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Explorations of University Physics in Abstract Contexts

From de Sitter Space to Learning Space

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Abstract

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This is a thesis which contributes to research in two different fields: theoretical physics and physics education research. The common link between these two research areas is that both involve explorations of abstract physics and mathematical representations, but from different perspectives.

The first part of this thesis is situated in theoretical physics. Here a cosmological scenario is explored where a de Sitter phase is replaced with a phase described with a scale factor a(t) ~ tq, where 1/3<q<1. This scenario could be viewed as an inflationary toy model, and is shown to open up the possibility of an information paradox. This potential paradox is resolved even in the worst case scenario by showing that the time scales involved for such a paradox to occur is of the order of magnitude of the recurrence time for the de Sitter space.

The second part of this thesis is situated in physics education research. A number of learning situations that are experienced as abstract by students are explored: probability in one dimensional quantum tunnelling; the mindsets that students adopt towards understanding physics equations used in typical teaching scenarios; and what students focus on when presented with physics equations. The results for the quantum scattering study are four phenomenographic categories of description, for the mind sets study, six epistemological components of mindsets and for the focus on physics equations study, three foci creating five levels of increasing complexity of ways of experiencing physics equations. Pedagogical implications of these results are discussed.

Keywords: phenomenography, cosmology, de Sitter space, quantum mechanics, conceptual understanding, epistemology, physics equations

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List of publications

This thesis consists of an introductory text and the following appended research papers, henceforth referred to as Papers I-IV:

I. Danielsson, U. H., Domert, D., & Olsson, M. E. (2003). Puzzles and resolutions of information duplication in de Sitter space. Physical Review D, 68, 083508

II. Domert, D., Linder, C., & Ingerman, Å. (2005). Probability as a conceptual hurdle to understanding one-dimensional quantum scattering and tunnelling. European Journal of Physics, 26, 47-59.

III. Airey, J., Domert, D., & Linder, C. (2006). Representing disciplinary knowledge? Understanding students' experience of the equations presented to them in physics lectures. Extended abstract accepted for presentation at EARLI SIG-2 2006 biannual meeting. University of Nottingham. 30 August – 1 September 2006, Nottingham, UK.

IV. Domert, D., Airey, J., Linder, C., & Lippmann Kung, R. (2006 submitted, revision in progress). An exploration of university physics students' epistemological mindsets towards the understanding of physics equations. NorDiNa (Nordic Studies in Science Education).

Contents

1. Introduction to the thesis

1.1 The overarching theme

At one level, my research presented in this thesis is in two very different fields of physics – theoretical physics and physics education research – so at first it might seem that these two areas have very little in common. However, from a learning perspective several common attributes could be brought to the fore, and I also believe that my research in theoretical physics provided me with essential insights into physics learning that helped inform my research in physics education.

There is however an explicit common denominator in this thesis that link my work in theoretical physics and my work in physics education research. This link is what I have chosen to call *potentially abstract physics*. What does this term mean? Let us look at the individual words of this term in order to get an understanding of the meaning of potentially abstract physics. I assume that the reader knows or at least has an idea of what physics means, so let us focus on the other two words: "abstract" and "potentially".

The word "abstract" can take on many different meanings. To many people it is simply a synonym of words such as "difficult" and "complex". This common usage is, however, not the meaning I would like to attribute to the word "abstract" in this thesis so let me for the moment focus on pinning down the meaning of "abstract" which I would like a reader to have in mind.

Let us for a moment turn our attention to a scene where the word "abstract" is perhaps most well-known: abstract art. What does "abstract" mean in this context? In the world of art "abstract" normally refers to art that is not a realistic representation of something concrete. Another way of putting it is to say that abstract art does not have a recognizable subject that relates to anything familiar. Or to put it even more simply: abstract art is art which depicts something that we cannot recognize from our everyday life and past experiences. Lifting out the word art from the last sentence brings out the meaning of "abstract" that I want the reader to bear in mind throughout this thesis:

abstraction is used to characterise something that we cannot recognize from our everyday life or past experiences.

So the common denominator of this thesis – potentially abstract physics – involves *physics that we cannot relate to our everyday life or past experiences*. However, I have not yet explained why I have decided to put "potentially" put in front of abstract physics. The reason for this is that whether something is perceived as abstract or not is dependent on the individual perceiver. Something perceived as abstract by one person is not necessarily perceived as abstract by another person. There are, however, parts of physics that are likely to be perceived as abstract and these parts are what I refer to as *potentially abstract physics* in this thesis.

Many branches of physics contain concepts or phenomena that could be classified as potentially abstract physics – such as electric and magnetic fields in electromagnetism – but potentially abstract physics is more prominent in some areas of physics than in others.

In classical mechanics, which deals with the motion of objects and the cause of these motions, we have a lot of earlier experience with these issues that sometimes help us and sometimes hinder us in dealing with aspects of this branch of physics. Velocity, acceleration and force are all familiar terms in our everyday life (even though some terms may take on different meanings in our everyday life compared to their meaning within physics). It is a completely different story when we get to other fields of physics such as, for instance, the special and general theory of relativity, quantum mechanics, cosmology and string theory.

Special theory of relativity contains some highly counter-intuitive phenomena and results, such as the relativity of times and lengths; different people can measure different times and lengths for an event depending on their relative motion. However, these effects only become noticeable when we are concerned with speeds close to the speed of light. This is not the case in most people's everyday lives – unless you happen to be a particle physicist working at an accelerator – and we therefore lack previous experience of this phenomenon.

General relativity, which may be characterised as a generalisation of special relativity, features a geometrical model of gravity that is almost unimaginable from everyday life experience. Within general relativity, space and time is no longer a passive background but rather constitute a four-dimensional space-time that is affected by the presence of matter.

Quantum mechanics is concerned with the particles that constitute matter in nature, and consists of plenty of counter-intuitive and seemingly absurd ideas that we have no experience with and cannot relate to our everyday life. One of the ideas is, for instance, that a particle can behave both as a particle and a wave. We clearly have no experience with such an object in our everyday lives. In fact, it is hard to imagine two things more different than a particle and a wave. I will return to the peculiarities of quantum mechanics in greater detail later in this thesis.

String theory (like quantum mechanics) is concerned with the building blocks of matter, but at an even unimaginably smaller scale. In string theory, matter is assumed, at the smallest scale, to consist of, in everyday terms, “ridiculously small” one-dimensional vibrating strings that interact with each other. Apart from the difficulties with imagining a one-dimensional object, string theory also needs nature to have more dimensions than those we can perceive. For string theory to be consistent, we need dimensions beyond our three familiar spatial dimensions and the fourth dimension corresponding to time. This is all clearly beyond our everyday experiences.

While string theory is concerned with nature at its smallest conceivable scale, cosmology is concerned with nature at its largest conceivable scale. In cosmology, we try to understand the origin, history, current state and further development of the entire Universe. The Universe as a whole and the extreme conditions in the earliest states of our Universe are once again things that we cannot relate to in our everyday life, and thus we have no previous experiences to rely on and draw from in our sense making.

There are many other areas of physics where we have a similar situation, but the examples given above should be sufficient to give the reader a sense of what I mean by potentially abstract physics – physics that potentially can not be related to our everyday life and earlier experiences. Besides the fact that these areas probably are the most important cases where abstract physics prevails, there is another reason for specifically mentioning these areas – these areas of physics have been fields of interest in my own research in different ways which I will later spell out in more detail.

1.2 Style of writing

I would like to seize the opportunity in this introduction to discuss how this thesis is written and why. It is my strong personal belief that this kind of publication should be accessible for as wide an audience as possible and I consider making my research accessible outside of the research community to be an important part of being a researcher. Therefore, this thesis provides relevant background material in a non-technical way. However, this thesis should also be of value for peer researchers, by presenting the contributions of my own research in more technical terms, relating it to previous research and giving some ideas on future investigations to which my research points. These two thesis ingredients are not mutually exclusive and I hope that I have managed to write this thesis in such a way that a wide breadth of readers can find things of interest and value.

1.3 The significance of the thesis

I would argue that this thesis is able to provide a significant contribution to the development of understanding abstraction's role in the discourse of physics. The research in theoretical physics corresponds to a description of potentially abstract physics from the perspective of an inside observer, and the research in physics education corresponds to a description of potentially abstract physics from the perspective of an outside observer.

More specifically, this thesis can be seen as making research contributions in three main areas:

- Understanding and investigating the nature of some potentially puzzling aspects of de Sitter space in relation to inflationary cosmological models.
- Contributing to the exploration of university students' understanding of quantum mechanics in the area of quantum tunnelling.
- Exploring university students' experiences of physics equations in terms of their focus when presented with equations and how they epistemologically view what it means to understand physics equations.

1.4 Research questions

The first part of this thesis involves theoretical physics research in cosmology. Most contemporary cosmological models involve an era where the expansion of our universe is accelerating. Such an era is called inflation and in Paper I we investigate some potentially peculiar properties that could occur in a cosmological toy model where an inflationary phase represented by a de

Sitter space is followed by a particular (and highly realistic) type of phase in the evolution of our Universe. In this toy model, we pose and investigate an interesting question:

- In a cosmological scenario, where a de Sitter phase is followed by a phase with a scale factor evolving as t^q, where $1/3<q<1$, is there a possibility of information being duplicated, violating the quantum Xerox principle?

The second part of this thesis consists of research in physics education. Since I was interested in theoretical physics, a natural step was to investigate from an educational perspective an area of physics containing some kind of experienced abstractness. Paper II looks at one area of potentially abstract physics by investigating students' understanding of quantum tunnelling. The main research question for this study was:

- What is the variation in university students understanding of probability in one-dimensional quantum tunnelling?

From this study, it was clear that it can be difficult to conceptually understand potentially abstract physics and that mathematical representations play an even more important role in this type of physics than in general. This initiated an interest in students' experience of mathematical representations in terms of physics equations, leading to Paper III and IV. In Paper III, we are interested in exploring the focus of students in relation to physics equations, and the main research question was:

- What do university students focus on when presented with physics equations?

The results of Paper III caused us speculate whether these focuses could be related to a view of what it means to understand physics equations. So for Paper IV we are interested in exploring students' epistemological views of what it means to understand equations. This is done by investigating three questions in relation to what it means to understand physics equations:

- When students say that they understand an equation, how do they describe what that means to them?
- How can these descriptions be characterised in terms of epistemological mindsets?
- Are similar epistemological mindsets prevalent across levels for students at various stages in their academic career?

1.5 Overview of the thesis

In this introduction I have presented the reader with a description of the overall theme and style of this thesis. I have also described the significance of this thesis and presented the main research questions that are explored. Chapter 2 presents the research reported in Paper I as well as a non-technical background to some of the key aspects of this research. In Chapter 3, a general description of physics education research and its relation to physics as a discipline is provided, as well as an overview of the literature relevant for my research presented in this thesis. Chapter 4 presents the methodologies used in my research, as well as a general discussion of the value and reliability of qualitative research. In Chapter 5, the results of Paper II are presented and discussed, and a non-technical description of quantum mechanics and quantum tunnelling is provided. Chapter 6 contains a discussion of why and how mathematics plays an important role in physics, as well as a presentation and discussion of the results of Paper III and IV. The thesis is concluded with Chapter 7, which contains some concluding remarks as well as an outlook and topics for future research, and Chapter 8 which provides a Swedish summary.

Part I

2. Exploring aspects of de Sitter space

2.1 Introduction

In this chapter, I will present the reader with a discussion of what my theoretical physics research in Paper I involve. I attempt to do this by providing the reader with very brief crash courses in my research areas and those areas significantly related. Section 2.2 presents the basic cosmology necessary to appreciate the discussion of Section 2.3, where I take the reader on a brief, and far from complete, historical journey through the history of our Universe. Section 2.4 adds an era to this history called inflation and explains why this "add-on" is necessary to deal with some problems related to the history of our Universe discussed in Section 2.3. In Section 2.5 I discuss some properties of de Sitter space that are relevant for my research in Paper I and in Section 2.6 and 2.7 I discuss the scenario explored in Paper I and the main results of that study. I conclude this chapter with some reflective remarks on my research in theoretical physics in Section 2.8.

2.2 Basic Cosmology

For a long time, the theories of physics involved what could be referred to as absolute time and space. In these theories spacetime is a passive background against which everything else takes place – a background which is unaffected by whatever takes place against that background. This view changed dramatically with Einstein's entrance into the physics arena. In Einstein's general theory of relativity, time and space are no longer a "passive" background. Energy (including the mass of objects) affects spacetime in ways which are dependent on the details of the energy distribution. This is governed by a mathematical relation known as Einstein's equation. In simple terms, Einstein's equation gives us the geometry of spacetime given the distribution of energy.

In cosmology we are interested in the geometry of our Universe as a whole, which means that the energy distribution we should put into Einstein's equation is the distribution of all of the energy in our Universe. How could this possibly be done? This would at first sight imply that we had to know the exact position and state of every single particle in the Universe. This is of course an impossible task (personally I find it hard enough to know the exact positions of important papers in my office!).

Fortunately, observations have indicated that our Universe is highly symmetric *on large scales*. On these scales our Universe seems to be *homogenous* (it has the same matter density everywhere) and *isotropic* (it looks the same in all directions). These observations already tell us a lot about the general geometric properties of our Universe without us having to solve Einstein´s equation.

Another crucial observation, first made by Edwin Hubble in the late 1920s, is that our Universe is expanding. As time passes, our Universe grows larger and larger. This is captured by what is known as the scale factor a(t) for our Universe. As time goes by, the scale factor becomes larger and larger, reflecting the fact that our entire Universe is expanding. The distance between two comoving points (two points moving with the expansion of the Universe) is proportional to this scale factor. It is not easy to visualize an expanding four-dimensional spacetime, but one way of at least getting a rough idea (even though the analogy should not be taken literally) is to imagine a balloon (representing our Universe) with small coins (representing galaxies) attached to its surface (see Figure 1). As the balloon inflates (analogous to the expansion of our Universe), the distance between these coins increases. The increase in distance between two coins could be described by a scale factor *a(t)* in the same way as for our entire Universe. One can imagine a comoving coordinate system drawn on the surface of the balloon. *In this coordinate system*, the coordinate distance *d* between two coins will stay the same during the expansion since the coordinate system expands as well. The actual physical distance between the two coins would then at each moment in time be given by *da(t)* where *a(t)* is the scale factor at that given time.

