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Vacua in String Theory

de Sitter Space and Stability in Flux Compactifications

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Abstract

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Our understanding of cosmology has evolved radically in the last decades. Current models demand the presence of dark energy in our universe and the most favored candidate behind this component is a small positive cosmological constant that characterizes a de Sitter (dS) spacetime. Simultaneously, theoretical physicists have stood up to the challenge of building a consistent theory of quantum gravity and string theory has raised as a strong contender.

In this thesis we present some explorations within supergravity, a low energy limit of string theory, studying non-supersymmetric vacua, its stability, and the possibility of finding dS.

We study the landscape of flux compactifications to produce dS with non-geometric fluxes. We find precise analytic procedures to find perturbatively stable dS near supersymmetric and no-scale Minkowski in a potential derived from type IIB compactifications. We also provide analytical evidence of naked singularities being produced in supergravity backgrounds after the introduction of anti-Dp-branes, at both vanishing and finite temperature.

In order to study the problem of semi-classical stability, we explore compactifications with anti-de Sitter as external space. We argue that truncations to closed-string-sector excitations of non-supersymmetric theories may be non-perturbatively protected by the existence of globally defined fake-superpotentials if they are perturbatively stable, a reasoning that goes in line with the standard positive energy theorems.

We find that non-supersymmetric solutions tend to manifest modes with masses under the Breitenlohner-Freedman bound once the open-string-sector is explored while supersymmetric solutions remain stable. We see this as a hint in the nature of the instabilities predicted by the weak gravity conjecture.

Keywords: String theory, Cosmology, Flux Compactifications, Landscape, Swampland

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Dedicated to my family and friends

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I J. Blåbäck, U. H. Danielsson, D. Junghans, T. Van Riet, and S. C. Vargas, *Localised anti-branes in non-compact throats at zero and finite T*, JHEP 1502 (2015) 018, arXiv:1409.0534 [hep-th].

II J. Blåbäck, U. H. Danielsson, G. Dibitetto and S. C. Vargas, *Universal dS vacua in STU-models*, JHEP 1510 (2015) 069, arXiv:1505.04283 [hep-th].

III U. H. Danielsson, G. Dibitetto and S. C. Vargas, *Universal isolation in the AdS landscape*, Phys. Rev. D94(12):126002, 2016, arXiv:1605.09289 [hep-th].

IV U. H. Danielsson, G. Dibitetto and S. C. Vargas, *A swamp of non-SUSY vacua*, JHEP 1711 (2017) 152, arXiv:1708.03293 [hep-th].

Contents

1. Introduction

An avalanche of ground breaking physical observations has deeply enriched our understanding of cosmology, general relativity (GR) and particle phenomenology in the last years and we expect many more to come in the near future. Simultaneously, theoretical physics has stood up to the challenge of building a consistent conceptual and mathematical framework that ideally would provide a quantum field theory (QFT) picture of gravity. It goes without saying that this is not a trivial problem, an inviting test, considering how effective both descriptions of nature, QFT and GR, are in their own range of energies and curvature.

In part, the success of QFT and GR has been an inconvenient, as testing quantum gravity models is therefore a challenge on its own. One possibility is exploring high energies in order to study and catalog whatever we can reach of the spectrum beyond the standard model. Another one is attempting to find consistency features of a unifying theory that one can extrapolate to low energy measurements. Along that line of thought, one comes to wonder what cosmology can tell us about quantum gravity.

Consider, for instance, the ΛCDM model or the correspondence model [R$^+$98, P$^+$99, SS00, WME$^+$13]. It is one of the most significant achievements of the past decades in observational cosmology. While it is under continuous scrutiny, it gives a reasonably good account of many observable properties of the universe, suggesting that about 70% of the present energy density corresponds to *dark energy*. The more favored suspect behind this component is a small positive cosmological constant, which characterizes a de Sitter (dS) universe. From the point of view of theoretical physics, establishing models that predict or at least consistently motivate this and other cosmological parameters represents a fundamental challenge.

A strong candidate for a description of quantum gravity is string theory. It has evolved drastically pervading all aspects of theoretical physics. As part of its features, it renders an interesting picture of spacetime physics. On the one hand, it comes with supersymmetry, a property that relates bosonic and fermionic degress of freedom. From the point of view of phenomenology, one expects this symmetry to be realized at energies higher than the ones so far probed, demanding the implementation and study of supersymmetry breaking mechanisms. On the other hand, it provides a constraint on the dimensionality of spacetime, requiring 6 additional dimensions to our 4-dimensional universe.

Observations demand this 6D extra space, often known as the *internal* or *compact* space, to have a finite and small volume. In our 4D *external* spacetime, our observed day-to-day world, we would currently be unable to detect these additional dimensions.

While these properties might sound discouraging, the fact that gravity appears in its formalism and constraints on spacetime are predicted, gives strength to the idea that string theory can eventually lead to a unifying formalism. In the present thesis we work fundamentally from the point of view of a low energy limit of this theory, *supergravity* (SUGRA). As we will see, by truncating the spectrum of excitations of strings in 10D, one reproduces the field content and dynamics of massless fields found in 10D supergravity. As a classical field theory, the hope is that one can capture the fundamental features of compactifications with supersymmetry breaking scenarios and produce models that are interesting from the point of view of our current understanding of cosmology.

In this work, we study some of the features of compactifications of supergravity with non-supersymmetric vacua. This includes solutions that contain 4D de Sitter as external spacetime as well as other vacua in which we explore the problems of stability and the presence of singularities. As we have mentioned, this requires us to deepen our understanding of field theories in the presence of gravity. The interplay of objects such as black holes and horizons with fields not too different from the electromagnetic, paints a rich and intriguing picture where one must go beyond perturbative analysis to see defining aspects of a theory of quantum gravity.

In supergravity, as in GR, gravity is seen as the interdependence between energy and the geometry of spacetime. In addition, there is a variety of fields and sources that enter its action. In particular, it turns out that 10D supergravity produces an enourmous amount of solutions. The term landscape has been used for those theories which could potentially provide a consistent effective field theory of quantum gravity and a good amount of effort has been put in exploring its extension, properties and phenomenology. At the center of these explorations is the question of whether supergravity vacua lives in the landscape or not.

In that spirit, there has been enormous progress in producing and cataloging solutions to 10D supergravity, with different amounts of supersymmetry, types of sources, fluxes and metric backgrounds, studied in different parameter regimes and understood at different orders in perturbation theory. These, and more to be found yet, are expected to shed some light on many of the challenges that high energy physics faces today.

In order to *see* the 4D world that one obtains in a given solution, one must perform a (consistent) dimensional reduction, in which some of the degrees of freedom of the 10D theory are integrated out while others remain as massless bosonic fields in the 4D theory. The massless scalars that are left in this process are called *moduli*. In a typical setup, the moduli include parameters of the internal space (like its volume), positions and field fluctuations of sources, and the value of the closed string dilaton.

If these scalar fields are not fixed, they manifest as massless particles in our 4D world and those are not favored by observations[1]. It is then necessary to find a mechanism by which they are constrained, a problem known as moduli stabilization [DK07, Gra06, BBCQ05]. The intention is then to generate a potential V, for these degrees of freedom, which effectively gives them a mass (see figure 1.1). In the context of dS, one would like to find positive masses such that fluctuations cost energy and hence their dynamics are effectively frozen. This would then fix the parameters of the compact space.

This was the major motivation that led to the field of flux compactifications. As we will see, besides gravitational degrees of freedom, 10D string theory allows for the existence of gauge fields. By using not trivial topologies in the internal space, fields can acquire (quantized) non-zero fluxes. Heuristically, one can expect the energy of these fields to couple these fluxes with parameters of the compact space. These couplings are indeed part of the 10D action of supergravity. Once we perform the process of dimensional reduction, a potential V is inherited as the remainder of the integrating process resulting in a 4D effective action with non-trivial dynamics for the moduli. We will see explicitly how V is directly produced by the metric and the fields of the internal space in this process. Among the many possible applications of this procedure, we will focus mostly on the study of (stable) critical points of this potential V as a function of the moduli (see figure 1.2). In these, not only the values of the moduli are then fixed but also the value of V acts precisely as a cosmological constant (in the 4D action).

This realization generated excitement, as it could then provide an interesting model with a dS universe. Unfortunately, it has been shown that, without the presence of non-perturbative effects, one usually does not find a stable universe with a positive cosmological constant. The underlying challenges for the possible candidates are common troublemakers: moduli stabilization, resilience against non-perturbative decays, the energy scale hierarchy problem and consistency of uplifting mechanism, among others. Despite the existence of some proposals, fundamental limitations still indicate important remaining questions that need to be answered satisfactorily. It should be mentioned that the

[1] Alternatively, one might be interested in inducing some dynamics for these fields to describe other phenomena like inflation, domain walls or false vacuum decays. The latter will be relevant in our discussion of stability.

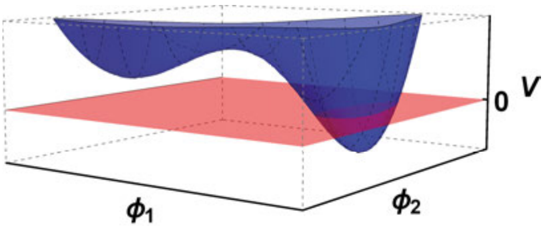

Figure 1.1. In flux compactifications, one can induce a potential V for the moduli, which here we have as the pair ϕ_1 and ϕ_2. V appears as a cosmological constant term in the 4D action and perturbatively stable points have positive moduli masses when $V \geq 0$. The hope is to find a stable dS vacuum in the landscape of string theory, similar to our universe.

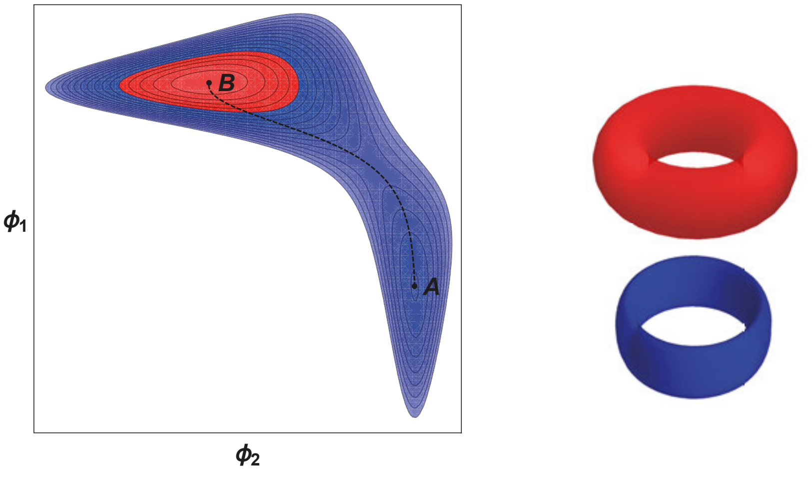

Figure 1.2. Different critical points are characterized by distinct values of the moduli and some of the moduli parameterize the properties of the compact space. On the left, we see the contour plot of the potential with a couple of critical points marked. By moving from a point A to a point B, the 4D cosmological constant changes as well as the internal space. On the right, we have a representation of this transition in the compact manifold. This transition can happen if it is possible for one vacuum to decay into the other.

de Sitter geometry presents particular complications when it comes to stablishing a clear picture of unitary evolution due to the fact that its assymptotics and finite number of observable degrees of freedom produce innate complications in defining scattering matrices or correlation functions [Wit01].

No-go theorems, in particular, have excluded a significant number of configurations that cannot achieve a positive cosmological constant and/or cannot attain perturbative stability [MN01, AB17]. This has produced a clearer picture of how and where to probe the landscape. It should also be mentioned that a lot of progress has been made recently in describing dS with non-linear realizations of supersymmetry, a proposal known as *de Sitter supergravity* [KQU15, BDK$^+$15, GdMPQZ17], which is not explicitly explored in this thesis. Nevertheless, in chapter 3 we present a no-go theorem that suggests the presence of undesirable singularities in constructions that are often used to produce dS. Since it was published, new evidence has appeared in this particular exploration which we also discuss in context in chapter 3.

In the field of flux compactifications, it is known that with the presence of available (although not well understood) *non-geometric* sources, moduli stabilization and uplifting can be achieved in twisted compactifications. Nevertheless, more satisfactory constructions can be performed, and recent proposals have suggested unexplored compact spaces that improve notably in consistency and provide reasonable scale hierarchies. In this thesis we provide some techniques to explore the space of fluxes systematically to produce examples of perturbatively stable dS near Minkowski vacua.

In this work we will also focus on the problem of stability. It is critical to find constructions whose lifetimes are consistent with our observable universe. The weaker type of stability, *perturbative* stability, is the one we have been discussing: there is a multitude of modes for which we have to induce a positive mass when it comes to a dS or Minkowski spacetime. It is also possible to explore other types of maximally symmetric spacetimes, known as anti-de Sitter (AdS) spacetimes, which have instead a negative cosmological constant. In this case, perturbative stability means having all masses above a negative value fixed by the Breitenlohner-Freedman (BF) bound [BF82b, BF82a]. Nevertheless, the fundamental idea stands: if a field has a mass below this value in a critical point, this is an unstable solution.

A stronger type of stability, non-perturbative or semi-classical stability, is even harder to establish for a specific configuration. Broadly speaking, non-perturbative decays include the possibility of quantum tunneling of a given critical point to other configurations with less energy. In order to claim that a point is semi-classically stable, one should be able to consistently define a Hamiltonian with a lower-bounded spec-

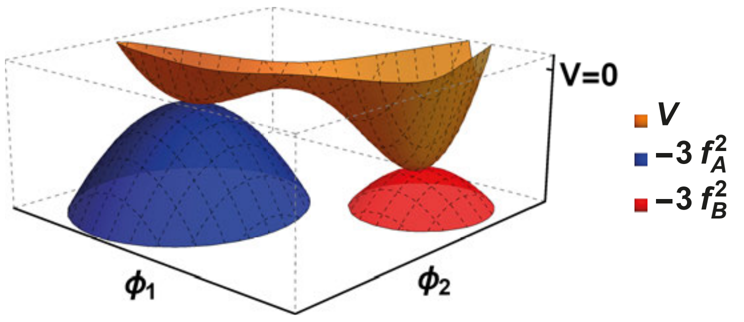

Figure 1.3. As we will see in chapter 5, there is a systematic mechanism that allows us to find fake-superpotentials for perturbatively stable points in AdS ($V < 0$). Here we plot a potential V and a representation of a couple of fake-superpotentials, f_A and f_B, for points A and B, respectively. The energy difference between $-3f_A^2$ and V can be read as the cost that a state in the minimum A must pay to reach other points in the moduli space. In this interpretation, one then concludes that A is stable. Identical arguments would then follow for point B and its fake-superpotential.

trum, whose vacua precisely matches the critical point. If this is possible, then one can classically define an energy in such set of configurations and state that *any* transition would cost energy.

In practice, this is a extremely challenging problem, more so with the presence of gravity. Usually, one instead attempts to find a decay channel semi-classically and establish its probability as a smoking gun for instabilities [CDL80, Wit82]. We will briefly explore some of the arguments that lead to suggest supersymmetry most likely withstands these processes, but, since it must be broken, it is necessary to improve our understanding of stability in non-supersymmetric solutions.

In order to probe the nature of these concerns we can explore compactifications with AdS as external space. A proposal that has been developed in the previous decades states that one may be able to establish semi-classical stability with the help of a positive energy theorem. It turns out that for some configurations one can define a lower-bounded quantity that matches the energy and whose minimum is achieved in the solution of interest. We will be more precise in chapters 3 and 4 but we can mention three basic features. First, constructing this quantity can be done in supersymmetric solutions with relative ease. Second, this idea has been extended for non-supersymmetric solutions [Bou84] but, as we will see, care must be taken before implying stability with this argument. Third, it has been shown that this construction is equivalent to the solution of a partial differential equation with specific boundary conditions [ST06]. The solution is often called fake-superpotential and we provide in this thesis a mechanism to find it (see figure 1.3).

