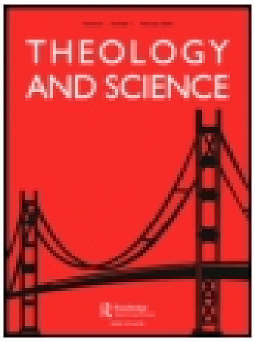

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ABSTRACT
This paper investigates some consequences of a mathematical understanding of infinity and connectedness for a panentheistic conception of God. Given the existence of God and an understanding of the world in terms of the finite, countable infinity, or uncountable infinity I argue, (a) that a panentheistic conception of God is supported given a mathematical understanding of the infinite, (b) that by applying the notion of uncountable infinity and the mathematical concept of connectedness a panentheistic God can be seen as unifying irrespective of whether the world includes a finite, countably infinite or an uncountably infinite number of entities.

KEYWORDS
Panentheism; cantor; mathematics; infinity; connectedness; unity; multiverses

In theism in general and thus also in panentheism concepts such as God's omnipotence, omnibenevolence or omniscience play a central role in theological and philosophical reasoning. At least if the "omni" in the above concepts is understood in terms of quantity, such concepts can be directly or indirectly related to a mathematical notion of the infinite. The infinite is also a common, widely used, and well-studied concept within mathematics. The development of mathematics in the nineteenth and twentieth centuries produced some important qualifications of this concept. In particular, the distinction between countable and uncountable infinite sets, going back to the works on set-theory by Georg Cantor (1845–1918), has been of great value and significance for modern mathematics. Already at the time Cantor developed his novel ideas about infinity, theologians were interested in Cantor's ideas, although mostly out of concern that his ideas might ultimately strengthen the position of pantheism.[1] More recently philosophers have argued that mathematical understandings of infinity can and presumably should be related to theological reasoning.[2] In this paper I will investigate possible consequences of the standard mathematical distinction between countable and uncountable infinite sets and the concept of connectedness for a panentheistic conception of God within the philosophy of religion.[3]

First I will shortly present one of the more significant problems for Cantor's set-theoretical ideas and its relation to issues and the discussion of the foundation of mathematics in general since it is possible that such issues may subsequently lead to problems for the applicability of mathematical ideas to theological reasoning. After a summary of the concepts of countable and uncountable infinite sets, and the mathematical concept of connectedness, which will be useful in the argument for unity, and how panentheism can be construed, I will consider three cases of how the universe or the multitude of universes is understood in relation to God. It should be noted that universe, multiverse and the *actual* world are here understood in the following sense: Generally, by "universe" I mean a possible world; that is, universe and world will here be used as synonyms. Thus the universe could be the world we are living in, but it could also be a world devoid of living creatures and not accessible for human observation. In the case of the *actual* world I mean the world or universe humans live in. Multiverse will be used, as the term indicates, to mean the multitude of all existing universes irrespective of whether they are accessible or not. With "universe in total" I denote the totality of universes in any given case. Thus it is possible that the multiverse consists of a great number of universes or worlds, possibly infinitely many, including the universe or world we are living in, the actual world being one of them. In the case where I assume that there is no multitude of universes, I will stick to the traditional term of "universe".

The three cases to be considered in this paper are, firstly, the case that the universe consists of a vast, but finite number of entities. Standard contemporary cosmology suggests that the observable universe had a beginning in the Big Bang and thus is finite *backward* in time. In the case of its spatial extension, the universe is possibly finite or infinite dependent on whether the geometry of the universe is flat or curved. Based on observations, the geometry of the universe is commonly believed to be flat which would amount to a spatially infinite universe. Likewise, the universe may expand infinitely in time, dependent on factors such as the geometry of the universe and the value of the so-called cosmological constant. Nevertheless, the *observable* universe is finite.[4] Thinking of the universe as vast but finite is presumably a common way of understanding the universe. The second case is a version of a multiverse in which the multiverse consists of a countably infinite number of entities, and possibly a countably infinite number of universes or worlds. Finally, the universe could consist of an uncountably infinite number of entities. The third **case**, although presumably unconceivable in the *actual* world, seems to be a possibility if modal realism, as suggested and argued for by David Lewis, is taken into consideration.[5] This position, I deem, has in versions with its roots in physics—recently received increased attention. Notably, ideas about multiverses, which strongly remind of modal realism, have been suggested and argued for by theoretical physicist Max Tegmark, in terms of four levels of multiverses.[6] Furthermore, following a generic definition of panentheism by Niels-Henrik Gregersen I will establish what is minimally required for an argument for panentheism.[7]

Following the above stated three cases of how the multiverse can be construed, and the minimal requirements for a successful argument for panentheism, I will present an argument based on the Anselmian idea of God as "[…] that than which no greater can be thought", and as an extension of a standard argument from infinity applied by Philip Clayton in *God and Contemporary Science*.[8] I will argue that in all cases a panentheistic conception of God in the sense to be defined below is certainly supported and perhaps even strongly suggested. In particular, in the discussion of the third case that involves an uncountably infinite number of entities, ideas by Yujin Nagasawa about "modal panentheism" will be taken into account.[9] It will also become clear that although panentheism is certainly a reasonable position for the reasons to be given here, it is possible to find specific relations between countable and uncountable infinite sets and cases of the relation between God and the universe, that may lead to the opposing suggestion that God could also be construed as wholly transcendent of the universe. Such possibilities require further assumptions before they can be excluded.

