Efficient Large-eddy Simulation for Wind Energy Applications

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Abstract

Modelling the interaction of wind turbines with the ambient flow is essential for almost all technical aspects of wind energy exploitation. Large-eddy simulation (LES) is the most detailed approach feasible to model this complex interaction of wind turbines with the atmospheric boundary layer and the wakes of upstream turbines. Despite more than twenty years of fundamental research on wind turbine modelling with LES, applications of the method remain limited to academic use cases to date. The main bottleneck hindering a broader adoption of LES in the industrial practice is the large computational demand of the method. Nevertheless, it holds enormous potential for addressing various modelling challenges arising from current trends in wind energy.

A promising alternative to classical numerical approaches for LES is the lattice Boltzmann method (LBM). In particular, GPU-based (graphics processing unit) implementations of the method provide significant performance gains and have enabled unprecedented computational efficiencies for LES in different fields of fluid dynamics. Still, the LBM’s potential for wind energy applications remains untapped due to open questions, some of which are specific to the field. This thesis addresses two specific problems in applications of LES to wind turbine and farm simulations. First, is the representation of wind turbines with the actuator line technique. And, second, is the modelling of the surface shear stress in simulations of atmospheric boundary layers. Both aspects are crucial to enable LES for wind energy applications with the LBM, as is usually done with conventional approaches.

As for the former, an LBM implementation of the actuator line model is applied in multiple studies on wind turbine wakes. Code-to-code comparisons and experimental validations show that the model can accurately capture the aerodynamic forces acting on the turbine blades as well as the wake characteristics. For the simulation of boundary layer flows a novel LBM-specific wall model is developed. The model, referred to as inverse momentum exchange method, imposes the surface shear stress at the first offwall grid points by adjusting the slip velocity in bounce-back boundary schemes. Simulations are compared to theoretical, numerical, and experimental reference data of isothermal boundary layer flows. It is consistently found that both mean quantities and higher-order turbulence statistics can be well-captured by wall-modelled lattice Boltzmann LES using the presented wall model and the employed cumulant collision scheme.

The results presented illustrate that the LBM is a suitable approach for state-of-the-art LES of wind turbine wakes and boundary layer flows. Moreover, the applied method is shown to be robust, and, above all, extremely computationally efficient. Based on the observed computational efficiencies, it is concluded that industry LES for wind energy applications is possible with GPU-based LBM solvers. Furthermore, additional studies presented in this thesis illustrate further potentials of the method. Such are applications of reinforcement learning to wind farm control or large-scale data generation for the training of deep learning models for wake predictions.

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URN urn:nbn:se:uu:diva-468558 (http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-468558)
You cannot ban the wind, but you can build wind turbines.

Supposedly Dutch proverb
List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.


Reprints were made with permission from the publishers. In part II of the printed version of the report the papers are included in full text. The appearance of the papers has been adjusted to the format of the thesis.

**The following publications are not included in the thesis:**


Division of the work among authors:

This work provides an initial presentation and analysis of the actuator line model in a lattice Boltzmann framework. The development and implementation of the actuator line model was done by Henrik Asmuth (HA). Hugo Olivares-Espinosa (HOE), Karl Nilsson (KN) and Stefan Ivanell (SI) provided guidance on the choice of test cases and data analysis. The simulations were set-up and analysed by HA. The manuscript was written by HA with input from HOE, KN and SI.

Paper II - Actuator Line Simulations of Wind Turbine Wakes Using the Lattice Boltzmann Method
In this study the wake characteristics of the lattice Boltzmann actuator line model were compared against a finite volume Navier-Stokes implementation in both laminar and turbulent inflow. For this work, HA implemented the parametrisation of the cumulant collision model, the Smagorinsky sub-grid-scale model and the turbulent inflow boundary condition. The simulations were planned, set-up and analysed by HA. HOE contributed to the analysis and discussion of the results. The manuscript was written by HA with input from HOE and SI.

Paper III - Assessment of weak compressibility in actuator line simulations of wind turbine wakes
This work was concerned with the analysis of compressibility effects on the wake and aerodynamic forces of the actuator line model in lattice Boltzmann frameworks. HA and Christian Janßen (CJ) formulated the underlying research questions. HA performed the simulations and data analysis. The manuscript was written by HA with input from CJ, HOE, KN and SI.

Paper IV - Exploring the application of reinforcement learning to wind farm control
This paper summarises the key findings of the master’s thesis by Henry Korb (HK) supervised by HA and SI. The initial research idea was formulated by HA. HK developed and implemented the coupling of the reinforcement learning model and the LES framework, and performed the simulations and data analysis with input from HA and Merten Stender (MS). The final paper was written by HK and HA and reviewed by MS and SI.
**Paper V - Wall-modelled Lattice Boltzmann Large-eddy Simulation of Neutral Atmospheric Boundary Layers**

This work discusses a novel wall-modelling approach for Lattice-Boltzmann Large-eddy simulation. HA developed the concept for the new wall-model with input from CJ. The implementation, simulations and data analysis were done by HA. The final paper was conceptualised and written by HA and reviewed by CJ, HOE, and SI.

**Paper VI - Wind Turbine Response in Waked Inflow: A Modelling Benchmark Against Full-scale Measurements**

This paper summarises the findings of an international modelling benchmark organised through the International Energy Agency. The overall benchmark study was conceptualised and administered by HA, KN and SI. HA and KN selected and processed the experimental reference data with the help of Helge Aa. Madsen (HM), Emmanuel Branlard (EB) and HA generated and processed the synthetic inflow data. HA, Gonzalo Navarro (GN), HM, EB, and Alexander R. Meyer-Forsting (AMF) performed the respective simulations of the participating institutions. HA post-processed and analysed the simulation data of the participating models. The paper was written by HA with input from EB, GN, HM, AMF and Jason Jonkman.

**Paper VII - Towards a complete model chain for ice accretion effects on wind turbines**

This paper discusses the impact of blade icing on the performance of wind turbines modelled by a chain of four sub-models. The study was conceptualised by Johan Revstedt (JR), SI and Heiner Körnich. The meteorological simulations were set-up and run by Metodija Shapkalije (MS). The model for the ice accretion and airfoil simulations was developed and run by JR and Robert Szász (RS). HA set-up and ran the BEM and LES model of the full turbine. JR and HA post-processed and analysed the data. The paper was written by JR and HA with input from RS, MS, and SI.

**Paper VIII - WakeNet 0.1 - A Simple Three-dimensional Wake Model Based on Convolutional Neural Networks**

This paper presents a surrogate model for wind turbine wakes based on convolutional neural networks, trained with LES-generated data. HA and HK developed the concept and implemented the model and data processing pipeline. HK set-up and ran the LES training cases. HA and HK post-processed and analysed the data. The final paper was written by HA with input from HK.
# Contents

Part I: Comprehensive Summary ................................................................. 11

1  Introduction ............................................................................................. 13
   1.1 Current Challenges in Wind Farm Modelling .......................... 14
   1.2 Large-eddy Simulation of Wind Farms ................................. 18
   1.3 Potentials of Lattice Boltzmann Methods for Wind Energy
       Applications .................................................................................. 20
   1.4 Aim of This Work ........................................................................ 22
   1.5 Outline ......................................................................................... 22

2  The Lattice Boltzmann Method ................................................................. 23
   2.1 Fundamentals .............................................................................. 23
   2.2 The Cumulant Collision Model .............................................. 27
   2.3 Boundary Conditions ................................................................ 30
   2.4 Sub-grid Scale Modelling ............................................................ 31

3  Actuator Line Simulations ...................................................................... 34
   3.1 Implementation of the Actuator Line Model ............................ 35
   3.2 Sensitivities of the Aerodynamic Forces ............................... 36
   3.3 Analysis of the Wake Characteristics ....................................... 39
   3.4 Aspects of Compressibility ......................................................... 44
   3.5 Validation Against Full-scale Measurements ......................... 51

4  Wall-modelled Boundary Layer Simulations ...................................... 62
   4.1 Lattice Boltzmann Wall Modelling ......................................... 63
   4.2 Estimating the Wall Shear Stress ............................................ 66
   4.3 Numerical Set-up and Case Description ................................ 68
   4.4 Impact of the Wall Shear Stress Model ................................. 68
   4.5 Grid Sensitivity ......................................................................... 71
   4.6 Concluding Remarks .................................................................. 78

5  Applications ............................................................................................ 79
   5.1 Exploring Reinforcement Learning for Wind Farm Control ..... 79
   5.2 Towards a Complete Model Chain for Ice Accretion Effects on
       Wind Turbines ............................................................................. 84
   5.3 A Three-dimensional Wake Model Based on Convolutional
       Neural Networks ........................................................................ 92

6  Computational Performance ................................................................. 100
   6.1 Comparison on Identical Grids .................................................. 101
6.2 Comparison on Solver-typical Grids ........................................ 103

7 Conclusion ................................................................................. 105

8 Sammanfattning .......................................................................... 108

Acknowledgements .......................................................................... 110

References ......................................................................................... 113
Part I:
Comprehensive Summary
1. Introduction

Almost every technical aspect of wind energy exploitation depends directly or indirectly on the interaction of the wind turbine with the ambient flow. Above all, the kinetic energy of the incoming flow determines the power capture of the turbine. Variations in the inflow thus transmit to power fluctuations. This in turn requires the intervention of the turbine’s mechanical control systems and power electronics and ultimately the reaction of the power grid. At the same time, the characteristics of the inflow are the dominant factor for both fatigue and extreme loads of mechanical components such as the rotor blades, bearings, or the tower. Modelling the interaction of wind turbines with the surrounding flow is thus crucial for all engineering aspects of wind energy projects. This starts with the design of the turbines, on to the planning of wind farms and predictions of future energy yields, and finally the operation, control and performance forecasting throughout the entire lifetime of the turbine. First of all, this requires the modelling of the unperturbed ambient flow: the atmospheric boundary layer (ABL). This includes the effects of thermal stratification, orography as well as other features of the terrain such as forest canopies. In addition to that, perturbations of the ABL by upstream turbines become a determining factor for the flow within wind farms and, thus, for the majority of today’s wind power projects. These so-called wakes, illustrated in Fig. 1.1, exhibit lower mean velocities and larger turbulence intensities than the unperturbed inflow. Their characteristics and downstream evolution depend both on the operational status of the upstream turbine and the ambient atmospheric flow conditions. The complexity of the aerodynamic interaction of wind turbines, wakes and the ABL render the topic as one the greatest challenges in wind energy research today, despite significant progress over the past three decades (Veers et al., 2019; Meneveau, 2019; Porté-Agel et al., 2020).

Today, large-eddy simulation (LES) is the most detailed approach feasible to model the aerodynamics of wind turbines and farms. LES resolve the large energy-containing turbulent structures (eddies) of the flow while only parametrising small-scale turbulence. Firstly, this implies substantially less empiricism than in common engineering models or Reynolds-averaged Navier-Stokes (RANS) approaches. Secondly, it enables investigations of aerodynamic effects that are directly associated with the transient nature of turbulent flows as found in the ABL and wind turbine wakes. However, spatially and temporally resolving these turbulent structures does come at an immense computational cost (Mehta et al., 2014). As a consequence, applications of LES remain limited to fundamental academic case studies, despite the great potential for various modelling challenges in the industrial practice.
Figure 1.1. Illustration of characteristic features of a wind turbine wake by means of iso-surfaces of the vorticity magnitude obtained from actuator line large-eddy simulations developed within this work. The unperturbed turbulent inflow is coming from the right ($\bar{u}$). The rotational direction of the rotor is indicated by the grey vector. Bound vortices emerge due to the circulation around the turbine blades. Trailing vortices are shed at the tip and root of the blades forming the typical tip and root vortex spiral of the near-wake. Eventually, instabilities triggered by the turbulent inflow initiate the breakdown of the tip (and root) vortex spiral and the transition of the near-wake to the turbulent far-wake.

1.1 Current Challenges in Wind Farm Modelling

Computational efficiency has always been a main objective in wind farm modelling research. While industrial applications have always been a main motivation, considerable academic research efforts have been dedicated to the development of simple fast modelling approaches. The key challenge in the development of such engineering models lies in finding suitable simplifications of the overall non-linear, multi-physics, multi-scale problem that accurately capture the essential aspects of interest. The most prominent examples are analytical wake models that describe the downstream evolution of the mean velocity deficit based on first principles. Even four decades after the pioneering work by Jensen (1983) the development of analytical wake descriptions still attains notable attention (Frandsen et al., 2006; Bastankhah and Porté-Agel, 2014; Ishihara and Qian, 2018), highlighting the continuous need for efficiency in wind farm modelling. Another important approach is the class of dynamic wake meandering (DWM) models. As opposed to the aforementioned steady-state wake models, DWM models seek to efficiently capture the transient behaviour of the wake. This becomes particularly crucial for load or control problems. The underlying assumption is that the lateral and vertical meandering of the velocity deficit is driven by the large-scale structures of the turbulent inflow (Larsen et al., 2008).

Traditionally, engineering models, as the ones outlined above, are the backbone for wind farm modelling. The Jensen model, for instance, is still extensively used for yield assessments in the majority of commercial software tools (Porté-Agel et al., 2020). DWM models, on the other hand, are the recommended approach for the assessment of loads in waked inflow conditions
following the IEC (International Electrotechnical Commission) standard for design requirements of wind turbines (IEC, 2019). Comparatively, non-linear computational fluid dynamics (CFD) approaches like RANS or LES have only played a minor role. Typical use-cases are the calibration and validation of engineering models (see, e.g., Madsen et al., 2010b; Jonkman et al., 2018) or academic case studies aiming for the fundamental understanding of specific phenomena. However, the current trends in the development of large-scale wind energy exploitation challenge established engineering models on multiple levels. The need for high-fidelity models such as LES is thus more pressing than ever, not only as a method to investigate and understand the underlying phenomena but even as a potential replacement for low-fidelity models.

### 1.1.1 Wind Turbine Size

One of these upcoming “grand challenges” (Veers et al., 2019) relates to the ever-increasing size of wind turbines. Driven by economies of scale and the access to higher wind speeds, the wind industry has continuously been pushing for larger and taller wind turbines. Today, the rotor diameters of the largest wind turbines exceed the 200 m mark with tip heights reaching more than 250 m (Ramírez et al., 2021). Oftentimes, modern turbines thus partially operate outside the surface layer, i.e. the lowest 10 – 20% of the ABL. An illustration of this trend is given in Fig. 1.2. However, various engineering models rely on the self-similarity of the surface layer. Examples thereof are analytical boundary layer models used in commercial site assessment software packages or synthetic turbulence models commonly used for load simulations (see, e.g., Mann, 1998). Moreover, as rotors sweep a substantial part of the ABL, Coriolis forces and thermal stratification become increasingly important.
Figure 1.3. Comparison of the Lillgrund wind farm (first offshore wind farm of Sweden and third-largest wind farm in the world at the time of commissioning) and the Hornsea 1 wind farm (world’s largest wind farm at the time of writing). The given power refers to the total rated capacity. The city area of the Swedish town of Visby (~24,000 inhabitants) and the German city of Hamburg (~1.8 million inhabitants) are shown for scale. Wind farm layouts and data retrieved from 4coffshore.com. City maps provided by Google Maps (2021).

for accurate predictions of the characteristic mean and turbulent features of the flow (Sanz Rodrigo et al., 2017). Other challenges arise from the higher flexibility of longer and more slender blades. Among others, the occurring large dynamic deflections can cause interactions of the blades with their own shed vorticity. Such effects are not captured by classical aerodynamic models based on blade element momentum (BEM) theory (Heinz et al., 2016; Sayed et al., 2019; Santo et al., 2020).

1.1.2 Growing Wind Farms

Not only wind turbines but also farms increase in size, as illustrated in Fig. 1.3. This provokes effects that have previously been deemed negligible. A whole range of phenomena is commonly summarised under the term wind farm blockage. It describes cumulative induction-related effects of clusters of wind turbines on the wind speed upstream. Recently, such blockage effects have been shown in wind-tunnel (Segalini and Dahlberg, 2020) and field measurements (Schneemann et al., 2021) as well as numerical studies (Bleeg et al., 2018). While RANS or LES naturally capture the underlying induction mechanisms, most engineering model frameworks do not explicitly account for them. Overpredictions of the power production can be the consequence, particularly in large wind farms. Closing this modelling gap motivated several recent studies (Branlard et al., 2020; Nygaard et al., 2020; Segalini, 2021). The growing size of wind farms combined with shorter inter-farm distances also increases the need to incorporate the effects of wind farm wakes in the planning process. Schneemann et al. (2020), for instance, report noticeable velocity deficits up to 55 km downstream of a 400 MW offshore wind farm. Fundamentally, farm
wakes describe the long-distance effect of merged individual turbine wakes. Nonetheless, Nygaard et al. (2020) or Stieren and Stevens (2021) show that common turbine wake models lack the sufficient tuning to capture the long-distance recovery of the farm wake. In addition, long wind farm wakes can be subject to notable lateral deflections due to Coriolis effects (van der Laan and Sørensen, 2017; Eriksson et al., 2019).

1.1.3 Wind Farm Control

Further modelling challenges are linked to the topic of wind farm control. To date, wind turbines follow individual control strategies aiming at the maximum aerodynamic efficiency in below-rated conditions. This approach disregards any effect on the performance of other turbines downstream and is therefore often referred to as greedy control. In recent years, alternative strategies have been proposed that aim at maximising the power of the farm as a whole by mitigating wake effects. Investigated approaches include the power derating of upstream turbines (Nilsson et al., 2015; Munters and Meyers, 2017), yaw- or tilt-induced wake deflection (Fleming et al., 2015) as well as wake-redirection techniques using individual pitch control (Fleming et al., 2014; Frederik et al., 2020). On the one hand, these approaches often imply off-design conditions that are not well-captured by standard wake models. One example is the prediction of the wakes of highly-yawed turbines that requires substantial modifications to existing descriptions (Gebraad et al., 2014) or even entirely new modelling approaches (Bastankhah and Porté-Agel, 2016; Martínez-Tossas et al., 2021). On the other hand, many of the approaches discussed in the literature involve dynamic changes to the power set-point, pitch, or yaw, making time-resolved models inevitable. So far DWM or even simpler dynamic models have been the basis for most of the aforementioned studies. As to that, Kheirabadi and Nagamune (2019) indicate that low-fidelity models have often overpredicted efficiency gains. More validations by high-fidelity models are therefore indispensable to gain further trust in potential farm optimisation strategies before implementing them in real wind farms.

1.1.4 Complex Terrain

Lastly, wind turbine developers are moving to sites of increasing aerodynamic complexity. This refers to areas with complex orography (like mountains or hills), heterogeneous aerodynamic roughness, forests or combinations of the three. This trend can particularly be seen in countries like Sweden or Finland, where the majority of wind power development is taking place in the northerly forested hilly regions due to the lower population density (Enevoldsen, 2016; Badman and Tengblad, 2021). These aerodynamic characteristics can lead to large uncertainties of linear wind resource assessment (WRA) models due to
multiple violations of the underlying model assumptions such as homogeneous roughness or gentle slopes (Palma et al., 2008; Sanz Rodrigo et al., 2020). Others have shown that load predictions based on standard synthetic turbulence models largely underestimate the damage equivalent loads at typical hub heights above forest canopies (Nebenführ and Davidson, 2017).

1.2 Large-eddy Simulation of Wind Farms

LES numerically solve the filtered governing equations of thermo-fluid dynamic problems. This allows to resolve turbulent structures larger than the filter scale while the effects of smaller scales need to be modelled. For problems described by the incompressible Navier-Stokes equations (NSE) this refers to solving the filtered conservation equations of mass and momentum, i.e.,

\[
\frac{\partial \tilde{u}_i}{\partial x_i} = 0 ,
\]

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + F_i ,
\]

where \(\tilde{\cdot}\) indicates a filtered quantity, \(u_i\) is the velocity, with \(i = \{1, 2, 3\}\), \(x_i\) is the respective spatial coordinate, \(p\) the pressure, \(\rho\) the density, and \(t\) time. \(\tau_{ij}\) represents the traceless part of the sub-grid scale stress tensor and \(F_i\) is a generic body force. As for wind energy applications, the latter can contain contributions due to thermal stratification (typically associated with an additional transport equation for the potential temperature), Coriolis effects, or the body forces of actuator models, as discussed later.

1.2.1 A Historical Overview

Indeed, the LES technique originated in the field of atmospheric sciences with the pioneering works of Smagorinsky (1963) and Lilly (1967). Other early studies are due to, e.g., Deardorff (1972) or Moeng (1984) simulating ABL flows over flat terrain with homogeneous roughness. Over the past five decades, the complexity of simulated cases has constantly grown, and LES now states one of the dominant methods for the investigation of fundamental micro-meteorological phenomena (Stoll et al., 2020). Today, simulations of diurnal cycles (Brown et al., 2002; Kumar et al., 2006), flows over forest canopies (Dwyer et al., 1997; Cassiani et al., 2008; Ivanell et al., 2018) or complex terrain (Uchida and Ohya, 2003; Chow and Street, 2009; Diebold et al., 2013) can be considered state of the art. Further developments include the introduction of novel sub-grid scale (SGS) models for the fluid momentum (Porté-Agel et al., 2000; Lu and Porté-Agel, 2010), as well as active and passive scalars (Stoll and Porté-agel, 2006; Abkar et al., 2016b).
Wind energy applications of LES largely build upon the existing experiences from the atmospheric sciences. However, simulating wind turbines in the ABL brings about an additional problem of scale that resulted in significant domain-specific research efforts, in particular, on so-called actuator models. Actuator models compute the forces acting on a rotor blade element via tabulated lift and drag coefficients of the respective airfoil section and the local inflow velocity sampled from the simulated flow field. The opposing force is then again applied as a body force in the CFD simulation. The most common are the actuator disc (ADM) and line model (ALM). The former applies the mean force contribution of all blades across the entire rotor-swept area while the latter applies the forces of each blade along individual lines. The use of actuator models avoids prohibitively expensive geometrically resolved simulations of the turbine and, historically, only made LES of wind farms possible (Mehta et al., 2014). Most importantly, crucial aspects such as the loads of the turbine or the main features of the wake remain well-captured when compared to geometrically resolved rotor simulations (Troldborg et al., 2015b; Lin et al., 2019). Various model-specific aspects have been intensively advanced since the introduction of the ADM (Ammara et al., 2002; Mikkelsen, 2003) and ALM (Sørensen and Shen, 2002) to wind power applications. Such are, among others, issues related to the projection of the rotor forces to volumetric body forces (Shives and Crawford, 2013; Martínez-Tossas et al., 2017; Meyer Forsting et al., 2019), the overcoming of time step constraints of the ALM by means of actuator sector models (Storey et al., 2015; Vitsas and Meyers, 2016) and, the coupling of actuator models to structural solvers (Churchfield et al., 2012b; Storey et al., 2016).

1.2.2 Recent Trends

Early LES studies in the field of wind energy were mostly concerned with the development and validation of the numerical models. Yet, in recent years, the use of LES increasingly shifted towards applied engineering-driven topics. This includes investigations of the dependency of fatigue loads on atmospheric conditions (Churchfield et al., 2012b; Storey et al., 2016; Meng et al., 2018), the impact of power-derating on wind farm performance (Nilsson, 2015; Fleming et al., 2015; Dilip and Porté-Agel, 2017) or the development and assessment of wind farm control approaches (Ciri et al., 2017; Munters and Meyers, 2018). With this shift, the computational demand of individual cases increased drastically in comparison to earlier studies. In some studies larger domains and/or higher grid resolutions led to larger grid sizes. Examples thereof are simulations of entire offshore wind farms (Churchfield et al., 2012a; Abkar and Porté-Agel, 2013; Nilsson, 2015) or large areas of complex orography (Ivanell et al., 2018; Fang et al., 2018). Other investigations require long times to be simulated. See, for instance, the studies by Abkar et al. (2016a) or Tian et al.
(2020) on the development of wind farm wakes throughout multiple diurnal cycles.

1.3 Potentials of Lattice Boltzmann Methods for Wind Energy Applications

The immense computational demand of LES is the main factor limiting the applicability of the method to the current challenges in wind energy and an implementation in the industrial practice. But also fundamental academic studies, for instance, in the field of boundary layer flows (see, e.g., Sullivan and Patton, 2011; Stevens et al., 2014b), constantly strive for higher resolutions, making computational demand a persistent bottleneck (Bou-Zeid, 2015; Stoll et al., 2020). From an industry perspective, Löhner (2019) labelled this evident issue as the “LES crisis”, i.e. “the inability to obtain overnight solutions” (cf. ibid.). According to the author, the ability to run typical cases overnight is pivotal for the applicability of a numerical method in most industries. RANS solvers, for instance, achieved this ability in the 1990s leading to a wide adoption in the industrial practice, particularly for automotive and aerospace applications. The first commercial RANS tools for wind energy applications followed in the early 2000s. With conventional numerical methods LES of automotive or aerospace applications still remain far from this milestone, with run times in the order of days or even weeks to obtain statistically converged solutions. The same applies to LES of wind farm flows.

