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From Symbolic Manipulations to Stepwise Instructions: A Curricular Comparison of Swedish School Algebra Content over the Past 40 Years

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ABSTRACT
Although there have been a huge number of attempts to improve school algebra teaching, several countries are still struggling to improve students algebraic skills. In this study, we focus on the specific case of Sweden where students for several decades have had major problems mastering algebra. In order to get a better understanding of the Swedish situation, we consider what constitutes Swedish school algebra by investigating the development of algebraic content in the Swedish mathematics curriculum documents over the past 40 years. The results reveal that the connection between arithmetic and algebra, the so-called generalized arithmetic, is almost absent in all three curricula although researchers argue that generalized arithmetic is one of the most relevant topics within early algebra. Instead, Sweden has chosen a unique approach as programming, with a specific focus on stepwise instructions and algorithms, recently has been implemented within the core content of algebra.

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Case study; content analysis; early algebra; intended curriculum; mathematics education

Introduction

Today it is well established that algebra serves as a critical gatekeeper for more advanced studies in both mathematics and science (Kaput, 2008). Several studies reveal that students' difficulties with algebra lead not only to school failure at the tertiary level, but also to limited career opportunities within the STEM-related fields (Brandell et al., 2008). In Western countries, arithmetic has traditionally been taught before algebra, and therefore algebra was not introduced in mathematics curricula until the secondary school level. However, research has disclosed that students have encountered serious problems when they did not begin to learn algebra until secondary school, and a gap between arithmetic and algebra has been documented (e.g., Filloy & Rojano, 1984; Kieran, 2018; Linchevski & Herscovics, 1996). In fact, several studies reveal that it is not only possible but also beneficial to start working with algebraic ideas in parallel with arithmetic from the earliest school grades (Carraher et al., 2006).

These findings have slowly made their way to the educational system, where several countries have revised their mathematics curricula in order to introduce students to algebra already in early grades (Blanton et al., 2015). In Sweden, which is the focus of the study reported in this paper, algebra has always been a part of mathematics that has caused students major difficulties. In the international evaluation TIMSS (Trends in International Mathematics and Science Study), Swedish students' results in algebra have been below average since the 1960s (Bråting et al.,

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2019). Even in the TIMSS test from 1995, in which Sweden's overall results were the best ever, the results in algebra remained below the international average. In the two TIMSS tests from 2007 and 2011, when Sweden's overall results decreased significantly, algebra was the topic that deteriorated the most (Yang Hansen et al., 2014). In the most recent TIMSS evaluation from 2015, Sweden's overall results in mathematics slightly increased, but those in algebra remained poor (Swedish National Agency for Education, 2016).

There have been various attempts to improve school algebra teaching in Sweden, for instance through in-service training projects for teachers and the "Algebra for All" movement, in which all students were ensured the opportunity to study algebra before graduating high school. In connection with the latter, a specific textbook (Bergsten et al., 1997) was compiled and used in teacher education at several universities in Sweden. However, it is not possible to discern a general positive effect of these efforts on Swedish students' algebraic skills; at least not if we consider the results of the TIMSS evaluations.

The study reported in this article is part of a research project (Hemmi et al., 2021) aiming at finding reasons for the failure to raise the quality of algebra teaching in Sweden, but also to find possible ways to improve the situation. Both diachronic and synchronic studies are being conducted in the formulation and realization arenas (Lindensjö & Lundgren, 2000). The Swedish case is interesting not only on a local level but also from an international point of view, as several countries are struggling to improve students' algebraic skills (e.g., Jupri et al., 2014; Prendergast & Treacy, 2018). The study reported in this article focuses on the formulation arena, investigating the Swedish mathematics curriculum documents from 1980, 1994, and 2011. Within the research field of early algebra, investigations of steering documents are few and their focus is almost exclusively on the most recent curriculum documents (e.g., Cai et al., 2010; Hemmi et al., 2021). In this paper, we aim to understand what constitutes Swedish school algebra and are therefore looking closer at how the algebraic content has developed over time.

The study uses both a synchronic and a diachronic perspective (Saussure, 1974), analysing the algebraic content of each curriculum document separately (the synchronic perspective) as well as comparing the algebraic content in the documents over time (the diachronic perspective). When analysing the content in the three curriculum documents, we have used Blanton et al.'s (2015) so-called big ideas of algebra, which are based on research from the last four decades. The study contributes to international research regarding the incorporation of algebra in school curricula, especially as it discusses possible ways to improve future curriculum reforms. The Swedish case is particularly interesting as (1) algebra has been the single most problematic topic for Swedish students over a long time period, (2) during the last 50 years the Nordic welfare state has been established as a unique model with a strong emphasis on access to high-quality education (Yang Hansen et al., 2014), and (3) Sweden has chosen a unique approach as programming was included within the core content of algebra in the recent curriculum revision (Swedish National Agency for Education, 2018).

In this study, the term curriculum should be interpreted as the intended curriculum (Valverde et al., 2002), which refers to the curriculum representing official intentions for instruction, national goals, and the description of content regarding what mathematics students are expected to learn and in what sequence. Therefore, the curriculum documents analysed in this study can only be regarded as frameworks for school mathematics in Sweden, that is, we cannot draw conclusions about the effects on students' performance. We aim to answer the following research question:

> Based on recent decades' research on early algebra, what are the characteristics of the algebraic content in the Swedish curriculum documents from 1980, 1994, and 2011 and how has the algebraic content developed over these years?

Early Algebra – Previous Research and Theoretical Frameworks

During the past decades, there has been an explosion of research on how to incorporate algebra in school mathematics already from early grades (for an overview see Kieran, 2007, 2018). It is by now generally agreed that we cannot simply push the secondary school algebraic content into elementary mathematics, instead the algebra content in early and middle grades needs to be adapted for young students (Blanton et al., 2015; Cai & Knuth, 2011; Filloy & Rojano, 1984). This idea has given rise to the emergence of the research field "early algebra" (Kieran et al., 2016). One of the most fundamental ideas of early algebra is to facilitate the transition from arithmetic to algebra by focusing on structures and generalizations of arithmetical expressions and relations (Blanton et al., 2015; Kaput, 2008; Linchevski & Herscovics, 1996). This idea is often referred to as generalized arithmetic (Kieran et al., 2016).