In this spirit, an important development in the last decade has been the formalization of the weak gravity conjecture (WGC) [AHMNV07]. Originally, this proposal starts as an observation on charged or Reissner-Nordström black holes. When its mass and charge match in magnitude ($M = |Q|$) (in Planck units) the solution is called extremal. But if a black hole acquires a mass below its charge, it becomes a naked singularity which is undesirable from the point of view of general relativity. The statement of the conjecture comes as a constraint for quantum theories of gravity. More explicitly, it says in a consistent theory in which there is a U(1) gauge field, there must exist a super-extremal state. These are, in the language of the Reissner-Nordström geometry, particles with mass below the magnitude of its U(1) charge, i.e. $|Q| < M$. These super-extremal objects provide a decay channel for charged (non-supersymmetric) black holes avoiding the fate of undesirable remnants or naked singularities.

This conjecture later evolved in a stronger statement, also known as the swampland conjecture [OV16]. This new version indicates that only supersymmetric states are and stay extremal. Hence, in a consistent theory, super-extremal objects must exist so that non-supersymmetric states can decay, rendering them unstable. In particular, this would apply to vacua built out of non-supersymmetric sources. In chapter 6 we will present some of the explorations that we have done with the objective of clarifying the nature of this conjecture. We find that, by studying specific examples in flux compactifications, it is possible to associate the appearance of perturbative instabilities of the open-string spectrum with non-supersymmetric vacua, which do not manifest in solutions that preserve supersymmetry [DD16].

The fundamental goal of this work is to study cosmological features of string theory vacua as well as readdress standard gravitational problems with the machinery of supergravity. To this purpose, we use both analytical and numerical methods corresponding to constructions of explicit supergravity solutions (flux compactifications, harmonic superpositions, instantons and probe excitations), applications of the embedding tensor formalism and implementations of integrability techniques in Hamilton-Jacobi systems. These can be used as top-down and bottom-up approaches to the identification and manufacture of models with signatures of phenomenological interest that intersect many of today's cosmology and quantum gravity challenges.

In summary, exploring these territories seems both accessible and relevant to advance our understanding of cosmology and its connection to high energy physics. This thesis is divided as follows: In chapter 2, we discuss some basic ideas behind string theory and specific flux compactifications of type IIA and type IIB supergravity that we will use repeatedly through this work. In chapter 3 we describe the basic setup involved in no go theorems for dS in supergravity, some of which were discussed in paper I. In chapter 4, we propose our own 4D dS vacua, in the context of non-geometric compactifications of type IIB. There we show some simple but interesting techniques found in paper II that treat the problem of perturbatively stable dS near Minkowski critical points. In chapter 5, we go through the mechanisms explored in paper III used to compute fake-superpotentials and discuss semi-classical stability of AdS vacua in twisted compactifications of type IIA. In chapter 6, we discuss the implications of the WGC and the role of open-string sector excitations in the nature of the instabilities it predicts. These were explored with specific examples in paper IV with vacua in AdS_4 and AdS_7. We conclude with an epilogue in chapter 7.

2. Flux compactifications

In the search for a description of quantum gravity, string theory is one of the most promising candidates. It is a formalism in which one-dimensional (open and closed) strings are the fundamental objects that, by propagating in time, draw two-dimensional surfaces known as *world-sheets*. The worldsheet can be seen as a hypersurface embedded in a d-dimensional spacetime, a *target* space, in which its dynamics are dictated by its induced metric. Alternatively, the target space coordinates can be seen as a set of d massless scalars living in the worldsheet (see figure 2.1). Let us provide a brief introduction to the basics of this topic and its connection to supergravity. While this can be found in any standard book on string theory, this summary goes along the line of [BLT12].

Building on this idea, one can put both bosonic and fermionic degrees of freedom evolving in the worldsheet. This allows for the construction of a supersymmetric theory, the Neveu-Schwarz-Ramond (RNS) superstring[1]. From the point of view of the target space, one can get rid of ghosts by picking the critical number of dimensions, $d = 10$. By making use of the multiple symmetries enjoyed by this theory and picking a gauge, the equations of motion can be reduced to those of a free wave and a free dirac equation, for bosonic and fermionic fields respectively.

Contrary to the case of bosons, for fermionic modes in a closed string one can consider periodic or anti-periodic boundary conditions. These are known as Ramond (R) or Neveu-Schwarz (NS) conditions, respectively, and can be implemented independently for left and right movers in the closed string. More precisely, the spectrum of the closed string can be obtained as the (level matched) tensor product of two open-string spectra. This leads to the RR, NSNS, RNS and NSR sectors in the closed string[2].

Furthermore, by performing a truncation, known as Gliozzi-Scherk-Olive (GSO) projection, one can get rid of tachyons and attain spacetime supersymmetry. This is done by defining a notion of fermionic parity and eliminating all the states in the spectrum being parity odd.

There are five supersymmetric string theories: Type IIA, Type IIB, Type I, Heterotic-SO(32) and Heterotic-$E_8 \times E_8$. As theories of closed

[1]This is only one of formalisms used in describing the superstring.
[2]These are sufficient for the description of the spectra of type II theories.

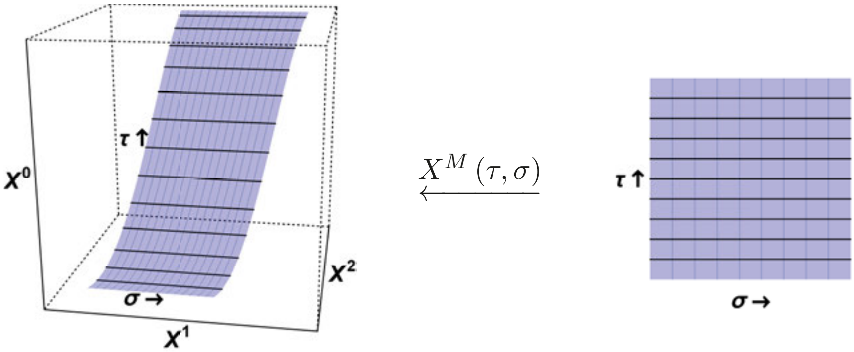

Figure 2.1. An open string worldsheet. The functions $X^M(\tau, \sigma)$ embed the worldsheet, parameterized by (τ, σ), into the target spacetime, whose coordinates are the X^M.

strings, their massless spectra contain a spin 2 field, g_{MN}, which corresponds to the graviton, an antisymmetric tensor field, B_{MN}, and a scalar, Φ, the dilaton. These are known as the universal bosonic sector of superstring theories and, for type II theories, they are all part of the NSNS sector.

Later we will see that massless RR fields may also enter in the actions of some of these superstring theories. Given the fact that they enjoy spacetime supersymmetry, it is to be expected that massless fermionic fields are also produced in the closed string. These include one or two gravitinos (spin $\frac{3}{2}$) and dilatinos (spin $\frac{1}{2}$). However, in classical solutions, we freeze and set these spacetime fermions to 0.

Let us write the bosonic action for the universal bosonic sector of the closed string. To do this, consider the scalar fields X^M, with $M = 0, ..., 9$ living in a worldsheet Σ with metric $h_{\alpha\beta}$, $\alpha = 0, 1$. Including terms with at most two world-sheet derivatives one finds

$$S_\Sigma = -\frac{1}{4\pi\alpha'} \int_\Sigma \mathrm{d}\sigma^2 \sqrt{-h} \left[h^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N g_{MN}(X) + \alpha' \Phi(X) \mathcal{R}(h) \right.$$
$$\left. + \epsilon^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N B_{MN}(X) \right], \qquad (2.1)$$

with $\mathcal{R}(h)$ the Ricci curvature of the worldsheet, the antisymmetric $\epsilon^{\alpha\beta}$ with $\epsilon^{01} = \frac{1}{\sqrt{-h}}$ and $\alpha' \sim l_s^2$, with l_s the string length scale. In reality, this action should be seen as the first order in a expansion in $\sqrt{\alpha'}/L = l_s/L$ where L is a typical length scale of the background geometry, e.g. its size or the radius of curvature. $\alpha' \ll L^2$ is a limit of low energy with respect to the string energy scale, in which strings *look* like point objects. Massive states in the superstring spectrum result with masses of the order l_s^{-1} and are, hence, decoupled in this limit.

The background fields g_{MN}, B_{MN} and Φ corresponding to massless excitations of the string can be seen as couplings in the conformal theory of scalars living in the 2D worldsheet. The requirement of conformal invariance imposes their β-functions to vanish. These can also be expanded in powers of $\sqrt{\alpha'}/L$ leading, at first order, to a set of equations of motion. These can be reproduced equivalently with the spacetime 10D action

$$S_{\text{eff}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x\sqrt{-g}\mathrm{e}^{-2\Phi} \left[R - \frac{1}{12} H_{MNO}H^{MNO} + 4\,\partial^M \Phi \partial_M \Phi \right] , \tag{2.2}$$

where R is the Ricci scalar of the 10D spacetime and $H = \mathrm{d}B$. One can also incorporate the remaining massless bosonic fields corresponding to each supertring theory consistently. The dynamics one reproduces with this formalism happen to be precisely reproduced by supergravity theories in 10D.

It is also worth mentioning that the superstring coupling, g_s, which plays a role in the string loop expansion, is then given by the vacuum expectation value of the dilaton $\langle\Phi\rangle$ with $g_s = \mathrm{e}^{\langle\Phi\rangle}$. The action we have derived should be seen as the tree level contribution in this expansion and hence one should be careful to work in the limit $g_s \ll 1$ in order to trust the result.

Let us focus now on type II supergravity theories. Both type IIA and type IIB enjoy maximal supersymmetry ($\mathcal{N} = 2$) and allow for the presence of RR fields. These are p-form gauge fields, usually denoted with $C^{(p)}$, with even (odd) values of p allowed for type IIB (IIA). The two supersymmetry generators are real and chiral (Majorana-Weyl) and can be chosen to have opposite chirality (type IIA) or the same chirality (type IIB). Their bosonic degrees of freedom can be arranged as

$$\begin{aligned} \text{IIA}\ :& \quad \left\{ g_{MN}, B_{MN}, \Phi, C^{(1)}_{M}, C^{(3)}_{MNO} \right\} \\ \text{IIB}\ :& \quad \left\{ g_{MN}, B_{MN}, \Phi, C^{(0)}, C^{(2)}_{MN}, C^{(4)}_{MNOP|\text{SD}} \right\} \end{aligned} \quad , \tag{2.3}$$

where the subscript SD on $C^{(4)}$ stands for self-dual.

In order to study 4D low energy descriptions from 10D string theory it is necessary to introduce compactifications, reducing the field content of 6D manifolds. As it turns out, background values for the internal space RR, NSNS fluxes and/or metric flux are fundamental in producing potentials for many of the resulting moduli. In the present chapter we will see some examples of 10D flux compactifications of massive IIA that will later play a role in our study of semiclassical stability. In chapter 3, we will also consider examples in type IIB to approach the problem of finding 4D dS in the landscape, and in this chapter we will also explore the basics of these solutions.

We work in type II theories and, for computational purposes, we follow the conventions of [Koe11], although with a change in the sign of H. This is a democratic string frame formulation of 10D supergravity. Chapter 3 is the exception as we work in Einstein frame. Here we quote some basic results to see this prescription in action. Using the formal sum $C = \sum_p C^{(p)}$ of the RR fields and introducing the corresponding RR forms $F^{(p+1)}$, we have (in the absence of sources)

$$F = \mathrm{d}C - H \wedge C + F^{(0)} \mathrm{e}^B , \tag{2.4}$$

where $F = \sum_p F^{(p+1)}$, e^B is to be interpreted as the formal exponentiation series of B and the $F^{(0)}$ term appears only in (massive) type IIA. This expression is to be understood as the family of order by order equations that one finds for each $(p+1)$-form. $F^{(0)}$ does not have propagating degrees of freedom and corresponds to a constant $m = F^{(0)}$, called the Romans mass. In the absence of sources and following this formulation, the equations of motion for the RR fields and the Bianchi identities take the form

$$(\mathrm{d} + H\wedge) *_{10} F = 0 , \tag{2.5}$$

$$(\mathrm{d} - H\wedge) F = 0 . \tag{2.6}$$

Even in the absence of sources, non-vanishing field strengths can manifest in manifolds with non-trivial homology groups. After integration over the corresponding non-trivial cycles, these (NSNS or RR) fluxes follow the same constraints implied by Dirac quantization. These quantized fluxes produce rich dynamics once one focuses on the dimensionally reduced theory.

The $p+1$-field strengths corresponding to these fluxes will remain invariant upon the addition of non-trivial harmonic p-forms to their p-form potentials. Physical degrees of freedom parameterize the deformations which are not gauge transformations. From the point of view of the 4-dimensional theory, these appear generically as scalar massless fields known as axions, which are a subset of the moduli of the theory. Other possible moduli are the dilaton, the metric moduli that parameterize deformations of the manifold and fields associated with the degrees of freedom of branes and other sources.

The contribution to the 10D action of the field strengths comes generically in the form of quadratic terms which then couple all types of moduli with the quantized fluxes. In addition, curvature terms will also manifest moduli dependence. Together, they produce dynamics in terms of an effective potential in the 4D theory and in the following sections we will explore some particular realizations of this process. Unfortunately, these massless degrees of freedom are usually not favored by phenomenology.

Finding a potential which fixes their value and gives them massess in an acceptable energy scale is the challenge of the moduli stabilization program.

Let us consider a simple example of the flux compactification process. We may pick the dynamics of an Einstein-Maxwell theory in six dimensions and apply the dimensional reduction process down to 4D to find the distinct contributions to the effective potential. This can be seen as a particular case of the solutions presented first in [FR80]. We can write the action as

$$S_{\text{EM}} = \frac{1}{16\pi G_{\text{N}}^{(6)}} \int \mathrm{d}^6x \sqrt{|g|} \left[R - \tfrac{1}{4} F_{(2)}^2 \right] , \tag{2.7}$$

where $G_{\text{N}}^{(6)}$ is the six-dimensional gravitational constant and $F_{(2)}$ is the standard electromagnetic field-strength. To make contact with the topic in question, we pick the geometry $\mathcal{M}_4 \times S^2$ and we add quantized flux limited to S^2 that satisfies

$$\int_{S^2} F_{(2)} = n_F , \tag{2.8}$$

with n_F an integer. We can be more precise and write the metric

$$\mathrm{d}s_6^2 = r^{-2}\,\mathrm{d}s_4^2 + r^2 \left(\mathrm{d}\xi_1^2 + \sin^2\xi_1 \mathrm{d}\xi_2^2 \right) , \tag{2.9}$$

where r is a moduli that controls the volume of the S^2, ξ_1 and ξ_2 parameterize the internal manifold and $\mathrm{d}s_4^2$ describes the external manifold. As we will see, the latter simply corresponds to AdS_4, the only type of vacua that we can reach with this compactification ansatz. $F_{(2)}$ can then be written as

$$F_{(2)} \propto n_F\, \mathrm{d}\xi_1 \wedge \sin\xi_1 \mathrm{d}\xi_2 . \tag{2.10}$$

While one can be interested in finding spacetime dependent dynamics for r, there is a simple solution to the equations of motion with a fixed value of this modulus. Both the Einstein and Maxwell equations can be satisfied with

$$r \propto n_F \;\text{ and }\; L \propto n_F^2 , \tag{2.11}$$

where L is the radius of AdS_4 and appropriate proportionality factors are omitted for presentation purposes. The same solution can be found consistently following an alternative interpretation. One may simply introduce the ansatz for the field strenght and metric in the 6D action and find that the resulting 4D effective Lagrangian is of the form

$$\mathcal{L}_{\text{4D}} \propto R^{(4)} - 2\,\Lambda , \tag{2.12}$$

where Λ is an effective potential for r that has the dependence

$$\Lambda \sim -\frac{1}{r^4} + \frac{n_F^2}{r^6} . \tag{2.13}$$

The first term in this expression comes from the contribution to the Ricci scalar from the internal manifold while the second comes from the square of the Maxwell field strength. The minimum of this potential precisely matches the result of the 6D equations of motion, producing a 4D vacuum with a cosmological constant that behaves as $\Lambda \propto -n_F^{-4}$. By introducing spacetime dependence for r, one can also compute the normalized mass for the modulus, $\frac{L^2 r^2}{4}\frac{1}{|\Lambda|}\partial_r^2\Lambda \propto n_F^4$, where the factor $\frac{L^2 r^2}{4}$ comes from the corresponding kinetic Lagrangian.