Subsequently and importantly, I argue that if God is construed in the sense that God transcends the universe by being at least uncountably infinite, and if the universe is understood as consisting of a finite, countably infinite set of entities, or even if the universe consists of uncountably infinite entities, and given that a panentheistic God is immanent, then God and the universe may be understood as *united* in the sense of being connected as described in mathematical topology. Thus by this argument at least a minimal form of unity is established irrespective of whether the universe in total is finite or infinite in a strict sense.

On Problems for Cantor's Set Theory

Although Cantor's set-theory, including his ideas about the infinite, is generally regarded as highly innovative and significant for contemporary mathematics, they were not without problems. One crucial question is whether such problems threaten the applicability of Cantor's theories about the infinite for theological reasoning. It is obviously beyond the scope of this article to discuss all issues related to set-theory and the history of the foundation of mathematics in detail. However, one of the paradoxes specifically generated by Cantor's set theory seemed to result in the inconsistency of Cantor's set-theory. This problem is the antinomy of the set of all sets, also known as Russell's paradox: Let A be the set of all sets that are not members of themselves. If A itself is a set which *is not* a member of itself, then by definition it *does* belong to the set of all sets that are not members of themselves. If A itself is a set which *is* a member of itself, then A according to the definition is not in the set of all sets that are not members of themselves.[10] Thus in both cases, the definition of A leads to a contradiction. This antinomy and the seeming inconsistency of Cantor's set-theory was part of a group of problems in mathematics which subsequently led to what Ernst Snapper denotes as "the three crises in mathematics". According to Snapper, the three crises refer to the attempts of logicism, intuitionism and formalism and their philosophical counterparts realism, constructivism and nominalism respectively to provide a foundation of mathematics.[11] In the first case, Russell's paradox was avoided, and thus the above problem was remedied, by introducing certain axioms, one of them being the axiom of infinity. All the same, Snapper concludes that this axiom in itself "[…] is not a logical proposition in the sense of logicism." (my emphasis)[12] Although free from contradictions intuitionism was unattractive to mathematicians since many theorems had to be rejected. Formalism finally ran into problems, in particular due to Gödel's theorem stating that no formal language can prove that a theory is without contradiction.[13] So, in summary, neither of the three approaches succeeded in a firm foundation of mathematics.

Returning to the initial question in this section, one may wonder whether these failures and the lack of a firm foundation of mathematics lead to problems for the applicability of Cantor's theories about the infinite for theological reasoning? At first glance, this may seem the case. If neither of the above positions of logicism, intuitionism or formalism is successful in founding mathematics, and if in particular, Cantor's set theory is inconsistent, then why should one believe that mathematical research or Cantor's set theory has any bearing on theological reasoning? However, mathematical research has flourished and made significant progress, and importantly, Cantor's distinction between countably and uncountably infinite sets is widely used despite the crises mentioned above. It seems that contemporary mathematics generally is pragmatic in relation to the question of providing a firm foundation to mathematics. If the theologian, therefore, adopts a more pragmatic stance towards the above problems or if one accepts the position of logicism and its related philosophical position of realism, which remedied the antinomy mentioned above, then it seems reasonable to believe that the Cantor's results regardless of the possible problems still have bearing on theological reasoning. Furthermore, the third case to be analyzed based on Tegmark's multiverses or Lewis' modal realism already *presupposes* a form of realism. Thus I believe that Cantor's ideas and also other ideas from mathematics can fruitfully be applied to theological questions. In support and agreement, Robert J. Russell similarly suggests that "[…] despite the profound problems caused by the antinomies associated with set theory and the eventual series of crises […] Cantor's work *is* applicable theologically […]".[14]

Infinity and Connectedness in Mathematics

The two mathematical notions which shall be applied to panentheism in this article are infinity and connectedness. Both are frequently applied in mathematical problems and can be regarded as fundamental to modern mathematical theory.

The Concept of Infinity in Mathematics

The notion of infinity may at first glance appear to be clear and unproblematic. Yet as the influential works of Cantor have shown, matters are not that simple. As part of his theory of transfinite numbers Cantor has shown that it is possible to make the important distinction between countable and uncountable infinite sets. According to Cantor the less common term of transfinite denotes anything which is neither finite nor absolutely infinite.[15] Since this article is not written for mathematicians, I shall make an attempt to describe these concepts and the ideas behind them in a non-technical way and I shall mainly focus on the distinction between countable and uncountable infinity.