Already in the 1980s, Usiskin (1988) introduced the term generalized arithmetic when he was elaborating on the characteristics of school algebra, arguing that it is not possible to study arithmetic sufficiently without algebra. The idea of focusing on arithmetical structures and relations can be understood as reasoning about properties of numbers and operations (e.g., Blanton et al., 2015; Fujii & Stephens, 2001). For instance, let us consider the task "Is the statement $46 + 23 = 47 + 22$ true or false?". One way to solve this problem is to calculate the sums on the left and right hand side of the equal sign, concluding that the statement is true since both additions are equal to 69. Another way of solving the problem is to reason about the numbers and operations involved; if we move 1 from 23 to 46 we get $47 + 22$, that is the same expression as on the right hand side. In the latter case no calculations are needed to solve the problem, the focus is on the arithmetical structure. This example can easily be *generalized* by increasing one term by 2 (3, 4, …) and decreasing the other term by 2 (3, 4, …). The numbers are then used in a way that reminds of the usage of variables, that is, the numbers can be seen as quasi-variables (Carraher et al., 2006). The way of using quasi-variables to understand algebraic generalization is sometimes referred to as a bridge between arithmetic and algebra (Fujii & Stephens, 2001).

Over the years, researchers have come up with frameworks aiming at conceptualizing algebraic thinking and learning (e.g., Bednarz et al., 1996; Kaput, 2008; Usiskin, 1988). In Kaput's (2008) well-known framework it is suggested that algebraic thinking involves two *core practices*: (a) making and expressing generalizations in increasingly formal and conventional symbol systems; and (b) reasoning with symbolic forms. These practices take place across the following three *content strands*: (1) algebra as the study of structures and relations arising in arithmetic (sometimes referred to as generalized arithmetic); (2) algebra as the study of functions; and (3) algebra as the application of a cluster of modelling languages (Kaput, 2008). In addition to generalized arithmetic, the study of functions, symbolic reasoning and modelling languages are highlighted not only in Kaput's framework but also in the frameworks suggested by Bednarz et al. (1996), Blanton et al. (2015) and Usiskin (1988).

Since the 1980s, two different (and sometimes competitive) views on school algebra learning have emerged. The *structural perspective* highlights the importance of developing the abilities to generalize, work abstractly using symbols, and follow procedures in a systematic way (Cai et al., 2010). The content typically includes simplifications of expressions, symbolic manipulations, factoring polynomial and rational expressions, and solving equations by using formal methods (Kieran, 2007). Meanwhile, *the functional perspective* emphasizes the terms change and variation, with the idea of representing various situations by means of relationships between variables (Kieran, 2007). The content is based on contextualized, real world problems, and attempts to solve these problems with methods other than manual symbolic manipulations.

Cai et al. (2010) compared different US curricula in relation to the functional/structural perspective. One of the differences between the curricula was how the variable concept was considered. In curricula with a functional perspective, variables were regarded as quantities that change and were used to represent relationships, and were not formally defined until Grade 7. Meanwhile, from the structural perspective, variables were treated predominantly as placeholders and represented unknowns in expressions and equations. In Grade 6, the variable concept was formally defined as a symbol used to represent a number (Cai et al., 2010). Nevertheless, as Kieran (2007) points out, most curricula take neither a strict functional nor a strict structural perspective.

Research on Early Algebra in Sweden

In the research project where this study is included, some investigations of Swedish school algebra on the formulation arena have already been conducted. Based on three of Blanton et al.'s (2015) big ideas, the algebraic content in the 2011 curriculum document and current Swedish mathematics textbooks for Grades 1–6 were analysed (Bråting et al., 2019). The results showed that the big ideas of functional thinking as well as inequalities, expressions, and equations are well represented in the textbooks. Meanwhile, the big idea of generalized arithmetic is poorly represented. In a comparative study, Hemmi et al. (2021) investigated the algebraic content in the most recent curricula in Estonia, Finland, and Sweden. The result revealed three quite different approaches: The Estonian approach showed influences of the Russian Davydov School while Finland resembled a more traditional approach addressing algebra first at the lower secondary level and then in a formal manner. In the Swedish curriculum, a transition to more formal sophisticated methods at the secondary level was not visible at all (Hemmi et al., 2021). The study in this article will complement these already conducted studies by analysing how the algebraic content has developed over time in Sweden.

There are some additional studies on early algebra in Sweden outside the formulation arena. For instance, in a case study, Madej (2021) investigated Swedish primary school students' knowledge of the equal sign. The results show that the students have good knowledge regarding the meaning of the equal sign, but at the same time have trouble applying this knowledge when solving problems. In another study, Nyman and Kilhamn (2015) analysed how Swedish teachers engage students in algebra. The results indicate that the teachers focused more on activities and social interaction than content related issues. Moreover, Lundberg and Kilhamn (2018) have studied the process of how algebraic "knowledge to be taught" was transposed into "knowledge actually taught", concerning a task on proportional relationships in a grade 6 Swedish classroom. The results showed that modelling word problems about everyday situations is limited and, in their case, made the algebraic problem unsolvable.

Currently, there is an international movement in school development to introduce programming, computational thinking and digital competence (Nouri et al., 2020) in the curriculum. As a result, the Swedish 2011 curriculum document was revised in 2018 and programming was included in the mathematics curriculum with the major part integrated within the core content of algebra (Swedish National Agency of Education, 2018). New concepts such as stepwise instructions, algorithms and programming are now included in the algebra content. The integration of programming in algebra makes the Swedish approach unique compared to other countries (Misfeldt et al., 2020). Researchers find this approach somewhat surprising considering that concepts such as functions and variables can have different meanings in programming and algebra (Bråting & Kilhamn, 2021).

Methodology and Material

We have conducted a qualitative content analysis (Bryman, 2012) of the Swedish mathematics curriculum documents from 1980, 1994, and 2011. By analysing and comparing the algebraic content in the three documents, we will provide valuable knowledge of what constitutes Swedish school algebra in the formulation arena. In this section, we first describe the synchronic and diachronic perspectives that constitute the methodological frame of the study. Then, we briefly characterize the Swedish national curriculum documents from 1980, 1994, and 2011. Finally, we describe the analytical tool and the analysis procedure. In the rest of the paper, the abbreviations Lgr80, Lpo94, and Lgr11 will be used for the three mathematics curriculum documents.