1.3.1 Computational Limitations of Established Solvers

Given today’s trends in high performance computing (HPC) hardware, it is questionable whether the required (affordable) computational performance can be achieved with the common LES frameworks used in the wind energy community today, i.e. CPU-based incompressible Navier-Stokes solvers. Firstly, Moore’s law appears to come to an end. For decades it correctly predicted that transistor density, and hence chip performance, doubles every other year (Moore, 1975). Yet, a notable slow-down began around the year 2000 and in 2018 processor performance lagged behind the prediction by a factor of 15 (Waldrop, 2016; Hennessy and Patterson, 2019). Secondly, Dennard’s scaling, predicting that power consumption per transistor drops as transistor density increases (implying an increase in energy efficiency; Dennard et al., 1974), ended around the year 2012 (Hennessy and Patterson, 2019). Both trends imply that further performance gains hinge more and more on the efficient parallelisation of algorithms instead of an increasing efficiency of individual processing units. However, the required large-scale parallelisation of the aforementioned class of solvers on modern multi- or many-core processors, such
as multi-core CPUs (Central Processing Units), GPUs (Graphics Processing Units) or FPGAs (Field-programmable Gate Arrays), is arguably challenging, while particular bottlenecks can depend on the specific numerical scheme (pseudo-spectral, finite-difference, spectral element, etc.). See, e.g., Vázquez et al. (2016), Merzari et al. (2020) or Verma et al. (2020).

1.3.2 The Efficiency of Lattice Boltzmann Methods

A promising alternative is the use of an utterly different numerical approach: the Lattice Boltzmann Method (LBM). Emerging from the lattice gas cellular automata (McNamara and Zanetti, 1988) in the early 1990s, the LBM gained popularity in various fields of fluid dynamics over the past three decades, chiefly, due to its high computational performance (Krüger et al., 2016). As opposed to classical Navier-Stokes-based approaches, the LBM is based on the discretisation of the kinetic Boltzmann equation. The efficiency of the method originates from an explicit time-stepping, a strict separation of non-linear and non-local terms, a direct advection (streaming) requiring no interpolation, and the excellent parallelisability due to these very features (Succi, 2015). This also renders the LBM perfectly suitable for efficient implementations on GPUs or FPGAs. Unprecedented cost-to-performance ratios of such implementations have been shown in numerous studies. See, for instance, Pasquali et al. (2016), Onodera and Idomura (2018) or Altoyan (2020), to name a few. Indeed, GPU-based LBM implementations already now allow for overnight LES for certain applications with existing hardware (Niedermeier et al., 2018; Lenz et al., 2019). However, key developments of the method that enabled the simulation of highly turbulent flows only occurred recently. In particular, novel collision operators remedied persisting numerical stability issues at high Reynolds numbers and provided sufficient robustness for turbulent engineering and environmental flow cases (Geier et al., 2015, 2017b; Dorschner et al., 2016; Jacob and Sagaut, 2018). Furthermore, wall models prescribing the shear-stress in under-resolved near-wall regions are still at an early stage of development, yet, inevitable for the simulation ABL flows.

To date, one of the most prominent applications of the LBM in the wider field of atmospheric sciences are urban flows in contexts like wind comfort assessment (King et al., 2017; Ahmad et al., 2017; Jacob et al., 2018; Lenz et al., 2019) or pollutant dispersion (Merlier et al., 2018, 2019). Others applied the LBM for investigations of forest canopy flows. See, e.g., Watanabe et al. (2020, 2021). Some of the first studies explicitly dedicated to the field of wind energy are due to Deiterding and Wood (2016), Khan (2018) or Zhiqiang et al. (2018) discussing simulations of geometrically resolved model-scale wind turbines. Various crucial modelling aspects for simulations of wind turbines and farms thus remain open questions. This applies particularly to the use of actuator models as well as the simulation of wall-modelled ABL flows. Prior to
the studies compiled in this work, only Rullaud et al. (2018) presented actuator line simulations of a vertical axis wind turbine. Yet, the set-up was limited to two-dimensions which effectively reduces the actuator lines to actuator points. Similarly scarce are the experiences with wall-modelled simulations of ABL flows, mainly due to aforementioned infancy of robust near-wall treatments.

1.4 Aim of This Work

The overall potential of the LBM for extremely efficient LES has been recognised across various fields of fluid dynamics. The aim of this thesis is to facilitate efficient LBM-based LES for wind energy applications and to evaluate the potentials resulting from the method. More specifically, this adoption for wind energy applications hinges on the closing of several modelling gaps. Two particular ones shall be addressed in this work, namely, the modelling of wind turbines by means of the actuator line model as well as LBM-specific wall modelling allowing for the simulation of ABL flows.

1.5 Outline

The majority of the studies compiled in this thesis are concerned with the development and fundamental investigation of the actuator line model and a LBM wall model, respectively. This includes the analysis of the accuracy, stability and numerical sensitivities of the developed methods as well as the comparison to Navier-Stokes-based approaches, and to theoretical and experimental reference data. In addition to that, the potential of the developed framework is exemplified by means of additional application-oriented studies dedicated to current problems in wind energy and wind farm modelling.

The first part of this thesis provides an introduction to the field and a comprehensive summary of the compiled studies. The utilised numerical methods are outlined in Chapter 2. This includes a brief introduction to the fundamentals of the LBM, as well as details upon the cumulant collision model, boundary conditions and sub-grid scale modelling for LES. In Chapter 3, we discuss the studies concerned with the implementation, verification and validation of the ALM. Chapter 4 summarises the development of a novel wall modelling approach for the LBM and its application to neutral ABL flows. Chapter 5 compiles three studies applying the numerical framework to problems in wind farm control, wind turbine icing and deep-learning-based surrogate modelling. Lastly, Chapter 6 discusses the computational performance of the framework in light of the overall motivation for this work. General conclusions are given in Chapter 7. The original publications are included in Part II in the printed version the thesis.
2. The Lattice Boltzmann Method

The objective of LES is to provide a numerical solution to a given flow problem described by the filtered Navier-Stokes equations. A vast amount of numerical techniques has been employed for this particular problem. Regardless of the numerical method (e.g., finite volume, finite difference, etc.), all rely on a discretisation of the terms of the Navier-Stokes equations. Despite the same motivation, the LBM solves the Boltzmann equation instead. Formulated by Ludwig Boltzmann in 1872, it originally describes the statistics of particle distributions of a thermodynamic system in a non-equilibrium state. The connection to the macroscopic description of the fluid, i.e. the Navier-Stokes equations, thus appears less intuitive and requires some algebra. Nonetheless, most of the advantages of the LBM result from this different background and enable the method’s superior computational efficiency. Interestingly, some of these numerical features such as uniform Cartesian meshes, explicit time-stepping, and weak compressibility now even see a growing popularity in Navier-Stokes-based approaches as they facilitate the efficient usage of GPUs and other many-core architectures (Sauer and Muñoz-Esparza, 2020; Matsushita and Aoki, 2021; Löhner et al., 2021). This chapter provides a brief introduction to the fundamentals of the LBM. Furthermore, more specific aspects vital to this work are discussed, namely, the cumulant collision operator, the basics of LBM boundary conditions and sub-grid scale modelling.

2.1 Fundamentals

The fundamental variable in kinetic theory is the particle distribution function (PDF) $f$. $f$ describes the distribution of particle densities in space $x$, time $t$ and velocity space $\xi$. In that sense, $f$ generalises the density $\rho$, which only describes the distribution of particles in space and time (Krüger et al., 2016). The consideration of particle densities locates the approach in between the microscopic view point of individual particles (as, for instance, in molecular dynamics) and the macroscopic continuum assumption of the Navier-Stokes equations. Kinetic theory and the LBM are therefore typically considered mesoscopic.

2.1.1 From Particle Distributions to Macroscopic Quantities

The link between PDFs and the macroscopic variables of interest can be made via velocity moments. The integration of $f(x, \xi, t)$ over velocity space (zeroth-
order moment) yields the macroscopic mass density
\[ \rho(x, t) = \int f(x, \xi, t) d^3 \xi \quad . \] (2.1)
Consequently, integrating the first-order moment with respect to \( \xi \) provides the macroscopic momentum density
\[ \rho(x, t) u(x, t) = \int \xi f(x, \xi, t) d^3 \xi \quad . \] (2.2)
Similarly, the total energy density can be obtained from the second-order moment.

2.1.2 Discretising the Boltzmann Equation
The Boltzmann equation is the transport equation of \( f \) given by
\[ \frac{\partial f}{\partial t} + \xi_i \frac{\partial f}{\partial x_i} + \frac{F_i}{\rho} \frac{\partial f}{\partial \xi_i} = \Omega(f) \quad . \] (2.3)
The left hand side can be inferred from the total derivative of \( f \) with respect to time. The second term thus refers to the advection of \( f \) with \( \xi \) while the third term represents the acting of an external body force \( F \). The right hand side \( \Omega(f) \) is the so-called collision operator. It describes the redistribution of particle distributions by means of particle collisions. Any form of \( \Omega \) should therefore adhere to the kinetic theory of particle collisions, namely the conservation of mass, momentum and energy. Furthermore, collisions tend to reduce the anisotropy of the momentum distribution. Hence, in the absence of external forces, particle distributions should relax towards an isotropic equilibrium state around a mean velocity by particle collisions. The most common description of this equilibrium state is the Maxwell-Boltzmann equilibrium, and the basic assumption of \( \Omega \) a relaxation towards it. Further details upon the exact form of collision operators in the LBM will be given later.

The main complexity of the Boltzmann equation relates to \( f \) being a function of space, time and velocity. Consequently, both physical and velocity space need to be discretised when solving the Boltzmann equation numerically. The velocity space discretisation rests upon the constraint that we are only interested in the macroscopic behaviour of the fluid. Thus, only the conserved moments of \( f \) need to be captured by a discrete set of PDFs \( f_{ijk} \) related to corresponding discrete velocities. Suitable forms of such discrete velocity lattices can be found from a Hermite series expansions of the Maxwell-Boltzmann equilibrium distribution \( f^{eq} \). The discrete form of the equilibrium distribution reads
\[ f_{ijk}^{eq} = w_{ijk} \rho \left( 1 + \frac{u \cdot e_{ijk}}{c_s^2} + \frac{(u \cdot e_{ijk})^2}{2 c_s^4} - \frac{u \cdot u}{2c_s^2} \right) \quad , \] (2.4)
where \( c_s \) is the speed of sound and \( e_{ijk} \) the abscissae of the velocity lattice with

\[
e_{ijk} = (ic, jc, kc)
\]

and \( i, j, k \in \mathbb{Z} \). The lattice speed \( c \) is typically chosen such that

\[
c = \Delta x / \Delta t \quad .
\]

Common lattices for three-dimensional problems are the D3Q19 and D3Q27 lattice with 19 and 27 discrete lattice velocities, respectively; see Fig. 2.1. The weights \( w_{ijk} \) result from the Gauss-Hermite quadrature rule and ensure the conservation of the moments up to second order on the respective lattice. Accordingly, the moments equivalent to Eqs. (2.1) and (2.2), etc., reduce to the weighted sums of the discrete PDFs

\[
m_{\alpha\beta\gamma} = \sum_{ijk} (ic)^{\alpha} (jc)^{\beta} (kc)^{\gamma} f_{ijk} \quad ,
\]

where \( \alpha, \beta \) and \( \gamma \) denote the order of the moment in the respective lattice direction and \( \alpha + \beta + \gamma \) the total order of the moment. The macroscopic pressure \( p \) can be found by employing the isothermal equation of state of an ideal gas, with

\[
p = c_s^2 \rho .
\]

In the classical LBM, the Boltzmann equation is discretised on uniform Cartesian grids. Employing the method of characteristics for the left-hand side and a forward-Euler scheme for the right-hand side (and exploiting the simplifications resulting from \( c = \Delta x / \Delta t \)) yields the lattice Boltzmann equation (LBE):

\[
f_{ijk}(t + \Delta t, x + \Delta t e_{ijk}) - f_{ijk}(t, x) = \Delta t \Omega_{ijk}(t, x) \quad .
\]
\[ \Omega_{ijk}(t,x) = \frac{1}{\tau} \left( f_{ijk}(t,x) - f_{ijk}^{eq}(t,x) \right). \] (2.10)

Algorithmically, it is standard practice to decompose and rearrange Eqs. (2.9) and (2.10) into two separate steps, as shown in Fig. 2.2. The first is the collision step
\[ f_{ijk}^{*}(t,x) = \left( 1 - \frac{\Delta t}{\tau} \right) f_{ijk}(t,x) + \frac{\Delta t}{\tau} f_{ijk}^{eq}(t,x), \] (2.11)
where \( f_{ijk}^{*} \) is the post-collision distribution function. The second is the streaming (or propagation) step
\[ f_{ijk}(t + \Delta t, x + \Delta t e_{ijk}) = f_{ijk}^{*}(t,x), \] (2.12)
advection \( f_{ijk}^{*} \) to the neighbouring nodes. In summary, the classical LBM given by Eqs. (2.11) and (2.12) states a simple and explicit numerical algorithm. Most importantly, non-linear terms are only found in the calculation of the equilibrium distribution, see Eq. (2.4), and solely depend on variables that can be computed locally, namely \( u \) and \( \rho \) using Eq. (2.7). The computationally intensive collision step is therefore perfectly parallelizable. The propagation step, on the other hand, as the only non-local operation, is exact as opposed to, e.g., the advection in the Navier-Stokes equations, and requires no interpolation. In practice, the streaming thus merely requires a pointer swap.

Eqs. (2.1) and (2.2) provide a brief idea of the correspondence between the mesoscopic description of fluids of the LBM and the macroscopic one of the Navier-Stokes equations. Equivalently, the conservation equations of the respective macroscopic variables can be established via the moments of the

**Figure 2.2.** The LBM collide-and-stream algorithm: Firstly, the collision of incoming distribution functions \( f_{ijk} \) (grey) at each grid point is modelled using a collision operator. Secondly, the post-collision distributions \( f_{ijk}^{*} \) (red) stream to the neighbouring grid points.

### 2.1.3 Definition of the Collision Operator

The final step to complete the numerical scheme is the definition of the collision operator. The most simple collision operator is the single-relaxation-time model (SRT), commonly referred to as lattice Bhatnagar-Gross-Kroog (LBGK) model (Bhatnagar et al., 1954). In the SRT, all PDFs are relaxed towards the equilibrium using a single constant relaxation time \( \tau \) with
Boltzmann equation or the discretised LBE, respectively. Hence, the zeroth-order moment yields the conservation of mass (the continuity equation) and the first-order moment provides the conservation of momentum. While the former is found to be independent of $f_{ijk}$, the latter is not. More specifically, the macroscopic stress tensor is depending on the non-equilibrium $f_{ijk}^{\text{neq}} = f_{ijk} - f_{ijk}^{\text{eq}}$. Further analysis is therefore required to transform the LBE to a complete macroscopic form. The most common method for this is the Chapman-Enskog analysis, independently developed by Sydney Chapman and David Enskog in 1917, and later combined by Chapman and Cowling (1952). The basic idea of the Chapman-Enskog analysis is a perturbation expansion of $f_{ijk}$ around the equilibrium distribution $f_{ijk}^{\text{eq}}$ in terms of a smallness parameter $\epsilon$. Combined with a Taylor expansion of the LBE and an additional perturbation expansion of the time derivative, each moment of the LBE can be separately analysed at each respective order in $\epsilon$. Specifically, it can be shown that the moment equations at $\mathcal{O}(\epsilon)$ recover the continuity and Euler equations, respectively. The missing description of the viscous stress tensor, complementing the latter to the Navier-Stokes equations, is found at second order in $\epsilon$ and reveals that

$$
\tau = \frac{3 \nu}{c_s^2} + \frac{\Delta t}{2},
$$

(2.13)

where $\nu$ is the kinematic viscosity. Generally, the Chapman-Enskog analysis shows that the LBE recovers the compressible Navier-Stokes equations. Still, the classical LBM only captures weakly compressible phenomena at low Mach numbers as the model remains isothermal. From an incompressible perspective, the remaining compressibility effects are typically referred to as the \textit{compressibility error}. Furthermore, the Chapman-Enskog analysis reveals a cubic velocity defect in the viscous stress tensor stating a violation of Galilean invariance. On the one hand, the limitation of the classical LBM to the weakly compressible regime thus relates to the underlying assumption of isothermality. On the other hand, $\text{Ma}^2 \ll 1$ is required from a numerical point of view to keep violations of Galilean invariance in reasonable bounds.

2.2 The Cumulant Collision Model

The simplicity of the classical SRT collision model is obviously attractive from a computational point of view. Yet, the limited numerical stability of the model renders it unsuitable for many applications, especially high Reynolds number flows. A considerable part of past LBM research has been dedicated to the development of alternative collision models. Many belong to the class of multiple-relaxation-time models (MRT). See, for instance, Lallemand and Luo (2000) or d’Humières et al. (2002). MRT models transform the pre-collision PDFs $f_{ijk}$ into raw velocity moments following Eq. (2.7). Each moment is
then relaxed towards a respective equilibrium moment $m^\text{eq}_{\alpha\beta\gamma}$ with an individual relaxation rate $\omega_{\alpha\beta\gamma} = 1/\tau_{\alpha\beta\gamma}$. The relaxation rates of non-hydrodynamic higher-order moments can thus be freely chosen in order to improve the numerical stability. After the relaxation, the moments are transformed back into particle distribution space and propagated as in the SRT model. Despite some stability improvements, moment-based models reveal other fundamental shortcomings. First of all, moments are no statistically independent quantities. For instance, third-order raw moments depend on first-order moments on finite lattices as used in the LBM. As a consequence, not only the chosen relaxation rate determines the effective relaxation but also the velocity. This states an additional violation of Galilean invariance and can introduce further viscosity errors. A possible solution is to choose central moments instead of raw moments that do not possess this defect (Geier et al., 2009, 2015). Yet, further dependencies of non-hydrodynamic and hydrodynamic moments exist for both raw and central moments. These can evoke hyper-viscosities, i.e. an additional damping of second-order moments through the relaxation of coupled higher-order moments. Deteriorations of the flow field through local instabilities can be the consequence as shown, for instance, by Gehrke et al. (2017).

### 2.2.1 Relaxation in Cumulant Space

A remedy to the aforementioned deficiencies is found in a formulation based on statistically independent observable quantities, *cumulants*, of the PDFs. These form the basis for the cumulant lattice Boltzmann method (CLBM) introduced by Geier et al. (2015). Based on the moment generating function of pre-collision PDFs

$$F(\Xi) = \mathcal{L}(f(\xi)) = \int_{-\infty}^{\infty} f(\xi) e^{-\Xi \cdot \xi} d\xi ,$$

(2.14)

with $\Xi = \{\Xi, Y, Z\}$ denoting the particle velocity $\xi = \{\xi, \nu, \zeta\}$ in wave number space, cumulants $c_{\alpha\beta\gamma}$ can be obtained as

$$c_{\alpha\beta\gamma} = c^{-\alpha-\beta-\gamma} \frac{\partial^{\alpha} \partial^{\beta} \partial^{\gamma}}{\partial \Xi^\alpha \partial Y^\beta \partial Z^\gamma} \ln \left(F(\Xi, Y, Z)\right) .$$

(2.15)

After a transformation of $f_{ijk}$ to cumulant space, cumulants are relaxed towards their respective equilibrium:

$$c^*_{\alpha\beta\gamma} = \omega_{\alpha\beta\gamma} c^\text{eq}_{\alpha\beta\gamma} + (1 - \omega_{\alpha\beta\gamma}) c_{\alpha\beta\gamma} ,$$

(2.16)

similarly to moment-based models. Here, $c^*_{\alpha\beta\gamma}$ denotes the post-collision cumulant and $\omega_{\alpha\beta\gamma}$ the respective relaxation rate. Geier et al. (2015) show that the statistical independence of cumulants eliminates the aforementioned deficiencies associated with a relaxation in moment space. Moreover, the authors
introduce an additional correction to the cubic velocity defect using locally approximated velocity derivatives. Consequently, violations of Galilean invariance are reduced even further. Note, however, that the concept of this correction is not specific to the CLBM. Similar corrections can be applied in other collision models, as shown in the same study or, e.g., by Hajabdollahi and Premnath (2018).

2.2.2 Choice of Relaxation Rates
The initial approach of the CLBM was to set all relaxation rates of higher-order cumulants to one, commonly referred to as *AllOne*, K15 or regularised CLBM. Hence, all non-hydrodynamic cumulants are directly set to their respective equilibrium. Higher-order perturbations are thus unconditionally damped providing an inherently stable numerical scheme. Numerous studies illustrate that the regularised CLBM can be readily applied to high Reynolds number flows eliminating any numerical stability issue (see, Geier et al., 2015; Far et al., 2016; Gehrke et al., 2017; Kutscher et al., 2019; Onodera and Idomura, 2018). This also includes the initial study of this work presented in Paper I and discussed later.

Later, Geier et al. (2017b) presented a further modification of the CLBM. Based on the theory of the so-called *magic parameter* (Ginzburg and Adler, 1994; Ginzburg et al., 2008) the authors derive a parametrisation of the higher-order relaxation rates eliminating the linearised leading error in diffusion. The resulting parametrisated CLBM exhibits fourth-order accuracy in diffusion under diffusive scaling (i.e., $\Delta t \propto \Delta x^2$), with negligible computational overhead. Furthermore, the parametrisation eliminates another inherent problem of the LBM, namely a deterioration under pure Mach-number scaling. In turn, the scheme no longer guarantees unconditional numerical stability. A practical compromise is the use of a stabilising limiter for the relaxation rates of the third-order cumulants. While the limiter reintroduces some ad-hoc tuning to the scheme, it is shown to be an effective measure to provide sufficient numerical stability without compromising the increased accuracy due to the parametrisation (Geier et al., 2017b, 2020; Gehrke et al., 2020).

Within this work, the parametrisation of the CLBM was implemented in the utilised numerical framework elbe¹ (Janßen et al., 2015) and applied in Paper II and all later studies. Moreover, along with the parametrisation the scheme was changed to so-called *well-conditioned* distribution functions $f_{ijk}^{\text{wc}} = f_{ijk} - w_{ijk}$ (Geier et al., 2015, 2017b). Macroscopically, this shift of the distribution functions subtracts the constant background density $\rho_0$, as can be inferred from Eq. (2.4). The well-conditioned zeroth order moment $m_{000}$ only refers to the fluctuating part of the density $\delta \rho$ with $\rho = \rho_0 + \delta \rho$. The measure significantly reduces round-off errors since $f_{ijk}^{\text{wc}}$ is typically several

¹www.tuhh.de/elbe
orders of magnitude lower than $f_{ijk}$. The numerical conditioning becomes particularly important in single-precision floating-point format that is commonly used in GPU-based LBM frameworks. For the simulations performed in this work well-conditioned PDFs proved to be vital for the numerical stability of the parametrised CLBM.

2.3 Boundary Conditions
The problem of formulating boundary schemes in the LBM lies in finding unknown incoming PDFs that adhere to a desired macroscopic boundary condition. LBM boundary schemes can be split into two main classes, i.e., wet-node and link-wise formulations. Both methods can be applied to prescribe a variety of macroscopic boundary conditions.

2.3.1 Wet-node Boundary Conditions
In wet-node boundary conditions (WNBCs) the boundary node is originally assumed to lie infinitesimally close to the boundary and is thus part of the simulated domain (hence wet). In most cases this implies that the number of unknown distributions is larger than the amount of macroscopic conditions to be prescribed. A variety of approaches emerged to solve the underlying under-specified problem. The most simple method is to explicitly impose an equilibrium distribution $f_{ijk}^{eq}(u_{bc}, \rho_{bc})$, with $u_{bc}$ and $\rho_{bc}$ being the imposed boundary velocity and density, respectively. However, neglecting velocity gradients (captured by the non-equilibrium) renders the method only first-order accurate. Various methods have therefore been proposed that also reconstruct $f_{ijk}^{eq}$ at the boundary node. A comprehensive overview can be found in Latt et al. (2008).

2.3.2 Bounce-back Schemes
Link-wise approaches, so-called bounce-back schemes (BBS), are a purely kinetic approach. The most basic method of this model family is the simple or half-way bounce-back (SBB) scheme illustrated in Fig. 2.3. It assumes that post-collision distributions $f_{ijk}^{*}$ propagating from an off-boundary node $x_1$ towards the boundary are reflected and return as $f_{ijk}$, with $e_{ijk} = -e_{ijk}$. Thus, we get

$$f_{ijk}(x_1, t + \Delta t) = f_{ijk}^{*}(x_1, t) + 2 w_{ijk} \rho \frac{u_w \cdot e_{ijk}}{c_s^2},$$

where the second term on the right-hand side accounts for the additional momentum exchange between the wall and $f_{ijk}^{*}$ due a wall velocity $u_w$. Despite its simplicity, the SBB is a second-order accurate boundary scheme if the boundary is grid aligned and lies half-way between $x_1$ and the next (solid) node.
Figure 2.3. Schematic of the simple bounce-back scheme: Post-collision distributions (red) hitting a solid boundary get reflected and return to their grid point of origin as pre-collision distributions of the next time step (grey).

outside the domain (Ginzburg and Adler, 1994). For other distances to the boundary, the point of origin $x_{BB}$ of reflected distributions reaching $x_1$ no longer coincides with $x_1$. In this case, $f^*_{ijk}(x_{BB})$ needs to be interpolated in order to maintain second-order accuracy (Bouzidi et al., 2001; Lallemand and Luo, 2003).