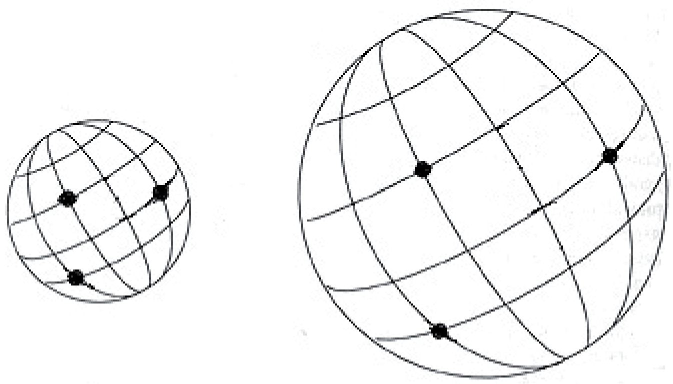

Figure 1. An illustration intended to give an insightful idea of the expansion of our universe in terms of coins attached to the surface of an inflating balloon. It can be seen from this picture that the coordinate distance between the coins stays the same since the coordinate system is expanding as well, while the actual physical distance between the coins increases.

Given the fact that our Universe expands, and that the expansion can be described by a scale factor *a(t)*, it is of course of great interest to determine what this factor looks like and how it has evolved and will evolve. The rate of expansion (how fast our Universe is expanding) is normally expressed in terms of the Hubble parameter $H = \dot{a}/a$, where $\dot{a} = da/dt$. If we can find the Hubble parameter, we know the rate at which our Universe is expanding.

Using Einstein's equation (and reasonable assumptions regarding the matter and energy in our Universe), it is possible to end up with an equation that gives us the scale factor a(t) if we know the matter density ρ in our Universe. This equation is known as the Friedmann equation:

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{K}{a^2}$$

In this equation Λ is the famous cosmological constant, G is the gravitational constant and K is a parameter related to the geometry of spacetime that can be deduced from the values of ρ and H today. If K=0 our Universe is flat, if $K > 0$ our Universe is closed and if $K < 0$ our Universe is infinite or open. (It is also infinite for K=0, but the term open is normally reserved for the case of $K < 0$). Observations indicates that our universe is spatially flat, corresponding to K=0.

Using the Friedmann equation one can calculate the scale factor and thereby the expansion rate of our Universe. We "only" have to plug in appropriate matter densities ρ. I put only in quotation marks since this is far from trivial. We have to make educated guesses and theorize the kind and amount of matter that existed in the early Universe based on cosmological observations and our knowledge of particle physics.

On top of that, to make the story slightly more intriguing, astronomical observations have indicated that there exists a vast amount of dark matter in our Universe, matter whose origin and nature still remains a puzzle. The mysteries do not end there. Einstein included the cosmological constant Λ in his original equation of general relativity to get a Universe which, according to his (and many others) belief, was eternal and had a fixed size. When the expansion of our Universe was discovered he abandoned the cosmological constant and is said to have called the inclusion of this parameter "his biggest blunder". However, in modern quantum field theories a cosmological constant arises naturally, corresponding to the energy of vacuum, which is far from an empty, boring place. In these theories vacuum contains a "boiling soup" of oscillating particles being constantly created and annihilated, giving vacuum an energy, even though the exact origin of and size of this constant continues to puzzle theoretical physicists. Recent observations [1, 2, 3, 4] indicate that our Universe is accelerating, indicating that there is a small but non-zero cosmological constant. This has turned the understanding of the origin and magnitude of the cosmological constant into one of the most extensively researched areas of theoretical physics. Many researchers view it as one of the most important issues to address in the field.

2.3 A (very) brief history of our Universe

After providing some basic cosmology I would like to give a brief, schematic description of selected moments of the history of our Universe to give the reader a sufficient background for Paper I.

The earliest time in the history of our Universe is an area which we know very little about. General relativity ceases to be useful at the enormous energy scale corresponding to this era corresponds. One could hope that string theory will provide us with valuable insights into this era in the history of our Universe. In fact, a lot of cosmological scenarios related to this era have been suggested by string theory, and string cosmology is a subfield of theoretical physics that has been extensively studied in the last couple of years. However, these cosmological scenarios are so far little but ideas and conjectures without firm theoretical and experimental bases and only time will tell whether string cosmology can take us closer to an understanding of this elusive era in the history of our Universe.

This is neither the time nor place for a discussion of the multitude of different ideas that string cosmology has provided for this earliest time of our Universe. Instead, I will move on to the parts of the history of our Universe where we believe that we understand the physics at least reasonably well. The common paradigm for this part of the history of our Universe is called the Big Bang theory (or Hot Big Bang theory to be precise) or sometimes the standard model of cosmology.

According to this common paradigm, the expansion of our Universe started billions of years ago, from a state with extreme physical conditions. At this stage, our Universe was an enormously hot, dense and small Universe that, as time passed, evolved to the Universe we live in today. This state occurred somewhere around 10^{-12} s to 10^{-24} s after the creation of our Universe. At this point in time, our Universe was a “boiling soup” of particles and radiation. The energy density of radiation was so much larger than the energy density of ordinary matter that we can neglect the contribution to the energy density from ordinary matter. Our Universe was in the so called radiation-dominated era.

For the next 10 000 years or so there was a rather complicated history involving our fundamental forces and particle species that I do not intend to describe here. One of the important events in this era was the formation of atomic nuclei from protons and neutrons, a major step towards the formation of the ordinary matter that fills our Universe today.

When the Universe was about 10 000 years old, the energy density of radiation became equal to the energy density of ordinary matter, and the radiation-dominated era ended, giving way to the matter-dominated era. During this stage, the expansion rate, and hence the cooling, of our Universe increased. At first, our Universe was still a soup of radiation and matter, but at the age of around 300 000 years, the shared life of matter and radiation started to end. At this point in time, the temperature of our Universe had reached a value where it is possible for the electrons to bind with nuclei to form atoms. Before this, the energy of the photons in the radiation was big enough to break up the newly forms atoms, but this ceased to be the case for the majority of the photons at this time. This process is called *recombination* (even though it is a strange name, since the electrons and the nuclei were never previously combined) and occurred to a larger and larger extent as the Universe expanded and the photons cooled off. Shortly a state where we had matter and freely propagating photons was reached. These photons are still observable today, and constitute what is called the cosmic microwave background radiation (CMBR). This radiation can be observed as a relic from this time as microwaves corresponding to a blackbody radiation spectrum with a temperature of around 2.7 K. Why microwaves? As our universe expands, so does the wavelengths of radiation. Waves with longer wavelengths correspond to lower energy and thus a lower temperature. So what we today observe as microwaves were once waves with much higher energy, which have since been drained of energy and become microwaves with a temperature of 2.7 K. This cosmic microwave background radiation is a very important source for hints about the physics involved in the early Universe, and I will come back to this radiation in next section.

Between this era where radiation and matter decouple from each other and our present time, structures must have started to form in our Universe. These structures were the origin of what we today can observe as galaxy clusters, galaxies and other large-scale astronomical structures. The origin of these structures is a very complicated and intriguing story and this is not the right time or place to further discuss these matters. For the interested reader there is an excellent discussion of the formation of structures in our Universe in [5].

2.4 Inflation

The history of our Universe described in Section 2.3 leaves some important questions unanswered that need to be addressed. I will not go into detail for all of these questions, but will, as an example, rather focus on one of these questions that motivate adding a new era to the history of our Universe, an era that was not included in the brief historical overview in last section. The question on which I will focus is often referred to as the horizon problem. For a discussion of some of the other problems with the standard cosmological model, the interested reader is referred to [5].

The cosmic background radiation mentioned in the last section, which reaches us from a time when our Universe was around 300 000 years old, has properties that present a serious problem for the standard cosmological model described earlier. If we look at this cosmic radiation in various directions, it turns out that this radiation is surprisingly uniform (to one part in 100 000). No matter in which direction we look, the temperature of the radiation agrees to an amazingly high degree. This is puzzling, since if we take a closer look at the situation in the standard cosmological model, many of the areas that exhibit this homogeneity could never have been in contact which each other before the time when the cosmic microwave background radiation was produced. If we take two regions of our Universe that are so far apart today that they can not communicate with each other, and track the history of this origin by using our standard model of cosmology, it turns out that they could never have communicated with each other. This is illustrated in Figure 2.

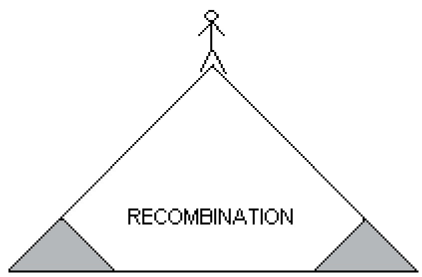

Figure 2. In this picture, the solid line at the bottom represents the birth of our Universe in the standard cosmological model. The grey triangles represent the light cone of a given point in spacetime – the part of spacetime with which the point can receive information. It can be seen in this picture that points observable by us but separated by a large distance could never have been able to communicate with each other.

How can regions of our Universe that have never been able to exchange information with each other, and that therefore have never been able to reach equilibrium with each other, have such identical properties? An analogous example would be an examination where 100 students write an examination in a large lecture hall. Due to an important call the lecturer in charge of the exam leaves the students unattended for a short period of time. When she, a few days later, starts to grade the examination, she finds that all of the students have scored exactly 93% on the examination, all with identical answers. That would be a truly astonishing result if there were no communication between the students while the lecturer was away.

One proposed theory is that our Universe started in such a homogenous state, but this would require extremely fine-tuned initial conditions and is hardly a satisfactory explanation. It would be preferable to have an explanation of why the Universe was in such a homogenous state. Such an explanation – which also addresses some other problems with the standard cosmological model – was proposed by Alan Guth in 1980. This involves adding an era to the description of the history of our Universe, an era where our Universe undergoes a rapid *accelerating expansion*. Such a period is called inflation. In cosmological models with inflation, such an event is proposed to have occurred very early in the history of our Universe, when our Universe was around 10^{-38} to 10^{-30} seconds old. I will not address inflation theory in any more detail here (the interested reader is referred to [5] which provides an excellent summary of inflationary models and ideas), but will move on to look at how such an era could solve the horizon problem. This concept is illustrated in Figure 3.

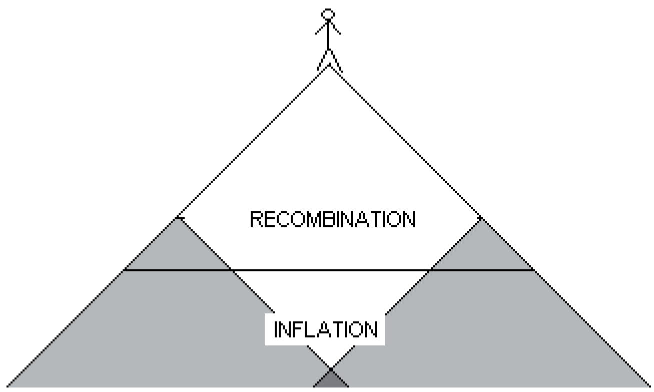

Figure 3. In the inflationary scenario the solid horizontal line no longer represents the birth of our Universe, but simply corresponds to the end of inflation. We can extend the light cones into the inflationary era, where they eventually overlap if inflation persists for sufficiently long time, making communication between distant points possible in this early era.

Before inflation, our Universe was extremely small and all points in our Universe could easily communicate with each other, exchanging radiation and thereby achieving a uniform temperature. This resulted in a Universe with homogenous properties – the same homogeneity that we can observe in the cosmic microwave background radiation today. When inflation occurred, our Universe expanded enormously and a many points became separated by such large distances that they could no longer communicate with each other, and no longer needed to do so in order to avoid the horizon problem, since they had already been communicating and creating the homogenous state of our Universe before the era of inflation.

So, a cosmological model where our Universe undergoes a period of inflation early in its lifetime provides a solution to the horizon problem. There are other problems with the cosmological standard model, such as the monopole problem, the flatness problem and the origin of our large-scale structures in our Universe which also seem to get satisfactory solutions by assuming an inflationary phase (details can be found in [5]). In summary, it seems like we need a period of inflation in the history of our Universe to get a satisfactory description of how our Universe has evolved into what we see today.

2.5 de Sitter space

As described in the previous section, inflation involves an accelerating expansion of our Universe. In Paper I we study a particular type of accelerating spacetime known as the de Sitter space. This spacetime has received a lot of attention lately for several reasons. First of all, observations [1, 2, 3, 4] indicate that our Universe currently has an accelerating expansion, which might be due to a positive cosmological constant, making this spacetime very interesting to take a closer look at. Moreover, de Sitter space plays a central role in many inflationary scenarios and research [6] has shown that due to inflation the cosmic microwave background radiation could provide us with valuable insights into the physics of our early universe at extremely high energy scales. A third reason for the interest in de Sitter space is that it turns out to be very tricky to implement string theory and quantum gravity in de Sitter space. In the case of a negative cosmological constant, a spacetime known as anti-de Sitter space (AdS), a lot of progress has been made in terms of holographic dualities (see e.g. the references listed in [7]) and the hope has been that similar ideas could be applied to de Sitter spac*e* as well, but so far de Sitter space continues to be elusive. A final reason for the interest in de Sitter space is that de Sitter space could have important parallels to the physics of black holes, due to the presence of cosmological horizons.