To start with, the infinite in mathematics can be defined in various ways. One possible definition could be based on finite sets as something that can be counted and assigned a number of elements. An infinite set would then be a set which is *not* finite.[16] Thus the simplest infinite set in mathematics that comes into one's mind is surely the set of *all* natural numbers (1,2,3, …). A countably infinite set would be an infinite set such that it is possible to sort the set by assigning a natural number to the members of the set. Again one may believe that this should always be possible, but consider the set of real numbers between 0 and 1, which should cover all numbers. These numbers can surely be written in decimal form. Now assume that these numbers were countable in the above sense; that is, they can be numbered, labeled with 1,2,3 … and so on. The first number would look like $0,a_{11}a_{12}a_{13}\ldots\ \ldots$, the second like $0,a_{21}a_{22}a_{23}\ldots$.. and the nth number would be $0,a_{n1}a_{n2}a_{n3}\ldots$.. with the digits "$a_{jk}$"s being a number between 0 and 9. According to an assumption of what the real numbers between 0 and 1 are, this listing is complete. Now construct a number by considering the digits from the numbers in the list in a diagonal. If the first digit in the first number, a_{11}, is 3 then let the first digit b_1 in the new number be 7, if it is not 3, then let it be 3. Continue doing this with every digit in the new number and this new number $0,b_1b_2b_3$ … which is surely also a real number but by construction different from *any* number in the list. Thus the *numbered* list is incomplete or in other words any numbered, that is, countably infinite, list of the numbers between 0 and 1 is incomplete and thus the set of *all* numbers between 0 and 1 is *uncountable*. In general, infinite sets, which have this property, are called *uncountably infinite*.[17] Thus there are at least two different types of infinity, countable and uncountable.[18]

As mentioned in the introduction, universe and world will be used as synonyms if not otherwise specified. Multiverse or universe in total will be used if it is considered that there may be more than one existing world or universe. Thus in Lewis' modal realism "the world in total", which he thinks consists of a multitude of worlds, would be the most all-encompassing form of a multiverse. So in the first case to be considered, there would only be one universe: the actual, observable world. In the second case to be considered there are already other worlds, which are beyond the reach of observation; thus this is in general a case of a multiverse. The case that the observed universe is connected to and extends to an unobservable possibly infinite part would also be covered by the second case. In the last case, the worlds existing may even be of a totally different kind, and may as in Tegmark's fourth level of multiverses or as in modal realism even include structures which in some sense are uncountably infinite. This case again amounts to a multiverse in which our world is just one world, namely the part to which we humans have access.

However, where could finite, countably infinite and uncountably infinite sets be found? Evidently, finite sets of anything existing in the actual, observable world can be constructed. Further, if physics is correct, present theories at least in some sense are taken to reflect the structure of the observable world, and if these theories are not interpreted as Tegmark interprets them, then the observable world consists of basic discrete entities. The question of whether these basic entities should be regarded as substances *or* processes is left open here since, I deem, it is not relevant in relation to the topic in question. In any case, if the world is made up of discrete basic entities, which it supposedly is, the total set of these is either finite or countably infinite. The *observable* world is thus surely made up of a *countable* set of discrete basic entities. If the multiverse is extended to include other worlds, as in modal realism or Tegmark's multiverse theory, then there could conceivably be cases of worlds incorporating an uncountably infinite number of entities.[19]

The Concept of Connectedness and its Relation to Infinite Sets

The intuition behind the mathematical or more specifically topological concept of connectedness is simple and can precisely be described as that of being *connected,* that of consisting of *one piece.* Of importance in relation to this concept is the fact that it describes *spaces* or more precisely, *topological spaces.* Such spaces are usually made up of sets, which—in terms of the above—could be finite, countably infinite or uncountably infinite. So more precisely a (topological) space is said to be *connected* iff it "[…] cannot be represented as the union of two disjoint non-empty open sets".[20] The qualification of "open" in this definition is of lesser importance in the argument to be given here and therefore I will not specify this concept any further. One significant result which is related to ideas about infinite sets is the following: Any n-dimensional space $\mathbf{R}^n$, based on the real numbers $\mathbf{R}$, which is an uncountably infinite set of numbers, is connected.[21] Thus for example our everyday space $\mathbf{R}^3$ or even four-dimensional space–time, under the assumption that time is continuous and described by real numbers, is connected. It should be noted here that the fact that connectedness has to be related to a topological space, suggests that the reasoning about God, infinity and connectedness further down should be restricted to space, space–time or at least an n-dimensional space, which is (topologically) equivalent to $\mathbf{R}^n$. Having introduced the concepts of countable and uncountable infinity, and connectedness, it is now time to briefly account for some basic ideas about panentheism.

Panentheism

As the word pan-en-theism—"all-in-God"—already says itself, God should be understood as both transcendent *and* immanent, and thus panentheism could, initially and in simple terms, be described as a position that claims that the universe is in God, referring to the "en" in panentheism, and that God nevertheless is not exhausted by the universe, but is "greater", transcendent. Yet, as is well-known, the idea of God's immanence in the universe can also be found in classical theism, for example in Thomas Aquinas' *Summa Theologica*.[22] Still, panentheists, for example Elizabeth Johnson, often point out that the transcendence in theism has often been overemphasized. She thus describes panentheism as follows:

> If theism weights the scales in the direction of divine transcendence and pantheism overmuch in the direction of immanence, panentheism attempts to hold onto both in full strength. […] At the root, this notion is guided by an incarnational and sacramental imagination that eschews any fundamental competition between God and the world in favor of the power of mutually enhancing relation.[23]

In her description it is possible to discern *three* features of panentheism, the first two being, as mentioned, the immanence and transcendence of God and the third being a "mutual relationship". These three features are captured in Niels-Henrik Gregersen's generic definition of panentheism:

> 1. God contains the world, yet is also more than the world. Accordingly, the world is (in some sense) 'in God'. 2. As contained 'in God', the world not only derives its existence from God but also returns to God, while preserving the characteristics of being a creature. Accordingly, the relations between God and world are (in some sense) bilateral.[24]

Clearly, part (1) describes precisely both the transcendence and immanence of God whereas part (2) establishes, contrary to classical theism, that God is bilaterally related to the world. Indeed, Gregersen thinks, with reference to Aquinas, that:

> […] the real demarcation line between panentheism and classical philosophical theism is neither the immanence of God nor the use of the metaphor of the world's being 'in' God.