2.4 Sub-grid Scale Modelling

With the development of sufficiently robust collision operators, the LBM has become a viable numerical approach for LES of highly turbulent environmental and engineering flows. To date, the application of common eddy-viscosity models is the most popular SGS modelling approach for the LBM (see, e.g., Premnath et al., 2009; Weickert et al., 2010; Jafari and Mohammad, 2011, to name a few). Typically, these SGS models are directly adopted from the original Navier-Stokes context with only minor LBM-specific adaptations. In this work both the standard Smagorinsky model (Smagorinsky, 1963; Lilly, 1967) as well as the anisotropic minimum dissipation model (AMD; Rozema et al., 2015) are employed. Details about their implementation will be given in the following. Furthermore, note that some of the initial studies compiled in this work refer to model-free LES. Further remarks thereupon will be given later.

2.4.1 The Smagorinsky Model

In the Smagorinsky model, the eddy-viscosity given by the well-known formulation

$$\nu_t = (C_s \Delta)^2 \tilde{S}$$  \hspace{1cm} (2.18)

where $C_s$ is the Smagorinsky constant, $\Delta$ the filter width (here referring to the grid spacing $\Delta x$) and $\tilde{S}$ the second invariant of the filtered strain rate tensor

$$\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right)$$

with $\tilde{S} = \sqrt{2 \tilde{S}_{ij} \tilde{S}_{ij}}$.  \hspace{1cm} (2.19)
From an LBM perspective, the Smagorinsky model is particularly advantageous since all components of $\tilde{S}_{ij}$ can be determined locally by means of the second-order moments of the local PDFs. The eddy-viscosity given by Eq. (2.18) is then incorporated in the relaxation rate by adding $\nu_t$ to the shear viscosity $\nu$. For the sake of completeness, it should be noted though that $\tilde{S}_{ij}$ in cumulant and most moment-based models is dependent on the total shear viscosity ($\nu_T = \nu + \nu_t$) and the bulk viscosity. It is therefore not possible to determine $\nu_t$ explicitly, as opposed to the LBGK. Hence, the eddy-viscosity $\nu_t(t)$ can either be approximated explicitly, using $\nu_t(t - \Delta t)$ or, needs to be computed iteratively. However, Yu et al. (2005) show that the error associated with the implicitness of $\nu_t$ is typically negligible due to the small time steps used in the LBM. In line with the majority of LBM eddy-viscosity approaches using MRT or cumulant models it is therefore refrained from implicitly solving for $\nu_t$ in this work.

### 2.4.2 The Anisotropic Minimum Dissipation Model

Minimum dissipation models seek for an eddy-viscosity that provides the minimum eddy dissipation preventing the accumulation of SGS kinetic energy within a certain filter box $\Omega_b$. In the AMD model the upper limit of the SGS kinetic energy is approximated by the Poincaré inequality

$$\partial_t \int_{\Omega_b} \frac{1}{2} \tilde{u}_i \tilde{u}_i' \, dx \leq C_t \int_{\Omega_b} \frac{1}{2} (\partial_i \tilde{u}_j) (\partial_j \tilde{u}_i) \, dx,$$

with $C_t$ being the Poincaré constant. The eddy viscosity is given by

$$\nu_t = \max \left\{ \frac{- (\hat{\partial}_k \tilde{u}_i)(\hat{\partial}_k \tilde{u}_j)}{(\partial_l \tilde{u}_m)(\partial_l \tilde{u}_m)} \bigg| 0 \right\},$$

where $\hat{\partial}_i = \sqrt{C_i} \Delta x_i$, $\partial_i$, $i \in \{1, 2, 3\}$ is the scaled gradient operator (Rozema et al., 2015, 2020). As opposed to the Smagorinsky model, the AMD model can not be realised in a strictly local manner as it requires all components of the velocity gradient tensor. The additional velocity gradients thus have to be computed via finite differences of the macroscopic velocity. Still, the additional memory accesses and floating point operations of the AMD model only amount to a minor computational overhead when compared to the standard Smagorinsky model. Furthermore, despite its overall simplicity, recent studies highlight the AMD as a cost-competitive alternative to, e.g., dynamic Smagorinsky models for ABL simulations (Abkar et al., 2016a; Gadde et al., 2020). This makes it particularly interesting for GPU-based frameworks where efficient implementations of the large non-local stencils of dynamic SGS models are challenging (Ren et al., 2018).

32
2.4.3 Model-free Large-eddy Simulation

Obviously, the main motivation for the use of SGS models is to account for the effects of non-resolved turbulence. However, the additional eddy-viscosity can also be beneficial from the point of view of numerical stability. For example, LBGK or MRT models often rely on the additional damping of the SGS model to remain numerically stable at high Reynolds numbers. A key feature of the CLBM is the model’s numerical stability outlined in Section 2.2. From an LES perspective, the numerical stability clearly sets the CLBM apart from many other collision operators that require an SGS model to stabilise the numerical scheme. Historically, therefore, the CLBM has almost exclusively been applied for model-free LES, i.e. without explicit turbulence model. For the sake of simplicity, the same approach was chosen in some of the initial actuator line studies discussed in Chapter 3. Moreover, the model-free CLBM has often been referred to as implicit LES (cf. Far et al., 2017; Lenz et al., 2019; Nishimura et al., 2019). Nonetheless, this is solely supported by the fact that the scheme remains numerically stable in under-resolved turbulent flows. And, to the author’s knowledge, a full understanding of the dissipation behaviour associated with the scheme’s truncation errors or the limiter, is still lacking. Following the general conception of implicit LES (see, e.g., Adams and Hickel, 2009), this, however, would be clearly required to reliably replace an explicit SGS-model. Admittedly, a recent case study by Geier et al. (2020) of a decaying Taylor-Green vortex shows that the performance of the CLBM with SGS model (in this case, the wall-adapting local eddy-viscosity model; WALE) not necessarily outperforms the model-free approach. On the contrary, pre-studies of model-free LES of boundary layer flows (as discussed in Chapter 4) have shown clearly inferior results compared to both Smagorinsky and AMD cases and were therefore not considered in the subsequent investigations in this work. Similar findings were made in full-scale validations of wind turbine wake simulations (see, Section 3.5).
3. Actuator Line Simulations

Replacing geometrically resolved rotor blades with representative body forces from actuator models has proven to be an appropriate compromise between accuracy and computational efficiency. Actuator models are therefore nowadays the standard method for modelling wind turbines in LES. Of the actuator model family, the ALM provides the closest representation of the actual rotor as each blade is modelled by an individual line. While typically coming at a larger computational cost than the ADM it brings about two main advantages. Firstly, at sufficiently high spatial resolutions, the ALM can resolve the tip and root vortices of the near-wake (as shown, for instance, in Fig. 1.1). Among others, this allows for fundamental investigations of phenomena such as tip vortex instabilities (Ivanell et al., 2010; Sarmast et al., 2014), or interactions of the blades with the near-wake as found for surging (or pitching) floating wind turbines (Cheng et al., 2019; Johlas et al., 2020) or deflecting slender blades. In comparison, the wake emerging from an ADM always refers to a continuous vortex sheet lacking these typical near-wake features. Secondly, the ALM facilitates studies of transient turbine loads because the modelled forces directly refer to the load distribution of each individual blade. Furthermore, dynamic deflections resulting from the loads can be incorporated in the position and velocity of the individual lines. This can be realised by a weak coupling of the ALM to a structural solver; see, for instance, Churchfield et al. (2012b). Both aspects are not straightforward in a disk representation. Indeed, the spatial resolutions used in simulations of entire wind farms are often even too coarse to resolve distinct tip vortices. In these cases, the use of the ALM (instead of an ADM) remains purely motivated by the fact that it facilitates transient load calculations (Churchfield et al., 2012a; Andersen et al., 2015; Foti and Duraisamy, 2019).

This chapter summarises the key findings related to the development, implementation and analysis of the ALM applied in lattice Boltzmann frameworks. The two main aspects of interest of the model analysis are the aerodynamic forces along the blade and the characteristics of the wake induced by the turbine. Both aspects have been extensively studied in Navier-Stokes frameworks over the past two decades. Paper I and II therefore largely build upon a comparison to solutions obtained from such a well-established model. The main findings are discussed in Sections 3.2 and 3.3, respectively. Paper III, summarised in Section 3.4, discusses multiple aspects of the impact of compressibility on the modelled aerodynamics. The study is motivated by recent discussions of the topic in the literature as well as the findings of Paper I. Lastly, an additional
validation study (Paper VI) is presented in Section 3.5, comparing power, loads and wake properties against full-scale measurements.

3.1 Implementation of the Actuator Line Model

Fundamentally, the ALM implementation used in this work closely follows the original description by Sørensen and Shen (2002). Each actuator line is represented by \( n \) discrete blade elements of length \( dr \). The forces acting on each blade element are determined using the local relative velocity \( u_{rel} \), as shown in Fig. 3.1, with

\[
u_{rel} = \sqrt{u_n^2 + (\omega r - u_\theta)^2},
\]

where \( \omega \) is the rotational velocity of the turbine and \( r \) the radial position of the blade element. The blade-normal (stream-wise) and tangential velocity components \( u_n \) and \( u_\theta \), respectively, are sampled in the center of each blade element in the computational domain. The local force per unit length is given by

\[
F = \frac{1}{2} \rho u_{rel}^2 c_\alpha (C_L e_L + C_D e_D),
\]

with \( e_{L,D} \) being the unit vector in the direction of the lift and drag force, respectively, and \( c_\alpha \) being the chord length of the respective airfoil section. The lift and drag coefficients \( C_L \) and \( C_D \) are obtained from tabulated airfoil data as functions of the local Reynolds number \( Re \) and angle of attack \( \alpha \) with

\[
\alpha = \arctan \left( \frac{u_n}{\omega r - u_\theta} \right) + \theta_t + \theta_p,
\]

where \( \theta_t \) is the twist of the airfoil section and \( \theta_p \) the pitch angle of the blade. The resulting forces of each blade element are subsequently projected to the computational domain by means of the convolution of \( F \) with a Gaussian regularisation kernel \( \eta_\epsilon \), given by

\[
\eta_\epsilon = \frac{1}{\pi^{3/2} \epsilon^{3/2}} e^{-(d/\epsilon)^2}.
\]
Here, \( \epsilon \) adjusts the spatial extent of the regularisation and \( d \) is the distance from the centre of the blade element to the point in space where the force is applied. First of all, the regularisation provides a simple practical way to map forces from one discrete space (the blade elements of the actuator line) to another (the computational grid of the CFD simulation). In addition to that, it avoids singularities in the applied force distribution and can therefore be crucial for the numerical stability. Further aspects of the regularisation will be discussed in Section 3.2. Finally, the resulting force is applied at each grid point in the CLBM kernel by adding the respective component of \( \Delta t \frac{F}{2} \) to the pre-collision first-order cumulants. Note, however, that the application of body forces in the LBM can depend on the collision model. See, e.g., Buick and Greated (2000) and Guo et al. (2008) for corresponding formulations in SRT and MRT frameworks, respectively.

The above description illustrates that the differences between ALM implementations in Navier-Stokes and LBM frameworks are somewhat small. After all, the link between the model and the flow solver simply refers to a loose coupling in terms of the macroscopic velocity and body forces. Still, from an implementation point of view it is worth mentioning that the locality of all aforementioned computations of the ALM allows for a perfect parallelisation. The developed LB-ALM is therefore efficiently parallelised on the GPU, in line with the general architecture of the utilised LBM solver using Nvidia’s CUDA toolkit.

### 3.2 Sensitivities of the Aerodynamic Forces

One of the most discussed aspects of the ALM is the impact of the regularisation kernel, Eq. (3.4), and its spatial extent set by the so-called smearing width, \( \epsilon \). The original motivation for the regularisation by Sørensen and Shen (2002) is to avoid singularities in the body force distribution in the flow simulation. Numerical stability is therefore often the determining factor for the choice of \( \epsilon \). For instance, \( \epsilon \geq 2 \Delta x \) is usually reported as a stability condition for common finite-volume Navier-Stokes solvers (Jha et al., 2013; Martínez-Tossas et al., 2015). At the same time, the magnitude of \( \epsilon \) affects the core size of the bound and trailing vortices of the actuator line. A larger \( \epsilon \) increases the viscous core of the induced vortices (Meyer Forsting et al., 2019). Consequently, the vorticity magnitude of the bound and shed vortices decreases with increasing \( \epsilon \). This has implications for both the wake of the modelled turbine and the computation of the forces along the actuator line. As for the wake, more dispersed tip-vortices represent a greater deviation from the actual flow features of the near-wake. However, these differences in the wake characteristics tend to become negligible after the transition to the turbulent far-wake. The waked inflow of other turbines at common downstream distances is therefore typically unaffected, rendering the problem negligible for many practical applications (Trolldborg et al.,
2015a; Weihs et al., 2017). More important is the impact of $\epsilon$ on the force computation by the ALM itself. This dependency has been widely discussed ever since the introduction of the ALM (see, e.g., Mikkelsen, 2003; Trolldborg et al., 2010; Martínez-Tossas et al., 2015). The main issue associated with the regularisation is an overprediction of the forces near the tip and root of the blade when compared to BEM or lifting line simulations. This overprediction generally increases with $\epsilon$. Numerous empirical correction methods have been presented to remedy this issue including the use of classical tip-loss corrections (Shen et al., 2005; Draper and Usera, 2015) or parametrised distributions of $\epsilon$ (Jha et al., 2013; Jha and Schmitz, 2018). Only recently, Meyer Forsting et al. (2019) and Martínez-Tossas and Meneveau (2019), independently, provided a comprehensive fundamental explanation of the underlying mechanism. In brief, the authors show that a trailing vortex with viscous core induces lower velocities than a corresponding inviscid vortex following lifting line theory. As a consequence, the tangential velocity $u_\theta$ sampled by the ALM near the root and tip is overpredicted, leading to an overprediction of $\alpha$ and therefore $C_L$ and $C_D$.

### 3.2.1 Case Set-up

In light of the above, the work presented in Paper I primarily focuses on the numerical sensitivity of the aerodynamic forces to the smearing width $\epsilon$. To this end, simulations of a canonical test case are performed, namely, the NREL 5MW reference wind turbine (Jonkman et al., 2009) operating at a constant tip-speed ratio of $\lambda = 7.55$ in a uniform laminar inflow of $u_0 = 8 \text{ m s}^{-1}$. As a reference, the same case is simulated in the incompressible Navier-Stokes finite volume framework EllipSys3D (Michelsen, 1994a,b; Sørensen, 1995). For the sake of comparison, the simulations are set-up in the most similar manner possible. This includes the use of the same uniform Cartesian grid in the turbine vicinity as well as the choice of boundary conditions. The time step in EllipSys3D is set to meet the common criterion of $\Delta t \leq \Delta x / (\lambda u_0)$ ensuring that the tip of the actuator line does not skip a cell in one time step. Hence, $\Delta t$ is smaller by a factor $1/\lambda$ than the CFL (Courant-Friedrich-Lewy) condition requires. Following Trolldborg et al. (2010), we can therefore assume that both bulk flow and the actuator line forces are time-step-independent. On the other hand, in the LBM, the time step is linked to the Mach number, $Ma$, which should generally lie well below the incompressible limit ($Ma < 0.3$). The aforementioned ALM time step requirement is thereby almost always fulfilled. More explicitly, this requirement can be translated to $Ma / \sqrt{3} < 1/\lambda$ (based on a lattice speed of sound $c_s = \sqrt{1/3} \Delta x / \Delta t$, and $Ma = u_0 / c_s$). For $Ma = 0.1$, for instance, the criterion is thus fulfilled up to $\lambda \approx 17$ which is far beyond the tip-speed ratios encountered during normal turbine operation. Still, as opposed to the solution of the Navier-Stokes solver, the LBM results
exhibit a notable dependency on the time step (or rather Mach number) that is specifically investigated in this study. Further details on the numerical set-ups can be found in the original publication.

3.2.2 Results

The mean tangential and normal blade forces obtained from the two solvers using a spatial resolution of $\Delta x = D/64$ ($D = 126$ m) are compared in Fig. 3.2. Results from a standard BEM method (see, e.g., Hansen, 2008) are provided as an additional reference. As for the largest investigated smearing width of $\epsilon = 3\Delta x$, both the NS- and the LB-ALM exhibit an acceptable agreement with the BEM reference. Both numerical approaches exhibit typical deviations, particularly in $F_t$, from the BEM results near the tip and root. These are primarily related to the viscous core of the trailing vortices, as outlined above. The Mach number dependency of the LBM results is found to be small. Expectantly, the agreement with the BEM reference increases using a smaller smearing width of $\epsilon = 2\Delta x$. On the other hand, the LBM results become more sensitive to the choice of Ma. As mentioned earlier, most Navier-Stokes approaches suffer from numerical instabilities if $\epsilon$ is chosen too low. The effect of such instabilities on the mean forces of the NS-ALM can be observed with $\epsilon = 1\Delta x$, where notable deviations from the BEM reference occur along the entire actuator line.
At the same time, spurious oscillations are found in the instantaneous forces as well as the surrounding flow field. As for the LB-ALM, severe deviations from BEM can also be observed for the largest Mach number (Ma = 0.1). However, the forces are again found to converge towards the BEM reference as Ma is lowered. Moreover, deteriorations of the flow field can not be observed at any Mach number.

3.2.3 Concluding Remarks

In summary, this initial investigation reveals two main aspects of the LB-ALM. Firstly, at sufficiently low Mach numbers and common choices for the smearing width (\(\epsilon \geq 2\Delta x\)) the model behaves similarly to the standard Navier-Stokes implementation. Also, it illustrates the general suitability of the CLBM for actuator line simulations. Note, for instance, that other collision models tested prior to this study (SRT and MRT) suffered from severe numerical instabilities. Secondly, the results show that \(\epsilon < 2\Delta x\) is numerically feasible when using the CLBM. Nonetheless, this comes at the cost of lower Mach numbers in order to keep deviations from BEM in reasonable bounds.\(^1\)

3.3 Analysis of the Wake Characteristics

A detailed analysis of the wake characteristics of the ALM is presented in Paper II. The analysis is again based on a code-to-code comparison to the incompressible Navier-Stokes solver EllipSys3D using the same test case as in Paper I. However, this study focusses on the parametrised CLBM. This choice is primarily motivated by the higher accuracy of the scheme when compared to the regularised version. In addition, a standard Smagorinsky SGS model is applied in both solvers with a model-constant of \(C_s = 0.08\), similar to previous investigations of the case (Martínez-Tossas et al., 2018; Deskos et al., 2019).

3.3.1 Wakes in Laminar Inflow

The first part of the study is concerned with the wake characteristics in uniform laminar inflow. The simplicity of this case eliminates various uncertainties associated with more complex, yet, possibly more realistic (e.g., sheared and/or turbulent) inflow conditions. Moreover, it facilitates the analysis of the effect of the numerical scheme on the downstream evolution of the wake,

\(^1\) Corrigendum to paper I: In the discussion in the original publication the occurring Mach number dependency was presumed to be related to compressibility effects. The findings from Paper III (see Section 3.4) indeed corroborate this conjecture. Yet, the term “compressibility error” was also inaccurately linked to the cubic velocity error, mentioned in Section 2.1, and used interchangeably.
and particularly the onset of turbulence (Abkar, 2018). Three grid resolutions $\Delta x = \{D/16, D/24, D/32\}$ are considered in the comparison. The smearing width is kept constant in all simulations with $\epsilon = 0.078125\, D$, referring to $\epsilon/\Delta x = \{1.25, 1.875, 2.5\}$, respectively. The Mach number in all CLBM cases is set to $Ma = 0.1$. Because of the sufficiently large $\epsilon$, differences in the blade forces between the three CLBM cases were found to be sufficiently low not to affect the wake comparison.

A general impression of the wake of all simulated cases can be obtained from the contour plots of the instantaneous stream-wise velocity $u$ shown in Fig. 3.3. For all investigated grid resolutions, the near-wake is given by a continuous vortex sheet. Some distance downstream, depending on the grid resolution and the numerical approach, the vortex sheet starts to meander and eventually transitions to the turbulent far-wake. Fig. 3.4 provides a comparison of selected cross-stream profiles of the mean stream-wise velocity $\bar{u}$. In the near-wake, the two approaches agree closely in terms of $\bar{u}$. Up until $x = 6\, D$ the differences between the two numerical approaches at each grid resolution amount to less than 1.5%, measured in terms of the $L^2$-relative error norm along the profiles. The differences in the tangential velocity (not shown here for the sake of brevity) are somewhat higher with about 5%. The latter relates to larger discrepancies in the tangential forces along the ALM between the two approaches. After the transition, the wake recovery accelerates while the mean stream-wise velocity approaches the typical self-similar Gaussian far-wake profile. In this part of the wake, the differences in $\bar{u}$ increase significantly due to the different downstream positions of the points of transition. Along with the transition of the wake, the turbulence intensity (Ti) grows by several orders of magnitude, as exemplified in Fig. 3.5. At every grid res-
Figure 3.4. Cross-stream profiles of the mean stream-wise velocity component, $\bar{u}$, at different downstream positions. Figure adapted from Paper II.

Figure 3.5. Stream-wise evolution of the turbulence intensity $T_i = \frac{1}{u_0} (\overline{u_i' u_i'})^{0.5}$ at $r/D = 0.625$. For the legend, see Fig. 3.4. Figure adapted from Paper II.
olution Ti increases earlier in the CLBM than in the NS case. Furthermore, the turbulence intensity grows faster with downstream distance the higher the spatial resolution.

In the absence of ambient turbulence, the perturbations triggering the transition of the wake grow within the wake itself, starting from infinitesimal magnitudes. Consequently, the point of transition depends on the growth of such perturbations and where they eventually reach a critical magnitude. Any damping of the perturbations therefore delays the onset of turbulence in the wake. This damping is affected by two main factors. One is the added dissipation due to the sub-grid scale model, as shown, e.g., by Sarlak et al. (2015) or Abkar (2018). And second, is the numerical diffusivity of the scheme itself. As for the latter, Martínez-Tossas et al. (2018) show that schemes with a lower numerical diffusivity (pseudo-spectral approaches in that study) lead to a notably faster growth of turbulence in the wake than lower-order schemes with a higher numerical diffusivity (second-order finite volume solver, equivalent to the Navier-Stokes approach used here). The same interpretation can be applied to the results presented here. The parametrised CLBM provides fourth-order accuracy in diffusion as opposed to the second-order accuracy of the Navier-Stokes solver. Furthermore, it is briefly shown in Paper II that the second-order accurate regularised CLBM shifts the transition further downstream when compared to the parametrised CLBM. In line with the literature, the occurring differences in the far-wake can thus be attributed to the different numerical diffusivities of the schemes.

3.3.2 Wakes in Turbulent Inflow

Based on the same set-up, a comparison in turbulent inflow is presented in the second part of the study. Generally, inflow turbulence accelerates the transition of the wake while reducing the dependency of the point of transition on the numerical diffusivity of the scheme (Sørensen et al., 2015; Martínez-Tossas et al., 2018). This facilitates a direct comparison of the statistics in the far-wake, complementing the initial comparison in laminar inflow. For the sake of brevity, only one case with a resolution of $\Delta x = D/32$ is compared in this part. The inflow turbulence is directly imposed at the inlet in both solvers. To this end, the uniform inlet velocity is perturbed by synthetic homogeneous isotropic turbulence that is pre-generated using the method by Mann (1998). The turbulence intensity at the inlet measures about $2.3\%$ and the turbulent length scale is $L = 40\, m = 0.317\, D$. In the Navier-Stokes simulation, the imposed turbulence is found to decay slightly faster. Yet, the overall differences throughout the domain are deemed sufficiently small for the intended comparison of the wakes.

Cross-stream profiles of the mean stream-wise velocity and turbulence intensity are depicted in Fig. 3.6. As for $\bar{u}$, an excellent agreement is found
between the two solutions. When compared to the laminar cases this not only applies to the near-wake but the entire domain. More specifically, the differences in $\bar{u}$ between the cross-stream profiles of the two approaches for $x \leq 12D$ amount to less than 1%. While steadily increasing with downstream distance, the maximum discrepancy measures 1.6% at the far end of the domain ($x = 24D$). A similar agreement is found for $Ti$. Most importantly, it can be observed that the transition of the wake is triggered at very similar downstream positions. This also explains the significantly better match in terms of the velocity. In contrast to the laminar inflow case, the wake is immediately affected by finite-size perturbations in the form of the inflow turbulence. Therefore, the impact of the dissipative characteristics of the numerical scheme becomes subordinate.

Lastly, the spectra of the turbulence kinetic energy (TKE) at three different downstream positions are compared in Fig. 3.7. With both numerical approaches a distinct peak can be observed at the blade-passing frequency $f_B = (3u_0\lambda)/(\pi D) = 0.458$ Hz and its higher harmonics in the near-wake ($x =
Figure 3.7. One-point turbulence kinetic energy spectra in the near-wake ($x = 1 \, D$, left), transition-region ($x = 6 \, D$, center) and far-wake ($x = 18 \, D$, right) in turbulent inflow. Vertical dashed-dotted line marks the blade-passing frequency $f_B$. For the legend, see Fig. 3.6. Figure adapted from Paper II.