The spacetime known as de Sitter space is the maximally symmetric vacuum solution to the Einstein equation, in the case of a positive cosmological constant. Geometrically, de Sitter space can be represented by a hyperboloid embedded in flat Minkowski space and in the case of a four-dimensional de Sitter space, which is our interest in Paper I, this hyperboloid is described by:

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \frac{3}{\Lambda} = R$$

The quantity R is known as the de Sitter radius and we will have more to say about the significance of this radius later on. From this basic expression, we can equip de Sitter space with many different coordinate systems. The choice of coordinate system depends on what we are interested in exploring as far as de Sitter space is concerned. For cosmological purposes and global questions, a suitable set of coordinates is the Friedmann-Robertson-Walker-coordinates, where the metric of the de Sitter space becomes:

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2),$$

where $a(t)=Re^{t/R}$ is the scale factor for the de Sitter space. Another useful set of coordinates is the static coordinates, which are useful when we want to adapt a local perspective such as when an observer is put in de Sitter space and we are interested in exploring the experiences of this observer. For these coordinates we have the metric:

$$ds^2 = -\left(1 - \frac{r^2}{R^2}\right)dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Perhaps the most interesting thing that becomes visible in these coordinates is that when r=R (i.e., the radial coordinate equals the de Sitter radius) we find that there is a cosmological horizon – the de Sitter horizon. What is the significance of this horizon? If we were to put an observer in a de Sitter space, this observer would all the time be surrounded by a cosmological horizon beyond which the observer can not get any information of what is going on. Even though de Sitter space itself is infinite, the observer is shielded from all but a finite portion of de Sitter space.

Another well known situation where we have a horizon, is when we consider black holes. For these there exists an event horizon which prevents anything that happens to end up inside of this horizon from ever making their way out again from the black hole. In de Sitter space, we seem to have the opposite situation: anything sent out through the horizon is doomed to be lost forever.

If this spacetime remained a de Sitter spacetime this would not be much of a problem, unless you happened to drop one of your favourite CDs and it vanished at the horizon. In Paper I, however, we present a scenario where a de Sitter space is replaced with another spacetime and now the existence of this horizon gives rise to a potentially puzzling situation, which is discussed in detail in the following section.

2.6 A possible information paradox

In our scenario in Paper I, a de Sitter spacetime is momentarily replaced by an era where the scale factor grows like t^q where $1/3<q<1$, referred to as the *post-de Sitter phase*. This can be viewed as a "toy model" for an inflation era and the replacement of this era with an era of matter or radiation domination. This kind of transition is an integral part of all inflationary theories and is known as reheating. During this transition the vacuum energy is transformed into ordinary matter and radiation through mechanisms which are thoroughly discussed in, for example, [5].

In this scenario a situation that threatens one of the fundamental principles of quantum mechanics can occur. As described in the previous section, anything that happens to end up beyond the de Sitter horizon is no longer in the causal patch of an observer situated in de Sitter space. An object which is released by an observer in de Sitter space that crosses the de Sitter horizon is lost forever, and so is any information contained in the object. However, if the de Sitter space is suddenly changed into a spacetime which expands at a slower rate than the lightcone for an observer in this spacetime, the object and any associated information are no longer gone forever. The object can actually return to the part of spacetime that an observer can access. This is still not a problem. The object and the information becomes accessible again – so what? The problem lies in the fact that horizons emit radiation. Stephen Hawking showed in a seminal paper that black holes in fact are not completely black. Radiation is emitted from the event horizon of the black hole. If we throw an object into a black hole, microscopic description of black holes makes it reasonable to believe that this radiation is able to carry the information contained in the object.

If black holes emit radiation from a horizon and that radiation carries information about what was thrown into the black hole, the same should be true for the horizon that is surrounding each observer in de Sitter space. In fact, analogous to the case for black holes it is possible to attribute a temperature and an entropy to the de Sitter horizon, so these horizons seem to have many things in common. The essence of this is that the information contained in an object that leaves the causal patch bounded by the horizon can be sent back towards the observer as radiation from the horizon. It is however important to stress that even though the radiation can carry information, it is by no means obvious that an observer can extract that information.

Assuming that the information about the object *is* sent back towards the observer as radiation, and assuming that the observer *is* capable of extracting this information from the radiation, we do have a problem. In our scenario there is the possibility for the object to become accessible to the observer again when the de Sitter phase is turned off. So the radiation from the de Sitter horizon carries information about the object and the object itself eventually becomes accessible to us again. This means that we apparently have managed to duplicate information, something that is strongly prohibited by the basic laws of quantum mechanics and often referred to as the quantum Xerox principle. Figure 4 illustrates this apparent threat to the quantum Xerox principle.

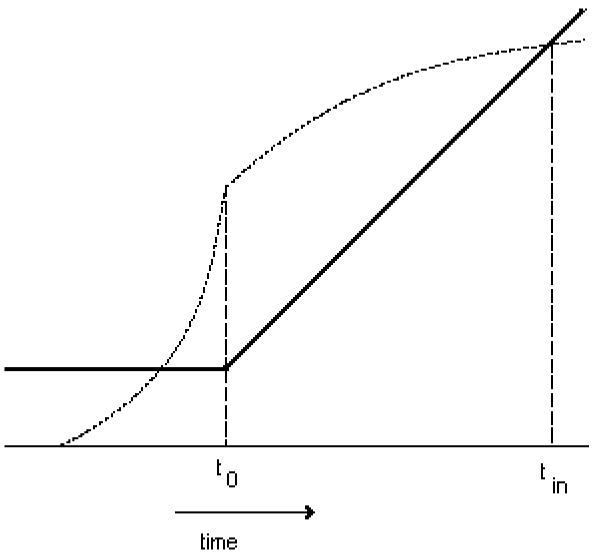

Figure 4. The thin line at the bottom represents the observer and the thick line describes the horizon from beyond which we can not get any information. During the de Sitter phase the horizon stays at a constant distance from the observer, and during the phase following the de Sitter phase the horizon grows linearly with time (corresponding to the fact that as time passes, information from regions further and further away has had the time to reach us). The dashed line describes the motion of an object that is sent out through the de Sitter horizon only to later on, when we no longer are in the de Sitter phase, return to within the area from which the observer can obtain information. At the moment when the object passes through the de Sitter horizon we put t=0, then t_o and t_{in} then correspond to the time at which the de Sitter phase is turned off and the time at which the object become accessible to the observer again, respectively.

2.7 Resolving the apparent paradox – results of Paper I

As discussed in Paper I, there are fortunately several ways of avoiding the possible duplication of information outlined in Section 2.6. One possibility is that the radiation does not carry any information about the object, which would eliminate our paradox, but which also would have severe implication for information in other situations, such as black holes, where horizons are present. Another related possibility is that even though the radiation carries information, it is not possible to extract this information from the radiation.

In Paper I we show that even if we try really hard to get a duplication of information, by assuming a worst case scenario: that the radiation *does* carry information, that we *can* extract this information from the radiation by using some kind of detector and that the object *does* become accessible again, we can avoid the apparent paradox. If we start a clock at the time the object crosses the horizon in the de Sitter phase, then the time it takes for the object to become accessible to the observer again, t_{in}, is shown to be:

$$t_{in} = \left(\frac{(1-q)x\,Re^{t_0/R}}{(qR)^q} + (qR)^{1-q} \right)^{1/(1-q)} + t_0 - qR$$

where x is the comoving coordinate of the object, R is the de Sitter radius and t_0 is the time at which the de Sitter phase is replaced by the post-de Sitter phase with the scale factor $a(t) \sim t^q$. We then provide an estimate of the minimum time, τ, needed to measure the information in the radiation from the de Sitter horizon (once again assuming that this is possible) which turns out to be

$$\tau \sim R^3$$

where R once again is the de Sitter radius.

To be able to extract information from the radiation, we thus need the time t_0 to be at least τ.
Replacing t_0 in the expression for t_{in} with τ and keeping the dominant term we get the time of return to be

$$t_{in} \sim e^{R^2} \sim e^S,$$

up to factors of the order of one in the exponential (where we have used that the entropy S in the de Sitter space is given by πR^2). This time is, however, nothing but the *Poincaré recurrence time* for our de Sitter space. This is the time it takes for a trajectory in phase space (a space whose dimensions corresponds to the positions and momentum for all the components of the system and where a trajectory represents a possible evolution of the system) for an isolated finite system to return arbitrarily close to its initial value. This means that discussing experiments lasting longer than the recurrence time is meaningless, since the system effectively has lost its memory. Since our detector obviously has less entropy than the entire de Sitter space, we are considering an experiment lasting far longer than the recurrence time of the detector, thereby indicating that the experiment does not make any sense! It makes the retrieval of information from the radiation an impossible task and prohibits our scenario from violating the quantum Xerox principle.

I would like to conclude my discussion of Paper I with some remarks. First of all, in our scenario, we have assumed a worst case scenario, where the radiation does carry information, that we can extract this information from the radiation by using some kind of detector and that the object does become accessible again. We showed that we can resolve the paradox in this worst case scenario, but there might be mechanisms that prevent this worst case scenario from occurring in the first place, e.g. that it is not possible for the observer to extract the information during the de Sitter phase (which however would mean that an observer in the de Sitter phase would not experience unitary evolution until the post-de Sitter phase).

Secondly, it would be wonderful if we could rigorously analyse this process in detail using e.g. string theory. However, as already mentioned, there is a long way to go before we (if we ever) have a happy marriage between string theory and de Sitter space. This means that any discussion of the physics in de Sitter space has to involve semi-classical arguments, general discussions and a certain amount of qualitative arguments. These explorations are however still valuable, by providing insights into the properties of de Sitter space that informs the quest for a realisation of a more fundamental theory in de Sitter space.

2.8 Experiencing potentially abstract physics

My research presented in this chapter and in Paper I has taken me on a journey through the landscape of theoretical physics. On this journey, I learnt a lot about general relativity, cosmology and string theory, but I also experienced some of the challenges of learning potentially abstract physics. Hence, I believe that, apart from subject knowledge, my research in theoretical physics has provided me with valuable insights into the process and dynamics of making the learning of potentially abstract physics possible. These insights are arguably invaluable to both my physics education research and to the development of my understanding of the teaching and learning of potentially abstract physics.

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Part II

3. Physics education research – overview and literature review

3.1 Introduction

In this chapter I attempt to provide the reader with a brief overview of what physics education research involves as well as a selection of some of the main findings of physics education research. I also dedicate a section of this chapter to a discussion of the relationship between physics education research and physics. After these introductory discussions, I proceed to a more detailed literature review of the research in physics education that is closely tied to the research presented in this thesis. The main purpose of this review is to situate my research in physics education and to provide some of the background to why we chose to conduct the studies included in this thesis.

3.2 Overview of physics education research (PER)

3.2.1 What is physics education research?

Physics education research (PER) is a relatively young but rapidly growing area of research. The main research focus of this area is to obtain a better understanding of the teaching and learning of physics and the factors that affect these processes. The growth of this research area reflects a dissatisfaction with the way physics is being learnt and taught, and represents a realisation that it is not only important to produce new physics knowledge, but also to make physics knowledge accessible in as fruitful, successful and stimulating a way as possible. In PER, detailed and systematic studies of the learning and teaching of physics are conducted that can influence the creation of more effective learning environments.

The first studies in PER originated in physics departments, from a concern about the knowledge (or rather lack of knowledge) students seemed to have acquired from their learning of physics. These studies were, due to being situated in physics departments, typically not grounded in any educational theoretical frameworks, but consisted mainly of empirical investigations of students' level of knowledge and understanding of specific concepts in physics. Today, research into the teaching and learning of physics has developed into a scholarly inquiry, and a full-fledged, diverse research area has emerged. The rest of this chapter aims at providing the reader with an overview of PER and the areas that are closely related to my own research presented in this thesis in particular.

3.2.2 The relationship between PER and physics

Every now and then, PER researchers have to answer to the question of whether PER should be considered a subfield of physics. I think you can get different answers to such a question depending on whom you ask and in what context the question is asked. I do not believe it to be the most relevant question to ask. Let me try to elaborate on why I believe this to be the case.

Educational research differs from research in physics in several important aspects. In physics, the main way to collect data is to conduct measurements which provide the researchers with quantitative results. In PER, a multitude of ways of collecting data exists (e.g. interviews, video-recording, in-class observations) and the results are most often qualitative. In physics, there is a strong agreement on the interpretation of concepts (such as an electron) while this is not necessarily the case in PER (for example, learning). These differences and others (see for instance Aalst, 2000) should not be alarming or surprising. First of all, PER is a young research field still in its infancy which has not had the time to reach the same mature state as physics. More important, though, is that the research questions of physics and physics education research are vastly different. Naturally enough, different types of research questions need different approaches and provide different types of results. In PER we study human individuals and human interaction and attempt to answer questions about the learning and teaching of physics for which quantitative results, in most cases, are neither relevant nor sufficient to provide satisfying answers.

Due to these differences, I believe that the relevant question might not be whether PER is a subfield of physics or not, but whether PER research should be conducted at physics departments. Aalst (2000) provides three reasons why this should be the case:

(1) PER contributes positively to the teaching of physics at all educational levels;
(2) curriculum development should take into account the context in which the physics is taught and should involve both practical and theoretical knowledge of the teaching and learning of physics. This practical knowledge is available in physics departments;
(3) subject matter knowledge (content and epistemological) is considered to be a very important factor as far as the improvement of physics education is concerned and physics departments are ideally suited to provide this knowledge.

In summary, I argue that PER research is most fruitfully conducted in physics departments, since both physicists and educational researchers can benefit from each other's expertise in a dynamic dialogue. It is important to stress that this does not mean that physicists should become educational researchers or vice versa. However, for this cross-fertilisation to be successful, educational researchers should have a solid knowledge of physics and physicists should have familiarity with PER and a willingness to reflect on their teaching and to try new instructional approaches. I believe this to be a possible scenario and therefore I strongly believe that PER research should be situated in physics departments as an integral part of physics.

3.2.3 A selection of main findings in PER

Physics education research is a relatively young research area, but has nonetheless been able to provide important insights into the teaching and learning of physics. In this section I attempt, in broad terms, to highlight and exemplify some of the most prominent findings. An excellent and extensive summary of many of the main contributions PER has made can be found in Redish (2003) and comprehensive overviews of physics education research, which provides a flavour of the various areas that have been studied, can be found in Duit (2004), McDermott and Redish (1999) and Thacker (2003).

