Can pairs trading be used during a financial crisis?

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Abstract

In this paper, it is investigated whether pairs trading is a suitable trading strategy during a financial crisis. It is written in the subject of financial statistics and aims to particularly focus on the statistical aspects of the strategy. The constituents of the S&P 500 index from the crisis of 2008 are used as empirical evidence for the study. The evaluation is made by comparing the yearly performance of the constructed pairs trading portfolios, to the performance of the S&P 500. The pairs trading methodology that is used is largely based on the statistical concepts of stationarity and cointegration. The tests that are chosen to check for these properties, are the ADF-test and Johansen’s test respectively. In the study it is found that in terms of return, the portfolio outperforms the S&P 500 for all of the years but one. Despite some flaws with regard to the limited extent of the study, it is concluded that the findings suggest pairs trading as profitable during a financial crisis.

Key words: Pairs trading, Cointegration, Stationarity, Market neutrality, Financial crises, Statistical arbitrage.
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1 Introduction

The consequences of a financial crisis are often devastating. When the financial market crashes, it affects everyone; banks and other financial institutes shut down, entire countries go bankrupt and the unemployment rate increases drastically. This results in people losing their houses, their jobs and their savings. The overall effect of a crisis could perhaps be broken down into people losing their wealth. There are plenty of vastly different investment strategies that are used around the world every day. If it would be possible to use one of these trading strategies to prevent losses, or maybe even generate profit, during a financial crisis, it would work as a safety net when the crash comes. It then becomes interesting to investigate the topic of market neutral strategies, strategies that in theory are independent of the market’s fluctuations. There are several different approaches and versions of market neutral strategies, one of which being pairs trading. This investment strategy was formally introduced in the 1980’s by a group of quantitative analysts led by Nunzio Tartaglia at Morgan Stanley (Investopedia, 2019a). They developed a trading strategy which considered the movement of stocks in pairs, rather than their individual fluctuation (Vidyamurthy, 2004, p. 73). For pairs trading to be a profitable strategy, two characteristics are required. Firstly, there has to exist pairs with cointegrating properties throughout the crisis. Secondly, the pairs must create a positive return.

The purpose of this research is to determine whether the pairs trading approach is efficient during financial crises. This is examined by looking at the financial crisis of 2008. Moreover, the universe of stocks that is considered for the portfolio is limited to be the constituents of the S&P 500 index. It is then evaluated whether the pairs trading strategy is appropriate to be withheld throughout the crisis. This is decided by whether a sufficient amount of cointegrated pairs can be found and whether a positive return is generated. In addition, the performance is compared to the return of the S&P 500 index. However, the statistical aspects of pairs trading are what primarily have been focused. The choice of hypothesis tests are, among other things, made with caution. In accordance with the objective of this paper, the research question is formulated as follows:

**Is pairs trading a profitable trading strategy during a financial crisis?**

In the following section, pairs trading will be presented to the reader in an intuitive and non-technical way. Thereafter, the statistical framework is presented in section 3, to lay the technical foundation for the strategy. Section 4 provides a more mathematical approach to the
financial theory. In section 5, the methodology will be discussed and in section 6 the results are presented. These results are then analyzed in section 7, followed, lastly, by a conclusion in section 8.

2 Financial background

2.1 Basic terms

*Short*
Selling short is a trading technique where a trader borrows a stock from one part to sell it to another part. The part from which the trader borrowed the stock is then paid back within a given time frame. The trader pays back his debt to the stock price that is active when the payment is performed. Hence it is the negative of the price change that is the trader’s return. It is therefore favorable to purchase a stock that is believed to be overvalued (Investopedia, 2019b).

*Long*
Buying a stock long could be explained as the opposite of selling short. The stock is bought with the intention of being sold later, at an expected higher price level (Investopedia, 2019c).

*Statistical arbitrage*
Statistical arbitrage is the usage of historical data whilst simultaneously purchasing and selling two stocks to generate profit (Erhman, 2006, p. 5).

2.2 Fundamentals of pairs trading
Pairs trading is a trading technique where the pricing of two stocks are analyzed simultaneously and compared (Ehrman, 2006, p. 2). The ways in which the pairs are matched can be divided into two main categories - fundamental and technical analysis (Ibid, p. 5). Fundamental analysis focuses the companies’ properties and current situation, the industry it acts within, and the economy as a whole (Ibid, p. 46). Technical analysis is instead solely based on the historical data of the stock prices, and is what primarily will be used in this paper (Ibid, p. 7). For the stocks to be considered an appropriate pair when using technical analysis, they must be cointegrated (Vidyamurthy, 2004, p. 83-84). The technicalities behind cointegration is further
explained in section 3.4. Once a cointegrated pair is found, the goal for the pairs trader is not to predict exactly how the stock prices will develop; it is merely to evaluate whether their current pricing is in line with how they have been priced relative to each other historically (Ibid). When performed correctly, pairs trading allows for a return of investment that is independent of the markets’ fluctuations (Ehrman, 2006, p. 4). Adding the features of arbitrage and the ability to use leverage (defined in section 2.1), pairs trading qualifies as a market-neutral strategy (Ibid, p. 3). Three different types of market neutrality are dollar neutrality, beta neutrality and sector neutrality (Ibid, p. 5). The first of the three refers to the monetary balance of the portfolio (Ibid, p. 64). It is achieved by having the total value on the short side of the portfolio the same as on the long side. This is given by the following formula:

$$\text{# of short shares} = \text{# of long shares} \times \frac{\text{Price of long stock}}{\text{Price of short stock}}.$$  \hspace{1cm} (1)

Beta neutrality on the other hand, relates to the risk, and the volatility of the stocks (Ehrman, 2006, p. 32-33). A stock that has a beta equal to 1 has a risk that is historically in sync with the market. To reach complete beta neutrality, the beta of the long side of the portfolio should be the exact same as the beta of the short side. Having betas that are exactly the same is both redundant and far from realistically attainable. Therefore this neutrality can be seen as achieved when the betas that are not too different from each other. The final neutrality, sector neutrality, instead emphasizes the importance of keeping each sector of the portfolio monetarily in balance (Ibid, p. 65-66). For it to be achieved, the value of the short and long side of each sector in the portfolio should be the same.

The arbitrage element of pairs trading originates in recognising when one of the two stocks has become over- or underpriced, relative to the other (Ehrman, 2006, p. 90-91). This finding can then be utilized by selling the relatively overpriced stock short, whilst taking a long position in the other. How short and long investments function is described in section 2.1 above. Using technical analysis, such a deviation is defined to have occurred when the stock prices have diverged from their historical relationship (Ibid, p. 75, 82-83). Since the pair that is used is found to be cointegrated, it is believed that the pair shall eventually converge back to what is suggested by their historical relationship. The reason for this belief is, in pairs trading, explained by the idea of implied convergence (Ibid). Implied convergence states that two related stocks that have diverged from their historical relationship should at some point return to that historical mean. This type of convergence is found in all types of mean reversion, where the convergence part of the process is called a reversion to the mean. While pairs trading is a type
of mean reversion strategy, the typical mean reversion trader bases their decision on the analysis of only one stock at a time.

What is important for the reader to keep in mind is that from a technical analyst point of view, any potential intuitive or fundamental explanation for temporary significant price discrepancies and their expected dissolving, is viewed as redundant (Ehrman, 2006, p. 81). This might clarify why a pairs trader is categorized as a speculative arbitrageur. However, it is crucial that the historical data of the stock prices identifies them as divergences and suggest that they shall eventually converge (Ibid). When is the deviation then sufficient to be classified as a divergence? There are several, quite different, indicators that can be used to help determine this, one of which being the Bollinger bands (Ibid, p. 106-107). The indicator targets deviation from the moving average of the spread of the pair. Spread is a measure of how the stocks relate to each other, and moving average is its average over a specified period of time (Ehrman, 2006, p. 106-107; Vidyamurthy, 2004, p. 8). This period can be defined as 10, 20 or 50 days, depending on the trader is seeking a short-, mid- or long-term strategy. Commonly, it is somewhere between 12-26 days. A more technically rigorous explanation of spread and moving average can be found in sections 4.1 and 4.3 respectively. The way that the Bollinger bands often define a divergence is when the difference between the spread and its moving average exceeds two standard deviations. Hence, when the spread surpasses this limit of deviation, the trades are initiated. The stock that is overvalued relative to the other should be sold short, while the other should be bought long. The mathematical details of the Bollinger bands indicator are discussed in section 4.3.

Having determined when the spread has diverged, and thereby when it is suitable for the trade to be initiated, what is yet to be decided is how convergence is defined, and when to exit the trade. As for divergence, there is no one absolute answer. One way of defining convergence, which is used by some traders, is when the moving average is crossed (Ehrman, 2006, p. 116-117). Using this method, the trade should be initiated when the spread has moved passed two standard deviations from its moving average, and ended when it finally crosses the moving average. Another definition is to instead await sufficient convergence (Ibid, p. 156). This boundary is quite arbitrarily set by the trader as a standardized distance from the moving average, often in terms of standard deviations. This could for example mean that the trade should be ended as the price ratio returns to a distance of one standard deviation to its moving average.

Despite being found as cointegrated, the stock pair does not always converge. Although
this might be rather obvious, it is perhaps less clear how exactly a non-convergence should be defined. Being the biggest risk faced within pairs trading, realizing for how long it on average is profitable to wait for convergence, is highly relevant (Ehrman, 2006, p. 130). There are two ways that non-convergence is defined to have occurred, the first being when the stop-loss limit has been reached (Ibid, p. 83-84). A stop-loss limit is a certain stock price level, set by the trader, where the trade should be ended. If, for instance, the price of the stock that has been sold short, rises above an upper bound stop-loss limit, it is an indication that the trade should be ended. The stop-loss level is often, similarly to the Bollinger bands, set in terms of standard deviations from the moving average.

There are several different types of elements that add insecurity not only to pairs trading, but to all market-neutral strategies. The potential flaws in the construction of the model are, quite intuitively, called model risk (Ehrman, 2006, p. 40). Any defect in the model might sabotage the entire strategy, and turn a potential profit into a loss. Hence it is critical for the trader to strive for the model to be well constructed. Another risk that any pairs trader faces is execution risk (Ibid). It can involve issues with liquidity, commission, margin ability and rules regarding short-sale. Risks in this category can be managed by having a trader that is aware of them, and understands how to avoid them. Lastly, there is the security selection risk, which typically is the risk of an unexpected news report or company announcement that severely affects the perception of one of the stocks in a pair (Ibid). This risk cannot be completely avoided, which is central in the general critique towards market-neutral strategies.

In this section, a brief introduction has been presented of the essential non-technical concepts and necessities of pairs trading. Attempting to deepen the readers intuitive understanding for the trading technique, a visual presentation of the strategy is given below, in Figure 1. It is highly relevant to acknowledge that this illustration is not intended to accurately replicate the technicalities of the strategy. Instead, the aim of the figure is to capture its elemental mechanisms, and comprehensively assist the reader to a better understanding of the actual trading. In the figure, the daily prices of two cointegrated stocks, Stock 1 and Stock 2, are visualised. The two stock prices in this example, are from historical data found to closely follow each other in the long run. That is, if their prices did drift apart, or diverge, they have eventually intersected and returned to follow each other. As can be seen in the figure, the stocks do drift apart. The dotted grey lines represent when a deviation is sufficient to be defined as a divergence. The divergence occurs at January 5th, where Stock 1 is relatively overvalued to Stock 2. Hence
Stock 1 is shorted, whilst Stock 2 is taken a long position in. The trade is ended when the stock prices have converged passed the green dotted lines, which is clearly pointed out at January 11th.

![Graph showing the trade strategy](image)

**Figure 1:** An illustrative theoretical example of the strategy, showing when a trade is initiated and ended respectively.

### 2.3 Financial crisis of 2008

In 2007, the first major warning signs regarding the condition of the U.S. economy arose. The US Federal Reserve continuously lowered the interest rates and American house prices experienced the largest yearly downfall in a hundred years (BBC News, 2009). The lending of the banks reached the highest levels since 1998 and S&P reduced its investments in monoline insurers (Ibid). At the end of the year, the crisis escalated and losses started to emerge (Ibid). The return of the S&P 500 index for 2007 was at 3.53 % (Macrotrends, 2019).

The following year, 2008, the financial crisis became a fact. The crisis brought down several giants, one of which being the investment bank Bear Sterns, that was purchased by JP Morgan (BBC News, 2009). Two other companies that were affected, were the financial institutes Fannie May and Freddie Mac (Ibid). Both institutes were bailed out by the U.S. government. Another famous example was the investment bank Lehman Brothers and its bankruptcy at September 15th (Ibid). Two other victims were the banks Washington Mutual and Wachovia, both of them collapsing at the the end of September (Ibid). The amount of people who lost
their jobs in 2008 was the highest recorded number since World War II (Ibid). In December of 2008, the United States officially entered a recession (Ibid). The return of the S&P 500 was a negative 38.49% (Macrotrends, 2019).

U.S. President Barack Obama signed an economic stimulus package of $787 billion in 2009, with the intention of getting the economy back on track (BBC News, 2009). Shortly thereafter, the U.S. car industry took a major hit. Consequently, two of the three leading car producers, Chrysler and General Motors, entered bankruptcy (Ibid). In the second quarter of 2009, Goldman Sachs and several other banks announce large profits (Ibid). Despite these signs of recovery, analysts warned that the crisis was not over yet. The return of the S&P 500 was at 23.45% at the end of the year (Macrotrends, 2019).

The U.S. economy had started to stabilize in 2010, something that could hardly be said about the European economies. What came to follow were several national economic crashes in Europe. In May, it was decided by the finance ministers of the Eurozone that Greece was to be bailed out (The Guardian, 2012). In November, the same decision was made for Ireland (Ibid). The return of the S&P 500 was at 12.78% at the end of the year (Macrotrends, 2019).

In 2011, the financial crisis in the U.S. was practically over. In Europe however, the aftermath of the U.S. crisis was still visible. Greece for example, received a second bailout (The Guardian, 2012). The yearly return of the S&P 500 index was at 0.00% (Macrotrends, 2019).

3 Statistical theory

3.1 Stationarity

A stochastic process, $Y_t$, is a covariance-stationary process if the process has a constant mean, a constant variance and if the covariance only depends on the lag $k$ (Cryer, Chan, 2008, p. 16). This can be expressed mathematically as

\[
E(Y_t) = \mu \quad \forall t,
\]
\[
V(Y_t) = \sigma^2 \quad \forall t,
\]
\[
Cov(Y_t, Y_{t+k}) = \gamma_k \quad \forall t, k.
\]

A property that a stationary process possesses, is that it is mean reverting. This means that after a shock to the process it will revert back to its unconditional mean (Asteriou, Hall, 2016, p. 277). If the process is not stationary, it will not revert back to an unconditional mean, and
the variance will depend on time and will approximate infinity as time goes to infinity (Ibid, p. 348).

### 3.2 Unit root

To understand how unit roots function, an AR(1) model could be considered, which is written as follows (Asteriou, Hall, 2016, p. 349):

\[ Y_t = \phi Y_{t-1} + e_t, \]  

where \( e_t \) is a white noise process. The potential values of \( |\phi| \) can be categorized into three different scenarios; it is either smaller, equal to or larger than 1. Firstly, if \( |\phi| < 1 \), the process is stationary. If, instead, \( |\phi| = 1 \), the process is nonstationary. Lastly, if \( |\phi| > 1 \), the process "explodes" (Ibid, p. 350). By taking equation 3 and substituting its lagged term by the day before that, it is possible to extend the AR(1) process to an infinite moving average process, MA(\( \infty \)):

\[
\begin{align*}
Y_t &= \phi Y_{t-1} + e_t, \\
Y_t &= \phi(\phi Y_{t-2} + e_{t-1}) + e_t, \\
Y_t &= \phi^2 Y_{t-2} + e_t + \phi e_{t-1}, \\
Y_t &= \phi^2(\phi Y_{t-3} + e_{t-2}) + e_t + \phi e_{t-1}, \\
Y_t &= \phi^3 Y_{t-3} + e_t + \phi e_{t-1} + \phi^2 e_{t-2}, \\
\vdots \\
Y_t &= e_t + \phi e_{t-1} + \phi^2 e_{t-2} + \phi^3 e_{t-3} \ldots.
\end{align*}
\]

In the final step of equation 4 it can be understood that if \( |\phi| < 1 \), the influence of the error terms will decay and the process will revert back to its equilibrium value. However, if \( |\phi| = 1 \), the influence of the error terms will never decay. All past error terms will influence today’s value equally, and this is what is called a unit root process, or more specifically, a Random walk in this case. If, in the example above, \( |\phi| = 1 \), there would be a unit root in the AR(1) process. This is solved by taking the first difference

\[
\begin{align*}
Y_t - Y_{t-1} &= Y_{t-1} - Y_{t-1} + e_t, \\
\Delta Y_t &= e_t.
\end{align*}
\]
Hence, \( \Delta Y_t \) is found to be a stationary process. In this example, the process was integrated of order 1, written mathematically as \( Y_t \sim I(1) \). However, a process might be integrated of a higher order than 1, implying that it taking the first difference is not sufficient for the process to become stationary. A generalized way of formulating this is that for a stochastic process, \( Y_t \), that is integrated of order \( d \) becomes stationary by taking the \( d \)th difference. This would be expressed mathematically as if \( Y_t \sim I(d) \), then

\[
\Delta^d Y_t = e_t. \tag{6}
\]

One approach to test for unit roots in the process, is to perform the augmented Dickey-Fuller’s test.

