

A GAME THEORETIC PERSPECTIVE ON THE UTTERANCE SELECTION MODEL FOR LANGUAGE CHANGE

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Many mathematical models for language change have recently been proposed and their interpretation is not always straightforward. In this paper, we take a closer look at the Utterance Selection Model with preferences and reinterpret its dynamics in terms of evolutionary game theory. The analysis demonstrates that the preference associated with a variant is formally equivalent to the payoff of using that variant in the associated game. Importantly, this payoff is subjectively perceived by speakers and evolves according to the current local use of the variant. Additionally, the accommodation to others' utterances can be encoded as a mutation term in a Replicator-Mutator equation. This analysis demonstrates how arbitrary variants can acquire fitness through usage, allowing selective processes to take place.

1. Introduction

In recent years, a number of mathematical models of language evolution have been developed (Smith, 2014; Pierrehumbert, Stonedahl, & Daland, 2014; Mitchener, 2009) and it is sometimes cumbersome to really understand how the models actually work. In this paper, we argue that reanalysing some of these models using evolutionary game theory (Weibull, 1997; Hofbauer, 1985) can shed light on the inner working of the models and provide an evolutionary interpretation of them. More specifically, we reanalyse the Utterance Selection Model (USM) (Baxter, Blythe, Croft, & McKane, 2006) and in particular its version with preferences (USMP) (Michaud, 2019) in order to demonstrate the power of the analogy with game theory.

The USM for language change models the change in the frequency of a set of competing variants due to interactions of speakers in a population. Speakers exchange utterances (biased sample of variants) to update their state or idiolect (frequency of variant usage). The different versions of the USM encode different updating rules for a speaker state. Although the USM is linguistically simplistic, (it considers a single trait that can be instantiated in a finite number of variants) it provides some fundamental insights into the emergence and change of conventions in a given population. The version with preferences (Michaud, 2019) in

focus in this paper has been used to understand self-actuation of language changes as well as their S-shaped trajectories.

The main idea underlying our analysis is that the state of a speaker in the USM, which encodes the probability of an agent to produce a given variant, can be interpreted as a mixed strategy to play a game associated with the interaction, hence, the dynamics of the USM models the evolution of players' strategies. By mapping the USMP dynamics onto a Replicator-Mutator (RM) dynamics (Hofbauer, 1985; Komarova, 2004) it is possible to reinterpret the USMP in terms of evolutionary game theory, where fitness functions and mutation matrices can be defined for every individual. It turns out that any individual's preferences used in the USMP can be interpreted as speaker-dependent fitnesses of variants in the RM dynamics, whereas accommodation to incoming utterances contributes to the mutation part of the RM dynamics.

This paper is organized as follows. In section 2, we recall the definition of the USMP. In section 3, we reanalyze the USMP through the lense of evolutionary game theory. In section 4, we discuss the implication of this interpretation for the understanding of the dynamics of the USMP and language change in general.

2. The utterance selection model with preferences (USMP)

The USM for language change and its various extensions (Baxter et al., 2006; Baxter, Blythe, Croft, & McKane, 2009; Blythe & Croft, 2012; Baxter & Croft, 2016; Stadler, 2016; Stadler, Blythe, Smith, & Kirby, 2016; Michaud, 2019) model the evolution of the use of a fixed number of variants V used in a population of N speakers connected through a static network. In every pairwise interaction a speaker i and a speaker j exchange utterances \mathbf{u}^1 and update their state \mathbf{x} and, in the USMP, their preferences $\boldsymbol{\pi}$ as illustrated in Figure 1.²

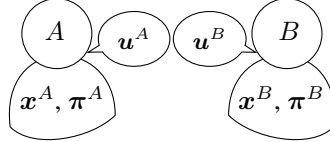


Figure 1. Illustration of the USMP interaction. Speakers A and B exchange utterances \mathbf{u} reflecting their state \mathbf{x} and update their state using their preferences $\boldsymbol{\pi}$.