Methodological Frame

This study's analysis uses both a synchronic and a diachronic perspective on the algebraic content. The terms synchrony and diachrony originate from the Swiss linguist Ferdinand de Saussure (1857–1913), who studied semiotic systems. Saussure (1974) suggested that the synchronic and diachronic perspectives are two different, but also complementary, ways of analysing languages. The diachronic approach refers to the development of a language along a time axis, while the synchronic approach considers a language at a specific moment in time, often the present, without taking into account its history.

The synchronic and diachronic perspectives can be used not only on languages but also on other disciplines, such as mathematics. In the same way as languages, mathematics can also be understood as a system of meaning (Fried, 2007). The synchronic and diachronic perspectives can therefore be used in regard to the history of mathematics, for instance in order to better understand the development of specific mathematical concepts (Fried, 2007) or to the history of mathematics education (Pejlare & Bråting, 2021).

In the study reported in this paper, the algebraic content in each curriculum document is investigated separately for the school levels 1–3, 4–6, and 7–9 without taking into account its history or future (the synchronic perspective). In addition, the algebraic content in these three curriculum documents are also compared with each other in order to find similarities and differences within Swedish school algebra (the diachronic perspective). In this way, the use of a diachronic perspective enhances the investigation of the algebraic content from a synchronic perspective. The contrasting effect occurring between different time periods clarifies the algebraic content today as well as how it has changed over the years.

The Material

Lgr80[1]- the Mathematics Curriculum

The 1980 mathematics curriculum consists of three parts: (1) aim, (2) goals, and (3) main content. In the first part, the aim of including mathematics in compulsory school is described in terms of "mathematics can be used to describe the reality" and "students mathematical skill should be built up gradually and the students will gain insight into how to use it" (Swedish National Board for Education, p. 98).

In the second part, two main goals of the mathematics teaching are prescribed. The first goal emphasizes that mathematics teaching should be based on the students' experiences and needs, and should prepare them for their future role as adult citizens. The second goal prescribes that students should also acquire mathematical skills usable for studying other subjects, as well as further studies after primary school.

The third part, the main content, is divided into nine topics, each of which is described for the grade levels 1–3, 4–6, and 7–9. These nine topics consist of problem solving, arithmetic, real numbers, percentage, measurement and units, geometry, algebra and basic functions, descriptive statistics and probability, and computer knowledge. The topic of algebra and basic functions, besides computer knowledge, is given the least space in this section. Moreover, it is pointed out that the topic of algebra and basic functions is less important for everyday life but that students should gain some orientation of the content. In this study only Part 3, the main content, is included in the analysis. We checked that the other two parts did not contain any explicit algebraic content.

Lpo94[2] – the Mathematics Curriculum

The 1994 mathematics curriculum consists of four parts: (1) the aim and role of school mathematics, (2) goals to strive for ("Strävansmål"), (3) the character and structure of mathematics, and (4) goals to achieve ("Uppnåendemål"). In the first part, it is clarified how the subject of mathematics

[1]http://ncm.gu.se/media/kursplaner/grund/LLlgr80.pdf

[2]In this study, the revised version from 2000 (Swedish National Agency for Education, 2000) is analysed, where knowledge criteria for Grades 3, 5, and 9 are added. https://www.skolverket.se/download/18.6bfaca41169863e6a653dcc/1553957116988/pdf745.pdf

contributes to the achievement of the curriculum goals, as well as how the subject of mathematics is motivated based on different social and civic needs.

The second part, goals to strive for, expresses the direction the teaching should have regarding the development of students' knowledge. These goals are written on a general level; that is, no specific topic or concept is mentioned.

The third part, the character and structure of mathematics, considers specific characteristics as well as significant perspectives that can be added to the teaching of mathematics. The fourth part, goals to achieve, indicates the minimum level of knowledge that all students are to achieve in Grades 3, 5, and 9, respectively. Unlike Lgr80, Lpo94 is not divided into specific core contents. However, in Part 4 the goals are formulated with respect to specific content. In this study, only Part 4 is included in the analysis. We checked that the other three parts did not contain any explicit algebraic content.

Lgr11[3]- the Mathematics Curriculum

The 2011 mathematics curriculum consists of the following three parts: (1) introduction to the subject, (2) core content, and (3) knowledge criteria. The first part is the same for all grades (1–9), and includes a historical background of the subject and a description of the aim of school mathematics, which is summarized in terms of five abilities: problem solving ability, conceptual ability, using methods' ability, reasoning ability, and communication ability. Although the terminology used is different, it is clear that the formulation of these five abilities is influenced by the development of theories about mathematical *competencies* during the beginning of the twenty-first century (see Boesen et al., 2014; Niss & Højgaard-Jensen, 2002).

The second and third parts are split between the grade levels 1–3, 4–6, and 7–9. Part 2, the core content, is divided into six topics: number sense and the use of numbers, algebra, geometry, probability and statistics, relationships and change, and problem solving. As mentioned, a major part of the recently included programming content has been integrated within the core content of algebra. Part 3, the knowledge demands, is based on the five mathematical competencies in Part 1 and does not refer to any specific content. In this study only Part 2, the core content, is included in the analysis. We checked that the other two parts did not contain any explicit algebraic content.

The Analytical Tool

In order to characterize the algebraic content, Blanton et al.'s (2015) so-called big ideas of algebra have been used as an analytical tool. The big ideas can be seen as a categorization of algebraic content adapted especially to elementary and compulsory school level, which was an important factor in our decision to use the big ideas. Another important factor was that the big ideas are based on research on algebraic thinking and learning from past decades, covering the time period relevant to our study.

The benefit of the big ideas is that they both "[…] provide rich contexts in which algebraic thinking can occur and they represent central components of algebra as a discipline" (Blanton et al., 2015, p. 44). This is also an important reason why we chose the big ideas as our analytical tool. Below, we briefly describe each big idea and how they have been interpreted in this study:

1. *Variable* (VAR) refers to "symbolic notation as a linguistic tool for representing mathematical ideas in succinct ways and includes the different roles variable plays in different mathematical contexts" (Blanton et al., 2015, p. 43). In algebra, the concept of a variable is broad and can be categorized as *unknown*, *variable* (in a more nuanced meaning), *placeholder,* and *parameter*. In this study the two former categorizations are considered. The term unknown corresponds to a

[3]In this study, the revised version from 2018 is analysed, which includes the implementation of digital tools and programming. https://www.skolverket.se/download/18.6bfaca41169863e6a65d48d/1553968042333/pdf3975.pdf

determinate quantity in an equation that remains to be solved, while variable is meant to correspond to a varying quantity, typically x and y in the equation $y = x^2 + 1$.