1 $D$). From the velocity profiles in Fig. 3.6 it can be inferred that the transition of the wake occurs between $x = 3 \, D$ and $x = 6 \, D$, characterised by the change from a typical near-wake to a Gaussian far-wake profile. In the spectra, this is reflected by an increase in the energy level across all resolved frequencies. Furthermore, the peak at $f_B$ vanishes. Moving further downstream ($x = 18 \, D$) the TKE decreases due to the continuous decay of both ambient and far-wake turbulence. When compared to the previous position, the energy content at smaller scales increases slightly relative to the larger scales. This relates to the continuous break-down of the turbulent structures of the wake along the energy cascade. This relative energy increase at higher frequencies appears to be more pronounced in the CLBM solution. Again, this might relate to the higher dissipation found in the NS solver inducing an earlier cut-off in the sub-inertial range.

3.4 Aspects of Compressibility

The standard LBM recovers the compressible Navier-Stokes equations but lacks an explicit formulation of the total energy.\(^2\) Temperature fluctuations can therefore not be captured, limiting applications to the quasi-isentropic regime and weakly compressible phenomena such as acoustics (Sagaut and Cambon, 2018). For incompressible problems ($\text{Ma} < 0.3$) the weak compressibility is usually seen as a mere numerical artefact and assumed to have negligible effects on the flow physics of interest. For instance, the statistics of the boundary layer flows discussed in Chapter 4 remain almost unaltered up to $\text{Ma} = 0.15$,

\(^2\)By standard we refer to the LBM on classical single-speed lattices. Note, however, that fully compressible extensions of the LBM exist, e.g., using multi-speed lattices (Frapolli et al., 2015) or hybrid approaches involving additional solvers for the energy conservation (Feng et al., 2019).
while the real-world Mach number of the problem is about one order of magnitude lower. In practice, this implies that incompressible problems are often simulated at even higher Mach numbers than the actual physical value. The simple motivation for this is the lower computational cost associated with the larger time steps at larger Mach numbers (Krüger et al., 2016). As opposed to plain flow simulations, the ALM (and particularly the results summarised in Section 3.2) exhibits a comparably strong sensitivity to the Mach number that can eventually also affect the wake flow. This strong sensitivity motivates further assessments of the effects of compressibility on the ALM that are outlined in Paper III.

An additional motivation for this investigation originates from a recent work by Yan and Archer (2018) that touches upon similar phenomena but from a different perspective. The study compares actuator line simulations of both compressible and incompressible solvers in order to assess the effects of compressibility on the turbine performance and wake properties. The authors report notably smaller power production, velocity deficits and TKE for the compressible framework, even under normal operational tip-speed ratios that do not imply high local Mach numbers in the blade vicinity. These differences are argued to be related to an upstream reduction of kinetic energy due to compression. However, with turbulent Mach numbers, $M_t = \sqrt{k}/c_s$ (where $k$ is the TKE), of $O(10^{-3})$ this interpretation is arguably bold. After all, both theory and fundamental studies of compressible turbulence associate $M_t < 0.1 \ldots 0.3$ with the quasi-isentropic regime where interactions of dilatational and solenoidal components are weak (Chassaing et al., 2002; Donzis and Jagannathan, 2013; Bull and Jameson, 2015; Sagaut and Cambon, 2018). An alternative assumption based on Paper I is that the turbine performance, and hence the wake, differ due to erroneous force calculations of the ALM in the compressible framework rather than physical compressibility effects.

3.4.1 Case Set-up

Due to the large sensitivity of the blade forces on both $\epsilon$ and $Ma$ it appears inevitable to investigate compressibility effects of the ALM and the wake separately. To this effect, we firstly perform actuator line simulations with different speeds of sound $c_s$ (hence, different $Ma$) while prescribing the aerodynamic forces along the actuator line. In this way, potential differences in the wake flow that originate from the force calculation of the ALM, and not from compressibility effects within the flow, can be avoided. In a second set of simulations we employ the standard ALM. This suite of simulations is then contrasted against the former in terms of the differences in the wake. Furthermore, it serves as a basis for further investigations of the Mach number sensitivity of the forces. In both simulation sets $c_s$ is varied between twice the physical speed of sound $c_s = 680\,\text{m}\,\text{s}^{-1}$ (serving as a near-incompressible reference,
with \( \text{Ma}_{0.5 \text{pc}} = u_0 / c_s = 0.017 \), a Mach-matched case where \( c_s \) refers to the physically correct (PC) speed of sound (\( \text{Ma}_{pc} \)), as well as additional cases with a higher compressibility with a half and a quarter the Mach-matched speed of sound (\( \text{Ma}_{2pc} \) and \( \text{Ma}_{4pc} \), respectively). The test case refers to a single turbine (NREL 5MW reference wind turbine) in a sheared turbulent inflow. With the chosen inlet velocity profile the turbine operates at rated power, referring to a hub height velocity of \( u_0 = 11.4 \text{m s}^{-1} \) and \( \lambda = 7.55 \). This point of operation implies the highest possible local Mach numbers in the blade vicinity excluding above-rated conditions. The turbulence intensity at hub height measures \( Ti_0 = 2.8\% \). All cases are simulated with two different smearing widths \( (\epsilon = \{1, 2\} \Delta x) \).

### 3.4.2 Mach Number Sensitivities With Prescribed Body Forces

Fig. 3.8 provides an exemplary illustration of the Ma-related differences in the wake for the cases with \( \epsilon = 1 \Delta x \) and prescribed blade forces. The most significant velocity differences can be found in the near-wake for the high Ma cases. In particular, the velocity in the upper (and lower) shear layer is lower (higher) than in the reference case. This feature gets more pronounced the higher the Mach number. Comparatively, no clear trend can be observed in the far-wake due to a larger scatter in the differences. The turbulence intensity in
Figure 3.9. Spatial mean of the difference in $\bar{u}$ (top) and $Ti$ (bottom) towards the reference $Ma_{0.5pc}$ across cross-sectional planes at different downstream positions. Figure adapted from Paper III.

The near-wake decreases with larger $Ma$, while the differences in the far-wake are similarly scattered as for $\bar{u}$. Only for $Ma_{4pc}$, a clear region of higher $Ti$ is found in the entrainment zone of the far wake.

A more quantitative picture of the occurring differences can be obtained from Fig. 3.9, showing the spatial mean of the velocity and turbulence intensity differences ($\langle|\Delta \bar{u}|\rangle$ and $\langle|\Delta Ti|\rangle$, respectively) over cross-sectional planes of $2D \times 2D$ encompassing the rotor-swept area. The same evaluation is also presented for corresponding cases without turbine to estimate $Ma$-related differences in the background flow. The latter could also be related to changes in compressibility or error terms scaling with $Ma$. These ambient flow cases (no turbine) show only a weak sensitivity to $Ma$. Even for the highest Mach number the velocity difference towards the near-incompressible reference remains below 0.5% in the entire domain, while the differences in $Ti$ are about one order of magnitude higher. In the presence of the ALM (with prescribed forces) the $Ma$-related differences grow noticeably throughout the near-wake while remaining somewhat constant in the far-wake. Furthermore, turbine-induced compression/expansion only appears to affect the flow downstream of the turbine. A significant sensitivity to $\epsilon$ is only found for the turbulence intensity differences in the rotor plane ($x/D = 0$), which increase with $Ma$. This is generally to be expected, since smaller $\epsilon$ values imply higher local pres-
ure gradients, which in turn lead to larger changes in density the higher the Mach number. It is noteworthy, however, that $\epsilon$ only mildly affects the flow downstream of the rotor.

The relative dilatation $a$ allows for a more explicit identification of flow regions that are strongly affected by compressibility. Given by

$$a = \frac{\partial u_i}{\partial x_i} \left( \frac{\partial u_n}{\partial x_m} \frac{\partial u_n}{\partial x_m} \right)^{-\frac{1}{2}},$$

(3.5)

it refers to the divergence of the velocity normalised by the magnitude of the velocity gradient tensor. Algebraically, $a$ is limited to $\{-\sqrt{3}, \sqrt{3}\}$, while negative values refer to compression (decrease in volume) and positive values to expansion (increase in volume). A selection of statistics of $a$ for the cases with prescribed rotor loads are shown in Fig. 3.10. In the wake, the mean relative dilatation, $\bar{a}$, exhibits only small departures from zero in regions of high turbulence intensity. Even for $\text{Ma}_{4pc}$, the maximal values are $O(10^{-3})$ which is arguably negligible with regards to fundamental studies of compressible turbulence (Lee et al., 2009; Suman and Girimaji, 2010). Thus, the wake flow falls consistently into the weakly compressible regime. In comparison, the mean dilatation in the rotor vicinity is distinctly higher but remains low in absolute terms. However, the PDF of $a$ reveals an intermittent occurrence of non-negligible dilatations. Particularly, the cases with $\text{Ma}_{4pc}$ show secondary peaks of relatively large positive and negative values. Moreover, the spectra of $a$ (not shown for the sake of brevity) confirm the obvious assumption that these peaks are associated with the passage of the blade. This also explains the lower magnitudes of the peaks with $\epsilon = 2\Delta x$.

In summary, the analysis reveals that non-negligible compressibility (that formally also violates the scope of the LBM) only occurs in the direct vicinity of the ALM in cases of very low artificial speeds of sound ($\text{Ma}_{4pc}$). Nonetheless, even these large local relative dilatations have only little effects on the wake, which can arguably be appreciated for applications of the ALM in weakly compressible frameworks. In addition, it is shown that under Mach-matched conditions even the flow in the direct blade vicinity remains in the weakly-compressible regime, including small to negligible differences in the overall flow-field towards (near-) incompressible simulations.

### 3.4.3 Mach Number Sensitivities of Standard Actuator Line Simulations

Using the standard (fully coupled) ALM and $\epsilon = 2\Delta x$ similar Ma-related changes in the flow can be observed as in the cases with prescribed forces (see Fig. 3.9, right column). On the other hand, with $\epsilon = 1\Delta x$ the differences in the rotor plane and near-wake grow significantly. In line with the findings from Paper I, the higher deviations in the wake go along with corresponding changes
Figure 3.10. Comparisons of the relative dilatation, $a$, with prescribed rotor loads. Top: vertical profiles of the mean relative dilatation $\bar{a}$ at different downstream positions. The turbulence intensity, $T_i$, with $Ma_{pc}$ is plotted with respect to the upper abscissa. The dashed grey line marks $z_{hub}$. The rotor-swept area is shaded in light grey. Bottom: Probability density function of $a$ at the upper edge of the rotor-swept area ($x = -\Delta x$, $y = 0$, $z = z_{hub} + R$). Note, that $\bar{a}$ is shown for $x = -\Delta x$ instead of the rotor plane ($x = 0$) as the latter corresponds to the zero-crossing of $\bar{a}$ in the stream-wise direction. Figure adapted from Paper III.
in the aerodynamic forces of the rotor. The relative difference between the tangential forces $F_t$, and the reference ($e_r(F_t) = F_t/F_{t,0.5pc} - 1$) are shown in Fig. 3.11 along with a corresponding comparison of the three determining parameters for $F_t$ (see Eq. (3.2)), i.e. the density $\rho$, the relative velocity $u_{rel}$ and the angle of attack $\alpha$. Expectantly, the influence of $\text{Ma}$ and $\epsilon$ on the magnitude of $u_{rel}$ is small as it is dominated by $\omega r$ which is prescribed via a constant tip-speed ratio in this case. Also the differences in the density only amount to a maximum of 0.2%. Hence, changes in the local angle of attack can be identified to be mainly responsible for the differences in the blade forces, based on both magnitude of $e_r(\alpha)$ and its correlation with $e_r(F_t)$. The well-known overpredictions of the ALM forces near the root and tip have been discussed in Section 3.2 as well as the responsible modelling issues, namely spurious velocity inductions of the trailing vortices causing changes in the angle of attack. For the cases discussed here, it is shown that the combination of small $\epsilon$ and large $\text{Ma}$ evokes an additional error in the angle of attack. However, this error is not concentrated at the tip or root. It therefore appears to originate from changes to the bound vortex of the actuator line. This in turn affects the sampled velocity away from the tip and root. Indeed, preliminary (unpublished) investigations of the vorticity along the actuator line seem to confirm this conjecture. In summary, the analysis provides an initial understanding of the mechanism causing the $\text{Ma}$-dependency of the actuator line blade forces firstly reported in Paper I. Moreover, it illustrates that investigations of compressibility effects in the context of actuator line simulations as presented by Yan and Archer (2018) should be taken with due care. After all, differences in

\[Figure 3.11.\] Relative difference $e_r$ of the tangential force $F_t$, angle of attack $\alpha$, density $\rho$ and relative velocity $u_{rel}$ with respect to the near-incompressible reference case. For the legend, see Fig. 3.10. Figure adapted from Paper III.
the wake or turbine performance do not necessarily relate to compressibility effects in the flow but rather compressibility-related errors in the ALM.

3.5 Validation Against Full-scale Measurements

Quantifying the accuracy of wind turbine modelling approaches is arguably challenging. On the one hand, this is due to the complex multi-physics, multi-scale nature of the problem itself. On the other hand, measurement campaigns that provide comprehensive data of the inflow and wake of utility-scale turbines, their power, and preferably structural response are costly, technically challenging, and therefore rare. Investigations of model accuracies or sensitivities thus often rely on the comparison to other numerical solutions, as presented in Sections 3.2 and 3.3, or wind-tunnel experiments (see, e.g., Porté-Agel et al., 2011; Krogstad et al., 2015; Lignarolo et al., 2016). Still, both typically imply various simplifications of real-world conditions and are prone to inaccuracies and/or scaling issues themselves. Comparisons against full-scale measurements thus remain inevitable for the assessment of wind farm modelling tools that are meant to capture the interplay of the ABL, wakes and the response of the turbine. To date, the majority of full-scale validations has been made against readily available SCADA (Supervisory Control And Data Acquisition) data such as power or wind speed measurements from nacelle anemometers (Nilsson, 2015; Sebastiani et al., 2020). The interpretation of such comparisons though is often difficult because of the lack of information about the undisturbed inflow or wake.

One of the most detailed datasets of a full-scale turbine available today was measured in the DanAero experiment at the Tjæreborg wind farm in Denmark. The experiment comprised a met mast within the farm as well as various sensors monitoring one of the eight 2.3-MW turbines. Over the past decade, the DanAero dataset served a multitude of validation studies covering aspects like aerodynamic blade loads (Madsen et al., 2018; Bangga and Lutz, 2021), aeroelastic effects (Grinderslev et al., 2021a, b), or transition modelling (Özçakmak et al., 2020), to name a few. Still, the dataset remains almost unexploited when it comes to the turbine response in waked inflow conditions. Indeed, to the author’s knowledge, detailed full-scale validations of such cases remain generally untouched. Nonetheless, simulating the response of a turbine in waked conditions represents one of the most stringent tests of the capabilities of a model framework. Eventually, it scrutinises the comprehensive accuracy in modelling the interaction of the ABL, the wakes of upstream turbines and the response of the turbine of interest. As part of a comprehensive model comparison, six different frameworks, including the one presented in this thesis, were benchmarked against two wake inflow cases from the DanAero experi-
ment. The results of the comparison are discussed in Paper VI.\textsuperscript{3} For the sake of brevity, only one of the cases will be summarised in this section. It should be noted, however, that the main conclusions drawn from both cases are similar, despite various small differences in the model performances. For further details it is referred to the original publication.

3.5.1 Experimental Data

The Tjæreborg wind farm, located at the west coast of Denmark, comprises eight 2.3-MW NM80 turbines that are placed in two rows of four turbines each. The turbine hub height and diameter are $H = 57$ m and $D = 80$ m, respectively. The surrounding terrain is mostly flat agricultural land with low roughness. A schematic of the farm is provided in Fig. 3.12. A brief description of the measured data used in the benchmark is given in the following.

The inflow is characterised by measurements of the sonic anemometers mounted at three heights $z = \{17, 57, 93\}$ m along the met mast. The lateral offset of the mast with respect to the wake region ensures mostly undisturbed inflow measurements. Still, blockage effects of the nearby turbines or influences of the wake of WT1 can obviously not be ruled out. Additional temperature measurements along the mast only served the initial case selection ensuring near-neutral atmospheric stability.

\textsuperscript{3}The benchmark was organised within the IEA (International Energy Agency) Wind Task 29 along with several other comparisons to the DanAero experiment. A comprehensive summary of the project can be found in Schepers et al. (2021).
The aerodynamic forces acting on four sections of one of the blades of WT2 are derived from local pressure measurements. In the DanAero project, 64 pressure taps were embedded along chord-wise rows at four radial stations \( r = \{13, 19, 30, 37\} \) m on the pressure and suction side of the blade. Integrating the pressure along the rows provides an estimation of the local aerodynamic force acting on the respective section (Madsen et al., 2010a). Furthermore, data of the local blade inflow are available at \( r = 20.3 \) m, measured by a Pitot tube about one chord length upstream of the leading edge.

In addition to the data from the DanAero instrumentation, the SCADA data of WT2 was recorded at 35 Hz. Unfortunately, only 10-min averages are available for the upstream turbine. For the benchmark, the electrical power logged by the SCADA system is converted to the mechanical power of the rotor \( P \) using the mechanical and electrical efficiency obtained from torque measurements on the main shaft reported by Bak et al. (2013).

### 3.5.2 Benchmark Definition and Case Set-up

Considering multiple criteria (near-neutral stability, suitable mean wind direction, etc.) individual 10-min bins are selected as benchmark cases. In the case discussed here, the mean wind direction at hub height is \( \theta = 251.08^{\circ} \) implying a partial wake inflow for WT2. The mean ambient wind speed at hub height is \( \bar{u}_{57} = 8.40 \text{ m s}^{-1} \) with a turbulence intensity of 6.98%. For the sake of comparability, a common turbulent inflow field was pre-generated for all participants of the benchmark using the PyConTurb toolbox (Rinker, 2018). PyConTurb-generated turbulence is based on the Kaimal spectrum with enforced exponential coherence following multiple user-defined constraints, in this case the velocities measured by the met mast. The only choice left to the modellers was a uniform scaling of the imposed turbulence. Such modifications are often required in LES to match the turbulence statistics at reference points within the domain (in this case the locations of the sonic anemometers) due to the downstream evolution and decay of imposed synthetic turbulence (Gilling and Sørensen, 2011; Keck et al., 2014a; van der Laan et al., 2019).

A further constraint was to simulate both turbines with a fixed \( \omega \) and yaw angle (referring to the respective mean of the measured values of the bin). This way, potential differences between the models originating from the implementations of the controller, or the controller response to differences in the inflow, can be ruled out.

A detailed schematic of the two turbines simulated in the case, including the definition of the coordinate system, is provided in Fig. 3.13. All remaining choices regarding the numerical set-up (such as grid resolution, domain size, boundary conditions, turbulence models, etc.) were left to the participants.

The parametrised CLBM is employed for the lattice Boltzmann simulations of the case. Sub-grid scales are modelled with the standard Smagorinsky model
Figure 3.13. Illustration of the case set-up. The coordinate origin is located at the turbine center of WT2 at \( z = 0 \) (ground level), with \( x \) being the stream-wise direction, \( y \) being the lateral direction and \( z \) pointing upwards (not shown here). The turbulence inflow planes (dashed black line) measure \( 7.5D \) in both lateral and vertical direction. The exact upstream position of the inlet depends on the model. Wake profiles are sampled along the solid grey lines upstream of WT2 (as well as corresponding vertical lines). Figure adapted from Paper VI.

\( (C_s = 0.1) \). A symmetry boundary condition is applied at the top of the domain (simple bounce-forward; Krüger et al., 2016). The bottom boundary condition is a SBB scheme coupled to the iMEM wall modelling approach outlined in Chapter 4. Periodic boundary conditions are applied in the lateral direction. At the outlet we employ a linear extrapolation boundary condition that is combined with a viscous sponge layer extending one \( D \) upstream. The grid resolution in the turbine vicinity and wake region is \( \Delta x/D = 1/32 \). The Mach number is \( Ma = 0.1 \). The smearing width in the ALM is set to \( \epsilon = 1.5\Delta x \). A brief overview of all participating models in given in Table 3.1. Further details can be found in Paper VI.

Table 3.1. Summary of the models participating in the benchmark, hereafter referred to by the given abbreviations. The subscripts of the abbreviations refer to the respective institution, i.e. Uppsala University (UU), Denmark’s Technical University (DTU), and the National Renewable Energy Laboratory (NREL). LES\(_{\text{UU,pc}}\) refers to the same model as LES\(_{\text{UU}}\) but using a precursort-generated inflow as opposed to the one defined for the benchmark. The given grid resolution \( \Delta x/D \) refers to the one of the wake region. Depending on the model, different grid topologies and spacings have been employed in the rest of the domain.

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Numerical Method</th>
<th>Turbine Model</th>
<th>Code</th>
<th>( \Delta x/D )</th>
<th>( \epsilon/\Delta x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWM(_{\text{NREL}})</td>
<td>DWM</td>
<td>BEM</td>
<td>FAST.Farm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DWM(_{\text{DTU}})</td>
<td>DWM</td>
<td>BEM</td>
<td>DTU-DWM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RANS(_{\text{UU}})</td>
<td>RANS</td>
<td>ADM</td>
<td>OpenFOAM</td>
<td>1/20</td>
<td>2.0</td>
</tr>
<tr>
<td>LES(_{\text{UU,LBM}})</td>
<td>LES (LBM)</td>
<td>ALM</td>
<td>elbe</td>
<td>1/32</td>
<td>1.5</td>
</tr>
<tr>
<td>LES(_{\text{DTU}})</td>
<td>LES</td>
<td>ALM</td>
<td>EllipSys3D</td>
<td>1/60</td>
<td>2.5</td>
</tr>
<tr>
<td>LES(<em>{\text{UU}}) / LES(</em>{\text{UU,pc}})</td>
<td>LES</td>
<td>ALM</td>
<td>OpenFOAM</td>
<td>1/32</td>
<td>2.0</td>
</tr>
</tbody>
</table>

54
Figure 3.14. Comparison of the mean stream-wise velocity, $\bar{u}$ (a), and resolved TKE (b) along the vertical at the met mast location normalised by the mean stream-wise velocity of the sonic at hub height, $u_0$. The error bars of the sonics denote the uncertainty estimated via the integral length scale of the respective velocity component using the method by Lenschow et al. (1994). Note, that the steady-state RANS simulation ($\text{RANS}_{\text{UU}}$) only provides sub-grid scale turbulent quantities that are not compared here for the sake of brevity. Figure adapted from Paper VI.

3.5.3 Results

A selection of the results from the benchmark case will be outlined in the following. The section follows a sequential downstream order starting from a comparison of the inflow, followed by the power and thrust of WT1 and finally the wake inflow, power and loads of WT2.

A comparison of the ambient flow along the met mast is shown in Fig. 3.14. The mean velocity and TKE in the DWM simulations closely match with the statistics of the imposed inflow and, hence, the measured values. This is generally to be expected since the inflow turbulence in DWM models is passively advected through the domain and only evolves when being superimposed by wakes. Also the mean velocity of the LES cases rather closely matches with the met mast measurements. The most notable deviations are somewhat elevated velocities near the ground in the LES$_{\text{DTU}}$ case which can be attributed to the symmetry boundary condition applied at the bottom. On the other hand, the TKE in the LES cases only falls within the bounds of the measurement uncertainty at hub height. This highlights the difficulty in matching the turbulence statistics within the domain when using synthetic turbulence in LES.

The mean power $\bar{P}$ and thrust $\bar{T}$ of WT1 along with the corresponding standard deviations are contrasted in Fig. 3.15. The mean values are plotted against the average of the mean stream-wise velocity at the positions of the three son-
Figure 3.15. Comparison of the mean power, $\bar{P}$ (a), and thrust force, $\bar{T}$ (c), of WT1 plotted against the averaged mean stream-wise velocity, $\langle \bar{u} \rangle_z$. The error bar of the measured velocity corresponds to the measurement uncertainty as described in Fig. 3.14. The corresponding standard deviation, $\sigma(P)$ (b), and $\sigma(T)$ (d), are plotted against the averaged stream-wise velocity variance, $\langle u' u' \rangle_z$. Note, that neither thrust measurements, nor $\sigma(P)$ is available from the experiment. Figure adapted from Paper VI.

ics $\langle \bar{u} \rangle_z$, serving as an estimate of the mean inflow across the rotor-swept area. The standard deviations are plotted against the corresponding average velocity variance $\langle u' u' \rangle_z$. All models predict a smaller mean power than the SCADA data, ranging from $-0.8\%$ relative error for $\text{LES}_{UU}$ to $-18.3\%$ for $\text{DWM}_{\text{NREL}}$. Generally, $\bar{P}$ and $\bar{T}$ show a weak correlation with the mean inflow velocity which potentially explains some of the differences towards the measurement. A slightly stronger correlation is found for the standard deviations and $\langle u' u' \rangle_z$.