> The real difference, according to Thomas is that the natures and activities of the creatures do not have a real feedback effect on God.[25]

So, it is of importance to realize that the third feature in panentheism together with the emphasis on immanence and the balance between transcendence and immanence, clearly distinguishes panentheism from classical theism in that it suggests a bilateral relationship between God and the world.

According to the above, in order to argue for panentheism one would minimally firstly have to establish the *transcendence* of God; secondly one would have to reason that God nevertheless in some sense is *immanent*; and finally it is necessary to argue for some form of *bilateral relationship* between God and the world.[26] It should be noted here that some panentheists rather think of panentheism in a metaphorical sense, as Sally McFague does in *The Body of God*.[27] In such cases, it may not be necessary to argue for the above three features on the level of ontology; it would instead be sufficient to argue that the metaphor —in this case of the world as God's body—has important theological advantages and positive consequences in the relation of humans to the world. This is undoubtedly done brilliantly by McFague.[28] Still I think that it is likewise fruitful to argue for panentheism on other possibly more abstract grounds, and then, based on the results of the argument, to consider the consequences of a panentheistic understanding of God. It is this path that I intend to follow in the reasoning below and thus it is time to turn to the three cases: (I) the universe consists of a finite number of discrete entities, (II) the multiverse consists of a countably infinite number of discrete entities, and (III) the multiverse consists of an uncountably infinite number of entities.

Case I: The Universe Consists of a Finite Number of Discrete Entities

In the first of the three cases the observable universe is made up of a finite number of discrete entities. Admittedly, this number is immensely large, but it is nevertheless finite. Now God, not necessarily in a panentheistic understanding, can be understood, based on Anselm's ideas, as "[…] that than which no greater can be thought".[29] In contrast to ontological arguments based on this idea, I will not argue for the *existence* of God, but rather for *panentheism*. Thus another premise is that God exists. If God then is "that than which no greater can be thought", then the transcendence is easily established, since firstly the universe is finite and secondly God by being greater cannot be finite. But *not* being finite by the previous definition—recall that an infinite set is a set which is *not* finite—amounts to God being infinite, thus transcending the universe.

The second part of establishing the immanence of God also appears to be straightforward in this first case. Here another premise needs to be specified and introduced, namely that God in a sense is *all-encompassing*. Surely, this premise is already included in the understanding of God as "that than which no greater can be thought". However, in order to establish the immanence of God, it is necessary to emphasize that "that than which no greater can be thought" also means all-encompassing. If this premise were not implicitly included in "that than which no greater can be thought" then one could think of something "greater"; something "more encompassing", thus contradicting the initial assumption. This premise of God as all-encompassing is also an essential part in Clayton's argument for panentheism from divine infinity.[30] Yet, although Clayton reasons that since God is *absolutely unlimited,* God must be all-encompassing, he does not use or involve the concept of uncountable infinity. Still he realizes the importance of Cantor's ideas about infinity.[31] Anyhow, if God is all-encompassing then clearly God cannot simply transcend the finite world, but everything must also be within God since God would otherwise not be all-encompassing. The importance of this further specification cannot be underestimated since the reasoning in Clayton's argument, that, if the finite is excluded from the infinite then the infinite would not be "truly" infinite, is not strictly correct in mathematics.[32] In measure theory there are for example cases in which even an uncountably infinite set with an infinite number of point-sized holes has the same measure as the same set without the holes.[33] In other words, even if God is construed as uncountably infinite it would be conceivable that a finite number of entities could be excluded from God *without* affecting the infinity of God, which would at least allow for the possibility of a totally *transcendent* God. Thus if God is to be construed as "that than which no greater can be thought" then it is important to realize and emphasize that this also includes the feature of God being all-encompassing. Otherwise God could be construed as transcendent without for example including such point-sized holes.

Finally, the bilateral relationship has to be established. If the immanence of God is understood in the above stronger sense of God as all-encompassing then *anything* that happens in the world, *any* process in the world would by the premise of God as all-encompassing also be within God and thus it would affect God. Further, any action of God in the world, which by the immanence of God is part of God, would obviously affect the world. Such interactions of God with the world need not be understood as dualistic.[34] Any cause in the world would by the immanence of God be part of God. The causes of God need not be understood as *external* to the physical world. Since the world is in God, God's causes could already be *internal* to the world. Thus immanence in a stronger sense seems to entail a bilateral relationship.

Case II: The Multiverse Consists of a Countably Infinite Number of Discrete Entities

It is nevertheless conceivable that the universe includes entities which are beyond human observation. There could be solar systems, galaxies or even entire worlds, which are not observable but which are still significantly similar to the observable world. There could conceivably even be a countably infinite number of entities which constitute the multiverse. This is basically what Tegmark claims in his level 2 and 3 multiverses.[35] In this case it may be tempting to think that there may be a problem in arguing for panentheism since the multiverse is already infinite. Nevertheless, it is possible to argue in parallel to the first case.