2. *Equivalence, expressions, equations, and inequalities* (EEEI) include relational understanding of the equal sign, representing and reasoning with expressions and equations, and relationships between and among generalized quantities (Blanton et al., 2015). An example of a task in this category is to solve the open number sentence $8 + 5 = \_\_ + 4$ and be able to reason based on the structural relationship in the equation. Number sentences such as $8 + 5 = \_\_$ have not been included in this category, as this kind of task involves the ability to calculate.
3. *Generalized arithmetic* (GA) involves reasoning about structures of arithmetic expressions (rather than their computational value) as well as generalizations of arithmetical relationships, which include fundamental properties of numbers and operations (e.g., the commutative property of addition). In this study, this category also includes relationships between operations, such as multiplication defined as repeated addition. Sometimes, the term generalized arithmetic is referred to as the bridge between arithmetic and algebra (Fujii & Stephens, 2001).
4. *Proportional reasoning* (PR) refers to opportunities "to reason algebraically about two generalized quantities that are related in such a way that the ratio of one quantity to the other is invariant" (Blanton et al., 2015, p. 43). In this study, some specific applications of proportional reasoning, such as scaling and similarity, are also included.
5. *Functional thinking* (FT) involves generalizations of relationships between covarying quantities, and representations and reasoning with relationships through natural language, algebraic notation, tables, and graphs (Blanton et al., 2015). This can mean generating linear data and organizing it in a table, identifying recursive patterns and function rules and describing them in words and using variables, as well as using a function rule to predict far function values.

Procedure of Analysis

First, a pilot study was conducted to test the unit of analysis and the interpretation of the categories within the big ideas (Bråting, 2019). The pilot study was limited to include the two curricula Lgr80 and Lgr11 but without the implementation of programming content in the revised version of Lgr11. The results were discussed at the conference ESU8 (*Eighth European Summer University of History and Epistemology in Mathematics Education*). Both the unit of analysis (see below) as well as the usage of the big ideas as an analytical tool turned out to be suitable. However, in order to continue with the broader study a methodological frame as well as more research studies from the 1980s and 1990s needed to be included.

The unit of analysis was a statement or part of a statement that addressed an issue connected to at least one of the big ideas. Most of the statements were connected to only one big idea, but a few suited two big ideas. For instance, the statement "Linear functions, especially those that indicate proportionality" (Swedish National Board for Education, 1980, p. 106) was categorized as both FT and PR.

The results of the categorization were recorded in five tables (one for each big idea) for each curriculum; that is, 15 tables altogether. Each table was divided into the three grade levels 1–3, 4–6, and 7–9, and consisted of all the statements connected to a specific big idea. In a few cases, it was not completely clear whether or not a statement represented a certain big idea. Some statements connected to the decimal system, such as "natural numbers and their properties as well as how the numbers can be divided and how they can be used to indicate quantity and order" (Swedish National Agency for Education, 2018, p. 55), were excluded because they were considered to belong to the development of number sense rather than algebra. In these cases the interpretation had to be reconsidered, which led to minor corrections.

After this procedure, the statements from the three curricula were compared for each big idea. In order to do this, the five times three tables from Lgr80, Lpo94, and Lgr11 were merged into five tables, one for each big idea. In each table, the horizontal axis represents the synchronic perspective; that is, the progression between the grade spans 1–3, 4–6, and 7–9. Meanwhile, the vertical axis represents the diachronic perspective; that is, the three curriculum documents from 1980, 1994, and 2011 (see Tables 1–5). Through this process, specific features and gaps within and between each of the three curricula were identified.

Although programming is included in the core content of algebra in Lgr11, it was not possible for us to classify it as any of the big ideas. However, we have presented the programming content separately at the end of the *Results* section.

Results

The results of the analysis are presented separately for each big idea. Every section begins with a table showing the authentic expressions identified for each grade span in the three curriculum documents.

VAR – Variable

Table 1 reveals that the big idea VAR appears in earlier grade levels over time; in Lgr80 it first appears in Grades 7–9, while in the two more recent curricula it appears already in Grades 4–6. We can also see that Lgr80 and Lpo94 only address the term unknown number, while Lgr11 addresses unknown number as well as variable. There is a progression in Lgr11 whereby students are to be introduced to unknown numbers in Grades 4–6 and thereafter to variables in Grades 7–9. Before 2011, the term variable was not introduced until upper secondary school in Sweden.

We also note a difference in how the concepts are addressed in the three curricula. While Lgr80 and Lpo94 prescribe unknown numbers in connection with equations and the determination of unknown numbers in simple formulas, Lgr11 addresses *properties* of unknown numbers and *the meaning* of the variable concept. In Tables 2–5 below, we return to differences regarding how mathematical concepts are addressed in the three curricula.

EEEI – Equivalence, Expressions, Equations, and Inequalities

Table 2 shows that EEEI is represented in all three curricula already in the earliest grade span. However, algebraic expressions have been moved down the grades over time. In Lgr80 algebraic expressions are introduced in Grades 7–9, while in Lgr11 they appear already in Grades 4–6. Lpo94 does not mention algebraic expressions; instead, it addresses unknown numbers in simple formulas beginning in Grades 4–6.

As in Table 1, there is a difference regarding how concepts are addressed in the three curricula. Let us first consider Grades 1–3 and 4–6. Lgr80 emphasizes *solving* simple equalities/equations by trial and error and on the basis of problems, while Lpo94 prescribes *handling* mathematical

Table 1. Distribution of content categorized as VAR with respect to the grade levels 1–3, 4–6, and 7–9.

	Grades 1–3	Grades 4–6	Grades 7–9
Lgr80			First-order equations, including unknowns on both sides of the equality sign and with parentheses and fractional numbers.
Lpo94		Determine unknown numbers in simple formulas.	
Lgr11		Unknown numbers and their properties. Situations in which there is a need to represent an unknown number with a symbol.	The meaning of the variable concept and its use in algebraic expressions, formulas, and equations.