One then finds that, with enough flux, the radius can be made large (in fundamental units) producing small curvatures in this particular solution. While this goes well in the spirit of validity regimes of supergravity, the curvatures of both the internal and external manifold are comparable. Quantum effects are then of similar magnitudes in both scales which makes it difficult to accept this classical computation as an effective 4D field theory. In addition, as we pointed out, this type of construction is limited to AdS vacua.

Methods have been developed in tailoring supergravity solutions according to amount of supersymmetry, types of sources and internal manifolds as well as many other parameters to obtain more theoretical consistency and/or more interesting phenomenology. In the following sections we briefly discuss some approaches that tackle some of the issues we find in the previous example. In particular, 10D supergravity offers plenty more fields and degrees of freedom, and controlling this amount of data without a more systematic approach would not be advisable. The following constructions explore some of the tools that have allowed to categorize and classify vacua in specific geometries and the moduli and fluxes involved.

2.1 Massive type IIA on $\mathrm{AdS}_4 \times T^6/\mathbb{Z}_2^2$

Let us briefly explore the basic ideas behind SU(3)-structure manifolds. Here we follow the approach of [DDG15]. It has been noticed that reducing type II SUGRA over such manifold can preserve $\mathcal{N}=2$ SUGRA in 4D (i.e. 8 supercharges). A 6D manifold with SU(3)-structure is characterized by the presence of two globally defined SU(3) invariant fundamental forms: a holomorphic 3-form Ω and a real 2-form J. J and Ω are not closed in general, and in fact, their failure to be closed is parametrized by the 5 torsion classes which source curvature and specify the SU(3) structure,

$$\mathrm{d}J = \frac{3}{2}\mathrm{Im}\left(\overline{W}_1\Omega\right) + W_4\wedge J + W_3\,, \tag{2.14}$$

$$\mathrm{d}\Omega = W_1 J\wedge J + W_2\wedge J + \overline{W}_5\wedge\Omega\,, \tag{2.15}$$

where W_1 is a complex 0-form, W_2 is a complex primitive 2-form (i.e $W_2 \wedge J \wedge J = 0$), W_3 is a real primitive 3-form (i.e. $W_3 \wedge \Omega = 0$) and W_4 and W_5 are real 1-forms. We will focus on the case $W_4 = 0 = W_5$ which will suffice for the compactification of interest.

In terms of the fundamental forms, one can subsequently introduce a metric. This can be accomplished with the use of the quantity

$$I_m{}^n = \lambda \epsilon^{m_1 m_2 m_3 m_4 m_5 n} (\Omega_R)_{m m_1 m_2} (\Omega_R)_{m_3 m_4 m_5} \,, \tag{2.16}$$

where $\Omega_R = \mathrm{Re}\,\Omega\,,\ \Omega_I = \mathrm{Im}\,\Omega$ and λ is a moduli-independent normalization factor that produces $I^2 = -1$. The metric can then be obtained as

$$g^{(6)}_{mn} = J_{mp} I_n{}^p \,. \tag{2.17}$$

The Ricci scalar $\mathcal{R}^{(6)}$ for such 6D manifold can then be expressed as a function of the torsion classes as

$$\mathcal{R}^{(6)} = 2 \star_6 \mathrm{d} \star_6 (W_4 + W_5) + \frac{15}{2}|W_1|^2 - \frac{1}{2}|W_2|^2 - \frac{1}{2}|W_3|^2 - |W_4|^2 + 4 W_4 \cdot W_5 \,, \tag{2.18}$$

where norms and inner products are understood to be contracted with the metric g^{mn} and are weighted with a factor of $\frac{1}{n!}$ with n the order of the forms involved. For the norm of a complex form, it is understood the contraction with the complex conjugate to produce a real quantity.

Here we use the string frame in the 10D description. We write a 10D metric of the form

$$\mathrm{d}s^2_{(10)} = \tau^{-2} \mathrm{d}s^2_{(4)} + \mathrm{d}s^2_{(6)} \tag{2.19}$$

allowing us to fix τ in order to reproduce an Einstein frame formulation of the 4D external space. Up to factors, $\mathcal{R}^{(6)}$ then becomes part of the 4D scalar potential. From this point of view, we see clearly how the torsion classes, which are generically moduli dependent, contribute to the 4D dynamics.

The presence of torsion manifests in non-trivial geometry. We can use a basis of left-invariant vielbein η_a to see this in the Maurer-Cartan equations

$$\mathrm{d}\,\eta^a + \frac{1}{2} \omega_{bc}{}^a\, \eta^b \wedge \eta^c = 0 \,. \tag{2.20}$$

Twisted orbifold compactifications are then generated with constant metric flux $\omega_{bc}{}^a$. In addition, we can turn on background fluxes along the internal space for the set of type IIA gauge potentials: (NSNS) $H_{(3)}$ and (RR) $F_{(p)}$ with p even.

For twisted $X_6 = T^6/\mathbb{Z}_2^2$ the reduction can give rise to $\mathcal{N} = 2$ supergavity, more explicitly, a so-called STU model. Modding out by an extra $\mathbb{Z}_2$ orientifold action further reduces to four supercharges ($\mathcal{N} = 1$). Upon reduction, the scalar sector of the $\mathcal{N} = 1$ effective action contains seven complex fields or moduli. It is convention to denote them by $T_A = (S, T_I, U_I)$ with $A = 1, ..., 7$ and $I = 1, 2, 3$. We can visualize their role in the parametrization of the complexified Kähler form J_c and complex 3-form Ω_c with

$$
\begin{aligned}
J_c &= B_{(2)} + iJ && (2.21)\\
\Omega_c &= C_{(3)} + i\mathrm{e}^{-\phi}\Omega_R\,, && (2.22)
\end{aligned}
$$

where $B_{(2)}$ is the NSNS 2-form, $C_{(3)}$ the RR form and ϕ the dilaton. We have then

$$
\begin{array}{rl}
J_c = \sum_{I=1}^{3} U_I \omega_I & \text{with } \omega_I \in H^{(1,1)}(X_6) \\
\Omega_c = S\alpha_0 + \sum_{I=1}^{3} T_I \beta^I & \text{with } \alpha_0 \in H^{(3,0)}(X_6), \beta^I \in H^{(2,1)}(X_6)\,.
\end{array}
\tag{2.23}
$$

Furthermore, we can write the $\mathcal{N} = 1$ 4D superpotential as

$$
W_{\mathrm{IIA}} = \int_{X_6} \mathrm{e}^{J_c} \wedge F + \int_{X_6} \Omega_c \wedge \left(H_{(3)} + \mathrm{d}J_c\right)\,, \tag{2.24}
$$

with F the formal sum $F = \sum_p F_{(p)}$ of RR fluxes and similar interpretation of e^{J_c}, such that the paring picks the 6-forms for integration. With the relations we have provided, one finds W_{IIA} to be a polynomial linear in the fluxes and generically cubic in the moduli. Here we use the identification in table 2.1 for the fluxes [DGR11, DDG15]. For instance, turning on the Romans' mass $F_{(0)}$ (or $-a_3$) produces a cubic term $-a_3 U_1 U_2 U_3$. We follow the convention in [DDG15] by which the axions are associated to $\mathrm{Re}(T_A)$ and dilatons to $\mathrm{Im}(T_A)$. Consequently we write the Kähler potential

$$
K = \sum_{A=1}^{7} \log\left[-i\left(T_A - \overline{T}_A\right)\right]\,, \tag{2.25}
$$

and the standard $\mathcal{N} = 1$ potential follows then from

$$
V = e^K \left(-3\,|W|^2 + K^{A\overline{B}}\, D_A W\, D_{\overline{B}}\overline{W}\right)\,, \tag{2.26}
$$

with $K^{A\overline{B}}$ the inverse of the Kähler metric and $D_A W = [\partial_A + (\partial_A K)]\, W$ the Kähler covariant derivative. The imaginary parts of the complex fields carry geometric meaning and appear in the above Kähler potential, being constrained to be strictly positive. The real parts, on the contrary, have no sign restriction. In the next section we will discuss more explicitly the characterization and dynamics of $\mathcal{N} = 1$ potentials as well as the role of the moduli in the Kähler geometry.

Here we will focus on the isotropic case that corresponds to $T_I = T$ and $U_I = U$ and similar identifications for the I-dependent fluxes. It is then possible to write a explicit realization of the $SU(3)$ structure and the set of fluxes in vielbien components. The details can be found in [DDG15]. Here we limit to write the general form of the resulting 10D background,

$$F_{(0)} = f_1\,,\; F_{(2)} = f_2\,J\,,\; F_{(4)} = f_3\,J\wedge J\,, \tag{2.27}$$

$$F_{(6)} = f_4\,\text{vol}_6\,,\; H_{(3)} = h_1\,\Omega_I + h_2\,\frac{W_3}{|W_3|}\,, \tag{2.28}$$

where $f_1, ..., f_4$ and h_1, h_2 are functions of the moduli, and the 10D dilaton being a constant as well. It was shown that critical points of the type IIA action of these form are also critical points of the effective 4D potential obtained after dimensional reduction. In particular, spacetime-filling sources are generically required in the form of smeared O6/D6 sources, demanding the need to add a local term to the source-less type IIA action of the form

$$S_{\text{loc}} = -\int \mathrm{d}^{10}x\,\mathrm{e}^{-\phi}j_{(3)}\wedge\Omega\wedge\text{vol}_4\,, \tag{2.29}$$

with

$$j_{(3)} \;=\; j_1\,\Omega_I + j_2\,\frac{W_3}{|W_3|} \tag{2.30}$$

and j_1, j_2 linear functions of $N_6^{\|} = N_{\text{O6}_\|} - N_{\text{D6}_\|}$ and $N_6^{\perp} = N_{\text{O6}_\perp} - N_{\text{D6}_\perp}$. These are, respectively, the number of O6/D6 sources parallel and orthogonal the orientifold directions and can be computed in terms of the fluxes with

$$N_6^{\|} \;=\; 3b_1a_2 - b_0a_3 \tag{2.31}$$

$$N_6^{\perp} \;=\; (2c_1 - \tilde{c}_1)a_2 + c_0a_3\,. \tag{2.32}$$

As a consequence, the Bianchi identity for the $F_{(2)}$ RR form takes the form

$$\mathrm{d}F_{(2)} - F_{(0)}H_{(3)} = j_{(3)}\,. \tag{2.33}$$

2.2 Type IIB on $\mathcal{M}_4 \times T^6/\mathbb{Z}_2^2$

The $T^6/\mathbb{Z}_2^2$ orbifold and orientifolds thereof are very interesting setups since the internal manifold is its own mirror. As a consequence, one can have low energy effective descriptions which are related by dualities and are still formally described by the same effective field theory, where only the fields and the couplings have been transformed. In particular, this means that everything that we have introduced in the context of type IIA compactifications with O6-planes can be reformulated or reinterpreted in the language of type IIB compactifications with O3- and O7-planes.

Type IIB compactifications on $T^6/\mathbb{Z}_2^2$ with O3/O7-planes and D3/D7-branes are $\mathcal{N}=1$ supergravity theories. In the isotropic case, we have again three complex moduli, $\Psi^\alpha \equiv (S, T, U)$, that enjoy an $\mathrm{SL}(2)^3$ global symmetry. The S scalar is the axiodilaton and the role of the T and U scalars is swapped with respect to the the type IIA case. This is, T and U moduli are interpreted as Kähler and complex structure moduli respectively.

The kinetic Lagrangian

$$\mathcal{L}_{\text{kin}} = \frac{\partial S \partial \overline{S}}{\left(-i(S-\overline{S})\right)^2} + 3\,\frac{\partial T \partial \overline{T}}{\left(-i(T-\overline{T})\right)^2} + 3\,\frac{\partial U \partial \overline{U}}{\left(-i(U-\overline{U})\right)^2}\ , \tag{2.34}$$

can be derived from the isotropic limit of the Kähler potential we wrote in the previous section. In our convention, it is the imaginary part of the moduli that manifests in the Kähler potential, so our choice for an *origin* of moduli space is given by

$$S_0 \;=\; T_0 \;=\; U_0 \;=\; i\ . \tag{2.35}$$

Let us consider the different mechanisms giving rise to scalar potentials for the (S, T, U) moduli. Since there are no vector fields available, a potential cannot be induced by means of a gauging procedure. However, allowed deformations are given by W_{IIB}, the superpotential, which induces a scalar potential, just as we saw in the Type IIA case. It is also possible to associate the scales of the gravitino and the spin-$\frac{1}{2}$ fermions masses with the modulus of the superpotential and its covariant derivative, respectively. This fact will play a role in the realization of a systematic procedure to separate and solve the equations of motion and constraints for perturbatively stable de Sitter vacua that we will present in chapter 4. In addition, we will be able to directly relate W_{IIB} and W_{IIA} with an explicit identification of their fluxes.

Perturbative and non-perturbative contributions appear in general the superpotential. In the perturbative category, we may include fluxes like those we have already, such as NSNS and RR gauge fluxes, which have a clear 10D interpretation. In our case, $H_{(3)}$ and $F_{(3)}$ fluxes are allowed by the combination of sources. Compactifications with only $F_{(3)}$ and $H_{(3)}$ fluxes were originally studied in [GKP02]. In chapter 3 we will discuss the role of this work in the context of de Sitter proposals. The corresponding superpotential is [GVW00]

$$\begin{aligned} W_{\text{GKP}} \;=&\; a_0 \,-\, 3a_1\, U \,+\, 3a_2 U^2 \,-\, a_3\, U^3 \\ &-\, S\,\left(b_0 \,-\, 3b_1\, U \,+\, 3b_2 U^2 \,-\, b_3\, U^3\right)\ . \end{aligned} \tag{2.36}$$

couplings	Type IIB	Type IIA	fluxes
1	F_{ijk}	F_{aibjck}	a_0
U	F_{ijc}	$-F_{aibj}$	a_1
U^2	F_{ibc}	F_{ai}	a_2
U^3	F_{abc}	$-F_0$	a_3
S	H_{ijk}	$-H_{ijk}$	$-b_0$
SU	H_{ijc}	$-\omega_{jk}{}^a$	$-b_1$
SU^2	H_{ibc}	Q_i^{bc}	$-b_2$
SU^3	H_{abc}	R^{abc}	$-b_3$
T	Q_k^{ab}	H_{ibc}	c_0
TU	$Q_k^{aj} = Q_k^{ib}$, Q_a^{bc}	$\omega_{ka}{}^j = \omega_{aj}{}^k$, $\omega_{bc}{}^a$	c_1 , $\tilde{c}_1$
TU^2	$Q_c^{ib} = Q_c^{aj}$, Q_k^{ij}	$Q_b^{ci} = Q_a^{jc}$, Q_k^{ij}	c_2 , $\tilde{c}_2$
TU^3	Q_c^{ij}	R^{ijc}	c_3
ST	P_k^{ab}		$-d_0$
STU	$P_k^{aj} = P_k^{ib}$, P_a^{bc}		$-d_1$, $-\tilde{d}_1$
STU^2	$P_c^{ib} = P_c^{aj}$, P_k^{ij}		$-d_2$, $-\tilde{d}_2$
STU^3	P_c^{ij}		$-d_3$

Table 2.1. *Mapping between unprimed fluxes and couplings in the superpotential in type IIB with O3 and O7 and type IIA with O6. The six internal directions are split into "$-$" labelled by $i = 1, 3, 5$ and "$|$" labelled by $a = 2, 4, 6$. This identification can be found in [DGR11]. Here we adapted it to our type IIA sign conventions.*

The $\mathcal{N} = 1$ potential produced by this superpotential has a so-called *no-scale* symmetry due to the absence of the Kähler modulus T. This implies the presence of massless directions in the scalar potential, as we will see in a more general context in chapter 4.

Models with generalized fluxes

Starting from a geometric STU-model, one can start acting with $\mathrm{SL}(2)^3$ transformations to obtain dual models. In this way, it becomes natural to conjecture the existence of a completely duality-covariant superpotential [STW05] containing all possible STU-terms up to linear in S and up to cubic in T and U.

