (Ambrose, 1999). There are some examples of second and third-year introductory quantum mechanics students inappropriately relating larger amplitude to higher energy (Bao, 1999; Falk, 2004; Sadaghiani, 2005).

There is also research showing that some students have problems discerning a potential diagram – describing the surrounding of a particle – with diagrams of potential energy – describing a property of the particle (McKagan & Wieman, 2006).[5]

3.5 Measurements

> *"I: If I asked you to draw the wave function of the particle before they make the position measurement, and then [do the same for the wave function] after they make the position measurement, would those wave functions be identical or would they be different?*
>
> *S: I would say they would be identical…. You measure [the position of the particle], it collapses its wave function, you get this delta function, because the particle was here. And then after you're done measuring it, if you will, I haven't done anything necessarily to the energy of the particle, so it would go back to its [original] state."*
>
> *(Ambrose, 1999, p. 230, student interview)*
>
> *[Quotation illustrating a student (S) using an inappropriate depiction of how a measurement process affects a wave function.]*

[5] An example of the difference between potential and potential energy may be illustrated by an imagined super-electron with charge -5e. In the same potential as an ordinary electron, it will have a different potential energy.

> *"Some students [...] felt that the collapse of the wavefunction upon measurement is a mathematical construct and they only half-heartedly believed that the collapse can actually change the wavefunction permanently."*
>
> *(Singh, 2006a, p. 3)*

3.5.1 An accepted depiction of measurements

Measurements play a crucial role in quantum mechanics. In the Copenhagen interpretation, measurements are represented by a discontinuous change of the wave function. Technically, any measurable quantity is represented by an Hermitian operator. Each such operator has a set of associated eigenfunctions, and any wave function may be expanded into a linear superposition such eigenfunctions. An actual measurement corresponds to projecting the wave function onto one of the eigenfunctions in the expansion. Also, the distribution of measurement outcomes is described by the amplitude squared of the coefficients of the expansion, while the measurement value corresponds to the eigenvalue of each eigenfunction.

The expectation value for a measurement is equal to a weighted mean value of the possible measurement outcomes.

It is worth pointing out the difference between applying an operator to a wave function, which is simply a mathematical operation, and performing a measurement, which means changing the wave function into one of the possible eigenfunctions.

In particular: measurements change the wave function, unless the wave function was already an eigenfunction of the operator corresponding to the measured quantity; only the first of a sequence of rapidly repeated measurements[6] causes a change of the wave function, and thus rapidly repeating a measurement produces the same outcome; an eigenfunction is in general time-dependent; and after a measurement, a wave function does not in general return to the state before the measurement.

3.5.2 Sub-topic: the effect of measurements on quantum systems

There have been a few qualitative and quantitative studies looking at students' depictions of quantum measurements.

Two of the quantitative studies looked at students' depictions of repeated energy measurements. One study, involving probably fourth-year intermediate quantum mechanics students at six US universities, showed that 68 out of 89 students correctly stated that rapidly repeated measurements will yield the same measurement outcome (Singh, 2001). In a similar question posed to 53 third-year introductory quantum mechanics students at the University of Washington, only 15 students answered correctly – but only five provided acceptable reasoning. The majority, 30 students, held that the same measurement outcome was one of several possible (Ambrose, 1999).

The latter study (henceforth called the Ambrose study) also posed two other related questions to the students. One question concerned consequent measuring of energy, position and energy, to which 21 out of the 53 students correctly stated that the final energy value may be the same as the initial, but only two provided acceptable reasoning.

[6] Theoretically, the "rapidness" of the measurements should be infinite for the ideal case, for a general system.

Fourteen stated inappropriately that the final energy value must be the same as the initial. The other question concerned three Stern-Gerlach apparatuses, sequentially measuring spin in z, x, and z-directions. Out of 81 students, 46 answered correctly that the final apparatus will yield two electron beams, but only 21 students provided acceptable reasoning (Ambrose, 1999). Inappropriate depictions of this process are discussed under the section on spin.

Parts of the explanation for the learning problems in the area of measurements in quantum systems could be found in students' inappropriate depictions of the change of the wave function caused by measurements. Ambrose (1999) presents multiple examples of students depicting some kind of relaxation time, after which the wave function returns to its original state. This is also reported in a study of 300 intermediate quantum level third-year and post-graduate students in the US, in particular when the ground state is involved in the problem (Singh, 2006a). (The latter study is henceforth called the Singh study.)

Relaxation time, meaning that a wave function after collapsing returns to the state as it was before the measurement, is inconsistent with quantum mechanics on the basis of ignoring the quantum mechanical descriptions of time development of a system.

There are also several examples of students, in both written exercises and interviews, referring to conservation of energy when inappropriately claiming that the energy value of the system does not change due to a position measurement. One can also find single examples of students claiming that every measurement disturbs the system – even for repeated energy measurements; and a student inappropriately depicting energy measurement as taking some energy away from the system, thus repeated energy measurements brings it closer and closer to the ground state (Ambrose, 1999).

All these depictions are inconsistent with quantum mechanics. Since the measurement process involves the measuring equipment, there may be energy exchange between the systems, and thus energy conservation is not applicable to the wave function. The two latter depictions do not agree with the wave function collapsing into one of its eigenfunction components, implying that rapidly repeated measurements make a wave function remain in the same state.

In the Singh study described above, 11 % out of 300 participating students depict operators in a quantum-mechanical equation as a measurement performance changing the wave function, instead of merely a mathematical expression (Singh, 2006b). Further reporting of this study reveals that in an equation $H\Psi = E\Psi$, some students see the left-hand side as input, and the right-hand side as output, thus making the equation signify a change in the wave function (Singh, 2006a). This is inconsistent with basic mathematical notation, since the equality sign would no longer signify equality.

3.5.3 Sub-topic: possible measurement outcomes, and probabilities

> *"If Q is not in an eigenstate then $Q|\psi\rangle = \lambda|\psi\rangle$ is not true… so if you measure Q you won't be able to get λ and your results will be different every time."*
>
> *(Singh, 2001, p. 888, student statement)*
>
> *[Quotation illustrating a student using an inappropriate depiction of how the measurement process affects the wave function.]*

There have been a few small to mid-sized studies looking at student understanding of possible measurement outcomes, some of which included looking at the related probabilities, and expectation values.

In a study with 202 post-graduates at probably intermediate quantum level at seven US universities, only 67 % correctly identified possible energy measurement outcomes of a simple linear combination of two energy eigenstates. Seven percent displayed a confusion of measurement outcome and expectation value (Singh, 2005).

In a study at the University of Washington, only 17 out of 53 third-year introductory quantum mechanics students correctly identified possible energy measurement outcomes of a simple linear combination of two energy eigenstates. Seven answered with the expectation value, and four answered with sum of possible energies, sometimes motivated by the superposition principle. Five answered with the energy of the only component with a real coefficient (Ambrose, 1999). The latter result is discussed further in the section dealing with complex-valued wave functions.

Since measurement changes the wave function into one of its eigenfunction components, the correct answer to the questions above is that energy measurements would yield the eigenvalue for one of the energy eigenfunction components.

In a study with 89 probably fourth-year intermediate quantum mechanics students at six American universities, 74 correctly answered that measurements on an ensemble of identically prepared systems may yield different outcomes. 38 students also managed to state probabilities for the measurement outcomes of an arbitrary wave function, in terms of eigenvalues and expansion coefficients (Singh, 2001). (The absolute number of students reported here is calculated from weighted averages in the study, and thus might not correspond to the actual number of students.)

3.5.4 Sub-topic: expectation values

> *"$|\Psi|^2$ gives the probability of the wavefunction being at a given position and if you multiply it by x you get the probability of* measuring *(student's emphasis) the position x"*
>
> *(Singh, 2006b, p. 4, student quote)*
>
> *[Quotation illustrating a student using an inappropriate depiction of calculating measurement outcomes, confusing probability density and expectation value.]*

The Singh study described in 3.5.2 also showed that 11 % out of the 300 participating students confused the expectation value with probability density (Singh, 2006b). This inappropriate depiction also emerged in a study of 156 second- and third-year students at Ohio State University, in the context of a position distribution described by a delta function (Sadaghiani & Bao, 2005; Sadaghiani, 2005). This depiction is inconsistent with quantum mechanics, since expectation value is a weighted mean value of measurement outcomes, while probability density describes the distribution of measurement outcomes of a large ensemble of identically prepared particles.

There are interview data indicating that even post-graduate students are sometimes not aware that the expectation value of a measurement is a weighted mean value (Singh, 2005), and a single example of a probably fourth-year student inappropriately claiming that single measurements are not possible in quantum mechanics (Singh, 2004).

3.5.5 Sub-topic: complex-valued wave functions and measurements

> *"The measured value would be ε... only real components of the wave function can be measured."*
>
> *(Ambrose, 1999, p. 225, student quote)*
>
> *"The 'real part' [of ψ(x)] means where a particle can physically exist, I think. If so, Re{ψ(x)} = 0 anywhere right of the barrier"*
>
> *(Ambrose, 1999, p. 121, student quote)*
>
> *[Two quotations illustrating students using an inappropriate depiction of the meaning of complex-valued wave functions.]*

A study with third-year introductory students at the University of Washington provided several examples of students inappropriately stating that only the real part of a wave function is measurable. The same study also reports on students inappropriately assuming that the real part of the wave function is zero inside classically forbidden regions (Ambrose, 1999).

This inappropriate depiction has not been reported in any other study covered by this literature review. The depiction is inconsistent with quantum mechanics, for example in the aspect that a global phase shift in the polar plan can be added to any quantum system without changing any physical aspects – thereby giving real and imaginary wave functions the same status.

3.6 Energy quantisation

3.6.1 An accepted depiction of energy quantisation

A typical way of introducing energy quantisation is through solving for energy eigenfunctions for an infinite square potential system. Calculating energy eigenfunctions, one finds that only certain values for the total energy yield wave functions that are physically acceptable – for example, it should be possible to normalise the wave function. Similarly, it can be shown that all bound particles – that have vanishing wave function amplitude at the boundaries of the systems' space – have discrete spectra of possible energies, while free particles have continuous energy spectra.

3.6.2 Depictions of energy quantisation

Studies on German pre-university, pre- or introductory quantum mechanics students indicate that students accept energy quantisation, in the sense that they refer to it and use it to explain other phenomena (Bethge, 1992; Bethge & Niedderer, 1996). This outcome also emerged in a study on Australian second-year to post-graduate students, where students accepted and used energy quantisation even when having problems explaining it (Fletcher, 2004).

However, studies conducted in the US show that some students have problems depicting energy quantisation: a study of approximately 200 second-year introductory quantum mechanics students at the University of Maryland showed that some students did not recognise that bound particles have quantised energies, nor were they able to explain what is meant by a bound state (Bao, 1999). A similar result was also found in another US study, which presented an exemplar case of an introductory quantum mechanics student having problems telling free particles from bound particles (Wittmann & Morgan, 2003).