Table 2. Distribution of content categorized as EEEI with respect to the grade levels 1–3, 4–6, and 7–9.

	Grades 1–3	Grades 4–6	Grades 7–9
Lgr80	Solving simple equalities by trial and error.	Solving simple equations mainly by trial and error and on the basis of problems.	Setting up, simplifying, and calculating algebraic expressions. Treatment of parenthetical expressions, factorization, and identities of binomial squares, based on maturity and interest. First-order equations, including unknowns on both sides of the equality sign and with parentheses and fractional numbers. Problem solving with simple equations. Systems of linear equations and simple second-order functions mainly in connection with problem solving and graphical solutions.
Lpo94	Handling mathematical equalities for integers between 0 and 20.	Determining unknown numbers in simple formulas.	Interpreting and applying simple formulas. Solving simple equations.
Lgr11	Mathematical equalities and the importance of the equal sign.	Methods for solving simple equations. Simple algebraic expressions and equations relevant to students.	Algebraic expressions, formulas, and equations in situations relevant to the students. Methods for solving equations.

equalities. Meanwhile, Lgr11 addresses *methods* of solving equations without pinpointing the character of the methods. Moreover, in Grades 1–3, Lgr11 introduces the term equality along with the *importance of the equal sign*, which is not mentioned in the other two curricula.

Let us continue to focus on the differences of addressing concepts in the three curricula. In Grades 7–9, equations and algebraic expressions are addressed in all three curricula. However, Lgr80 includes more detailed descriptions of the concepts than the other two curricula and emphasizes computational aspects such as *setting up*, *simplifying*, and *calculating* algebraic expressions. Meanwhile, Lpo94 refers to *interpreting* and *applying* different formulas, and Lgr11 again refers to *methods* for solving equations but does not pinpoint the character of the methods.

Finally, Lgr80 and Lgr11 emphasize that the mathematical content should be relevant to students and connected to their interests. Lgr80 also states that students' maturity and needs should be taken into account. These kinds of statements are not included in Lpo94.

GA – Generalized Arithmetic

As Table 3 reveals, GA is poorly represented in all three curricula. In fact, in Lgr80 we found no content connected to generalized arithmetic. One item was found in Lpo94 and Lgr11: in Grades 1–3, the relationships between the four arithmetical operations are included. However, there is no progression in Lpo94 or Lgr11 with respect to generalized arithmetic.

PR – Proportional Reasoning

Table 4 reveals that PR is represented in all three curricula already in the earliest grade span. We found no PR-related content in Grades 4–6 in Lpo94, but it should be mentioned that percentage

Table 3. Distribution of content categorized as GA with respect to the grade levels 1–3, 4–6, and 7–9.

	Grades 1–3	Grades 4–6	Grades 7–9
Lgr80			
Lpo94	Relationships between the four arithmetic operations by means of, e.g., concrete material or images.		
Lgr11	Properties and relationships between the four arithmetical operations and their use in different situations.		

Table 4. Distribution of content categorized as PR with respect to the grade levels 1–3, 4–6, and 7–9.

	Grades 1–3	Grades 4–6	Grades 7–9
Lgr80	Simple and practical examples of enlargements and reductions, for instance in connection with maps and handicraft objects.	Treatment of percentage in connection with practical problems and other school subjects. Calculations with percentages. Relationships between fractions, decimals, and percentages. Treatment of scale in everyday life.	Calculations with percentages, parts, and the whole. Usage of scale, mainly in practical contexts. Treatment of congruence and uniformity. Linear functions, especially those that indicate proportionality.
Lpo94	Compare and numerically rank integers between 0–1,000. Describe, compare, and name simple fractions.		Calculations with decimal numbers and percentage. Proportionality by means of mental arithmetic, written calculations, and technical equipment.
Lgr11	Different proportional relationships, including doubling and halving. Apply and give examples of simple proportional relations in situations relevant to students. Scale with simple enlargements and reductions.	Proportions, percentage, fractions and decimal numbers, and their relationships. Graphs for expressing different types of proportional relations in simple investigations. Scale and the use of scale in situations relevant to students.	Percentage to express change (factor). Calculations with percentage in everyday situations. Scale, enlargement and reduction of 2- and 3-dimensional objects. Uniformity in the plane.

Table 5. Distribution of content categorized as FT with respect to the grade levels 1–3, 4–6, and 7–9.

	Grades 1–3	Grades 4–6	Grades 7–9
Lgr80		The function concept is introduced through practical experiments. Interpretations of simple functions in the first quadrant in a coordinate system. Calculations of function values by inserting them into formulas, connected to everyday life or other school subjects.	Interpretations and constructions of graphs in the whole coordinate system. Linear functions, especially those that indicate proportionality.
Lpo94	Describe patterns in simple number sequences. Continue and construct simple geometric patterns.	Discover number patterns. Describe some properties of geometric patterns.	Interpret and use graphs of functions that describe real-life conditions.
Lgr11	How simple patterns in number sequences and simple geometrical forms can be constructed, described, and expressed.	How patterns in number sequences and geometrical patterns can be constructed, described, and expressed. The coordinate system and strategies for scaling coordinate axes. Tables and graphs.	Functions and linear equations. How functions can be used to investigate change, rate of change, and other relationships.

is included in the "goals to strive for" part, which was not included in our analysis. As in the previous tables, we can see that some concepts have been moved down the grades over time. In Lgr80, the term scale first appears in Grades 4–6, while in Lgr11 it appears already in Grades 1–3. Furthermore, proportional relationships are included already in Grades 1–3 and 4–6 in Lgr11, while in Lgr80 and Lpo94 the term proportion(al) first appears in Grades 7–9. It is also notable that the term relationship is used more in Lgr11 compared to the other two curricula.

As in Tables 1 and 2, there are differences in how concepts are addressed in the three curricula. While Lgr80 and Lpo94 emphasize *calculations* with percentages, Lgr11 also connects percentage to *express change*. In Lgr11, the word express is also used in connection with graphs, which are prescribed as a way of *expressing* proportional relations. In Lgr80, the word treatment is repeatedly addressed in connection with percentage, scale, congruence, and uniformity. Treatment is not used in the other two curricula. Finally, Lpo94 uses the words describe and compare in connection with fractions.