### 3.3 Augmented Dickey-Fuller’s test

In 1979, David Dickey and Wayne Fuller developed a method for testing non-stationarity in a process, called the Dickey-Fuller’s test. Their insight was that testing for non-stationarity is the same thing as testing for a unit root. However, this test assumes that the error term is white noise, which it is unlikely to be (Asteriou, Hall, 2016, p. 357). Therefore they expanded their test. The new, augmented version eliminates autocorrelation by adding lagged terms of the dependent variable (Ibid). This is called the Augmented Dickey-Fuller’s test, often referred to as the ADF-test. There are three forms of the ADF-test, and the characteristics of the data determines which one that should be used (Ibid). The models for these three forms can be written as

\[
\Delta y_t = \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t,
\]

\[
\Delta y_t = a_0 + \gamma y_{t-1} + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t, \tag{7}
\]

\[
\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t,
\]

where \( a_0 \) is a constant in the random walk-process, \( \gamma = (\phi - 1) \), \( a_2 t \) is a non-stochastic time trend, \( \sum_{i=1}^{p} \beta_i \Delta y_{t-i} \) is a lagged term of the dependent variable to remove autocorrelation and \( u_t \) is an error term. In the ADF-test, the null hypothesis is that \( \phi = 1 \), or equivalently that \( \gamma = 0 \), implying that there is a unit root. The alternative hypothesis is that \( \phi < 1 \), or equivalently that
\( \gamma < 0 \), implying that there is not a unit root (Ibid, p. 356). The test statistic used is

\[
ADF_{obs} = \frac{\hat{\gamma}}{\hat{\sigma}_\gamma}.
\]

(8)

### 3.4 Cointegration

The concept of cointegration is perhaps most comprehensive when exemplifying with the cointegrated relationship of a pair. Consider two stochastic processes that both are found to be nonstationary. If a linear combination of the two is found to be stationary, then the processes are cointegrated (Asteriou, Hall, 2016, p. 368). To describe the concept more formally, consider the processes \( Y_t \sim I(d) \) and \( X_t \sim I(b) \), where \( d \geq b > 0 \). If \( d = b \), it is possible that the linear combination of the two processes would be \( I(0) \), which is stationary (Ibid, p. 369). This follows from that it is given that \( d - b = 0 \). What is concluded is that the process lacks a unit root. It is therefore clear that covariance-stationarity, rather than strict stationarity, is considered sufficient for the purpose of cointegration.

To reach a deeper and more technical understanding, cointegration can be explained mathematically with an example of two nonstationary processes, \( Y_t \) and \( X_t \). This pair would be cointegrated if both of these variables are integrated of the same order and if there exist two vectors, \( \theta_1 \) and \( \theta_2 \), which make the linear combination of \( Y_t \) and \( X_t \) stationary (Asteriou, Hall, 2016, p. 369). This can be written as

\[
\theta_1 Y_t + \theta_2 X_t = u_t \sim I(0).
\]

(9)

Hence, the linear combination possesses the characteristics of a stationary process. From a mathematical point of view, this is arguably the very foundation of the used pairs trading methodology. It is possible to rewrite equation 9 as

\[
Y_t = -\frac{\theta_2}{\theta_1} X_t + e_t,
\]

(10)

where \( e_t = \frac{u_t}{\theta_1} \). Then, \( Y^* = -\frac{\theta_2}{\theta_1} X_t \) is the equilibrium value of \( Y_t \). This is the value that \( Y_t \) will have in the long term (Asteriou, Hall, 2016, p. 369). However, if a shock hits the cointegrated process, it would cause the process to shift away from its equilibrium. In such a scenario, the error-correction model, ECM, corrects this deviation, and shifts the process back to its equilibrium (Ibid, p. 371).
### 3.5 The error-correction model

To understand the error-correction model, consider again the two processes $Y_t$ and $X_t$, where both are integrated of order 1. If then $Y_t$ is regressed upon $X_t$, the true model is given by the equation

$$Y_t = \beta_1 + \beta_2 X_t + u_t. \quad (11)$$

If $Y_t$ and $X_t$ would both be integrated of order 0, this would be a legitimate estimation and equation 11 could be written as

$$Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_t = \hat{u}_t \sim I(0). \quad (12)$$

However, due to the fact that $Y_t$ and $X_t$ are both are integrated of order 1, a problem arises. Spurious correlation estimates are given, and the $\hat{\beta}$’s in equation 12 are not consistent estimators (Asteriou, Hall, 2016, p. 370). This problem is solved by taking the first difference, making $\Delta Y_t \sim I(0)$ and $\Delta X_t \sim I(0)$. The regression model would then be

$$\Delta Y_t = \hat{b}_1 + \hat{b}_2 \Delta X_t + \Delta u_t. \quad (13)$$

This solution solves the problem with spurious correlations, and $\hat{b}_1$ and $\hat{b}_2$ can be estimated correctly, but a new problem appears with the model. This model only includes the short-term relationship between the two variables (Ibid, p. 370). The ECM can express both the short- and long-term relationship between the variables. It is implied from the cointegrated relationship between $Y_t$ and $X_t$, that $u_t$ is stationary. Recall that $\Delta u_t = u_t - u_{t-1}$. From these insights, it is clear that the term $u_t$ could be excluded from the model given by equation 13. The true model can then be written as (Ibid, p. 371)

$$\Delta Y_t = a_1 + a_2 \Delta X_t - \pi (Y_{t-1} - \hat{\beta}_1 - \hat{\beta}_2 X_{t-1}) + e_t. \quad (14)$$

Here, the advantage of the ECM is shown clearly, as the model includes both short- and a long-term elements. An additional perk of the ECM is that it often removes time trends, as a result of using the technique of taking first differences (Ibid, p. 371).

### 3.6 The vector autoregressive model

A vector autoregressive model, or the VAR model, is an extension of the regular AR-model which allows for more than one stochastic variable. In the VAR model, no distinction is made
between exogenous and endogenous variables, meaning that all the variables should be treated as endogenous (Sims, 1980, p. 2). This means that in its reduced form, the same regressors are used for all equations (Asteriou, Hall, 2016, p.334). To understand the VAR-model, consider a bivariate model with one lag. Such a model could be written as

\[ Y_t = \phi_{10} - \phi_{12}X_t + \psi_{11}Y_{t-1} + \psi_{12}X_{t-1} + u_{Yt}, \]  
\[ X_t = \phi_{20} - \phi_{21}Y_t + \psi_{21}Y_{t-1} + \psi_{22}X_{t-1} + u_{Xt}, \]  

(15)

where both \( Y_t \) and \( X_t \) are assumed to be stationary and \( \{u_{Yt}, u_{Xt}\} \sim iid(0, \sigma^2) \). By writing equation 15 in matrix form, the following is obtained

\[
\begin{pmatrix}
1 & \phi_{12} \\
\phi_{21} & 1
\end{pmatrix}
\begin{pmatrix}
Y_t \\
X_t
\end{pmatrix}
=
\begin{pmatrix}
\phi_{10} \\
\phi_{20}
\end{pmatrix}
+
\begin{pmatrix}
\psi_{11} & \psi_{12} \\
\psi_{21} & \psi_{22}
\end{pmatrix}
\begin{pmatrix}
Y_{t-1} \\
X_{t-1}
\end{pmatrix}
+
\begin{pmatrix}
u_{Yt} \\
u_{Xt}
\end{pmatrix}.
\]  

(16)

This can be written as

\[ BZ_t = \Gamma_0 + \Gamma_1Z_{t-1} + u_t, \]  

(17)

where \( B = \begin{pmatrix} 1 & \phi_{12} \\ \phi_{21} & 1 \end{pmatrix} \), \( Z_t = \begin{pmatrix} Y_t \\ X_t \end{pmatrix} \), \( \Gamma_0 = \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} \), \( \Gamma_1 = \begin{pmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{pmatrix} \) and \( u_t = \begin{pmatrix} u_{Yt} \\ u_{Xt} \end{pmatrix} \). If \( B^{-1} \) is multiplied to both sides of equation 17, then

\[ Z_t = A_0 + A_1Z_{t-1} + e_t, \]  

(18)

where \( A_0 = B^{-1}\Gamma_0 \), \( A_1 = B^{-1}\Gamma_1 \) and \( e_t = B^{-1}u_t \). The standard form of the VAR is then

\[ Y_t = a_{10} + a_{11}Y_{t-1} + a_{12}X_{t-1} + e_{1t}, \]
\[ X_t = a_{20} + a_{21}Y_{t-1} + a_{22}X_{t-1} + e_{2t}. \]  

(19)

The two new error terms, \( e_{1t} \) and \( e_{2t} \), are mixtures of the two shocks \( u_{Yt} \) and \( u_{Xt} \) (Ibid, p.335). The two error terms are in fact

\[ e_{1t} = (u_{Yt} + \phi_{12}u_{Xt})/(1 - \phi_{12}\phi_{21}), \]
\[ e_{2t} = (u_{Xt} + \phi_{21}u_{Yt})/(1 - \phi_{12}\phi_{21}). \]  

(20)

Because of the fact that \( u_{Yt} \) and \( u_{Xt} \) are white noise processes, the two new error terms \( e_{1t} \) and \( e_{2t} \) are also white noise (Ibid, p. 335). Since each process in a VAR-model is explained by a lagged version of itself, it is important to set an appropriate lag length for the model. When doing this, \( k \) VAR models with up to \( k \) lags, are constructed and then estimated (Ibid, p.}
The estimated models are then compared by their estimated AIC-score. The estimated model with the lowest AIC-score is the model with the appropriate length (Ibid, p. 383). The test statistic for the AIC-test is the following

$$AIC = -2 \log \left[ \text{maximum likelihood} \right] + 2k,$$

where $k$ is the number of estimated parameters (Wang, Liu, 2006, p. 223).

### 3.7 Johansen’s test

Johansen’s test is a test for cointegration that uses a multiple equation system, allowing for more than two variables to be tested (Asteriou, Hall, 2016, p. 380). If there are $n$ number of processes being tested, there can be maximally $n - 1$ found cointegrated vectors (Ibid, p. 380). Johansen’s test is derived from the VAR model. If equation 18 is extended to $k$ lags and no intercept, using three variables, $X_t$, $Y_t$ and $W_t$, it can be written as

$$Z_t = A_1 Z_{t-1} + A_2 Z_{t-2} + \ldots + A_k Z_{t-k} + e_t,$$

where $Z_t = [X_t, Y_t, W_t]$. This VAR-model can then be written as an vector error-correction model (Ibid, p. 380)

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \Gamma_2 \Delta Z_{t-2} + \ldots + \Gamma_{k-1} \Delta Z_{t-k-1} + \Pi Z_{t-1} + e_t,$$

where $\Gamma_i = (I - A_1 - A_2 \ldots - A_k)$, $(i = 1, 2, \ldots, k - 1)$ and $\Pi = -(I - A_1 - A_2 \ldots - A_k)$ (Ibid, p. 380). The $\Pi$ matrix is the information of the long-run relation. $\Pi$ can be written as $\alpha \beta'$ where $\alpha$ is the speed of adaption to the equilibrium whilst $\beta'$ is the matrix of the long run coefficients (Ibid). For a simple explanation, there will only be two lagged terms. The model will then be

$$\begin{pmatrix} \Delta Y_t \\ \Delta X_t \\ \Delta W_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta W_{t-1} \end{pmatrix} + \Pi \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ W_{t-1} \end{pmatrix} + e_t.$$

Or, of course

$$\begin{pmatrix} \Delta Y_t \\ \Delta X_t \\ \Delta W_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta W_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ W_{t-1} \end{pmatrix} + e_t. $$

16
Then, if only the error-correction part of the first equation is considered
\[
\Pi_1 Z_{t-1} = \left( [\alpha_{11}\beta_{11} + \alpha_{12}\beta_{12}] [\alpha_{11}\beta_{21} + \alpha_{12}\beta_{22}] [\alpha_{11}\beta_{31} + \alpha_{12}\beta_{32}] \right) \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ W_{t-1} \end{pmatrix}, \quad (26)
\]
where \( \Pi_1 \) is the first row of \( \Pi \). Rewriting equation 26 gives
\[
\Pi_1 Z_{t-1} = \alpha_{11}(\beta_{11}Y_{t-1} + \beta_{21}X_{t-1} + \beta_{31}W_{t-1}) + \alpha_{12}(\beta_{12}Y_{t-1} + \beta_{22}X_{t-1} + \beta_{32}W_{t-1}), \quad (27)
\]
where two cointegrating vectors are clearly represented with their respective \( \alpha_{11} \) and \( \alpha_{12} \), which is the rate of adjustment to the equilibrium (Ibid, p. 381). This is the theory for Johansen’s test to find cointegrating vectors. Of course, if \( \alpha_{11} = 0 \) and \( \alpha_{12} = 0 \) in the example above, there would not exist any cointegrating vectors.

The specific type of VAR model that is relevant for the cointegration tests of this paper, estimates the cointegration of two processes. Hence, equation 25 could instead written as
\[
\begin{pmatrix} \Delta Y_t \\ \Delta X_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + e_t, \quad (28)
\]
which could be further explicated to
\[
\begin{pmatrix} \Delta Y_t \\ \Delta X_t \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \end{pmatrix} + \begin{pmatrix} \alpha_{11} \beta_{11} & \alpha_{11} \beta_{12} \\ \alpha_{21} \beta_{21} & \alpha_{21} \beta_{22} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} e_Y_t \\ e_x_t \end{pmatrix}. \quad (29)
\]
Deriving this to two separate equations then gives
\[
\Delta Y_t = \gamma_{11}\Delta Y_{t-1} + \gamma_{12}\Delta X_{t-1} + \alpha_{11}\beta_{11}Y_{t-1} + \alpha_{11}\beta_{12}X_{t-1} + e_Y_t, \quad (30)
\]
\[
\Delta X_t = \gamma_{21}\Delta Y_{t-1} + \gamma_{22}\Delta X_{t-1} + \alpha_{21}\beta_{11}Y_{t-1} + \alpha_{21}\beta_{12}X_{t-1} + e_X_t,
\]
or
\[
\Delta Y_t = \gamma_{11}\Delta Y_{t-1} + \gamma_{12}\Delta X_{t-1} + \alpha_{11} (\beta_{11}Y_{t-1} + \beta_{12}X_{t-1}) + e_Y_t, \quad (31)
\]
\[
\Delta X_t = \gamma_{21}\Delta Y_{t-1} + \gamma_{22}\Delta X_{t-1} + \alpha_{21} (\beta_{11}Y_{t-1} + \beta_{12}X_{t-1}) + e_X_t.
\]
Here, there is instead only one vector per equation, with \( \alpha_{11} \) and \( \alpha_{21} \) as their respective rates of adjustment.
4 Financial theory

4.1 Spread

The spread of the pair is used to define how the current relationship between the stocks differs from its historical mean (Vidyamurthy, 2004, p. 8-9). Once the distance to the mean is considered substantial, it is defined to be a divergence. The spread of a cointegrated pair is asserted to be stationary and mean reverting (Ibid). For the stocks $A$ and $B$, the spread is defined as (Ibid, p. 82)

\[
\text{Spread}_t = \log(p_A^t) - \gamma \log(p_B^t),
\]  

(32)

Here however, the model does not allow for an intercept. To get a better understanding of the spread, it should be clarified that it is based on a regression of the processes. Allowing for an intercept, regressing the log-price of stock A on the log-price of stock B, gives

\[
\log(p_A^t) = \mu + \gamma \log(p_B^t) + e_t,
\]  

(33)

which could be rewritten as

\[
\log(p_A^t) - \mu - \gamma \log(p_B^t) = e_t.
\]  

(34)

In this scenario, the residual term of the model, $e_t$, is defined as the spread at point $t$. As was mentioned in section 3.4, cointegration is when the linear combination of two nonstationary processes is stationary. It is therefore logical that the spread is stationary.

4.2 Sharpe ratio

Since it was first introduced, in 1966, the Sharpe ratio has grown widely in popularity as a way of estimating the reward-to-variability ratio for a certain portfolio (Sharpe, 1994). To understand the intuition behind the measure, the following equation could be considered (Investopedia, 2019d)

\[
\text{Sharpe ratio} = \frac{r_p - r_f}{\sigma_p},
\]  

(35)

where $r_p$ is the return of portfolio, $r_f$ is the risk-free return rate and $\sigma_p$ is the standard deviation of portfolio. To in greater detail grasp how the ratio functions, the following mathematical definition of the measure can be used to in retrospect determine the Sharpe ratio of a portfolio.

The difference between the risky and risk-free returns for a given point in time is

\[
D_t \equiv r_p - r_f.
\]  

(36)
The mean difference in returns then becomes

\[ \bar{D} \equiv \frac{1}{T} \sum_{t=1}^{T} D_t, \quad (37) \]

and its standard deviation is given by

\[ \sigma_D \equiv \sqrt{\frac{\sum_{t=1}^{T} (D_t - \bar{D})^2}{T - 1}}. \quad (38) \]

Thus the Sharpe ratio is given by

\[ \text{Sharpe ratio} \equiv \frac{\bar{D}}{\sigma_D}. \quad (39) \]

The perhaps simplest way of interpreting the measure is: the higher the Sharpe ratio, the better. It is maximized when the return from the selected portfolio, \( r_p \), is high, while its volatility, \( \sigma_p \), as well as the return from the alternative risk free asset, \( r_f \), are low. Another important aspect of the ratio is that it is uninterpretable when taking a negative value. The reason for this is that an increase in volatility for the same difference in returns, would move the ratio closer to zero. This would mean that added risk increases the ratio, which arguably dismantles the entire purpose of the ratio.