To get a picture of how this works, we can start with a generic 3-form flux on some 3-cycle, H_{ijk}, in T^6 [STW05]. Under a single T-duality in one direction, metric flux is generated which produces twisting of the topology of the form $\mathrm{d}x^i\mathrm{d}x^i \to \left(\mathrm{d}x^i - \omega_{jk}{}^i x^k \mathrm{d}x^j\right)^2$. This type of twisted torus manifolds appear in the context of the Scherk-Schwarz construction [SS79] and have been widely used in the context of flux compactifications. It is possible to pick a second direction that remains an isometry of the metric and implement a new T-duality. This produces a dual "torus" that possesses only a locally geometric description. This new type of flux is often denoted as Q-flux. Despite the fact that after this second T-duality there are no isometries left, persisting with a third one produces a new configuration which lacks even a locally geometric description and produces R-flux,

$$H_{ijk} \xleftrightarrow{T_i} {\omega_{jk}}^{i} \xleftrightarrow{T_j} Q_k^{ij} \xleftrightarrow{T_k} R^{ijk} \ . \tag{2.37}$$

One can study these fluxes from the point of view of the couplings they generate in the superpotential, where contributions coming from non-geometric fluxes generically entail terms with higher powers of the moduli. Nevertheless, it should be pointed out that in order to maintain T-duality between type IIA and type IIB, non-geometric fluxes are generally required. Similarly, invariance of the superpotential under S-duality requires the introduction of P-flux, which partners together with Q-flux when suffering the action of an SL(2) transformation.

Here we present the correspondence between generalized isotropic fluxes and superpotential couplings appearing in the $\mathcal{N} = 1$ effective 4D description. The complete generalized flux-induced superpotential can be written as

$$\begin{aligned} W_{\text{pert.}} = & \left(P_F - P_H\, S\right) + 3\, T \left(P_Q - P_P\, S\right) \\ & + 3\, T^2 \left(P_{Q'} - P_{P'}\, S\right) + T^3 \left(P_{F'} - P_{H'}\, S\right) \,, \end{aligned} \tag{2.38}$$

where[3] the couplings in

$$\begin{aligned} P_F &= a_0 - 3a_1 U + 3a_2 U^2 - a_3 U^3, \quad & P_H &= b_0 - 3b_1 U + 3b_2 U^2 - b_3 U^3, \\ P_Q &= c_0 + c_1 U - c_2 U^2 - c_3 U^3, & P_P &= d_0 + d_1 U - d_2 U^2 - d_3 U^3, \end{aligned} \tag{2.39}$$

are introduced and explained in table 2.1, whereas the details of the couplings in

$$\begin{aligned} P_{F'} &= a'_3 + 3a'_2 U + 3a'_1 U^2 + a'_0 U^3, \quad & P_{H'} &= b'_3 + 3b'_2 U + 3b'_1 U^2 + b'_0 U^3, \\ P_{Q'} &= -c'_3 + c'_2 U + c'_1 U^2 - c'_0 U^3 & P_{P'} &= -d'_3 + d'_2 U + d'_1 U^2 - d'_0 U^3, \end{aligned} \tag{2.40}$$

can be found in paper II. The first half of the terms (see table 2.1) are characterised by lower powers in T, i.e. up to linear, and represent fluxes

[3]In principle, the truncation to the isotropic sector gives rise to 40 fluxes. All the fluxes transforming in the mixed symmetry representations of GL(6) (i.e. Q, P and their primed counterparts) have in fact two fluxes $(c_1, \tilde{c}_1)$ etc. giving rise to one single coupling $(2c_1 - \tilde{c}_1)$ etc.. Without loss of generality, we set $\tilde{c}_1 = c_1$ etc..

which admit a locally geometric interpretation in type IIB (unprimed fluxes). The remaining ones (primed fluxes) appear with quadratic and cubic behaviour in T, and represent additional generalized fluxes which do not even admit a locally geometric description [ACFI06].

2.3 Massive type IIA on $\mathrm{AdS}_7 \times S^3$

In the study of stability of non-supersymmetric vacua, we will make use of compactifications with AdS as external spacetime. Here we will briefly mention the characteristics of an $\mathrm{AdS}_7 \times S^3$ solution of interest [PRT15]. While it is possible to turn on other bosonic fields, we will focus on a theory with only a metric and a scalar X that fully parameterizes the internal manifold. In this solution, we have localized 6D-brane charge that fills AdS_7. Despite this apparent simplicity, it allows for the presence of two critical points, one supersymmetric and one that is not. Stability relays then on the interaction between the brane charge at the background fluxes.

In particular, we will explore in the following chapters several situations in which these setups of sources will have to be explored carefully and we will try to follow distinct approaches to the identification of instabilities in flux compactifications. In this particular case, the dynamics of this scalar field will be enough to establish the appearance of tachyonic modes in the open-string-sector of the non-supersymmetric solution.

The solution can be described in 10D massive IIA supergravity with a background including the RR 1-form $C_{(1)}$, the Romans' mass $F_{(0)}$, the NSNS $B_{(2)}$ field, the dilaton Φ and the metric g. We write these as [PRT15]

$$\mathrm{d}s^2_{10} = \frac{1}{16} X^{-1/2} e^{2A} \mathrm{d}s^2_{\mathrm{AdS}_7} + X^{5/2} \mathrm{d}r^2 , \tag{2.41}$$

$$+ X^{5/2} e^{2A} \frac{1-\xi^2}{16w} \left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\psi^2\right) , \tag{2.42}$$

$$\mathrm{d}s^2_{\mathrm{AdS}_7} = e^{4z/L} \mathrm{d}s^2_{\mathrm{Mkw}_6} + \mathrm{d}z^2 , \tag{2.43}$$

$$B_{(2)} = -\frac{1}{8} e^A \cos\theta\, \mathrm{d}r \wedge \mathrm{d}\psi + \frac{e^{2A}\xi\sqrt{1-\xi^2}\sin\theta}{32\left[\xi^2 + X^5\left(1-\xi^2\right)\right]} \mathrm{d}\theta \wedge \mathrm{d}\psi, \tag{2.44}$$

$$\Phi = \Phi_0 + \frac{1}{2}\log\left(\frac{X^{5/2}}{w}\right) , \tag{2.45}$$

$$C_{(1)} = \frac{1}{4}\cos\theta\, e^{A-\Phi_0}\sqrt{1-\xi^2}\, \mathrm{d}\psi , \tag{2.46}$$

$$F_{(0)} = m , \tag{2.47}$$

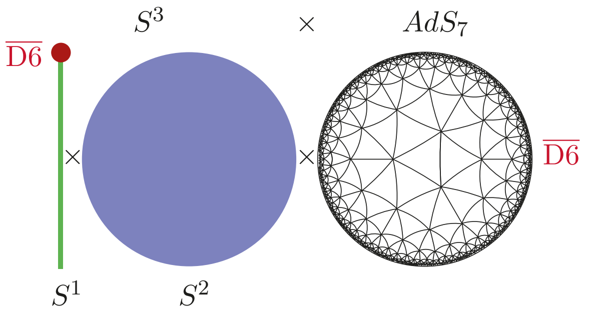

Figure 2.2. Geometry and sources of the $\mathrm{AdS}_7\times\mathrm{S}^3$ type IIA compactification. There is localized $\overline{\mathrm{D6}}$ charge in the north pole of the S^3.

where

$$w \equiv \xi^2 + X^5\left(1 - \xi^2\right) , \tag{2.48}$$
$$L \equiv 8\sqrt{15}\,X^4\left(8X^{10} + 8X^5 - 1\right)^{-1/2} . \tag{2.49}$$

It is useful to change from the coordinate r to a coordinate y via

$$\mathrm{d}r = \frac{9}{16}\frac{e^{3A}}{\sqrt{\beta}}\,\mathrm{d}y , \tag{2.50}$$

which allows one to analytically describe a family of solutions in terms of a function $\beta = \beta(y)$. In terms of the y coordinate, the north pole of S^3 is located at $y = -2$. A family of solutions was found in [PRT15] defined by this system that introduce distinct amounts of positively and negatively charged objects localized in the internal manifold.

Our case of interest is the AdS vacuum supported by a stack of anti-D6-branes ($\overline{\mathrm{D6}}$) located at $y = -2$, which corresponds to the following choice

$$\beta = \frac{8}{m}\,(y-1)\,(y+2)^2 , \tag{2.51}$$
$$\xi^2 = -\frac{y\beta'}{4\beta - y\beta'} , \tag{2.52}$$
$$e^A = \frac{2}{3}\left(-\frac{\beta'}{y}\right)^{1/4} , \tag{2.53}$$
$$e^{\Phi_0} = \frac{1}{12}\left(4\beta - y\beta'\right)^{-1/2}\left(-\frac{\beta'}{y}\right)^{5/4} . \tag{2.54}$$

The above background is a complete solution to the set of 10D field equations, provided that the scalar X satisfies

$$1 - 3X^5 + 2X^{10} = 0 , \tag{2.55}$$

which holds for $X = 1$ (SUSY extremum), and $X = 2^{-1/5}$ (non-SUSY extremum).

Since the above solutions are supported by *spacetime-filling* $\overline{\text{D6}}$-branes, they require the inclusion of a source term on the right hand side of one of the Bianchi identities to yield something of the form of

$$dF_{(2)} - F_{(0)}H_{(3)} = N_{(\text{D6})}j_{(3)} \ , \tag{2.56}$$

where $j_{(3)}$ denotes a 3-form current. Such $\overline{\text{D6}}$-branes would then fill AdS_7 and be fully localized at $y = -2$ inside S^3 (see figure 2.2).

In paper IV and chapter 6 of this work we review the 7D effective description of the above AdS vacua within $\mathcal{N} = 1$ gauged supergravity, where we see it as the coupling of a gravity multiplet with three extra vector multiplets.

3. 4D de Sitter: No Go's

There have been several approaches to the production of 4D vacua with positive cosmological constant. Several constraints in the space of higher dimensional constructions have been found [MN01, GKP02, IP01, GMPW04, GMW04], showing that the majority of well understood classical vacua is not simultaneously dS and perturbatively stable (see also [And18, ABVR17, AB17]). From the point of view of attempts to produce a quantum theory in a dS space-time, these difficulties can be expected, since definitions for a S-matrix or correlation functions are in a natural conflict with the asymptotics and measurable properties of this geometry [Wit01, Ban01].

In string theory, one of the most popular alternatives was presented in [KKLT03], in which supersymmetry is broken and an arbitrarily small and positive 4D cosmological constant is produced. However, this description does leave some questions unanswered. A first issue to look into is related to the supersymmetry breaking process involved. Breaking supersymmetry can be achieved by putting together sources which preserve different supercharges. With branes of opposite (brane) charge, one generically expects a perturbative instability of the construction associated to the possibility of sources moving towards each other and annihilating. An analogous instability with fluxes and branes of opposite charge (anti-branes) is harder to establish. A brane-flux annihilation process would start first with the nucleation of branes from the fluxes which then annihilate with the anti-branes. While meta-stability of such configurations is up for debate in many cases, it can be unquestionably useful: constructing de Sitter vacua [KKLT03], as a channel for brane inflation [KKL$^+$03], in holography [KPV02, ABFK07, KP11] and in the construction of non-extremal black hole micro-states [BPV12].

3.1 The KKLT construction

Let us focus on applications for dS constructions of [KKLT03]. The standard example uses the Klebanov-Strassler (KS) model [KS00] and places an spacetime-filling anti-D3-brane ($\overline{\mathrm{D3}}$) at its tip. Let us go through each of the basic ingredients.

The KS background [KS00] contains the geometry of a deformed conifold with a base of topology $S^2 \times S^3$ as internal space. F_5, F_3 and H_3

fluxes live in this six-dimensional non-compact Calabi-Yau manifold and are enough to support this background.

The work in [GKP02] showed how the insertion of negatively charged objects (such as an O3-plane) allows for the satisfaction of the tadpole conditions even in the case of a compact manifold. To be more precise, let us consider the Bianchi identity for the F_5 flux,

$$\mathrm{d}F_5 \;=\; H_3 \wedge F_3 \;+\; Q_3^{\mathrm{loc}}\,\delta_6\,(\mathrm{D3/D7/O3})\;, \tag{3.1}$$

where Q_3^{loc} is the net induced D3 charge of spacetime filling point sources. By integrating over the internal manifold, the left hand side of this relation vanishes for a compact manifold. Inserting D7-branes, O3-planes (or anti-D3-branes) gives then more freedom to balance out the contribution coming from the background fluxes. Unfortunately, even in this compact presentation, the Kähler moduli remain as runaway directions, a fact we observed previously in their effective 4D description.

In [KKLT03] stabilization is achieved through the introduction of non-perturbative terms in the superpotential. As a consequence, a supersymmetric configuration with a negative cosmological constant is attained. This correction is motivated with phenomena such as gaugino condensation or with D-brane instantons producing a correction of the form

$$\delta W \;\propto\; P_Z\,\mathrm{e}^{i\alpha T}\,, \tag{3.2}$$

with T the Kähler modulus, P_Z a holomorphic function of the remaining moduli and $\alpha > 0$. In the spirit of treating the stability of T explicitly, it is argued in [KKLT03] that P_Z can be considered constant. Finally, the uplift to de Sitter is made with the addition of the anti-D3-brane charge. The estimated raise in the vacuum energy goes as

$$\delta V \;\propto\; \frac{D}{\mathrm{Im}\,[T]^3}\,, \tag{3.3}$$

with D a quantity that depends on the number of anti-D3-branes and on the warp factor at the end of the KS throat.

3.2 The singularity

Supergravity solutions [MSS11, BGH10] revealed that the infra-red region of the KKLT construction has a diverging 3-form flux density

$$\mathrm{e}^{-\phi}|H_3|^2 \to \infty\,. \tag{3.4}$$

In the last years, a significant effort has been put in discerning the nature of these potential instabilities and establishing how hazardous they really are. Earlier literature had evidence of $\overline{\text{D3}}$-branes in the KS background to be present beyond the linearisation [Mas12, BGKM13a] and partial smearing [GJZ13] limits. Other setups with anti-branes in flux backgrounds led to similar results [GGO12, BGH11, Mas11, BDJ$^+$11, BDJ$^+$12, GOP13, CGH13, Blå13, BGKM14].

Heuristically, a singularity can be expected as the counterbalance to the pile-up of flux produced by gravitational and electromagnetic attraction between opposite charge fluxes and sources [DKV04, BDJ$^+$10, BDJ$^+$11, BDVR13]. If it is strong enough to generate a balance of forces, one would expect an increased but finite flux density. On the other hand, in the case of $\overline{\text{D3}}$-branes, the energy density is integrable [BGH10] and does therefore not immediately invalidate anti-branes as an uplifting mechanism [Jun14]. Furthermore, the solution is well-behaved in the UV and stands some very non-trivial tests [Dym11, DM13].

In order for the solution to be physical, the singularities arising at the classical level should be resolved in string theory by some mechanism. A solution comes with the Myers effect [Mye99] in which a Dp-brane polarizes into a higher dimensional object: a $(p+2)$-brane. The infinite result that we had with a point-like anti-Dp-brane might then be avoided with finite pile-up of the bulk fluxes once attraction is spread out over additional directions [PS00].

While some works have found some polarization channels to be forbidden [BJK$^+$12, BGKM13b, BGKM14], the work of [KPV02] explored the possibility of the $\overline{\text{D3}}$-branes in the KS geometry polarizing into spherical NS5 branes. This 5-brane is wrapped on a S^2 inside the S^3, leaving a transverse direction (in the S^3) which parameterizes the contribution of the source in a probe computation. Initially, at one extreme of this segment, say the *south pole*, the anti-D3 charge remains but if the source moves to the *north pole*, the background fluxes are reduced in brane-flux annihilation, effectively becoming a supersymmetric state with only D3 charge. This less energetic state would also lose the desired uplifting of the cosmological constant. Nevertheless, [KPV02] computed an effective potential for this dynamics showing that for a small enough anti-D3 charge the NS5 has a local minimum in the south pole configuration and for any quantum tunneling the decay rate is highly suppressed.

Recent evidence [CMDvRV16b] has shown that this polarization channel is indeed viable if only with a resulting geometry that deviates from the one predicted by [KPV02]. Despite the fact that the radius of the NS5 scales faster with the anti-brane charge than in the probe result, this polarized state still seems to be meta-stable avoiding the infinite pile-up. This is even more surprising considering the fact that further analysis of backreacted solutions [DGVR17] has provided evidence that singularities persist for the cases of $\overline{\text{D6}}$, $\overline{\text{D5}}$ and $\overline{\text{D4}}$-branes.