A study involving 202 post-graduate students at seven American universities revealed a "common" inappropriate depiction – "that all allowed energies for the infinite square well were possible and the ground state is the most probable because it is the lowest energy state" (Singh, 2005, p. 70). This is inconsistent with quantum mechanics, since not all energies are possible for a bound particle, nor need the ground state be the most frequent measurement outcome.

3.7 Time-dependence

> *"Although students in advanced quantum mechanics courses may have learned to solve the Schrödinger equation with complicated potentials and boundary conditions, many have difficulties with conceptual understanding of quantum measurements and time development. […] Students often used conflicting justifications throughout the test, and there was a lack of discrimination between related concepts."*
>
> *(Singh, 2001, p. 892)*

3.7.1 An accepted depiction of time-dependence

Time-dependence in quantum mechanics can be described through the Schrödinger equation, stating that the Hamilton operator acting on a wave function equals the time derivative of the wave function times an imaginary factor. In the cases where the wave function is an eigenfunction of the Hamilton operator, that is, the particle has a definite total energy, the time dependence becomes a simple exponential times a stationary spatial wave function. The time-dependent exponential has a complex argument involving the total energy.

In the cases where the wave function is not an eigenfunction of the Hamilton operator, the time dependence can be obtained through expanding the wave function into a linear combination of such eigenfunctions, and multiplying each eigenfunction by the appropriate time-factor.

In particular: any wave function may be described as a linear combination of eigenfunctions of the Hamilton operator; not all wave functions are eigenfunctions of the Hamilton operator; a sum of eigenfunctions of the Hamilton operator is in general not a new eigenfunction; wave functions that are not eigenfunctions of the Hamilton operator do not have a shared simple time-factor; eigenfunctions of an arbitrary operator are not stationary; systems with discrete and continuous energy spectra are dealt with in essentially the same way; and, also, expectation values of energy are constant in time when the Hamilton operator is time-independent.

If energy quantisation is a relatively trouble-free topic for a majority of the students, time-dependence, ironically, is very poorly understood – time and energy being complementary units. This is a finding consistently emerging from studies where students neglect time-dependence or misinterpret time-dependence, and are unable to calculate time-dependence,

3.7.2 Inappropriate depiction: neglecting time-dependence

Neglecting time-dependence is reported in Singh study described in 3.5.2: when asked to give the most fundamental equation of quantum mechanics, 48 % of the students answered the time-independent Schrödinger equation, while 32 % answered the time-dependent equation (Singh, 2006a). Since the time-independent Schrödinger equation may be derived from its time-dependent version – but not vice versa – and the time-dependent Schrödinger equation also describes larger parts of quantum mechanics, it may be argued that the time dependent Schrödinger equation is more fundamental than the time-independent version.

This somewhat inappropriate honouring of the time-independent Schrödinger equation may also shed light on why 39 % of the participants claimed that $H\Psi = E\Psi$ for any quantum mechanical state, while 29 % correctly stated that it is only true for stationary state (Singh, Belloni, & Christian, 2006).

Also, in probably a sub-investigation of the same study, "many" of the 202 participating post-graduates at seven US universities inappropriately used the time-independent Schrödinger equation $H\Psi = E\Psi$ to investigate whether or not a wave function is acceptable. Some students only allowed energy eigenstates while other also accepted linear combinations of energy eigenstates – but ruled out other continuous functions that they did not find a way of composing linearly from energy eigenstates (Singh, 2005). This is inconsistent with quantum mechanics, since all continuous functions may be written as a linear sum of energy eigenfunctions, and all such functions are physically acceptable wave functions.

3.7.3 Inappropriate depictions of time-dependence

There are some examples of inappropriate interpretations of time-dependence.

There are examples of probably third-year US intermediate quantum mechanics students hold that eigenstates to an arbitrary operator are stationary (Singh, 2001). In a study on 35 third-year introductory quantum mechanics students at Ohio State University, 15 claimed that a sum of energy eigenfunctions becomes a new energy eigenfunction (Sadaghiani, 2005), while this is not true in the general case.

In the Singh study described in 3.5.2, some students inappropriately related the validity of the time-independent Schrödinger equation to whether or not the Hamilton operator was time-dependent (Singh, 2006b). This is inconsistent with quantum mechanics, since the time-independent Schrödinger equation is valid for obtaining energy eigenstates at any single moment – even if the Hamilton operator is time-dependent.

There is also an example of a student inappropriately making a distinction between time development of discrete and continuous spectra, inappropriately depicting them as continuous spectra of eigenvalues giving rise to a slow, gradual change with time, while a discrete eigenvalue spectrum allows drastic and quick changes (Singh, 2001).

3.7.4 Inability to calculate time-dependence

There are also several examples of students' inability to calculate time-dependence.

In a study with 146 students at Pennsylvania State University and Arizona State University, participants answered a fairly complex multiple-choice question involving time dependence at the level of guessing. The study included students from second year to post-graduates, at pre-quantum to intermediate quantum level, but there were no differences between the groups concerning this question (Cataloglu, 2002).

A study of 89 probably third-year, intermediate quantum mechanics students at six US universities also showed great problems with calculating time-dependence and expectation values, in this case through the Ehrenfest theorem (Singh, 2001). Further investigations, on probably the same student population, revealed examples of students squaring each time-dependent term individually when calculating expectation values – instead of their linear sum – thereby inappropriately eliminating, for example, all time dependence (Singh, 2004).

Of 202 post-graduates at seven US universities, above introductory quantum level, 31 % inappropriately used a common time factor when asked to write down the time development of a simple linear combination of energy eigenstates. Nine percent answered with a wave function without any time parameter. 43 % gave a correct answer. When asked for the time-dependence of the energy expectation value, 39 % gave a correct answer – many using "brute-force" calculations rather than recognising that the expectation value of energy is time independent (Singh, 2005).

Students using a common phase factor for time-dependence have also been reported on by a study at Ohio State University. Out of 48 third-year introductory students, 23 chose a common phase factor in a multiple-choice test (Sadaghiani, 2005).

Intermediate and advanced quantum level

There have been very few studies on students' depictions of topics in intermediate or advanced quantum education. This section presents research into depictions of spin, and then presents a list of other studies in intermediate or advanced quantum topics.

3.8 Electron spin

> *"The spin of electrons is commonly interpreted in the form of the physical rotation or circular motion of an electron – which is viewed as a classical particle rather than as a [quantum particle] with very different properties."*
>
> *(Taber, 2002a, p. 162)*

> *"Only one beam would emerge: the first apparatus separated the spin up and spin down electrons, so repeating the process (even in a different direction) would not split the beam further."*
>
> *(Ambrose, 1999, p. 233, student quote)*
>
> *[Quotation from a student discussing a Stern-Gerlach experiment setup, illustrating an inappropriate depiction of the direction-dependence of spin measurements.]*

The spin topic is difficult to place in one of the categories pre- to advanced quantum education. The reason for this is that while describing spin technically demands mathematics at intermediate level or above, it could also be included in the pre-quantum descriptions of atoms and orbitals. Since there are only a few studies made into students' depictions of spin, they are presented together rather than under separate categories.

3.8.1 An accepted depiction of electron spin

Quantum mechanical spin is an additional, intrinsic form of angular momentum, which is not described through any rotation in space. As with spatial angular momentum, spin is usually described through the magnitude of the spin, and the spin along a z-axis.

For an electron, the total spin quantum number is ½, resulting in a z-component quantum number of either +½ or -½.

In particular: spin does not correspond to a rotating extended entity; and spin measurements along different axes are non-commuting measurements, thus affecting each other's measurement outcomes.

3.8.2 Depictions of electron spin

Some research that has been undertaken in this topic has been with pre-university chemistry students, who described quantum spin as a spatial movement, associated with the macroscopic meaning of the word. This was either depicted through an electron spinning around its own axis in a particular direction (Taber, 2002a), or an electron that "moves about this volume of space that's called the orbital in *one particular direction*" (Taber, 2002b, p. 155).

There are also examples of students not appreciating that spin in different directions are incompatible entities; for example, that measurement of spin in x-direction affects the z-component of the spin (Ambrose, 1999; Singh, 2006b). A related reported inappropriate depiction is failure to recognise that the spin measurement is related to a particular direction (Ambrose, 1999).

3.9 Other topics from intermediate and advanced quantum mechanics

Other intermediate or advanced topics that research has touched upon include: the Dirac formalism, basically a notation system convenient for quantum mechanics; dimensionality of quantum states, described in a Hilbert space; and non-locality and the EPR paradox, which deals with action-at-distance, seemingly super-light communication, and local hidden variables.

A study on 89 third- or fourth-year intermediate quantum mechanics students, at six US universities, revealed as a side-point that students have problems with the Dirac formalism. "[R]esponses suggest that many advanced students are uncomfortable with the Dirac formalism and notation, even though it was used in all of the classes in this study" (Singh, 2001, p. 887).

Two separate studies indicate that students have difficulties differentiating between "real" three-dimensional space and a Hilbert space of quantum mechanical states, in particular concerning orthogonality of spin states (Sadaghiani, 2005; Singh, 2006b). There are also reports on students adding dimensionality instead of multiplying, for example, when constructing a Hilbert space for multiple spin-particles (Singh, 2006a).

A study that included 37 German pre-university physics-teacher students showed that five students had some idea of what non-locality and the EPR paradox meant. Another five had heard about non-locality and the EPR paradox, but did not know what they were. 19 of the student teachers had previously encountered quantum mechanics in school (Müller & Wiesner, 1999). It is not clear whether the students who had encountered quantum mechanics in school also were the ones who recognised non-locality and the EPR paradox.

Also, a list of some other conceptual problems at the intermediate or advanced level have been proposed (Styer, 1996), but not supported by more evidence than collected teaching experience.

3.10 Instruments for investigating depictions of quantum mechanics

There have been several attempts at designing questionnaires for investigating students' depictions of quantum mechanics. The different questionnaires are designed with different purposes, and naturally contain questions with different nature and contents. The general goal has been to develop instruments that allow quick and easy investigations of students' depictions. Questionnaires as research instruments are discussed further in 4.2.2.

The questionnaires are presented below.

- (Mashhadi, 1996): a 54-item questionnaire developed to investigate students' depictions of the wave-particle duality, in particular electrons and photons, in relation to conceptions of models and the ontological and epistemological status of electrons and photons.
- QMVI: Quantum Mechanics Visualization Instrument (Cataloglu, 2002): a 24-item questionnaire, weighted to match curricula and designed primarily as an achievement test for introductory and intermediate quantum mechanics.
- QMCI: Quantum Mechanics Concept Inventory (Falk, 2004): a nine-item questionnaire developed to investigate some inappropriate depictions related to potential energy barriers.
- QMCS: Quantum Mechanics Conceptual Survey (McKagan & Wieman, 2006): an online questionnaire investigating inappropriate depictions related to mainly one-dimensional quantum mechanics. The questionnaire is being developed continuously.
- QPCS: Quantum Physics Conceptual Survey (Wuttiprom et al., 2006): a questionnaire currently being developed, mainly focusing on students' depictions of the wave-particle duality.