4.3 Bollinger bands

Bollinger bands are used in pairs trading to define a divergence from the conditional mean, or the moving average. The interval is constructed by estimating the moving average and standard deviation of the spread, given by

\[ \mu_{\text{spread}_i} = \frac{1}{T} \sum_{t=1}^{T} \text{spread}_i, \]

\[ \sigma_{\text{spread}_i} = \sqrt{\frac{\sum_{t=1}^{T} (\text{spread}_i - \mu_{\text{spread}_i})^2}{T - 1}}. \quad (40) \]

The limits are then constructed as

\[ \text{Upper limit} = \mu_{\text{spread}_i} + S \times \sigma_{\text{spread}_i}; \]

\[ \text{Lower limit} = \mu_{\text{spread}_i} - S \times \sigma_{\text{spread}_i}; \quad (41) \]

where \( S \) is the number of standard deviations from the conditional mean. This number is set by the trader.
5 Methodology

5.1 Data - Thomson Reuters Eikon

All presented information regarding the companies of the S&P 500 and their stock prices are collected from Thomson Reuters Eikon database. The only exception is the price data of the S&P 500 index, which is taken from Yahoo Finance. Thomson Reuter was founded in 1851 and operates within the printing and publishing industry and its services are directed towards businesses (Forbes, 2019). Eikon is a product that originates from the Trading segment of the company. It is a product that provides the financial community with access to information such as news feeds and financial data (Ibid). The data that has been gathered is all based on the constituents of the S&P 500 index from 2005. Besides the companies’ names, it consists of stock prices for each individual stock between 2005-01-01 and 2011-12-31. There was a significant number of stocks that were part of the S&P 500 in 2005, but had stock prices that could not be easily fetched. An explanation for some extent of this loss, is a change of ticker during the intended trading period. It would perhaps be possible to go through all companies individually to include a larger share of the constituents of the index. Nevertheless, due to the restricted extent of this study, it has been concluded that the 398 remaining stocks are sufficient for the purpose of this paper.

Besides the stock prices, the General Industry Classification (GIC) of each company was fetched via Eikon. The classification divides the companies into the following six industries:

1. Industrial
2. Utility
3. Transportation
4. Bank/Savings & Loan
5. Insurance
6. Other Financial

The distribution of the stocks from each industry is quite uneven, as can be found in Table 2. What might be slightly troubling about this is that having only 7 stocks in the Transportation industry, it is not quite certain that cointegrated pairs will be found for that industry every year. A remedy for this issue however, is to not include pairs from this industry in the portfolio for that year. Although this might compromise the sector neutrality of the portfolio, it is worth
to acknowledge that pairs from all of the five remaining industries would be included. Hence, there is no excessive weighting of one single industry. Therefore it is concluded that sector neutrality is still sufficiently fulfilled for the purpose of this paper. Another potential issue that this loss of pairs in the portfolio might bring, is a weakened risk diversification. Nonetheless, it is not the main focus of this paper to optimize risk diversification. It is therefore decided that having one or two less pairs in the portfolio is a potential flaw that is considered acceptable. It should be pointed out that, of course, the definition of sector differs to the definition of industry. Nonetheless, Ehrman mixes the terms whilst defining the necessities that sector neutrality requires (2006, p. 65). This choice of categorization is therefore considered appropriate for its cause.

5.2 Stock universe and portfolio selection

As is explained in the introduction, this paper aims to evaluate how well the pairs trading strategy performs during a financial crisis. It is considered desirable to constrain the universe of stocks such that it limits the risk for potential bankruptcies of companies that are included in the universe. Therefore, the universe is reasonably reduced only to include large-cap companies. It is necessary for the universe to be more precisely defined, as well as manageable in size. Therefore, the stocks that are selected for the study are the constituents of the S&P 500 index in 2005. For every year, the pairs from each of the GIC industries that are found to be cointegrated are ordered by their level of significance. The top two pairs from each industry are what constitute the portfolio for that year.

It is difficult to set a definite start and an end date for the financial crisis of 2008. Therefore, the years 2007 - 2011 is used in this paper to ensure that the entire crisis is covered in the trading. The first trading period is initiated 2007-01-01, and the test for cointegration is based on the data from the two years prior to the start of the period. For the first period that would be between 2005-01-01 and 2006-12-31. After one year, the pairs are revised and a new test for cointegration is performed on all of the stocks in the S&P 500 to find appropriate pairs for the new trading period of 2008. Similarly to the previous period, the cointegration testing is based on the two prior years, 2006 and 2007. This procedure is then repeated for 2009, 2010 and 2011. The testing- and trading periods are visualised in Figure 2, where the black lines are the testing periods, and the orange lines are the trading periods.
Figure 2: Testing- and trading periods illustrated for the years 2007-2009.

The version of the S&P 500 that is used is from 2005-01-01, from the starting point of the data that is used. A rather obvious improvement to the stock universe could be achieved by adjusting the stock universe to be updated for every trading year. That is, updating the stocks that are considered for the portfolio to being the constituents at the start of that testing period, instead of using the list of 2005-01-01 for every year. This would perhaps increase the relevance of the stock universe. However, it shall again be referred to what the focus of the study lies on. Although this brief relevance related weakness could be regulated, it would not contribute considerably to the focus and purpose of this paper. It is therefore considered sufficient to only use the list of constituents of the S&P 500 from the beginning of 2005.

Both in the portfolio construction and trading technique that is used, the aspects and criteria discussed in section 2.2, are taken into consideration. Some of these aspects are the different types of market neutrality. When a pair diverges, the relatively overvalued stock is shorted to a value that is equal to the stock where a long position is taken. This way, dollar neutrality can be said to be achieved. Moreover, it is made sure that both stocks in each pair are from the same industry. Sector neutrality is partially achieved from the very procedure with which the portfolio is constructed. Consider the scenario of a fatal altering in one of the industries, without having a substantial impact on the others. Hence, a severe risk arises for the part of the portfolio that is constituted by stocks from the affected industry. Selecting two pairs from each of the six industries regulates the plausibility for such an event to have a severe effect on the entire portfolio. This indicates an existence of sector neutrality to some extent. Of course, all industries do not have trades open simultaneously, meaning that the criterion is not perfectly fulfilled. However, the complicated task of having it completely fulfilled would, among other
things, require a larger stock universe, as well as a severely larger portfolio. This is considered to be beyond what can be claimed to be a manageable limitation for this study. It is therefore decided that despite not having sector neutrality entirely achieved, it is considered sufficient. From the limitation of only looking at the stocks from the S&P 500 as contestants for the portfolio, the volatility of the stocks within each industry should be quite similar. The reason for this is that all of the included companies in the index all are among the companies with the highest market capitalization. Hence they should have a fairly similar $\beta$ (also explained in section 2.2), meaning that beta neutrality should be somewhat fulfilled.

5.3 Selecting stationarity test

As will be further explained in section 5.4, the processes are designated to be $I(1)$. Some suggest the hypothesis of stock prices being random walks, and thereby integrated of order one. However, since the statistical accuracy is regarded as crucial for this paper, stationarity tests are performed to verify that the stocks that are considered for each year's portfolio are in fact $I(1)$. This raises the question of which stationarity test that should be used.

The ADF-test was presented in section 3.3 as a viable option for testing a stochastic process for stationarity. However, there are several other tests that can be used, for example the KPSS-test. In contrary to the ADF-test, the KPSS-test uses the null hypothesis that the process is stationary. Hence, the ADF-test either rejects, or fails to reject the null hypothesis of non-stationarity. With the KPSS procedure, a rejection of the null is instead a rejection of stationarity. To further clarify the meaning of this distinction, the difference between Type I and Type II errors is considered. A Type I error is a false rejection of the null hypothesis, while a Type II error is failing to falsely to reject the null. The implications for the two considered stationarity tests, can be written as follows in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Type I error</th>
<th>Type II error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF-test</td>
<td>Reject nonstationarity falsely</td>
<td>Falsely failing to reject nonstationarity</td>
</tr>
<tr>
<td>KPSS-test</td>
<td>Reject stationarity falsely</td>
<td>Falsely failing to reject stationarity</td>
</tr>
</tbody>
</table>

It can from the table be interpreted that, in practice, the errors for the two tests are switched. From this insight, it is obvious that the Type I and II error rates, $\alpha$ and $\beta$, become relevant. The Type I error is typically known to be the fatal error, whereas making a Type II error normally
is considered more acceptable. It is common for $\alpha$ to be restricted, normally to a rate of 0.05, or a 5% significance level. Although $\beta$ too can be somewhat restricted, it is rarely as firmly as $\alpha$. Connecting this understanding to the stationarity tests in question, it becomes vital what the purpose of the test is. The stationarity test are performed in two separate steps. In the first step, the processes that are found to be stationary are excluded, since nonstationarity is a necessity for cointegrated pairs to be found (explained further in section 5.4). In the second step, the first difference versions of the processes are instead tested for stationarity. In this step however, it is the processes that are found to be nonstationary that are excluded. Hence all remaining processes are concluded to be integrated of the same order, namely of order one. In the first step, the fatal error would be to falsely state that a process is nonstationary. The reason for this is that it is the nonstationary processes that are of interest, and that are further examined. Thus it is worse to commit the error of keeping a stationary process, believing that it is nonstationary, than to get rid of a nonstationary process, thinking that it is stationary. This reasoning can than be associated to the categorical outcome, presented in Table 1. Keeping in mind that it is $\alpha$ that is restricted to the selected significance level, it becomes clear that the KPSS-test would be best suited. In the second step, it is the processes with first difference versions which are found to be stationary that are kept. It is therefore the ADF-test that better suits the latter part of the procedure. Hence, the two conclusions are conflicting with regard to which of the tests that preferably should be used, based on how the hypotheses are formulated. However, in research by Shin and Schmidt, the ADF-test is found to be the superior of the two unit-root tests (1992, p. 387). For this reason, the ADF-test is the test for stationarity that is chosen for this paper, with a standardized significance level of 5%. Due to the characteristics of the data, the version of the test that is used is

$$\Delta y_t = a_0 + \gamma y_{t-1} + a_2 t + \sum_{i=1}^{p} \beta_i \Delta y_{t-i} + u_t.$$  (42)

### 5.4 Selecting cointegration test

Whether two time series are cointegrated can be tested and shown in several ways. The original test for cointegration was introduced in 1987, by Engle and Granger (Asteriou, Hall, 2016, p. 376). Asteriou & Hall suggest Johansen’s test as a viable alternative approach when there are more than two possibly cointegrated vectors (Ibid, p. 380). It is however also possible to use Johansen’s test when testing for cointegration between only two vectors, as is the case in this
paper. In a paper by Bilgili from 1998 (p. 10), some evidence is presented, which suggests Johansen’s test being superior to Engle-Granger’s. A mentioned tangible difference between the methodologies is that the Engle-Granger test relies on a two step estimator, whereas the maximum likelihood estimators from Johansen’s test only require one step. The additional step in the process of estimation leads to the errors from the first step being brought into the estimation of the second step. This added insecurity is then not taken into account. Bilgili points out this to be a rather obvious flaw of the Engle-Granger methodology. Brooks further points out the limitation in testability regarding the cointegration relationship between the processes in the Engle-Granger methodology (2008, p. 343). Moreover, he highlights how instead using Johansen’s test is a remedy for the issue. Due to the mentioned advantages, Johansen’s approach is the selected cointegration test for this paper. In Johansen’s approach to test for cointegration, Asteriou and Hall presents six steps (2016, p.328-387). In this paper however, only the first four steps are of interest, since the two final steps are used for estimation and intuitive understanding of the data (Ibid, p.386-387).

1) Test the order of integration of the processes.

The necessary criterion is that all processes are $I(1)$ (Asteriou, Hall, 2016, p.383). If a process is $I(0)$, it forms an independent vector with itself (Ibid, p. 383). This is a problem since the goal is to find cointegrated pairs. However, all processes that are found to be integrated of order 0 are removed in the test for stationarity. Another problem arises if some processes are $I(2)$, since a specific combination of two $I(2)$ variables might cointegrate to an $I(1)$ (Ibid., p. 383). This problem is solved by performing an second ADF-test, described in section 5.3.

2) Set the appropriate numbers of lags in the model.

Setting the appropriate numbers of lags in the model is done by estimating $k$ VAR models, explained in section 3.6 (Asteriou, Hall, 2016, p. 383). The first estimated model has $k$ lags. The number of lags is then decreased for the estimation of the second model to $k - 1$, and to $k - 2$ for the third. The procedure is then repeated until zero lags, $k - k$, is reached. The model with the lowest AIC-score is then chosen (Ibid). For this paper, an upper limit of 20 lags is selected.

3) Choose a fitting model with regards to the components in the system

25
It is important to correctly specify the model with constants and trends (Asteriou, Hall, 2016, p. 384). The equation that includes all possible scenarios is

\[
\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \Gamma_2 \Delta Z_{t-2} + \ldots + \Gamma_{k-1} \Delta Z_{t-k+1} + \alpha (\beta Z_{t-1} \mu_1 \delta_1 t) + \mu_2 + \delta_2 t + u_t,
\]

where, again, \(\Gamma_i = (I - A_1 - A_2 - \ldots - A_k)\), \((i = 1, 2, \ldots, k - 1)\). \(\mu_1\) is the coefficient for the constant and \(\delta_1\) is the coefficient for the trend \(t\) in the long term model, while \(\mu_2\) is the coefficient for the constant and \(\delta_2\) is the coefficient for the trend \(t\) in the short term model.

Asteriou and Hall presents three different models that are the practically relevant combinations of these coefficients (Ibid, p. 384). 1) A model with only an intercept in the long term model, \(\mu_2 = \delta_1 = \delta_2 = 0\). 2) A model with only intercepts included, no trend, \(\delta_1 = \delta_2 = 0\), and 3) a model with with intercept included in both short- and long term models, only trend in long term model, \(\delta_2 = 0\). A problem then arises: which one of these three models would be appropriate to use in this paper? The approach for testing this is called the Patula principle, where all three models are estimated (Ibid, p. 385). It is utilized by moving from the most to the least restrictive model. The model that is chosen is the first where the null hypothesis of no cointegration is rejected (Ibid, p. 385). However, for this paper, it is not one potential cointegrated relationship that is to be tested, but many thousands. Applying the Patula principle would thereby mean that the type of cointegration would differ from pair to pair. It was priorly mentioned that the pairs that are found to be cointegrated are to be ordered by their level of significance. Thereafter, the top two from each industry would be selected. Using the Patula principle, this ranking methodology would arguably become rather problematic as the different cointegration tests would not be fully comparable. Instead, an alternative approach is used, which keeps the mentality behind the principle, but alternates the problematic aspect of the outcome. With this procedure, all pairs are initially tested with the most restrictive model. If a sufficient amount of cointegrated pairs are found for the strategy to function, that model is selected. Otherwise, the model that is the second most restrictive is attempted. Lastly, if the result for the second model was the same as for the first, the third and least restrictive model is used.

4) **Determine the number of cointegrated vectors.**

There are mainly two different approaches when testing for cointegration, the maximum eigenvalue statistic and the trace statistic (Asteriou, Hall, 2016, p. 385). In this paper, the maximum eigenvalue statistic is going to be used based on the research of Lütkepohl et al., which con-
cludes that the tests are very similar (2000, p. 0). The maximum eigenvalue statistic tests the \( \text{Rank}(\Pi) \) (Asteriou, Hall, 2016, p. 385). The null hypothesis of the test is that \( \text{Rank}(\Pi) = r \), versus the alternative hypothesis that \( \text{Rank}(\Pi) = r + 1 \) (Ibid, p. 385). The test statistic is based on eigenvalues, retrieved in the estimation process. The test orders the eigenvalues from the largest to the smallest: \( \lambda_{(\text{max})} > \lambda_{(\text{max}-1)} > \lambda_{(\text{max}-2)} > \ldots > \lambda_{(\text{min})} \) and the goal is to find how many of these eigenvalues that are statistically significant different from zero. Then the following test statistic is used:

\[
\lambda_{\text{max}}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1}).
\] (44)

Since only two stocks are tested at a time, this can be written as

\[
\lambda_{\text{max}}(0, 1) = -T \ln(1 - \hat{\lambda}_1).
\] (45)

To understand this test, consider a scenario where no cointegrating vectors are found. Then \((1 - \hat{\lambda}_1) = 1\), and since \(\ln(1) = 0\), the test result will be that no cointegrating vectors are found (Ibid, p. 385). If a cointegrating vector is found however, \(\lambda_1\) will be between 0 and 1, and \(\ln(1 - \hat{\lambda}_1) \leq 0\). The test result will then show that there exists a cointegrating vector (Ibid, p. 385).

5.5 The trading algorithm

The trading algorithm consists of three main parts. Firstly, there has to be a signal for when sufficient divergence has occurred. This is where the algorithm would short the relatively over-valued stock and long other. There also has to be a signal for sufficient convergence, that is when the trade is to be ended. Lastly, a signal that detects non-convergence is needed.

5.5.1 Sufficient divergence

In this paper Bollinger bands, explained in section 4.3, is used to define a sufficient divergence from the conditional mean. There are other trade signals that could be used, such as RSI, Stochastics and Volume (Ehrman, 2006, p.99-111). These methods are unadvised to use for the inexperienced trader since it complicates the process significantly when using combined trade signals.