One of the most important things that can be concluded from the amassed body of physics education research is the limited success of the traditional teacher-centred transmission approach to teaching, as well as the limited success of the traditional way of assessing students' knowledge. As far as assessment is concerned, research in PER has shown that very little insight into students' actual understanding is provided by the traditional way of assessing physics knowledge and that there is very weak correlation between the ability to solve examination-type problems and an understanding of the physics involved. Several studies have shown that although successful on the exam, students' still have many difficulties in relation to many important concepts in physics. Furthermore, the traditional role of the lecturer as a *transmitter of knowledge* has, in many contexts, been shown to be inferior and less successful than more interactive ways of learning physics, as far as successful student learning is concerned. Many studies have also investigated the effectiveness and fruitfulness of laboratory work and tutorials as well as suggested new ideas for curriculum development. Other findings have suggested alternative ways of presenting physics that are not as heavily rooted in mathematics as it traditionally is. In summary, PER suggests that in many contexts the traditional way of presenting and assessing physics knowledge needs a transformation towards a student-centred, interactive environment which promotes successful learning and assesses the things we want our students to learn.

Hake (1998) provides a good illustrative example of this area of research, which highlights the fact that traditional methods of presenting physics might not be the most efficient approach as far as students' understanding of physics is concerned. In a survey involving more than 6 500 introductory physics students, the results of two conceptual tests which probed students' conceptual understanding of key concepts in mechanics were analysed together with the results of a test of problem solving in mechanics. The most interesting aspect of this analysis was that Hake compared the results for students who had been taught in a traditional way with the results for students who had experienced a more interactive learning environment. The results showed, in Hake's own words, that, "the use of IE [interactive-engagement, author's remark] strategies can increase mechanics-course effectiveness well beyond that obtained with traditional methods" (Hake, 1998, p. 71).

Another important main finding in PER is that many students have negative experiences of the culture of physics. Narrow, abstracted, boring, "male", and irrelevant are some of the epithets assigned to physics by students. On top of that, teachers are experienced as uncaring, disdainful and impersonal and perceived to teach in ways which are disconnected from real life with a lot of things taken for granted. Taken together with the fact that in many universities, fewer and fewer students want to study physics, the significance of these results cannot be underestimated. We need to make the culture of physics appealing to the students, so that they feel that learning physics is an exciting endeavour in a stimulating environment.

A very good illustrative account of students' experiences of the culture of physics, which I could recommend to anyone involved in the teaching of physics, is provided by Thomas (1990). This study is first and foremost a gender study, but it also excellently explores students' experiences of being a science student. Looking at Thomas' discussions with students about their experiences of studying science, and physics in particular, it is clear that much remains to be done in order to create a stimulating learning environment in physics. Something that can be found in many students' experiences is a sense of physics as a subject which does not require thought and understanding, and where there rarely or never are any *discussions* of physics going on. Illustrative examples are provided below with quotes from students in Thomas' study.

> I went to all the lectures and they're easy to go to, because you're spoon-fed, they don't sit back and they don't philosophize, a lot of it, it's all material on the board. (Thomas, 1990, p.57)

> It's just proved my ability to learn chunks of knowledge, chunks of pages and books, and reproducing them the day after, then forgetting it, then you have to learn it again for next year's exams. (Thomas, 1990, p.57)

One of the most interesting things found in Thomas' (1990) study of physics students' experiences of studying physics was students' comparison between how they experience physics and how they experienced *studying* physics. This highlights a need for making the physics culture more stimulating. Tomas found that most students

> …were very enthusiastic about physics. Physics was perceived as exciting, progressive and fundamental in contrast to other disciplines which were perceived as routine, static and lacking substance. Yet their experience of studying physics was far from exciting. (Thomas, 1990, p. 56-57)

Apart from the unsuccessfulness of the traditional ways of teaching physics and the negative experiences of the culture of physics discussed earlier in this section, a third major finding in PER involves student and teacher epistemology. Epistemology is a concept which refers to how knowledge and the acquisition of knowledge (i.e., learning) are viewed. Research has shown that both teachers and students exhibit a wide variety of different views of what physics is and what it means to learn something, both in general and in physics in particular. Moreover, these views have been shown to affect how physics is taught and learned. Thus it seems that apart from the content knowledge *per se*, we also have to pay attention to epistemological views, in order to get as full a picture of the mechanisms and factors that affect student learning as possible, in order to be able to provide a successful learning environment.

Paper IV in this thesis explores epistemological views, and a detailed overview of the research in PER involving epistemology is given in Section 3.5.

Here, I will only provide the reader with an illustrative example of this area of research in PER – a study by Lising and Elby (2004). In their study, they investigated the influence of epistemology on learning by conducting a case study on an introductory university physics student. By analysing videotaped class work, written work and interviews they could conclude that there is a direct, causal relationship between a student's epistemological views and her learning of physics. In particular, her reasoning was either formal/technical or everyday/intuitive, with very few links between these ways of reasoning that could nurture a cross-fertilisation of ideas. The activation of either of these seem to be dependent on her beliefs about what type of reasoning were "appropriate", i.e., her beliefs about what physics and learning physics involves. Drawing on these results, Lising and Elby (2004) suggest that "curriculum materials and teaching techniques could become more effective by explicitly attending to students' epistemologies" (p. 1).

This section has provided the reader with a flavour and broad overview of some of the main findings of PER. In the rest of this chapter I will go into much more detail and try to situate my own research in PER, as well as review areas of PER which are closely related to or have been important for my own research

3.3 Conceptual understanding

3.3.1 Summary, history and theoretical development

One important ingredient in developing a good understanding of the critical issues in the teaching and learning of physics is to explore how students perceive and understand central concepts. This area, known as *conceptual understanding*, was one of the first areas to be explored within PER and still continues to be an important area of PER. Here, the nature and prevalence of students understanding of particular physics concepts as well as difficulties and "misconceptions" (also referred to as "alternative conceptions" or "alternative frameworks") associated with these concepts are explored and compared to what could be called appropriate disciplinary concepts.

Studies dealing with conceptual understanding can be found in many areas of physics (see McDermott and Redish, 1999, for a comprehensive overview) but the most extensively studied area of physics is classical mechanics, where most of the initial studies took place. Here, studies have explored students' conceptions of kinematical quantities (velocity and acceleration) and their graphical representations (e.g. Trowbridge & McDermott, 1980;

Peters, 1982; Bowden et al., 1992), while others have explored force, energy and momentum (e.g. Halloun & Hestenes , 1985; Gunstone, 1987; Thornton 1997). This research led to an appreciation of the fact that students do not enter their learning of physics as a *tabula rasa*. Based on their experiences, students bring prior conceptions to their learning of physics. So, if the students' minds are not a blank sheet, what do they bring in terms of previous knowledge? As more and more studies were conducted it became apparent that what students bring are more or less consistent fragments of knowledge and concepts which, to varying degrees, differ from the conceptions of experts.

The further exploration of these issues led to several important contributions to the development of PER. One of these contributions involved considerations of how to best address the conceptual difficulties found. One approach to this, which could be described as a primitive model of learning, is known as the conceptual change model (e.g. Hewson, 1981; Hewson, 1982; Posner, Strike, Hewson & Gertzog, 1982). In this model, the conceptual structures held by the students are restructured in order to allow the acquisition of science concepts. Duit (1999) describes this as learning pathways from pre-instructional conceptions to the science concepts to be learned. In this model, this restructuring occurs when a student becomes dissatisfied with his/her prior conception and an alternative conception exists that is perceived as intelligent (non-contradictory and understandable), plausible (believable) and/or fruitful. These criteria are seen to be mediated by a set of epistemological commitments referred to as the *conceptual ecology* (e.g. metaphysical beliefs, prior experience, analogies).

The change of conception that occurs if a competitive conception is deemed to be more intelligent, plausible and fruitful than the conception previously held, is seen as (adapting ideas from Piaget) involving either *accommodation* (also known as strong/radical knowledge restructuring or conceptual exchange) or *assimilation* (also known as weak knowledge restructuring or conceptual capture). Assimilation refers to the recognition that an event fits an existing conception as well as a selective ignorance of discrepancies that are not prominent. Thus, the existing conception is not changed dramatically, but rather enlarged by incorporating new aspects. Accommodation is more drastic and involves a change in beliefs about how the world works. This enables an event to be assimilated that could not have been assimilated until accommodation occurred.

The conceptual change model is considered to be a powerful framework for improving science teaching and learning and has, since its birth, been continuously debated and modifications, alternatives and additions have been suggested (for example, Linder, 1993). Duit and Treagust (2003) provide a good summary of this development.

Another example of an approach towards understanding and addressing conceptual difficulties is known as *elicit, confront and resolve* (Shaffer and McDermott, 1992). Here, a clash between a student's conceptual understanding and what could be referred to as an expert's conceptual understanding are deliberately introduced, making it necessary for the student to somehow resolve this mismatch, thereby replacing their inappropriate conceptual understanding with a, from a disciplinary point of view, more accurate understanding.

These two examples illustrate the fact that the interest in a deeper understanding of the conceptions held by students, and how these conceptions could be changed, initiated a more theoretical orientation in PER. At the same time an awareness that there were other, previously unexplored (at least in PER) factors that influenced the learning of physics (such as epistemology – discussed in detail in Section 3.5) arose. Moreover, a lot of ideas from psychology and other areas of science education started to make their way into PER. Taken together, these were the seeds of an emerging theoretical interest in PER at this time.

One major contribution to this theoretical orientation resulted from critical scrutiny of the dominant views of students' conceptual understanding. This dominant view involved viewing students' conceptions as "misconceptions" that had to be confronted and replaced with more appropriate conceptions. At this time researchers such as Smith et al. (1993) and diSessa (1993) started to approach student conceptions from another viewpoint. Students' conceptions were no longer seen as erroneous conceptions that needed to be replaced by conceptions mimicking those of experts, but seen as learning seeds – resources for learning which could be developed through instruction.

Apart from this shift in how student conceptions were viewed, another interesting theoretical development started to emerge at this time – a more fine-grained view of conceptions taking into account the context and dynamics of conceptions. In a seminal paper, diSessa (1993) suggested a model where introductory physics students' conceptions do not form a coherent, organised structure, but manifest themselves as loosely related, highly context-dependent pieces of knowledge based on students' prior experience that are applied in a context-dependent manner. These loosely connected pieces of knowledge are called phenomenological primitives, or p-prims for short. In diSessa's model these p-prims are refined and developed – not replaced – in the process of learning. These p-prims are a theoretical construct that is viewed by many researchers as a more efficient way of approaching students' conceptions than by looking at the conceptions themselves, due to the fine-grained nature of the p-prims as well as the possibility of taking context and dynamics into account.

The introduction of p-prims by diSessa (1993) was the starting point for a growing interest in the mechanisms of student learning and initiated an increasing theoretical interest in cognitive models for student learning. To give an example, in Hammer (2000) and Hammer and Elby (2000) a more fine-grained structure of epistemological beliefs was introduced known as resources – in many ways analogous to the introduction of p-prims for student conceptions.

Building on the ideas that emerged during the development of cognitive mechanism of learning that followed diSessa's introduction of p-prims, as well as on ideas from psychology, sociolinguistics and neuroscience, Redish (2004) proposed a "supertheory" for students' learning of physics. This theory is built on various cognitive mechanisms with one of the key concepts being resources – which are now seen as the building blocks for *both* students' epistemological views and student's conceptions. These resources can be seen as a development of diSessa's ideas about p-prims, consisting of loosely connected intuitive ideas that are activated or deactivated depending on the context. In this model, the key to successful learning of physics is to activate the appropriate resources in a given context. Hence, successful teaching should aim to provide a learning environment which activates these appropriate resources.

In summary, the interest in students' conceptual understanding has provided PER with a lot of theory development apart from the obvious contribution to our understanding of students' content-knowledge. Today, many areas of physics have been explored as far as conceptual understanding is concerned and explorations of students' conceptions in previously unexplored areas continue to interest many PER researchers. At the time of this writing, areas where students' conceptual understanding has been explored is electromagnetism (e.g. Cohen, Eylon & Ganiel, 1983; Rainson & Viennot, 1997), relativity (e.g. Hewson, 1982), light and optics (e.g. Watts, 1985; Goldberg & McDermott, 1986; Grayson, 1995), thermodynamics (e.g. Erickson, 1979; Rozier & Viennot, 1991) and waves and sound (e.g. Linder & Erickson, 1982; Linder, 1993; Wittmann, Steinberg & Redish, 1999). The common denominator in all of these studies is the fact that they might no longer play as prominent a role as the early studies of conceptual understanding in mechanics which initiated several important theoretical orientations in PER. However, they all make important contribution by identifying student difficulties with dealing with various aspects of key concepts in physics, thereby informing the teaching and learning of physics.

There is one area where conceptual understanding has been explored which is not mentioned above. This area is quantum mechanics. Paper II of this thesis belongs to this category and in the following section I attempt to provide the reader with a detailed overview of research into students' conceptual understanding of quantum mechanics.

3.3.2 Conceptual understanding of quantum mechanics

Despite the unquestionable importance of quantum mechanics in many areas of modern physics and technology – implying the importance of both the teaching and the learning of quantum mechanics – physics education research in the area of quantum mechanics has not been given the same attention as it has in classical areas of physics such as mechanics, electromagnetism and thermodynamics. There has, however, been a steady increase in interest in research in this area over the last few years and one can only hope that this increasing interest will persist.

The study in Paper II of this thesis deals with university students' conceptual understanding of quantum mechanics and most of the early research carried out in this area has involved pre-university students. Some examples are Niedderer, Bethge and Cassens (1990) and Mashhadi (1995) where secondary school students' view of the atom has been investigated. Other examples are Müller and Wiesner (1999) (included in Zollman, 1999) which explores conceptions of quantum physics and the work of Ireson (2000) where pre-university students understanding of quantum mechanics has been explored in broad terms.

In the realm of university physics, conceptual understanding has looked at a range of areas and concepts in quantum mechanics. One such area, which seems to be conceptually challenging to students, is the wave-particle duality and the wave nature of matter. A study by Johnston, Crawford and Fletcher (1998) concluded that the university students involved in the study had difficulty describing what characterises a particle or a wave. Another study by Vokos et al. (2000) investigated university students' understanding of the wave nature of matter in the context of interference and diffraction of particles and concluded that students had difficulty interpreting interference and diffraction in terms of a wave model and furthermore, that students lacked a proper understanding of the de Broglie wavelength.