4 DISCUSSION

4.1 General conclusions

4.1.1 Problems in learning quantum mechanics

This literature review leaves no doubt that students have great problems learning quantum mechanics. With very few exceptions, the research reports on finding problems with students' depictions in all topics.

Problems in depicting quantum mechanics have arguably severe consequences for learning the subject. In quantum mechanics, everything a student relies on when trying to learn the subject is abstract physical and mathematical models. When these are misinterpreted, there is no intuition, no common sense, and no every-day experience to fall back on. A student with inappropriate depictions of a quantum mechanics topic stands a very small chance of building a sound understanding of the topic, whatever is meant by understanding, let alone applying knowledge about the topic when learning more advanced quantum mechanics.

4.1.2 Focus of research

Most education research in quantum mechanics has been in introductory and pre-quantum education. More than half of the articles included in this review deal with pre-quantum topics, and more than a quarter deal with student depictions of atoms. More than a third of the articles deal with introductory quantum mechanics, and one out of four deal with tunnelling or one-dimensional square potentials.

In contrast, little research has been carried out in intermediate or advanced quantum education: in total eight articles – approximately one out of ten – deal with any of the intermediate or advanced topics. If the spin topic is removed from this category, which may be appropriate in some respects[7], only five articles remain. No research into students' depictions of topics beyond advanced quantum mechanics education, for example in relativistic quantum mechanics, was encountered when doing this literature review.

Measuring research in numbers of articles like this is of course an extremely crude way of comparing research results and efforts, but the extreme differences in number of articles still shows that more advanced quantum mechanics topics have received very little attention.

For a few topics – atoms and tunnelling – there is now enough research to give a fairly good description of what inappropriate depictions exist among students' depictions, and also to tell how common some inappropriate depictions are. A number of topics – such as wave-particle duality, uncertainty, measurements and spin – have been investigated enough to give some descriptions of inappropriate depictions, but it is highly unlikely that current research has captured all aspects of students' inappropriate depictions. In many topics, particularly in intermediate and advanced quantum mechanics, research is only scratching the surface, merely indicating that problems exist. There are still many topics in quantum mechanics that are left completely uninvestigated by education researchers.

4.1.3 Frequency of some inappropriate depictions

Six inappropriate depictions have been investigated quantitatively in more than one study. Their results may be summarised as follows:

- 20–60 % of pre-quantum mechanics students use the classical orbits depiction for describing an atom, even after being introduced to a quantum mechanical model;
- 10–20 % of pre-quantum mechanics students depict the wave-particle duality as classical particles moving in wave-shaped trajectories;
- 20–60 % of introductory quantum mechanics students depict tunnelling as causing energy loss;
- 20–50 % of introductory quantum mechanics students depict probabilities for transmission and reflection through a range of possible energies for the particle;

[7] See 3.8 for a brief discussion on this.

- 30–50 % of introductory quantum mechanics students depict a probability peak as an indivisible entity, similar to a classical particle;
- 30–50 % of introductory or intermediate quantum mechanics students use a common phase factor when asked for the time-dependence of a wave function.

4.1.4 Gender in quantum mechanics education research

In doing this review, only one study reporting on gender aspects of quantum mechanics education was found. This Norwegian study, described in 2.1.1, found only small and non-significant differences between male and female students' attitudes towards quantum mechanics (Olsen, 1999). The literature research revealed no study focusing on gender issues in quantum mechanics education, though there are some philosophical discussions centred around quantum mechanics from a feminist perspective (see, for example, Barad, 1996).

As has been pointed out by Olsen (1999), education research in quantum mechanics has been carried out primarily by male researchers. Although the amount of research completed and publications written by women has increased, the research area is still dominated by men.

4.1.5 Classical depictions: a theme among inappropriate depictions

One common theme among students' inappropriate depictions of quantum mechanics is that students tend to apply classical depictions to interpret quantum mechanics. This is apparent at pre-quantum education, in the classical orbit model of the atom and interpreting the wave-particle duality as an oscillating classical particle; but it is also apparent at introductory quantum education in the energy loss conception, in using a range of energies to explain transmission and reflection, and in interpreting a probability peak as an entity.

This naturally leads to a question of whether classical mechanics hinders students' learning of quantum mechanics, or if this broad theme simply is the result of that students' struggling with quantum mechanics rely on whatever depictions they have – in this case from classical mechanics. In any case, the relation between physical models used in classical mechanics and quantum mechanics appears to be an interesting focus for future research.

4.2 Suggestions for future research

This review clearly shows that education research in quantum mechanics is growing, both geographically and concerning span of topics. Still, there are only a few topics where students' depictions are explored beyond the basic level, and several topics where no investigations of students' depictions have taken place at all.

4.2.1 Interpretations

A crucial aspect missing in quantum education research is what interpretations of quantum mechanics are actually employed in teaching. Even if the formalism of quantum mechanics is clear-cut, the existing interpretations give diverse explanations to what the formalism *means*.

When investigating how students depict quantum mechanics, it is of course necessary for the researcher to know quantum mechanics well, in order to notice important subtleties and nuances that someone unfamiliar with quantum mechanics is likely to miss

– or lose in the noise of all possible unimportant subtleties. Indeed, being able to contrast students' depiction to the intended learning is what makes physics a prerequisite for conducting physics education research. Similarly, a researcher in quantum mechanics education should know what interpretation is employed in teaching, in order to facilitate the interpretation of students' depictions of quantum mechanics.

Another possible aspect of quantum mechanics education research involving interpretations is whether learning difficulties are different when the teaching employs different interpretations.

4.2.2 Large-scale studies

The lack of large-scale studies in quantum mechanics education research makes it very difficult to make conclusions on how common inappropriate depictions are. Such knowledge would inform future education research, not only by providing a baseline to compare student populations, but also to make an argument on where education researchers should focus their efforts.

Large-scale studies would enable contrasting different inappropriate depictions and student populations, to see, for example, whether or not certain difficulties are correlated, and if advanced level quantum mechanics students have fewer problems with some topics than do introductory level students.

One way to make large-scale studies possible is to develop questionnaires to be used as research instruments. As described earlier in this review, there are several ongoing attempts at developing such questionnaires, and of course these instruments should collectively try to encompass as many inappropriate depictions as possible.

Large-scale studies and questionnaires to quantitatively investigate inappropriate depictions would not only be a resource to education researchers, but would also constitute useful tools for quantum mechanics teachers: knowing what inappropriate depictions are common, and which are present in a particular class, would help in developing curricula and course outlines.

4.2.3 Closer investigations of quantum mechanics topics

A number of vital topics in introductory quantum mechanics, for example measurements and time development, should be more closely investigated to provide a richer description on students' inappropriate depictions of the topics. Investigating different aspects of inappropriate depictions will hopefully make it possible to form conclusions on why these difficulties exist, how to cope with them in teaching, and also to include them in large-scale studies on students' depictions of quantum mechanics.

There are also some introductory quantum topics left completely uninvestigated, such as students' depictions of angular momentum.

4.2.4 Inclusion of more advanced topics

There is a surprising and alarming lack of education research on what in this review is called intermediate and advanced quantum topics. Only a few topics – spin, Dirac formalism, dimensionality of Hilbert spaces, non-locality and the EPR paradox – have been empirically investigated, and our knowledge about students' depictions of these topics is scarce, at best.

Future education research in quantum mechanics must include not only pre-quantum and introductory quantum topics, but also more advanced topics. In doing this research, one initial task will be identifying and discerning the most appropriate topics, and also finding suitable approaches to investigating students' depictions of them.

One challenging and interesting task is also to expand quantum education research beyond what is called advanced quantum education in this review. This will probably require even larger efforts in finding suitable approaches for investigating students' depictions and trends among these.

Of course, education research in topics beyond introductory quantum level will require expert skills in quantum mechanics from the researcher's side.

4.2.5 Classical physics interfering with quantum mechanics?

The inappropriate depictions presented in this review show a trend of students applying classical mechanics to depict quantum topics, as described in 4.1.5. This trend leads to a number of interesting and important questions for the future.

One possibly beneficial question for future research is whether depicting quantum topics by applying classical mechanics is a trend persistent between different topics: that is, whether some students are apt to apply classical mechanics in several different quantum topics, or if it is restricted to isolated topics.

Another possibly rewarding question for future research is to investigate how students who do *not* apply classical models depict certain quantum aspects, such as quantum particles, the wave-particle duality, and the quantum wave function.

Hopefully, similar research questions can lead to a better understanding of not only how students depict certain topics, but also of how they understand and interpret quantum mechanics. It may also lead to new ideas of how to teach quantum mechanics.

4.2.6 Future reviews

This review only includes empirical studies into students' depictions of quantum mechanics topics. No doubt, a comprehensive review of research into teaching methods and aids would be very useful for quantum mechanics teachers, and of course also for education researchers.

Also, if quantum mechanics education research continues to grow, there will soon be reasons to update this review.

5 REFERENCES

Ambrose, B. S. (1999). *Investigation of student understanding of the wave-like properties of light and matter.* Unpublished doctoral thesis, University of Washington.

Ambrose, B. S., Shaffer, P. S., Steinberg, R. N., & McDermott, L. C. (1999). An investigation of student understanding of single-slit diffraction and double-slit interference. *American Journal of Physics, 67*(2), 146-155.

Bao, L. (1999). *Dynamics of Student Modeling: A theory, algorithms and application to quantum mechanics.* Unpublished doctoral thesis, University of Maryland.

Bao, L., & Redish, E. F. (2002). Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics. *American Journal of Physics, 70*(3), 210-217.

Barad, K. (1996). Meeting the universe halfway: Ambiguities, discontiuities, quantum subjects and multiple positionings in feminism and physics. In L. H. Nelson & J. Nelson (Eds.), *Feminism, science and the philosophy of science: A dialogue* (pp. 161-194). Boston: Kluwer Press.

Bethge, T. (1988). *Aspekte des Schülervorverständnisses zu gundlegenden Begriffen der Atomphysik.* Unpublished doctoral thesis, University of Bremen, Bremen.

Bethge, T. (1992). Schülervorstellungen zu grundlegenden Begriffen der Atomphysik. In H. Niedderer (Ed.), *Quantenphysik in der Schule* (pp. 215-233): Kiel: IPN.