For the usage of Bollinger bands in this paper, two standard deviations are going to be used to define a sufficient divergence from the conditional mean. The number of two standard
deviations can be seen as an arbitrarily set number, this could be set to a lower or higher number as well. The risk of using a larger standard deviation is that the trading algorithm would potentially miss trades that would have been profitable. The risk of using a smaller standard deviation is that the trading algorithm would potentially trade too often and trade when sufficient divergence has not occurred for the trade to be profitable. The usage of ±2 standard deviations is the standard measure and is therefore chosen (Bollinger, 1992, p.2). The limits for sufficient divergence in this paper will then be

\[
\begin{align*}
\text{Upper divergence limit} & = \mu_{\text{spread}_i} + 2 \times \sigma_{\text{spread}_i}, \\
\text{Lower divergence limit} & = \mu_{\text{spread}_i} - 2 \times \sigma_{\text{spread}_i}.
\end{align*}
\]  

(46)

5.5.2 Sufficient convergence

After a trade has been initiated, a level of sufficient convergence must be defined such that the algorithm knows when to exit the trade. This limit is derived in quite the same way as the sufficient level for divergence, by using Bollinger bands. The difference, of course, being that a lower number of standard deviations is used. This limit can vary, and is set somewhat arbitrary by the trader. If the limit is set too high, the algorithm might exit the trade too soon, not maximizing the profit from the trade. If, on the other hand, the limit is set too low, the convergence limit might be difficult to reach, resulting in holding on to the stock for too long. Hence, it could become an issue in terms of alternative cost or even reach the stop-loss limit, which is yet to be explained. In this paper, the limit of sufficient divergence is set to a conservative ±1 standard deviation. This is shown in the following equations

\[
\begin{align*}
\text{Upper convergence limit} & = \mu_{\text{spread}_i} + 1 \times \sigma_{\text{spread}_i}, \\
\text{Lower convergence limit} & = \mu_{\text{spread}_i} - 1 \times \sigma_{\text{spread}_i}.
\end{align*}
\]  

(47)

5.5.3 Non-convergence

Even though cointegrated pairs have been found, and a trade has been entered, it does not necessarily mean that the pair is going to converge within a given time period. A risk is that after the trade has been initiated, the pair continues to diverge. Therefore there has to exist a limit for when this continued divergence is no longer considered acceptable. Thus it functions as a sort of roof, or insurance, for how big the loss of an individual trade is allowed be. This limit is often referred to as the stop-loss limit. Just as the limits for sufficient divergence and convergence, the stop-loss is set somewhat arbitrarily by the trader. If it is set too high, it is

28
made possible for substantial individual losses to occur. However, if the limit is set too low, the spread of a pair might not have enough room to oscillate after surpassing the divergence limit. Hence it is possible for trades that could have ended up being profitable, to be exited too early as losing trades. In this paper, $\pm 3$ standard deviations is going to be used to define the stop-loss level, which is given by

$$
\begin{align*}
\text{Upper stop-loss limit} &= \mu_{\text{spread}_i} + 3 \cdot \sigma_{\text{spread}_i}, \\
\text{Lower stop-loss limit} &= \mu_{\text{spread}_i} - 3 \cdot \sigma_{\text{spread}_i}.
\end{align*}
$$

Another factor that the trader has to take into account is the time limit of the trade. This time limit is set somewhat arbitrarily by the trader. If the time limit is set too short, the trader does not allow the pair to converge. If the time limit is set too long, the trader loses money due to not having them invested in other trades. In this paper, a time limit of 40 days is used. Although it is selected quite arbitrarily, it is set to a level that grants the spread to oscillate between the convergence and stop-loss limit for a somewhat lengthy period, whilst not allowing the deal to be open for a too large share of the year.

5.5.4 Summarizing the algorithm

To summarize the trading algorithm, Figure 3 is presented. The plot is quite comparable to Figure 1, described in section 2.2. A tangible difference however, is that this figure reflects the trading mechanisms more accurately with regard to the technical aspects of the strategy. The red line shows the how the distance between the current spread and its moving average evolves over time. The distance is measured in standard deviations. From the figure, it is possible to distinguish that initially, the spread gradually diverges. At January 5th, when the spread is larger than two standard deviations from the moving average, a trade is initiated. If the spread is given by $\log p_t^{\text{Stock}_1} - \gamma \log p_t^{\text{Stock}_2}$, it would mean that a short position is taken in Stock 1 and a long position is taken in Stock 2. As the days pass, the spread begins to converge to its moving average. At January 11th, when the spread is smaller than one standard deviation from its moving average, the trade is ended. If, instead of converging after the trade was initiated, the spread continued to diverge, the trade would have been ended if the spread exceeded three standard deviations.
5.6 Calculating the Sharpe ratio

In section 4.2, the Sharpe ratio was introduced and equation 35 was given. When calculating the Sharpe ratio with a risk-free rate that is set to 0, the following equation is used

\[
\text{Sharpe ratio} = \frac{r_p - 0}{\sigma_p}.
\]  

This means that the calculated Sharpe ratio is the return of the portfolio divided by the standard deviation of return of the portfolio. The results from this equation are compared to the Sharpe ratios of the S&P 500 index, which is added as an evaluating measure for the portfolio.

5.7 Calculating returns

The calculation of returns that are performed in this paper are based on logarithmic approximations. This is in line with the theoretical framework presented in section 4.1. As mentioned priorly, it is of interest to keep the portfolio balanced at all times to achieve dollar neutrality. For this to be fulfilled, the value on the short and long side of the portfolio must be equal. The example of stock A and stock B is used to explained how the return is calculated.

A divergence occurs, where stock A is to be shorted and B is longed. To achieve dollar neutrality the value of A and B stocks should thus be equal, or

\[
\# \text{ of Stock}^A \cdot p^A = \# \text{ of Stock}^B \cdot p^B.
\]
Now the number of A stocks that is shorted, is set to 1. For a balance in the portfolio, the number of B stocks that are longed is given by the price ratio at the point in time, \( t \), when the trade is initiated which is given by

\[
\text{# of Stock}_B = \frac{p_t^A}{p_t^B} = PR. \tag{51}
\]

If the trade is ended \( j \) days after the initiation, the return of stock A is given by

\[
R_{\text{Short}} = \log(p_t^A) - \log(p_{t+j}^A). \tag{52}
\]

Using the price ratio from earlier, the return of stock B is approximated by

\[
R_{\text{Long}} = \log(PR \cdot p_{t+j}^B) - \log(PR \cdot p_t^B),
\]

which can be written as

\[
R_{\text{Long}} = [\log(p_{t+j}^B) + \log(PR)] - [\log(p_t^B) + \log(PR)],
\]

\[
= \log(p_{t+j}^B) - \log(p_t^B). \tag{53}
\]

Due to the nature of log-approximation, the return can be calculated by adding the log-differences.

An approximation of the return for the trade is therefore given by

\[
R_{\text{Total}} = R_{\text{Short}} + R_{\text{Long}} = \log(p_t^A) - \log(p_{t+j}^A) + \log(p_{t+j}^B) - \log(p_t^B), \tag{54}
\]

or submitted in percentage form

\[
R_{\text{Total}} = (\log(p_t^A) - \log(p_{t+j}^A)) \times 100 + (\log(p_{t+j}^B) - \log(p_t^B)) \times 100. \tag{55}
\]

If, instead, B is shorted and stock A longed, the return is given by

\[
R_{\text{Total}} = (\log(p_t^B) - \log(p_{t+j}^B)) \times 100 + (\log(p_{t+j}^A) - \log(p_t^A)) \times 100. \tag{56}
\]

The returns from all trades, from all pairs, are then added up to get the cumulative for the entire portfolio for that year. It is noteworthy that this approximation of the return by no means gives an exact replication of the return of the portfolio. This is largely due to its impreciseness for approximating larger changes, which is a commonly known property. However, the focus in this paper is not to submit the exact virtual return that the portfolio could produce. Besides, it is still an unbiased approximation which should not only favor, nor disfavor, the return of the portfolio.
6 Results

As priorly mentioned, the entire trading period reaches from 2007 to 2011. The data for each year of trading is divided into two sub-periods, hereinafter referred to as the testing period and the trading period. The testing period refers to the two preceding years to the year of trading, or the trading period. It is during the testing period where the stationarity- and cointegration tests are performed, and where the trading portfolio is constructed. The trading period on the other hand is when the strategy is put to practice, and when the actual trading is done.

The results will be presented sequentially, in the order of constructing of the portfolio, performing the trading and calculating the returns. Firstly, the results from the ADF-tests and Johansen’s tests for each year are gone through. Thereafter, the portfolio for each year is presented. Finally, returns are presented accompanied with a comparison between the performance of the portfolio and the S&P 500 index.

6.1 The stationary stocks

The results from the ADF-tests in the first step, where the log-prices that are found to be stationary are excluded, can be interpreted from Table 2. As can be seen in the table, the number of stocks that were found to be stationary was quite similar for all years, except 2011. For the first four years, between 7 and 13 of the 398 processes per year had the the null hypothesis of nonstationarity rejected on a 5 % significance level. Hence between 385 and 391 were kept for the second step. However, in 2011 this number increased drastically, dropping the number of stocks that were considered contenders for the portfolio to 329.

In the second step, the processes which were found to be integrated of an order higher than one were excluded by testing the first difference version of the remaining processes for stationarity. However, all processes were found to be $I(1)$ every year. Expressing this in a more statistical manner, the null hypothesis of nonstationarity was rejected for all processes, each year. This implies that the frequencies and distributions between the industries are the exact same as after the first step. Therefore, the results from these steps can also be interpreted from Table 2.
6.2 The cointegrated pairs

For each testing period, a Johansen’s test was performed for every combination of pairs within each industry. Only the pairs that were found to be cointegrated were kept. This could also be expressed as that all pairs where the null hypothesis of $r = 0$ was not rejected, were excluded. For all five years, the least restrictive model was sufficient for the strategy to function. From Table 3 it can be seen that the number of cointegrated pairs that were found, varied quite freely. This variation was found partly in the total number of cointegrated pairs that were found each year, but also in the number of pairs in several of the industries. Analyzing the table, a general increasing trend can be distinguished in the total number of cointegrated pairs that were found for each year of trading. In the testing period for 2007, that number was 1878, which is noticeably lower than the 4179 pairs that were found for the last year, 2011. There were no occasions when an industry had no cointegrated pairs. There were however two occurrences, in 2008 and 2009, where the Transportation industry only had one pair that was found to be cointegrated.

6.3 Portfolio selection

For every testing period, the two cointegrated pairs from each industry that were found to be most highly significant were included in the portfolio. As mentioned above, both in 2008 and 2009, there was only one cointegrated pair found in the Transportation industry. Therefore, the portfolio for those years consisted of only 11 pairs. The pairs that were selected are presented in Table 4 and Table 5. In these tables, the name of the stocks, the industry they belong to and their level of significance in Johansen’s test is presented. A vast majority of the pairs are shown to be significant on the 1% level in the four latter years. For 2007 however, merely a third of the pairs surpass the 1% limit, whereas the remaining eight are significant on the 5% level. The exact test statistics for the pairs of each years portfolio, with their respective critical values, are found in Table 6. Moreover, it is notable that some stocks were part of two pairs of one year’s portfolio. For example, WMI Holdings Corp and MBIA Inc are both part of two different pairs in the 2008 portfolio.
6.4 Performance of the portfolio

The returns from each year’s portfolio were positive, yet distinctively varying. This is shown in Table 7 below, where the cumulative return from each pair in the portfolio is presented for every year. The table further presents Sharpe ratios as well as standard deviations for each years return. The measures are presented both for the pairs trading portfolio and for the S&P 500 index. What might stand out is that for all years but 2010, the returns are distinctively higher for the portfolio. This is particularly the case for 2008, where the portfolio produces over 640% in return, while the S&P 500 index has a return that is close to -40%. It can be seen in the table that the return of 2010 is slightly higher for the S&P 500 than for the portfolio, being the only year where the index outperforms the portfolio in terms of return. Continuing the walkthrough of Table 7, it can be seen that the standard deviation of the return is higher for the portfolio each year. It is also shown that the Sharpe ratio is larger for the portfolio for all years except 2010.

Table 7: A comparison of the performance of the pairs trading portfolio and the S&P 500 index.

As comparative measures, the table presents the cumulative return, the standard deviation of the daily return and the standard deviation of the daily return.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Return</td>
<td>99.06 %</td>
<td>644.90 %</td>
<td>257.36 %</td>
<td>12.70 %</td>
<td>104.94 %</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.341</td>
<td>0.112</td>
<td>0.102</td>
<td>0.005</td>
<td>0.098</td>
</tr>
<tr>
<td>St. dev</td>
<td>0.028</td>
<td>0.220</td>
<td>0.097</td>
<td>0.094</td>
<td>0.041</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative Return</td>
<td>3.53 %</td>
<td>-38.49 %</td>
<td>23.45 %</td>
<td>12.78 %</td>
<td>0.00 %</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.015</td>
<td>-0.084</td>
<td>0.063</td>
<td>0.041</td>
<td>0.000</td>
</tr>
<tr>
<td>St. dev</td>
<td>0.009</td>
<td>0.024</td>
<td>0.014</td>
<td>0.011</td>
<td>0.013</td>
</tr>
</tbody>
</table>

In addition to each years overall performance, Figures 5-9 give a graphical presentation of how the portfolios’ return evolved throughout every year. The corresponding plots for the S&P 500 are found in Figures 17-21. As was priorly found from Table 7, the standard deviation of the return was found to be greater for the portfolio than for the S&P 500. Looking at the scale of the return axis, this difference in volatility could perhaps aswell be distinguished from the appearance of these figures.

Some additional plots, shown in Figures 10-14, present a more nuanced picture of how the
different pairs of each years portfolio performed. Interpreting these figures, it can be clarified which of the years the performance was more dependent on the individual return of one or two pairs, and which years were more stable. Again, 2008 stands out, with Pair 11 generating quite an extreme positive individual performance. The pair consisted of Ambac Financial Group Inc and WMI Holdings Corp, found in Table 4. In 2010, Pair 12 instead generates a strongly negative return. This pair was formed by Prologis and, again, WMI Holdings Corp.

7 Analysis

The analysis will follow the same structure as the results. Firstly an analysis of the stationary stocks and the cointegrated pairs will be presented. Thereafter, the portfolio selection will be discussed, followed by a discussion of the performance of the portfolio.

7.1 The stationary stocks

The amount of stocks that was removed due to stationairity was fairly similar years 2007-2010. This number however increased for year 2011 and almost 60 stocks more were removed that year. This was considered unexpected since it is difficult to find a logical explanation to why more stocks would be stationary in the testing period for 2011 than any of the other years. One reason could be that the financial crisis had stabilized in USA by 2010, leading to a lower volatility in the stocks, and therefore more stocks are showing stationary properties. By looking at the industries in table 2, it can be seen that the largest amount of stocks dropped is in the Industrial industry. This could be solely because it is the largest industry by number of stocks in the original data. This could also be because of the fact that when the financial crisis stabilized, the industrial companies were the types of companies that was invested in to get the economy back on track.

In the second ADF-test, it was found that all processes were integrated of order 1. It is both interesting and expected that all stocks that were not stationary was \( I(1) \) processes. This aligns with financial theory which states that stock prices are integrated of order 1.

7.2 The cointegrated pairs

The amount of cointegrated pairs that was found within each industry was fairly different for all years except for the industries "Utilities" and "Transportations". For the other industries,
this number fluctuated.

What is noteworthy is that the total number of cointegrated pairs increased from year to year for all years except for 2009 - 2010. A possible explanation for this is the fact that 2010 has the testing period of 2008-2009, two well-known turbulent years. As can be seen in Table 7, both the standard deviations and returns, in absolute values, were at their highest for the S&P 500 during these two years. This indicates quite a volatile market, which is confirmed by the turn of events of the period. This is presented in the timeline of the financial crisis, which can be found in Appendix C. Lehman brothers declaring bankruptcy at the 15th of September 2008 was without doubt one of the important triggers of the crisis. The crash of WMI and Wachovia followed shortly after, shaking the economy additionally and contributing to the outbreaking chaos. The effect of these falls can easily be distinguished from Figure 18.

It is notable that in 2009, there was an increase of over 2000 pairs compared to 2008. This is perhaps less expected as 2008 uses a less volatile testing period. It should however be mentioned that the very nature of such an expectation is rather speculative. Ehrman speaks in his book from 2006 of some hazards that a pairs trader might face, describing price trends as one of them (p. 120). Despite not showing a perfect trend, the return S&P 500 index of 2006-2007 can be said to follow a positive trend in general. This is illustrated in Figures 15-16.

Another cointegration related aspect that should be pointed out, is the absence of a second pair in the Transportation industry in 2008 and 2009. Although this threatens the sector neutrality of the strategy and risk diversification, it was accounted for beforehand, and a flaw that was considered manageable. This is mentioned in section 5.1.

### 7.3 Portfolio selection

In two of the years, only 11 pairs was included in the portfolio due to the fact that there existed only one cointegrated pair in the transportation industry. This does not seem to have negatively impacted the performance of the portfolio, the years are still profitable with the strategy. This would not have been an acceptable solution for pairs trading in the real world, since sector neutrality is not fulfilled. For the purpose of this paper however, this is an acceptable flaw.