A peculiar property of quantum mechanics is the occurrence of indeterminacies (often referred to as uncertainties). These indeterminacies are a fundamental "built-in" part of quantum mechanics and Johnston, Crawford and Fletcher (1998) have found that students find it difficult to distinguish the quantum indeterminacies from measurement uncertainties.

Other examples of studies of conceptual understanding in quantum mechanics are Vokos et al. (2000), where university students' understanding of the wave nature of matter in the context of interference and diffraction of particles has been investigated, Singh (2001), which deals with how undergraduate students deal with concepts related to quantum measurements and time development, Steinberg et al. (1999), which investigate the influence of students' understanding of classical physics when learning quantum mechanics and Bao and Redish (2002), which is a study of university students understanding of classical probability and the implications of this understanding for teaching quantum mechanics.

Besides these empirical explorations, listings of "misconceptions" from the researchers' own experience can also be found. An example is Styer (1996), who lists 15 common "misconceptions" related to the measurement process in quantum mechanics and the nature of quantum states as well as miscellaneous misconceptions.

It is not only researchers in physics education who have explored students' conceptual understanding of quantum mechanics. There has also been research on students understanding of quantum mechanics in chemistry education, although a lot of the research consists of pedagogical suggestions and inventions (for an overview, see Fletcher, 2004). One of the areas that have, however, been explored in a systematic way is orbitals. In Tsaparlis (1997), successful physics students' understanding of orbitals were explored and the results showed that students have a number of difficulties associated with orbitals such as confusing orbital representations and failing to recognise the approximate nature of orbitals in many-electron atoms.

Apart from looking at individual concepts or phenomena in quantum mechanics, there has also been research aimed towards constructing a quantum mechanics concept inventory – similar to the FCI (Halloun & Hestenes, 1985) in mechanics – to probe students understanding of basic quantum mechanical concepts. One such survey is the Quantum Mechanics Concept Inventory developed by Falk (2004), which is "currently not being refined nor extended" (Falk, personal communication) and another one is the Quantum Mechanics Conceptual Survey (QMCS) developed and continued to be developed by McKagan and Wieman (2006).

There is also a large body of PER research in quantum mechanics that suggests various ways of presenting the material that are supposed to enhance the learning of the students. However, many of these publications make very little or no reference to supporting empirical research. An overview of this research as well as a comprehensive general literature review as far as PER and quantum mechanics is concerned can be found in Fletcher (2004).

Since a lot of the physics education research in the area of quantum mechanics had been carried out at the pre-university level and with some areas of quantum mechanics being under-explored, we felt that there was a need for further exploration of university students' conceptual understanding of quantum mechanics. For Paper II we chose to focus on a phenomenon known as quantum tunnelling (described in non-technical terms in Section 5.3), an area of quantum mechanics where, to our surprise, no previously published research could be found at the time we started our study. Quantum tunnelling is an area very well suited for an exploration of students' conceptual understanding of quantum mechanics, since it deals with many of the basic concepts of quantum mechanics and involves a number of counter-intuitive results.

In the last few years, quantum tunnelling has started to receive well-deserved attention from the PER community and a number of interesting results have been found as far as students conceptual understanding of quantum tunnelling is concerned. One such result is that many students carry a conception that tunnelling causes particles to loose energy. This conception has been reported and explored in Paper II of this thesis and in Wittman (2003) as well as Wittman et al. (2005) and has been confirmed in two quantitative surveys (Falk, 2004; McKagan & Wieman, 2006). Several studies (Paper II in this thesis; Wittman, 2003; Wittman et al., 2006) have made it plausible that one likely source of such a conception is the way diagrams of quantum tunnelling are drawn.

Other findings related to quantum tunnelling involve conceptions where students believe that a wave packet is either reflected or transmitted as an entity (Paper II in this thesis; Falk, 2004), that only particles with "high enough energy" are transmitted (Paper II in this thesis; Falk, 2004) and that probabilities for different energies determine whether a particle manage to tunnel or not (Paper II in this thesis; Falk, 2004; McKagan & Wieman, 2006). McKagan and Wieman (2006) have also shown that students have difficulty interpreting what the potential energy in potential energy diagrams (such as the ones used to represent a barrier in quantum tunnelling) means.

In summary, quantum tunnelling seems to be conceptually challenging in a variety of ways for the students, thereby providing a suitable "laboratory" for exploring students' conceptual understanding of quantum mechanics.

The results of the study in Paper II of this thesis, which explores students' understanding of probability in quantum tunnelling (of which some results briefly have been mentioned here) is presented and discussed in detail in the last two sections of Chapter 5.

3.4 Physics education research related to physics equations

Looking at mathematics, there has been significant progress in understanding students' use of mathematics in the context of mathematics courses. These results are both useful and interesting, but situated in pure mathematics. In order to research the role of mathematics in physics, studies are likely to be most fruitfully conducted in the context of physics. For these reasons, the rest of this section focuses on research on mathematics *in the context of physics*.

As described in detail in Section 6.3, physics can be viewed as a discipline concerned with describing the world by constructing models – the end product of this modelling process often being equations. These equations encode the relationships inherent in the modelling process in the language of mathematics and therefore play an important role in the representation of knowledge of physics in most situations where physics is taught or learned. Therefore, physics equations are an important aspect to study as far as the role of mathematics in physics and the learning of physics are concerned.

Despite their importance, physics equations have received surprisingly little attention in educational literature. Looking at physics equations, there are several things that come to one's mind as potentially interesting aspects to explore. As far as the symbolic structure of equations is concerned, Herscovics and Kieran (1980) and later Kieran (1981) have investigated students' interpretations of the equal sign, concluding that many students view the equal sign as a symbol meaning "do something", although it is not clear from this research whether this is a harmful interpretation. Another study which has looked at signs inherent in physics equations is provided by Govender (1999). This study conducted a phenomenographic study of university physics students' experiences with sign conventions for quantities such as displacement, acceleration and force. One of the findings is that students do not realise the arbitrary nature of sign conventions. The main finding is however that the transition from one-dimensional motion to two- and three-dimensional motion poses some difficulties to the students. Govender attributes this to an incomplete understanding of the relationship between a vector component and a scalar component. Related to this is a conception found among the students that only vectors are associated with algebraic signs. A suggested way of dealing with this is to start with two- and three-dimensional motion and consider one-dimensional motion as a special case in order to get a more coherent introduction to vectors in kinematics.

There has also been research which has looked at students understanding of the variables involved in physics equations. Clement, Lochhead and Monk (1981) videotaped college students solving word problems and identified difficulties in translation from a verbal representation to an mathematical representation in terms of algebraic symbols. Rozier and Viennot (1991) showed in the context of algebraic relationships in thermodynamics that some students find it hard to parse the relationships between variables in problems which involve multiple variables and their relationships. Problems with multiple variables have also been identified by Steinberg, Wittmann and Redish (1997) who found – in the context of mechanical waves – that students have difficulty understanding the meaning and internal structure of equations involving functions of more than one variable. Based on these findings, a tailored tutorial aiming at addressing these difficulties was developed.

Other studies in PER have looked students' *use* of physics equations. A lot of studies have indirectly investigated equations while looking at problem solving (for an overview of research in problem solving, see the review in Hsu et al., 2004) and Tuminaro (2004) has discussed a cognitive framework for analyzing and describing introductory physics students' use of mathematics in physics, which uses cognitive mapping between knowledge elements and student reasoning as the analytical framework. Another significant study as far as the use of physics equations is concerned is the work of Sherin (2001), who has examined students' ability to construct equations appropriately describing a given physics situation, claiming that much of the reasoning related to equations draws on different kinds of equation templates that carry specific meanings for the students.

All of these studies have a common denominator: they either focus on the structural elements of physics equations or on students' *use* of physics equations. Looking at this research it occurred to us that there were several important aspects of physics equations that remained unexplored. Three such aspects that we became interested in exploring were what students focus on when they are presented with physics equations, how students view what it means to understand an equation and how well students understand physics equations. The first two of these questions: "What do students focus on when presented with physics equations" and "What does it mean for students to understand physics equations" have been explored in Paper III and Paper IV respectively.

3.5 Epistemology, attitudes and beliefs

As mentioned earlier, early PER into student learning at the university level traditionally had a strong focus on students' difficulties and "misconceptions" associated with specific physics concepts. As described by Hammer (2000) this work "has been productive for curriculum development as well as in motivating the physics teaching community to examine and reconsider methods and assumptions, but it is limited in what it can tell us about student knowledge and learning" (2000, p.52).

In order to complement this work, and in line with the growing interest in constructivism that has emerged, studies on university students' epistemology (beliefs about knowledge and knowing) and the relationship between epistemology and learning have started to become an important and increasingly explored area of educational research. For example, Linder (1992) illustrated how teacher-reflected epistemology could be a further source of conceptual difficulty for students. Hammer (1994) showed that introductory physics students can be characterized as having beliefs about knowledge and learning and that these beliefs affect their work in physics courses, i.e., the way they learn physics. Similar results have been found by Redish, Saul and Steinberg (1998) and Roth and Roychoudhury (1994). Lising and Elby (2004) found a causal relationship between students' epistemology and learning behaviour and May and Etkina (2002) found correlations between students' conceptual understanding and epistemological beliefs.

Research in epistemology within physics education has generated several areas of research, apart from exploring and establishing the relationship between epistemological beliefs and learning while highlighting the importance of taking student epistemology into account. Two of these are a closer and more fine-grained look at what epistemological beliefs involve and the development and implementation of surveys to probe students' epistemological beliefs in physics.

The first of these areas involves a closer look at the notion of epistemological beliefs. In Hammer (2000) and Hammer and Elby (2000), a more fine-grained description of epistemology than "beliefs" is suggested. It is argued that by using "beliefs", epistemological perspectives in certain situations run into the same difficulties as descriptions of students' conceptual learning in terms of "misconceptions". In both cases there is no account of how "misconceptions" or "beliefs" evolve and how different contexts affect these "misconceptions" and "beliefs". It is argued that an exploration of such situations would need more fine-grained elements, similar to diSessa's

(1993) introduction of p-prims (phenomenological primitives) as a more fine-grained account of conceptions. In this spirit, Hammer and Elby argue for and present examples of "epistemological resources" – a more fine-grained account of epistemological beliefs (Hammer & Elby, 2000, 2002, 2003; Elby & Hammer 2001). In this model these resources are more basic epistemological units that get activated in a context-sensitive manner – sometimes appropriately, sometimes not. An example of such a resource is "*Knowledge as propagated stuff"* where someone who invokes this resource treats knowledge as something that is transformed from a source to a recipient. The idea of epistemological resources is only one piece of a larger anatomy of epistemological views, and Hammer and Elby (2002) elaborate of the idea of resources by introducing another concept – "epistemological anchors" – concepts or analogies that cue the activation of appropriate resources. The search for a more fine-grained and detailed view of epistemological beliefs is beyond doubt some very interesting research in progress.

There has also been research which looked at epistemology from different perspectives and provided useful insights into the taxonomy of epistemological beliefs. An interesting concept as far as student epistemology is concerned is the notion of *pedagogical commitments,* which was introduced and further developed by Hewson (1981, 1985). Pedagogical commitments involve what an individual believes counts as a successful approach or explanation in a given field, as well as the individual's more general view of what knowledge involves. As described in Hewson and Hewson (1984), such epistemological commitments may be a very important component as far as students' learning is concerned. An example of this importance can be found in relation to my own research: consider a student whose epistemological commitment to understanding an equation involves being able to use the equation to solve problems. This student might only be focusing on that particular aspect of an equation, thereby overlooking other important features of the equation.

An interesting question in relation to student epistemologies is the robustness of these epistemologies, i.e., can we tailor our teaching of physics so that students develop more sophisticated and appropriate epistemological views? May and Etkina (2002) attempted such an approach and concluded that this indeed seems to be a difficult task. However, Linder and Marshall (1998) investigated the possibility of affecting students' epistemological views of science and of learning through stimulating students to adopt a meta-cognitive perspective of science and of their own learning in an introductory physics course. They concluded that such an intervention led to a more appropriate view of science and learning, and that it could "profoundly influence students' conceptions of science and conceptions of learning" (Linder and Marshall, 1998, p. 116). Thus, affecting students' epistemologies seems to be a delicate matter, likely to be heavy dependent on the context and the ways of attempting to achieve this.

In the same way as the amassed indications of students' conceptual difficulties in mechanics led to the construction of large scale surveys to probe this further, the PER movement towards a larger focus on epistemological issues has led to the development and implementation of several epistemological survey instruments whose purpose is to assess and probe students' attitudes and beliefs about physics, physics knowledge and learning in physics. These include the MPEX (Redish, Saul & Steinberg, 1998), EBAPS (Elby, n.d.), VASS (Halloun, 2004) and CLASS (Adams et al., 2004) surveys, which all have shown that students carry a range of epistemological beliefs as far as physics and learning of physics is concerned.

It is not only the epistemologies of students that are of importance and consequently have been explored. As interesting as students' view of what knowledge and learning involves is teachers' views of the same issues. Already at the end of the 1980's Hewson and Hewson (1987, 1988) highlighted the importance of taking into account teachers' conceptions and beliefs of teaching and learning. Several educational researchers have since concluded that teachers' beliefs affect their instructional practice (Nespor, 1987; Pajares, 1992; Abd-El-Khalick, Bell & Lederman, 1998; Lederman 1992, 1999). An important contribution to the research on teachers' epistemologies was provided by Linder (1992) who could establish a relationship between teachers' epistemological beliefs and student learning, arguing that "metaphysical realism overtones in physics teaching not only affects how we teach but also affects how students view the learning and understanding of physics" (Linder, 1992, p. 111). In summary, research which explores the structure of teachers' epistemologies and the possible effects of these on teaching practice, students' epistemologies and student learning has provided important insights into the relation between epistemology and learning, and has become an important integrated part of epistemological research.