Bethge, T., & Niedderer, H. (1996). Students' conceptions in quantum physics.
Brookes, D. T., & Etkina, E. (2005, 10-11 August, 2005). *Do Our Words Really Matter? Case Studies From Quantum Mechanics.* Paper presented at the 2005 Physics Education Research Conference, Salt Lake City, Utah.
Budde, M., Niedderer, H., Scott, P., & Leach, J. (2002). "Electronium": a quantum atomic teaching model. *Physics Education, 37*(3), 197-203.
Cataloglu, E. (2002). *Development and Validation of an Achievement Test in Introductory Quantum Mechanics: The Quantum Mechanics Visualization Instrument (QMVI).* Unpublished doctoral thesis, The Pennsylvania State University.
Cataloglu, E., & Robinett, R. W. (2002). Testing the develpment of student conceptual and visualization understanding in quantum mechanics through the undergraduate career. *American Journal of Physics, 70*(3), 238-251.
Cervellati, R., & Perugini, D. (1981). The Understanding of the Atomic Orbital Concept by Italian High School Students. *Journal of Chemical Education, 58*(7), 568-569.
Cros, D., & Chastrette, M. (1988). Conceptions of second year university students of some fundamental notions in chemistry. *International Journal of Science Education, 10*(3), 331-336.
Domert, D. (2005). *An exploration of abstract physics in the spirit of the scholarship of teaching.* Unpublished licentiate thesis, Uppsala University, Uppsala, Sweden.
Domert, D. (2006). *Explorations of University Physics in Abstract Contexts - From de Sitter Space to Learning Space.* Unpublished doctoral thesis, Uppsala University, Uppsala, Sweden.
Domert, D., Linder, C., & Ingerman, Å. (2005). Probability as a conceptual hurdle to understanding one-dimensional quantum scattering and tunnelling. *European Journal of Physics, 26*(1), 47-59.
Euler, M., Hanselmann, M., & Müller, A. (1999, March 1999). *Students' views of models and concepts in modern physics.* Paper presented at the Research on Teaching and Learning Quantum Mechanics.
Falk, J. (2004). *Developing a quantum mechanics concept inventory.* Unpublished master thesis, Uppsala University, Uppsala.
Fischler, H., & Lichtfeldt, M. (1991). *Learning quantum mechanics.* Paper presented at the Research in Physics Learning, Theoretical issues and Empirical Studies (International workshop), Bremen, Germany.
Fischler, H., & Lichtfeldt, M. (1992). Modern Physics and Students' Conceptions. *International Journal of Science Education, 14*(2), 181-190.
Fletcher, P. R., & Johnston, I. (1999, March 1999). *Quantum mechanics: exploring conceptual change.* Paper presented at the Reserach on Teaching and Learning Quantum Mechanics.
Fletcher, P. R. (1997). *How Students Learn Quantum Mechanics.* Unpublished master thesis, University of Sydney, Sydney.
Fletcher, P. R. (2004). *How Tertiary Level Physics Students Learn and Conceptualise Quantum Mechanics.* Unpublished doctoral thesis, University of Sydney, Sydney.
Gil, D., & Solbes, J. (1993). The introduction of modern physics: overcoming a deformed vision of science. *International Journal of Science Education, 15*(3), 255-260.
Griffiths, A. K., & Preston, K. R. (1992). Grade-12 students' misconceptions relating to fundamental characteristics of atoms and molecules. *Journal of research in science teaching, 29*(6), 611-628.
Göritz, G., & Wiesner, H. (1984). Wie verstehen Schüler die Probleme der Quantenmechanik? *Praxis der Naturwissenschaften, 33*(3), 90-92.
Harrison, A. G., & Treagust, D. F. (1996). Secondary students' mental models of atoms and molecules: implications for teaching chemistry. *Science Education, 80*(5), 509-534.
Ireson, G. (1999a). A multivariate analysis of undergraduate physics students' conceptions of quantum phenomena. *European Journal of Physics, 20*(3).
Ireson, G. (1999b). On the quantum thinking of physics undergraduates, *Second International Conference of the European Science Education Research Association (ESERA)*. Kiel, Germany.
Ireson, G. (2000). The quantum understanding of pre-university physics students. *Physics Education, 35*(1), 15-21.
Johnston, I. D., Crawford, K., & Fletcher, P. (1998). Student difficulties in learning quantum mechanics. *International Journal of Science Education, 20*(4), 427-446.
Kalkanis, G., Hadzidaki, P., & Stavrou, D. (2003). An instructional model for a radical conceptual change towards quantum mechanics concepts. *Science Education, 87*(2), 257-280.
Ke, J.-L., Monk, M., & Duschl, R. (2005). Learning Introductory Quantum Physics: Sensori-motor

experiences and mental models. *International Journal of Science Education, 27*(13), 1571-1594.
Koopman, L., & Ellermeijer, T. (2005, June 2005). *Understanding student difficulties in first year quantum mechanics courses.* Paper presented at the The First European Physics Education Conference (EPEC-1), Bad Honnef, Germany.
Lawless, C. (1982). Arts Students and Quantum Theory in an Open University History of Science Course. *Studies in Higher Education, 7*(2), 133-139.
MacKinnon, G. R. (1999). Students' understanding of Orbitals: A Survey.
Mannila, K., Koponen, I. T., & Niskanen, J. A. (2002). Building a picture of students' conceptions of wave- and particle-like properties of quantum entities. *European Journal of Physics, 23*(1), 45-53.
Mashhadi, A. (1994, September 22-27 1994). *Students' Conceptions of Quantum Physics.* Paper presented at the Thinking Physics for Teaching, Rome, Italy.
Mashhadi, A. (1995). Advanced level physics students' conceptions of quantum physics, *Singapore Educational Research Association 9th Annual Conference*. Singapore.
Mashhadi, A. (1996). *What is the nature of the understanding of the concept of "wave particle duality" among Advanced level Physics student?* Unpublished doctoral thesis, Oxford University, Oxford, United Kingdom.
Mashhadi, A., & Woolnough, B. (1997). Dualistic Thinking Underlying Students' Understanding of Quantum Physics, *Annual International Conference on Thinking* (pp. 23). Singapore.
Mashhadi, A., & Woolnough, B. (1999). Insights into students' understanding of quantum physics: visualizing quantum entities. *European Journal of Physics, 20*(6), 511-516.
McKagan, S. B., & Wieman, C. E. (2006, February 14 2006). *Exploring Student Understanding of Energy through the Quantum Mechanics Conceptual Survey.* Paper presented at the Physics Education Research Conference.
Morgan, J. T., Wittmann, M. C., & Thompson, J. R. (2003, 9 September 2003). *Student Understanding of Tunneling in Quantum Mechanics: Examining Interview and Survey Results for Clues to Student Reasong.* Paper presented at the Physics Education Research Conference.
Muller, D. A., & Sharma, M. D. (2006). Quantum tunneling: a gateway to understanding student conceptions of quantum mechanics (pp. 25): School of Physics, University of Sydney.
Müller, R. (2003). *Quantenphysik in der Schule* (Vol. 26): Logos.
Müller, R., & Wiesner, H. (1999). Students' conceptions of quantum physics. In D. Zollman (Ed.), *Research on Teaching and Learning Quantum Mechanics* (pp. 20-22): National Association for Research in Science Teaching (NARST).
Müller, R., & Wiesner, H. (2002). Teaching quantum mechanics on an introductory level. *American Journal of Physics, 70*(3), 200-209.
Nakiboglu, C. (2003). Instructional misconceptions of Turkish prospective chemistry teachers about atomic orbitals and hybridization. *Chemistry Education Research and Practice, 4*(2), 171-188.
Nakiboglu, C., & Benlikaya, R. (2001). Misconceptions about orbital concept and modern atom theory (in Turkish). *Kasamonu Egitim Dergisi, 9*(1), 165-174.
Olsen, R. V. (1999). *Kvantefysikk i skolen: En undersøkelse av fysikkelevers forståelse av kvantefysikk og en analyse av dette emnets status i skolefysikken.* Unpublished master thesis, Oslo University, Olso, Norway.
Olsen, R. V. (2001). A study of Norwegian upper secondary physics specialists' concetion of atomic models and the wave particle duality, *ESERA*. Thessaloniki.
Olsen, R. V. (2002). Introducing quantum mechanics in the upper secondary school: A study in Norway. *International Journal of Science Education, 24*(6), 565-574.
Petri, J., & Niedderer, H. (1998). A learning pathway in high-school level quantum atomic physics. *International Journal of Science Education, 20*(9), 1075-1088.
Rebello, N. S., & Zollman, D. (1999, March 1999). *Conceptual understanding of quantum mechanics after using hands-on and visualization instructional materials.* Paper presented at the Reserach on Teaching and Learning Quantum Mechanics.
Redish, E. F., Bao, L., & Jolly, P. (1997). Student Difficulties with Energy in Quantum Mechanics, *AAPT winter meeting*. Phoenix, Arizona.
Redish, E. F., Wittmann, M. C., & Steinberg, R. N. (2000). CF14: Affecting Student Reasoning in the Context of Quantum Tunneling, *AAPT summer meeting 2000*. Guelph, Ontario.
Sadaghiani, H. R., & Bao, L. (2005, 10-11 August, 2005). *Student Difficulties in Understanding Probability in Quantum Mechanics.* Paper presented at the 2005 Physics Education Research

Conference, Salt Lake City, Utah.

Sadaghiani H. R., (2005). *Conceptual and mathematical barriers to students learning quantum mechanics.* Unpublished doctoral thesis, Ohio State University.

Singh, C. (2001). Student understanding of quantum mechanics. *American Journal of Physics, 69*(8), 885-895.

Singh, C. (2004, 16 September 2005). *Transfer of Learning in Quantum Mechanics.* Paper presented at the 2004 Physics Education Reserarch Conference, Melville, New York.

Singh, C. (2005, 14 February 2006). *Assessing and improving student understanding of quantum mechanics.* Paper presented at the 2005 Physics Eduaction Research Conference.

Singh, C. (2006a). Helping students learn quantum mechanics for quantum computing, *Physics Education Research Conference*.

Singh, C. (2006b). Student difficulties with quantum mechanics formalism, *2006 Physics Education Research Conference*.

Singh, C., Belloni, M., & Christian, W. (2006). Improving students' understanding of quantum mechanics. *Physics Today*, 43-49.

Stapp, H. P. (1972). The Copenhagen Interpretation. *American Journal of Physics, 40*(8), 1098-1116.

Steinberg, R. N., Oberem, G. E., & McDermott, L. C. (1996). Development of a computer-based tutorial on the photoelectric effect. *American Journal of Physics, 64*(11), 1370-1379.

Styer, D. F. (1996). Common misconceptions regarding quantum mechanics. *American Journal of Physics, 64*(1), 31-34.

Taber, K. S. (2002a). Compounding quanta: Probing the frontiers of student understanding of molecular orbitals. *Chemistry Education: Research and Practice in Europe, 3*(2), 159-173.

Taber, K. S. (2002b). Conceptualizing quanta: Illuminating the ground state of student understanding of atomic orbitals. *Chemistry Education: Research and Practice in Europe, 3*(2), 145-158.

Taber, K. S. (2005). Learning quanta: Barriers to stimulating transitions in student understanding of orbital ideas. *Science Education, 89*(1), 94-116.