By looking at Tables 4 and 5 it is clear that few stocks are used to trade with for more than one year. This shows the importance of switching stocks frequently during a financial crisis. When the market is volatile, there exists no perfect pairs that are the best during several years in a row. By changing the pairs used to trade, it allows for the trader to use the "best pairs"
during that timeframe. It could be discussed whether it would be even better to switch these pairs more frequently, maybe twice every year, or even more frequently every quarter. In this paper however, switching the pairs on a yearly basis seems to be a good alternative since the returns are larger than index for all, except one, year.

From further investigation of the same tables, it can be found that a not too seldomly occurring phenomenon, is the same stock being used in both pairs of an industry. By using the same stock in more than one pair, the trading strategy becomes less intuitive than if one stock was limited to only one pair. For example, the same stock could be bought long and sold short at the same time. Another example would be that it could be sold short twice at the same time. This is not a problem for the pairs trading strategy, and this could be extended to a case where the same stock is included in more than two pairs.

7.4 Performance of the portfolio

Looking at Table 7, it appears as if the more volatile the stock market was, the better the portfolio performed in terms of percentage return. This can to some degree be further confirmed by looking at the scale of the return axis in Figures 18-19, and comparing them to Figures 17, 20 and 21. It is obvious that these two years are the most volatile, whilst also generating the highest returns. The presented results suggest that the strategy peaks in performance when the market is declining.

Aside from producing the highest returns when the market is volatile and declining, it is highly relevant to acknowledge that the return is found to be positive for all five years. In fact, all years but 2010 have positive returns that are undisputedly quite extreme, especially 2008. Attempting to explain these returns is certainly not an simple task, but some of the years might be slightly easier to understand.

To understand the 104.94 % return of 2011’s trading, a graphical representation of its cumulative return, found in Figure 9, could be considered. As is clearly shown in the plot, there are no sudden giant leaps. Hence, it can be concluded that no individual trade is individually responsible for the largely positive result. It is rather many small trades that add up to over 100 % return, when combined. To get further grasp of the return, Figure 14 displays the individual performance of all 12 pairs. It appears from the plot that the spread between each pair’s return is rather limited. Moreover, a vast majority of the stocks end the year at above 0. It can therefore be concluded that neither was it the performance of one individual pair that decided the
entire return.

An even higher return, 644.90 %, was generated from the 2008 trading period. Unlike the return of 2011, this result was highly dependent of not just a pair, but a certain stock - WMI Holdings corp. WMI was part of both pairs of the "Other Financial" industry for 2008, the very year that the savings bank crashed at the end of September, shortly after the fall of Lehman Brothers. Looking at how the return of the portfolio develops throughout the year, shown in Figure 6, it is obvious that something occurs around that occasion. What makes it even more interesting is how the return of the S&P 500 evolves at the same point in time, displayed in Figure 18. While the index takes a steep downturn, the portfolio skyrockets. Tracking this down, it is discovered that WMI is shorted in both of the pairs it was part of, number 10 and 11. The trades were open between the 22nd-25th and 19th-26th respectively. Figure 11 displays the effect that the short positions had on the return of these two pairs. The return that was gained from these two trades was 73.77 % for Pair 10 and 304.47 % for Pair 11. Due to the nature of log-approximations, these returns can be subtracted from the total return of 644.90 % to get an estimation of what the return would be if these two trades were not included. Since the result after the adjustment is no less than 266.67 %, it still utterly high. Hence, it can be said that the return of the portfolio for 2008 was not solely dependent on these trades, but that they had a strong impact on the extent of how largely positive the return turned out to be.

Analyzing the volatility of the portfolio it is obvious that for all years, the standard deviation is larger for the portfolio than for the S&P 500 index. However, due to the overall sovereignty of the returns of the portfolio, the portfolio’s Sharpe ratio is larger for all years except 2010. This could be interpreted as that the portfolio generates a higher profit for a given level of risk. What can be observed is that in 2010, the returns were almost equal for the portfolio and the S&P 500 index. The Sharpe ratio however, differs largely, being eight times larger for the S&P 500 index compared to the portfolio. Looking instead at the outcome of 2009, the return of the pairs trading portfolio is more than ten times larger than the index. However, the Sharpe ratio is only 50 % larger. This highlights the extent of the general difference in volatility between the portfolio and the S&P 500.

Apart from reflecting upon where the return originates and its volatility, it is highly relevant to question the details of the algorithm. From investigating individual trades, it was discovered that some quite beneficial trades were made from an interesting moving pattern in the pair’s spread. To understand this pattern, it should be remembered that the daily closing price of the
stocks is used as price data for this study. What was notable about these trades was that the spread went from within the Bollinger bands to past a stop-loss limit in just one day. Thereafter, a gradual convergence was initiated, which resulted in a successful trade. One of these cases is illustrated in Figure 4 below. This example is, again, found in the 11th pair from the 2008 portfolio, reaching from the 11th to the 24rd of January. The leap in the spread occurred between the 15th and 16th. Since it is found to be within two standard deviations the 15th and past the lower stop-loss limit the 16th, no trade is initiated for either of the days. However, as the spread began to converge and reached a position inside of the stop-loss boundaries at the 18th, a trade was initiated. After achieving sufficient convergence on the 23rd, the trade was ended, with a generated profit of 62.04 %. These types of trades might not be the theoretically typical trades in pairs trading. It is however clear that in these cases, the spread diverges sharply and then shows signs of a commencing convergence. It is therefore considered more reasonable to include, than to exclude these opportunities as trades.

![Figure 4](image)

**Figure 4**: An example of a notable stop-loss related trade. In a similar manner to Figure 3, the spread of Pair 11 from the trading year of 2008 is illustrated.

### 8 Conclusion

Recall that the purpose with this paper was to investigate whether pairs trading is a profitable strategy during a financial crisis. The aim was to answer the research question that follows:
Is pairs trading a profitable trading strategy during a financial crisis?

To evaluate the suitability of the strategy, the constituents of the S&P 500 during the financial crisis of 2008 was selected as empirical evidence. Firstly, for a pairs trading strategy to be efficient, it is necessary that cointegrated pairs can be found throughout the entire period. For the sake of sector neutrality, it is desirable that the number of pairs in each industry is sufficiently large for the share of every industry to be equal in the portfolio. Connecting these criteria to the outcome of this study, it was found for both 2008 and 2009 that the Transportation industry had only one pair. The imbalance that this flaw could cause was considered to be rather insignificant, and was therefore accepted. Hence it is concluded that enough cointegrated pairs were found throughout the period.

What might be the most obvious requirement for a trading strategy to be considered profitable, is that it generates positive returns. The returns that were made from the strategy were mostly unexpectedly large. Moreover, the returns were positive for all years and larger than the S&P 500 for all years except 2010, when they were approximately equal. Regarding the volatility of the strategy, it was found to be quite large. For all years, the standard deviation of the return was larger for the pairs trading portfolio than for the S&P 500. The higher standard deviations are reflected in the Sharpe ratios, where the difference between the portfolio and the S&P 500 index is not as large in the returns. However, the Sharpe ratios were higher for the trading strategy for all years, except 2010.

What can be concluded from the performance of the strategy, is that high returns were generated, although sometimes at the cost of a higher volatility. Considering that the strategy has elements of statistical arbitrage, which is commonly known to be associated with risk, a higher volatility was quite expected. That being said, the reasonable conclusion appears to be that the evidence supports pairs trading as a suitable strategy during a financial crisis. To which extent the conclusion is viable can however be questioned. As repeatedly mentioned, the empirical evidence that is used is from the crisis of 2008. Hence, it is not at all fully granted that the strategy would have worked during any of the earlier financial crises, let alone a future crisis. It was nonetheless not the intention for such a conclusion to be drawn. Neither was it plausible to obtain findings which could motivate such a conclusion, due to the very nature of this study.

With this paper, it was intended to evaluate whether pairs trading is a suitable strategy during a financial crisis. The results showed only positive yearly returns, some of which being
extremely high. For four out of the five trading years, it was severely higher than the S&P 500 index, and during the outbreak of the crisis in 2008, the return of the portfolio peaked at over 640%. Thus, despite the mentioned limitations of the study, it is considered reasonable to conclude that the implications of the findings of this paper answer the asked research question affirmatively. Yes, the results suggest that it is profitable during a financial crisis.
9 Further research

In this paper, only "large cap" companies are included in the stock universe. It would be interesting to investigate the performance of pairs trading during a financial crisis for smaller companies. Furthermore, it would be interesting to use other stock markets.

The findings in this paper found pairs trading to be well-suited for a financial crisis. It would be interesting to find out whether it is solely for the crisis of 2008 that these returns are generated, or if similar results are found for earlier crises. This would enhance the consistency of the findings in this paper.

In this paper, the ADF-test was used to test for stationarity and Johansen’s test was used to test for cointegration. Since there are numerous different tests that one could use, a topic that would be interesting to explore is whether other statistical tests that are better suited to use during a financial crisis. Some of the empirical evidence from this study could be well-suited for an evaluation of the stationarity test, as over 60 stocks were found to be stationary for the period 2009-01-01 - 2010-12-31. It could then be investigated whether other tests find as many stocks to be stationary. Another interesting topic that could be investigated in such a study would be to examine whether a SARMA model could be found, which makes the process stationary.
10 References


Wang, Yanjun. & Liu, Qun. 2006. *Comparison of Akaike information criterion (AIC) and Bayesian information criterion (BIC) in selection of stock–recruitment relationships*. Fisheries Research 77.2: 220-225.
11 Appendix A - Tables

Table 1: Type 1 and 2 errors for the ADF- and KPSS-tests.

<table>
<thead>
<tr>
<th>Test</th>
<th>Type I error</th>
<th>Type II error</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADF-test</td>
<td>Reject nonstationarity falsely</td>
<td>Falsely failing to reject nonstationarity</td>
</tr>
<tr>
<td>KPSS-test</td>
<td>Reject stationarity falsely</td>
<td>Falsely failing to reject stationarity</td>
</tr>
</tbody>
</table>

Table 2: Showing the distribution of the stocks that were found to be nonstationary each year, with regard to the industry that the companies are associated with. The column "Original data" displays the number of stocks in each industry for all 398 stocks.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Original data</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial</td>
<td>288</td>
<td>284</td>
<td>280</td>
<td>285</td>
<td>286</td>
<td>253</td>
</tr>
<tr>
<td>Utility</td>
<td>37</td>
<td>36</td>
<td>36</td>
<td>37</td>
<td>37</td>
<td>25</td>
</tr>
<tr>
<td>Transportation</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Bank/Savings &amp; Loan</td>
<td>19</td>
<td>17</td>
<td>19</td>
<td>18</td>
<td>19</td>
<td>12</td>
</tr>
<tr>
<td>Insurance</td>
<td>24</td>
<td>24</td>
<td>24</td>
<td>21</td>
<td>24</td>
<td>15</td>
</tr>
<tr>
<td>Other Financial</td>
<td>23</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>Total</td>
<td>398</td>
<td>389</td>
<td>388</td>
<td>391</td>
<td>385</td>
<td>329</td>
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</table>

Table 3: Industry distribution of the pairs that were found to be cointegrated. The columns 2007-2011 present the number of pairs in the different industries that were found to be cointegrated that year. The column "Original data" displays the number of stocks in each industry for all 398 stocks.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Original data</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
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<td>Industrial</td>
<td>288</td>
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<td>1771</td>
<td>3779</td>
<td>3746</td>
<td>4030</td>
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<tr>
<td>Utility</td>
<td>36</td>
<td>38</td>
<td>23</td>
<td>48</td>
<td>48</td>
<td>57</td>
</tr>
<tr>
<td>Transportation</td>
<td>7</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<tr>
<td>Bank/Savings &amp; Loan</td>
<td>19</td>
<td>3</td>
<td>35</td>
<td>4</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Insurance</td>
<td>24</td>
<td>4</td>
<td>23</td>
<td>41</td>
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<td>46</td>
</tr>
<tr>
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<td>7</td>
<td>36</td>
<td>84</td>
<td>37</td>
<td>48</td>
</tr>
<tr>
<td>Total</td>
<td>392</td>
<td>1878</td>
<td>1889</td>
<td>3957</td>
<td>3869</td>
<td>4189</td>
</tr>
</tbody>
</table>
Table 4: The stock pairs that are selected for the portfolio for the years 2007-2009. The last column, named "*/ **", specifies the level of significance for each pair. "*" indicates that the pair is cointegrated on a 5 % significance level, whereas "**" indicates significance on the 1 % level.

<table>
<thead>
<tr>
<th>Year</th>
<th>Industry</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>*/ **</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Anadarko Petroleum Corp</td>
<td>National Semiconductor Corp</td>
<td>**</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Moody’s Corp</td>
<td>Stanley Black &amp; Decker Inc</td>
<td>**</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Constellation Energy Group Inc</td>
<td>Progress Energy Inc</td>
<td>**</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Duke Energy Corp</td>
<td>Verizon Communications Inc</td>
<td>**</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Norfolk Southern Corp</td>
<td>United Parcel Service Inc</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>Motors Liquidation Co</td>
<td>United Parcel Service Inc</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>Comerica Inc</td>
<td>Marshall &amp; Ilsley Corp</td>
<td>*</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>Comerica Inc</td>
<td>Wells Fargo &amp; Co</td>
<td>*</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>Aetna Inc</td>
<td>Unum Group</td>
<td>*</td>
</tr>
<tr>
<td>10</td>
<td>5</td>
<td>Hartford Financial Services Inc</td>
<td>Prudential Financial Inc</td>
<td>*</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>Altaba Inc</td>
<td>Apartment Investment and Management Co</td>
<td>*</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>American Express Co</td>
<td>Merrill Lynch &amp; Co Inc</td>
<td>*</td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Allegheny Technologies Inc</td>
<td>E I Du Pont de Nemours and Co</td>
<td>**</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Kohls Corp</td>
<td>VF Corp</td>
<td>**</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Ameren Corp</td>
<td>Sprint Communications Inc</td>
<td>**</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Allegheny Energy Inc</td>
<td>Dominion Energy Inc</td>
<td>**</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Southwest Airlines Co</td>
<td>Unoin Pacific Corp</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>First Horizon National Corp</td>
<td>U.S. Bancorp</td>
<td>**</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>PNC Financial Services Inc</td>
<td>Suntrust Banks Inc</td>
<td>**</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>Anthem Inc</td>
<td>MBIA Inc</td>
<td>**</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>LOEWS CORP</td>
<td>MBIA Inc</td>
<td>**</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>Charles Schwab Corp</td>
<td>WMI Holdings Corp</td>
<td>**</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>Ambac Financial Group Inc</td>
<td>WMI Holdings Corp</td>
<td>**</td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Altria Group Inc</td>
<td>Hospira Inc</td>
<td>**</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>Nucor Corp</td>
<td>United States Steel Corp</td>
<td>**</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>Centerpoint Energy Inc (CNP.N)</td>
<td>Sprint Communications Inc</td>
<td>**</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>Centurylink Inc</td>
<td>Frontier Communications Corp</td>
<td>**</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>Motors Liquidation Co</td>
<td>United Parcel Service Inc</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>Regions Financial Corp</td>
<td>Wells Fargo &amp; Co</td>
<td>**</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>Bank Of America Corp</td>
<td>Wells Fargo &amp; Co</td>
<td>**</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>American International Group Inc</td>
<td>Metlife Inc</td>
<td>**</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>American International Group Inc</td>
<td>Lincoln National Corp</td>
<td>**</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>Lehman Brothers Holdings Inc</td>
<td>Prologis</td>
<td>**</td>
</tr>
<tr>
<td>11</td>
<td>6</td>
<td>Federal National Mortgage Assocation</td>
<td>Janus Capital Group Inc</td>
<td>**</td>
</tr>
</tbody>
</table>
Table 5: The stock pairs that are selected for the portfolio for the years 2010-2011. The last column, named "*/\*\*", specifies the level of significance for each pair. "\*" indicates that the pair is cointegrated on a 5 % significance level, whereas "\*\*" indicates significance on the 1 % level.

<table>
<thead>
<tr>
<th>Year</th>
<th>Industry</th>
<th>Stock 1</th>
<th>Stock 2</th>
<th>*/**</th>
</tr>
</thead>
<tbody>
<tr>
<td>2010</td>
<td>1</td>
<td>Staples Inc</td>
<td>Tenet Healthcare Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Time Warner Inc</td>
<td>Unisys Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Exelon Corp</td>
<td>Verizon Communications Inc</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Constellation Energy Group Inc</td>
<td>El Paso LLC</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Norfolk Southern Corp</td>
<td>Southwest Airlines Co</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>CSX Corp</td>
<td>Southwest Airlines Co</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>M&amp;T Bank Corp</td>
<td>U.S. Bancorp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>Bank Of America Corp</td>
<td>Huntington Bancshares Inc</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Lincoln National Corp</td>
<td>Metlife Inc</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Metlife Inc</td>
<td>Principal Financial Group Inc</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Lehman Brothers Holdings Inc</td>
<td>Prologis</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>Prologis</td>
<td>WMI Holdings Corp</td>
<td>**</td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
<td>Johnson Controls International PLC</td>
<td>Textron Inc</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Mylan NV</td>
<td>Walmart Inc</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>AT&amp;T Inc</td>
<td>CMS Energy Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>CMS Energy Corp</td>
<td>PG&amp;E Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>Motors Liquidation Co</td>
<td>Union Pacific Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>CSX Corp</td>
<td>Motors Liquidation Co</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>Bank Of America Corp</td>
<td>JPMorgan Chase &amp; Co</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>JPMorgan Chase &amp; Co</td>
<td>Wells Fargo &amp; Co</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>Allstate Corp</td>
<td>Lincoln National Corp</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>Allstate Corp</td>
<td>Johnson Controls International PLC</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>Goldman Sachs Group Inc</td>
<td>Prologis</td>
<td>**</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>Goldman Sachs Group Inc</td>
<td>T. Rowe Price Group Inc</td>
<td>**</td>
</tr>
</tbody>
</table>
Table 6: A brief summary of the cointegration tests for the pairs that are included in the portfolio each year. The summary consists of the test statistics and their respective critical values, for both the 5 % and 1 % significance level.