Vertical and horizontal profiles of the wake characteristics of WT1 are depicted in Fig. 3.16. With increasing downstream distance the wake center shifts laterally as the mean wind direction is not aligned with the $x$-axis (see Fig. 3.13). WT2 is eventually subject to a half-wake inflow in all simulations. Four diameters upstream of WT2, all models except $\text{RANS}_{UU}$ exhibit a typical near-wake velocity profile. A clear correlation between the magnitude of the velocity deficit and the thrust of WT1 cannot be observed. Other differences between the models, e.g., the smearing width of the ALMs, thus seem to dominate these discrepancies. At $x = -2D$, the velocity profile in all cases approaches a Gaussian far-wake state. Hence, in all LES and DWM models the laminar-turbulent transition of the wake occurs at a similar downstream distance. In the $\text{RANS}_{UU}$ case the wake recovery remains faster than in the other simulations, which points towards a higher numerical and/or sub-grid scale dissipation. One diameter upstream of WT2, all LES models and $\text{DWM}_{\text{NREL}}$ are
Figure 3.16. Wake characteristics upstream of WT2. First row: horizontal profiles at hub height of the mean stream-wise velocity, $\bar{u}$. Second row: vertical profiles of $\bar{u}$ through the hub. Third and fourth row: Corresponding profiles of the stream-wise variance, $\overline{u' u'}$. The rotor-swept area is shaded in grey. Note, that not all models could sample velocities in the rotor plane. The measurements of the cup anemometer and Pitot tube are therefore compared against data at $-1 \, D$. Figure adapted from Paper VI.
in quite close agreement in terms of $\bar{u}$, while DWM_{DTU} and RANS_{UU} predict a somewhat larger and smaller velocity deficit, respectively. Unfortunately, the experimental data of the wake is limited to the velocity measurements from the nacelle cup anemometer and the blade-mounted Pitot tube. Thus, the reference data points are not only limited in space, but also entail comparatively large uncertainties. Still, it can be appreciated that most models are in close agreement with the nacelle anemometer as well as the Pitot tube at an azimuth angle of $\phi = 270^\circ$ (positive $y$-direction). At $\phi = 90^\circ$ (negative $y$-direction) and in the vertical profiles, a considerably larger velocity deficit is measured by the Pitot tube. The discrepancies between the models in terms of $\overline{u' u'}$ are generally larger, particularly in the near wake. However, further downstream, both magnitude and shape of the $\overline{u' u'}$ profiles converge. Furthermore, they eventually match somewhat closely with $\overline{u' u'}$ measured by the Pitot tube.

Similarly to Fig. 3.15, we compare the power and thrust of WT2 in Fig. 3.17. However, the means and standard deviations are now plotted against the statistics of the blade-normal velocity $u_n$, at the Pitot tube location. These represent the most suitable inflow data for WT2 that are available from both simulations and measurements. When compared to Fig. 3.15, the modelled mean power correlates significantly better with the inflow metric $\bar{u}_n$. The consistent over-prediction of $\bar{P}$ by most models consolidates that the modelled wake velocities are indeed too large, as previously indicated by the results in Fig. 3.16. Similarly to WT1, a good correlation is found for $\sigma(P)$ (and $\sigma(T)$) and $\overline{u'_n u'_n}$. 

Figure 3.17. Comparison of the mean power, $\bar{P}$ (a), and thrust force, $\bar{T}$ (c), of WT2 vs. the mean blade-normal velocity, $\bar{u}_n$. The corresponding standard deviations, $\sigma(P)$ (b) and $\sigma(T)$ (d), are plotted against the stream-wise velocity variance, $u'_n u'_n$. Figure adapted from Paper VI.
Furthermore, the $\sigma(P)$ from the SCADA data agrees closely with the trend emerging from the simulations.

Lastly, we turn to the aerodynamic forces along the blade of WT2. The mean tangential and normal component are compared against the data of the pressure probes in Fig. 3.18. In the mid third of the blade, DWM_{NREL} and LES_{UU,LBM} show the best agreement with the measured tangential force. Near the tip, all LES and RANS models except LES_{DTU} clearly overestimate $F_t$. Generally, LES_{DTU} predicts a smoother decrease in $F_t$ towards the tip, which better reflects the overall trend of the measurements. Referring to the discussions by Meyer Forsting et al. (2019), this behaviour can be attributed to the more advanced tip correction model. The relative differences between the models in the normal force are considerably smaller. At the same time, $F_n$ is consistently overestimated in comparison to the measurements.

An exemplary impression of the azimuthal variation of $F_n$ can be obtained from the Fig. 3.19. Here, we subtract the overall mean $F_n$ from the mean of each azimuthal bin $\bar{F}_n^\phi$. This way, the comparison focuses on force variations due to the wake shape while excluding potential offsets related to, e.g., differences in the mean inflow or sensor calibrations. The mean force exhibits a clear maximum at the right-hand side of the rotor plane (seen from upstream), referring to about 90° azimuth, and a corresponding minimum at $\phi \approx 270°$. The phase of this azimuthal oscillation is quite closely captured by most models, meaning the minima and maxima are found at similar azimuth angles. In contrast, the magnitudes of both minima and maxima differ significantly. In addition to the previous comparison of the wake velocity, this facilitates further assessments of how well the wake shape is captured by the models. As opposed to the mean, the standard deviation of the measured normal force exhibits less azimuthal variation and does not follow a similarly distinct sinusoidal curve.
Figure 3.19. Difference of the mean normal force per azimuth bin $\bar{F}_n^{\phi}$ and the overall mean $\bar{F}_n$ (a), and corresponding standard deviation $\sigma(\bar{F}_n^{\phi})$ (b) at $r/R = 0.75$. Both statistics refer to azimuthal bins of $\Delta \phi = 12^\circ$. Figure adapted from Paper VI.

Generally, the simulation results deviate more significantly from the experimental data in terms of magnitude and characteristic shape. Furthermore, all models consistently underestimate $\sigma(\bar{F}_n^{\phi})$ which seems contradictory to the comparison of the velocity variances shown earlier (see Fig. 3.16). Unfortunately, a conclusive explanation for this issue can not be found based on the available data.

3.5.4 Discussion and Concluding Remarks
A concise evaluation of the overall model performance in wake inflow cases is arguably difficult because of the various interdependencies of individual modelling aspects. Possibly the most informative single quantity is the mean relative error in the azimuthal variation of the aerodynamic forces on the blade of WT2 (given the objective of simulating the wind turbine response in a waked inflow). For the two cases of this benchmark these errors lie in the range of $15 – 20\%$. Other comparisons against the DanAero experiment in undisturbed inflow conditions report errors in the range of $3 – 9\%$ for this metric. In light of the higher complexity of the cases discussed here, such error magnitudes can certainly be appreciated. Nevertheless, it is difficult to clearly identify weaknesses in the models that could be targeted by future improvements efforts. A few aspects have been discussed above, such as the deficient tip-loss correction of some of the models, or the overly strong dissipation in $\text{RANS}_{\text{UU}}$. Additional conclusions though remain rather speculative due to the limitations of the experimental data. Primarily, this relates to missing free-stream measurements of the wake, time-resolved SCADA and load data of WT1 as well as additional upstream velocity measurements. Ultimately, modelling errors in the response of WT2 can originate from the turbine model itself, the undisturbed inflow, the
modelling of the upstream turbine and its wake or, presumably, a combination of all. One illustration of this issue is that even a good match with the met mast data or power of WT1 does not necessarily correlate with a good agreement in terms of the power or loads of WT2.

Still, in summary, the comparison shows that the majority of the models are able to capture the crucial characteristic features of the wake and the resulting aerodynamic response of the downstream turbine. This finding is generally encouraging, especially given the simplifications in the benchmark definition (neglecting stratification, constant rotational speed and yaw). As for the LBM framework, this study serves as a first formal validation for typical wind energy applications. And, it provides evidence that the framework also performs reliably in more complex cases than initially discussed in Sections 3.2 and 3.3. Furthermore, it stands out that both mean and second-order statistics are not consistently better capture by the models with the highest fidelity, namely LES. To some extent, this can be related to the difficulties in matching the ambient flow conditions at the met mast with LES. Similar problems have been found in similar studies and can be stated as a persistent challenge for LES in general (Muñoz-Esparza et al., 2015; Vasaturo et al., 2018; Doubrawa et al., 2020). Moreover, it can be concluded that validation studies should not be limited to integrated quantities like power or bending moments. Such quantities can hide local model deficiencies, for example, in the wake flow or aerodynamic forces. Over- and underpredictions can also compensate one another and remain undetected. Similar problems can occur if only time-averaged quantities are considered.
4. Wall-modelled Boundary Layer Simulations

The modelling of the ambient flow field can be just as vital for the quality of wind farm simulations as the modelling of the turbines and wakes. This applies particularly to LES, where the ambient conditions need to be prescribed as transient high-resolution flow data at the inlet and maintained throughout the domain. A common approach for wind turbine simulations is to prescribe synthetically generated turbulent flow fields based on certain theoretical and empirical assumptions using, e.g., the method by Mann (1998). Undoubtedly, the use of synthetic turbulence has several practical advantages, chiefly, the low computational cost of generating the inflow, an easy tunability, and the possibility to incorporate constraints as discussed in Section 3.5. On the other hand, synthetic turbulence often implies various simplifications, particularly when compared to stratified boundary layers. And, even more importantly, it is usually not in equilibrium with the LES solution and therefore tends to decay throughout the domain (Gilling and Sorensen, 2011; Keck et al., 2014b; Olivares-Espinosa et al., 2018). This limits the approach to studies of relatively short fetches. A more consistent, but computationally more demanding alternative are complementary simulations of the ambient atmospheric flow, so-called precursor simulations. Inflow data sampled from such simulations typically provides the best possible representation of the ABL physics and can explicitly capture flow features related to stratification or Coriolis effects (see, e.g., Stevens et al., 2014a; Munter et al., 2016). Furthermore, the ambient turbulence statistics will be naturally sustained in the main (successor) simulation when using the same numerical set-up (grid resolution, SGS model, etc.). These advantages become particularly important given the trends outlined in Section 1.1, such as larger wind farms and turbines. Precursors, and thus simulations of ABLs in general, have therefore become a crucial part of wind energy research.

The very large Reynolds numbers encountered in ABL flows make wall-resolved LES practically impossible (Piomelli, 2008; Stoll et al., 2020). A parametrisation of the interaction of the surface with the bulk flow therefore becomes imperative. Wall models intend to capture this interaction by prescribing the shear stress at the surface based on empirical formulations. Ever since the first LES of boundary layer flows almost 50 years ago (e.g., Schumann, 1975), wall modelling has been an active field of research and remains a major challenge for LES in general. In the LBM, on the other hand, the topic only emerged recently with the pioneering work by Malaspinas and Sagarut (2014). To date, LBM wall modelling remains at an early stage when compared to classical LES, despite a growing attention to the topic over the past
years (see, e.g., Pasquali et al., 2017; Nishimura et al., 2019; Haussmann et al., 2019, 2020; Wilhelm et al., 2018, 2020).

Paper V documents the development of a novel LBM wall modelling approach and its validation for neutral ABL flows. The new model is motivated by a variety of deficiencies and limitations of existing LBM wall models for atmospheric applications.

4.1 Lattice Boltzmann Wall Modelling

The majority of LES wall modelling approaches relies on empirical formulations that provide the wall shear stress $\tau_w$ as an implicit or explicit function of the bulk velocity $u_{el}$ sampled at some point off the wall $x_{el}$ (hereafter referred to as exchange location), with a corresponding wall-normal distance $z_{el}$. While this also applies to LBM wall models, the main challenge is the incorporation of $\tau_w$ into existing boundary schemes. The concepts of the two most common approaches are outlined in this section, followed by a description of the novel iMEM approach developed in this work. An overview of the methods is provided in Fig. 4.1.
4.1.1 Wall Models Based on Wet-Node Boundary Conditions

The initial approach by Malaspinas and Sagaut (2014) as well as later developments of the method (Haussmann et al., 2019; Wilhelm et al., 2018, 2020; Maeyama et al., 2020) are based on WNBCs (see Section 2.3.1). In this model family the wall stress $\tau_w$ is based on the velocity sampled at $z_{cl} > z_1$, with $z_1$ being the wall-normal distance of the first off-wall node $x_1$. The wall stress, however, is only used to compute the velocity $u_1$ at the boundary node $x_1$ (using the same wall function initially employed to compute $\tau_w$) which is then enforced via the reconstructed distribution functions. Thus, the wall stress is never explicitly prescribed at the boundary. Many of the aforementioned studies show that WNBC-based wall models can be a robust approach for many LES applications. However, spurious oscillations due to the interpolation of the macroscopic quantities at the boundary node are a persistent issue (Wilhelm et al., 2018, 2020; Cai et al., 2021). Furthermore, the approach inherently suppresses resolved turbulent shear stress at the first off-wall node $x_1$ because $f_1$ is only reconstructed using wall-tangential velocity components. This is also reflected in the choice of RANS turbulence models in the near-wall region in conjunction with this method. Generally, such hybrid RANS-LES approaches can be a viable option for ABL simulations (see, Senocak et al., 2007; Bechmann and Sorensen, 2010). Nonetheless, plain LES in combination with equilibrium stress wall models clearly is the dominant numerical approach in the field (Stoll et al., 2020).

4.1.2 Slip-velocity-based Wall Models Using Bounce-back Schemes

Another concept builds upon BBSs and imposes the shear stress by means of a first-order approximation of the wall-normal gradient of the wall-tangential velocity $u_t$:

$$\tau_w = \rho \nu_T \frac{\partial u_t}{\partial n} \left|_{z_{cl}} \right. \approx \rho \nu_T \frac{u_{cl,t} - u_w}{z_{cl}},$$  \hspace{1cm} (4.1)

where $u_w$ is the wall velocity of the BBS (see Section 2.3.2). Given a certain $\tau_w$, Eq. (4.1) can be solved for $u_w$, which is then applied in the BBS. Promising results of the method are discussed by Nishimura et al. (2019) and Hayashi et al. (2021). However, the dependency of $u_w$ on $\nu_T$ can become problematic. After all, SGS-models with appropriate near-wall scaling tend to yield vanishing eddy-viscosities near the wall. This can result in large velocity gradients that lead to reversed wall velocities and spurious oscillations. Similar problems have been reported for slip-velocity-based wall models in finite-volume Navier-Stokes solvers (see, for example, Jaegle et al., 2010) and were similarly found in pre-studies of this work.
4.1.3 The Inverse Momentum Exchange Method

The emergence of LBM wall modelling approaches clearly marks a crucial step for the applicability of the method to a broader range of flow problems and particularly for LES of ABLs (see, e.g., Feng et al., 2020). Nonetheless, the aforementioned deficiencies and limitations of existing approaches motivate the development of novel wall models that do not rely on the concepts of the existing approaches described above. Some recent examples of other new concepts are the works by Kuwata and Suga (2021) and Han et al. (2021).

The aim of the method proposed here is to impose a wall-stress by manipulating the wall velocity \( u_w \) in BBSs, while avoiding the problematic dependency of \( u_w \) on the eddy-viscosity. The starting point is the momentum exchange method (MEM), which is commonly used to compute the force acting between a solid boundary and the fluid in the LBM. At an off-wall node \( x_1 \), the momentum exchanged between the surface and the fluid along a lattice direction \( e_{ijk} \) can be computed as

\[
\Delta p_{ijk} = e_{ijk} f_{ijk}(x_1, t) - e_{ijk} f_{ijk}(x_1, t + \Delta t) = e_{ijk} \left( f_{ijk}(x_1, t) + f_{ijk}(x_1, t + \Delta t) \right). \tag{4.2}
\]

Thus, the total force acting at the fluid-solid interface of \( x_1 \) amounts to

\[
F = \frac{\Delta x_1^3}{\Delta t} \sum_{ijk \in \Gamma} \Delta p_{ijk}, \tag{4.3}
\]

where \( \Gamma \) is the set of lattice directions intersecting the solid boundary. For the sake of generality, we recast \( f_{ijk} \) in the form

\[
f_{ijk}(x_1, t + \Delta t) = f_{BB} + 2 \omega \rho \frac{u_w \cdot e_{ijk}}{c_s^2}, \tag{4.4}
\]

where \( f_{BB} \) are the post-collision distributions reflecting at the wall, namely \( f_{ijk}(x_1, t) \) in the case of SBB, or \( f_{BB}^*(x_{BB}, t) \) for interpolated BBSs. Inserting Eqs. (4.2) and (4.4) into Eq. (4.3), we can split the total force into the contribution directly related to the bounce-back of distributions \( F^f \) and a contribution due to the wall velocity \( F^{uw} \), respectively:

\[
F^f = \frac{\Delta x_1^3}{\Delta t} \sum_{ijk \in \Gamma} e_{ijk} \left( f_{ijk}(x_1, t) + f_{BB} \right), \tag{4.5}
\]

\[
F^{uw} = \frac{\Delta x_1^3}{\Delta t} \sum_{ijk \in \Gamma} e_{ijk} \left( 2 \omega \rho \frac{u_w \cdot e_{ijk}}{c_s^2} \right), \tag{4.6}
\]

with \( F = F^f + F^{uw} \). \tag{4.7}

65
The general objective of a wall model is to prescribe the local wall shear stress $\tau_w$. Hence, the wall-tangential component of the total force $F_t = F - (F \cdot n)n$, with $n$ being the wall-normal unit vector, should obey

$$F_t = F^{wm} = \tau_w A_1,$$  

(4.8)

where $A_1$ is the surface area intersecting the unit cell of $x_1$. To this effect, we propose a simple algorithm, hereafter referred to as the inverse momentum exchange method (iMEM). The main idea is to set a wall velocity $u_w$ such that Eq. (4.8) is satisfied. Initially, we perform a bounce-back and compute the shear force exerted on a wall at rest, i.e. $F_t^f$ following Eq. (4.5). From Eq. (4.7) we find that $u_w$ should satisfy

$$F_{uw} = F^{wm} - F_t^f.$$  

(4.9)

Inserting Eq. (4.6) into Eq. (4.9) we can now solve for $u_w$. Lastly, we add the momentum due to $u_w$ to $F_{ijr}^f$ (second term on the right-hand side of Eq. (4.4)), completing this modified bounce-back scheme. A closer look at Eq. (4.6) reveals that the required expression $v_w(x_1, F_{uw}) = u_w$ depends on the lattice type and $\Gamma$. Thus, $v_w$ can differ between different boundary nodes in the case of complex geometries.

In summary, the iMEM wall modelling approach can be readily coupled to any BBS, as it merely requires the computation of $u_w$ based on a given wall shear stress. Moreover, it does not inherently suppress wall-normal velocity fluctuations (as in WNBC-based methods), while avoiding the aforementioned problematic dependency on $\nu_T$.

### 4.2 Estimating the Wall Shear Stress

In LES of ABLs, the wall shear stress $\tau_w$ is commonly computed via Monin-Obukhov similarity theory (MOST), i.e.

$$u(z) = \frac{u_s}{\kappa} \ln \left( \frac{z}{z_0} + \phi_M \right),$$  

(4.10)

where $u$ is the stream-wise velocity, $u_s = \sqrt{\tau_w/\rho}$ is the friction velocity and $\phi_M$ the momentum stability correction (Stoll et al., 2020). To this end, the wall-tangential velocity and the corresponding basis vector are obtained from

$$u_{cl, t} = u_{cl} - (u_{cl} \cdot n)n$$  

and

$$e_t = \frac{u_{cl, t}}{\|u_{cl, t}\|},$$  

(4.11)

(4.12)

respectively. Inserting $u_{wm} = \|u_{cl, t}\|$ and $z_{cl}$, Eq. (4.10) can be solved for $\tau_w$, providing the local wall stress $\tau_w = \tau_w e_t$ to be applied at the surface. This
general procedure is practically the same in all LES studies of ABLs. Still, crucial differences in the estimation of $\tau_w$ can result from the choice of the wall-normal distance of the exchange location $z_{el}$ and the filtering/averaging of the sampled velocity $u_{el}$.

As for $z_{el}$, a preferable choice is to sample at the first off-wall node ($z_{el} = z_1$; see, e.g., Porté-Agel et al., 2000; Bou-Zeid et al., 2005). Primarily, this is due to the fact that the correlation of the bulk velocities with the surface fluxes decreases with the distance to the wall. This can compromise the validity of the wall function, particularly in stratified flows. On the other hand, the turbulent fluxes near the wall are often under-resolved and therefore prone to numerical and SGS-modelling errors. Thus, surface layer modelling errors become additionally coupled to the outer layer flow via the wall model, causing problems such as log-layer mismatch (LLM). Based on the discussion by Kawai and Larsson (2012) it is therefore common practice in various numerical models to sample the velocity at the second off-wall node or beyond (see, e.g., Frère et al., 2017; Maronga et al., 2020; Owen et al., 2020). This also includes most of the wall-modelled LBM approaches mentioned earlier.

Originally, the need for an averaging of $u_{el}$ arises from the fact that MOST was developed and validated in an averaged sense. However, in LES $u_{el}$ fluctuates. Thus, the use of instantaneous locally sampled velocities leads to an overestimation of the mean shear stress since $\langle \tilde{u}^2 \rangle > \langle \tilde{u} \rangle^2$ (Schwartz inequality), where $\langle \cdot \rangle$ denotes a spatial averaging (Bou-Zeid et al., 2005). A common approach to overcome this problem is the Schumann-Grötzbach (SG) model (Schumann, 1975; Grötzbach, 1987). The model computes the average wall shear stress $\langle \tau_w \rangle$ based on the planar-averaged velocity $\langle u(z_1) \rangle$. Locally, the wall model then applies the instantaneous deviation from $\langle \tau_w \rangle$, which is approximated as a linear function of the local resolved velocity

$$\tilde{\tau}_w(x, y, t) = \langle \tau_w \rangle - \frac{\tilde{u}(x, y, z_1, t)}{\langle u(z_1) \rangle} \cdot \langle u(z_1) \rangle. \tag{4.13}$$

Maronga et al. (2020) recently modified the SG model by incorporating other exchange locations than $z_1$, referred to as the elevated SG (ESG) model. Based on the arguments for $z_{el} > z_1$, the planar-averaged velocity $\langle u(z_{el}) \rangle$ is used to compute $\langle \tau_w \rangle$, while retaining the local scaling using $\tilde{u}(x, y, z_1, t)$. Still, the applicability of planar averages as in the SG or ESG model is limited to flat homogeneous surfaces. Yang et al. (2017) therefore replace the planar average by a temporal exponential filter:

$$u_{wm}(x, y, t) = (1 - \epsilon) u_{wm}(x, y, t - \Delta t) + \epsilon \tilde{u}(x, y, z_1, t), \tag{4.14}$$

where the time decay is defined as $\epsilon = \Delta t / T_f$, with $T_f$ being the filter time scale. The study shows similarly beneficial effects as with a planar averaging for time scales $T_f > \Delta t_c$, where $\Delta t_c = \Delta x / \langle u(z_{el}) \rangle$ is the convective time scale at the exchange location.
4.3 Numerical Set-up and Case Description

For the analysis of the proposed iMEM approach we perform a series of simulations of the canonical test case of a simplified neutral ABL in an open-channel flow set-up, i.e., an isothermal turbulent boundary layer above a horizontally homogeneous rough surface. For the sake of simplicity, Coriolis effects are neglected in this study and the flow is driven by constant pressure gradient $\partial p/\partial x = \rho u_*^2/H$, where $H = 1000$ m is the domain height and $u_* = 0.4$ m s$^{-1}$. The roughness length is set to $z_0/H = 10^{-4}$. The domain measures $L_x = 6H$, $L_y = 4H$ and $L_z = H$ in the stream-wise, lateral and vertical direction, respectively, and is periodic in $x$ and $y$. At the top we apply a zero-stress no penetration boundary condition ($\partial \tilde{u}/\partial z = \partial \tilde{v}/\partial z = 0$, $\tilde{w} = 0$) by means of a simple bounce-forward scheme (Krüger et al., 2016). A SBB is applied at the bottom and coupled to the iMEM wall model. The sub-grid scales are modelled by the AMD model with a model constant of $C_{x,y,z} = 1/12$. The Mach number in all simulations is $Ma = 0.1$. The flow field is initialised with the equilibrium mean velocity profile including sinusoidal perturbations in $\tilde{w}$. Each case is initially run for 50 eddy-turnover times $T_* = H/u_*$. Subsequently, data is collected for another $150 T_*$. 

4.4 Impact of the Wall Shear Stress Model

The sensitivity of boundary layer simulations to $z_{el}$ and the averaging of $u_{el}$ can largely depend on the numerical scheme. For the presented numerical set-up we therefore discuss the impact of the SG and ESG approach. However, for the sake of locality, the planar averaging of the original methods is replaced by a temporal averaging following Eq. (4.14). For the following case study we use a moderate spatial resolution of $n_z = 96$ grid points in the vertical direction. As a reference, we also include the vanilla version of the wall model with $z_{el} = z_1$ and no averaging of $u_{wm}$, referred to as the instantaneous logarithm method (IL). Following the recommendations by Yang et al. (2017), the filter time scale in the SG and ESG model is set to $T_f = 10 \Delta t_c$. The exchange location in the ESG model is set to $z_{el} = 2 \Delta x$.