Thacker, B. A. (2003). A study of the nature of students' models of microscopic processes in the context of modern physics experiments. *American Journal of Physics, 71*(6), 599-606.

Tsaparlis, G. (1997). Atomic orbitals, molecular orbitals and related concepts: conceptual difficulties among chemistry students. *Research in Science Education, 27*(2), 271-287.

Wittmann, M. C. (2003). Understanding Coordinated Sets of Resources: An Example from Quantum Tunnelling, *Fermi School in Physics Education Research*. Varenna, Italy.

Wittmann, M. C., & Morgan, J. T. (2003, 9 September 2004). *Understanding Data Analysis from Multiple Viewpoints: An Example from Quantum Tunneling.* Paper presented at the Physics Education Research Conference.

Wittmann, M. C., Morgan, J. T., & Bao, L. (2005). Addressing student models of energy loss in quantum tunnelling. *European Journal of Physics, 26*(6), 939-950.

Vokos, S., Shaffer, P. S., Ambrose, B. S., & McDermott, L. C. (2000). Student understanding of the wave nature of matter: Diffraction and interference of particles. *American Journal of Physics, 68*(S1), S42-S51.

Wuttiprom, S., Chitaree, R., Soankwan, C., Sharma, M., & Johnston, I. (2006, September 28, 2006). *Developing a prototype conceptual survey in fundamental quantum physics.* Paper presented at the Assessment in Science Teaching and Learning Symposium, University of Sydney.

Appendix I: Resource list for research into quantum mechanics teaching

Concerning this appendix

This appendix contains a non-exhaustive list of articles concerning teaching quantum mechanics. The intention of this appendix is mainly to be a tool for teachers looking for valuable insights to inform their teaching of certain quantum mechanics topics. To make the resource list more accessible, the resources have been categorised into quantum mechanics topics similar to the categorisations used in the review in Paper 1. Of course, this appendix may not only be of use for teachers, but also for education researchers interested in understanding learning.

With three exceptions, only articles from journals are listed. The reason for mainly listing journal articles is to ensure that the items are accessible to the reader. The three exceptions – one book and two doctoral theses – are included due to their usefulness in presenting empirical research into the teaching of quantum mechanics. I want to stress that there is empirical research into teaching quantum mechanics in conference proceedings that is not included here.

I have not read all articles listed in this appendix extensively, but have to a large extent relied on the abstract and the introduction of the articles. Apart from the list of recommended articles, I have no ambition to comment on what teaching suggestions I agree or disagree with. Also, I have no intention of pointing out any inaccuracies in arguments presented. I am aware that some of the articles contain inaccuracies, but believe that those articles may inspire teaching. In whole, I leave it to the reader to decide what she or he consider useful or less useful suggestions.

A few articles are listed twice, since they cover more than one category. These are items 1, 3, 4, 22, and 81 (also 72, 9, 96, 65 and 91, respectively). Some articles appear both in Paper 1 and in this appendix, since they report on both student depictions and quantum mechanics teaching.

Some recommended articles

Below is a list of articles I found particularly informative or useful.

1. French, A. P., & Taylor, E. F. (1971). Qualitative Plots of Bound State Wave Function. *American Journal of Physics, 39*(8), 961-962.
2. Müller, R. (2003). *Quantenphysik in der Schule* (Vol. 26): Logos.
3. Müller, R., & Wiesner, H. (2002). Teaching quantum mechanics on an introductory level. *American Journal of Physics, 70*(3), 200-209.
4. Styer, D. F. (1996). Common misconceptions regarding quantum mechanics. *American Journal of Physics, 64*(1), 31-34.

Empirical studies into quantum mechanics teaching

Below are lists of empirical studies into the teaching of quantum mechanics. The articles are presented in themes.

Concerning atoms

5. Budde, M., Niedderer, H., Scott, P., & Leach, J. (2002a). "Electronium": a quantum atomic teaching model. *Physics Education, 37*(3), 197-203.
6. Budde, M., Niedderer, H., Scott, P., & Leach, J. (2002b). The quantum atomic model 'Electronium': a successful teaching tool. *Physics Education, 37*(3), 197-203.
7. Fischler, H., & Lichtfeldt, M. (1992). Modern Physics and Students' Conceptions. *International Journal of Science Education, 14*(2), 181-190.
8. Kalkanis, G., Hadzidaki, P., & Stavrou, D. (2003). An instructional model for a radical conceptual change towards quantum mechanics concepts. *Science Education, 87*(2), 257-280.
9. Müller, R., & Wiesner, H. (2002). Teaching quantum mechanics on an introductory level. *American Journal of Physics, 70*(3), 200-209.
10. Trindade, J., Fiolhais, C., & Gil, V. (2005). Atomic orbitals and their representation: can 3-D computer graphics help conceptual understanding? *Revista Brasileira de Ensino Física, 27*(3), 319-325.

Concerning other pre-quantum topics, or how to introduce quantum mechanics

11. Ambrose, B. S. (2004). Investigating student understanding in intermediate mechanics: Identifying the need for a tutorial approach to instruction. *American Journal of Physics, 72*(4), 453-459.
12. Escalada, L. T. (1997). *Investigating the applicability of activity-based quantum mechanics in a few high school physics classrooms.* Unpublished doctoral thesis, Kansas State University, Manhattan, Kansas.
13. Gil, D., & Solbes, J. (1993). The introduction of modern physics: overcoming a deformed vision of science. *International Journal of Science Education, 15*(3), 255-260.
14. Lawless, C. (1982). Arts Students and Quantum Theory in an Open University History of Science Course. *Studies in Higher Education, 7*(2), 133-139.

15. Wittmann, M. C., Morgan, J. T., & Feeley, R. E. (2006). Laboratory-Tutorial activities for teaching probability. *Physical Review Special Topics Physics Education Research*, 1-26.

Concerning one-dimensional square potentials

16. Ambrose, B. S. (2004, 16 September 2005). *A Repeat Performance? Challenges In Developing Robust Conceptual Understanding in Quantum Mechanics*. Paper presented at the 2004 Physics Education Research Conference (PERC).
17. Bao, L., & Redish, E. F. (2002). Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics. *American Journal of Physics, 70*(3), 210-217.
18. Wittmann, M. C., Morgan, J. T., & Bao, L. (2005). Addressing student models of energy loss in quantum tunnelling. *European Journal of Physics, 26*(6), 939-950.

Non-empirical articles concerning teaching quantum mechanics

Below are lists of articles where the authors discuss or present ways of teaching quantum mechanics. The articles are presented in themes.

Concerning how to introduce quantum mechanics

19. Barad, K. (1995). A Feminist Approach to Teaching Quantum Physics. In S. V. Rosser (Ed.), *Teaching the Majority: Breaking the Gender Barrier in Science, mathematics and engineering* (pp. 43-75). New York: Teachers College Press.
20. Cuppari, A., Rinaudo, G., Robutti, O., & Violino, P. (1997). Gradual introduction of some aspects of quantum mechanics in a high school corriculum. *Physics Education, 32*(5), 302-308.
21. Fedak, W. A., & Prentis, J. J. (2002). Quantum jumps and classical harmonics. *American Journal of Physics, 70*(3), 332-344.
22. Greca, I. M. (2003). Does an Emphasis on the Concept of Quantum States Enhance Students' Understanding of Quantum Mechanics? *Science & Education, 12*, 541-557.
23. Hadzidaki, P., Kalkanis, G., & Stavrou, D. (2000). Quantum mechanics: a systemic component of the modern phyics paradigm. *Physics Education, 35*(6), 386-392.
24. Henry, R. C. (1990). Quantum mechanics made transparent. *American Journal of Physics, 58*(11), 1087-1100.
25. Hobson, A. (1996). Teaching quantum theory in the introductory course. *The Physics Teacher, 34*(4), 202-210.
26. Holbrow, C. H., Amato, J. C., Galvez, E. J., & Lloyd, J. N. (1995). Modernizing introductory physics. *American Journal of Physics, 63*(12), 1078-1090.
27. Ireson, G. (2000). The quantum understanding of pre-university physics students. *Physics Education, 35*(1), 15-21.
28. Lawrence, I. (1996). Quantum physics in school. *Physics Education, 31*(5), 278-286.

29. Michelini, M., Ragazzon, R., Santi, L., & Stefanel, A. (2000). Proposal for quantum physics in secondary school. *Physics Education, 35*(6), 406-410.
30. Ogborn, J. (1974). Introducing quantum physics. *Physics Education, 9*(7), 436-443.
31. Rüdinger, E. (1976). On the teaching of introductory quantum mechanics. *American Journal of Physics, 44*(2), 144-148.
32. Strnad, J. (1981). Quantum physics for beginners. *Physics Education, 16*(2), 88-92.
33. Taylor, E. F. (1998). The boundaries of nature: Special and general relativity and quantum mechanics, a second course in physics. *American Journal of Physics, 66*(5), 369-376.
34. Teixeira-Dias, J. J. C. (1983). How to Teach the Postulates of Quantum Mechanics without Enigma. *Journal of Chemical Education, 60*(11), 963-965.
35. Wise, M. N., & Kelley, T. G. (1977). Fundamental quantum mechanics -- a graphic presentation. *American Journal of Physics, 45*(4), 384-394.
36. Zollman, D. A., Rebello, N. S., & Hogg, K. (2002). Quantum mechanics for everyone: Hands-on activities integrated with technology. *American Journal of Physics, 70*(3), 252-259.

Concerning introducing quantum mechanics earlier in school

Several of the items in the section concerning how to introduce quantum mechanics also argues for introducing quantum mechanics earlier in school.

37. Barlow, R. (1992). Particle physics: from school to university. *Physics Education, 27*(2), 92-95.
38. Martin, J. L. (1974). Quantum mechanics from the cradle? *Physics Education, 9*(7), 448-451.
39. Shabajee, P., & Postlethwaite, K. (2000). What happened to modern physics? *School Science Review, 81*(297), 51-55.

Concerning the wave-nature of matter

40. Berstein, J., & Shaik, S. (1988). The wave-particle duality: teaching via a visual metaphor. *Journal of Chemical Education, 65*(4), 339-340.
41. Donati, O., Missiroli, G. P., & Pozzi, G. (1973). An experiment on electron interference. *American Journal of Physics, 41*(5), 639-644.
42. Hobson, A. (2005). Electrons as field quanta: A better way to teach quantum physics in introductory general physics. *American Journal of Physics, 73*(7), 630-634.
43. Hood, G. C. (1993). Teaching about quantum theory. *Physics Teacher, 31*(5), 290-293.
44. Jones, D. G. C. (1991). Teaching modern physics - misconceptions of the photon that can damage understanding. *Physics Education, 26*(2), 93-98.
45. Jones, D. G. C. (1994). Two slit interference - classical and quantum pictures. *European Journal of Physics, 15*(4), 170-178.
46. Merli, P. G., Missiroli, G. F., & Pozzi, G. (1976). On the statistical aspect of electron interference phenomena. *American Journal of Physics, 44*(3), 306-307.