<table>
<thead>
<tr>
<th>Pair no.</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>5 % level</th>
<th>1 % level</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>35.184</td>
<td>36.959</td>
<td>44.776</td>
<td>64.325</td>
<td>55.139</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>2</td>
<td>34.728</td>
<td>35.330</td>
<td>44.038</td>
<td>44.493</td>
<td>48.996</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>3</td>
<td>25.176</td>
<td>28.800</td>
<td>29.675</td>
<td>26.790</td>
<td>32.842</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>5</td>
<td>17.275</td>
<td>18.172</td>
<td>18.670</td>
<td>20.579</td>
<td>24.482</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>7</td>
<td>16.474</td>
<td>27.038</td>
<td>21.940</td>
<td>20.249</td>
<td>32.091</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>9</td>
<td>18.593</td>
<td>26.037</td>
<td>37.847</td>
<td>50.953</td>
<td>63.758</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>10</td>
<td>16.051</td>
<td>33.938</td>
<td>48.149</td>
<td>33.916</td>
<td>59.901</td>
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<td>20.2</td>
</tr>
<tr>
<td>11</td>
<td>17.789</td>
<td>32.888</td>
<td>41.139</td>
<td>36.647</td>
<td>48.300</td>
<td>15.67</td>
<td>20.2</td>
</tr>
<tr>
<td>12</td>
<td>17.230</td>
<td>-</td>
<td>-</td>
<td>35.535</td>
<td>40.501</td>
<td>15.67</td>
<td>20.2</td>
</tr>
</tbody>
</table>

Table 7: A comparison of the performance of the pairs trading portfolio and the S&P 500 index. As comparative measures, the table presents the cumulative return, the standard deviation of the daily return and the standard deviation of the daily return.

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portfolio</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Return</td>
<td>99.06 %</td>
<td>644.90 %</td>
<td>257.36 %</td>
<td>12.70 %</td>
<td>104.94 %</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.341</td>
<td>0.112</td>
<td>0.102</td>
<td>0.005</td>
<td>0.098</td>
</tr>
<tr>
<td>St. dev</td>
<td>0.028</td>
<td>0.220</td>
<td>0.097</td>
<td>0.094</td>
<td>0.041</td>
</tr>
<tr>
<td>S&amp;P 500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cumulative Return</td>
<td>3.53 %</td>
<td>-38.49 %</td>
<td>23.45 %</td>
<td>12.78 %</td>
<td>0.00 %</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.015</td>
<td>-0.084</td>
<td>0.063</td>
<td>0.041</td>
<td>0.000</td>
</tr>
<tr>
<td>St. dev</td>
<td>0.009</td>
<td>0.024</td>
<td>0.014</td>
<td>0.011</td>
<td>0.013</td>
</tr>
</tbody>
</table>

49
Table 8: Number of trades within the portfolio for each year, both for each pair and in total. The numeration of the pairs correspond to Tables 4 and 5.

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pair 1</td>
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<td>12</td>
<td>14</td>
<td>12</td>
</tr>
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<td>Pair 2</td>
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<tr>
<td>Pair 3</td>
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<td>9</td>
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<td>Pair 4</td>
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<td>Pair 5</td>
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<td>Pair 6</td>
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<td>12</td>
<td>9</td>
</tr>
<tr>
<td>Pair 7</td>
<td>8</td>
<td>14</td>
<td>10</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>Pair 8</td>
<td>15</td>
<td>10</td>
<td>10</td>
<td>10</td>
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<tr>
<td>Pair 9</td>
<td>13</td>
<td>12</td>
<td>9</td>
<td>11</td>
<td>12</td>
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<tr>
<td>Pair 10</td>
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<td>12</td>
<td>11</td>
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<tr>
<td>Pair 11</td>
<td>10</td>
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<td>10</td>
<td>12</td>
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<tr>
<td>Pair 12</td>
<td>11</td>
<td>-</td>
<td>-</td>
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<td>10</td>
</tr>
<tr>
<td>Total</td>
<td>143</td>
<td>127</td>
<td>116</td>
<td>134</td>
<td>135</td>
</tr>
</tbody>
</table>
Appendix B - Figures

Figure 1: An illustrative theoretical example of the strategy, showing when a trade is initiated and ended respectively.

Figure 2: Testing- and trading periods illustrated for the years 2007-2009.
Figure 3: A similar example to Figure 1, but more technically accurate.

Figure 4: An example of a notable stop-loss related trade. In a similar manner to Figure 3, the spread of Pair 11 from the trading year of 2008 is illustrated.
Figure 5: The return of the portfolio for 2007. The line represents the cumulative return of the entire portfolio throughout the year.

Figure 6: The return of the portfolio for 2008. The line represents the cumulative return of the entire portfolio throughout the year.
Figure 7: The return of the portfolio for 2009. The line represents the cumulative return of the entire portfolio throughout the year.

Figure 8: The return of the portfolio for 2010. The line represents the cumulative return of the entire portfolio throughout the year.
Figure 9: The return of the portfolio for 2011. The line represents the cumulative return of the entire portfolio throughout the year.

Figure 10: The return of each pair in the portfolio for 2007. The lines represent the cumulative return of every individual pair throughout the year. The legend displays which pair each line belongs to. The numeration matches the presentation of the portfolio found in Table 4.
Figure 11: The return of each pair in the portfolio for 2008. The lines represent the cumulative return of every individual pair throughout the year. The legend displays which pair each line belongs to. The numeration matches the presentation of the portfolio found in Table 4.

Figure 12: The return of each pair in the portfolio for 2009. The lines represent the cumulative return of every individual pair throughout the year. The legend displays which pair each line belongs to. The numeration matches the presentation of the portfolio found in Table 4.
Figure 13: The return of each pair in the portfolio for 2010. The lines represent the cumulative return of every individual pair throughout the year. The legend displays which pair each line belongs to. The numeration matches the presentation of the portfolio found in Table 5.

Figure 14: The return of each pair in the portfolio for 2011. The lines represent the cumulative return of every individual pair throughout the year. The legend displays which pair each line belongs to. The numeration matches the presentation of the portfolio found in Table 5.
Figure 15: The return of the S&P 500 index for 2005. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.

Figure 16: The return of the S&P 500 index for 2006. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.
Figure 17: The return of the S&P 500 index for 2007. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.

Figure 18: The return of the S&P 500 index for 2008. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.
Figure 19: The return of the S&P 500 index for 2009. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.

Figure 20: The return of the S&P 500 index for 2010. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.
Figure 21: The return of the S&P 500 index for 2011. The line represents the cumulative return of the entire portfolio throughout the year. The return is estimated in the same manner as the portfolio, by using log-approximations.
13 Appendix C - Timeline of the crisis

4 Sep 2007 Banks lending rises to the highest levels since December 1998.
18 Sep 2007 US Federal Reserve reduces its main interest rate from 5.25% to 4.75%.
19 Dec 2007 Standard and poor’s reduced the amount of investment in monoline insurers.
22 Jan 2008 US Federal Reserve reduces interest rates from 4.25% to 3.5%.
24 Jan 2008 The largest single-year drop of housing prices in the last century is announced by analysts.
14 Mar 2008 JP Morgan purchases the investment bank Bear Sterns.
7 Sep 2008 Fannie May and Freddie Mac gets bailed out by the US government.
15 Sep 2008 Lehman Brothers goes bankrupt.
1 Dec 2008 Recession in the United States is announced by the National Bureau of Economic Research.
16 Dec 2008 US Federal Reserve reduces interest rates from 1% to 0.25%.
17 Feb 2009 President Obama signs a stimulus package of $787bn.
1 May 2009 Chrysler goes bankrupt.
1 June 2009 General Motors goes bankrupt.
14 July 2009 Goldman Sachs announce profits.
2010 The US economy stabilizes.
2011 The financial crisis in the US is over.
14 Appendix D - R Script

```r
setwd("D:/Dokument/Ekonomie kandidat/Statistik C/Examensarbete/Eikon")

library(xts)
library(zoo)
library(quantmod)
library(urca)
library(tseries)
library(dplyr)
library(tidyverse)
library(roll)
library(rapportools)
library(fUnitRoots)
library(vars)
library(readxl)
library(ggplot2)
library(reshape2)

### For all data

# Fetch closing price
alldata <- read_excel("S&P 500 klar - close.xlsx", col_names = TRUE)
idx <- as.Date(as.matrix(alldata[,1])) # date column set as index
mat.data <- as.matrix(alldata[,,-1]) # date column excluded from stock data
xts.data <- xts(mat.data, order.by = idx)
# xts object created with stock prices as core data and date column as index

# Fetch industry and company names
industry <- read_excel("S&P 500 klar - Industry.xlsx", col_names = TRUE)
mat.industry <- as.matrix(industry)

# Make the columns identifiable
colnames(xts.data) <- as.character(mat.industry[,1])
# column containing a companys’ prices labeled with companys’ RIC (a type of ticker)

# Filter such that stocks containing NA’s are excluded
vec.filt <- vector() # vector where the placement of the stock is stored
for(i in 1:ncol(xts.data)){
  if(colSums(is.na(xts.data[,i]) != 0)){ # if there are NA’s
    vec.filt[i] <- i # identify which stock it is
  }
}
if(sum(vec.filt) != 0){ # if any of the stocks contained NA’s
  vec.filt <- vec.filt[!is.na(vec.filt)] # filter vector for NULL’s
  filtereddata <- xts.data[,!vec.filt] # remove NA stocks
} else{ # if no NA stocks
  filtereddata <- xts.data # make no changes
  print("No NA’s") # state that there are no NA’s
```

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# Take log(prices)

logfiltereddata <- filtereddata
for(i in 1:ncol(filtereddata)){
  logfiltereddata[,i] <- log(filtereddata[,i])
}

## General

signlvl <- 0.05 # significance level for hypothesis testing

### For each year

# Start loop such that all trading periods (2007-2011) are run individually
for(g in 2007:2010){ # select which year that should be run
testyear1 <- as.character(g-2)
# select the starting year of the period where the testing and portfolio construction is made

testyear2 <- as.character(g-1)
# select the ending year of the period where the testing and portfolio construction is made

tradeyear <- as.character(g)
# select the year where the model is applied

# period which is used if there are any ongoing trades at the end of the trading period

# Adjust data to the desired periods
testdata <- logfiltereddata[c(testyear1,testyear2)]
tradedata <- logfiltereddata[tradeyear]
# endtradedata <- head(logfiltereddata[endstradeyear], n=40)

# Those stocks that have infinite values of that have no variance are removed
for(i in 1:ncol(testdata)){ # if there are any infinite values
  for(j in 1:length(index(testdata))){
    if(any(is.infinite(testdata[j,i])) != "FALSE"){
      testdata[j,i] <- NA # identify them as NA's
    }
  }
}

vec.nas <- vector() # vector where NA columns are identified
j <- 1
for(i in 1:ncol(testdata)){
  if(colSums(is.na(testdata[,i]))!=0){ # if there are NA's in column
    vec.nas[j] <- i # store the placement of that column
    j <- j+1
  } else if(sd(testdata[,i]) == 0){ # if variation is 0
    vec.nas[j] <- i # store the placement of that column
    j <- j+1
  }
}
# Those that have no variance
#testdata <- testdata[, colSums(is.na(testdata))==0] # filter for NA's
if(sum(vec.nas) != 0){ # if there are NA's in data from test period
testdata <- testdata[, -vec.nas]
# filter testing data for infinite values and nonvariance stocks
tradedata <- tradedata[, -vec.nas]
# filter trading data for infinite values and nonvariance stocks
#endtradedata <- endtradedata[, -vec.nas] # filter data for ending unfinished trades for NA's
}

# Add 19 days from prior year to be able to calculate moving average from the first trading day
tradedata_tail <- rbind(tail(testdata, n=19), tradedata)

### Test for stationarity

# Identify stationary processes by setting them to NA
vec.stat <- vector() # used to identify stationary processes
adf.list1 <- list() # list of results from all ADF-tests
for(i in 1:ncol(testdata)){
  adf.list1[[i]] <- adf.test(testdata[,i]) # store ADF-tests
  if(adf.test(testdata[,i])$p.value<signlvl){
    vec.stat[i] <- i # p-value smaller than sign.lvl -> stock should be removed
  }
}

length(vec.stat[!is.na(vec.stat)])

if(sum(vec.stat) != 0){ # if there are any stationary processes
  vec.stat <- vec.stat[!is.na(vec.stat)]
  # remove nulls from vector containing the placements of those processes
  nonstat <- testdata[,vec.stat] # remove stationary processes
  tradedata <- tradedata[,vec.stat]
  # remove those processes from the trading data which will be used later
  tradedata_tail <- tradedata_tail[,vec.stat]
  # remove those processes from some other trading data which will be used later
  #endtradedata <- endtradedata[,vec.stat]
  # remove from data for ending unfinished trades aswell
} else{ # if there are no stationary processes
  nonstat <- testdata # don't make any changes
  tradedata <- tradedata # nor to trading data
  tradedata_tail <- tradedata_tail #
  #endtradedata <- endtradedata #
  print("All nonstationary") # state that all logprice processes where stationary
}

# See if there were any nonstationary processes which were integrated of any other order than one
vec.inot1 <- vector() # vector identifying those processes
adf.list2 <- list() # store results from ADF-tests
for(i in 1:ncol(nonstat)){
  adf.list2[[i]] <- adf.test(diff(as.numeric(nonstat[,i])))
  # stationarity tests performed on first difference of logprices
  if(adf.test(diff(as.numeric(nonstat[,i])))$p.value>signlvl){

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# if first difference is found to be nonstationary
vec.inot1[i] <- i  # identify
}
}
# In that case, identify the names of the processes which are I(d), where d > 1
if(sum(vec.inot1) != 0) {  # if any stocks were I(d), d>1
vec.inot1 <- vec.inot1[!is.na(vec.inot1)]  # filter vector for NULL's
ndata <- nonstat[,-vec.inot1]  # remove those stocks
tradedata <- tradedata[,-vec.inot1]
tradedata_tail <- tradedata_tail[,-vec.inot1]
endtradedata <- endtradedata[,-vec.inot1]
} else {  # if all remaining stocks were I(0)
ndata <- nonstat  # make no changes
tradedata <- tradedata
tradedata_tail <- tradedata_tail
endtradedata <- endtradedata
print("All I(1)")  # state All I(1)
}

## Categorize after the 6 industries which the data is divided into
xts.ind1 <- xts()  # create object where stock logprice data is stored for industry 1
xts.ind2 <- xts()  # create object where stock logprice data is stored for industry 2
xts.ind3 <- xts()  # ... 3
xts.ind4 <- xts()  # ...
xts.ind5 <- xts()  #
xts.ind6 <- xts()  #
list.ind <- list()  # create list where all data is stored, categorized by industry
k <- 1  # stock indexing for industry 1
l <- 1  # stock indexing for industry 2
m <- 1  # ... 3
n <- 1  # ...
o <- 1  #
p <- 1  #
colnames1 <- vector()  # store colnames for each industry to be able to identify them later
colnames2 <- vector()
colnames3 <- vector()
colnames4 <- vector()
colnames5 <- vector()
colnames6 <- vector()
for(i in 1:ncol(ndata)){
  for(j in 1:nrow(mat.industry)){
    if(colnames1[i] == mat.industry[j,1] & mat.industry[j,2] == "1"){
      # if the stock is of industry 1
      xts.ind1 <- cbind(xts.ind1, ndata[,i])
      # place in object containing stocks of industry 1
    }
  }
}
colnames1[k] <- colnames(i1data[,i])  # store name of stock
indnumber <- as.numeric(mat.industry[j,2])
# identify which industry it is to place it in the list accordingly
list.ind[[indnumber]] <- xts.ind1  # place stock in list
k <- k+1  # increase index by one manually
if(colnames(i1data[,i]) == mat.industry[j,1] & mat.industry[j,2] == "2"){
  # do the same for every other industry
  xts.ind2 <- cbind(xts.ind2, i1data[,i])
  colnames2[l] <- colnames(i1data[,i])
  indnumber <- as.numeric(mat.industry[j,2])
  list.ind[[indnumber]] <- xts.ind2
  l <- l+1
}
if(colnames(i1data[,i]) == mat.industry[j,1] & mat.industry[j,2] == "3"){
  xts.ind3 <- cbind(xts.ind3, i1data[,i])
  colnames3[m] <- colnames(i1data[,i])
  indnumber <- as.numeric(mat.industry[j,2])
  list.ind[[indnumber]] <- xts.ind3
  m <- m+1
}
if(colnames(i1data[,i]) == mat.industry[j,1] & mat.industry[j,2] == "4"){
  xts.ind4 <- cbind(xts.ind4, i1data[,i])
  colnames4[n] <- colnames(i1data[,i])
  indnumber <- as.numeric(mat.industry[j,2])
  list.ind[[indnumber]] <- xts.ind4
  n <- n+1
}
if(colnames(i1data[,i]) == mat.industry[j,1] & mat.industry[j,2] == "5"){
  xts.ind5 <- cbind(xts.ind5, i1data[,i])
  colnames5[o] <- colnames(i1data[,i])
  indnumber <- as.numeric(mat.industry[j,2])
  list.ind[[indnumber]] <- xts.ind5
  o <- o+1
}
if(colnames(i1data[,i]) == mat.industry[j,1] & mat.industry[j,2] == "6"){
  xts.ind6 <- cbind(xts.ind6, i1data[,i])
  colnames6[p] <- colnames(i1data[,i])
  indnumber <- as.numeric(mat.industry[j,2])
  list.ind[[indnumber]] <- xts.ind6
  p <- p+1
}
}
colnames(list.ind[[1]]) <- colnames1  # Set the name of the stock as colname
colnames(list.ind[[2]]) <- colnames2  # for each industry
colnames(list.ind[[3]]) <- colnames3
# Calculate number of lags for cointegration test with AIC
list.VARres <- list()  # create list to store optimal number of lags categorized by industry
res <- data.frame()  # create data frame to store the values in
for(h in 1:length(list.ind)){
  testing <- list.ind[[h]]  # select the industry which the company operates within
  list.VARres[[h]] <- data.frame()
  for (i in 1:ncol(testing)-1){
    for(j in (i+1):ncol(testing)){
      res[1,1] <- VARselect(testing[,c(i,j)], lag.max=20, type="both")$selection[[1]][[1]
      # find optimal number of lags for the cointegration test of the pair
      list.VARres[[h]] <- rbind(list.VARres[[h]], res)
      # store value in the list which separates by industry
    }
    # order in which the optimal lags are stored matches the
  }
  # order in which the cointegration test are performed
}