Closely related to the area of epistemology is metacognition, which is a rather new and emerging area in PER. A good overview of metacognition in the more general setting of science education is provided by Georghiades (2004) and an example of a study in the area of physics is Koch (2001), where students' understanding of physics texts is explored from a metacognitive perspective. So why is this interest in metacognition emerging? While epistemology could be seen as students' views of what knowledge is and how knowledge should be obtained, metacognition involves a reflection on and evaluation of one's own learning process, i.e., metacognition can be seen as an extension of epistemology. Gunstone (1991) states, drawing on a constructivist perspective, that if "learners ideas and beliefs about learning/teaching etc. are in conflict with the notion that learners must recognize, evaluate and reconstruct their existing physics ideas, then little progress is possible" (Gunstone, 1991, p.135). Thus, metacognitive reflection seems to be an important factor to take into account in order to achieve as effective learning as possible.

In summary, epistemology has turned into one of the main research areas in PER and has provided, and continues to provide, invaluable insights into students' learning of physics and how to improve learning outcomes, teaching and curriculum. Paper IV in this thesis, which is described in detail in Chapter 6, can be seen to contribute to this epistemologically oriented area of PER, by exploring university students' epistemological views of what it means to understand physics equations.

4. Method and methodology

4.1 Introduction

As in any other area of research, PER involves a spectrum of approaches to deal with the research questions that emerge. Once a research question has been formulated, finding a suitable way to explore that research question is the next logical task. In PER this generically involves choosing a method and a methodology. A method is simply a technique or way of collecting data, while a methodology is a theoretical lens that is used for the analysis of the gathered data. In physics such a distinction is generally not made or considered important since physicists in most cases share an implicit view of the world, as well as a view of what constitutes knowledge and how this knowledge is obtained. This is not the case in PER, where different theoretical lenses can corresponds to different views of the world as well as different views of knowledge and learning. In this chapter I will described the methods and methodologies that were used for my studies in Paper II, III and IV.

4.2 Methods of data collection

In all of my educational studies (Paper II, III and IV) the main method for collecting data has been interviews. The strength of interviews is that they permit a rather detailed exploration while being able to be tailored according to the overarching research question(s). The difficulties as a researcher are, apart from attempting to create a friendly interview atmosphere, to stay away from pre-conceptions or leading questions, and to know at what stage the information gathered is sufficient to make a plausible interpretation of what the interviewee has told me.

In all of the interviews for Paper II, III and IV, interview protocols were created as a starting point for the interviews. These protocols proved to be very useful since they kept the interviewer on track and made sure that no part of the interviews relevant for exploring the research questions was omitted. In most cases the initial interview protocol were tested using pilot interviews, where a small number of interviewees were used to assess the usefulness of the protocol. Many times these pilot interviews led to modifications of the original protocol, reflecting the difficulty of anticipating how the interview discourse could be best established as a research tool.

In all of the interviews for the studies in this thesis, either the entire interview or selected parts of the interviews were transcribed verbatim. In some cases the interviews were conducted in Swedish, and in those cases the interview excerpts that we chose to use in the publications were translated to English. The transcribed interviews were used as the main data source for the analysis process.

There are several possible ways to structure an interview and for Paper II and III the interviews were semi-structured and open-ended, involving a few main questions to keep the interviews on track but providing generous space for the interviewees to express whatever experiences they carried in relation to the interview questions. In Paper IV, the research focus was much narrower, and in this case, the interviews were much more structured and centred around one main question and associated follow-up questions.

In the empirical study presented in Paper II, 12 intermediate physics students from two Swedish universities were interviewed. The students were selected to render a mixture of typical second and third year physics students who had successfully completed at least one major-level quantum mechanics course. Students were interviewed while interacting with a specially selected computer simulation of quantum scattering and tunnelling. The interviews lasted between 30 and 60 minutes and were semi-structured to explore students' understanding in a focused, yet open-ended way. A large part of the interviews asked students to predict what would happen in a particular simulation scenario and then to comment on what actually happened. This allowed us to probe students' understanding of probability in quantum tunnelling in an efficient way that allowed the students to reflect on many aspects of probability, which was exactly what we were trying to achieve.

Paper III is based on interviews with thirty undergraduate physics students from three Swedish universities. These students attended a wide range of physics courses, such as electromagnetism, classical mechanics and modern physics. In audio-recorded interviews, the students were asked to discuss various equations they had been presented with during the courses. To be able to capture on what students were focusing, we asked the students questions like "What do you see here?", "What does this equation mean to you?" and "What does this equation tell you?". In order to get as wide a variation as possible, equations of different type and complexity from several different areas of physics were discussed, ranging from simple equations such as the definition of angular frequency, $\omega = 2\pi f$, to the time-dependent Schrödinger equation. During this interview process we both interviewed students for whom the equations were relatively novel and students for whom the equations should be familiar to be able to compare and contrast the data for these different student groups.

In Paper IV, twenty voluntary physics students from three different Swedish universities were interviewed using a mixed mode semi-structured interviewing strategy involving both face-to-face and e-mail interviews. A good discussion of the latter type of interviews in qualitative research can be found in Meho (2006). Seven of the interviewees were first year undergraduate students, nine were second or third year undergraduate students, and four were PhD students. Each interview lasted approximately twenty minutes and began with some introductory discussion centred around the nature of physics and the role of mathematics in physics. The purpose of this introductory discussion was to set the context to physics and physics equations. After this introductory discussion, the main question that we asked the students and were interested in exploring was: “When you say feel/say that you understand an equation, what does that mean?”. Associated follow-up questions were used for clarification and to allow students to elaborate on their answers.

4.3 Choosing an appropriate methodology

In any research, it is important to choose an appropriate methodology once the research questions have been formulated. In other words: what is the best way to deal with and attempt to answer the research questions? Depending on the nature of these questions, some research methodologies are often more adequate and fruitful than others.

In Paper II, we were interested in exploring the variation in students’ understanding of probability in quantum tunnelling. This meant that we were not interested in describing the individual students, but to map the collective variation of the group of students as a whole. This focus on the collective variation made it natural to choose phenomenography (described in detail in Section 4.4) as our research approach, since phenomenography is explicitly aimed at capturing the collective variation in how people experience, understand or perceive a phenomenon or situation. The same motivation can be given for Paper III, where we also were interested in mapping the collective variation, this time of what students focus on when they are presented with physics equations.

For Paper IV, where we explore students' views of what it means to understand equations, we are interested in the variation in this understanding. While this could have been an appropriate situation to use phenomenography, we decided to conduct a case study to hold focus on the individual (described in Section 45).

The reasons for this choice are twofold. First of all, not only were we interested in capturing the variation in students' views of what it means to understand an equation, but we were also interested in comparing the understandings of students at various levels in their academic career. This meant that, apart from looking at the variation, we were also interested in comparing individual students, which made us question whether phenomenography would be the best research approach, since phenomenography does not aim to describe or compare individual experiences or understandings. Surely, this comparison could be added on top of a phenomenographic study, but there was another reason for why we felt that a case study might be more appropriate.

In phenomenography, the experiences or understandings are removed from the context and from the individuals. All the individual experiences or understandings of the phenomenon or situation that are studied are identified, removed from whatever context that might be surrounding them and put into a collective "pool" from which the collective variation is discerned. In our case, we felt that the descriptions of the individual students were too rich to be cut up and stripped of context. For these reasons we decided to frame the study in Paper IV as a case study.

4.4 Phenomenography

In the study for Paper II, we were interested in exploring the variation in students' understanding of probability in quantum tunnelling which made it natural to choose phenomenography as our analytical approach. Phenomenography is a research approach which was developed in the Department of Education at Gothenburg University in Sweden in the early 1970's, with Ference Marton as one of the principal pioneers. It originated with the observation that some people learn better than others. This apparently trivial observation led the research group to consider research questions such as: What does it mean that some people are better at learning than others? Why are some people better at learning than others? (see Marton, 1993 for more details). The attempts to answer these questions paved the way for what would eventually become phenomenography. Phenomenography is now a well respected framework and a large number of educational studies that use phenomenography in different areas can be found. It has, for example, been used to examine students' understanding of the fundamentals of force and motion (Johansson, Marton & Svensson, 1985), sound (Linder & Erickson, 1989; Linder, 1993), central concepts in computer programming (Booth, 1992), concepts in electricity (Millar, Prosser & Sefton, 1989; Prosser, 1994), concepts in special relativity (Bantom, 1999) and displacement, velocity and frames of reference (Bowden et al., 1992).

4.4.1 The main ideas of phenomenography

To describe phenomenography, it is useful to start out by discussing the key elements of this research approach. These key elements have been neatly summarised by Trigwell (2000), who states that phenomenography:

> …takes a relational (non-dualist), qualitative, second-order perspective, that it aims to describe the key aspects of the variation of the experience of a phenomenon rather than the richness of individual experiences and that it yields a limited number of (internally related), hierarchical categories of description of the variation. (p. 1).

This is indeed a very compact summary of the main ideas of phenomenography. Let us dissect this quote and describe phenomenography in a more detailed manner by taking a closer look at the meaning of some of these different key aspects of phenomenography.

Phenomenography is a relational (non-dualist) perspective
This statement refers to the ontological basis of phenomenography. In some research perspectives, such as cognitivism, a separation is made between an individual's mind and the outside world. In phenomenography there is no such thing as an independent reality existing without someone perceiving it. Reality is seen as being constituted as a relation between an individual and what the individual experiences. This also means that phenomenography does not separate the learner from what is learnt and that learning takes on a particular meaning: experience the world in a different way.

Phenomenography is a second-order perspective
A distinction is often made between first-order and second-order research perspectives. In a first-order perspective, the researcher describes how he or she experiences the phenomenon or situation that is studied while, in a second-order perspective, the main focus is the experience of the phenomenon or situation as described by others.

Phenomenography describes the key aspects of the variation of the experience

The fundamental aim of phenomenography is to capture and describe the qualitatively different ways of experiencing a phenomenon, concept or situation. It is important to start out by stressing that in phenomenography, the word "experience" can be used interchangeably with "understand", "perceive", "apprehend", "conceptualize" etc. There are obviously differences between these terms, and phenomenography does not try to deny these differences – they are simply not important. As described by Marton (1993):

> The point is not to deny that there are differences in what these terms refer to, but to suggest that the limited number of ways in which a certain phenomenon appears to us can be found, for instance, regardless of whether they are embedded in immediate experience of the phenomenon or in reflected thought about the same phenomenon (p. 4427).

So, since phenomenography is aimed towards capturing the variation, we do not need to worry about whether that experience is a result of unreflected thought or reflected thought, or about whatever subtle epistemological differences exist between e.g. understanding and apprehending.

The units of analysis of phenomenography are the qualitatively different ways of experiencing a phenomenon, concept or situation. As a consequence, phenomenography does not produce accounts of the experiences of single individuals. All the individual experiences are collected in a pool of experiences, and from this pool the collective variation is extracted.

Something else that merits a discussion is the meaning of *qualitatively different ways* of experiencing. To answer this question, we need to examine what phenomenography has to say about what it means to experience something. In phenomenography, to experience something is to be aware of something. We are all the time aware of a lot of things, but we are also aware of these things to various extents. Some of the things might be in the foreground, i.e., in our focal awareness, while others might be in the background. Marton and Booth (1997) have an elaborate discussion of this in terms of an *anatomy of awareness* which involves looking at the structure of the awareness in terms of the whole and the parts and how these are internally related. An interested reader is referred to Marton and Booth (1997) for a full description, but the main point is that experiences can be classified as qualitatively different due to structural differences in the anatomy of awareness.

Phenomenography yields a limited number of categories of description of the variation

So far, and there is no reason to believe this will change in the future, every phenomenographic study has yield a *limited* number of qualitatively different ways of experiencing a phenomenon or situation. These qualitatively different ways of experiencing are referred to as *categories of description*. There is never just one such category and rarely more than five. Of course, if we simply were to collect individual experiences, we could end up with as many different experiences as individuals. In phenomenography this is not the case since we do not look at individual experiences *per se* but collect all of the individual experiences in a pool and then map the collective, qualitatively different variation.

4.4.2 Data collection

The data for a phenomenographic study can be gathered in a variety of different ways. For example, there are phenomenographic studies where group interviews, drawings and written responses have been used. The dominant way of gathering data in a phenomenographic study is, however, the individual interview and as described in the previous section, this is also the way of collecting data that I have used in my phenomenographic studies.

Since the phenomenographic researcher is trying to find the variation in how a concept, phenomenon or situation is experienced, the researcher focuses on trying to obtain a dialogue where as many aspects of the concept, phenomenon or situation as possible become reflected upon by the interviewee. These interviews are normally semi-structured, meaning that the researcher knows what he or she wants the interviewee to discuss but the details or exploratory direction of the interview have not been determined in advance. Many of the questions follow from what the interviewee brings up, and the aim is to enrich the dialogue to thematise as many aspects of the concept, phenomenon or situation as possible, as well as to clarify the interviewee's meaning.

4.4.3 Data analysis

In phenomenography, all the individual experiences of a concept, situation or phenomenon are collected into a common "pool of experiences", from which the researcher constructs "categories of description" which correspond to the possible qualitatively different ways of experiencing the concept, situation or phenomenon. So how do we get from the pool of data to the categories?

The first step in the analysis process is to identify overall themes in the interview data and to tentatively group similar pieces of data into categories in relation to the research question. In the next phase of the analysis these categories are re-examined in relation to the interview data and, if necessary, modified, replaced, split or merged. In practice, the two steps in the data analysis process are carried out simultaneously in iterative cycles. This process continues until the categories stabilize into an appealing bigger picture that gives a satisfactory answer to the research questions and can be supported by illustrative examples from the data.

As far as the categories are concerned, it is important to stress that there is *not* a one-to-one relation between these categories and the individuals whose experiences constitute the data from which the categories are constructed. An individual may belong to several categories, experiencing the concept, phenomenon or situation in several different ways. It is exploring the variation in the "collective pool of experiences" that is the aim – not to collect numerous different individual experiences.

In some situations it may be possible and fruitful, even desirable, to take these categories a step further by looking for logical relations between the categories. This is normally done in terms of an inclusive hierarchical ordering of the categories, where categories lower down in the hierarchy are included in categories higher up in the hierarchy.