The number of entities in the multiverse, and possibly even the number of constituting universes is countably infinite. Now the assumption that God is understood as "that than which no greater can be thought" again includes the assumption of God as all-encompassing. In order to establish the transcendence of God, a panentheistic God would in some sense have to be greater than the above-stated *countably* infinite number of entities. This is easily established by thinking of God as uncountably infinite. Since God is all-encompassing and "that than which no greater can be thought", God in relation to any part and aspect of the universe in total, in this case a countably infinite multiverse, would have to be "more"; and it is possible, thinkable, and conceivable that there is a being more infinite than countably infinite; namely uncountably infinite. Thus transcendence is established with the help of the concept of uncountable infinity.

The second part of establishing the immanence of God also runs in parallel to the first case. Again, if God is all-encompassing then clearly God cannot merely transcend the finite world, but everything must also be within God since God would otherwise not be all-encompassing. The above-mentioned problem that a countable—possibly infinite—number of entities are excluded from God in the multiverse reoccurs. What if, for the sake of argument, the computer I am writing on is *not* part of God? God would still be uncountably infinite in relation to the objects in the universe or the multiverse and thus would exceed a multiverse consisting of a countably infinite number of discrete countable entities. However, God would *not* be all-encompassing. Thus, if God is all-encompassing, is construed as "that than which no greater can be thought"; then it is not possible that *any* objects, entities, points or what have you are excluded. So again immanence is established.

Finally, the bilateral relationship runs entirely in parallel to the first case. *Anything* that happens in the world, *any* process, and *any* causal chain initiated in the world would be within God and thus it would affect God. Likewise, any action of God in the world would affect the world.

Case III: The Multiverse Consists of an Uncountably Infinite Number of Entities

In the final case, if the universe in total consists of *any* conceivable kind of world—this is the thesis of modal realism –, and if these universes would consequently include all forms of mathematical structures, as in level 4 of Tegmark's theory, then at least some of them would also include structures made up of an *uncountably* infinite number of entities.[36] To be sure, it is hard, if not impossible to *imagine* such universes, but here the point is that given modal realism or Tegmark's level 4 multiverse, this kind of universe would at least be conceivable.

Now, Nagasawa defines a form of "modal panentheism" in parallel to modal realism as follows:

(1) God is the totality of all possible worlds.

(2) All possible worlds exist to the same extent that the actual world does.[37]

Surely, in this third case the requirements for modal panentheism are fulfilled even though the totality of all possible worlds is uncountably infinite. One possible objection is that such modal panentheism is actually a form of pantheism, that the transcendence of God is *not* established in relation to the multiverse; or, as Nagasawa writes, the totality of all possible worlds. Nagasawa deals with this problem in two ways. Firstly, he points out that "[…] it is a matter of definition whether modal panentheism is a version of pantheism or panentheism".[38] Secondly, he emphasizes that transcendence is granted in relation to the actual world. God transcends the world humans can observe or encounter. It would also have been possible to argue in these lines in case 2. Nevertheless, Nagasawa's second reply would still amount to a form of *global* pantheism, which could *locally* be construed as panentheism.

However, it is also possible to form a reply to this objection from Cantor's theory of the infinite. Cantor himself addresses this problem by making the distinction mentioned above between the absolute infinite and the actual transfinite. The former is reserved for God, the latter for any actual infinite such as the creation of God, the infinite or transfinite numbers used in mathematics.[39] The former also exceeds the latter. In this third case, the actual transfinite, as Cantor denotes it, would include uncountable infinity. As R.J. Russell explicates, God would still transcend this form of infinity by God's absolute infinity. R.J. Russell applies the reflection principle, which "[…] states that properties of the class of all sets are shared with (or 'reflected down to') the properties of the sets in that class".[40] In his interpretation of Wolfhart Pannenberg's theological reasoning about infinity, R.J. Russell suggests that on the one hand the transfinite, which includes uncountable infinity, by the reflection principle bears properties of the absolute infinite, but that on the other hand, God by being absolute infinite transcends the transfinite.[41] In a similar reasoning Cantor pointed out that the problem of ending up in pantheism can be avoided.[42] Pannenberg expresses this view by writing, "The infinite is first truly infinite if it at the same time exceeds and embraces its own opposition to the finite" (my translation).[43] The second and third parts of establishing the immanence of God and a bilateral relationship run in parallel to the first two cases and shall not be repeated here.

Connectedness and the Unity of God and the World

Especially in forms of panentheism or pantheism, the question of the unity of God seems to raise problems. Nagasawa writes:

> Panentheism is known to face the problem of unity: How can we consider the world a single unified entity given that it consists of uncountably many individual objects? Modal panentheism faces a parallel problem: How can we consider the totality of all possible worlds a single unified entity given that it consists of uncountably many individual objects?[44]

At first glance the consideration of "*uncountably* many individual objects" may seem odd. How could anything be uncountably infinite and yet individualized? The standard example given of an uncountable set of numbers is the set of real numbers which forms a *connected* set, and which is commonly construed as a continuum. Still, in mathematics there are objects which are uncountably infinite and yet individualized. One famous example is the Cantor set, which is uncountable but "totally disconnected" in the mathematical sense. Surely, there is a problem if the universe in total is considered to consist of an uncountably infinite number of *individual* objects. If the universe in total consisted of this type of set, for example a Cantor set, as could be the case in *pantheism*, then the *pantheist* would indeed have a problem of unity since the world and thus also God would consist of *no other* objects than these uncountably many individual objects, which would be disconnected. However, in panentheism the situation is different. *Even* if the universe in total or the multiverse consisted of an uncountably infinite number of *individual* objects, as Nagasawa suggests, then God as all-encompassing and by the transcendence of God would include and exceed this uncountably infinite number of *individual* objects. Indeed, God by being "that than which no greater can be thought", would in relation to any space at the least include the whole *continuum* of this space.