47. Olsen, R. V. (2002). Introducing quantum mechanics in the upper secondary school: A study in Norway. *International Journal of Science Education, 24*(6), 565-574.
48. Schneider, M. B., & LaPuma, I. A. (2002). A simple experiment for discussion of quantum interference and which-way measurement. *American Journal of Physics, 70*(3), 266-271.
49. Strnad, J. (1981). Pitfalls in the teaching of introductory quantum physics. *European Journal of Physics, 2*(4), 250-254.

Concerning photons

50. Johnson, S. C., & Gutierrez, T. D. (2002). Visualizing the photon wave function. *American Journal of Physics, 70*(3), 227-237.
51. Marburger, J. H. (1996). What is a photon? *The Physics Teacher, 34*(8), 482-486.
52. Steinberg, R. N., Oberem, G. E., & McDermott, L. C. (1996). Development of a computer-based tutorial on the photoelectric effect. *American Journal of Physics, 64*(11), 1370-1379.

Concerning quantum uncertainty

53. Muiño, P. L. (2000). Introducing the uncertainty principle using diffraction of light waves. *Journal of Chemical Education, 77*(8), 1025-1027.
54. Sauer, G. (1979). Simulation of quantum-mechanical measurements with programmable pocket calculators. *American Journal of Physics, 47*(12), 1038-1041.

Concerning atoms

55. García-Castañeda, M. (1984). An abuse with the wave properties of matter. *American Journal of Physics, 53*(4), 373-374.
56. Morwick, J. J. (1979). Should Orbitals Be Taught in High School? *Journal of Chemical Education, 56*(4), 262-263.
57. Mott, N. (1964). On teaching quantum phenomena. *Contemporary Physics, 5*, 401-418.
58. Nielsen, L. D. (1978). A simplified quantum mechanical model of diatomic molecules. *American Journal of Physics, 46*(9), 889-892.
59. Shiland, T. W. (1997). Quantum mechanics and conceptual change in high school chemistry textbooks. *Journal of Research in Science Education, 34*(5), 535-545.
60. Stweart, A. (1976). Simulation of the Schrodinger equation on SHAC. *Physics Education, 11*(2), 104-110.
61. Summers, M. K. (1975). Solution of the Schrodinger equation for the hydrogen atom using a modular analogue computing system. *Physics Education, 10*(6), 424-429.
62. Taber, K. S. (2001). When the analogy breaks down: modelling the atom on the solar system. *Physics Education, 36*(3), 222-226.

Concerning quantum mechanics interpretations in teaching

63. Bettany, L. (1998). David Bohm and the implicate order: a new paradigm for physics teachers. *Physics Education, 33*(6), 419-424.
64. Bowman, G. E. (2002). Bohmian mechanics as a heuristic device: Wave packets in the harmonic oscillator. *American Journal of Physics, 70*(3), 313-318.
65. Greca, I. M. (2003). Does an Emphasis on the Concept of Quantum States Enhance Students' Understanding of Quantum Mechanics? *Science & Education, 12*, 541-557.
66. Müller, R., & Wiesner, H. (2002). Include interpretation in introductory quantum mechanics courses. *American Journal of Physics, 70*(9), 887.

Concerning one-dimensional square potentials

67. Andrews, M. (1998). Wave packets bouncing off walls. *American Journal of Physics, 66*(3), 252-254.
68. Burge, E. J. (1984). Over the barrier in wave mechanics. *Physics Education, 19*(1), 43-50.
69. Cedillo, A. (2000). Quantum Mechanical Tunneling through Barriers: A Spreadsheet Approach. *Journal of Chemical Education, 77*(4), 528-531.
70. Doncheski, M., & Robinett, R. W. (1999). Anatomy of a quantum 'bounce'. *European Journal of Physics, 20*(1), 29-37.
71. Doncheski, M. A., & Robinett, R. W. (2000). Comparing classical and quantum probability distributions for an asymmetric infinite well. *European Journal of Physics, 21*(3), 217-228.
72. French, A. P., & Taylor, E. F. (1971). Qualitative Plots of Bound State Wave Function. *American Journal of Physics, 39*(8), 961-962.
73. Goldberg, A., Schey, H. M., & Schwartz, J. L. (1967a). Computer-Generated Motion Pictures of One-Dimensional Quantum-Mechanical Transmission and reflection Phenomena. *American Journal of Physics, 35*(3), 177-186.
74. Goldberg, A., Schey, H. M., & Schwartz, J. L. (1967b). Edge Effects in One-Dimensional Scattering. *American Journal of Physics, 35*(8), 777-779.
75. Hasson, R., & Manners, J. (1995). A CAL package for teaching elementary quantum mechanics. *Physics Education, 30*(1), 33-37.
76. Jolly, P., Zollman, D., Rebello, S. N., & Dimitrova, A. (1998). Visualizing motion in potential wells. *American Journal of Physics, 66*(1), 57-63.
77. Robinett, R. W. (2000). Visualizing the collapse and revival of wave packets in the infinite square well using expectation values. *American Journal of Physics, 68*(5), 410-420.
78. Singh, C., Belloni, M., & Christian, W. (2006). Improving students' understanding of quantum mechanics. *Physics Today*, 43-49.
79. Styer, D. F. (2001). Quantum revivals versus classical periodicity in the infinite square well. *American Journal of Physics, 69*(1), 56-62.

Concerning other introductory quantum topics

80. Frederick, C. (1978). A mechanical model for teaching quantum mechanics. *American Journal of Physics, 46*(3), 242-243.
81. Kinderman, J. V. (1990). A computing laboratory for introductory quantum mechanics. *American Journal of Physics, 58*(6), 568-573.

82. Krass, A. S. (1978). Quantum measurements on a computer: An interactive teaching program. *American Journal of Physics, 46*(2), 152-158.
83. Roper, L. D., Braden, R. A., & Head, J. T. (1977). PSI quantum mechanics with optional lectures on videocassettes. *American Journal of Physics, 45*(5), 476-484.
84. Styer, D. F. (1990). The motion of wave packets through their expectation values and uncertainties. *American Journal of Physics, 58*(8), 742-744.
85. Summers, M. K. (1976). The analogue computer as an aid to teaching elementary quantum mechanics. *Physics Education, 11*(4), 296-302.
86. Summers, M. K. (1977). Numerical solution of the Schrodinger equation for a quantum harmonic oscillator (for teachers). *Physics Education, 12*(6), 387-389.
87. Summers, M. K. (1978). Programmable calculators as an aid in physics teaching. *Physics Education, 13*(4), 246-250.

Concerning intermediate quantum topics

88. DeWeerd, A. J. (2002). Interaction-free measurement. *American Journal of Physics, 70*(3), 272-275.
89. Ford, G. W., & O'Connell, R. F. (2002). Wave packet spreading: Temperature and squeezing effects with applications to quantum measurement and decoherence. *American Journal of Physics, 70*(3), 319-324.
90. Hussar, P. E., Kim, Y. S., & Noz, M. E. (1980). Quark model in the quantum mechanics curriculum. *American Journal of Physics, 48*(12), 1043-1049.
91. Kinderman, J. V. (1990). A computing laboratory for introductory quantum mechanics. *American Journal of Physics, 58*(6), 568-573.
92. Leubner, C. (1980). Pedagogical aspects of time-dependent rotation operators. *American Journal of Physics, 48*(7), 563-568.
93. Roussel, M. R. (1999). Redesigning the quantum mechanics corriculum to incorporate problem solving using a computer algebra system. *Journal of Chemical Education, 1999*(76), 1373-1377.
94. Shadmi, Y. (1978). Teaching the exclusion principle with philosophical flavor. *American Journal of Physics, 46*(8), 844-848.
95. Strnad, J. (1986). Photons in introductory quantum physics. *American Journal of Physics, 54*(7), 650-652.
96. Styer, D. F. (1996). Common misconceptions regarding quantum mechanics. *American Journal of Physics, 64*(1), 31-34.
97. de la Torre, A. C. (2002). Relativity of representations in quantum mechanics. *American Journal of Physics, 70*(3), 298-300.

Concerning the non-locality and the EPR paradox

98. Aravind, P. K. (2001). The magic tesseracts and Bell's theorem. *American Journal of Physics, 69*(2001), 348-353.
99. Carlson, J. A., Olmstead, M. D., & Beck, M. (2006). Quantum mysteries tested: An experiment implementing Hardy's test of local realism. *American Journal of Physics, 74*(3), 180-186.
100. Dehlinger, D., & Mitchell, M. W. (2002). Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory. *American Journal of Physics, 70*(9), 903-910.

101. Eberly, J. H. (2002). Bell inequalities and quantum mechanics. *American Journal of Physics, 70*(3), 276-279.
102. Galvez, E. J., Holbrow, C. H., Pysher, M. J., Martin, J. W., Courtemanche, N., Heilig, L., et al. (2005). Interference with correlated photons: Five quantum mechanics experiments for undergraduates. *American Journal of Physics, 73*(2), 127-140.
103. Holbrow, C. H., Galvez, E., & Parks, M. E. (2002). Photon quantum mechanics and beam splitters. *American Journal of Physics, 70*(3), 260-265.
104. Kwiat, P. G., & Hardy, L. (2000). The mystery of the quantum cakes. *American Journal of Physics, 68*(1), 33-36.
105. Mermin, D. N. (1994). Quantum mysteries refined. *American Journal of Physics, 62*(10), 880-887.
106. Mermin, N. D. (1981). Bringing home the atomic world: Quantum mysteries for anybody. *American Journal of Physics, 49*(10), 940-943.
107. Pospiech, G. (1999). Teaching the EPR paradox at high school? *Physics Education, 34*(5), 311-316.

Concerning other advanced topics

108. Havel, T. F., Cory, D. G., Lloyd, S., Boulant, N., Fortunato, E. M., Pravia, M. A., et al. (2002). Quantum information processing by nuclear magnetic resonance spectroscopy. *American Journal of Physics, 70*(3), 345-362.

Appendix II: Review of research into teaching and learning atoms, electrons and orbitals

Disclaimer

This review was written while still developing the review method. Terms used in this review are not the same as in Paper 1.

Though the terms may signal a cognitivistic approach to understanding quantum mechanics learning, the intention was never to impose this framework onto the literature study.

Education research on electrons, atoms and orbitals

This review summarizes quantum education research relating to orbitals and spin. The main point is that students often retain a classical view of electrons in general, and in particular a view of electrons orbiting the nucleus. There is also a comparison with orbital education in chemistry, and a presentation of teaching research related to orbitals.

1: Preface

1.1 The orbital

It might not be clear to all readers what is meant by orbitals. In this literature review, orbitals are the mathematical solutions to the Schrödinger equation associated with an atom or, in some cases, a complex of atoms.