4.5 Case studies

As described previously, Paper IV in this thesis draws on case study methodology in order to get as full an account as possible of students' epistemological views of what it means to understand an equation. Case studies have been used in a multitude of disciplines as early as the 1930s, and have become an important approach in educational research where a detailed investigation is required. A completely comprehensive account of case studies is beyond the scope of this thesis and an interested reader is advised to take a look at for example Merriam (1988) or Stake (2005), which provides good in-depth descriptions of case studies. In this section I limit myself to providing the reader with some key aspects of case studies and the background necessary to appreciate the methodological basis of Paper IV.

4.5.1 The main ideas of case studies

It is important to begin this description of case studies by stressing that there is no such thing as a generic blueprint for how a case study should look. There exists a wide range of various types of case studies, but they all have at least one thing in common. As described by Merriam (1988), a case study always involves "a detailed examination of one setting, or a single subject, a single depository of documents, or one particular event". The main features of this description are that a case study allows a detailed, in-depth exploration of a well-defined unit of study.

Data for a case study can be collected in almost any conceivable way. Data could, for instance, be collected as field notes, observations, questionnaires, letters, interviews or any combination of these and other methods. The important thing is to choose a method of data collection that allows a detailed investigation of the unit of study. Similar to the data collection, the analysis of the data can proceed along many different paths. The main idea is to use an analytical approach that takes into account the richness of the gathered data, thereby producing as comprehensive a description of the unit of study as possible.

The notion of detail in case studies means that the researcher should attempt to account for as many aspects of the unit of study as possible. This means that case studies normally involve a thick description, i.e., as complete a description as possible of the entire research process, i.e., the origin and motivation for the research questions, the data collection process, the analysis of the data and the interpretation and validation of the results.

When the results have been presented and the process and interpretations that lead to the results have been scrutinized, the ultimate judge of the value of case study research is the reader. Since a case study involves a study of a well-defined unit of study, someone might object that it is difficult to see any value of this research beyond the particular case. This is where, for case studies, it is important to have a think description. By providing as detailed an account as possible, the researcher makes it possible for a reader to understand the particular case in depth, thereby being potentially able to transfer the knowledge acquired from the case to new and foreign contexts. This is known as "naturalistic generalization" (Stake & Trumbull, 1982). This and the level of detail of case studies are two of the most attractive features of case studies which make them an important and most useful research approach.

4.5.2 Conducting a case study

In order to explore the research questions involved in Paper IV in as fruitful a way as possible, we decided to carry out an exploratory case study. In our study, we examined university students' epistemological mindsets towards the understanding of equations in a group of twenty students. We decided to frame this study as a case study and to use interviews (described in detail in Section 4.2) because we wanted a detailed exploration of the experiences of equations of each individual student participating in the study. This would enable us to map, characterise and further analyze students' epistemological mindsets towards what it means to understand an equation.

To add an additional dimension to the study, we included students at various levels in their education, ranging from first-year undergraduate students to PhD students. By having students from various educational stages, a cross-sectional case study was created, where we could both characterise students' epistemological mindsets towards the understanding of equations and analyse whether there is a difference in the mindsets for students at different stages in their academic career.

The principal aim of the data analysis process was to characterise students' descriptions of what it means to understand a physics equation when they feel that they have understood it, and then to take a closer look at these results. For this purpose we used what could be described as a standard qualitative data analysis (data-based inductive analysis). As described by Bogdan and Biklen (1982, p.145) this involves "working with data, organizing it, breaking it into manageable units, synthesizing it, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others". The process is generally carried out inductively, i.e., patterns and themes originate from the pool of data.

The first phase of the analysis process involved identifying overall themes in the raw data – a process characterised as "open coding" (cf. Strauss and Corbin, 1990) – and tentatively grouping pieces of data into descriptive categories corresponding to different characterisations of students' descriptions of what it means to understand an equation. The categories were given a descriptive heading and each piece of data was coded with an identification tag involving the origin of the data and to which of the tentative categories it was assigned.

In the next phase the characterisations were iteratively compared to modify, replace, split or merge the characterisations until saturation occurred. During this phase there was also a continual cross reference to the full transcripts and the two described steps in the data analysis process were essentially carried out simultaneously in iterative cycles. This process continued until the characterisations stabilised into a comprehensive set of outcomes that well captured the content and richness found in the data. The results of this analysis are presented in Section 6.6.

4.6 Trustworthiness and value in PER

As in any area of research, it is important to consider the trustworthiness and value of the research. In qualitative research, Lincoln and Guba (1985) have described this as being able to answer the question: "How can an inquirer persuade his or her audiences that the finding of an inquiry are worth paying attention to, worth taking account of?" (1985, p.301). Lincoln and Guba (1985) present a number of criteria for judging the value and trustworthiness of a qualitative research: credibility, transferability, dependability and confirmability. Let us take a closer look at what these criteria involve.

Credibility refers to the truth value of a qualitative study. Is the construction and interpretation made by the researcher an adequate and believable map of the experiences and statements made by the interviewees? There is really only one legitimate judge of this: the interviewees themselves, and there are several methods to enhance the credibility of a study (for an overview, see Lincoln and Guba, 1985). One of the most widely used methods is called member checks, where the researcher presents the interviewees with the data record, interpretations and findings of the study, asking them to review the interpretations made by the researcher and to judge whether their perspectives have been adequately mapped by the researcher. This may or may not be possible to do, depending on the available time of the researcher, the accessibility of the interviewees and the context and aim of the study.

Transferability refers to the extent to which the findings can be transferred to other contexts and situations that stretch beyond the boundaries of the study. In qualitative research, the researcher's role in allowing for transferability is different from that in quantitative research. In a quantitative study it is the obligation of the researcher to ensure that findings can be generalised, while in a qualitative study it is the readers who are the ultimate judges of the level of transferability. The obligation of the researcher is to ensure and enhance the *possibility* of transferability. One way to achieve this is to provide a thick description. A thick description involves detailed descriptions of the data, context, analysis and assumptions of the study. By using a thick description, the researcher makes it possible for a reader to judge the transferability to their particular context or situation. For all of the studies in this thesis the aim has been to provide the reader with a thick description.

In quantitative research, *reliability* is an important criteria that hinges on assumptions of replicability or repeatability. This basically means that we should obtain the same results if we observe the same thing twice in the same setting. However, in qualitative research, it is not possible to repeat a study in the same way as in quantitative research. Qualitative research deals with human interactions and "repeating" a study would correspond to investigating a new context. In qualitative research, we could talk about dependability instead of reliability. The idea of dependability involves accounting for the consistency of the inquiry processes used over time. This involves examining whether the researcher has been careless or made mistakes in framing the study, collecting data, interpreting data or reporting findings.

Confirmability refers to the extent to which the findings reflect the focus of the inquiry and not of the biases of the researcher, i.e., the extent to which the findings could be confirmed or corroborated by others. There are several ways to increase the likelihood that findings in qualitative research will be confirmable. Two such methods are to actively search for contradictions of prior conclusions during the analysis process and to critically examine the data collection and analysis processes for potential biases or distortions. Another powerful method is peer debriefing. Peer debriefing involves letting peer researchers not directly involved in the study scrutinize the data as well as the interpretations of the data and the conclusions drawn from the data, to play a “devil’s advocate” and to establish that the interpretations and conclusions are viable and well grounded in the data. This is a process which has been used for all of the studies presented in this thesis.

To conclude this section, it should be mentioned that the criteria listed above are not the only ones that could be considered. They may not all be applicable to a particular study and there are several other criteria that could or should be considered when the trustworthiness and quality of qualitative research is judged. Examples are meaningfulness (is the study addressing a meaningful issue?), ethical treatment (have the participant been treated ethically?) and how the study is written (is what the researcher has learnt from the study communicated clearly enough?).

5. Exploring university students' understanding of probability in one-dimensional quantum tunnelling

5.1 Introduction

Part I of this thesis has described my research in theoretical physics and since I have always been fascinated by the world of abstract and theoretical physics, I wanted my research in physics education to involve similar physics. An increasingly growing interest in students' learning and understanding of quantum mechanics had begun to emerge at that time in physics education research and quantum mechanics. This thus constituted a natural starting point for my physics education journey – eventually leading to the study of student's understanding of probability in quantum tunnelling which is presented in Paper II.

This chapter is intended to present and discuss the results of Paper II as well as to give a reader not familiar with quantum tunnelling some relevant non-technical background. I will therefore give a short description of some of the main ideas of quantum mechanics that are highly relevant for Paper II in Section 5.2 and continue by discussing the particular area of quantum mechanics – quantum tunnelling – that is the topic of Paper II in Section 5.3. In Section 5.4 I will start to get closer to the actual content of Paper II, by motivating why quantum tunnelling was chosen as a topic of research. Finally, this chapter is concluded with a summary and discussion of the main results and implications from Paper II in Section 5.5 and 5.6.

5.2 What is quantum mechanics?

At the end of the 19^{th} century, many physicists believed that physics – apart from some minor details – was an almost completed science; that we knew practically all there was to know about physics. This turned out to be far from the truth. Several observations and experiments at the beginning of the 20th century indicated that the established physical theories were not capable of accounting for the results of these experiments. Sometimes theories and experiments gave very different results and sometimes the theory could simply not explain or predict what was going on in the experiments. At this time, two major innovations in the field of physics were born, leading to a completely new paradigm in physics. One of them eventually led to Einstein's *special theory of relativity*, and the second one to the birth of *quantum mechanics*. This is neither the time nor the place to discuss the theory of relativity, but I will spend some time on quantum mechanics, since it is the area of physics where my research in Paper II is situated.

Quantum mechanics deals with matter and radiation at an atomic level, and intuitively it seems reasonable to believe that it is difficult to use our everyday knowledge and experiences to fully understand what is going on at such small scales. Even to someone keeping this in mind, a lot of the conceptual fundamentals of quantum mechanics still come as a complete surprise to anyone studying physics. To give quantum mechanics justice I would need a lot more space than a thesis to describe all of its intriguing properties. Here, I limit the description of quantum mechanics to a few of the basic properties of quantum mechanics that are relevant for my research presented in Paper II.

In quantum mechanics, every particle is associated with a mathematical function called the wave function. What does the fact that a particle is "associated with a mathematical function called the wave function" really mean? It is hard to find two things more conceptually different then a wave and a particle as we know of them in everyday life.

In quantum mechanics – contrary to the case in classical physics - there are no such things as well-defined paths that particles follow, where we at each instant in time can measure the position and velocity of the particle. The picture is far more subtle. In fact, quantum mechanics declares that it is *impossible* to determine the exact position and velocity of a particle at the same time. If we try to measure the position with greater and greater precision, the velocity will become more and more uncertain and vice versa. In quantum mechanics we can only talk about the probability of finding a particle in a certain position or with a certain velocity. In quantum mechanics probabilities are the best we can accomplish and classical determinism – in the sense that from knowing the current properties of a system we can predict how these properties will evolve – is no longer possible. This is a radical departure from the ideas of classical physics that is hard, not only for learners but also for physics scholars, to accept. For example, "God does not play dice" is a famous quote from Einstein, reflecting his concerns about this probabilistic nature of quantum mechanics.

Accepting this bizarre twist in the modelling of reality, the logical question to pose next is: “Alright, I do believe in this probability stuff, but how do we get these probabilities?” The answer is that we get the probabilities from the wave function previously mentioned. If we know the wave function of the particle, then we just have to take the magnitude of the wave function and square this magnitude to get the probability density of finding a particle in a certain position at a certain time. By integrating this probability density over a certain volume we get the probability of finding a particle in this volume at a certain time. From the wave function we can also get information about the velocity, energy, angular momentum and other properties of the particle. This means that the wave function is the single most important thing to get hold of in a quantum mechanical system. To get this important and informative wave function we specify in what kind of surroundings the particle is located, and then we use one of the most important equations of quantum mechanics – the *Schrödinger equation* – to calculate the wave function for the particle and how this wave function evolves with time. Once the wave function has been found, we theoretically know all there is to know about the particle.

Quantum mechanical ideas, involving wave functions and probabilities, give rise to many peculiar and counter-intuitive results concerning the behaviour of particles at atomic scales, and one of them is the *tunnelling effect* (which is the topic of Paper II in this thesis and which I therefore intend to discuss briefly in Section 5.3).

It is hard to fully appreciate the tremendous impact that quantum mechanics has had on physics and technology. Quantum mechanics has led to discoveries and inventions that have caused a revolution in both these areas. All of the theories of the fundamental building blocks of matter in modern physics – such as String Theory – contain quantum mechanics as a crucial ingredient and a fair portion of our modern technology, such as computer chips and transistors, is based on the implications and predictions that quantum mechanics has provided. Even though quantum mechanics may seem counter-intuitive and sometimes just plain weird, there is currently not a single piece of evidence that would make us believe that quantum mechanics is not a good model for the physics that takes place at atomic and sub-atomic scales. Quantum mechanics has withstood all the tests and challenges it has been given. It seems like God does play dice after all.

5.3 What is quantum tunnelling?

Quantum tunnelling is a quantum mechanical phenomenon that is a perfect illustration of just how counter-intuitive the quantum world can be. Imagine that you ran towards you front door, and instead of crashing into the door as you would expect, you suddenly find yourself on the other side of the door, and the door shows no sign of the fact that you passed straight through it. Or imagine that you rolled a ball towards the end of a table and instead of falling off the table, the ball bounced right back at you. These would be tricks worthy of applause from even the best magicians.

Even though these are just "generative metaphors"[1] (Schön, 1983), type analogies that involve aspects of our everyday world that we know well but should not be taken too literally, similar things happen in the microscopic world. Particles may face obstacles that they do not have sufficient energy to pass, but somehow they occasionally manage to pass the obstacle anyway. In a similar fashion, particles may have more than enough energy to pass an obstacle but sometimes they even so are incapable of passing the obstacle.

The first phenomenon of those described above, where a particle is able to overcome an obstacle even though the energy according to classical physics is too low, is called *quantum tunnelling*. The name stems from the fact that since the particle, according to classical physics, should not be able to pass the obstacle, it is as though there is a secret tunnel through the obstacle that the particle may use. This quantum tunnelling is not just an esoteric phenomenon from quantum mechanical theory, but a very important process in nature which plays a significant role for such things as chemical bonds, radioactive decay, and behaviour of semiconductors as well as for operation of the *scanning tunnelling microscope* (STM), by which we can actually observe individual atoms on surfaces. So once again, no matter how bizarre and counter-intuitive the quantum world may be, this seems to be the way in which nature behaves!