It should also be noted that imagining the world or the multiverse as a number of *individual* objects is not uncommon. Indeed, it is presumably a very commonsensical view. However, transcending such individual objects does not entail that these objects cannot be connected by the infinity which transcends them and thus form a unity. One may be tempted to imagine that there is "a gap" between the individual objects and that which transcends them. In contrast, the mathematical understanding of connectedness would mean that for any point outside the object, however close to the object, it is possible to find points "in between" which nevertheless belong to the transcending and including totality. There are no gaps. So, using the above-introduced concept of connectedness this would mean on the one hand that the objects, individual or not, are in some sense part of an n-dimensional space; and on the other hand, God would include and even exceed this space which in total would be connected. This idea of connectedness is loosely reflected in Nagasawa's and Khai Wager's ideas about cosmopyschism in the context of the problem of consciousness. They describe cosmic consciousness as "smooth, continuous and unified".[45] Understanding all God's infinities, all aspects in which God is infinite in terms of an all-encompassing, uncountable, absolute infinity grants connectedness in the above sense. Consequently, a minimal form of unity is established irrespective of whether the universe in total is finite or infinite in any stronger sense.

However, one may think that such unity would mean that the individuality of the objects in the world is lost. To see that this is not the case, an example about God's consciousness may be required. Other examples considering other aspects of God can surely be found. Here the following example may suffice. Imagine that God's consciousness encompasses and includes all individual objects in the world. On the one hand, according to the understanding of connectedness above, these objects form a unity. On the other hand, this unity need not entail that the *individual* objects "know of" each other. The *connecting* and *unity-forming* consciousness of God may only be in the part of God which is *not* the objects. Thus the objects themselves may still be regarded as individuals. I believe that such ideas, pointing in the same direction as the above-named ideas from Nagasawa and Wager, may be helpful in addressing the individuation problem as recently described by Joanna Leidenhag.[46] However, this would need further elaboration.

Summary and Final Reflection

Given that God is "that than which no greater can be thought", and is all-encompassing, the above analysis has led to the conclusion that God should be construed as panentheistic *irrespective* of whether the universe in total consists of a finite, countably infinite or uncountably infinite number of entities. Involving the topological concept of connectedness together with an understanding of God as all-encompassing leads to a minimal form of unity. Most surprising is presumably the result that understanding the world or the multiverse as infinite, even though humans usually only encounter finite entities, need not cause problems in relation to the infinity of God. Imagining the universe in total or the multiverse as infinite does not threaten the all-encompassing infinity of a panentheistic God. It has also become clear that understanding God as all-encompassing plays a vital role in all of the above cases. Uncountable infinity alone does not suffice to establish transcendence or immanence in some of the above cases since it is possible to conceive of cases of uncountable infinity which do not include all possible objects.

Since these relatively novel mathematical concepts have been successfully related to and fruitfully incorporated in philosophical and theological thinking, I suggest a further consequence, namely that innovative thinking in one discipline—here mathematics—could and presumably should be related to, and incorporated into disciplines, which in some sense use and apply similar concepts—in this case philosophy of religion. This meta-perspective opens up the way for similar attempts to engage in a dialogue between philosophy of religion, and, depending on the concepts involved, mathematics or the natural sciences.

In general the notion of infinity is surely difficult to grasp with a human finite mind. The fact that *finite* humans can conceptualize, analyze, and discuss various forms of infinity seems in some sense to be a mystery. As humans our brains are made of a finite number of neurons. Yet, we can imagine and reason about concepts like infinity, connectedness, Cantor sets or an all-encompassing God. Isn't it wonderful that this is possible although human brains *are* finite? Can this ability to positively and creatively relate to infinity be read as or understood as an example for humans or creation as an image of God?