4.4.1 Log-layer Mismatch

The mean stream-wise velocity $\langle \tilde{u} \rangle$, non-dimensional velocity gradient $\phi = \kappa_z d \langle \tilde{u} \rangle / dz$, and the mean resolved and modelled shear stresses $\langle u'w' \rangle$ and $\langle \tilde{r}_{xz} \rangle$ are depicted in Fig. 4.2. The IL slightly underpredicts the velocity at the first grid point when compared to the phenomenological velocity profile. At the same time, a positive LLM is found in the bulk, that is accompanied by a significant overshoot in the velocity gradient at the second grid point. In comparison, the SG model has only little effect on $\langle \tilde{u} \rangle$ and $\phi$. The use of instantaneous
velocities in the shear stress calculation typically results in an overprediction of $\tau_w$, as discussed in Section 4.2. In light of the fixed driving pressure gradient, the small underprediction of the velocity at the exchange location (the first grid point) with the IL can be inferred from the argument of global momentum conservation. In turn, since the velocity is only slightly underestimated with the IL, it can be expected that the averaging of the SG model has only little effect on the results. Most of the studies mentioned in Section 4.2 show that an averaging in the wall model can be a remedy for problems such as LLM and the gradient overshoot. On the other hand, others report similarly negligible effects as here and even refrain from averaging despite the theoretical argument for it (see, e.g., Frère et al., 2017; Whitmore et al., 2020; Owen et al., 2020). Considering Schwartz inequality, it can be expected that the impact of the averaging will depend on the amount of resolved turbulence at the exchange location. Or, in other words, the larger the stream-wise velocity fluctuations at $x_{el}$, the stronger the overprediction of the wall shear stress without averaging.

The ESG model lowers the entire velocity profile in comparison to the IL. Thus, the LLM in the surface layer ($z/H \lesssim 0.1$) is considerably reduced, while the velocity at the first grid point is now underpredicted. The velocity gradient is only marginally affected. Small deviations from the log-law in the surface layer therefore remain even above the first grid point. In this sense, the effect of the ESG model is consistent with the original motivation to sample above the first grid point, following Kawai and Larsson (2012): the approach is no remedy for the underlying problem of the LLM, i.e., a poorly resolved flow at the first grid point leading to an overshoot of the velocity gradient. But, it can

Figure 4.2. Vertical profiles of mean quantities with different wall shear stress approaches. (a) Mean stream-wise velocity, black line marking the theoretical log-law following MOST (Eq. (4.10)). (b) Non-dimensional vertical velocity gradient, black line marking the theoretical value of 1. (c) Mean resolved shear stress $\langle \bar{w}'w' \rangle$ (filled markers), modelled shear stress $\langle \bar{\tau}_{xz} \rangle$ (empty markers) and total shear stress $\langle \tau_T \rangle = \langle \bar{u}'w' \rangle + \langle \bar{\tau}_{xz} \rangle$ (full lines), black line marking the theoretical profile $\tau_T/u^*_w = z/H - 1$. Figure adapted from Paper V.
be a practical measure to circumvent the problem and reduce the LLM in the bulk.

4.4.2 Statistics of the Wall Shear Stress

For a better understanding of the near-wall effects of the wall model we turn towards the PDFs of the instantaneous friction velocity \( u_{*w} \) computed by the wall models; see Fig. 4.3. The corresponding mean \( \mu \), standard deviation \( \sigma \), skewness \( S \) and flatness (kurtosis) \( F \) are provided in Table 4.1.

With the IL model the mean is lower than the theoretical equilibrium value of \( u_* \), while the distribution is positively skewed. The flatness is close to 3 implying near-Gaussian occurrences of extreme values. With the SG model \( \mu \) is closer to \( u_* \), the distribution is less skewed, and the flatness is slightly lower. The ESG model leads to a further decrease of both skewness and flatness. The positive skewness of the IL can be explained by its linear dependency on \( \tilde{u}_1 \), which itself is positively skewed (see, for instance, Fig. 4.7). The filtering applied in the SG and ESG approach reduces the likelihood of extreme values. A narrower and less skewed distribution is therefore expected. The differences between the SG and ESG model can again be related to the different statistics of the sampled velocity, i.e. \( u(z_1) \) and \( u(z_2) \), respectively. With increasing distance to the wall the skewness and flatness of the stream-wise velocity tend

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**Figure 4.3.** PDF of the predicted friction velocity \( u_{*w} \) using different wall shear stress models. Figure adapted from Paper V.

**Table 4.1.** Statistics of the predicted friction velocity using different wall shear stress models.

<table>
<thead>
<tr>
<th></th>
<th>( \mu/u_* )</th>
<th>( \sigma/u_* )</th>
<th>( S )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IL</td>
<td>0.966</td>
<td>0.169</td>
<td>0.386</td>
<td>3.088</td>
</tr>
<tr>
<td>SG</td>
<td>0.996</td>
<td>0.130</td>
<td>0.189</td>
<td>2.913</td>
</tr>
<tr>
<td>ESG</td>
<td>1.004</td>
<td>0.128</td>
<td>0.034</td>
<td>2.601</td>
</tr>
</tbody>
</table>
to decrease. Thus, despite the filtering these trends remain visible in the characteristics of $u_{wm}^*$. Surprisingly, the near-wall resolved turbulence is only mildly affected by the differences in the statistics of $u_{wm}^*$. See, for instance, the resolved turbulent shear stress in Fig. 4.2c. Similarly low effects can be observed for the velocity variances (not shown for the sake of brevity). Referring to the discussion by Brasseur and Wei (2010), the negligible impact of the wall shear stress model on $\langle \overline{u'w'} \rangle$ could also explain the negligible impact on $\phi$. Among others, they show that the gradient overshoot depends strongly on the ratio $\Re = \langle \overline{u'w'} \rangle / \overline{\tau}_{xz}$. Thus, increasing $\langle \overline{u'w'} \rangle$ in order to minimise the overshoot problem appears to require other measures, e.g., the use of different SGS-models.

### 4.5 Grid Sensitivity

Further details of the turbulence statistics are analysed in a grid sensitivity study. Based on the findings discussed in Section 4.4 we only consider the ESG model with the settings given earlier. The study comprises four grid resolutions $n_z = \{64, 96, 128, 160\}$. As a complement to MOST and other scaling laws, the results are compared with two LES solutions of the same test case. Both are performed with a staggered pseudo-spectral-finite-difference (PSFD) solver using a pseudo-spectral discretisation in the horizontal directions and a second-order finite difference scheme in the vertical direction. PSFD solvers are among the most well-established numerical frameworks for boundary layer simulations due to their low numerical dissipation, and are often the go-to benchmark for other numerical approaches (Giacomini and Giometto, 2021). The first reference case by Gadde et al. (2020), referred to as PSFD$_{144}^{AMD}$, has a vertical resolution of $n_z = 144$ and uses the same SGS-model as in this study, the AMD model. In the second case by Stevens et al. (2014b) the vertical resolution is significantly higher with $n_z = 256$, and the more advanced Lagrangian-averaged scale-dependent (LASD) dynamic Smagorinsky is employed (Bou-Zeid et al., 2005). Further details of the reference simulations are contrasted against the CLBM set-up in Table 4.2.

#### 4.5.1 Mean Velocities and Shear Stress

The mean stream-wise velocity, non-dimensional velocity gradient, and the resolved and modelled shear stresses of the CLBM simulations are contrasted against PSFD$_{144}^{AMD}$ in Fig. 4.4. All CLBM cases are in reasonable agreement with the log-law in the surface layer. The underprediction of $\langle \overline{u} \rangle$ at the first grid point persists for all grid resolutions. Fig. 4.4b shows that the overshoot of the velocity gradient even increases with the grid resolution. Still, in all cases the overshoot remains limited to the second grid point. Further aloft,
Table 4.2. Summary of the CLBM and PSFD reference cases: vertical resolution $n_z$, aspect ratio $\Delta x/\Delta z$, domain dimensions and details of the SGS-model.

<table>
<thead>
<tr>
<th></th>
<th>CLBM</th>
<th>PSFD$^{\text{AMD}}_{144}$ (Gadde et al., 2020)</th>
<th>PSFD$^{\text{LASD}}_{256}$ (Stevens et al., 2014b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_z$</td>
<td></td>
<td>{64, 96, 128, 160}</td>
<td>144</td>
</tr>
<tr>
<td>$\Delta x/\Delta z$</td>
<td>1</td>
<td>$2\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$L_x$, $L_y$, $L_z$</td>
<td>6, 4, 1</td>
<td>${2\pi, 2\pi, 1}$</td>
<td>${4\pi, 2\pi, 1}$</td>
</tr>
<tr>
<td>SGS-model</td>
<td>AMD</td>
<td>C$_{x,y,z}$ = 1/12</td>
<td>C$_{x,y}$ = 1/12</td>
</tr>
<tr>
<td></td>
<td>AMD</td>
<td></td>
<td>C$_z$ = 1/5</td>
</tr>
</tbody>
</table>

Figure 4.4. Vertical profiles of mean quantities with different grid resolutions. (a) Mean stream-wise velocity. (b) Non-dimensional vertical velocity gradient. (c) Resolved shear stress $\langle u'w' \rangle$ (filled markers), modelled shear stress $\langle \tau_{xz} \rangle$ (empty markers) and total shear stress $\langle \tau_T \rangle$ (full lines). Figure adapted from Paper V.

$\phi$ quickly reapproaches the theoretical surface layer value of 1, while closely following the curve of PSFD$^{\text{AMD}}_{144}$. In PSFD$^{\text{AMD}}_{144}$ only a small overshoot can be observed at the second grid point.

At the first grid point the resolved shear stress lies well below the reference case regardless of the grid resolution. Nonetheless, the ratio of resolved to modelled shear stress grows significantly faster along the vertical in the CLBM cases than in PSFD$^{\text{AMD}}_{144}$. This can be attributed to the larger aspect ratio in the reference case as well as the different SGS model coefficient $C_i$.

Discussions of the overshoot problem can be found in the context of various numerical aspects of LES (see, e.g., Porté-Agel et al., 2000; Bou-Zeid et al., 2005; Frère et al., 2017; Whitmore et al., 2020). The attempt for a more general explanations can be found in the work by Brasseur and Wei (2010) mentioned earlier. They show that the velocity gradient overshoot in LES of rough-wall boundary layers is due to similar physics as the natural peak in the gradient in the viscous sub-layer of smooth-wall boundary layer flows. In the latter, this peak coincides with the transition from viscous to turbulent stress dominated flow regions. In LES of rough-wall boundary layers the eddy viscosity (or
numerical dissipation) can cause an artificial viscous layer with similar characteristics. Analogously, the overshoot is commonly found at $z \left( \overline{\langle u'w' \rangle} \right) \simeq \bar{\tau}_{xz}$. Therefore, a necessary criterion to reduce the overshoot is $\overline{\langle u'w' \rangle} \geq \bar{\tau}_{xz}$ at $z_1$, pushing the transition region, and thus the overshoot, below the first grid point. Indeed, the trends in the gradient overshoot of the CLBM cases (an overshoot at second grid point that increases in magnitude with $n_z$) and the shear stresses (transition to $\overline{\langle u'w' \rangle} > \bar{\tau}_{xz}$ between first and second grid point) are in line with this hypothesis. This also applies to the lower overshoot in PSFD$^{144}_{\text{AMD}}$. In light of the comparably large fraction of resolved turbulent shear stress above $z_2$ (as well as further results discussed later), it does not appear that the bulk scheme of the CLBM approach is significantly more dissipative than the PSFD case. It is therefore assumed that the reasons for the rather low $\overline{\langle u'w' \rangle}(z_1)$ lie elsewhere. One conjecture relates to the fact that the wall stress $\tau_w$ in the LBM is directly encoded in the distribution functions reflected at the wall. Thus, both horizontal and vertical velocity components at $x_1$ are directly affected by the modelled shear stress. On the other hand, on the staggered grid of the PSFD solver $\tau_w$ is only directly incorporated on the $w$-nodes that coincide with the wall. The horizontal velocity components at the first grid points in the bulk (at $z = \Delta z/2$) are thus only affected by $\tau_w$ via the stress divergence term.

### 4.5.2 Variances

The velocity variances are shown in Fig. 4.5 in comparison to PSFD$^{144}_{\text{AMD}}$ and PSFD$^{256}_{\text{LASD}}$. The stream-wise variance of the lower grid resolutions ($n_z = \{64, 96\}$) closely matches with PSFD$^{144}_{\text{AMD}}$ in the outer layer, while the higher resolutions approach the PSFD$^{256}_{\text{LASD}}$ solution. The trends in the surface layer are less clearly visible in Fig. 4.5a. Yet, a further analysis will given later. The most significant difference to the PSFD cases is $\overline{\langle u'w' \rangle}$ at the first grid node. Furthermore, it is worth mentioning that simulations by Gadde et al. (2020)
with the same resolution but using the LASD model (omitted here for the sake of clarity) exhibit a much closer agreement with PSFD_{256}^{LASD} in the outer layer. We can therefore assume that the disagreements are largely due to the different SGS model rather than the grid resolution or other numerical differences.

A somewhat close agreement is found for all cases in terms of the lateral variance. Again, the deviation between the CLBM and PSFD cases is greatest near the surface.

As for the vertical variance, the strongest impact of the grid resolution on the CLBM results is an increasing maximum in the surface layer. This peak is generally more pronounced than in PSFD_{256}^{LASD}. Interestingly, the magnitude of $\langle w'w' \rangle$ in PSFD_{144}^{AMD} is consistently lower than in PSFD_{256}^{LASD}. Moreover, the profile of $\langle w'w' \rangle$ in the CLBM cases and PSFD_{144}^{AMD} is slightly concave in the lower outer layer. In comparison, PSFD_{256}^{LASD} and other cases of even higher resolution discussed in Stevens et al. (2014b) exhibit an almost linear decrease of the vertical variance in this region.

In addition to the velocity log-law, Townsend (1976) and others predicted a logarithmic behavior for the stream-wise velocity variance in the surface layer based on the attached-eddy hypothesis. They propose

$$\frac{\langle u'u' \rangle}{u_*^2} = B_1 + A_1 \log \left( \frac{z}{\delta} \right),$$

where $\delta$ is an outer length scale and $A_1 = 1.25$ is the Townsend-Perry constant. Over the past two decades highly resolved experimental data and LES finally confirmed parts of this hypothesis, namely $A_1 \approx 1.25$ for $0.04 \lesssim z/H \lesssim 0.23$ (see Meneveau and Marusic, 2013; Stevens et al., 2014b). As for the constant $B_1$, though no particular self-similarity could be found. In Fig. 4.6 we compare $\langle u'u' \rangle$ in the surface layer against Eq. (4.15). As an additional reference, we compare against smooth-wall experimental data from Meneveau and Marusic (2013) obtained at $Re_\tau = 19000$ at the Melbourne open wind tunnel (referred to as WT_{19k}). The high resolution case PSFD_{256}^{LASD} and the experimental data exhibit a clear $z^{1.25}$-slope; see Fig. 4.6a. The CLBM results show a positive curvature throughout the upper part of the self-similarity region. For $z/H < 0.1$ the slope is more constant, but notably lower than the expected 1.25. Also in PSFD_{144}^{AMD} a positive curvature is found for $z/H > 0.1$ while an inflection point occurs at $z/H \approx 0.1$. In Fig. 4.6b we explicitly compare the local slope $A_1$. The profiles confirm that neither the CLBM cases nor PSFD_{144}^{AMD} develop a clear logarithmic region with the appropriate $A_1$. However, with increasing grid resolution the 1.25-crossing of the CLBM cases shifts towards the wall. This trend also agrees with the findings by Stevens et al. (2014b).

4.5.3 Higher-order Moments

Recent discussions by Bou-Zeid (2015) and others emphasise the importance of a correct reproduction of higher-order moments for the assessment of LES
quality. This concerns not only fundamental studies of, e.g., boundary layer turbulence, but also engineering aspects such as wind turbine loads (Berg et al., 2016; Schottler et al., 2017). On that account, Giacomini and Giometto (2021), for instance, report notable discrepancies between second-order finite-volume and PSFD approaches in the higher-order moments due to the overly strong numerical dissipation of the former.

Fig. 4.7 compares the skewness $S$ and flatness $F$ of the three velocity components. Near the surface, we generally find a positive skewness of $u'$, implying an increased likelihood of negative velocity fluctuations. Throughout the surface layer $S(u')$ decreases almost linearly in $\log(z/H)$, becoming consistently negative in the outer layer. Referring to Marusic et al. (2010) this change of sign of $S(u')$ occurs due to large-scale motions (LSMs) in the surface layer. These super-structures, characterised as elongated streaks of negative velocity fluctuations, evoke an amplitude modulation that dampens small-scale fluctuations near the wall while amplifying them further aloft. The zero-crossing of $S(u')$ in most CLBM cases and PSFD$^{\text{AMD}}_{144}$ is slightly elevated in comparison to PSFD$^{\text{LSD}}_{256}$. Moreover, with increasing grid resolution, the CLBM cases have a tendency for lower near-surface skewness than the PSFD results. In line with experimental observations (e.g., Marusic et al., 2010; Vallikivi et al., 2015) PSFD$^{\text{LSD}}_{256}$ exhibits an almost constant skewness of $S(u') \approx -0.1$ in the transition from the surface to the outer layer ($0.08 \lesssim z/H \lesssim 0.3$). In the CLBM cases and PSFD$^{\text{AMD}}_{144}$, on the other hand, it decreases more continuously. Nonetheless, the general level of agreement in $S(u')$ can arguably be appreciated, especially for the low resolution CLBM cases. The flatness of the stream-wise velocity is consistently sub-Gaussian in all cases for $z/H > 0.01$. Close to the wall $F(u')$ is somewhat larger, reaching Gaussian to super-Gaussian values depending on the case. According to Mathis et al. (2009),

Figure 4.6. (a) Stream-wise variance compared against the PSFD cases and wind tunnel data (WT$_{19k}$, grey diamonds) as well the theoretical slope of $A_1 = 1.25$ (grey dashed line). The grey shaded area marks the expected logarithmic region $0.04 \lesssim z/H \lesssim 0.23$. (b) Local slope $A_1$ of the $\langle u'w' \rangle$-profiles of the LES cases computed with second-order finite differences. Grey dashed line indicating the expected $A_1 = 1.25$. For the legend see Fig. 4.5. Figure adapted from Paper V.
super-Gaussian flatnesses near the wall are again associated with the strong intermittency resulting from the amplitude modulation of LSMs.

The skewness of the lateral component is unexpectedly close to zero in all cases due to the span-wise symmetry of the flow. In PSFD\textsuperscript{LASD}\textsubscript{256} the corresponding flatness is super-Gaussian, in agreement with experimental data shown in the original publication (Stevens et al., 2014b). With increasing resolution, $F'(v')$ in the CLBM approach converges to similar values in the surface layer as in the high-resolution PSFD reference.

The skewness of the vertical velocity consistently increases throughout the boundary layer. Only the first grid point in the CLBM cases deviates from this trend, which might be an artifact of the wall model. In line with other LES studies (Sullivan and Patton, 2011; Stevens et al., 2014b), the zero-crossing of $S(w')$ in the surface layer approaches the wall with increasing grid resolution. Moreover, a skewness of $S(w') \approx 0.3$ at $z/H \approx 0.3$ is in agreement with field measurements of near-neutral ABLs discussed by Lenschow et al. (2012). The study also reports flatnesses of 3 to 4 in the outer layer that tend to grow with height. In terms of $F'(w')$, it is again the first grid point in the CLBM cases that stands out with higher flatness than in the surface layer aloft.
4.5.4 Velocity Spectra

The stream-wise one-dimensional spectra of the stream-wise and vertical velocity, $E_{uu}$ and $E_{uw}$, respectively, are shown in Fig. 4.8. For the sake of brevity, we limit the following discussion to the lowest and highest resolution CLBM case. The spectra are obtained from the Fourier-transform of the respective velocity component, averaged in space and time and normalised by $u_a^2 z$. When plotted against the stream-wise wavenumber $\kappa_x$ multiplied by $z$, we anticipate a collapse of $E_{uu}$ in the inertial subrange ($\kappa_x z > 1$) with the phenomenological $\kappa^{-5/3}$-scaling as well as $\kappa^{-1}$-scaling in the production range ($\kappa_x z < 1$; Porté-Agel et al., 2000).

![Figure 4.8](image URL)

*Figure 4.8.* Stream-wise one-dimensional spectra of the stream-wise (a-c) and vertical (d-f) velocity. Vertical position $z$ increasing from dark red $(z = z_1)$ to dark blue $(z \approx 0.5H)$, with $z = \{z_1, z_2, 0.05 H, 0.1 H, 0.2 H, 0.3 H, 0.4 H, 0.5 H\}$. Dashed lines mark the expected Kolmogorov scaling of the inertial subrange ($\kappa^{-5/3}$) and the production range ($\kappa^{-1}$), respectively. Figure adapted from Paper V.

For $n_z = 64$, $E_{uu}$ agrees with the Kolmogorov scaling only in a small wavenumber range. Thus, the overlap in the inertial subrange is somewhat limited. Generally, we find that the decay of the larger scales in the inertial subrange is too small. On the other hand, the spectra peel off at higher wavenumbers with significantly larger slopes than $-5/3$ due to the non-negligible numerical dissipation. In PSFD$_{144}^{AMD}$ this behaviour is not observed because of the spectral accuracy of the solver. Close to the wall, $n_z = 64$ also underpredicts the production-range scaling.
The spectra of the vertical velocity (Fig. 4.8d) in the inertial subrange collapse slightly better due to a weaker peel-off at high wavenumbers. At lower wavenumbers $E_{ww}$ flattens out in agreement with both theory (Townsend, 1976) and experiments (Katif and Chu, 1998). The energy at the lowest grid point is noticeably larger when compared to the higher levels. Typically, we would expect a more continuous increase in $E_{ww}$, and eventually a collapse of the spectra at low wavenumbers. This trend can partially be observed in PSFD$_{14}$ (see, Fig. 4.8f) or other experimental (Katif and Chu, 1998) and numerical data (Lu and Porté-Agel, 2010; Stevens et al., 2014b).

In the high resolution case ($n_z = 160$, Fig. 4.8b), the collapse of $E_{ww}$ in the inertial subrange is notably better due to a steepening of the slopes at most heights. Similar trends are also found in the production range. As for $E_{ww}$, the energy difference between the first and second node reduces. Furthermore, steeper slopes and an improved overlap can be seen in the larger scales of the inertial subrange.

4.6 Concluding Remarks

In summary, the novel iMEM wall-model was applied to the canonical test-case of an isothermal pressure-driven rough-wall boundary layer capped by a solid lid. The initial comparison of the different models for the wall shear stress show that exchange locations beyond the first off-wall grid point can be an effective measure to reduce the LLM. Nonetheless, the persistent velocity-gradient overshoot at the second grid point calls for further investigations of potential remedial measures. These might be found in the use of different SGS models or near-wall corrections to the employed AMD model. Whitmore et al. (2020), for example, show that the eddy viscosity of the AMD model near the wall still exceeds that of dynamic Smagorinsky models. Thus, near-wall velocity fluctuations are damped too heavily, which can be a cause for the gradient overshoot (Brasseur and Wei, 2010; Chatterjee and Peet, 2017). Other methods to increase the resolved near-wall turbulence can be stochastic backscatter models (Mason and Thomson, 1992) or wall models with transpiration (Bose and Moin, 2014; Bae et al., 2019).

The detailed comparison of the CLBM simulations with the phenomenological, numerical and experimental references shows that the scheme is a promising method for LES of ABL flows. Particularly, the good agreement with the PSFD cases in terms of the higher-order moments and velocity spectra can be appreciated. We mainly attribute this good performance to the low numerical diffusivity of the scheme. Note, for instance, that pre-studies with the second-order accurate regularised CLBM show significantly worse results with regards to all turbulence statistics. Generally, this study illustrates the feasibility of wall-modelled LES of boundary layer flows with the LBM, marking an important step for future applications to wind farm flows.
5. Applications

In addition to the fundamental investigations of the developed methods, this work comprises three application-oriented studies. Concerned with the topics of wind farm control (Paper V), icing effects (Paper VII), and deep-learning-based surrogate models (Paper VIII), these studies cover a variety of current issues in the field of wind energy. Besides the investigated topics itself, they highlight the readiness and potential of LBM-LES for current wind energy applications. In particular, the approaches presented in Paper V and VIII would practically not be feasible with conventional LES models due to the extreme computational cost of the simulated cases.

5.1 Exploring Reinforcement Learning for Wind Farm Control

Maximising the power capture of wind farms is one of the main drivers for the development of wind farm control strategies. This typically involves two aspects, namely the choice of a wake mitigation approach (e.g., power-derating, yaw-based wake-steering, etc.) and an optimisation technique tuning the settings of the former for the scenarios of interest. This optimisation can be particularly challenging for dynamic control strategies due to the non-linearity of the problem and the strong mutual dependencies of the parameters controlling the individual turbines (Kheirabadi and Nagamune, 2019).