Renewed interest in the physics of antibranes has lead to contributions like [MMP$^+$15] proposing a brane-effective action for describing the physics of antibranes in the weak coupling regime opposite to that of [BGKM13a, BGKM13b, BGKM15]. In addition, several works [KQU15, BDK$^+$15, GdMPQZ17] propose an alternative scenario that could scape the instability of antibranes: an anti-D3-brane placed on top of an orientifold plane. In these, it is suggested that the low energy limit of this configuration can be described with the help of a Volkov-Akulov mechanism that has acquired the name of de Sitter supergravity.

3.3 A no-go for finite T

Generically, it is hard to compute fully back-reacted solutions consistent within the validity limits of supergravity. Nevertheless, it is often unnecessary to have the complete solutions to establish the presence of 3-form singularities. One can extract fundamental features of a solution and use the equations of motion to proof that, under certain assumptions, a divergent behavior must exist. These are 'no-go theorems', which have been established in different settings of solutions in type IIA [BDJ$^+$11, BBDVR13, BGKM13a, GJZ13] and M-theory [Blå13].

In paper I we presented a no-go theorem for finite temperature $T \neq 0$ arguing for the presence of singularities. In there the assumptions were laid clearly and it was pointed out that while all the hypothesis were well motivated, one of these could not be taken for granted. This was emphasized later in [CMDvRV16b], in relation to the results in [Har15]. While this is not considered a settled issue, this could potentially give a loophole for the case of $\overline{\text{D3}}$-branes. Nevertheless, it should be pointed out that [DGVR17] does establish the possibility of distinct arguments signaling the instability of $\overline{\text{D3}}$-branes in the KS throat. Their reasoning follows Smarr-like relations for the Arnowitt-Deser-Misner (ADM) mass, suggesting that one may be able to establish instabilities in the antibrane proposal given the innate unbalance between the gravitational and electromagnetic contributions to the on-shell action.

Keeping in mind the above caveats, in paper I we present analytic arguments showing that, for fully localised anti-branes, turning on a finite temperature does not resolve the singularities. The fundamental starting point is the work done in [GJZ13]. By using a general approach to the dynamics of localized sources and the 10D equations of motion, very interesting relations were found between the cosmological constant of the spacetime corresponding to the word-volume of the sources and their corresponding on-shell actions. This was accomplished by computing suitable combinations of the equations of motion and picking a specific gauge that gets rid of additional contributions from the background fluxes. In [GJZ13] this was done in the context of a compact external space. This master equation allowed them to produce a no-go for configurations of this type, by indicating that a positive cosmological constant is associated with a divergent behavior of the 3-form flux.

On the other hand, in paper I, we instead considered the case of a non-compact geometry and we explored both the vanishing and finite T cases. In doing so, an additional boundary term appears in the master equation. By integrating over specific regions and under specific assumptions about the background, one can still reproduce the divergent behavior found in the compact case. Even more, we were able to relate the boundary term to the ADM mass in the case of a vanishing cosmological constant, which points to the possibility of using charges of the geometry to study the fully non-linear back-reacted solution and establish similar results.

4. 4D de Sitter: Type IIB and non-geometric fluxes

In the present chapter we summarize some explorations of the landscape of dS vacua in (non-geometric) compactifiactions of 10D SUGRA. After dimensional reduction, one may use the dynamics of the remnant scalar degrees of freedom to explore potential cosmology-inspired models. Recent efforts have been made in developing systematic tools to explore perturbatively stable critical points in the resulting effective potentials for the moduli. Here we display some of the mechanisms that have been developed in this same spirit.

Previous studies of dS in this context [dCGM10, DD13, BDD13, DD14, BRZ14] have established a feature of critical points in the landscape with positive cosmological constant. In the space of parameters (such as fluxes or superpotential couplings), one tends to find localized *thin* regions of stable dS near Minkowski solutions. Deforming these Ricci-flat solutions is then a natural starting point for the generation and characterization of dS vacua.

4.1 Solving constraints linearly

In [KLVW14, MVW14] a procedure was established to approach the problem of dS vacua more systematically. Due to the form of the potential (2.26), finding critical points by directly exploring the moduli space is a non-linear problem. Fortunately, in some cases one can transform this system into an linear set of equations in the fluxes. This is possible for any supergravity model in which the scalars span a homogenous space. In those, a general non-compact $SL(2)^3$ duality transformation takes any point in moduli space to the origin (2.35) (see figure 4.1). Due to the general form of the scalar potential, the formulation of its extremality conditions at the origin is just given by a set of six algebraic quadratic equations in the superpotential couplings.

But one can go even further. In [KLVW14], it was shown that one can use all derivatives of the superpotential evaluated in the origin up to third order in a several-step-procedure to turn this into a linear problem. Specifically, if one starts out with the following arbitrary cubic superpotential

$$W(\Phi^\alpha) \; = \; W_0 + W_\alpha \, \Phi^\alpha + \frac{1}{2!} W_{\alpha\beta} \, \Phi^\alpha \Phi^\beta + \frac{1}{3!} W_{\alpha\beta\gamma} \, \Phi^\alpha \Phi^\beta \Phi^\gamma \, , \quad (4.1)$$

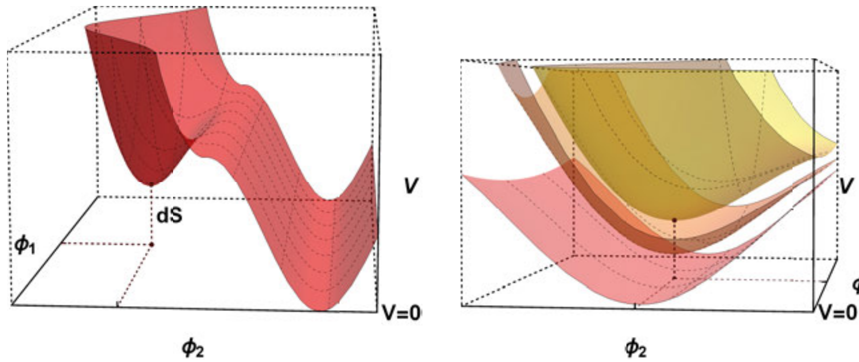

Figure 4.1. In the plot on the left, we have a potential V with a perturbatively stable dS point and a family of critical Minkowski points with a flat direction. In this picture, one would have to move in moduli space to go from critical Minkowsi to critical dS through a unique potential function, i.e. the fluxes in the compact space (that parameterize V) stay fixed. On the right we present an alternative picture that we use in our work. We stay in a fixed point of the moduli space, say $(\phi_1, \phi_2) = (0, 0)$, and then we variate the fluxes (and hence the potential function) in such a way that we stay critical through the whole trajectory in $(\phi_1, \phi_2) = (0, 0)$. By using our techniques, in this process the potential becomes positive and the point becomes perturbatively stable in all directions. This picture is known as the *going-to-the-origin* formulation.

where $\Phi^\alpha \equiv (S - i, T - i, U - i)$, and all W derivatives appear as arbitrary complex numbers, the problem becomes tractable. This is done by

- Choosing W_0, which fixes the gravitino mass scale,
- Choosing the W_α, which fix the SUSY-breaking scale parameters F_α, with
$$F_\alpha = D_\alpha W = W_\alpha + K_\alpha W, \tag{4.2}$$
- Solving the equations of motion, $D_\alpha V = 0$, which are linear in the $W_{\alpha\beta}$'s, needing at most 6 real parameters to be solved,
- Tuning the (non-normalized) mass matrix
$$\left(m^2\right)^I{}_J = \begin{pmatrix} K^{\alpha\bar{\gamma}}\, V_{\bar{\gamma}\beta} & K^{\alpha\bar{\gamma}}\, V_{\bar{\gamma}\bar{\beta}} \\ K^{\bar{\alpha}\gamma}\, V_{\gamma\beta} & K^{\bar{\alpha}\gamma}\, V_{\gamma\bar{\beta}} \end{pmatrix}, \tag{4.3}$$
to become positive, using the fact that it is linear in the $W_{\alpha\beta\gamma}$'s.

Due to the homogeneity constraint and the form of the Kähler potential (2.25), we must restrict ourselves to superpotentials that are first order polynomials in S, i.e. $W_{SS} = W_{SSS} = W_{SST} = W_{SSU} \stackrel{!}{=} 0$. This implies that the most general parameterization of the form (4.1) compatible with the class of STU-models presented in chapter 2 counts 16 complex parameters, of which 1 is given by W_0, 3 by W_α, 5 by $W_{\alpha\beta}$ and the remaining 7 by $W_{\alpha\beta\gamma}$.

<table>
<tr><td></td><td colspan="2">Perturbatively stable dS near</td></tr>
<tr><td></td><td>SUSY Minkowski</td><td>No-scale Minkowski</td></tr>
<tr><td rowspan="3">Properties</td><td>$W_0 = 0 = W_\alpha$
$\alpha = S\,,T\,,U$</td><td>W_0 arbitrary
$W_S = -K_S\,W_0$
$W_U = -K_U\,W_0$
$W_T = W_{T\alpha} = 0$
$W_{T\alpha\beta} = 0$
$\alpha = S\,,T\,,U = \beta$</td></tr>
<tr><td colspan="2">These Mkw points satisfy $D_\alpha V = 0$ and $m^2 \geq 0$</td></tr>
<tr><td>2 massless directions</td><td>2 or 3 massless directions</td></tr>
<tr><td>Moving to dS</td><td>$W_0 = \kappa_0\,\epsilon$
$W_\alpha = \kappa_\alpha\,\epsilon$</td><td>$W_S = -K_S\,W_0 + \kappa_S\,\epsilon$
$W_T = \kappa_T\,\epsilon$
$W_U = -K_U\,W_0 + \kappa_U\,\epsilon$</td></tr>
<tr><td colspan="3">$D_\alpha V = 0$ uses generically 6 real components out of 10 $W_{\alpha\beta}$</td></tr>
<tr><td>Uplifting mass</td><td>1) $V_{\alpha\beta} = 0$
(degeneracy condition)
2) $m^2_{\rm sG} > 0$ if
$n_{\rm eff} > 3(1+\gamma)$
(inequality for the κ's)</td><td>$V_{\alpha\beta} = 0$ only consistent
with a Mkw with
2 massless directions</td></tr>
<tr><td colspan="3">$V > 0$ becomes an inequality for the κ's
$V_{\alpha\beta} = 0$ uses generically 12 real comps out of 14 $W_{\alpha\beta\gamma}$ (if used)</td></tr>
</table>

Table 4.1. *Characterization of SUSY and no-scale Minkowski and their deformations to perturbatively stable dS.*

In total these are 32 real parameters, a number we found already when exploring the set of real superpotential couplings in chapter 2, when we discussed the most general duality-invariant superpotential for our STU-model. The mapping relating generalized fluxes to complex superpotential derivatives is, in fact, linear and invertible. This in particular implies that, whenever a stable dS solution is found for a certain superpotential derivative configuration, this will always admit an STU-realisation in terms of 32 generalized perturbative fluxes.

4.2 Perturbatively Stable dS near Minkowski vacua

In paper II, we focused on dS vacua near SUSY Minkowski and no-scale Minkowski. In table 4.1, we have the fundamental ideas of this work.

Further applications and explicit examples can be found in the paper itself.

In the context of these polynomial superpotentials with generalized fluxes, we have very explicit parameterizations of these Minkowski vacua. On one hand, for SUSY Minkowski, we set vanishing supersymmetry breaking parameters. In addition, due to the form of the potential, the superpotential must also vanish in order to have a vanishing cosmological constant. These conditions are only possible if the first derivatives vanish. On the other hand, no-scale Minkowsi is characterized by the vanishing of all the T-derivatives as well as vanishing supersymmetry breaking parameters for S and U. Both of them are solutions of the equations of motion and, in our context, SUSY Minkowski always has two massless directions while no-scale Minkowski possesses either 2 or 3 massless directions.

Deforming these solutions to obtain a non-vanishing cosmological constant can be achieved with a single perturbation parameter ϵ, introducing deviations in the zeroth and first order derivatives of the superpotential, as shown in table 4.1. Here we denote these deformations with κ's, which enter as a quadratic polynomial in the potential. The second derivatives $W_{\alpha\beta}$ will contain 10 real parameters of which we can use 6 to solve the linear problem corresponding to $D_\alpha V = 0$. By following this procedure, the equations of motion will remain solved for arbitrary values of ϵ, although in principle we are mostly interested in nearby perturbatively stable de Sitter. It is to be expected that for large ϵ, the solution turns unstable or attains a non-positive cosmological constant.

The positivity of the mass matrix can be approached in several ways. One possibility is to make use of a degeneracy condition, $V_{\alpha\beta} = 0$, which makes the mass-matrix block diagonal and also turns out to enforce a pairwise organization of the mass spectrum. This condition is then irreconcilable with no-scale Minkowski containing 3 massless directions. Nevertheless, in the case of SUSY vacua, it is a very useful tool. Enforcing it is relatively simple since this is a linear system in the $W_{\alpha\beta\gamma}$'s. The outcome is a family of solutions with only 3 distinct masses. The massless pair corresponds to the sGoldstini, whose mass can be uplifted by using a bound for their average (see paper II). In doing so, the result is a stable dS parameterically close to a SUSY Minkowski.

5. Semi-classical stability: Positive Energy Theorems

Establishing semiclassical stability for a specific configuration of fields in curved space is not a trivial task. In practice, one may be able to compute whether a system is perturbatively stable if one has access to all the possible field excitations around a critical point. Even if that can be a challenge, in principle this reduces to finding no tachionic masses in whichever potential V describes their dynamics. This is, in Minkowski or de Sitter, finding that the eigenvalues of V_{IJ} are all above or equal to 0. Or in anti-de Sitter, finding that the eigenvalues of the normalized mass^2 matrix $|V|^{-1}K^{IJ}V_{JK}$ are all above or equal the Breitenlohner-Freedman (BF) bound [BF82b, BF82a], $-\frac{D-1}{2(D-2)}\kappa^2$, with K_{IJ} the kinetic matrix and D the spacetime dimension[1].

Still, this will not guarantee the absence of non-perturbative decays to distinct configurations. It was first suggested for the case of supergravity theories that supersymmetric vacua with asymptotic anti-de Sitter or Minkowski enjoyed full stability. This was argued with the help of a positive energy theorem: in these solutions one finds that the time component of the ADM momentum is semi-positive and vanishes only at the critical point. Any other state is then at a higher energy and semiclassical stability is secured.

It was later found that this theorem still has caveats. For instance, the conclusion heavily relies on comparing systems with the same asymptotic geometry. Later examples [Wit82] show configurations in which decays through KK instantons are possible to states with distinct topology to the original vacuum. Even more, there is also the possibility of vacua with negative energy failing the criteria presented in [BD68]. Caution for arguments in favor of this approach is then necessary: the validity of a positive energy theorem should be studied carefully and, quite possibly, case by case.

The prospect for non-supersymmetric theories is even more complicated. The expectation from the point of view of [Bou84] is that under certain circumstances it is possible to extend the protection enjoyed by SUSY. With the help of a Witten spinor, a semi-positive quantity can be built in the cases when the vacuum enjoys perturbative stability. In

[1]This particular expression applies for scalar fields, which will be the focus of our discussion, but for general states it acquires spin dependence.

[AHHM07] this argument was further explored, stressing that one can only relate this quantity to the energy if additional boundary conditions are taken into account that bound it from below.

Given the potential effectiveness of this criterion, it was established in [ST06] the equivalence between this approach and the possibility to construct a function of the fields called a fake superpotential. This is a solution f to a partial differential equation of the form,

$$V \;=\; -\,3\,f^2 \;+\; 2\,K^{IJ}\,\frac{\partial f}{\partial \phi^I}\,\frac{\partial f}{\partial \phi^J}\,, \tag{5.1}$$

where $I, J...$ run over the scalar field content and K_{IJ} is the corresponding kinetic matrix. As we will see, this expression can be read as a relaxed version of the $\mathcal{N} = 1$ supergravity potential V and it is automatically satisfied if f is built out of a holomorphic superpotential. The given vacuum would then appear supersymmetric in the sense that f (together with V) would be critical at that point. As it turns out, this equation also appears in the study of flat domain walls between vacua via the Hamilton-Jacobi equation, a field that also pondered the questions of stability.