Notes

1. Joseph Warren Dauben, *Georg Cantor* (Harvard University Press, 1979), 142–144; Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind (Springer Verlag berlin-Heidelberg, 1932): 385–387.
2. Eric C. Steinhart, "A Mathematical Model of Divine Infinity," *Theology and Science* 7:3 (August 2009): 261–274; Wolfgang Achtner, "Infinity in Science and Religion," *Neue Zeitschrift für systematische Theologie und Religionsphilosophie* 47 (2005): 392–411.
3. To be sure, the infinite in relation to *time* has been extensively discussed in the philosophy of religion. The timelessness of God would undoubtedly incorporate a specific understanding of the infinite. Examples of how God's relation to time is discussed can notably be found in works by for example William Lane Craig, *Time and Eternity* (Wheaton, IL: Crossway, 2001); Robert J. Russell, *Time in Eternity: Pannenberg, Physics, and Eschatology* (University of Notre Dame Press, 2012). Be that as it may, in this article, I will not consider infinity in relation to time. Also, in the significant anthology edited by Michael Heller and W. Hugh Woodin, eds., *Infinity: New Research Frontiers* (Cambridge University Press, 2011) Russell discusses God in relation to Cantor's ideas about infinity. In this specialized discussion he does not explore the distinction between countably and uncountably infinite sets. However, in Russell, *Time in Eternity*, chap. 3, he elaborates details about Cantor's theory of the transfinite.
4. Andrew Liddle, *An Introduction to Modern Cosmology* (Wiley, 2015): Chap 4,5,7.
5. David Lewis, *On the Plurality of Worlds* (Basil Blackwell Inc., 1986).
6. Although, Tegmark's ideas have their starting point in concepts and reasoning from theoretical physics, the arguments for a multiverse should not be regarded as based on *empirical* evidence. They are rather the outcome of philosophical reasoning about theories in physics and can sometimes presumably be regarded as speculation. This, I think, is an important observation since many ideas and problems with a multiverse theory have already been discussed and analyzed in analytical philosophy especially in relation to David Lewis' modal

realism. See Max Tegmark, "Varieties of Multiverse," in *The Mystery of Existence*, ed. John Leslie and Robert Lawrence Kuhn (Wiley-Blackwell, 2013), 194–206; Max Tegmark, *Vårt Matematiska Universum, e-book* (Volante, 2014).

7. Niels-Henrik Gregersen, "Three Varieties of Panentheism," in *In Whom We Live and Move and Have Our Being*, ed. Philip Clayton and Arthur Peacocke (Wm. B. Eerdmans Publishing Co., 2004), 22.
8. Anselm, *Monologion and Proslogion* (Hackett Publishing Company, 1995) Proslogion, originally 1077. Clayton, God and Contemporary Science, 99–100.
9. Yujin Nagasawa, "Modal Panentheism," in *Alternative Concepts of God*, ed. Yujin Nagasawa and Andrei A. Buckareff (Oxford University Press, 2016), 91–105.
10. Ralph P. Grimaldi, *Discrete and Combinatorial Mathematics: An Applied Introduction*, 4th ed. (Addison, Wesley, Longman Inc., 1999), 139.
11. Ernst Snapper, "The Three Crises in Mathematics: Logicism, Intuitionism, and Formalism," *Mathematics Magazine*, 52:4 (1979): 207–216.
12. Ibid., 208.
13. Ibid., 210–215, see also Russell, *Time in Eternity*, 205–207.
14. Ibid., 207.
15. Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind, 378.
16. In more technical terms finite and infinite sets can be defined as follows:

    "Any set A is called a *finite* set if $A = \emptyset$ or if $A \sim \{1,2,3, \dots, n\}$ for some $n \in \mathbf{Z}^+$. When $A = \emptyset$ we say that A has no elements and write $|A| = 0$. In the latter case A is said to have n elements and we write $|A| = n$. When a set A *is* not finite then it is called infinite" Ralph P. Grimaldi, Discrete and Combinatorial Mathematics: An Applied Introduction, 4th ed., appendix A-26.

    This definition implies that there is a one-to-one correspondence between $\{1,2,3, \dots, n\}$ and A for some n. Thus a definition of the infinite could also be based on a one-to-one correspondence, which was Cantor's original definition. He simply defined an infinite set as a set which is isomorphic to a proper subset of itself Leonard M. Wapner, *The Pea and the Sun* (A K Peters Ltd., 2006), 107.
17. Grimaldi, Discrete and Combinatorial Mathematics: An Applied Introduction, appendix A-30; George F. Simmons, *Topology and Modern Analysis* (Robert E. Krieger Publishing Company, Inc., 1963), 36–37.
18. In more strict and mathematically rigorous terms this can be formulated as follows:

    "The set $(0,1] = \{x|\ x \in \mathbf{R}$ and $0 < x \leq 1\}$ is not a countable set.

    Proof: If (0,1] were countable, then we could write this set as a sequence of distinct terms. $(0,1] = \{r_1,r_2,r_3, \dots\}$. To avoid[+] two representations we agree to write real numbers in (0,1] such as 0.5 as 0.4999 … – so no element in (0,1] is represented by a decimal expansion that terminates. Writing such decimal expansions for $r_1,r_2,r_3, \dots$ we get

    $r_1 = 0.a_{11}a_{12}a_{13} \dots$
    $r_2 = 0.a_{21}a_{22}a_{23} \dots$
    $r_3 = 0.a_{31}a_{32}a_{33} \dots$

    $r_n = 0.\mathrm{a}_{\mathrm{n}1}\mathrm{a}_{\mathrm{n}2}\mathrm{a}_{\mathrm{n}3} \dots$
    where $a_{ij} \in \{0,1,2, \dots ,9\}$ for all $i,j \in \mathbf{Z}^+$.
    Now consider the real number $r = 0.b_1b_2b_3 \dots$ where for each $k \in \mathbf{Z}^+$,
    $b_k = 3$, if $a_{\mathrm{kk}} \neq 3$ and $b_k = 7$, if $a_{\mathrm{kk}} = 3$.