# The minimum number of lags is two, so all optimal lags below 2 are set to 2
for(h in 1:length(list.ind)){
  for(i in 1:nrow(list.VARres[[h]])){
    lag <- list.VARres[[h]][i,1]
    if(lag<2){  # if the optimal number of lags < 2
      list.VARres[[h]][i,1] <- 2  # it is set = 2
    }
  }
}

### Johansen’s test
johtest <- 0
list.pairs <- list()  # create list where price data is stored, categorized by category
list.pairstick <- list()  # create list where the RIC’s of the
  # companies of each cointegrated pair are stored
list.pairtestsums <- list()
  # create list where the summary of each cointegrated pair is stored
list.place <- list()
  # create list where the placement in its xts object
  # of each stock in every cointegrated pair is stored
for(h in 1:length(list.ind)){
  k <- 1
  l <- 1
  test.ind <- list.ind[[h]]  # select industry
  list.pairs[[h]] <- xts()
  list.pairstick[[h]] <- list()
  list.pairtestsums[[h]] <- list()
  list.place[[h]] <- data.frame()
  for(i in 1:(ncol(test.ind)-1)){

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for(j in (i+1):ncol(test.ind)) {
  johtest <- ca.jo(test.ind[,c(i,j)], type="eigen", ecdet='const',
  spec="longrun", K=list.VARres[[h]][k,1])
  # perform Johansen’s test with optimal number of lags which is determined above
  if(johtest@teststat[2] > johtest@cval[4]) {
    # if the results are statistically significant on the 5 % level
    list.pairs[[h]] <- merge.xts(list.pairs[[h]], list.ind[[h]][,c(i,j)])
    # create xts file for the cointegrated pair
    list.pairstick[[h]][[1]] <- colnames(list.ind[[h]][,c(i,j)])
    # store the RIC’s of the companies
    list.pairtestsums[[h]][[1]] <- summary(johtest)
    # save the results from the test
    list.place[[h]][1,1] <- i  # store the placements of first stock
    list.place[[h]][1,2] <- j  # store the placements of second stock
    l <- l+1
  }
  k <- k+1
}

# Store the information of the cointegrated pairs in a list
list.coint <- list()
for(h in 1:length(list.ind)) {
  stock1 <- data.frame()
  place1 <- data.frame()
  stock2 <- data.frame()
  place2 <- data.frame()
  teststat <- data.frame()
  alpha_01 <- data.frame()
  alpha_005 <- data.frame()
  alpha_001 <- data.frame()
  for(i in 1:length(list.pairstick[[h]])) {
    stock1[i,1] <- list.pairstick[[h]][[i]][1]  # name of first stock
    place1[i,1] <- list.place[[h]][i,1]  # set the place of it for identification
    stock2[i,1] <- list.pairstick[[h]][[i]][2]  # name of second stock
    place2[i,1] <- list.place[[h]][i,2]  # set the place of it for identification
    teststat[i,1] <- list.pairtestsums[[h]][[i]]@teststat[2]
    # test statistic from Johansen’s test
    alpha_01[i,1] <- list.pairtestsums[[h]][[i]]@cval[2,"10pct"]
    # c. value on 10 % lvl
    alpha_005[i,1] <- list.pairtestsums[[h]][[i]]@cval[2,"5pct"]
    # c. value on 5 % lvl
    alpha_001[i,1] <- list.pairtestsums[[h]][[i]]@cval[2,"1pct"]
    # c. value on 1 % lvl
  }
  list.coint[[h]] <- data.frame(  # store information in one data frame per industry
    stock1 = stock1,
    place1 = place1,
    stock2 = stock2,
    place2 = place2,
    teststat = teststat,
    alpha_01 = alpha_01,
    alpha_005 = alpha_005,
    alpha_001 = alpha_001
  )
}

# Store the information of the cointegrated pairs in a list
list.coint <- list()
stock2 = stock2,
place2 = place2,
industry = h, # set the industry which the companies of the pair operate within
teststat = teststat,
alpha_01 = alpha_01,
alpha_005 = alpha_005,
alpha_001 = alpha_001
}
colnames(list.coint[[h]]) <- c("Stock 1", "Place 1", "Stock 2", "Place 2",
"Industry", "Test stat", "10 %", "5 %", "1 %") # set column names
list.coint[[h]] <- list.coint[[h]][order(-list.coint[[h]]$"Test stat"),]
# order by test statistic
}

# Select the 2 pairs with the highest level of significance from each industry
df.portfolio <- data.frame() # create data frame where
for(i in 1:length(list.ind)){
if(length(list.coint[[i]]) > 1){
df.portfolio <- rbind(df.portfolio, head(list.coint[[i]], n=2))
# take the 2 pairs from each industry with highest test stat
} else if(length(list.coint[[i]]) == 1){ # if one sector only has 1 cointegrated pair
df.portfolio <- rbind(df.portfolio, head(list.coint[[i]], n=1))
# take that pair from each industry with highest test stat
}
}

### Start trading
# Identify the stocks in the portfolio pairs
# Store data for selected portfolio. The data used includes the last 19 days
# from the year before to be able to calculate the rolling moving average and
# moving standard deviation from day 1 of trading
rollportstocks <- list()
# create list where the logprice data is stored for every stock pair in the portfolio
for(i in 1:nrow(df.portfolio)){
# industry <- as.numeric(df.portfolio[[i,5]])
# identify which industry the companies of the stock pair operate within
stock1 <- df.portfolio[i,1] # identify stock 1
stock2 <- df.portfolio[i,3] # identify stock 2
rollportstocks[[i]] <- tradedata_tail[,c(stock1, stock2)] #
}

# Calculate and store spread, moving average and moving standard deviation
spreadsres <- list() # create list where each place represents a pair,
# and where the spread for the entire trading period, is stored
movingAvg <- list() # same as list above, but for moving average
movingStd <- list()
# same as list above, but for moving standard deviation
pair <- list() # create list where the information is finally stored for each pair
for(i in 1:length(rollportstocks)){
    spreadres[[i]] <- residuals(lm(rollportstocks[[i]][,1]~rollportstocks[[i]][,2]))
    # estimate the spread for a pair, which is the same as the residuals from a linear regression
    movingAvg[[i]] <- roll_mean(spreadres[[i]], 20) # estimate the moving average for the pair
    movingStd[[i]] <- roll_sd(spreadres[[i]], 20) # estimate the moving average for the pair
    spreadres[[i]] <- tail(spreadres[[i]], n=-19)
    # remove the days used to calculate the rolling moving average and standard deviation
    movingAvg[[i]] <- tail(movingAvg[[i]], n=-19)
    movingStd[[i]] <- tail(movingStd[[i]], n=-19)
    pair[[i]] <- merge.xts(spreadres[[i]], movingAvg[[i]], movingStd[[i]])
    # store the data in the list
colnames(pair[[i]]) <- c("spreadres", "movingAvg", "movingStd") # name the columns
    index(pair[[i]]) <- index(tradedata) # index by date
}

# Identify the position indicators of the pairs' spread, relative to its moving average
# If indicator = 3 -> 3 or more std above moving average
# If indicator = 2 -> between 2 and 3 std above moving average
# If indicator = 1 -> between 1 and 2 std above moving average
# If indicator = 0 -> between 1 std below and 1 std above moving average
# If indicator = -1 -> between 1 std below and 1 std above moving average
# If indicator = -2 -> between 2 and 3 std below moving average
# If indicator = -3 -> 3 or more std below moving average
list.signport <- list() # create list where this information is stored for every pair
for(h in 1:length(rollportstocks)){
    x <- xts(rep(0, length(index(pair[[h]]))), order.by = index(pair[[h]])) # create empty column
    portpair <- merge.xts(pair[[h]], x)
    # merge with object containing spread, moving average and moving standard deviation
    for(i in 1:nrow(portpair)){
        stdfromma <- (portpair[i,1][[1]]-portpair[i,2][[1]])/portpair[i,3][[1]]
        # calculate standard deviations between spread and moving average (stdfromma)
        if(stdfromma > 2 & stdfromma < 3){ # if stdfromma between 2 and 3 above
            portpair[i,4][[1]] <- 2 # Possible divergence where stock 1 is relatively overvalued
        } else if(stdfromma < -2 & stdfromma > -3){ # if stdfromma between 2 and 3 below
            portpair[i,4][[1]] <- -2 # Possible divergence where stock 2 is relatively overvalued
        } else if(stdfromma > 3){ # if stdfromma over 3 above
            portpair[i,4][[1]] <- 3 # Stop-loss limit reached where stock 1 is relatively overvalued
        } else if(stdfromma < -3){ # if stdfromma over 3 below
            portpair[i,4][[1]] <- -3 # Stop-loss limit reached where stock 2 is relatively overvalued
        } else if(abs(stdfromma) < 1){ # if stdfromma between 1 below and 1 above
            portpair[i,4][[1]] <- 0 # Possible convergence
        } else if(stdfromma > 1 & stdfromma < 2){ # if stdfromma between 1 and 2 above
            portpair[i,4][[1]] <- 1
        } else if(stdfromma < -1 & stdfromma > -2){ # if stdfromma between 1 and 2 above
            portpair[i,4][[1]] <- -1
    }
}
```r
# Create trade signals!

## Create initiation signals

### In column 1

- If trading signal = 1 -> trade of type 1 initiated
- If trading signal = 10 -> trade of type 1 held
- If trading signal = 2 -> trade of type 2 initiated
- If trading signal = 20 -> trade of type 2 held
- If trading signal = 0 -> Not in trade

### In column 5

- If trading signal = 1 -> trade of type 1 initiated

### In column 6

- If trading signal = 2 -> trade of type 2 initiated

idxtrade <- c(tail(index(testdata), n=1), index(tradedata))

# add final day of test period to be able to index j-1
list.xts.trade <- list() # create a list where data for all pairs are stored
for(h in 1:length(pair)){
  x <- cbind(rep(0, length(idxtrade)), rep(0, length(idxtrade)),
             rep(0, length(idxtrade)), rep(0, length(idxtrade)),
             rep(0, length(idxtrade)), rep(0, length(idxtrade)))
  xts.trade <- xts(x, order.by = idxtrade)

  # create object where the initiation signals are stored
  colnames(xts.trade) <- c("Position 1", "Position 2", "logprice1", "logprice2",
                          "Trade signal 1", "Trade signal 2")

  # column names for information that will be used
  j <- 2
  signport <- list.signport[[h]] # select pair
  logprice <- tail(rollportstocks[[h]], n=-19)

  # store logprices for each pair to later calculate return
  for(i in 1:length(index(tradedata))){
    if(signport[i,4][[1]] == 2 & xts.trade[(j-1),1] != 1 & xts.trade[(j-1),1] != 10){
      xts.trade[j,1][[1]] <- 1 # position in type 1 taken
      xts.trade[j,5][[1]] <- 1 # initiate type 1 trade
      xts.trade[j,3][[1]] <- logprice[i,1][[1]]
      xts.trade[j,4][[1]] <- logprice[i,2][[1]]
      j <- j+1
    } else if(signport[i,4][[1]] == 2 & xts.trade[(j-1),1] == 1){
      xts.trade[j,1][[1]] <- 10 # position in type 1 held
      xts.trade[j,3][[1]] <- logprice[i,1][[1]]
      xts.trade[j,4][[1]] <- logprice[i,2][[1]]
      j <- j+1
    } else if(signport[i,4][[1]] == 2 & xts.trade[(j-1),1] == 10){
      # today 2/stdfrommax3 above, yesterday trade initiated
      xts.trade[j,1][[1]] <- 10 # position in type 1 held
      xts.trade[j,3][[1]] <- logprice[i,1][[1]]
      xts.trade[j,4][[1]] <- logprice[i,2][[1]]
      j <- j+1
    } else if(signport[i,4][[1]] == 2 & xts.trade[(j-1),1] == 10){
      # today 2/stdfrommax3 above, yesterday trade initiated
      xts.trade[j,1][[1]] <- 10 # position in type 1 held
      xts.trade[j,3][[1]] <- logprice[i,1][[1]]
      xts.trade[j,4][[1]] <- logprice[i,2][[1]]
      j <- j+1
    }
  }
}
```

```r
# today 2<stdfromma<3 above, yesterday trade held
xts.trade[j,1][[1]] <- 10 # position in type 1 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == 1 & xts.trade[(j-1),1] == 1){
# today 1<stdfromma<1 above, yesterday trade initiated
xts.trade[j,1][[1]] <- 10 # position in type 1 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == 1 & xts.trade[(j-1),1] == 10){
# today 1<stdfromma<2 above, yesterday trade held
xts.trade[j,1][[1]] <- 10 # position in type 1 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == -2 & xts.trade[(j-1),2] != 2 & xts.trade[(j-1),2] != 20){
# today 2<stdfromma<3 below, yesterday trade not initiated or held
xts.trade[j,2][[1]] <- 20 # position in type 2 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == -2 & xts.trade[(j-1),2] == 2){
# today 2<stdfromma<3 below, yesterday trade initiated
xts.trade[j,2][[1]] <- 20 # position in type 2 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == -2 & xts.trade[(j-1),2] == 20){
# today 2<stdfromma<2 below, yesterday trade held
xts.trade[j,2][[1]] <- 20 # position in type 2 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == -1 & xts.trade[(j-1),2] == 2){
# today 1<stdfromma<1 below, yesterday trade initiated
xts.trade[j,2][[1]] <- 20 # position in type 2 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else if(signport[i,4][[1]] == -1 & xts.trade[(j-1),2] == 20){
# today 1<stdfromma<2 below, yesterday trade held
xts.trade[j,2][[1]] <- 20 # position in type 2 held
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
    j <- j+1
} else{ # otherwise
```
xts.trade[j,1][[1]] <- 0 # Not in trade
xts.trade[j,5][[1]] <- 0 # Not in trade
xts.trade[j,2][[1]] <- 0 # Not in trade
xts.trade[j,6][[1]] <- 0 # Not in trade
xts.trade[j,3][[1]] <- logprice[i,1][[1]]
xts.trade[j,4][[1]] <- logprice[i,2][[1]]
j <- j+1
}
list.xts.trade[[h]] <- xts.trade
}