The connection between the mathematical content and students' interests across all grades (1–9) are pointed out in both Lgr80 and Lgr11. Lgr11 uses the phrase "in situations relevant to students", while Lgr80 uses the terms "practical contexts" and "everyday life". In Lgr80, the connection to other school subjects is also mentioned. Lpo94 is the only curriculum that mentions technical equipment in this category.

FT – Functional Thinking

As Table 5 reveals, the big idea FT is not represented at all in Grades 1–3 in Lgr80, while Lpo94 and Lgr11 include patterns in Grades 1–3. The introduction of the coordinate system and graphs has moved down the grades over time, from Grades 7–9 in Lgr80 and Lpo94 to Grades 4–6 in Lgr11. Regarding the function concept, we note that the movement over time is the opposite compared to our previous tables. The function concept is introduced already in Grades 4–6 in Lgr80, but not until Grades 7–9 in Lpo94 and Lgr11. However, it is possible that the purpose of including patterns in Grades 1–3 in Lpo94 and Lgr11 is to introduce the fundamental ideas of the function concept. We return to this issue in the *Discussion* section.

As in the previous tables, there are differences in how concepts are prescribed in the three curricula. Compared to the other two curricula, Lgr80 includes more detailed descriptions of what kinds of functions are intended, as well as how they are to be used. Lgr80 prescribes computational aspects such as *calculations* of function values, while Lgr11 prescribes how functions can be used to *investigate* rate of change and other relationships. Finally, Lpo94 and Lgr11 repeatedly use the word describe in connection with different patterns.

The connection to everyday life, practical experiments, and other school subjects is pointed out in Lgr80. Lpo94 highlights real-life conditions, while Lgr11 does not include these kinds of statements in this category.

The Inclusion of Programming

Finally, it is important to mention that the new content connected to programming, included within the core content of algebra in the revised version of Lgr11, was not possible to classify as any of the five big ideas. This content consists of:

(1) How unique stepwise instructions can be constructed, described, and followed as a basis for programming (Grades 1–3);
(2) The use of symbols in connection with stepwise instructions (Grades 1–3);
(3) How algorithms can be created and used in programming (Grades 4–6 and 7–9);
(4) Programming in visual/different programming environments (Grades 4–6 and 7–9).

Discussion

The results are discussed in relation to the research question and previous research considered earlier in this article. The discussion is centred around the following findings: (1) the earlier introduction of algebra in Swedish curriculum documents over time, (2) the low representation of GA in all three curriculum documents (3) the movement from a structural towards a functional approach, (4) the higher emphasis on conceptual and verbal skills over time, (5) the great emphasis on everyday mathematics, and (6) the recent inclusion of programming in the algebra content.

Our results clearly confirm that algebra has come to be introduced earlier in the curriculum documents over time, which reflects the international trend of integrating algebra in school mathematics already in primary school (Kieran et al., 2016). In our study, an exception to this movement seems, at first, to be the function concept, which is addressed earlier in Lgr80 compared to the two more recent curricula. However, a closer look at this result reveals that Lgr11 prescribes patterns already in Grades 1–3, which are often used in early algebra as a first step in introducing the fundamental ideas behind the function concept (Carraher et al., 2006). Within the scope of this article it is not possible to answer whether this is the intention in Lgr11, but the statement is clearly supported by our previous study of recent Swedish elementary textbooks, which shows that functional thinking with respect to patterns is a common topic in the textbooks for Grades 1–3 (Bråting et al., 2019).

Although algebra has moved down the grades during the time period, the results reveal that the representation of the big ideas differ. For instance, the big idea EEEI is well-represented in the three curriculum documents but has come to be introduced in earlier grades through the years, while GA is virtually absent in each curriculum. This absence is perhaps the most important insight from this study, especially considering that GA is emphasized in several frameworks (e.g., Bednarz et al., 1996; Kaput, 2008; Usiskin, 1988) and research studies (Blanton, 2015; Cai et al., 2011; Filloy & Rojano, 1984; Linchevski & Herscovics, 1996) on algebra learning from different time periods. In fact, the terms “generalize” and “generalization” do not appear in any of the three curriculum documents included in this study. This is surprising, considering that generalizing is central in all topics in mathematics and constitutes a bridge between arithmetic and algebra (Fujii & Stephens, 2001). It is interesting to note that in Finland and Estonia, both neighbouring countries to Sweden with educational systems similar to Sweden’s, GA is a well-represented topic in recent curriculum documents throughout all grade levels (Hemmi et al., 2021). Considering that Sweden has not yet detected any positive effect of the various attempts to improve school algebra teaching and that Estonia and Finland have considerably higher results on international evaluations in algebra than Sweden (Hemmi et al., 2021), more emphasis on generalized arithmetic could be one possible way to improve the Swedish situation.

In contrast to the absence of GA throughout the years, our results show that FT and PR are represented in all three curricula, but have come to be introduced in earlier grades over the years. We also note that the function concept is applied differently in the various curricula. The Lgr80 document prescribes *calculations* of function values, while Lgr11 emphasizes the use of functions in investigating different *relationships*. One reason for this difference is likely that “Relationships and change” constitutes a new, separate category of the mathematical content in Lgr11 (see *Methodology* section). This is apparent not only in connection with the function concept, but is also visible in connection with PR, whereby both relations and change are emphasized considerably more in Lgr11 compared to the previous curricula. We can also note that the variable concept is introduced in earlier grades in Lpo94 and Lgr11 compared to Lgr80. The emphasis on relations and variables is likely an effect of a recent international trend by which “Relationships and change” has been identified as one of the four broad mathematical content categories in the PISA framework for school mathematics (OECD, 2010). In terms of the distinction between the structural and functional perspective of algebra learning (Cai et al., 2010; Kieran, 2007), this result indicates a movement towards the functional perspective over the years. The movement becomes even more visible considering that the prescription of skills such as calculating, simplifying and manipulating with symbols, all representative of the structural approach, have decreased considerably during the years. Again it is interesting to compare with the recent curricula in Estonia and Finland, which both emphasize calculating and manipulating with symbols within several of the big ideas (Hemmi et al., 2021).