An alternative to classical control and optimisation approaches is reinforcement learning. Reinforcement learning mimics the trial-and-error learning process observed in humans and animals. Thus, the control strategy evolves by learning about cause and effect when interacting with an environment, and is not constrained by any predefined parametrisation as in many classical control approaches. Over the past decade, reinforcement learning has been successfully applied to a wide range of non-linear optimisation problems. This includes super-human behaviour in playing the game Go (Silver et al., 2016) or Poker (Moravéčik et al., 2017) as well as applications in robotics (Ibarz et al., 2021), self-driving cars (Pan et al., 2017), and fluid flow control (Garnier et al., 2021).

In Paper IV we discuss an exploratory study on the application of deep reinforcement learning (DRL) for the power optimisation of a row of wind turbines. In a first verification test case, the DRL approach is employed to optimise a single parameter in an existing dynamic wake mitigation method, the so-called
helix approach (Frederik et al., 2020). In further test cases, the DRL approach is left to freely control the generator torque in order to find utterly new control strategies. In the following we provide a brief introduction to DRL, outline the coupling of the DRL approach and the LES framework simulating the row of turbines, and discuss the results of one of the generator torque control cases.

5.1.1 Deep Reinforcement Learning

Based on experience, humans and animals constantly perform sequential decision making tasks in uncertain environments in order to achieve certain goals. Reinforcement learning tries to formalise the underlying concepts in a mathematical framework and apply them to stochastic decision-making problems (Francois-Lavet et al., 2018). The general idea is that an artificial agent is able to learn by interacting with an environment given a pre-defined objective. Formally, this agent takes an action $A$ following a policy $\pi$. The environment responds by transitioning to a state $S$ and provides a feedback that depends on the objective, the reward $R$. Given the new state $S$, the agent takes a new action $A$, etc. For continuing control problems, this process is split into finite sequences of length $T_E$, referred to as episodes. The aim of the learning process is a maximisation of the cumulative reward of an episode by optimising the actions taken by the agent. To that end, the expected discounted future return $G_t$ for each time step $t$ in the episode is defined as

$$G_t = \sum_{t'=t}^{T_E} \gamma^{t'-t} R_{t'},$$

with $\gamma \in [0, 1]$ being the discount rate, and $R_{t'}$ the reward of future time steps $t'$. Low values of $\gamma$ thus emphasise short-term effects while high values also consider effects in the far future (Sutton and Barto, 2018).

One class of reinforcement learning approaches are policy gradient methods. In policy gradient methods, the parameters $\theta$ of a policy $\pi$ are directly optimised such that a performance measure $J(\theta)$ is maximised. In the case of DRL, the function $\pi$ is parametrised by an artificial neural network (ANN) and $J$ is maximised by optimising $\theta$ via stochastic gradient decent or related gradient-based optimisations. A crucial aspect for the convergence of the problem is the explicit form of $J$. In this work we apply the proximal policy optimisation (PPO; Schulman et al., 2017) where $J$ is an approximation of the improvement in expected return of a policy update.

5.1.2 Coupling to an LES Environment

When applying DRL to wind farm control, the agent acts as a controller of the wind turbines, while the action is the controlled variable, for instance, the
generator torque or yaw angle. The environment comprises the flow field affecting the wind farm as well as the response of the turbines (rotor speed $\omega$, power $P$, etc.).

In this study, we consider a row of three turbines with a longitudinal spacing $L_x = 5D$. The wind turbines are represented by the ALM. For the sake of simplicity, the mean inflow is spatially uniform and perturbed with synthetic Mann turbulence. The ANN is a fully-connected neural network with three layers of long short-term memory cells (Hochreiter and Schmidhuber, 1997). Further details of the LES set-up and ANN can be found in the original publication. The reward is defined as the total power of the wind farm. The state vector passed to the ANN comprises the stream-wise velocity $u$ sampled by a set of velocity probes in front of each turbine, and $\omega$. The agent interacts with the environment with a frequency of 1 Hz in simulated time. In order to accelerate the convergence of the model, the gradients $\nabla_\theta J$ of multiple episodes are averaged before updating the policy. Following the suggestion by Rabault and Kuhnle (2019), these episodes are simulated in several parallel environments, meaning parallel simulations of the same case (6 in this study).

In the following section, one of the three control cases considered in the original study shall be discussed. In this case (referred to as M-short in Paper IV) the agent controls the generator torque. The strategy to be developed thus refers to axial induction control. The discount rate and length of the episode are $\gamma = 0.95$ and $T_E = 500$ s (referring to about 2.3 domain flow-trough times $T_h$), respectively.

5.1.3 Results
The progression of the running average of the generator torques $M$ set by the agent and the resulting power during the training is shown in Fig. 5.1. Both are normalised with the corresponding means $M_{\text{greedy}}$ and $P_{\text{greedy}}$, respectively, obtained with a standard greedy controller for each turbine under the same inflow conditions. Starting from relatively small magnitudes of $M$, the generator torque converges to similar values as in the greedy case. In the end of the training, the power of the first turbine is close to that of the greedy case, while the second and third turbine exhibit a somewhat smaller power. The total power of the farm is slightly less than in the greedy case. Furthermore, the power improves at a slower rate the further downstream the turbine. These different improvement rates can be explained by the definition of the reward. As the power of the first turbine is generally the largest, a relative increase yields a larger absolute increase in total power, the quantity used as the reward.

Details of the development of the control strategy are illustrated by means of exemplary time series of the instantaneous generator torque and power, shown in Fig. 5.2. While applying a somewhat dynamic strategy in the beginning of the training, the agent ceases to react to turbulent fluctuations and merely sets
Figure 5.1. Running average of the generator torque $M$ (top) and power $P$ (bottom) of the three turbines (— Total, — Turbine 1, — Turbine 2, — Turbine 3). Figure adapted from Paper IV.

Figure 5.2. Instantaneous generator torque $M$ (top) and power $P$ (bottom) of the three turbines (— Total, — Turbine 1, — Turbine 2, — Turbine 3) at the start of the training (left), after half the training (center) and at the end of the training (right). Figure adapted from Paper IV.
a constant torque after half of the training. Further training only adapts the magnitude of the constant torque applied to the turbines. The only differences between the turbines are the evolution rates of this constant value, as already observed in Fig. 5.1. Analogously, the power of the first two turbines is already in a similar range as the greedy case after half of the training. At the end of the training, also the third turbine exhibits a torque and power close to the greedy case. However, the goal of the DRL approach was not to redevelop the greedy controller, but to find new, possibly dynamic, strategies that improve the total power of the farm.

5.1.4 Discussion

Interpreting the behaviour of deep learning approaches is generally difficult due to the black-box character of neural networks (Ghorbani et al., 2019). Assuming a correct implementation of the PPO-algorithm, the reasons why the agent is not able to discover new strategies are sought in the overall set-up of this work.

Potentially the biggest issue relates to the correlation of action and reward in the configuration of the case. Generally, every single action yields a return and the higher the correlation between the two, the more effective the optimisation. In the case of this study, the action comprises three components, namely the generator torque set at each respective turbine. The return, on the other hand, combines the impact of the three into a single quantity. In addition to that, each action can affect the reward at several points in time since the impact of the action on the flow-field advects downstream to the other turbines (e.g., in the form of a lower velocity deficit due to a sudden reduction in thrust). The decay of the weights of future rewards in the calculation of the return is determined by the discount rate $\gamma$ (see Eq. (5.1)). Therefore, we can generally only capture the effects of an action applied to the first turbine on another turbine if $\gamma$ is large enough with respect to the the time-lag between the events. However, large discount rates also imply that we include many time steps in the return that are not directly affected by the action. This again decreases the correlation of action and return. This is particularly problematic in turbulent flows, where the stochastic nature of the environment already introduces noise into the reward signal.

One remedy to the problem could be a rephrasing of the overall problem. Instead of one agent controlling the entire farm, each turbine could be controlled by an individual agent, each with its own definition of the reward. The individual rewards can be formulated such that they only include information that can actually be influenced by the action. For instance, the reward for the agent of the first turbine would be defined as

$$ R_t^{(1)} = P_t^{(1)} + P_{t+T_{\text{travel}}}^{(2)} + P_{t+2T_{\text{travel}}}^{(3)} , $$

(5.2)
where $T_{\text{travel}}$ is the time required by information to advect from one turbine to the next (e.g., $L_x/u_0$ as a crude approximation), and $P^{(i)}$, is the power of the respective turbine. This approach could allow for lower discount rates that decrease the noise in the reward signal while still taking into account the wake interaction between the turbines.

5.1.5 Conclusion

DRL has been shown to be a promising approach for fluid flow control problems (see, e.g., Rabault and Kuhnle, 2019; Tang et al., 2020; Xu et al., 2020). Nonetheless, to date, applications of the method remain limited to laminar or weakly turbulent cases (Garnier et al., 2021; Viquerat et al., 2021). To our best knowledge, this study represents the first application of DRL to a highly turbulent flow problem and to wind farm control in general.

The study shows that the employed DRL approach is generally able to optimise the power of the row of turbines. Yet, only with a predefined dynamic wake mitigation approach (the helix approach), the agent is able to outperform the standard greedy control case. When left to develop a new induction-based control strategy, the agent merely recovers the power of the greedy reference case. More importantly, the agent does not develop a dynamic behaviour as expected from other flow control applications of DRL. The latter is assumed to be related to the general difficulties associated with optimisations in extremely stochastic environments like turbulent flows as well as the time-lag between action and reward in the formulation of the problem. Despite the modest optimisation results of this study, we anticipate that DRL can be a viable optimisation technique for wind farm control if the aforementioned problems can be addressed.

5.2 Towards a Complete Model Chain for Ice Accretion Effects on Wind Turbines

Low population densities and promising atmospheric conditions have led to significant wind power developments in cold climate regions over the past decade (Wallenius and Lehtomäki, 2016; Badman and Tengblad, 2021). A serious challenge for the operation of wind turbines in such regions is the risk of blade icing. On the one hand, this relates to safety issues due to ice throw. On the other hand, ice accretion can lead to aerodynamic degradations of the blades that bring about production losses and increased turbine loads. Recent estimations assume that a quarter of today’s global wind turbine fleet is prone to the risk of icing (Stoyanov and Nixon, 2020). Thus, modelling and forecasting the impact of icing events on power production and turbine loads is becoming a crucial aspect for wind farm planning and operation.
5.2.1 The Model Chain

Modelling the impact of blade icing on multi-megawatt wind turbines involves a multitude of different aspects. Due to the different scales and physics of the involved individual processes, it is common practice to employ a model chain approach. These model chains typically represent a one-way coupling of different sub-models. Thus, outputs from one model are used as inputs to the next modelling step. The model chains found in the literature differ largely in terms of model fidelity, simplifications, and number of sub-models (see, e.g., Homola et al., 2012; Etemadfar et al., 2014; Molinder et al., 2018). The study discussed in Paper VII combines four individual steps:

1. Simulations of the ambient conditions affecting the ice accretion (wind speed, temperature, liquid water content) using a numerical weather prediction (NWP) model.
2. Ice accretion simulations of individual airfoil sections of the blade using a 2D RANS model combined with Lagrangian particle tracking of the water droplets and a morphing model to account for the change in airfoil shape due to the build-up of rime ice.
3. 2D RANS simulations computing the altered lift and drag coefficient of the iced airfoil.
4. Actuator line LES providing estimates of the icing impact on the power, loads and wake of the turbine.

When compared to previous studies, this model chain implies notably less empiricism and/or a higher model fidelity for the individual modelling steps. Furthermore, investigations of the effects of icing on the wake have so far often been neglected (Jin et al., 2014).

Unfortunately, the available experimental datasets only allow for a validation of the ice accretion model (step 3). Therefore, the main focus of the study is a sensitivity analysis of the individual modelling steps on the final results of the entire chain. To this end, different potential icing scenarios are considered using real-world meteorological data. Based thereupon we model the impact of icing on the NREL 5M turbine and discuss the sensitivities of various aspects such as the duration of the icing event, changes in the surface roughness of the blade as well as the operational regime of the turbine.

In the following, we firstly provide a brief description of the outcome of the first three model steps for one of the considered icing scenarios. We then turn to a discussion of the impact of the iced blades on the response of the full turbine. The latter involves a preliminary analysis using a BEM model as well as the actuator line LBM-LES runs of several exemplary operational conditions. For further details on the numerical set-ups of the individual models we refer to the original publication.
Figure 5.3. Time history of wind speed (a) and LWC (b) at 100 m elevation at Site A. Figure adapted from Paper VII.

Figure 5.4. Total ice mass along the blade for some instances in time at Site A. Figure adapted from Paper VII.

5.2.2 Results

The discussed results are based on meteorological input data for a site in the North of Sweden, referred to as Site A in Paper VII. Fig. 5.3 shows the time history of the wind speed and LWC at hub height \((z = 100 \text{ m})\) provided by the NWP model. The highest LWC is found in the first 12 h of the event. In combination with the fairly high wind speed this causes most of the ice accretion to occur in the first 10 h, as shown in Fig. 5.4. Expectantly, most of the ice is found on the outer half of the blade. An exemplary impression of the local ice accretion is shown in Fig. 5.5 for the outermost airfoil section. The majority of the ice accumulates close to the forward stagnation point, with somewhat more accretion on the pressure side of the aerofoil.

Despite some growth of ice mass after 12 h (see, Figs. 5.4 and 5.5), the aerodynamic effects of this accretion are found to be negligible. For the sake of brevity, we therefore only discuss the aerodynamic impact of the accretion state at 12 h. Fig. 5.6 shows the lift coefficient \(C_L\) and the glide ratio \(C_L/C_D\) as a function of angle of attack \(\alpha\) after 12 hours for the outermost section of the blade. In the comparison we include a case without consideration of additional surface roughness of the airfoil \((r_{0.0})\) as well as cases with 0.1 mm and 1.0 mm surface roughness. These two roughnesses correspond to typical values found
Figure 5.5. Blade profiles after 12 and 43 hours compared to the clean aerofoil at section 17. Figure adapted from Paper VII.

Figure 5.6. Lift coefficient (a) and glide ratio (b) after 12 hours of ice accretion for section 17 for several values of the roughness height. Note, that similar icing effects are found for the other airfoil sections in the outer third of the blade. Figure adapted from Paper VII.

for rime ice and have been incorporated in the wall model of the 2D RANS simulations. The largest aerodynamic effects of the ice are found at very low and very high angles of attack. The glide ration is mostly affected by a substantial increase in drag although the lift is also decreased. The most notable effect on the lift is found at negative angles of attack where the ice shape causes separation. Moreover, we can observe a decrease in the stall angle. Most importantly, for $r_{0.0}$ the lift coefficient and the glide ratio are almost unchanged at typical angles of attack in normal operation, i.e. around 5 degree. In this region, the additional surface roughness generally has a more significant effect on the aerodynamic forces than the change in airfoil shape.

Before moving to the LES, we investigate the sensitivity of the turbine response to the altered aerodynamic data using a BEM model coupled to the standard controller of the turbine. Steady-state results of different operating regions are depicted in Fig. 5.7. As expected from the lift and glide ratio curves, both power and thrust are only mildly affected by the ice in the zero-roughness
Figure 5.7. Steady-state BEM results of the turbine response with clean and iced airfoil properties after 12h of accretion using different surface roughness. (a) power curve, (b) thrust curve, (c) pitch curve, (d) tip-speed ratio TSR. Figure adapted from Paper VII.

It (case \( r_{0.0} \)). In region 1.5 to 2.5 power losses range from 1 to 2%, while a small increase in thrust can be observed in region 3. With \( r_{0.1} \) and \( r_{1.0} \), power losses of up to 21% and 39%, respectively, are found in region 2. Above rated wind speed, increases of similar magnitude occur in the thrust. Generally, we find that the ice accretion leads to power losses (and smaller thrust) in below-rated conditions. Beyond region 2, the iced turbine still reaches the rated power, but only at larger-than-rated wind speeds and at the cost of a higher thrust. In variable-speed pitch-regulated wind turbines, the controller tries to maximise the power capture in region 2 while retaining a fixed pitch angle \( \theta_0 \). In this region, the generator-torque controller follows a pre-defined torque curve \( M_G = K_G \omega^2 \), with a controller constant \( K_G \) that is tuned for the optimal TSR and, thus optimal power capture. As \( K_G \) is optimised for the clean turbine but the aerodynamic torque is lower, the iced turbine operates at consistently lower TSRs than intended in the controller design (see, Fig. 5.7d). Beyond rated wind speed the controller objective is to limit the generator speed while retaining the rated power. To that end, the pitch controller increases the pitch, thereby decreases \( \alpha \) and reduces the overall torque and TSR. Throughout region 2, the iced turbines operate at a lower constant TSR. Therefore, the maximum rotor speed is reached at higher wind speeds and the pitch controller steps is later (see Fig. 5.7c).

The discussed power-thrust characteristics of the iced rotors suggest that the impact of icing on the wakes strongly depends on the operational regime
of the turbine. Two exemplary cases are therefore further investigated with LES. In the first case, we impose an inflow with a mean hub-height velocity $u_0 = 8.8 \text{ m s}^{-1}$, referring to region 2 operation. In the second case, we choose $u_0 = 15.5 \text{ m s}^{-1}$ which is well within region 3. The hub height turbulence intensity in both cases is about 9%. The generator torque and pitch in the ALM are again set by the standard controller. The mean tangential and normal forces along the blade of the first turbine are shown in Fig. 5.8. In the outer two-thirds of the blade both $F_t$ and $F_n$ exhibit relative differences of similar magnitude as the total power and thrust, respectively. Indeed, the resulting power losses and thrust increases closely agree with the BEM results discussed earlier.

In Figs. 5.9 and 5.10 we compare the vertical profiles of the mean streamwise velocity $\tilde{u}$ and turbulence kinetic energy (TKE) in the wake of the turbine for the two cases. In the region 2 case, the highest velocity deficit and TKE are found for the clean turbine and $r_{0,0}$ due to the significantly larger thrust. With larger surface roughnesses both velocity deficit and TKE decrease, particularly in the near-wake. In the region 3 case, the blade condition has less impact on the wake characteristics. In the near-wake the clean turbine and $r_{0,0}$ exhibit a slightly larger velocity deficit and TKE downstream of the root of the blade, reflecting the higher normal forces found in this part of the blade. Near the tip, the velocity deficit and TKE increase with blade surface roughness. A slightly elevated TKE in the outer part of the wake can still be observed until $6D$ downstream, while differences in the mean velocity become negligible.
Figure 5.9. Mean stream-wise velocity $\bar{u}$ (top) and TKE (bottom) in the wake of the turbine at different downstream positions $x$ in the region 2 case. Same line colors as in Fig. 5.7. Figure adapted from Paper VII.

Figure 5.10. Mean stream-wise velocity $\bar{u}$ (top) and TKE (bottom) in the wake of the turbine at different downstream positions $x$ in the region 3 case. Same line colors as in Fig. 5.7. Figure adapted from Paper VII.
Lastly, we discuss two additional cases of a row of two turbines in the same inflow conditions. The second turbine is situated 5 $D$ downstream of the first turbine. The combined power of the two turbines is compared in Fig. 5.11. As for the power of the second turbine, we find relative differences to the clean case of +1%, -10% and -22% for $r_{0.0}$, $r_{0.1}$ and $r_{1.0}$, respectively. It should be noted, however, that the slightly larger power in the $r_{0.0}$ case is only due to the smaller velocity deficit discussed earlier, rather than a better aerodynamic performance. Moreover, the differences in the combined power are still similar to the relative power differences of the single turbine. In the region 3 case, the power differences are negligible with less than 1% for all roughnesses. As shown earlier, all cases exhibit very similar velocity deficits in the far-wake. At the same time, the second turbine also still operates in region 3 with all roughnesses (with a mean hub height velocity of about 12.8 m s$^{-1}$) and therefore reaches the rated power, as expected from the BEM analysis. A notable impact of the icing on the second turbine is thus only seen in the blade loads (not shown here for the sake of brevity), similarly to the single turbine case.

5.2.3 Conclusion

In the icing events considered in this study the rime ice predominantly accretes at the stagnation point of the airfoil. It is therefore found that the mere change in shape has only little effect on the aerodynamic properties of the blades in the range of angles of attack encountered during normal turbine operation. More severe effects, particularly reductions in lift, are only observed when considering additional surface roughness in the simulations of the iced airfoils. To this
point, this only indicates that surface roughness has a significant impact on the aerodynamic degradation due to ice accretion. However, future explorations are clearly required, for instance, investigations where and to what degree rime ice generates rough surfaces. If the change in shape was consistently less important, the focus of future model chains could be to correlate the degree of roughness to the meteorological conditions instead of performing computationally demanding ice accretion simulations.

More generally, it can also be concluded that further validations of individual steps within the model chain are inevitable to pinpoint specific weaknesses. However, the lack of validation data remains an issue for both wind tunnel and full-scale data, the latter often being scarce due to confidentiality issues (Gao and Hong, 2021).

5.3 A Three-dimensional Wake Model Based on Convolutional Neural Networks

Fast engineering wake models are the workhorse of wind farm modelling, particularly in the industrial practice. Nonetheless, established modelling techniques, such as analytical models, are facing various challenges due to the trends outlined in Section 1.1. In recent years, novel approaches for efficient fluid dynamics modelling have emerged from the field of deep learning. This includes turbulence (Jiang et al., 2021; Mohan et al., 2020) and wall models (Yang et al., 2019; Balasubramanian et al., 2021) for high-fidelity simulations but also surrogate models predicting entire flow fields for case-specific problems (Han et al., 2019; Santos et al., 2020; Jin et al., 2021). A promising approach for such flow field predictions are convolutional neural networks (CNNs). CNNs are based on filter kernels (convolutions) mapping input data with a grid-like topology to multiple feature maps. Passing the stacked feature maps of one convolutional layer to the next leads to a progressively more complex encoding of larger features. Compared to classically fully connected neural networks, CNNs comprise significantly fewer tunable parameters because the kernel size of each layer is usually a lot smaller than the size of the input data (LeCun et al., 2015). CNNs are therefore particularly efficient in processing large multi-dimensional data such images, videos, but also flow field data (see, e.g., Guo et al., 2016; Bhatnagar et al., 2019; Santos et al., 2020).

Paper VIII discusses a first feasibility study of a CNN-based surrogate wake model referred to as WakeNet. The model is trained with the statistics obtained from actuator line LES runs.

5.3.1 WakeNet

The objective of WakeNet is to predict the time-averaged three-dimensional flow field downstream of a single turbine, while the input is required to be eas-
ily specifiable, i.e. mean inflow conditions, aerodynamic data of the turbine (radial distributions of thickness, chord length, etc., as well as airfoil polars) and a tip-speed ratio (TSR). The overall modelling concept and a brief description of the CNN model architecture is outlined in the following. An illustration of the workflow and model architecture is provided in Fig. 5.12.

The input data of the model is two-dimensional and comprises the undisturbed mean inflow velocities ($\bar{u}, \bar{v}$ and $\bar{w}$ in the stream-wise, lateral and vertical direction $x$, $y$ and $z$, respectively) and the turbulence intensity $T_i$ on an $x$-normal plane of $4D \times 4D$ upstream of the turbine. In addition, the model is fed with an estimation of the mean normal and tangential aerodynamic forces of the turbine ($\bar{p}_n$ and $\bar{p}_t$) that are mapped onto the same plane with an actuator-disk-like approach. To this end, we employ the BEM method and the prescribed inflow velocity $\bar{u}$, aerodynamic data of the turbine, and a user-specified TSR. $\bar{p}_n$ and $\bar{p}_t$ are thus only non-zero in the rotor-swept area of the turbine (see Fig. 5.12). Furthermore, it should be stressed that the mapped forces only serve as an input to the model that shall facilitate the prediction of the wake. It does not represent an output. The accuracy of the forces, for instance, with regards to the forces in the LES-generated training data, is therefore secondary. The mapped force distribution should only be well-correlated with the wake flow. In summary, the input data comprises $n_y = n_z = 96$ points in the lateral and vertical direction, respectively, as well as $C_{in} = 6$ input channels.

The output of the model is the time-averaged three-dimensional flow field downstream of the turbine ($\bar{u}, \bar{v}, \bar{w}$ and $T_i; C_{out} = 4$) with the same in-plane resolution $n_y$ and $n_z$, and $n_x = 40$ grid points in $x$. With a resolution of $\Delta x = 6 \Delta y$ the output data extends $10D$ in the stream-wise direction. It should be emphasised that one key idea of the model is the compatibility of the model output and input in terms of in-plane dimensions and features. This way, the output of one model instance could serve as the input to another instance, allowing for the modelling of a wind farm at a future stage (given a sufficient training of the model with wake inflow data). For a better conditioning of the data, all velocity scales and the forces are normalised with the mean hub height velocity $u_h$ and $\rho u_h^2 D$, respectively (with $\rho$ being the density).