¹The utterances \mathbf{u} are obtained from \mathbf{x} by a biased sampling process.

²Bold quantities denotes column vectors of length V and are indexed to the speakers by an upper bracketed index.

The learning rule of the USMP (Michaud, 2019) is given by

$$\delta \mathbf{x}^{(i)} = \lambda \left[\left(1 - h^{(ij)} \right) S \left(\mathbf{x}^{(i)}, \mathbf{u}^{(i)} \right) + h^{(ij)} A_p \left(\mathbf{x}^{(i)}, \mathbf{u}^{(j)}, \boldsymbol{\pi}^{(i)} \right) \right], \quad (1)$$

where λ is a learning parameter, $h^{(ij)}$ a parameter controlling the weight of the incoming utterance from speaker j with respect to speaker i own utterance, S is the self-monitoring function and A_p is the accommodation function with preferences. The self-monitoring and accommodation functions are given in vectorial form by³

$$\begin{cases} S \left(\mathbf{x}^{(i)}, \mathbf{u}^{(i)} \right) & := \mathbf{u}^{(i)} - \mathbf{x}^{(i)}, \\ A_p \left(\mathbf{x}^{(i)}, \mathbf{u}^{(j)}, \boldsymbol{\pi}^{(i)} \right) & := \mathbf{u}^{(j)} \left(1 - \boldsymbol{\pi}^{(i)} \cdot \mathbf{x}^{(i)} \right) \\ & \quad - \text{diag} \left(\mathbf{x}^{(i)} \otimes (\mathbf{1} - \boldsymbol{\pi}^{(i)}) \right). \end{cases} \quad (2)$$

The dynamics is complemented by the updating rule for the preferences

$$\delta \boldsymbol{\pi}^{(i)} = \mu (\mathbf{U}^{(i)} - \mathbf{x}^{(i)}), \quad \mathbf{U}^{(i)} := \frac{1}{|V_i|} \sum_{j \in V_i} \mathbf{u}^{(j)}, \quad (3)$$

where μ controls the speed of change of preferences, V_i is the set of neighbors of speaker i and $\mathbf{U}^{(i)}$ the average uttered frequency distribution of the neighbors of a speaker i . Equations (1), (2) and (3) fully determine the dynamics of the USMP. The standard USM version is recovered when $\boldsymbol{\pi} = 0$ and $\mu = 0$ for all speakers.

The understanding of the general dynamics is fairly straightforward, at each time step, two speakers interact and their state is updated by a weighted average of their own behavior and the behavior of their interlocutor. The self-monitoring function encodes the change in the state of a speaker towards the experienced frequency of the variants. The accommodation rule is more complicated and will be the object of the game theoretic interpretation provided in this paper. In parallel with the change in the state of the speaker, the preferences are also updated and encode a social alignment process (Gaissmaier & Schooler, 2008) that occurs in addition of the interaction process; speaker i increases the preference for the variants she uses less often than her neighbors and decreases the preference for the variants she uses more often. The change in preferences has consequences on subsequent changes in the state of the speakers and enables a differential accommodation to variants.

3. Game theoretic perspective

3.1. The USMP as a game

Each interaction of the USMP can be conceived as a strategic interaction that can be understood in the framework of game theory. The possible variants correspond

³In Equation (2), vectors are denoted by bold symbols, a dot represents the scalar product between two vectors and the combination of the diag and the tensor product \otimes is a mathematical way of encoding the elementwise multiplication of two vectors.

to the actions of the game, the state of the speakers correspond to a mixed strategy to play that game. Each time the game is played between two speakers, a variant is chosen. If the game is repeated, multiple variants can be observed with different frequencies. The utterances in the USMP, therefore, correspond to the estimated mixed strategy of the other player after L games, where L is the length of the utterance. With this interpretation, the USMP dynamics models the change in the mixed strategies of the players.