In contrast with the lower emphasis on calculations and manipulations with symbols during the years, the results show that conceptual and verbal skills have gained increased attention over time. This is likely a consequence of the inclusion of abilities (Boesen et al., 2014) in Lgr11, whereby for instance reasoning, communicating, and conceptual understanding are highlighted (Swedish National Agency of Education, 2018). In this study, we can draw no conclusions as to how this may have affected students' algebraic knowledge. However, there are studies showing that the inclusion of abilities has caused problems; some teachers find it difficult to identify the meaning of the competence message and it is claimed to be vague and formulated in complex wording (Bergqvist & Bergqvist, 2017; Boesen et al., 2014, 2018). It is also interesting to note that the Swedish students in Madej's (2021) study could describe the meaning of the equal sign but had great problems operationalizing this knowledge when solving problems. This indicates that the balance between conceptual and verbal skills on the one hand and skills such as calculating and manipulating with symbols on the other needs to be clarified in future Swedish curriculum documents.

Let us move on to discuss the great emphasis on practical and everyday mathematics in the three curricula. In a historical perspective, the increased attention of practical and everyday mathematics in Lgr80 has been viewed as a reaction to the New Math failure during the 1970s (Skovsmose, 1990). In Lgr80 it is prescribed that mathematics teaching should be based on students' experiences and needs, and in particular, the topic of algebra and basic functions is less important for everyday life and students should therefore only gain some orientation of the content (see the *Methodology* section; Swedish National Board for Education, 1980). Within the research field of early algebra, practical and everyday mathematics are not emphasized in any greater extent, for instance, it is not highlighted at all in Kaput's (2008) or Usiskin's (1988) frameworks on algebra learning, nor in recent published overviews of the research field early algebra (Kieran et al., 2016; Kieran, 2018). In fact, Lundberg and Kilhamn's (2018) results reveal that problems about everyday situations in algebraic contexts certainly have limitations and can even make an algebraic problem unsolvable. Moreover, it is interesting to take into account that in comparison with the recent Finnish curriculum document, the Swedish curriculum has significantly more emphasis on everyday mathematics (Hemmi & Ryve, 2015). We find it somewhat surprising that the great emphasis of practical and everyday mathematics within the Swedish curriculum's core content of algebra has remained over the years. Based on the last decade's research on early algebra, we doubt that a high emphasis on everyday mathematics is a successful way to improve students' algebraic skills.

Finally, let us consider the programming content implemented in Lgr11. Even though it is included in the core content of algebra, it was not possible for us to classify this content as any of the big ideas. As far as we know, programming is not highlighted in any research literature within the field of early algebra (e.g., Kieran, 2018; Kieran et al., 2016). We do not know why programming has been integrated with algebra in the Swedish curriculum, but an interesting future study would be to investigate the political background to this decision. Bearing in mind that algebra has been a problematic topic for Swedish students in several decades (Hemmi et al., 2021), it is difficult to see any reason for including a whole new topic within the core content of algebra. Especially, considering that fundamental concepts such as variables, algorithms and equality often have different meanings in programming compared to algebra, which may cause confusion for the students (Bråting & Kilhamn, 2021). Perhaps the newly awoken interest in algorithms may encourage studies of the structures embedded in algorithms, and as such afford the development of algebraic thinking in terms of increased focus on structure and generalizations? Even if that is a bit far-fetched, it could be a possible way of introducing generalized arithmetic in Swedish school algebra. However, we do believe that there are more convenient ways of emphasizing generalizations in the core content of algebra in the Swedish curriculum that instead follow the results of the last decades' research on early algebra.

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References

Bednarz, N., Kieran, C., & Lee, L. (1996). Approaches to algebra: Perspectives for research and teaching. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 3–12). Kluwer.

Bergqvist, E., & Bergqvist, T. (2017). The role of the formal written curriculum in standards-based reform. *Journal of Curriculum Studies*, *49*(2), 149–168. https://doi.org/10.1080/00220272.2016.1202323

Bergsten, C., Häggström, J., & Lindberg, L. (1997). *Algebra för alla*. NCM.

Blanton, M., Stephens, A., Knuth, E., Murphy Gardiner, A., Isler, I., & Kim, J. (2015). The development of children's algebraic thinking: The impact of a comprehensive early algebra intervention in third grade. *Journal for Research in Mathematics Education*, *46*(1), 39–87. https://doi.org/10.5951/jresematheduc.46.1.0039

Boesen, J., Helenius, O., Bergqvist, E., Bergqvist, T., Lithner, J., Palm, T., & Palmberg, B. (2014). Developing mathematical competence: From the intended to the enacted curriculum. *The Journal of Mathematical Behavior*, *33*, 72–87. https://doi.org/10.1016/j.jmathb.2013.10.001

Boesen, J., Lithner, J., & Palm, T. (2018). Assessing mathematical competencies: An analysis of Swedish national mathematics tests. *Scandinavian Journal of Educational Research*, *62*(1), 109–124. https://doi.org/10.1080/00313831.2016.1212256

Brandell, G., Hemmi, K., & Thunberg, H. (2008). The widening gap – a Swedish perspective. *Mathematics Education Research Journal*, *20*(2), 38–56. https://doi.org/10.1007/BF03217476

Bråting, K. (2019). Development of school algebra – a comparison between the Swedish curricula from 1980 and 2011. In U. T. Jankvist, T. Tzanakis, & B. Smestad (Eds.), *Eighth European summer university of history and epistemology in mathematics education, ESU8* (pp. 711–725). Oslo Metropolitan University.

Bråting, K., & Kilhamn, C. (2021). Exploring the intersection of algebraic and computational thinking. *Mathematical Thinking and Learning*, *23*(2), 170–185. https://doi.org/10.1080/10986065.2020.1779012

Bråting, K., Madej, L., & Hemmi, K. (2019). Development of algebraic thinking: Opportunities offered by the Swedish curriculum and elementary mathematics textbooks. *Nordic Studies in Mathematics Education*, *24*(1), 27–49.

Bryman, A. (2012). *Social research methods*. Oxford University Press.

Cai, J., & Knuth, E. (Eds.). (2011). *Early algebraization: A global dialogue from multiple perspectives*. Springer.

Cai, J., Nie, B., & Moyer, J. C. (2010). The teaching of equation solving: Approaches in *standards*-based and traditional curricula in the United States. *Pedagogies: An International Journal*, *5*(3), 170–186. https://doi.org/10.1080/1554480X.2010.485724

Carraher, D. W., Schliemann, A., Brizuela, B., & Earnest, D. (2006). Arithmetic and algebra in early mathematics education. *Journal for Research in Mathematics Education*, *37*(2), 87–115. https://doi.org/10.2307/30034843

Filloy, E., & Rojano, T. (1984). From an arithmetical thought to an algebraic thought. In J. Moser (Ed.), *Proceedings of PME-NA Vol. VI* (pp. 51–56). Madison.