# Create ending signals
for(h in 1:length(pair)){
  trading <- list.xts.trade[[h]]
  j <- 2
  signport <- list.signport[[h]]
  for(i in 1:length(index(signport))){
    if(signport[i,4][[1]] == 3 & trading[(j-1),1][[1]] == 1){
      # if today is past upper stop-loss & yesterday a type 1 trade was initiated
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == 3 & trading[(j-1),1][[1]] == 10){
      # if today is past upper stop-loss & yesterday a type 1 trade was open
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == 0 & trading[(j-1),1][[1]] == 1){
      # if today is within 1 standard deviation & yesterday a trade was initiated
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == 0 & trading[(j-1),1][[1]] == 10){
      # if today is within 1 standard deviation & yesterday a trade was open
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == -1 & trading[(j-1),1][[1]] == 1){
      # if today is between 1 and 2 standard deviations below & yesterday a trade was initiated
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == -1 & trading[(j-1),1][[1]] == 10){
      # if today is between 1 and 2 standard deviations below & yesterday a trade was open
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == -2 & trading[(j-1),1][[1]] == 1){
      # if today is between 2 and 3 standard deviations below & yesterday a trade was initiated
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    } else if(signport[i,4][[1]] == -2 & trading[(j-1),1][[1]] == 10){
      # if today is between 2 and 3 standard deviations below & yesterday a trade was open
      trading[j,5][[1]] <- 2 # end trade
      j <- j+1
    }
  }
}
j <- j+1
} else if(signport[i,4][1] == -3 & trading[(j-1),1][1] == 1){
  # if today is past 3 standard deviations below & yesterday a trade was initiated
  trading[j,5][1] <- 2 # end trade
  j <- j+1
} else if(signport[i,4][1] == -3 & trading[(j-1),1][1] == 10){
  # if today is past 3 standard deviations below & yesterday a trade was open
  trading[j,5][1] <- 2 # end trade
  j <- j+1
} else if(signport[i,4][1] == -3 & trading[(j-1),2][1] == 2){
  # same for type 2 trades
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == -3 & trading[(j-1),2][1] == 20){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 0 & trading[(j-1),2][1] == 2){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 0 & trading[(j-1),2][1] == 20){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 1 & trading[(j-1),2][1] == 2){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 1 & trading[(j-1),2][1] == 20){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 2 & trading[(j-1),2][1] == 2){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 2 & trading[(j-1),2][1] == 20){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 3 & trading[(j-1),2][1] == 2){
  trading[j,6][1] <- -2
  j <- j+1
} else if(signport[i,4][1] == 3 & trading[(j-1),2][1] == 20){
  trading[j,6][1] <- -2
  j <- j+1
} else{
  trading[j,5][1] <- trading[j,5][1]
  trading[j,6][1] <- trading[j,6][1]
  j <- j+1
}
}
list.xts.trade[[h]] <- trading
# End trade if time limit is past
list.xts.trade.time <- list()
for(h in 1:length(list.xts.trade)){
  trading <- list.xts.trade[[h]]
  for(i in 1:(nrow(trading)-41)){
    if(trading[i,1][[1]] == 1 & sum(trading[(i+1):(i+40),1]) == 400){
      # if trade has been open for 40 days
      trading[(i+41),4][[1]] <- 2 # end trade
      print("Type 1 time limit")
    }
    if(trading[i,1][[1]] == 2 & sum(trading[(i+1):(i+40),1]) == 800){
      # same for type 2 trade
      trading[(i+41),4][[1]] <- -2
      print("Type 2 time limit")
    }
  }
  list.xts.trade.time[[h]] <- trading
}

# Calculate returns
list.returns <- list()
for(h in 1:length(list.xts.trade.time)){
  trading <- list.xts.trade.time[[h]]
  x <- cbind(rep(0, nrow(trading)), rep(0, nrow(trading)))
  returns <- xts(x, order.by = idxtrade)
  colnames(returns) <- c("Return 1", "Return 2")
  for(i in 1:(length(index(trading))-1)){
    for(j in (i+1):length(index(trading))){
      if(trading[i,5][[1]] == 1){ # if a type 1 trade was initiated day i
        if(trading[j,5][[1]] == 2){ # and ended on day j, where j>i
          returns[j,1][[1]] <- trading[i,3][[1]]-trading[j,3][[1]]+
                               trading[j,4][[1]]-trading[i,4][[1]]
          # add log-differences to calculate returns
        } else{ # otherwise
          returns[j,1][[1]] <- as.numeric(0) # set returns for that day to 0
        }
      }
    }
    if(trading[i,6][[1]] == -1){ # same for type 2 trade
      if(trading[j,6][[1]] == -2){
        returns[j,2][[1]] <- trading[j,3][[1]]-trading[i,3][[1]]+
                             trading[i,4][[1]]-trading[j,4][[1]]
        # here the days are switched in calculation of returns
      } else{
        returns[j,2][[1]] <- as.numeric(0)
      }
    }
  }
  list.returns[[h]] <- returns
}
# Remove first row from prior trading year (which was added for technical reasons)
list.returns.final <- list()  # create list to store final returns from each pair
for(i in 1:length(list.returns)){
  list.returns.final[[i]] <- list.returns[[i]][-1,]
}

# Calculate cumulative return
cumsum <- 0
for(i in 1:length(list.returns.final)){
cumsum <- sum(list.returns.final[[i]][,c(1,2)]) + cumsum
  # sum returns from both types of trades for all pairs
}

## Sharpe’s ratio and standard deviation of
# Create xts file with portfolios return for each day
xts.portret <- xts(rep(0,length(index(list.returns.final[[1]]))),
  order.by = index(list.returns.final[[1]]))
for(j in 1:length(index(list.returns.final[[1]]))){
  portret <- 0
  for(i in 1:length(list.returns.final)){
    portret <- sum(list.returns.final[[i]][j,]) + portret
    # sum the return from all pairs for that day
  }
  xts.portret[j,1][[1]] <- portret  # set it as the portfolio return of the day
}
sdport <- sd(xts.portret[,1])  # calculate standard deviation of return
meanport <- mean(xts.portret[,1])  # mean return
sharpeport <- meanport/sdport  # calculate sharpe’s ratio

y2010 <- as.vector(index(xts.cumret_one))

## S&P 500 data
# Do the same calculations to calculate the standard
deviation and Sharpe ratio for the S&P500 index
year <- as.character(g)
year_tail <- as.character(g-1)
SP500_year <- SP500[year]
SP500_year_tail <- SP500[year_tail]
SP500_year_done <- rbind(tail(SP500_year_tail, n=1), SP500_year)
SP500cumret <- 0
xts.SP500.ret <- xts(rep(0, length(index(SP500_year))), order.by = index(SP500_year))
xts.SP500.cumret <- xts(rep(0, length(index(SP500_year))),
  order.by = index(SP500_year))
j <- 2
for(i in 1:length(index(SP500_year))){
  xts.SP500.ret[i,1][[1]] <- log(SP500_year_done[j,1][[1]]) - log(SP500_year_done[i,1][[1]])
  xts.SP500.cumret[i,1][[1]] <- xts.SP500.cumret[i,1][[1]]
SP500cumret <- SP500cumret + log(SP500_year_done[j,1][[1]]) - log(SP500_year_done[i,1][[1]])
  j <- j+1
}

SP500std <- sd(xts.SP500.ret[,1])
SP500mean <- mean(xts.SP500.ret[,1])
SP500sharpe <- SP500mean/SP500std
place <- g-2006
df.values[place,1] <- SP500std
df.values[place,2] <- SP500mean
df.values[place,3] <- SP500sharpe

## Save workspace
filename <- paste0(as.character(g))
save.image(file = filename)

### For plotting
## Intuitive plot of spread
Stock1 <- as.vector(c(0, 0.5, 1.2, 1.7, 2.1, 2.5, 2.0, 1.4, 1.2, 1.1, 0.7, 1.2))
Stock2 <- Stock1*(-1)
dates <- seq(as.Date("2007-01-01"), by = "day", length.out = length(Stock1))
xts.vals <- xts(cbind(Stock1, Stock2), order.by = dates)
df.example <- data.frame(
  dates,
  stack(as.data.frame(coredata(xts.vals)))
)

names(df.example)[1] <- "Dates"
names(df.example)[2] <- "Values"
names(df.example)[3] <- "Stock"
ggplot(df.example, aes(x = Dates, y = Values, colour = Stock)) +
geom_point(size = 3) +
geom_line(size = 1, linetype = 4) +
geom_hline(yintercept = 0, colour = "black", linetype = 1, size = 1) +
geom_hline(yintercept = 1, colour = 43, linetype = 2, size = 1) +
geom_hline(yintercept = 2, colour = "dark grey", linetype = 2, size = 1) +
geom_hline(yintercept = -1, colour = 43, linetype = 2, size = 1) +
geom_hline(yintercept = -2, colour = "dark grey", linetype = 2, size = 1) +
geom_vline(xintercept = index(xts.vals[5,]), linetype = 1, colour = "dark grey") +
geom_vline(xintercept = index(xts.vals[11,]), linetype = 1, colour = "dark grey") +
annotate("point", x = index(xts.vals[1,]), y = 0, colour = 113, size = 3) +
annotate("point", x = index(xts.vals[5,]), y = 2.1, colour = "black", size = 6, shape = 10) +
annotate("point", x = index(xts.vals[5,]), y = -2.1, colour = "black", size = 6, shape = 10) +
annotate("point", x = index(xts.vals[11,]), y = 0.7, colour = "black", size = 6, shape = 10) +
annotate("point", x = index(xts.vals[11,]), y = -0.7, colour = "black", size = 6, shape = 10) +
theme_classic() +
scale_y_continuous(name = "", breaks = 0, label = "Start")
## Rolling portfolio illustrated

df.years <- data.frame(Years = c(2005:2011),
                      Portfolio = c(1:7))

ggplot(df.years, aes(x = Years)) +
  theme_classic() +
  # Testing periods
  geom_segment(aes(x = df.years[1,1], xend = df.years[3,1], y = 2, yend = 2), size = 2) +
  geom_segment(aes(x = df.years[2,1], xend = df.years[4,1], y = 4, yend = 4), size = 2) +
  geom_segment(aes(x = df.years[3,1], xend = df.years[5,1], y = 6, yend = 6), size = 2) +
  # Trading periods
  geom_segment(aes(x = df.years[3,1], xend = df.years[4,1], y = 2, yend = 2), size = 2, colour = "coral") +
  geom_segment(aes(x = df.years[4,1], xend = df.years[5,1], y = 4, yend = 4), size = 2, colour = "coral") +
  geom_segment(aes(x = df.years[5,1], xend = df.years[6,1], y = 6, yend = 6), size = 2, colour = "coral") +
  scale_y_continuous(name = "Portfolio", breaks = FALSE) +
  annotate("text", x = 2006.5, y = 2.3, label = "2007", size = 5) +
  annotate("text", x = 2007.5, y = 4.3, label = "2008", size = 5) +
  annotate("text", x = 2008.5, y = 6.3, label = "2009", size = 5) +
  annotate("point", x = 2007, y = 2, size = 3, shape = 15) +
  annotate("point", x = 2008, y = 4, size = 3, shape = 15) +
  annotate("point", x = 2009, y = 6, size = 3, shape = 15)

## Technical plot of spread diverging and converging

Stock1 <- as.vector(c(0, 0.5, 1.2, 1.7, 2.1, 2.5, 2.0, 1.4, 1.2, 1.1, 0.7, 1.2))  # example vector
dates <- seq(as.Date("2007-01-01"), by = "day", length.out = length(Stock1))  # example dates
xts vals <- xts(Stock1, order.by = dates)  # example dates

df.example <- data.frame(dates,
                        stack(as.data.frame(coredata(xts vals))))

names(df.example)[1] <- "Dates"
names(df.example)[2] <- "Values"

ggplot(df.example, aes(x = Dates, y = Values)) +
  geom_point(size = 3, colour = "coral") +
  geom_line(size = 1, linetype = 4, colour = "coral") +
  geom_hline(yintercept = 0, colour = "black", linetype = 1, size = 1) +
  geom_hline(yintercept = 1, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -1, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = 2, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -2, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = 3, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -3, colour = "dark grey", linetype = 2, size = 1) +
  geom_vline(xintercept = index(xts vals[5,]), linetype = 1, colour = "dark grey") +
  geom_vline(xintercept = index(xts vals[11,]), linetype = 1, colour = "dark grey") +

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annotate("point", x = index(xts.vals[1,]), y = 0, colour = 113, size = 3) +
annotate("point", x = index(xts.vals[5,]), y = 2.1, colour = "black", size = 6, shape = 10) +
annotate("point", x = index(xts.vals[11,]), y = 0.7, colour = "black", size = 6, shape = 10) +
scale_y_continuous(breaks = 5) +
ylab("Distance from MA") +
theme_classic()

## Example stop-loss to divergence from 11th pair 2008
deviation <- (pair[[11]][,1]-pair[[11]][,2])/pair[[11]][,3]
                          "2008-01-23", "2008-01-24")]
df.stoploss <- data.frame(Dates = index(deviation),
                          Spread = as.data.frame(coredata(deviation)))

names(df.stoploss)[1] <- "Dates"
names(df.stoploss)[2] <- "Spread"
ggplot(df.stoploss, aes(x = Dates, y = Spread)) +
  geom_point(size = 3, colour = "coral") +
  geom_line(size = 1, linetype = 4, colour = "coral") +
  geom_hline(yintercept = 0, colour = "black", linetype = 1, size = 1) +
  geom_hline(yintercept = 1, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -1, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = 2, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -2, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = 3, colour = "dark grey", linetype = 2, size = 1) +
  geom_hline(yintercept = -3, colour = "dark grey", linetype = 2, size = 1) +
  geom_vline(xintercept = index(deviation[6,]), linetype = 1, colour = "dark grey") +
  geom_vline(xintercept = index(deviation[9,]), linetype = 1, colour = "dark grey") +
  annotate("point", x = index(deviation[1,]), y = 0, colour = 113, size = 3) +
  annotate("point", x = index(deviation[6,]), y = deviation[6,1][[1]],
            colour = "black", size = 6, shape = 10) +
  annotate("point", x = index(deviation[9,]), y = deviation[9,1][[1]],
            colour = "black", size = 6, shape = 10) +
scale_y_continuous(name = "Distance from MA") +
scale_x_date(breaks = pretty_breaks(10)) +
theme_classic()

## Plotting the cumulative return of the portfolio

cumret <- 0
xts.cumret <- xts(rep(0,length(index(list.returns.final[[1]]))),
                   order.by = index(list.returns.final[[1]]))
for(j in 1:length(index(list.returns.final[[1]]))){
  for(i in 1:length(list.returns.final)){
    cumret <- sum(list.returns.final[[i]][j]) + cumret
  }
  xts.cumret[j,1][[1]] <- cumret
}
df.cumret <- data.frame(index(xts.cumret), as.data.frame(coredata(xts.cumret)))
names(df.cumret)[1] <- "Date"
names(df.cumret)[2] <- "Return"

plot.total <- ggplot(df.cumret, aes(x = Date, y = Return)) + geom_line(size = 1)
plot.total <- plot.total + ylab("Return (%)") + xlab("Date")
plot.total <- plot.total + theme(text = element_text(size=16))

# The same for the S&P 500
SP500cumret <- 0
xts.SP500cumret <- xts(rep(0,length(index(xts.SP500.ret))), order.by = index(xts.SP500.ret))
for(i in 1:length(index(xts.SP500.ret))){
  SP500cumret <- xts.SP500.ret[i,1][1]*100 + SP500cumret
  xts.SP500cumret[i,1] <- SP500cumret
}
df.SP500cumret <- data.frame(index(xts.SP500cumret), as.data.frame(coredata(xts.SP500cumret)))
names(df.SP500cumret)[1] <- "Date"
names(df.SP500cumret)[2] <- "Return"

plot.total <- ggplot(df.SP500cumret, aes(x = Date, y = Return)) + geom_line(size = 1)
plot.total <- plot.total + ylab("Return (%)") + xlab("Date")
plot.total <- plot.total + theme(text = element_text(size=16))

# Plotting the individual cumulative return of each pair
listofreturnsforeachpair <- list()
for(h in 1:length(list.returns.final)){
  listofreturnsforeachpair[[h]] <- xts()
  cumret_one <- 0
  xts.cumret_one <- xts(rep(0,length(index(list.returns.final[[h]]))),
                        order.by = index(list.returns.final[[h]]))
  for(i in 1:length(index(list.returns.final[[h]]))){
    cumret_one <- sum(list.returns.final[[h]][i,]) + cumret_one
    xts.cumret_one[i,1] <- cumret_one
  }
  listofreturnsforeachpair[[h]] <- xts.cumret_one
}
allpairs <- xts()
for(i in 1:length(listofreturnsforeachpair)){
  allpairs <- merge.xts(allpairs, listofreturnsforeachpair[[i]][,1]*100)
  colnames(allpairs)[i] <- paste0("Pair", i)
}
df.allpairs <- data.frame(index(allpairs), stack(as.data.frame((coredata(allpairs)))))
names(df.allpairs)[1] <- "Date"
names(df.allpairs)[2] <- "Return"
names(df.allpairs)[3] <- "Pair"
cols71011 <- c("Pair1" = "dark green",
               "Pair2" = "green",
               "Pair3" = "red",
               "Pair4" = "blue",
               "Pair5" = "purple",
               "Pair6" = "green",
               "Pair7" = "red",
               "Pair8" = "blue",
               "Pair9" = "purple",
               "Pair10" = "green",
               "Pair11" = "red",
               "Pair12" = "blue",
               "Pair13" = "purple")

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cols89 <- c("Pair1" = "dark green",
            "Pair2" = "green",
            "Pair3" = "blue",
            "Pair4" = "dark blue",
            "Pair5" = "red",
            "Pair6" = "orange",
            "Pair7" = "yellow",
            "Pair8" = "purple",
            "Pair9" = "pink",
            "Pair10" = "brown",
            "Pair11" = "dark grey",
            "Pair12" = "black")

plot.allpairs <- ggplot(df.allpairs, aes(x = Date, y = Return, colour = Pair))
plot.allpairs <- plot.allpairs + ylab("Return (%)") + xlab("Date")
plot.allpairs <- plot.allpairs + geom_line(size = 1)
plot.allpairs <- plot.allpairs + scale_color_manual(values = cols71011)
plot.allpairs <- plot.allpairs + theme(text = element_text(size=16))

### Calculate number of trades
vec.pairtrades <- vector()
for(h in 1:length(list.xts.trade.time)){
  ntradespair <- 0
  for(i in 1:length(index(list.xts.trade.time[[h]]))){
    if(list.xts.trade.time[[h]][i,5] == 2){
      ntradespair <- ntradespair + 1 # Count type 1 trades for pair h
    } else if(list.xts.trade.time[[h]][i,6] == -2){
      ntradespair <- ntradespair + 1 # Count type 2 trades for pair h
    }
  }
  vec.pairtrades[h] <- ntradespair # Submit nr of trades for pair h in vector
}
vec.pairtrades
sum(vec.pairtrades) # Calculate total nr of trades