The definition of a game is incomplete without the specification of payoffs associated with the different outcome of the game. In order to extract the payoff structure associated with the USMP game, we map the accommodation rule onto a RM dynamics. By doing so, fitness functions and mutation matrices can be identified. Furthermore, using an additional assumption, one can obtain payoff matrices from the derived fitness functions.

3.2. Link with Replicator-Mutator dynamics

In order to explicitize the payoff matrix associated with the USMP game, we will focus on the accommodation part of the dynamics. We set $h^{(ij)} = 1$ for all i, j , scale $\lambda = \delta t$ and take the limit $\delta t \rightarrow 0$ in Equation (1). We obtain the following vectorial equation

$$\dot{\mathbf{x}}^{(i)} = A_p(\mathbf{x}^{(i)}, \mathbf{u}^{(j)}, \boldsymbol{\pi}^{(i)}), \quad (4)$$

which, component-wise, reads

$$\dot{x}_v^{(i)} = u_v^{(j)} \left(1 - \sum_{w=1}^V \pi_w^{(i)} x_w^{(i)} \right) - x_v^{(i)} (1 - \pi_v^{(i)}). \quad (5)$$

This equation is reminiscent of the RM dynamics given by

$$\dot{x}_v = \sum_w x_w f_w(\mathbf{x}) Q_{wv} - \phi(\mathbf{x}) x_v, \quad \phi(\mathbf{x}) = \sum_w x_w f_w(\mathbf{x}), \quad (6)$$

where x_v is the fraction of the population in state v , $f(\mathbf{x})$ is the fitness function and Q is a mutation matrix.

In the case of the USMP game, each speaker has a different RM dynamics associated with her and we have the following correspondence

$$f_v^{(i)}(\mathbf{x}) := \pi_v^{(i)} - 1, \quad (7a)$$

$$Q_{wv}^{(ij)} := x_v^{(i)} - u_v^{(j)} + \delta_{vw}, \quad (7b)$$

where δ_{vw} is the Kronecker delta that equals 1 if $v = w$ and 0 otherwise, and where the fitness function and the mutation matrix are indexed to the speaker i . We will, therefore, refer to these fitness functions as *subjective*. Interestingly, we

observe that the preferences define the fitness of a variant and that accommodation to incoming utterances defines the mutation matrix. Note that if the uttered variant frequency $u_v^{(j)}$ equals the expected variant frequency $x_v^{(i)}$, then there is no mutation. In general, the components of the mutation matrix reflect the fact that the larger the difference between the uttered variant frequency and the expected variant frequency, the larger the mutation rate towards the uttered variant.

From the RM dynamics, it is possible to extract an associated payoff matrix, provided that the subjective fitness functions are linear, that is we can rewrite $f(x) = Px$. For Equation (7), we obtain the following subjective payoff matrices

$$P^{(i)} = \begin{bmatrix} \pi_1^{(i)} - 1 & \dots & \pi_1^{(i)} - 1 \\ \vdots & & \vdots \\ \pi_V^{(i)} - 1 & \dots & \pi_V^{(i)} - 1 \end{bmatrix}. \quad (8)$$

The subjective payoff matrices $P^{(i)}$ have the property to be constant along the rows, which means that the behavior of a speaker is independent of the behavior of its interlocutor. In the USMP, there is no adaptation or strategic thinking that depends on who is talking to whom.

Under this interpretation, a rational player who tries to maximise her payoff should always choose the variant she prefers. In the USMP, the speakers play the game according to their mixed strategies, and not to the optimal rational choice, but if all speakers prefer the same variant, this variant will be used most of the time and, therefore, be conventional.

In the original version of the USM (Baxter et al., 2006), which corresponds to setting all preferences to 0, every variant is equally fit and the dynamics is purely driven by the mutation term of the RM dynamics. This explains why the original version of the USM produces more stochastic time series of variants' frequency.

Unlike conventional evolutionary games, in the USMP the payoff structure is dynamic since the preferences are updated during the dynamics. As discussed above, the change in preferences is driven by social alignment, which means that subjectively fit variants are the one used by the speech community of a speaker. Furthermore, the mutation matrices are changing at each interaction and depend on the current state of a speaker and the utterance of her interlocutor in that specific interaction.