The utilised CNN architecture is a standard encoder-decoder structure as traditionally used for image-processing tasks. The task of the encoder is to sequentially increase the number of features while reducing the in-plane dimensions. On that account, each encoder layer consists of a convolution followed by a batch normalisation (Ioffe and Szegedy, 2015), ReLU activation, and a final down-sampling via MaxPooling. The size of each convolution kernel $k_y \times k_z$ is $3 \times 3$ with a stride $s_{y,z} = 1$. The decoder sequentially upsamples the encoded data by means of transpose convolutions. In WakeNet, the upsampling not only extends the in-plane dimension ($n_y$ and $n_z$), but also the data in the stream-wise direction. After each transpose convolution of the first two decoder layers (Conv4 and Conv5; see Fig. 5.12), we apply the same operations as in the encoder. The last layer (Conv6) only consists of a convolution
transforming the data to the final output dimensions. The kernel size of the convolutions in the decoder is $3 \times 3 \times 3$ with a stride $s_{x,y,z} = 1$.

For the training of the model we define the loss $J$ as the mean squared error (MSE) of $\bar{u}, \bar{v}, \bar{w}$ and $\bar{T}$ (L2 loss). The model parameters are optimised by minimising $J$ with mini-batch gradient-descent using the Adam optimiser with decoupled weight decay (AdamW; Loshchilov and Hutter (2019)).

### 5.3.2 The Dataset

For the training and testing of the model we generate a comprehensive dataset of flow cases of a single turbine in various inflow and operating conditions with actuator line LBM-LES.

The domain in the LES runs measures $L_x = 14\, D$ and $L_y = L_z = 6\, D$ in the stream-wise, lateral and vertical direction, respectively. The grid resolution is $\Delta x/D = 1/24$. A mean logarithmic velocity profile is prescribed at the inlet and perturbed with synthetic Mann turbulence. The turbine is situated at $x = \{3\, D, 3\, D, z_h\}$ while the hub height $z_h$ varies between the cases. After an initial spin-up, the flow statistics serving as training data are gathered for $10\, T$ (with $T = L_x/u_h$) in the respective cross-stream planes downstream of the turbine. A corresponding plane sampled $2\, D$ upstream of the turbine serves as the inflow model input for each case. Further numerical details of the simulations can be found in the original publication.

In all cases we simulate the NREL 5 MW reference wind turbine (Jonkman et al., 2009). Five parameters are varied which notably affect the wake properties, i.e., the TSR, $z_h$, $u_h$, the shear (parametrised via the roughness length $z_0$ in the logarithmic inflow velocity profile) and the inflow turbulence intensity. The values of all parameters are summarised in Table 5.1. In total, the dataset contains $n_c = 900$ cases, resulting from all possible combinations of the parameters.
Table 5.1. Parameters covered in the simulations of the training dataset.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>n</th>
</tr>
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<tbody>
<tr>
<td>TSR</td>
<td>5, 6.0, 7.0, 7.5, 8.0</td>
</tr>
<tr>
<td>$z_h/D$</td>
<td>3, 0.71, 0.95, 1.19</td>
</tr>
<tr>
<td>$u_{ik}/m,s^{-1}$</td>
<td>5, 5.0, 7.5, 10.0, 12.5, 15.0</td>
</tr>
<tr>
<td>$z_0/m$</td>
<td>3, $10^{-3}$, $10^{-2}$, $10^{-1}$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>4, 0.01, 0.05, 0.10, 0.15</td>
</tr>
</tbody>
</table>

Figure 5.13. Box plots of the $L_2$ relative test error of the model trained with different amounts of training data $n_{\text{train}}/n_c$. The whiskers represent 1.5 times the interquartile range. The mean is given by blue dots. Figure adapted from Paper VIII.

5.3.3 Training

We evaluate the overall accuracy of the model in terms of the $L_2$ relative error for each case $i$ of the test data:

$$L_2 = \sqrt{\frac{\sum_{j=1}^{n_p} (Y_{\text{WN},i}(x_j) - Y_{\text{LES},i}(x_j))^2}{\sum_{j=1}^{n_p} Y_{\text{LES},i}(x_j)^2}},$$

(5.3)

where $Y_{\text{WN}}$ and $Y_{\text{LES}}$ are the model outputs and the LES reference value, respectively, and $n_p$ is the number of grid points of the output data.

As a starting point, we examine the sensitivity of the model to the number of training cases $n_{\text{train}}$. In each training run, the model accuracy is evaluated with the same randomly chosen test data, referring to 20% ($n_{\text{test}} = 180$) of the cases of the dataset. A comparison of $L_2$ of the model trained with different fractions of the dataset $n_{\text{train}}/n_c$ is shown in Fig. 5.13.

The accuracy of all output parameters notably improves the larger the number of training cases. Only the accuracy of $T_i$ seems to converge already with
60% training data. Furthermore, larger amounts of training data reduce the spread of the test error of $\bar{u}/u_h$ and $T_i$. The model trained with 80% training data exhibits a mean $L_2(\bar{u}/u_h)$ of 0.56%. The largest mean relative error is found for the lateral and vertical velocity with 27.95% and 27.41%, respectively. $T_i$ is predicted with a mean relative error of 17.01%. The observed errors indicate that the model concept is generally able to parametrise the wake flow. In particular, the accuracy of the stream-wise velocity can arguably be appreciated. Moreover, it is noteworthy, that the larger relative errors in the other outputs seem to be mainly attributed to their lower magnitudes, as the absolute errors are comparable to $\bar{u}$ or even lower (see Section 5.3.4).

5.3.4 Flow Field Predictions

Fig. 5.14 shows a selection of vertical profiles of $\bar{u}$, $\bar{v}$, and $T_i$ of three exemplary cases of the test data. The three cases correspond to three different TSRs but the same inflow conditions. In all cases, the model excellently predicts the downstream evolution of the stream-wise velocity. This refers to both the characteristic shape and strength of the velocity deficit as well as the velocity of the ambient flow. In absolute terms, a similarly good agreement is found for the lateral velocity. The model also reasonably predicts the general characteristics of $T_i$ throughout the wake. Yet, somewhat larger deviations from the LES solution can be observed, particularly at $x/D = 5$. Generally, the results illustrate that characteristic dependencies of the wake on the thrust and power of the turbine (e.g., magnitude of the velocity deficit, wake rotation and the downstream recovery rate) are successfully encoded in the model.

For a more explicit analysis of the spatial variation of the model accuracy, we provide contour plots of the local mean absolute error

$$\text{MAE}(x) = \frac{1}{n_{\text{test}}} \sum_{i=1}^{n_{\text{test}}} |Y_{\text{WN},i}(x) - Y_{\text{LES},i}(x)| \tag{5.4}$$

of the test data in Fig. 5.15. The highest MAE of the stream-wise velocity can be observed in the near-wake ($x/D = 1$) as a clear outline of the outer edge of the rotor swept-area of the turbine. Further downstream, the error decreases and becomes more diffused. The largest errors of $\bar{v}$ and $\bar{w}$ are also found in the near-wake, but are more concentrated in the rotor center. In contrast, the MAE of $T_i$ is largest at $x/D = 5$. In general, we find that larger errors tend to occur in parts of the flow field with a larger variability of the output variables in the training data. For instance, the turbulence intensity is generally small in the near-wake, irrespective of the inflow conditions or TSR of the turbine. A drastic increase of $T_i$ is then typically found after the transition of the wake (see, e.g., Fig. 5.14). The downstream distance of this transition point strongly depends on the inflow conditions and thrust of the turbine. Therefore, the
Figure 5.14. Vertical profiles of $\bar{u}$, $\bar{v}$, and $T_i$ as predicted by WakeNet (full lines) compared against the corresponding LES results (dashed lines). All cases refer to the same inflow conditions ($z_h/D = 0.95$, $u_h = 7.5$ m s$^{-1}$, $z_0 = 0.1$ m and $T_i = 0.05$). Figure adapted from Paper VIII.
magnitude and local distribution of Ti vary mostly between the training cases at $3 \leq x/D \leq 8$, while they are relatively similar closer to the turbine.

5.3.5 Conclusion

This study shows that CNNs are a promising deep learning method to encode high-fidelity simulation data into computationally efficient surrogate models. On a single core of a local CPU (Intel i7-7800X) a single prediction of the trained model took about 1326 μs, or alternatively 33 μs on an Nvidia RTX 2080 Ti GPU. The presented results illustrate that the model is able to accurately predict detailed features of the wake under different inflow and operating conditions. This combination of high efficiency and accuracy makes it a potential contender for current low to mid fidelity models such as analytical wake models or even RANS.

Although the model yields acceptable results with less than the maximum of 720 training cases (80% of the dataset), it is shown to be preferable to utilise the maximum or potentially even more cases for the training. This large amount of required data can also be stated as the main downside of the modelling approach. On the other hand, the computational demand of generating the dataset
(26 min wall time per simulated LES case on a single GPU) can be considered acceptable due to the efficiency of the utilised LBM-LES solver.

Furthermore, it should be stressed that the discussed model refers to the simplest CNN architecture possible, i.e. a shared encoder-decoder. Judging from other studies of CNNs for fluid flow problems, further improvements can be anticipated using, e.g., an encoder with separate decoders for each output variable (Bhatnagar et al., 2019), ResNet (Chung et al., 2020) or ResUNet-type architectures (Santos et al., 2020) and/or spatially weighted loss functions, to name a few. Other aspects worth investigating are the sensitivities of the model to wake conditions not covered in the training data. This includes other inflow or operating conditions as well as different turbine types.
6. Computational Performance

The excellent computational performance of the LBM is the main motivation for the use of method in this work. Over the past decade, numerous publications documented the computational potential of GPU-based LBM frameworks including detailed performance analyses of single- and multi-GPU implementations (e.g., Schönherr et al., 2011; Obrecht et al., 2013; Januszewski and Kostur, 2014; Hong et al., 2016; Onodera and Idomura, 2018). A common metric for the performance of LBM solvers is the number of node (grid point) updates per second, typically given in MNUPS (Million Node Updates Per Second). For the cases discussed in this work, the performance ranges from 750 to 1000 MNUPS on an Nvidia RTX 2080 Ti GPU and up to 1200 MNUPS on an Nvidia RTX 3090. This is well in line with the values found in the literature for applications of the CLBM on similar hardware (Lenz et al., 2019; Latt et al., 2021). By means of several performance metrics, we illustrate the potential of this efficiency of the LBM for two of the cases discussed earlier in this work. Furthermore, we compare the LBM performance with the corresponding simulations of finite-volume Navier-Stokes (FV-NS) solvers. The comparison is intended to put the performances of the LBM into perspective to one of the common numerical approaches used in the field for LES. For the sake of completeness, it should be emphasised that these cases are only exemplary for the performance gains that can be achieved with the LBM. Other cases, numerical set-ups (e.g., different grids topologies or number of subiterations in the FV-NS solvers, or different collision operators or Mach numbers in the LBM) and/or hardware configurations (e.g., different GPU models or other numbers of CPUs for the FV-NS solvers) will obviously yield somewhat different comparison results.

Specifically, we compare four metrics. The former is the plain wall time $T_{\text{wall}}$ for the simulation. Secondly, we compare the ratio of simulated time $T$ to wall time. $T/T_{\text{wall}} > 1$ thus refers to simulations faster than real-time and has become a crucial metric for the efficiency of fluid dynamics simulations (Harwood et al., 2018; Onodera and Idomura, 2018; Lenz et al., 2019). As for wind energy applications, Bauweraerts and Meyers (2019), for instance, discuss that beyond real-time LES would facilitate high-fidelity real-time forecasting of wind farm power and loads. Third, we compare the processor time. Note, that we hereby refer to the product of the wall time and the number of processors (GPU or CPU), not the total number of cores within the respective processing units (referring to core hours). Lastly, we contrast estimates of the energy consumption of the simulations, commonly referred to as energy to
solution. The energy consumption of HPC applications is becoming a considerable sustainability concern given the enormous electricity demand of modern HPC clusters (García-Martín et al., 2019; Portegies Zwart, 2020). Moreover, the financial cost for running large cases correlates strongly with the electricity consumption, making energy efficiency a key challenge for future HPC applications and one of the most important metrics for computational performance (Kamil et al., 2008; O’Brien et al., 2017; Hennessy and Patterson, 2019).

As for the energy consumption of the two FV-NS cases, it should be noted, that direct power measurements of the utilised compute nodes on the HPC cluster Tetralith were not available. We therefore base our estimate for the power per processor on the estimated power usage of the cluster during runs of the LINPACK benchmark (Dongarra et al., 2003) as reported in the Top500 list\(^1\), i.e. 188.68 W per processor\(^2\). The LINPACK benchmark serves as a good estimate for a high load scenario involving large matrix operations. Furthermore, as opposed to the theoretical thermal design power (TDP) stated by the CPU supplier, the figure also comprises the power required to run other hardware components and the cooling. The power for the LBM GPU case was directly measured at the local workstation and thus also includes the power of other hardware such as the CPU and cooling.

### 6.1 Comparison on Identical Grids

The first example case is from Paper II, dealing with simulations of the wake of a single turbine (see Section 3.3). A summary of the utilised hardware for the FV-NS and the CLBM solver and the main numerical aspects of the case set-ups are given in Table 6.1.

Most important for this comparison is the fact that the case was simulated on identical uniform Cartesian grids with the two solvers. Hence, the comparison serves as an example to contrast the computational performance when delivering a solution with the same spatial resolution. It should be noted, however, that this type of grid was dictated by the LBM solver. A larger aspect ratio of stream-wise to lateral/vertical resolution, and thus smaller grid, might be sufficient to obtain similar accuracies in the FV-NS solver for this particular case. See, e.g., Martínez-Tossas et al. (2018).

The aforementioned metrics of the two simulations are shown in Fig. 6.1. The wall time of the CLBM simulation is almost a factor of 10 lower than the FV-NS case, referring to near real-time simulation. Most striking, however, is the significantly larger energy consumption of the FV-NS case. This is particularly important when considering the scenario of achieving similar wall times

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1\(^{www.top500.org}\)

2\(^{Note, that the power consumption is not stated in the published LINPACK results of Tetralith, but was kindly provided by the support team of the National Supercomputer Center (NSC) at Linköping University.}\)
Table 6.1. Comparison of the utilised hardware and case set-up for the FV-NS and CLBM simulations of the cases with the highest grid resolution ($\Delta x = D/32$) discussed in Paper II.

<table>
<thead>
<tr>
<th>Solver</th>
<th>EllipSys3D (FV-NS)</th>
<th>elbe (CLBM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing platform</td>
<td>Intel Xeon Gold 6130 (CPU)</td>
<td>Nvidia RTX 2080 Ti (GPU)</td>
</tr>
<tr>
<td>Number of processors</td>
<td>66</td>
<td>1</td>
</tr>
<tr>
<td>Cores per processor</td>
<td>16</td>
<td>4352 (CUDA cores)</td>
</tr>
<tr>
<td>Total power /kW</td>
<td>12.45</td>
<td>0.44</td>
</tr>
<tr>
<td>Number of grid points /$10^6$</td>
<td>34.6</td>
<td>34.6</td>
</tr>
<tr>
<td>Time step $\Delta t$/s</td>
<td>0.065</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Figure 6.1. Comparison of the wall time $T_{wall}$, real time ratio $T/T_{wall}$, processor time and energy to solution for the high-resolution cases of Paper II. All metrics refer to the entire simulation with $T = 17.52 \ L_x/\ u_0 = 8000$ s.
with the FV-NS solver. Theoretically, this would obviously be possible using more CPUs in parallel. Yet, even with an ideal strong scalability of the solver, the energy demand of the simulation would eventually remain similarly large.

### 6.2 Comparison on Solver-typical Grids

The second example is from the modelling benchmark discussed in Paper VII. Here, we compare the LBM performance to the FV-NS solver OpenFOAM (referring to LESUU,LBM and LESUU in Section 3.5, respectively). In these two cases, the same uniform Cartesian grid is used in the wake region (with $\Delta x = D/32$). However, the grid in the rest of the domain refers to the typical topologies used in the respective solvers. For instance, in OpenFOAM the grid above the turbines is continuously stretched towards the top of the domain. In elbe, on the other hand, a uniform background grid is used, while a nested refinement is placed in the wake region. This also explains the somewhat larger number of grid points in the LBM case. Unfortunately, a notably higher resolution was used in the wake region in the corresponding EllipSys3D case. A direct performance comparison to this solver is therefore difficult. A summary of the hardware and numerical details is provided in Table 6.2.

The performance metrics of the two simulations are contrasted in Fig. 6.2. The wall time with the LBM solver is again significantly lower than with the FV-NS solver. For this particular case the LBM simulation even runs 67% faster than real time. Similarly to the previous case, the energy consumption of the FV-NS case is about 300 times larger than with the LBM solver. Another interesting fact is that the energy to solution of the LBM case is indeed close to that of the steady-state RANS case also presented in Paper VII (RANSUU ).

<table>
<thead>
<tr>
<th>Solver</th>
<th>OpenFOAM (FV-NS)</th>
<th>elbe (CLBM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Processing platform</td>
<td>Intel Xeon Gold 6130 (CPU)</td>
<td>Nvidia RTX 2080 Ti (GPU)</td>
</tr>
<tr>
<td>Number of processors</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Cores per processor</td>
<td>16</td>
<td>4352 (CUDA cores)</td>
</tr>
<tr>
<td>Total power / kW</td>
<td>1.13</td>
<td>0.44</td>
</tr>
<tr>
<td>Number of grid points /10$^6$</td>
<td>6.2</td>
<td>7.4</td>
</tr>
<tr>
<td>Time step $\Delta t / s$</td>
<td>0.030</td>
<td>0.018</td>
</tr>
</tbody>
</table>

Table 6.2. Comparison of the utilised hardware and case set-up for the FV-NS (OpenFOAM) and CLBM simulations of the DanAero benchmark discussed in Paper VII.
Figure 6.2. Comparison of the wall time $T_{\text{wall}}$, real time ratio $T/T_{\text{wall}}$, processor time and energy to solution for the FV-NS and CLBM cases of Paper VII. All metrics refer to the entire simulation with $T = 900$ s.
7. Conclusion

An efficient bulk scheme and excellent parallelisability have made the LBM a promising alternative to classical computational fluid dynamics approaches. For various use cases, that were previously limited to lower fidelity models, the LBM has enabled affordable utilisations of LES in both industry and academia. This includes applications such as external car aerodynamics (Niedermeier et al., 2018), urban flows (Lenz et al., 2019), or dam breaks (Miliani et al., 2021), to name a few. The aim of this work was to facilitate efficient lattice Boltzmann LES for wind energy applications. To this end, two particular modelling gaps have been addressed, i.e. the representation of wind turbines with the actuator line technique and the modelling of the wall shear stress in simulations of ABL flows.

Four of the studies compiled in this work address the numerical sensitivities, verification and validation of lattice Boltzmann actuator line simulations. The initial verifications (Paper I and II) demonstrate a close agreement between the developed model and a well-established implementation of the ALM in an incompressible Navier-Stokes solver. This refers to both the aerodynamic forces along the blade and the wake characteristics. Furthermore, the studies reveal a characteristic sensitivity of the aerodynamic forces of the model to the compressibility of the scheme. This becomes particularly noticeable at small widths of the regularisation kernel ($\epsilon < 2\Delta x$). Further investigations (Paper III), indicate that the observed sensitivity originates from changes of the bound vortex along the actuator line, evoking errors in the angle of attack. Potentially, future compressibility corrections to the ALM could address the observed issues. Nonetheless, it should be emphasized that the observed phenomena only become relevant for $\epsilon$ that lie below the values typically applied in Navier-Stokes solvers. Lastly, the model has been validated together with five other wind turbine modelling frameworks against full-scale measurements of a utility-scale turbine in waked inflow conditions (Paper VII). The validation shows that the model is able to reproduce various characteristic features of the incoming wake and the resulting response of the impacted turbine. Matching the measured ambient flow conditions downstream of the inlet is identified as a main difficulty for LES in general, in line with previous full-scale validation studies (Doubrawa et al., 2020). Moreover, the study highlights the need for more detailed full-scale measurement campaigns that allow for a more precise identification of model deficits.

Motivated by different deficiencies of existing LBM near-wall treatments, a novel wall modelling approach was developed: the inverse momentum exchange method. The application of the model to simulations of neutral ABL
flows has been discussed in Paper VI. Among others, the simulation results are compared against theoretical, numerical and experimental references. The comparison shows that both mean quantities and higher-order turbulence statistics can be well-captured by wall-modelled lattice Boltzmann LES using the iMEM wall model and cumulant collision scheme. This generally renders the set-up as a suitable approach for simulations of boundary layer flows. One deficiency of the discussed set-up is an under-prediction of the velocity at the first grid point. It is therefore necessary to sample the input velocity of the wall model beyond the first grid point in order to avoid a notable log-layer mismatch in the bulk.

With the exception of some initial pre-studies (see, e.g., Appendix A in Paper II) this work exclusively employed the cumulant collision model, mostly in its parametrised fourth-order accurate form (Geier et al., 2017a). This choice was initially motivated by the favourable numerical stability properties of the model. Conclusively, however, it should be emphasised that the unprecedented accuracy of the parametrised CLBM (compared to other single-speed LBM schemes) proves to be a similarly valuable asset. After all, the quality of both wake and boundary layer statistics benefits heavily from the low diffusivity of the scheme. This further justifies the computational overhead of the model compared to more simple approaches such as SRT or MRT collision models.

It can be concluded that the methods presented in this thesis adequately meet the targeted modelling goals. First and foremost, this shows that two essential building blocks of wind farm simulations are feasible with the LBM: the modelling of wind turbines with the actuator line technique, and wall-modelled boundary layer simulations. Furthermore, the studies illustrate that typical use cases of LES in wind energy can be simulated both significantly faster and at a fraction of the computational cost of conventional numerical approaches. At the same time, it can be appreciated that the gains in computational efficiency do not come at the expense of accuracy, at least when compared to second-order finite-volume solvers. Hereby, we can return to one of the starting points of this work. For the discussed cases, GPU-based LBM solvers do provide the sufficient efficiency to enable affordable overnight runs of typical wind energy LES cases. Thus, the method is able to overcome the LES crisis formulated by Löhner (2019). With the given performances we can even confidently project this to larger cases of, e.g., modern offshore wind farms, that have not been covered in this work. Cheaper and faster simulation alternatives obviously benefit fundamental academic studies that constantly strive for more demanding simulations. At the same time, this can facilitate high-fidelity simulations for typical industrial modelling tasks like power or load predictions that are so far limited to RANS or engineering models such as dynamic wake meandering models. And, even utterly new applications of LES can be envisioned. Real-time forecasts are one of them (Bauweraerts and Meyers, 2019). Other potentials have been shown in Paper V and VIII, i.e. applications of rein-
forcement learning or massive data generation for the training of deep learning models.
8. Sammanfattning

de är robusta och framför allt extremt beräkningseffektiva. Med tanke på den
observerade beräkningseffektiviteten dras slutsatsen att LES för industriella
vindkraftstillämpningar är möjliga med GPU-baserade LBM-lösare, vilket har
visats på andra områden. Dessutom illustrerar ytterligare studier som presen-
teras i denna avhandling, ytterligare potentialer hos metoden. Exempel på
sådana är tillämpningar av reinforcement learning för vindkraftverkskontroll
eller storskalig datagenerering för träning av deep learning modeller för vak-
enprognoser.
Seven years ago, during my Erasmus semester here in Visby, I came across wind turbine wakes for the first time, a phenomenon which immediately fascinated me. About the same time, I heard about the LBM, about GPUs, and that there seemed to be a way to accelerate CFD simulations significantly. Without knowing much about wind energy, nor ever having heard of LBM details like a collision operator, I had this gut feeling that the two might be a good match. Even two years later, suddenly returning to Visby to start a PhD, I did not expect that this should become what I would spend the next four years on. And here I am, writing these last lines of this thesis, and I can happily say that it actually all worked out. It all worked out because this gut feeling must have been not too bad after all – not to mention a good amount of luck along the way – and, above all, thanks to the support of many great people who knowingly or unknowingly helped me get this work done.

First of all, I want to thank my supervisor Stefan Ivanell. You actually lit the spark for my fascination with wakes back then in the master’s lectures on wind resource assessment. Who could have known that we would actually meet again for this PhD? Thank you so much for your guidance and for giving me the opportunity to work in such an excellent research environment. Most importantly, thank you for putting all this trust in my abilities and letting me freely follow my curiosity despite the uncertainty of the outcome.

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Special thanks go to Christian F. Janßen, who initially introduced me to the LBM during my master’s thesis and who played a crucial role in awakening my fascination for research and computational fluid dynamics. I am very glad that our collaboration did not end when I left Hamburg. Thank you for the many online coffee breaks, discussions on LBM issues, and for your motivating enthusiasm.

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the most gentle and curious mind, who taught me patience, persistence, and 
finding joy in the small things: I wish you were here today.

And finally, my deepest gratitude goes to Myri, my partner, accomplice, 
and most loyal companion. Thank you for always accepting my decisions, 
for giving me freedom and still being there for me, for enduring intangibly 
long times of spatial separation. Thanks for trying to share my enthusiasm for 
obscure things such as grids, eddies or wall shear stress. And, thank you for 
reminding me of the comical aspects of terms like slip velocity or smearing 
width. Your love, laughs and support were essential for this work.
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