In the most basic case, orbitals are the solutions to the Schrödinger equation for a one-electron atom where the nucleus is approximated by a point charge. These degenerated solutions are then separated according to different angular momenta, and the projection of this angular momentum on the z-axis (by convention).

In more complex (and realistic) forms of orbitals, they can be adjusted according to one or several of the following conditions:

- repulsive electric forces between electrons bound to the same nucleus;
- a non-point nucleus;
- electric forces from other nuclei in a molecule, sometimes resulting in *molecular orbitals*;
- disturbances from external atoms or molecules.

Note that orbitals are not the same thing as electrons bound to nuclei. Orbitals are a way of expanding the wave function associated with these electrons. For example, according to theories of identical particles, a multi-electron system must be in an anti-symmetrical superposition of several orbitals.

1.2 If you are new to physics education research

This document is a part of a literature review in physics education research, especially education research in quantum mechanics. Education research contains many different kinds of research; learning environment, curriculum research, learning problems and so on.

1.2.1 Understanding

A central and problematic idea in education research is what we generally call *understanding*. The problem with this word is that it cannot readily be defined, without either losing essential parts of the phenomenon, or making the definition to wide to be useful.

To get around this problem, education research often focuses on describing understanding rather than defining it. When it comes to quantum mechanics, for example, understanding the subject could be described as being able to discuss the theories, solve problems, being able to describe quantum mechanical processes, or to recognize when quantum mechanics is suitable to apply to a situation.

As you will see, this document and large parts of quantum education research focuses on *conceptual understanding*. Conceptual understanding is, briefly, being able to describe ideas or elements in what is taught. Examples of physics concepts are force, motion, wave functions, or orbitals.

Conceptual understanding is certainly not the only way of understanding physics, but it has shown some important advantages. Compared to problem-solving skills (for example), conceptual understanding has been shown to yield a more multi-faceted view of what is studied, and to enable the learner to apply the knowledge in a wider set of situations. Conceptual understanding is also likely to be more permanent. However, conceptual understanding must be complemented with other types of knowledge to be useful; for example problem-solving skills and experimental skills.

One part of research on conceptual understanding focuses on so-called *alternative conceptions*. Alternative conceptions (sometimes called misconceptions, pre-conceptions or naïve conceptions) are conceptual models that do not match, or even contradict, the accepted scientific models. One good example of alternative conceptions is that a moving object must have force acting on it.

It has been argued more than shown that alternative conceptions are obstacles for learning the accepted models. As we will see, quantum mechanics has some alternative conceptions to deal with as well. We will also see that one student may use different models for one and the same phenomenon, even when the models contradict each other. This is in line with education research in other areas.

2: Conceptual problems

2.1 The classical electron

When it comes to quantum mechanics and orbitals, the most frequently reported conceptual problem is that students tend to retain a classical view of electrons.

This means that students describe electrons as very small objects, usually moving about around the nucleus. The way students' describe this can be divided into three different categories, as described below.

2.1.1 Trajectories

A common and very broad conceptual problem is that electrons move in the atom (for example Bethge, 1992; Griffiths & Preston, 1992; Taber, 2002b). It seems that students tend to hold that electrons are moving about, even when the atom is a stable, non-changing state (for example Budde, Niedderer, Scott, & Leach, 2002a).

Even students who accept and use a probabilistic description of the electron's position may have trouble when it comes to electron trajectories, which could for example be described as a random zigzag between possible positions (Bethge, 1992).

2.1.2 The tiny planetary system

There are no large-scale surveys covering student understanding of orbitals, but the most common conceptual problem appears to be the view of classical electrons orbiting the nucleus (Bethge, 1992; Cervellati & Perugini, 1981; Nakiboglu, 2003). Sometimes this is explicitly compared to a tiny planetary system (for example Cros & Chastrette, 1988; Euler, Hanselmann, & Müller, 1999; Reiner Müller & Wiesner, 1999; Taber, 2005), and sometimes the orbits are expressed in more general ways. This difference could be explained through the researchers' questions and ways of presenting the student models, and does not necessarily reflect a difference in the students' ways of describing the atom.

As no large-scale surveys have been conducted, it is not possible to say how common this conceptual problem is. Mid-sized surveys (at least 100 participants) report of as much as 50% of pre-university students and 20% of chemistry teacher students at university level displaying this way of thinking about atoms (in Nakiboglu, 2003; Nakiboglu & Benlikaya, 2001). A French study from 1988 reports 16% of students explicitly referring to a planetary model. The study covered 199 first-year university students, who had studied both Bohr and Schrödinger models of the atom (Cros & Chastrette, 1988).

Minor studies (Bethge, 1992; Cervellati & Perugini, 1981; Euler et al., 1999; Griffiths & Preston, 1992; MacKinnon, 1999; Reiner Müller & Wiesner, 1999; Taber, 2005) also suggest that this conceptual problem is significant, even among students who have taken quantum mechanics courses. More than one author has reported that the planetary view of atoms seems particularly robust to change (Bethge & Niedderer, 1996; Reiner Müller & Wiesner, 1999).

Some education researchers suggest that this conceptual problem is strengthened by the atom concept sometimes being introduced by an analogy with the planetary system (Nakiboglu, 2003; Taber, 2001). The planetary model is also being used in some pre-university textbooks (Shiland, 1997). Taber (2001) has criticized the planetary model of the atom on the following basis:

- There are too many significant differences between an atom and the solar system.
- The differences between the solar system and an atom will cause conceptual problems in later physics and chemistry education.
- It is unlikely that students are more familiar with the solar system than with the concept of atoms (to a degree where it is useful as an analogy).

2.1.2.1 Quantized orbits

There are examples of students using general classical orbits – both circular and elliptic – when describing electrons in atoms (Bethge & Niedderer, 1996). However, among students who have studied quantum mechanics or dealt with orbitals in chemistry, it seems to be more common to hold that only certain orbits are allowed (Bethge & Niedderer, 1996; Reiner Müller & Wiesner, 1999; Taber, 2002b).

Some students describe the orbits as blurred, or talk about probabilities of different orbits (Bethge & Niedderer, 1996). This could be viewed as a semi-classical description of the atom, including both classical particles and the quantum probability.

2.1.3 Classical spin

There has been very little education research concerning spin, even when compared to relatively new field of quantum education research.

The research that has been done shows that some students describe quantum spin as a spatial movement, associated to the macroscopic meaning of the word. This could either be that an electron "moves about this volume of space that's called the orbital in *one particular direction*" (Taber, 2002b p.

155) or is rotating around its own axis in a particular direction (Taber, 2002a). (These studies concerned mainly chemistry students.)

Taber also reports a chemistry student explaining spin by the repulsive forces between electrons: “because they’re all going to be repelling each other and circling, always trying to get as far apart, ‘cause that’s why they’re always spinning” (2002b p. 155).

Wiesner (in Rainer Müller, 2003; , 1996) reports that 6 of 27 interviewed quantum mechanics students mentioned spin when asked for significant properties of quantum particles.

2.2 Hydrogenic[1] or one-electron orbitals only

Tsarpalis (1997) has conducted a mid-size study where about 10 % of the students describe orbitals as hydrogenic orbitals only, or as linear combinations of hydrogenic orbitals. The same author suggests that the reason for this is that in elementary courses, only hydrogen like orbitals are considered. This study was conducted with mainly chemistry students.

On top of this, Scerri (1991) has argued that the difference between orbitals and bound electrons are seldom pointed out for students, and also that the one-to-one correspondence only is valid for one-electron atoms.

2.3 Fragmented conceptions

Studies of students’ conceptions of orbitals have shown that many students have fragmented models for understanding: Students use different, even contradicting models in different situations (for example Bethge, 1992; Reiner Müller & Wiesner, 1999; Taber, 1998; Tan, Taber, Goh, & Chia, 2005). This is in line with general physics education research, where reports of fragmented and even contradicting models are rather frequent.

A illustrative example of this can be found in (Taber, 2002b p. 150–151), where a student discusses the atom structure:

> ”you could put energy into it, [of] the correct frequency, which a particular electron would absorb, it would absorb a photon of energy and be promoted to another vacant orbital...energy equals Planck’s constant times the frequency of the radiation.”

The same student explains that atoms do not collapse in the following way:

> “it’s something to do with, like the planetary motion, they had some initial kinetic energy, that’s why they don’t - well attract each other [i.e. move to-

[1] A hydrogenic atom is here used to refer to atoms with only one bound electron. In hydrogenic atoms, the bound electrons are mathematically treated in similar ways, regardless of the charge of the nucleus.

gether due to attraction]. Perhaps in creation they were given some initial kinetic energy, and some rotational energy, and that's why they rotate"

An even more explicit example can be found in a report by Bethge & Niedderer (1996 p. 13):

> Karl: I used to think of stability in such a way that Coulomb force and centrifugal force are exactly equal and balance each other. ... Of course, that doesn't fit at all to quantum mechanics.
>
> Interviewer: Do you see any connection between your ideas about probabilities we just discussed, and the stability of atoms?
>
> Karl: No, I don't. For me there is, on the one hand, this idea of stability and, on the other hand, the matter of probability. I completely keep them apart. For me these are two different things. ... I have two completely different ideas. With respect to stability, I should say, it moves on an orbit. In the other case I would speak of probability of finding.

Another fascinating example can be found in *I can't think about physics in chemistry* (Taber, 1998). A student presented with the left-hand picture below and asked about the attractive forces, correctly stated that case B would yield a greater attractive force. When subsequently asked to describe the attractive forces involved in the right-hand picture (describing an atom), "he thought that *all* electrons would be attracted *equally* despite some clearly being shown as further from the nucleus" (Taber, 1998 p. 1009).

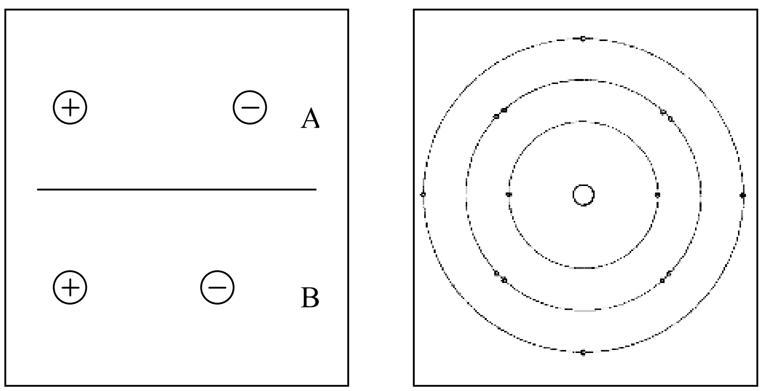

Figure 1. Two diagrams shown to a student in an interview study.

2.4 Conceptual problems in physics and chemistry

Orbitals is a concept that is used in both physics and chemistry. This brings some difficulties when summarizing education research on the topic: Since chemistry and physics students use orbitals in different ways, it is reasonable to believe that students have some different ways of describing and dealing with orbitals.