Since an interesting overlap comes from the combined approach, here we discuss their interplay in a generic scenario. The basic setup is well known since [CGS93]. We can write a domain wall ansatz interpolating between two solutions,

$$\begin{aligned}
\mathrm{d}s^2 \;&=\; g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu && (5.2)\\
&=\; \mathrm{d}r^2 \;+\; \mathrm{e}^{2a(r)}\;\tilde{g}^{(3)}_{bc}\;\mathrm{d}y^b\mathrm{d}y^c && (5.3)\\
&=\; \mathrm{d}r^2 \;+\; \mathrm{e}^{2a(r)}\left[-\,\mathrm{d}t^2 \;+\; S(t)^2\left(\frac{\mathrm{d}\rho^2}{1-k\rho^2} \;+\; \rho^2\mathrm{d}\varphi^2\right)\right]\,. && (5.4)
\end{aligned}$$

This can be used for domain walls with Minkowski solutions as well, but we will focus on AdS_4 critical points. Flat domain wall solutions extend through the whole interval $r \in (-\infty, \infty)$, but domain walls with positive curvature start from a finite r_- and terminate at $r \to \infty$ (domain walls with negative curvature can be covered as well, see [CGS93]). We will see this later explicitly, as has also been observed in the literature (e.g. [BEFP14]).

The dynamics we intend to describe is simply given by Einstein-Hilbert gravity plus n scalars. We intentionally avoid making explicit use of supersymmetry as we intend to describe potential decays from unstable (non-supersymmetric) solutions. We have the Gibbons-Hawking-York boundary terms $\mathcal{S}_{GHY}$ and counter terms,

$$\mathcal{S} \;=\; \mathcal{S}_{EH} \;+\; \mathcal{S}_{GHY} \;+\; \mathcal{S}_{ct} \;+\; \mathcal{S}_{\phi}\,, \tag{5.5}$$

with

$$\mathcal{S}_{EH} \;=\; \frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^4x \,\sqrt{-g}\; R\,, \tag{5.6}$$

$$\mathcal{S}_{GHY} \;=\; \int_{\Sigma-} \mathrm{d}^3y\,\sqrt{|\gamma|}\;\Theta_\Sigma \;-\; \int_{\Sigma+} \mathrm{d}^3y\,\sqrt{|\gamma|}\;\Theta_\Sigma\,, \tag{5.7}$$

$$\mathcal{S}_{ct} \;=\; \int_{\Sigma+} \mathrm{d}^3y\,\sqrt{|\gamma|}\;(\alpha \;+\; \beta\; R_\Sigma)\;, \tag{5.8}$$

$$\mathcal{S}_\phi \;=\; \int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{-g}\;\left[-\,\frac{1}{2}\,g^{\mu\nu}\;K_{IJ}\;\partial_\mu\phi^I\;\partial_\nu\phi^J \;-\; V(\phi)\right]\,, \tag{5.9}$$

$$\;=\; \int_{\mathcal{M}} \mathrm{d}^4x\,\sqrt{-g}\;\left[-\;g^{\mu\nu}\;K_{A\bar{B}}\;\partial_\mu\Phi^A\;\partial_\nu\bar{\Phi}^{\bar{B}} \;-\; V(\Phi,\bar{\Phi})\right]. \tag{5.10}$$

$\Sigma-$ and $\Sigma+$ are timelike hypersurfaces defined by $h(r) \;=\; r \;=\; r_\pm$ with (r_-, r_+) the interval covered by r. We denote with $\mathcal{M}$ the volume surrounded by these boundaries. $\mathcal{S}_{ct}$ contains, up to this point, the standard minimal AdS_4 counter terms for a flat domain wall. The remaining notation goes as follows: A normal vector to any of those hypersurfaces can be written as $n^\mu \;=\; g^{\mu\nu}\;\nabla_\nu\; h \;=\; \delta^\mu_r$. The induced metric in Σ is $\gamma_{bc} = \mathrm{e}^{2a(r)}\;\tilde{g}^{(3)}_{bc}$ or, alternatively, we may work with $\gamma_{\mu\nu} \;=\; \gamma_{bc}\;\delta^b_\mu\;\delta^c_\nu$. We write the extrinsic curvature as $\Theta_{\Sigma\;\mu\nu} = -\;\gamma^\tau{}_\mu\;\nabla_\tau\;n_\nu$ and direct computation shows that $\Theta_{\Sigma\;bc} \;=\; -\;a'\;\gamma_{bc}$. It is interesting to see the full variation of the action,

$$\begin{aligned}\delta\mathcal{S} \;=\;& \frac{1}{2}\int_{\mathcal{M}} \mathrm{d}^4x\sqrt{-g}\,(G_{\mu\nu} - T_{\mu\nu})\,\delta g^{\mu\nu} + \int_{\mathcal{M}} \mathrm{d}^4x\frac{\delta\mathcal{S}_\phi}{\delta\phi^I}\delta\phi^I \\ &+\frac{1}{2}\int_{\Sigma+} \mathrm{d}^3y\sqrt{|\gamma|}\left(\Theta^{bc}_\Sigma - \Theta_\Sigma\gamma^{bc} + \alpha\gamma^{bc} - 2\beta G^{bc}_\Sigma\right)\delta\gamma_{bc} \\ &-\frac{1}{2}\int_{\Sigma-} \mathrm{d}^3y\sqrt{|\gamma|}\left(\Theta^{bc}_\Sigma - \Theta_\Sigma\gamma^{bc}\right)\delta\gamma_{bc}\;,\end{aligned} \tag{5.11}$$

where we have written, as usual,

$$G_{\mu\nu} \;=\; R_{\mu\nu} \;-\; \frac{1}{2}\,g_{\mu\nu}\;R\,, \tag{5.12}$$

$$G_{\Sigma\;bc} \;=\; R_{\Sigma\;bc} \;-\; \frac{1}{2}\,\gamma_{bc}\;R_\Sigma\,, \tag{5.13}$$

and, following the prescription in [BK99], we introduce the stress energy tensors

$$
\begin{aligned}
T_{\mu\nu} &= -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{S}_\phi}{\delta g^{\mu\nu}}\,, &(5.14)\\
T^{bc}_{\Sigma+} &= \frac{2}{\sqrt{|\gamma|}}\left.\frac{\delta\mathcal{S}}{\delta\gamma_{bc}}\right|_{\Sigma+} = \Theta^{bc}_{\Sigma} - \Theta_\Sigma\,\gamma^{bc} + \alpha\,\gamma^{bc} - 2\,\beta\,G^{bc}_{\Sigma}\,, &(5.15)\\
T^{bc}_{\Sigma-} &= -\frac{2}{\sqrt{|\gamma|}}\left.\frac{\delta\mathcal{S}}{\delta\gamma_{bc}}\right|_{\Sigma-} = \Theta^{bc}_{\Sigma} - \Theta_\Sigma\,\gamma^{bc}\,. &(5.16)
\end{aligned}
$$

At this point it is always possible to discuss this problem in what is known as the thin wall approximation. This is done by solving the equations in motion in two distinct regions, assuming a fixed critical value of the moduli, and then connecting the two solutions with the standard junction conditions. In doing so, one relates the discontinuity in the metric with the surface stress-energy tensors we have computed (up to counter-terms). With the help of the latter, it is possible to directly assign a tension to the domain wall that unfolds in this discrete jump. While this has been done in the past, the intention in this work is to treat the complete problem of a continuous solution between two critical points.

In order to obtain consistent equations of motion, one must enforce the condition

$$\frac{\ddot{S}}{S} = q_0 = \frac{k+\dot{S}^2}{S^2}\,. \tag{5.17}$$

The solutions for the distinct possible values are summarized in [CGS93]. The fundamental cases of interest are, up to diffeomorphisms,

$$q_0 = 0\,,\; k = 0\,,\; S = 1\,, \tag{5.18}$$

and

$$q_0 > 0\,,\; k = q_0\,,\; S = \cosh(\sqrt{q_0}t)\,. \tag{5.19}$$

The former is a flat static wall and the latter is a time dependent bubble. Due to this condition we find [BK99]

$$
\begin{aligned}
T^{bc}_{\Sigma+} &= \gamma^{bc}\left(2\,a' + \alpha + 2\,\beta\,q_0\,\mathrm{e}^{-2a}\right)\,, &(5.20)\\
T^{bc}_{\Sigma-} &= \gamma^{bc}\left(2\,a'\right)\,. &(5.21)
\end{aligned}
$$

In the thin wall limit, [CGS93] discusses how solutions with $q_0 > 0$ could represent false vacuum decay (this will be the case if the tension violates the Coleman-De Luccia bound [CDL80]), while flat walls with $q_0 = 0$ represent static and stable configurations (which saturate the Coleman-De Luccia bound). Hence, in this framework, one can describe the transition from an unstable critical point towards a different solution via a curved domain wall, consistent with the intuition of true vacuum bubbles nucleating.

For future reference, we write here how the equations of motion look like after the previous identifications have been made,

$$
\begin{aligned}
0 &= -2q_0 \mathrm{e}^{a} + \frac{d}{dr}\left(a' \mathrm{e}^{3a}\right) + \mathrm{e}^{3a}\left(\frac{1}{2}R - \frac{1}{2}K_{IJ}\phi'^{I}\phi'^{J} - V\right), && (5.22)\\
0 &= 3q_0 \mathrm{e}^{-2a} - 3a'^{2} + \frac{1}{2}K_{IJ}\phi'^{I}\phi'^{J} - V\,, && (5.23)\\
0 &= \phi''^{I} + 3a'\phi'^{I} + \Gamma^{I}{}_{JL}\phi'^{J}\phi'^{L} - K^{IJ}\partial_J V\,, && (5.24)
\end{aligned}
$$

with $\Gamma^{I}{}_{JL} = \frac{1}{2}\, K^{IM}\left[\partial_J K_{LM} + \partial_L K_{JM} - \partial_M K_{JL}\right]$.

We can also perform a basic analysis of the required holographic renormalization [Ske02, BK99] that provides some intuition of the dynamics that distinguish flat and curved walls. Here we only aim for a minimal addition of counterterms that take care of the most obvious divergences, but notice that in general other contributions can appear in a more complete case by case treatment. Following that logic, we can pick the counterterms to be $\alpha = -\frac{2}{l_+}$ and $\beta = -\frac{l_+}{2}$, with $\Lambda_\pm = -\frac{3}{l_\pm^2}$ the cosmological constant on each side of the domain wall. As we will see, this choice is consistent since we may write the on-shell action also as

$$
\begin{aligned}
\mathcal{S}_{\text{On-Shell}} &= 2\, q_0 \int \mathrm{d}^3y\, \sqrt{|\tilde{g}^{(3)}|}\, \left\{\int_{r_-}^{r_+} \mathrm{e}^{a}\, dr \;-\; l_+\, \mathrm{e}^{a(r_+)}\right\} && (5.25)\\
&\quad + \int_{\Sigma+} \mathrm{d}^3y\, \sqrt{|\tilde{g}^{(3)}|}\, \mathrm{e}^{3a}\, \left(2\, a' - \frac{2}{l_+} - l_+\, q_0\, \mathrm{e}^{-2a}\right)\\
&\quad - \int_{\Sigma-} \mathrm{d}^3y\sqrt{|\tilde{g}^{(3)}|}\, \mathrm{e}^{3a}\, \left(\, 2\, a'\,\right).
\end{aligned}
$$

We can now analyze this result assuming *optimal* conditions. With optimal we mean two things. First, a smooth potential which may require minimal or none V-dependent counter terms. Second, a solution in which a approaches the $V = \Lambda_\pm$, $\phi' = 0$ limit fast enough[2]. In a way, we are exploring the *thick* domain wall as a perturbation from the thin domain wall, which is already well understood.

In the case of a flat domain wall (5.25) produces a finite result. On the one hand, with $q_0 = 0$, the bulk term in the previous equation vanishes. On the other, we can see that the asymptotic behavior of the boundary terms is acceptable as long as e^{2a} approaches e^{2r/l_+} when $r \to \infty$ and

[2]This limit is the solution of (5.23) for a in the thin domain wall limit which can be computed analytically in a straightforward way. Close to the ends of the domain wall, the functional form of the solution should depend only on whether the wall is flat or curved.

e^{2r/l_-} when $r \to -\infty$ fast enough. This is, the expected limit for pure AdS.

For $q_0 > 0$, this regularization also cures divergences. The expected asymptotic behavior of a in the IR and UV is given by the solution of the EOMs with $V = \Lambda_\pm = -3/l_\pm^2$, $\phi' = 0$, i.e.

$$e^a \sim l_\pm \sqrt{q_0} \sinh\left[\frac{r - r_0}{l_\pm}\right] . \tag{5.26}$$

This expression vanishes when $r = r_0$ and therefore, as we have discussed, there is a finite lowest value for r, r_-, which defines the IR limit of the curved domain wall. At this point, Σ_- contracts to a point. Assuming smoothness, in the on-shell action we are then left only with the UV boundary contribution and the bulk integral. In turns out that, with this asymptotic behavior of a, the bulk integral will be convergent in the IR. In the UV the integral diverges but the combination inside the brackets in (5.25) is finite. On the other hand, for the UV boundary, it turns out that both a' and the counter terms have finite contributions in the UV limit and put together they vanish for smooth and converging solutions.

5.1 The Hamilton-Jacobi Equation

Here we will briefly discuss how the Hamilton-Jaccobi formalism is extremely useful, not only in the computation of domain wall solutions, but also in the construction of fake superpotentials. This intimate relation has been explored in the last decades and here we will use a systematic procedure for solving this equation in the present context, and connecting it to the discussion of stability.

The effective action can also be written as

$$\mathcal{S} = \mathcal{S}_{EH} + \mathcal{S}_{GHY} + \mathcal{S}_{ct} + \mathcal{S}_\phi = \int_{\mathcal{M}} d^4x \sqrt{|\tilde{g}^{(3)}|}\, L + \mathcal{S}_{ct} , \tag{5.27}$$

with

$$L = e^{3a}\left(\frac{1}{2}R - \frac{1}{2}K_{IJ}\phi'^I\phi'^J - V\right) + e^{3a}\nabla_\mu\left[(-\Theta_\Sigma)\, n^\mu\right] . \tag{5.28}$$

This Lagrangian density is just

$$L = 3a'^2 e^{3a} + 3q_0 e^a - \frac{1}{2}e^{3a}K_{IJ}\phi'^I\phi'^J - e^{3a}V , \tag{5.29}$$

and the conjugate momenta to a and ϕ^I are then

$$p_a \;=\; \frac{\partial L}{\partial a'} \;=\; 6\; a'\; \mathrm{e}^{3a}\,, \tag{5.30}$$

$$p_{\phi\; I} \;=\; \frac{\partial L}{\partial \phi'^I} \;=\; -\;\mathrm{e}^{3a}\; K_{IJ}\; \phi'^J\,, \tag{5.31}$$

while the corresponding Hamiltonian is

$$\begin{aligned} H \;&=\; p_a\, a' \;+\; p_{\phi\; I}\, \phi'^I \;-\; L \\ &=\; \frac{1}{12}\,\mathrm{e}^{-3a}\, p_a^2 \;-\; \frac{1}{2}\,\mathrm{e}^{-3a}\; K^{IJ}\; p_{\phi\; I}\, p_{\phi\; J} \;-\; 3\; q_0\; \mathrm{e}^{a} \;+\; \mathrm{e}^{3a}\; V. \end{aligned} \tag{5.32}$$

Once on-shell, this is a vanishing function due to the equation of motion (5.23), what is often known as the zero energy condition. The Hamilton-Jacobi equation for Hamilton's principal function $S_{HPF} \;=\; F(a,\phi) \;-\; \Psi\; r$ is then

$$\Psi \;=\; \frac{1}{12}\,\mathrm{e}^{-3a}\left(\frac{\partial F}{\partial a}\right)^2 \;-\; \frac{1}{2}\,\mathrm{e}^{-3a}\; K^{IJ}\;\frac{\partial F}{\partial \phi^I}\;\frac{\partial F}{\partial \phi^J} \;-\; 3\; q_0\; \mathrm{e}^{a} \;+\; \mathrm{e}^{3a}\; V.$$

The solutions of interest correspond to $\Psi \;=\; 0$. For non-zero values of q_0, this equation is *not* separable with ansatz of the form $F \;=\; \mathrm{e}^{3a}\,(f(\phi) \;+\; h(a))$. On the other hand, the equation is easily separable if $q_0 = 0$.