4. Discussion

The game theoretic perspective on the USMP provided in this paper allows us to interpret the accommodation rule of this model in terms of subjective fitnesses and mutation matrices, where the preferences play the role of subjective fitnesses, while the incoming utterances drives mutations in the RM dynamics.

The RM dynamics obtained by analyzing the USMP is not standard. The fitnesses are usually negative and the mutation matrices do not satisfy usual properties

of standard mutation matrices⁴, since some of their elements can be greater than 1 or negative. This encodes the fact that a variant can mutate into itself in some kind of duplication process, while conserving the probability distribution character of the state of the speaker.

The advantage of the RM formulation is that it clearly distinguishes the role of the accommodation to the incoming utterance and the role of preferences. In the standard USM, there are no preferences and the dynamics are purely driven by the mutation part of the RM dynamics, while in the USMP the preferences act as subjective fitnesses that evolve according to local usage of variants, which grounds this model in a usage-based theory of language (Bybee, 2006). The parameter μ controlling the change of preferences is usually chosen small in order to better reflect reality. The role of this parameter is to model to speed of change in preferences. It can be thought of as a parameter controlling the importance of new utterances with respect to older ones. If new utterances have a great influence, then it is easier to change from one variant to another, whereas if the new utterances have a small influence, it will take more time for a new innovation to be adopted. It has been demonstrated by Michaud (2019) that such dynamics can explain the self-actuation of language changes.

Following the mapping from language change to ecology provided by Blythe and McKane (2007), one could make the analogy that every speaker is an island on which the variants, which are the analog to species, compete for being used. Their fitness evolves from observation of neighboring speakers/islands⁵ and the mutation encodes a migration process, since conversation is the analog of migration from island to island. Following this interpretation, the idiolect of a speaker can be thought of as an ecosystem of competing variants whose fitnesses are dependent on the speaker. Furthermore, a population of speakers can be thought of as a network of ecosystems, where each linguistic ecosystem has its own dynamic fitness landscape. These linguistic ecosystems are not independent and exchange variants through conversation/migration and their fitness landscape evolves by observation of neighboring ecosystems/speakers.

In terms of evolutionary forces, the perspective given in this paper suggests that a neutral evolutionary model is not sufficient to account for language change. Such a neutral model can be used as a null model (Blythe, 2012) for language change, but in order to better account for the observed time series of change, variants should have different fitnesses. However, these fitnesses are not externally defined, they depend on the history of variants' usage. The dynamic properties and

⁴A standard mutation matrix has all its components positive and is row stochastic, which means that the sum of the elements of a row sum up to one.

⁵In ecology, such type of observation would be absent. This is an example where the analogy don't fully work. This is of course not a problem, since analogical reasoning has its limits and should not be applied blindly.

the fact that each speaker develops her own fitness landscape make the fitnesses subjective. This subjectivity of fitnesses allows us to reconcile the fact that variants are arbitrary (de Saussure, 1959), but their fitness evolves as they are being used.

Different versions of the USM for language change have proven useful to understand the dynamics of language change, but there are still some limitations to these models that should be addressed in future work. For instance, the USM assumes that communication is *always successful*, since every variant can be used. This is a strong assumption that should be relaxed in the future to account for failure in communication. Another limitation of this model illustrated by the subjective payoff matrices derived in this paper is that the speaker usage of the variants is *selfish*, since there is no adaptation to the identity of the interlocutor. While such an assumption is needed to keep the model tractable, it does not account for the richness of human interactions. This issue should be addressed in future development of these models. The relation to game theory outlined here suggests a path to achieve such a generalization, since game theory is the science of strategic interactions. In addition, the type of game with dynamic strategies and dynamic payoff is also a novel contribution to game theory itself, which may open new paths of cross-fertilization between language evolution and game theory.

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