    Thus $r \in (0,1]$, but for every $k \in \mathbf{Z}^+$ we have $r \neq r_k$ – so $r \notin \{r_1,r_2,r_3, \dots\}$. This contradicts our assumptions that $(0,1] = \{r_1,r_2,r_3, \dots\}$. The technique employed in this proof […] is generally known as Cantor's Diagonal Construction in honor of the (Russian-born) German mathematician Georg Cantor (1845–1918) who introduced the idea in December of 1873" (Grimaldi, Discrete and Combinatorial Mathematics: An Applied Introduction, appendix A-30. See also Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind, 115–199).

19. Tegmark, Vårt Matematiska Universum, chap. 6,8,12; Tegmark, "Varieties of Multiverse," 203–205; Lewis, *On the Plurality of Worlds*, 133.
20. Simmons, *Topology and Modern Analysis*, 143. For a less modern definition of this concept by Cantor see Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind, 190–194.
21. Simmons, *Topology and Modern Analysis*, 145.
22. Aquinas discusses the following four questions related to God's immanence: Is God really present in all things? Is God everywhere? Is God everywhere by his essence, power and presence? Does to be everywhere belong to God alone? His answers to these four questions are positive in all four cases. See Thomas Aquinas, Summa Theologica, ed. *übersetzt von Dominikanern und Benediktinern Deutschlands und Österreichs* (Verlag Anton Pustet, Salzburg Leipzig, 1988), ST I Q8.
23. Elizabeth A. Johnson, *She Who Is* (New York: The Crossroad Publishing Company, 1992), 231.
24. Gregersen, "Three Varieties of Panentheism," 22.
25. Ibid., 24.
26. Surely, it is possible to argue for panentheism on other grounds. Charles Hartshorne for example introduces two forms of dipolarity in relation to God. The first dipolarity is expressed by God as the universal cause (C) and the whole of reality (W). The second dipolarity states that God can be understood in terms of being supremely abstract (A) and supremely concrete (R). Hartshorne then argues that God reasonably can be construed as having all four features which according to him amounts to panentheism. One advantage with Hartshorne's approach with the double dipolarity is that it results in a 3×3 matrix, which in turn provides a conceptual tool for describing other forms of theism. See Charles Hartshorne and William L. Reese, *Philosophers Speak of God* (Humanity Books, 2000), 503–510.
27. Sallie McFague, *The Body of God* (Fortress Press, 1993).
28. Ibid.; Sallie McFague, *A New Climate for Theology* (Fortress Press, 2008).
29. Anselm, Monologion and Proslogion Proslogion, originally 1077.
30. See Clayton, God and Contemporary Science, 99–100.
31. See Clayton, God and Contemporary Science, 90; Philip Clayton, *The Problem of God in Modern Thought* (Eerdmans Publishing Co, 2000), 69–70.
32. See Philip Clayton, *God and Contemporary Science* (Eerdmans Publishing Co, 1997), 99.
33. This observation is based on my interpretation of the concept of "almost everywhere" in measure theory, which allows that certain points can be "omitted" in for example the process of integration. Indeed there are even uncountably infinite sets such as the Cantor set which have measure of zero. The concept of "almost everywhere" surely is an important tool in integration theory. For further reading on this advanced mathematical topic see standard textbooks on measure theory or integration theory, for example Gerald B. *Folland, Real Analysis* (John Wiley and Sons, 1999); Jürgen Elstrodt, *Maß- Und Integrationstheorie*, 4th ed. (Springer-Verlag Berlin Heidelberg, 2005).
34. Mikael Leidenhag has argued in the direction that panentheism leads to dualistic ontology. Mikael Leidenhag, *Naturalizing God? A Critical Evaluation of Religious Naturalism* (Uppsala University, 2016), 181–184; Mikael Leidenhag, "The Relevance of Emergence Theory in the Science-Religion Dialogue," *Zygon®* 48:4 (2013): 977–979.
35. Tegmark, Vårt Matematiska Universum, chap. 6,8; Tegmark, "Varieties of Multiverse," 197–203.
36. Lewis, *On the Plurality of Worlds*, 2–3; Tegmark, Vårt Matematiska Universum, chap. 12.
37. Nagasawa, "Modal Panentheism," 91.
38. Ibid., 94.

39. Dauben, *Georg Cantor*, 144–146; Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind, 378–440.
40. Russell, *Time in Eternity*, 203.
41. Ibid., 219–221.
42. Cantor, Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. Mit erläuternden Anmerkungen sowie mit Ergänzungen aus dem Briefwechsel Cantor-Dedekind, 385–387, 399.
43. Wolfhart Pannenberg, *Systematische Theologie Band 1* (Vandenhoeck & Ruprecht, 2015): 432 The original text in German is: "Wahrhaft unendlich ist das Unendliche erst, wenn es seinen eigenen Gegensatz zum Endlichen zugleich übergreift."
44. Nagasawa, "Modal Panentheism," 94.
45. Yujin Nagasawa and Khai Wager, "Panpsychism and Priority Cosmopsychism," in *Godehard Brüntrup and Ludwig Jaskolla*, ed. Panpsychism (Oxford University Press, 2017), 122.
46. Joanna Leidenhag, "Unity Between God and Mind? A Study on the Relationship Between Panpsychism and Pantheism," Sophia, 2018.

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