In the case $q_0 = 0$, we may write

$$F \;=\; \pm\; 2\; \mathrm{e}^{3a}\; f(\phi)\,, \tag{5.33}$$

and hence

$$\frac{\partial F}{\partial a} \;=\; \pm\; 6\; \mathrm{e}^{3a}\; f \;=\; p_a \;=\; 6\; a'\; \mathrm{e}^{3a}\,, \tag{5.34}$$

$$\frac{\partial F}{\partial \phi^I} \;=\; \pm\;\; 2\; \mathrm{e}^{3a}\;\frac{\partial f}{\partial \phi^I} \;=\; p_{\phi\; I} \;=\; -\;\mathrm{e}^{3a}\; K_{IJ}\; \phi'^J\,. \tag{5.35}$$

These equations lead to

$$f \;=\; \pm\; a'\,, \tag{5.36}$$

$$\frac{\partial f}{\partial \phi^I} \;=\; \mp\;\frac{1}{2}\; K_{IJ}\; \phi'^J\,, \tag{5.37}$$

and the Hamilton-Jacobi equation becomes

$$V \;=\; -\;3\; f^2 \;+\; 2\; K^{IJ}\;\frac{\partial f}{\partial \phi^I}\;\frac{\partial f}{\partial \phi^J}\,. \tag{5.38}$$

Notice also that assuming this for V, an on-shell version of the Lagrangian written in (5.28) in the case $q_0 = 0$ can be written as *squares* plus a boundary term,

$$L_f = -\frac{1}{2}K_{IJ}\mathrm{e}^{3a}\left(\phi'^I \pm 2K^{IL}\frac{\partial f}{\partial \phi^L}\right)\left(\phi'^J \pm 2\mathrm{e}^a K^{JM}\frac{\partial f}{\partial \phi^M}\right) +3\mathrm{e}^{3a}\left(f \mp a'\right)^2 + \frac{d}{dr}\left(\pm 2f\mathrm{e}^{3a}\right) . \tag{5.39}$$

Enforcing the remaining equations of motion, the only term remaining in the previous expression is the boundary term. Adding the contribution from $\mathcal{S}_{ct}$, we obtain the boundary term found in (5.25).

Now we can readily connect this result with $\mathcal{N} = 1$ supergravity. In the complex scalar description, the Hamilton-Jacobi equation with $q_0 = 0$ becomes

$$V = -3 f^2 + 4 K^{A\bar{B}} \partial_A f \, \partial_{\bar{B}} f . \tag{5.40}$$

Let us consider a specific form for f. We write

$$f = \mathrm{e}^{K/2} |\omega| , \tag{5.41}$$

where $K = K(\Phi, \bar{\Phi}) \in \mathbb{R}$ and $\omega = \omega(\Phi, \bar{\Phi}) \in \mathbb{C}$. We write

$$\omega = \mathrm{e}^{i\theta} |\omega| , \tag{5.42}$$

where $\theta = \theta(\Phi, \bar{\Phi}) = \arg(\omega) \in [0, 2\pi)$ and we use the notation

$$D_A\omega = \partial_A\omega + \omega \, \partial_A K , \tag{5.43}$$

$$\overline{D}_{\bar{B}}\overline{\omega} = \partial_{\bar{B}}\overline{\omega} + \overline{\omega} \, \partial_{\bar{B}} K . \tag{5.44}$$

It can be shown that the general case gives

$$\partial_A f = \frac{1}{2} \mathrm{e}^{K/2} \left(\mathrm{e}^{-i\theta} D_A\omega + \mathrm{e}^{i\theta} \partial_A\overline{\omega}\right) , \tag{5.45}$$

$$\partial_{\bar{B}} f = \frac{1}{2} \mathrm{e}^{K/2} \left(\mathrm{e}^{i\theta} \overline{D}_{\bar{B}}\overline{\omega} + \mathrm{e}^{-i\theta} \partial_{\bar{B}}\omega\right) . \tag{5.46}$$

The Hamilton-Jacobi equation then becomes

$$V = \mathrm{e}^K \left\{-3|\omega|^2 + K^{A\bar{B}} \left(\mathrm{e}^{-i\theta} D_A\omega + \mathrm{e}^{i\theta}\partial_A\overline{\omega}\right)\left(\mathrm{e}^{i\theta}\overline{D}_{\bar{B}}\overline{\omega} + \mathrm{e}^{-i\theta}\partial_{\bar{B}}\omega\right)\right\} . \tag{5.47}$$

As it can be seen, when ω is a holomorphic function of Φ and $q_0 = 0$, ω satisfies the same equation as the superpotential W. In addition, the condition of criticality for f becomes precisely the vanishing of the covariant derivative $D_A\omega$. Nevertheless, a non-holomorphic ω can produce a solution f critical at a given point in moduli space. We will see that such a critical point is also a critical point of V. This is not a point in which the covariant derivative vanishes, but still we call f a fake-superpotential since it still satisfies the more relaxed $\partial_A f = 0$.

5.2 Solving the Hamilton-Jacobi equation

5.2.1 Expanding around a critical point

Let us consider again equation (5.38). Critical points (i.e. points where $\partial_I V = 0 \;\forall\; I$) as well as domain walls in between, are the focus of our study. Static domain walls are characterized by having constant fluxes at each side. In terms of (5.37), this translates into demanding $\partial_I f = 0 \;\forall\; I$ in each critical point. To be more precise, let us consider a theory with n scalars.

Taking into account these two conditions, we proposed in paper III the following scheme to study the types of solutions that can be found for (5.38) in-between AdS critical stable points. We consider a solution $\pm f$ written as an expansion in powers of $\Phi^I = \phi^I - \phi_0^I$ around a critical point ϕ_0^I. We expand the potential V in a similar fashion. Equating term by term, and, in consideration of the previous constraints, one finds

$$
\begin{aligned}
V &= V^{(0)} + \frac{1}{2} V^{(2)}_{LM} \Phi^L \Phi^M + \frac{1}{3!} V^{(3)}_{LMN} \Phi^L \Phi^M \Phi^N + \dots \,, \qquad (5.48) \\
\pm f &= f^{(0)} + \frac{1}{2} f^{(2)}_{LM} \Phi^L \Phi^M + \frac{1}{3!} f^{(3)}_{LMN} \Phi^L \Phi^M \Phi^N + \dots \,, \qquad (5.49)
\end{aligned}
$$

with

$$
f^{(0)2} = -\frac{1}{3} V^{(0)} \,, \qquad (5.50)
$$

$$
f^{(2)} = \frac{3}{4} f^{(0)} K + \frac{1}{2} K^{1/2} \sqrt{-\frac{3}{4} V^{(0)} + K^{-1/2} (V^{(2)} + C) K^{-1/2}}\, K^{1/2} \,, \qquad (5.51)
$$

where the elements of the matrix C are given by

$$
C_{LM} \;=\; -\,4 \lim_{\phi^N \to \phi_0^N} \; K^{IJ} \; (\partial_I f) \; (\partial_J \partial_L \partial_M f) \,. \qquad (5.52)
$$

Let us discuss signs and square root conventions. Distinct solutions are classified up to a global sign, since f appears only quadratically in (5.38). Thanks to this, one may pick $f^{(0)}$ to be positive without any loss of generality in (5.50). Once this sign is fixed, in order to consider all the distinct solutions for $f^{(2)}$ in (5.51), it is enough to consider: (1) the unique positive-semidefinite square root of K for $K^{1/2}$ and (2) all the (generically) 2^n square roots of $-\frac{3}{4} V^{(0)} + K^{-1/2} (V^{(2)} + C) K^{-1/2}$.

If one fixes $f^{(0)}$ following the previous prescription and if C vanishes, then each of the generically 2^n solutions for $f^{(2)}$ is fixed and well defined. In some way, this stays as a feature of the remaining system of equations: as long as the product of $\partial_I f$ with $(M+1)$-th order derivatives goes to zero when $\phi^N \to \phi_0^N$, the equations for $V^{(M)}$ form a linear system for the $f^{(M)}$ with equal number of equations and unknowns, namely $\binom{n+M-1}{M}$, with $M \geq 3$. This does not guarantee that the unknowns will be fully determined as degeneracies can still happen and, most importantly, the system of equations will, by definition, fail if there are any non-analytic behaviors.

While using a perturbative expansion does not sound as an ideal approach to a partial differential equation in several variables, it is important to point out that the nature of this non-linear problem produces a high number of analytic branches that overlap being all critical at the point of interest. A distinct numerical approach can easily produce a *Frankenstein monster* solution that is not ideal for our computation. The price we pay, or course, is the need to reach high orders in the expansion to explore the solution far from the critical point. Nevertheless, one can identify some important boundary behaviors as we will discuss later. It turns out this is still a tractable problem with a regular computer for the potentials we explored.

5.2.2 Fake superpotentials in Type IIA

Let us show the potential of these constructions in twisted compactifications of massive type IIA on $(S^3 \times S^3)/\mathbb{Z}_2^3$, which can be absorbed in the solutions described in chapter 2. From the point of view of a supergravity description, we explored in paper III those critical points that can be embedded in $\mathcal{N} = 4$ after a SO(3) truncation of the scalar coset. As we mentioned in chapter 2, they can be described with a $\mathcal{N} = 1$ potential, in particular, they live in the isotropic limit. These critical points can be grouped into two families of distinct flux values, each family with 4 critical points in the moduli space. Their fundamental properties are summarized in table 5.1, where they are organized according to two properties. First, we indicate whether they are supersymmetric with respect to the superpotential we found in chapter 2 and second, whether they are perturbatively stable. For the latter property, it is possible to go beyond the truncation and explore all the $2 + 6^2$ scalar degrees of freedom (DOFs) that one can expect from the coset of the closed-string sector excitations,

$$\frac{SL(2)}{SO(2)} \times \frac{SO(6,6)}{SO(6) \times SO(6)} \,. \tag{5.53}$$

There is only one SUSY point which is a member of the first family. It is, as expected, perturbatively stable. In addition, there are 2 non-SUSY points that are perturbatively unstable, one in each family. But then there are 5 non-SUSY points that are perturbatively stable, 2 living in the first family and 3 in the second. These are, as we have mentioned, the most interesting ones and establishing whether these are semi-classically stable is a core objective of the studies we have performed.

Family A		
Sol.	SUSY	BF bound
A1	✔	✔
A2	✘	✘
A3	✘	✔
A4	✘	✔

Family B		
Sol.	SUSY	BF bound
B1	✘	✔
B2	✘	✘
B3	✘	✔
B4	✘	✔

Table 5.1. *Properties of the 8 solutions in the $\mathcal{N}=4$ critical points of the twisted compactifications of massive type IIA on $(S^3 \times S^3)/\mathbb{Z}_2^3$. SUSY relates here to the $\mathcal{N}=1$ superpotential and stability refers to perturbative stabilty of the closed-string sector excitations.*

As seen in the previous sections, we have developed a systematic approach to the computation of fake superpotentials in this setup. In paper III we were able to compute a function f for each perturbatively stable critical point regardless of supersymmetry for the family that includes the SUSY point. In figure 5.1 we can see a graphical representation of the resulting curves for an analogous scenario coming from 11D supergravity with 3 critical points (one SUSY), all perturbatively stable. The plot for the curves corresponding to the case explained in this chapter can be found in paper III. The curve above is the projection of the potential in the 3D space defined by the 3 perturbatively stable critical points. The curves below correspond to $-3f^2$ for each f. These functions, as one expects from the Hamilton-Jacobi equation, have a maximum in the corresponding critical point and then drop in every direction.

Despite the fact that these solutions were computed numerically, we were able to study their boundary conditions in the moduli space. One finds that these functions only intersect the potential in the critical point. This is a fundamental property as one can deduce that a second intersection would provide a possible non-perturbative decay channel for the state. If these fake-supertentials are to be covered by the positive energy theorem, then one can read the difference between the curves V and $-3f^2$ as the energy excess that any other point in the moduli space contains with respect to the corresponding critical point, rendering it semi-classically stable. Given the fact that we found a systematic procedure to compute these solutions where the fundamental constrain is having all masses satisfying the BF bound, one would then conclude that under the assumption of a positive energy theorem, all AdS perturbatively stable points admit a fake-superpotential and are semi-classically stable.

Nevertheless, we shall be cautious regarding this and similar statements as we have discussed here and later in the next chapter. In par-

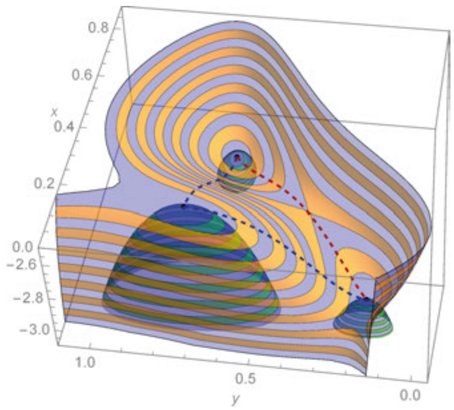

Figure 5.1. Potential and fake-superpotentials in a family of 3 solutions found in an effective description of a compactification of 11D supergravity. These are projected over the 3D space that contains the 3 perturbatively stable critical points. We plot $-3f_i^2$ to connect with positive energy theorems for each of the i-th critical point, $i = 1, 2, 3$. We were able to compute two flat domain walls that interpolate between a pair of critical points each. Here we plot the straight paths between these solutions, with the blue dashed lines indicating strictly monotonic paths that allowed for this construction. A red dashed line shows a non-monotonic path between two critical points for which we were not able to construct a flat domain wall.

ticular, we explore these finer issues in the same set of solutions in paper IV and we will find that there was a missing chapter in this story.

As a complementary gift, we of course can use some of these fake-superpotentials as solutions of the Hamilton-Jacobi equation and construct flat domain walls between the critical points. From the point of view of the "domain wall picture", these flat domain walls can be seen as static configurations that connect distinct stable critical points. In the thin-wall limit, flat domain walls saturate the Coleman-De Lucia bound and correspond to a infinite on-shell euclidean action impeding decay in both directions. In that sense, they are further signs of stability of such critical points. Nevertheless, as far as we know, there is no reason to expect that a single domain wall between two solutions would impede decays to other critical points or to the boundary of the moduli space. In that sense, one again resorts to the positive energy theorem in order to say something about complete stability.

On the other hand, finding a curved time-dependent domain wall with a finite euclidean action will certainly provide a decay channel for a solution and in that sense, the "domain wall picture" is very much relevant as a smoking gun for unstable solutions. In fact we will come back to this useful tool in the next chapter.

6. Semi-classical stability: Swampland

We have established the importance of finding criteria to identify semi-classically stable vacua. In the last decades interest has been drawn to another, although ultimately connected, story on instabilities in general relativity solutions. More explicitly, in [Are11a, Are11b] it has been argued that the near horizon geometry of the Reissner-Nordström black hole is unstable with respect to scalar field perturbations. Following a distinct although interconnected line of reasoning, [AHMNV07] discussed, from a more phenomenological point of view, how consistency in quantum theories of gravity imply the presence of decay channels for *sick* extremal states. More explicitly, in a consistent theory in which there is a U(1) gauge field, there must exist a super-extremal state. These are, in the language of the Reissner-Nordström geometry, particles with mass below the magnitude of its U(1) charge (in planck units). This became known as the weak gravity conjecture (WGC).

These arguments have evolved and have been extended to more general setups. Following the same line of reasoning, the need for super-extremal objects allowing for the decay of extremal charged black branes was stressed already in [HRR16], later followed for what was known as the strong WGC [OV16]: The weak gravity bound is saturated if and only if the theory is supersymmetric and the corresponding state is BPS. A consistent gravity theory with a $p-$brane whose $(p+1)-$form is not in the supergravity multiplet must contain a super-extremal $p-$brane.

The reasoning behind this stronger version of the conjecture comes from the study of curvature and compactifications. More explicitly, in [KMP07] it was found that higher derivative corrections to the effective action send non-SUSY extremal states into super-extremality and in [HRR16] a similar phenomenon was observed upon dimensional reduction. As a consequence, [OV16] suggests that non-susy holography could be inconsistent in the case of a finite number of matter fields coupled to gravity. In addition, it is concluded that non-susy vacua built out of non-BPS objects are unstable, or at most meta-stable, with a vanishing life time in the near horizon limit.