Fried, M. N. (2007). Didactics and history of mathematics: Knowledge and self-knowledge. *Educational Studies in Mathematics*, *66*(2), 203–223. https://doi.org/10.1007/s10649-006-9025-5

Fujii, T., & Stephens, M. (2001). Fostering an understanding of algebraic generalisation through numerical expressions: The role of quasi-variables. In H. Chick, K. Stacey, & J. Vincent (Eds.), *Proceedings of ICMI-12* (pp. 258–264). Melbourne University.

Hemmi, K., Bråting, K., & Lepik, M. (2021). Curricular approaches to algebra in Estonia, Finland and Sweden – a comparative study. *Mathematical Thinking and Learning*, *23*(1), 49–71. https://doi.org/10.1080/10986065.2020.1740857

Hemmi, K., & Ryve, A. (2015). Effective mathematics teaching in Finnish and Swedish teacher education discourses. *Journal of Mathematics Teacher Education*, *18*(6), 501–521. https://doi.org/10.1007/s10857-014-9293-4

Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710. https://doi.org/10.1007/s13394-013-0097-0

Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (pp. 5–18). Lawrence Erlbaum.

Kieran, C. (2007). Learning and teaching algebra at the middle school through college levels. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 707–762). Information Age Publishing.

Kieran, C. (Ed.). (2018). *Teaching and learning algebraic thinking with 5- to 12-year-olds*. Springer.

Kieran, C., Pang, J., Schifter, D., & Ng, S. F. (2016). *Early algebra. Research into its nature, its learning, its teaching*. Springer Open.

Linchevski, L., & Herscovics, N. (1996). Crossing the cognitive gap between arithmetic and algebra: Operating on the unknown in the context of equations. *Educational Studies in Mathematics*, *30*(1), 39–65. https://doi.org/10.1007/BF00163752

Lindensjö, B., & Lundgren, U. P. (2000). *Utbildningsreformer och politisk styrning* [Educational reforms and political governance]. HLS Publisher.

Lundberg, A. L. V., & Kilhamn, C. (2018). Transposition of knowledge: Encountering proportionality in an algebra task. *International Journal of Science and Mathematics Education*, *16*(3), 559–579. https://doi.org/10.1007/s10763-016-9781-3

Madej, L. (2021). Primary school students' knowledge of the equal sign – the Swedish case. *International Journal of Science and Mathematics Education*. https://doi.org/10.1007/s10763-020-10144-z

Misfeldt, M., Jankvist, U. T., Geraniou, E., & Bråting, K. (2020). Relations between mathematics and programming in school: Juxtaposing three different cases. In A. Donevska-Todorova, E. Faggiano, J. Trgalova, Z. Lavicza, R. Weinhandl, A. Clark-Wilson, & H.-G. Weigand (Eds.), *Proceedings of the 10th ERME topic conference on mathematics education in the digital era (MEDA 2020)* (pp. 255–262). Johannes Kepler University.

Niss, M., & Højgaard-Jensen, T. (2002). *Kompetencer och matematiklæring* [Competencies and mathematical learning]. Uddannelseministeriet.

Nouri, J., Zhang, L., Mannila, L., & Norén, E. (2020). Development of computational thinking, digital competence and 21st century skills when learning programming in K-9. *Education Inquiry*, *11*(1), 1–17. https://doi.org/10.1080/20004508.2019.1627844

Nyman, R., & Kilhamn, C. (2015). Enhancing engagement in algebra: Didactical strategies implemented and discussed by teachers. *Scandinavian Journal of Educational Research*, *59*(6), 623–637. https://doi.org/10.1080/00313831.2014.965790

Organization for Economic Co-operation and Development (OECD). (2010). *Draft PISA 2012 mathematics framework*. OECD Publishing.

Pejlare, J., & Bråting, K. (2021). Writing the history of mathematics: Interpretations of the mathematics of the past and its relation to the mathematics of today. In B. Sriraman (Ed.), *Handbook of the mathematics of the arts and sciences* (pp. 2395–2420). Springer.

Prendergast, M., & Treacy, P. (2018). Curriculum reform in Irish secondary schools – a focus on algebra. *Journal of Curriculum Studies*, *50*(1), 126–143. https://doi.org/10.1080/00220272.2017.1313315

Saussure, F. (1974). *Cours de Linguistique Générale* (C. Bally & A. Sechehaye, Eds.). Payot.

Skovsmose, E. (1990). Perspectives on curriculum development in mathematical education. *Scandinavian Journal of Educational Research*, *34*(2), 151–168. https://doi.org/10.1080/0031383900340206

Swedish National Agency for Education. (2000). *Grundskolan. Kursplaner och betygskriterier* [Compulsory school. Syllabi and grading criteria]. Revised version of Lpo94. Fritzes.

Swedish National Agency for Education. (2016). *TIMSS 2015 – huvudrapport. Svenska grundskoleelevers kunskaper i matematik och naturvetenskap i ett internationellt perspektiv* [TIMSS 2015 – main report. Swedish primary school students' knowledge of mathematics and science in an international perspective]. Kluwer.

Swedish National Agency for Education. (2018). *Läroplan för grundskolan, förskoleklassen och fritidshemmet, Lgr11* [Syllabus for compulsory school, preschool class and after-school center]. Revised version of Lgr11. Elanders.

Swedish National Board for Education. (1980). *Läroplan för grundskolan* [Syllabus for compulsory school]. Liber.

Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. Coxford & A. Shuite (Eds.), *The ideas of algebra, K-12. 1988 yearbook* (pp. 8–19). National Council of Teachers of Mathematics.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book*. Kluwer.

Yang Hansen, K., Gustafsson, J.-E., Rosén, M., Sulkunen, S., Nissinen, K., Kupari, P., Ólafsson, R. F., Björnsson, J. K., Grønmo, L. S., Rønberg, L., Mejding, J., Borge, I. C., & Hole, A. (2014). *Northern lights on TIMSS and PIRLS 2011: Differences and similarities in the Nordic countries*. Nordic Council of Ministers.