On the other hand, both physics and chemistry students have some shared educational background. It may also be noted that quantum mechanics and chemistry both deal with abstract phenomena, and thus rely heavily on models for building conceptual understanding. This, and other factors, makes it probable that physics and chemistry students share some conceptual problems when it comes to orbitals.

The similarities and differences between physics and chemistry makes comparisons between the two disciplines interesting, and orbitals is a good opportunity for such a comparison.

In this document, there are both results from chemistry and physics education research. In the cases where conceptual problems in chemistry and physics seem to agree, I have presented them without further comments. (This is the case of spin and several forms of classical views of electrons.) In some cases, results from chemistry education research is presented along with the physics education research. (This is pointed out in the relevant sections.)

In this section, a number of studies from chemistry education only are presented. The results may or may not agree with educational problems in physics, which may be an interesting question for future education research.

2.4.1 Dissimilarities from physics

Some conceptual problems in chemistry teaching seem not to be shared with physics at all. This emphasises the importance of the context where something is learnt; even though students obviously deal with the same real world phenomena, they apply different ways of thinking about it.

Some of the conceptual problems isolated to chemistry are:

- *The octet framework:* Some students explain chemical processes as a way of atoms to fill their electron shells, instead of applying orbital theory and energy levels. (For example Nakiboglu, 2003.)
- *Sharing out of nuclear attraction:* Some students hold that if one electron is removed from an atom the remaining will be attracted with greater force, claiming that the attractive force from the nucleus is shared between the nearby electrons. (Taber, 2001, 2005)
- *Social explanation to processes:* In chemistry, students tend to describe processes in anthropomorphical terms – the atom *wants* to give away an electron, *wants* to have a filled electron shell, et cetera. (Taber, 1997)

Tan *et al.* (2005) has conducted a cross-cultural study on the octet framework and the sharing out of nuclear attraction. The results show quite low internal validity, but quite high similarities between the two studied groups.

In all, it shows some evidence that the two conceptual problems are not associated with a specific university or country. The study covered about 150 students in Singapore and Great Britain.

2.4.2 More classical electrons

On top of the classical behaviour of electrons described in previous sections, there are also chemistry students having trouble with inter-atomic delocalization of atoms (for example in benzene compounds). Instead of a delocalized electron, as quantum theory describes it, some students interpret the bondings as electrons going back and forth very fast, or flickering between the two positions in a frequency making it more convenient to describe it as a probability distribution (Taber, 2005).

2.4.3 The closed volume orbital

Some students seem to be confusing the orbital with the common probability envelope diagrams (Taber, 2002b; Tsaparlis, 1997).

Taber (2002b) has reported on students describing the electron being located inside this orbital volume, while Tsarpalis has reported on student descriptions somewhat closer to the accepted scientific model: In a study covering 212 students passing a quantum chemistry course, 19% of the students described orbitals as a volume where there is (for example) 90% probability of encountering an electron (Tsaparlis, 1997). Some chemistry textbooks actually support this definition (see Tsaparlis, 1997 for a list of examples).

Both these studies were conducted with mainly chemistry students.

2.4.4 Electron shells

Some chemistry students also seem to describe orbitals as surfaces, either exclusively spherical surfaces or surfaces of the same shape as the equiprobability surfaces mentioned above (Taber, 2002b).

Students describe electrons either as fixed on the surfaces, moving around freely on the surfaces or sometimes – in case of atoms forming molecules – bound to the region where two surfaces intersect (Taber, 2002a, 2002b).

There are also multiple reports on students using the terms orbitals and (electron) shells interchangeably (MacKinnon, 1999; Taber, 2002b).

3: Research on teaching models

3.1 Textbooks

In an attempt to partly explain the poor understanding of the quantum model, Shiland (1997) has studied a number of chemistry books introducing the quantum model of atoms. To investigate how well the books helped students going from the Bohr model to the quantum model, he used a number of factors proposed to promote conceptual change (Posner, Strike, Hewson, & Gertzog, 1982):

- Students should be made aware that the old model is unable to describe certain phenomena or solve certain problems.
- The presented model should be comprehensible.
- It should be made clear that the presented model is plausible.
- The presented model should enable the student to explain previously unexplained phenomena.

All eight books investigated scored low or very low on all these factors.

3.2 Pictures and illustrations

Several studies on students' conceptions of orbitals suggest that pictures and diagrams play an important role in conceptual understanding (Bethge, 1992; Budde et al., 2002a; Niedderer, 1997; Taber, 1997), and sometimes the students' interpretations of images go beyond what is intended (Budde, Niedderer, Scott, & Leach, 2002b; Nakiboglu, 2003; Taber, 2005).

Seen in this light, the quantum model of the atom has a disadvantage compared to the classical model, which is comparatively often visualized – even outside a teaching environment.

In order to make orbitals and the atomic structure more visually accessible, some research has been made on tools for visualization (Niedderer, 1997; Trindade & Fiolhais, 2003; Trindade, Fiolhais, & Gil, 2005). These computer-aided visualizations have shown some improvements on students' conceptions of atoms and orbitals, but only small-scale studies (less than 50 students) have been reported (Niedderer, 1997; Trindade et al., 2005).

Some examples of computer programs developed to aid visualization of orbitals can be found in Trindade *et al.* (2005).

3.3 Electronium

In an attempt to find models that should be easier for students to accept, Budde *et al.* (2002a) have developed *the electronium model*, in short a model that describes electrons as a liquid shaped around the nucleus. See for example Niedderer & Deylitz (1998) for a closer description.

Evaluations of this model show an improved understanding among the students, even two years after the course. Students who have been introduced to the model don't seem to describe electrons as classical particles, nor that electrons move in stable atoms. One possible drawback of the model is students finding nodes in orbitals (in a non-s state) unintuitive, and the model also seems to strengthen the belief that electrons are built up by smaller particles (Budde *et al.*, 2002b).

References

Bethge, T. (1992). Schülervorstellungen zu grundlegenden Begriffen der Atomphysik. In H. Niedderer (Ed.), *Quantenphysik in der Schule* (pp. 215-233): Kiel: IPN.

Bethge, T., & Niedderer, H. (1996). Students' conceptions in quantum physics.

Budde, M., Niedderer, H., Scott, P., & Leach, J. (2002a). "Electronium": a quantum atomic teaching model. *Physics Education, 37*(3), 197-203.

Budde, M., Niedderer, H., Scott, P., & Leach, J. (2002b). The quantum atomic model `Electronium': a successful teaching tool. *Physics Education, 37*(3), 197-203.

Cervellati, R., & Perugini, D. (1981). The Understanding of the Atomic Orbital Concept by Italian High School Students. *Journal of Chemical Education, 58*(7), 568-569.

Cros, D., & Chastrette, M. (1988). Conceptions of second year university students of some fundamental notions in chemistry. *International Journal of Science Education, 10*(3), 331-336.

Euler, M., Hanselmann, M., & Müller, A. (1999, March 1999). *Students' views of models and concepts in modern physics*. Paper presented at the Research on Teaching and Learning Quantum Mechanics.

Griffiths, A. K., & Preston, K. R. (1992). Grade-12 students' misconceptions relating to fundamental characteristics of atoms and molecules. *Journal of research in science teaching, 29*(6), 611-628.

MacKinnon, G. R. (1999). Students' understanding of Orbitals: A Survey.

Müller, R. (2003). *Quantenphysik in der Schule* (Vol. 26): Logos.

Müller, R., & Wiesner, H. (1999). Students' conceptions of quantum physics. In D. Zollman (Ed.), *Research on Teaching and Learning Quantum Mechanics* (pp. 20-22): National Association for Research in Science Teaching (NARST).

Nakiboglu, C. (2003). Instructional misconceptions of Turkish prospective chemistry teachers about atomic orbitals and hybridization. *Chemistry Education Research and Practice, 4*(2), 171-188.

Nakiboglu, C., & Benlikaya, R. (2001). Misconceptions about orbital concept and modern atom theory (in Turkish). *Kasamonu Egitim Dergisi, 9*(1), 165-174.

Niedderer, H. (1997). *Teaching quantum atomic physics in college and research results about a learning pathway*.

Niedderer, H., & Deylitz, S. (1998). *Introduction to Atomic Physics: A concept based on the Schrödinger equation*: Institute of Physics Education, University of Bremen.

Posner, G. J., Strike, K. A., Hewson, P. W., & Gertzog, W. A. (1982). Accomodation of a scientific conception: Toward a theory of conceptual change. *Science Education, 62*, 211-227.

Scerri, E. R. (1991). The Electronic Configuration Model, Quantum Mechanics and Reduction. *The British Journal for the Philosophy of Science, 42*(3), 309-325.

Shiland, T. W. (1997). Quantum mechanics and conceptual change in high school chemistry textbooks. *Journal of Research in Science Education, 34*(5), 535-545.

Taber, K. S. (1997). Understanding chemical bonding---the development of A-level students' understanding of the concept of chemical bonding. *School Science Review, 78*(285), 85-95.

Taber, K. S. (1998). Sharing out nuclear attraction or "I can't think about physics in chemistry". *International Journal of Science Education, 20*(8), 1001-1014.

Taber, K. S. (2001). When the analogy breaks down: modelling the atom on the solar system. *Physics Education, 36*(3), 222-226.

Taber, K. S. (2002a). Compounding quanta: Probing the frontiers of student understanding of molecular orbitals. *Chemistry Education: Research and Practice in Europe, 3*(2), 159-173.

Taber, K. S. (2002b). Conceptualizing quanta: Illuminating the ground state of student understanding of atomic orbitals. *Chemistry Education: Research and Practice in Europe, 3*(2), 145-158.

Taber, K. S. (2005). Learning quanta: Barriers to stimulating transitions in student understanding of orbital ideas. *Science Education, 89*(1), 94-116.

Tan, K.-C. D., Taber, K. S., Goh, N.-K., & Chia, L.-S. (2005). The ionisation energy diagnostic instrument: a two-tier multiple-choice instrument to determine high school students' understanding of ionisation energy. *Chemistry Education Research and Practice, 6*(4), 180-197.

Trindade, J., & Fiolhais, C. (2003). *Students' Visualization and Conceptual Understanding of Atomic Orbitals Using a Virtual Environment.* Paper presented at the Proceedings of the 3rd IEEE International Conference on Advanced Learning Technologies.

Trindade, J., Fiolhais, C., & Gil, V. (2005). Atomic orbitals and their representation: can 3-D computer graphics help conceptual understanding? *Revista Brasileira de Ensino Física, 27*(3), 319-325.

Tsaparlis, G. (1997). Atomic orbitals, molecular orbitals and related concepts: conceptual difficulties among chemistry students. *Research in Science Education, 27*(2), 271-287.

Wiesner, H. (1996). Verständnisse von Leistungskursschülern über Quantenphysik. Ergebnisse mündlicher Befragungen. *Physik in der Schule*(34), 95.