A very explicit example was presented in [OS17]. This is a solution of 11D SUGRA, with a background given by $\mathrm{AdS}_5\times\mathrm{CP}^3$ and the 4-form flux F_4 begins as $\omega\wedge\omega$, with ω the Fubini-Study Kahler 2-form of CP^3. Previous studies had been able to establish that while the solution is not supersymmetric, it is perturbatively stable, although a KK mode saturates the BF bound exactly.

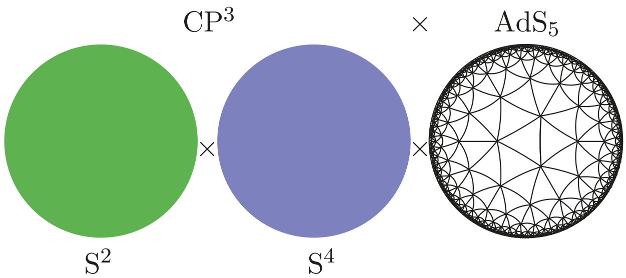

Figure 6.1. 11D background geometry in [OS17] with CP^3 as a S^2 fibration over S^4 and AdS_5 as external space. No spacetime-filling sources are required in this solution. S^2 collapses with positive and finite on-shell euclidean instanton action.

By studying the dynamics of the euclidean action, in [OS17] they decompose the geometry in vielbein that manifestly describe CP^3 as a S^2 fibration over S^4 (see figure 6.1). By allowing dependence of the geometry and F_4 on the radial coordinate of AdS_5, they find a solution in which S^2 collapses while F_4 locates itself fully in S^4, hence preserving flux conservation. In addition, upon subtraction of the AdS_5 volume, the on-shell euclidean action is finite and positive. This then corresponds to a non-perturbative decay channel for the original 11D non-SUSY and perturbatively stable solution.

6.1 Probing the Open String Sector

While the WGC poses a potential criterion to segregate unstable solutions, efforts have to be made in discerning the precise nature of these decay channels. There have been several proposals and here we will consider the reasoning presented in [DD16]. In this process, we must stress some fundamental ideas behind the construction of string theory vacua that become particularly relevant in this discussion.

SUGRA has provided us with two distinct paths to obtain lower-dimensional AdS solutions. On the one hand, one may put together stringy sources in a configuration such that at the near-horizon (NH) limit we reproduce a geometry of the form $AdS_d \times \mathcal{M}_{D-d}$. A list of examples of these constructions can be found in [CLPVP00] and with less supersymmetry in [KLPT07]. In [DD16] this is denominated as the *brane picture* of AdS vacua. On the other hand, one can limit oneself to find a flux compactification, that is, a set of "God-given" 10D/11D fluxes that satisfy the equations of motion. This, as we discussed in the previous sections, can generate a lower dimensional potential upon compactification consistent with the $\mathrm{AdS}_d \times \mathcal{M}_{D-d}$ geometry. This is denoted as the *flux picture* of vacua.

The first conjecture in [DD16] is that these two pictures generate the same vacua. To do this, one makes emphasis on the distinction of two types of sources that are used in the construction of vacua via the brane picture. These are the background branes which source fluxes in the AdS vacua (which typically fill $d-1$ spacetime dimensions) and then we have those that fill spacetime in order to comply with charge conservation, if they happen to be needed. From the point of view of the flux picture, the former ones manifest as dynamical terms in the equations of motion while the latter branes appear as tadpole-cancellation terms. This, as is known, leaves aside the problem of their backreaction which has then to be studied separately. The expectation in [DD16] is that, through this identification, one should be able to realize a one-to-one correspondence with vacua built through each picture. This then equates to identifying flux picture vacua as the limit in which we leave aside the dynamics of the background branes which source them.

The stereotypical example of an extremal object is the 4D Reissner-Nordström (RN) black hole with matching mass and charge. The NH geometry, $\mathrm{AdS}_2\times\ \mathrm{S}^2$ was shown to be unstable with respect to perturbations of a (massive or massless) charged scalar [LMRT13, Are11a, Are11b]. The consequence is the creation of a trapped surface, turning the black hole in a geodesically incomplete space time. If this is the case, what looks as a perturbative decay at the level of the NH limit, manifests as a non-perturbative effect of the extended geometry, as the one predicted by the WGC. Even more, the coupling between gravitational and electromagnetic degrees of freedom is crucial in capturing these phenomena.

In fact, this reasoning is further extrapolated to 10D. There one can find examples of non-supersymmetric vacua that enjoys full perturbative stability when truncated to closed-string sector excitations. Nevertheless, once one considers the coupling to the degrees of freedom of the space time filling sources, perturbative instabilities manifest. These can be captured with probe computations or alternatively one can consider the efective gauged supergravity model in which one couples extra vector multiplets to the closed-string sector description. In paper IV we showed this explicitly for examples in AdS_4 and AdS_7 for the case of a single brane probe.

The setup is again based on the compactifications we described in chapter 2 and 5. In chapter 5 we described a effective mechanism that builds fake-superpotentials for a family of critical points that can be embedded in a $\mathcal{N} = 4$ description with AdS_4 as external space. There we found non-SUSY solutions that are perturbativley stable when one considers the excitations of the closed-string sector. We may account for the degrees of freedom coming from open-string modes by adding extra vector multiples corresponding to the coupling of the background with the spacetime-filling sources. At first approximation, one may consider an abelian coupling consistent with a single $\overline{\text{D6}}$ brane. These adds 6 degrees of freedom per brane to the scalar coset description which is then enhanced to

$$\frac{SL(2)}{SO(2)} \times \frac{SO(6,6+n)}{SO(6) \times SO(6+n)}\,, \tag{6.1}$$

with n the number of branes and $2+6^2+6n$ DOFs in total. When computing the effective potential with the help of the embedding tensor description, one finds that most of the non-SUSY solutions acquire masses below the BF bound. The SUSY solution, on the other hand, remains stable.

We considered as well the case of AdS_7 whose background was described in chapter 2. Warped compactifications of massive IIA on a squashed S^3 with spacetime-filling O6/D6 sources are known to admit a gauged $\mathcal{N}=1$, $D=7$ supergravity description. The theory that captures all of the closed-string zero modes is the one obtained through the coupling of the gravity multiplet with 3 extra vector multiplets. Such a supergravity model enjoys

$$G_0 = R^+ \times SO(3,3) \tag{6.2}$$

as a global symmetry, where its 64 bosonic degrees of freedom are arranged into the metric (14), 6 vectors (6×5), one 3-form (1×10) and 10 scalars. Let us consider again the addition of additional vector multiplets to describe the coupling to n $\overline{\text{D6}}$ branes. The scalar coset is then

$$\mathbb{R}^+ \times \frac{\text{SO}(3,3+n)}{\text{SO}(3)\,\times\,\text{SO}(3+n)}\,, \tag{6.3}$$

with $10+3n$ DOFs. We again restrict ourselves to an abelian coupling and compute the effective potential to find that the non-SUSY solution develops a mass below the BF bound while the SUSY solution stays stable. It is also possible to perform the same computation with a probe potential using the 10D background to find the same result, as we show in paper IV.

A more realistic approach to these phenomena should be reached by studying the full non-abelian effect of a finite $n>1$ number of spacetime-filling probe branes. From the point of view of the effective gauged supregavity, this means the introduction of couplings reproducing this enhanced gauging. In a way, this will require an identification non dissimilar to the one done for 10D fluxes in a previous section. This is part of ongoing and future work. There is also interesting physics in the infinite n limit where interesting phenomena as brane polarization has been seen in other compactifications [JSZ14] which does not seem to manifest in the non-supersymmetric solutions we have studied.

The final conjecture in [DD16] is logically motivated by previous studies on instabilities of 10D backgrounds of mixed brane charge [DGVR17]. The expectation is that while supersymmetric vacua might require (at most) spacetime-filing sources of pure brane charge, non-supersymmetric vacua would introduce other sources, that is, anti-brane charge to cancel tadpoles. This mixed background would then decay thanks to the exchange open-string degrees of freedom leading to brane/anti-brane annihilation. Consequently, a truncation to the closed sector of these solutions would not see this channel of gravitational tunneling, allowing for apparent meta-stable non-supersymmetric vacua. In situations where no spacetime-filling sources are required, it is argued [DD16] that apparent stability is only established thanks to an incomplete description of SUGRA that lacks the full low energy spectrum coming from string theory [GOS07].

7. Epilogue

We have studied some realizations of dS in supergravity and we have explored the challenges that one faces when establishing stability of a gravitational theory. Whether we talk about instantons or interaction of D-branes and fluxes, at the very end we see how non-perturbative aspects of field theory in curved space still require more attention, particularly in situations without supersymmetry.

We have done these explorations from the point of view of the classical theory. In that regard one may ponder what are the paths to follow in this landscape. In the case of the explorations done in paper I, there is hope that one can go beyond case by case compactifications and find general relations between the ADM mass and the on-shell action of sources [CMDVRV16a, CMDvRV16b]. The fact that one can relate objects of both the UV and IR limits of a theory with the cosmological constant is, at its core, one of the hopes of the string cosmology (and quantum gravity) paradigm. On the other hand, explorations of the SUGRA vacua, like those in paper II, are being refined and expanded at an accelerated rate, with more sophisticated approaches that slowly but steadily satisfy more phenomenological criteria [MRW18, GMVW17].

We have provided mechanisms and ideas on the problem of stability in papers III and IV. In the explorations of domain walls and fake-supergravity, there has been follow-up work that indicates non-analytical branches of the Hamilton-Jacobi problem can be relevant in establishing the properties of the holographic RG flow [NSPS17]. In addition, there is renewed interest in refining the constraints that have to be imposed, in terms of both boundary conditions and the bulk flow itself, to be able to confirm a positive energy argument granting stability [DDTVR17, TVRV12].

Then again, this is a tale of two cities. Evidence for the WGC is increasing, with literature offering potential proofs [CLR18, Hod17]. There are some hints of exceptions or potential loopholes as well [GP18, DDS17]. Either of these could trigger a change in the way we approach the landscape, at least for the purpose of doing cosmology. The hope is that one way or another supergravity still has something to say about these problems, as we explore uncharted territories of the connection between non-perturbative and perturbative instabilities.

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Svensk sammanfattning

ΛCDM-modellen eller korrespondensmodellen är ett av de viktigaste resultaten under de senaste decennierna i observationell kosmologi. Medan modellen ständigt är satt på prov kan den med god noggrannhet redogöra för många av universums observerbara egenskaper, bland annat att cirka 70% av energitätheten motsvarar *mörk energi*. Den mest troliga orsaken bakom denna komponent är en liten men positiv kosmologisk konstant, som kännetecknar ett de Sitter (dS) universum. Ur den teoretiska fysikens perspektiv är det en stor utmaning att skapa modeller som kan förutsäga eller åtminstone beskriva dessa och andra kosmologiska parametrar.

Under de senaste åren har problemet med mörk energi gett upphov till en uppsjö av möjliga lösningar med en kosmologisk konstant som utvecklats och testats på olika sätt. Med tanke på att strängteori är en potentiell kandidat för en kvantteori för gravitationskraften, skulle man kunna förvänta sig att supergravitation (SUGRA) vid låga energier kan ge oss ledtrådar till detta problem. Ändå ger de enklaste och mer välförstådda modellerna inte upphov till ett stabilt dS-rum. Trots att det finns många förslag på modeller har de alla fundamentala begränsningar som behöver adresseras på ett tillfredsställande sätt.

I supergravitation, precis som i Einsteins allmänna relativitetsteori, ses tyngdkraften som det ömsesidiga beroendeet mellan energi och rumtidens geometri. Dessutom finns det en mängd olika fält och källor och det visar sig att en enorm mängd lösningar kan produceras. Termen "landskap" har använts för de teorier som har potentialen att ge oss en korrekt bild av gravitation och kvantmekanik. Under de senaste decennierna har man utvecklat verktyg för att utforska detta landskap, vilka man sedan använder för att söka efter en lösning (vakuum) som liknar vårt universum. Förutom de egenskaper som kosmologin kräver, finns teoretiska villkor som måste uppfyllas. Teorier som är orimliga sägs höra till "swampland" ("träsket"). I denna avhandling bidrar vi till denna klassificering av teorier genom att utforska och förhoppningsvis förfina gränsen mellan dessa världar.

Stabiliteten hos vakuum i supergravitation är ett viktigt ämne i sig. Ur ett kosmologiskt perspektiv måste vi hitta lösningar som överensstämmer med våra observationer. Ändå är det inte ett trivialt problem att skapa en semi-klassisk stabilitet för en specifik lösning. Svårigheterna härrör inte bara från behovet av att utforska en mängd olika lägen som kommer från kompaktifierings-frihetsgrader utan också möjliga icke-störande kanaler och deras motsvarande sönderfallshastigheter. Det är därför nödvändigt att vår förståelse av stabilitet i icke-supersymmetriska lösningar förbättras.

Men är det verkligen möjligt att hitta en modell med alla önskade egenskaper som beskriver supergravitation? Vid första anblicken verkar det som om det finns en motsägelse mellan de olika kraven: stabilitet kontra icke- supersymmetriska teorier. Forskning som genomförts under de senaste decennierna har lett fram till en hypotes som kallas "Weak gravity conjecture" (WGC) ("den svaga gravitations-hypotesen"). I sin senaste version verkar slutsatsen vara att vakuum skapade av icke-supersymmetriska källor har en mycket kort livstid.

I denna avhandling studerar vi några av egenskaperna hos konstruktioner i supergravitation med icke-supersymmetriska vakuum. Detta inkluderar lösningar som innehåller dS i fyra dimensioner samt andra vakuum där vi utforskar problemen med stabilitet och singulariteter. Som tidigare nämnt kräver detta en djupare förståelse av fältteorier där tyngdkraften tas med i beräkningen. Samspelet mellan objekt som exempelvis svarta hål och horisonter med fält som liknar elektromagnetiska fält, målar en rik och komplex bild där man måste gå bortom störningsanalys för att se viktiga aspekter av en teori om kvantgravitation.

I det här arbetet ger vi analytiska bevis för att nakna singulariteter produceras i en supergravitations-bakgrund efter introduktionen av anti-Dp-bran, vid låg temperatur. Detta är ett viktigt steg i arbetet med att hitta stabila konstruktioner av dS-vakuum. För närvarande verkar situationen vara oklar för anti-D3-fallet, men för $p > 3$ verkar en fluxklumpningsprocess som produceras via Myers-effekten omöjlig att förhindra och gör lösningen instabil.

Vi bidrar också med våra egna förslag genom att studera det ännu outforskade landskapet av flux-kompaktifieringar för att producera dS. Tidigare arbete inom detta område utnyttjade ofta icke-störande effekter som gaugino-kondensation för att erhålla dS-punkter nära Minkowski-vakuum. I stället använder vi de så kallade icke-geometriska flödena. Vi utvecklar exakta analysmetoder för att hitta störnings-stabila dS nära supersymmetriska Minkowski-vakuum. Dessa metoder har utökats och vidare tillämpats i andra sammanhang för att studera inte bara dS vakuum utan också inflationsmodeller och tidsberoende dynamik.

För att undersöka dessa problems natur kan vi istället utforska mer välförstådda teorier, exempelvis kompaktifieringar med anti-de Sitter (AdS) som externt rum. Vi argumenterar för att sektorn med slutna strängar i icke-supersymmetriska teorier kan vara skyddad mot sönderfall av falska superpotentialer. Vi tillhandahåller också ett systematiskt förfarande för att lösa motsvarande Hamilton-Jacobi problem när det gäller modulrum i flera dimensioner som också kan implementeras för att numeriskt skapa “domän-väggar” och instantoner.

Icke desto mindre har motexempel hittats med störnings-stabila teorier som har en icke-försvinnande sönderfallshastighet genom källförstöring. I vårt arbete undersöker vi därför rollen som excitationer i sektorerna med öppna strängar har, för att fastställa en möjlig orsak till dessa instabiliteter. Vi fann att icke-supersymmetriska lösningar tenderar att ha massor under Breitenlohner-Freedman-gränsen när sektorn med öppna strängar utforskas för enkla bran medan supersymmetriska lösningar förblir stabila. Även om detta inte är den fullständiga bilden, ger den oss en antydan om naturen hos de instabiliteter som förutspås av WGC.

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