5.4 Background to Paper II

In the previous sections I tried to provide the reader with at least a brief non-technical overview of quantum tunnelling. This next section will move closer to the actual research presented in Paper II by motivating why we chose the particular topic for that paper – how university students understand

[1] For Schön a generative metaphor generates "new perceptions, explanations and inventions" (Schön, 1983, p. 185)

probability in quantum tunnelling. It can also provide an insightful inside description of the unpredictable paths that research always travels along.

Quantum tunnelling is a phenomenon within quantum mechanics that is very interesting to study from a physics education researcher's perspective. It is a phenomenon that lies far from people's everyday experiences – it therefore belongs to the realm of abstract physics which I am interested in exploring – and a solid understanding of the phenomenon involves many of the most fundamental concepts of quantum mechanics, such as wave functions, probability and energy. Together with the fact that no previously published qualitative research could be found on the understanding of this phenomenon, these provided the main reasons for choosing quantum tunnelling as our object of research.

Why probability then? Initially our interest in students' understanding of quantum tunnelling was aimed towards a completely different goal. We were interested in exploring the general interplay and relationship between mathematics and physical concepts, and we believed that quantum tunnelling could be a suitable conceptual environment for this kind of investigation. However, it soon became clear to us that the mathematics of this phenomenon and the abstract nature of the phenomenon itself made that kind of study a very difficult, if not impossible, task.

But this is not the end of the story. From the initially obtained raw data, we noticed that there were interesting things brought to the fore by the students that were particularly related to probability and energy, so we changed our focus to investigate how students' dealt with these concepts in the context of one-dimensional quantum scattering and tunnelling. Working with this new focus and analysing our continuously increasing pool of data, we started to realize that even though some very interesting issues emerged centred around the notion of energy, the most interesting concept to explore further was probability since it was the most problematic aspect to the students. This led us to pose our main question: What is the variation in students' understanding of probability in the context of quantum tunnelling?

5.5 Results of Paper II

In Paper II, we carry out a phenomenographic study of how students understand probability in one-dimensional quantum scattering and tunnelling. The analysis revealed that it was possible to construct four qualitatively different categories to map the understanding of probability, and that many student-held views of quantum tunnelling and quantum mechanical concepts can, from a physics point of view, be classified as erroneous or incomplete. In Section 5.5.1 and 5.5.2 I present a summary of these results and in Section 5.6 the results and associated pedagogical implications are presented.

5.5.1 Categories of students' understanding of probability

By analysing the data from our interviews providing the data for Paper II using a phenomenographic approach, we could identify four different categories, corresponding to four different ways of understanding probability in quantum tunnelling. These categories are described in detail in Paper II so I will only briefly discuss the different categories here.

Category 1 – Understanding probability in terms of reflection and transmission

In this category, probability is seen to correspond to the same kind of probability involved in flipping a coin. For a coin, there is a certain probability for heads and a certain probability for tails. In the scattering process there is a certain probability for the wave packet to pass the energy barrier and a certain probability for the wave packet to be reflected. Comparing this category to the phases discussed earlier, it can be seen that this notion of probability is focused on the outcome of analysis of the scattering process – the transmission and reflection coefficients.

Category 2 – Understanding probability in terms of energy

In this category, probability is discussed in terms of energy. Either the wave packet itself is seen as having a probability of having a certain energy or the discussion is carried out in terms of the energy of the individual waves constituting the wave packet. In the latter case it is common to advocate that those individual waves that have an energy higher than the maximum of the energy barrier will be transmitted and the rest of the individual waves, with an energy lower than this maximum, will be reflected.

Category 3 – Understanding probability in terms of finding a particle

In this category, probability is discussed in terms of the probability of finding the particle represented by the wave packet at a certain place at a certain time – even though the temporal aspect is often overlooked in comparison to the spatial aspect.

Category 4 – Understanding probability in terms of an ensemble

In this category the notion of ensembles emerges. Probability is discussed in terms of repeated experiments with identically prepared systems.

5.5.2 Conceptual difficulties

Apart from identifying the different ways of understanding probability in quantum tunnelling presented in the previous section, several conceptual difficulties were identified during the interviews or in relation to the categories presented in Section 5.5.1. One such difficulty was that for some students, tunnelling implies that a particle loses energy when tunnelling, which is not the case. This conception has also been identified by Wittmann (2003) and Wittmann et al (2005) and has later been confirmed in two quantitative surveys (Falk; 2004; McKagan & Wieman, 2006).

We could also identify conceptions where students believe that the wave packet representing the particle is either completely reflected or transmitted, and that only particles with "high enough energy" are able to tunnel. Both of these conceptions have been identified in Falk (2004). Yet another conception identified involves probabilities for particles to have a certain energy as a determining factor whether tunnelling is possible or not. Once again this conception has been confirmed by the surveys of Falk (2004) and McKagan and Wieman (2006)

5.6 Discussion of the results of Paper II

From the discussion of the different phases of the tunnelling process in Paper II and the different facets of probability that are important to interpret and analyse the scattering process, these categories described in Section 5.5.1 might look promising. All the important facets of probability involved in quantum tunnelling can be found in the four categories. However, if one starts to look at individual students, it can be noted that none of them takes *all* these facets of probability into account. We would like our students to understand and use all these facets, in order to be able to switch between and link the different facets depending on what phase of the process is being considered. Lacking one or several of these facets makes it difficult to interpret and analyze the full scattering process.

To improve on the incoherent understanding of probability reported in Paper II, we postulate that it is important to explicitly raise focal awareness of the different facets of probability. This could be done in terms of in-class discussions of the different facets of probability and their relation to each other and the different phases of the process. This implicitly means a need for going beyond the “standard” treatment of this process, which normally involves individual waves and snapshots of the process before and after the scattering complemented by a mathematical analysis.

In light of these incomplete understandings of probability and the conceptual difficulties identified in Paper II, it seems to be important to go beyond the “standard” treatment to get a deeper conceptual understanding. From our study we have come to believe that the use of a computer simulation of one-dimensional quantum scattering and tunnelling of a wave packet could be a way of achieving this. This approach has successfully been carried out for many areas of quantum mechanics in the Visual Quantum Mechanics Project[2]. Such a computer simulation could be shown during a lecture accompanied by a discussion with the students or it could be implemented as a tutorial with conceptual questions relating to wave packets, probability and tunnelling. A simulation of this kind naturally cues a discussion of conceptual issues and could be a fruitful and more alive alternative or complement to the standard treatment of this process, to help the students to get a firmer grip of one-dimensional scattering and tunnelling and probability in particular.

[2] This is a project developed by the Physics Education Research group at Kansas State University. The project has an excellent web site worth visiting: http://www.web.phys.ksu.edu/vqm/.

6. Exploring the role of mathematics in physics

6.1 Introduction

Looking at the research which has been presented thus far in this thesis – theoretical physics research involving the de Sitter space and educational research exploring students' understanding of quantum tunnelling – there is a common denominator, as promised in the introduction of this thesis. Both of these studies involve the abstract experienced in physics. If we recall that the main characteristic of such physics is that it is difficult to relate the physics to everyday situations, this means that it is hard to use everyday analogies as a tool to get a better understanding of the physics and that it is cumbersome to find concrete everyday examples. In such situations, mathematics becomes the main actor and an understanding of the physics equations involved becomes even more important than in less abstract areas of physics. Therefore I became interested in exploring how students deal with mathematics in physics, and to explore students' experiences of physics equations in particular.

In this chapter I aim to present the reader with results from two empirical studies (Paper III and Paper IV) which investigate students' view of what it means to understand physics equations. Before getting to those results, I would like to set the stage with a discussion in Section 6.2 of *why* mathematics plays an important role in physics and in Section 6.3 with a discussion of *how* mathematics plays an important role in physics.

6.2 Why does mathematics play an important role in physics?

As any physicist or physics student know, mathematics is a crucial ingredient and an important and useful tool in the field of physics. Mathematics is often even considered to be "the language of physics". From a pragmatic point of view this might seem unproblematic, but if we take a step back and reflect on the immense importance of mathematics in physics, it starts to become far from trivial. The basic quandary is that physics is an endeavour devoted to describing objects and phenomena in the world we live in, while mathematics on the other hand is a field that deals with abstract, mental constructions and their relations. Why should two such disparate fields have anything in common and why does mathematics play such an important role in such a vastly different field as physics?

It is beyond doubt a very subtle question and many well known physicists have given it a considerable amount of thought. One of these physicists is Dirac (1939), who claims that there are two basic methods for a physicist to make progress when studying nature:

a) experiments and observations; and,
b) mathematical reasoning.

The first of these methods involves the process of collecting data from given experiments and situations, while the latter, for some reason, lets us draw conclusions about experiments that still haven't been performed and about situations that still haven't been studied. As Dirac states, there is no logical reason whatsoever for the second method to be possible, but we know from our experience and from the history of science that it works like a charm in practice and that it has led to tremendous advances and successes in science.

According to Dirac mathematicians play a game where they set the rules, while physicists play a game where nature sets the rules. However, as time goes by, it becomes more and more clear that the rules that mathematicians consider to be interesting are the same rules that nature has chosen. Dirac furthermore believes that this has to be attributed to "some mathematical quality in nature, a quality which the casual observer of nature would not suspect, but which nevertheless plays an important role in nature's scheme".

Wigner (1960) also addresses the question about why mathematics is important in physics in a paper with the expressive title "*The unreasonable effectiveness of mathematics in the natural sciences*". In this paper Wigner points to the fact that mathematical concepts often turn up in completely unforeseen contexts and then often give rise to an unexpectedly detailed and precise description of the phenomena. Wigner talks about abstract, advanced mathematical concepts which do not in any natural way derive from the physical world and describe these constructs as advanced constructs chosen by mathematicians since they allow them to practice "mental acrobatics" and perform "brilliant manipulations". The unreasonable aspect of these, according to Wigner – as indicated by the title of his paper – is that they, for some reason, turn out to be efficient tools in science and physics in particular. In the same way as Dirac, he considers this usefulness of mathematics unexpected in the sense that we lack a rational explanation for such usefulness.

A partial "explanation" of why mathematics is important in physics, can be found in the historical development of mathematics and physics. A large part of modern mathematics was developed in symbiosis with physics. Such great mathematicians as Leibniz, Gauss, Laplace, Legendre, and Fourier are also well known for ground-breaking contributions to the field of physics. A lot of mathematics has been developed from considerations of physics problems. Let me give some examples to illustrate this:

- Calculus was mainly developed by Newton and Leibniz. Leibniz's motivation was mainly geometrical problems, while Newton wanted a mathematical tool to describe the motion of physical bodies. Influenced by Euler, Lagrange, Hamilton and others, mechanics evolved into analytical mechanics, where we can find the origin of the calculus of variations.

- The foundations of Fourier analysis originate in Fourier's attempts to solve one of the classical equations in physics: the heat conduction equation.

- The Hamilton-Jacobi theory of mechanics was one of the sources of inspiration for the theory of partial differential equations, which in turn led Lie to the concept of continuous groups.

This list could easily be extended, and one is led to speculate whether the mathematical concepts have been constructed in such a way that they suit physics and the physics problems that currently are studied. This may be true in certain cases, and the history of science involves several examples of this as the above list illustrates, but it is not a generally valid statement. As an example, the mathematics that constitute the backbone of the quantum mechanical theory was developed far earlier. So this historical "cross-fertilization" between physics and mathematics can at best only be considered a partial attempt to explain the role of some mathematics in some areas of physics and does not address the general question of why mathematics is important in physics.

The surprising usefulness of mathematics in physics has also attracted the attention of the Danish physicist Niels Bohr. In Bohr's view, mathematics is a science of structures that in principle supplies us with all possible structures. Physics is involved with finding structures in nature, and according to Bohr (1958) it is therefore not surprising that we find the structures that are adequate for physics in the realm of mathematics. A possible objection is hidden in the words "in principle" above. Theoretically, mathematics has the possibility to supply us with all possible structures, but in practice this is not the case. The structures studied by mathematicians are only a tiny fraction of all the possible structures, and according to Wigner those structures that are most useful to physicist are already part of the existing structures.

Where do the thoughts of so many great physicists take us? We can conclude this section by stating that the question of why mathematics is a successful and useful tool in physics is subtle and far from simple to answer. Maybe we have to resort a pragmatist's point of view and say that we do not know why mathematics is important in physics, but somehow nature is constructed in such a way that mathematics turns out to be an important tool to describe nature, so let us use this fortunate coincidence to our benefit. This is however a banal and, in my opinion, dissatisfying conclusion and the connection between mathematics and physics and the process of describing nature is probably far more deep than what such a statement involves.

6.3 What is the role of mathematics in physics?

As discussed in the last section, it is difficult to answer the rather philosophical question why mathematics plays an important role in physics. However, it is still possible to discuss in detail *what* the role of mathematics is in physics, which is the topic of this section.

Physics is a branch of science dedicated to describing various features of and phenomena in nature in order to help us get a better understanding of nature[3]. nature is beyond doubt a very complex system so this might seem like a daunting task. Fortunately, it turns out that nature is not an unstructured mess. We can, and do, find patterns and regularities in the complexity, and the primary focus of physics is to identify these patterns and regularities and organize them into larger, coherent structures.

The identification of these patterns normally involves several stages of approximation and idealization. We need to be able to decide what the important factors in the phenomena we are studying are, and what features of the situation we can neglect for our particular investigation. Otherwise the pattern may be hidden behind too many different factors and influences. At the same time, what is important in one situation might be neglected in another

[3] The word "various" is actually more important than it might seem. Physics is not a science that attempts to answer every imaginable question about nature. While it is well suited to answer some questions about the world we live in it is for example not very useful when discussing whether Harold Pinter is a worthy Nobel Laurate, why people have phobias and how to remedy them or the factors that led to the Second World War.

situation. It all depends on what kind of questions about nature we are posing and trying to answer.

For physicists, patterns and regularities in nature can be found by observations, experiments, theoretical modelling or a combination of some of or all of these methods. When we, by using these techniques, have gathered enough information we might be able to construct a *physical theory* – a condensed set of identified patterns and relations that can describe and unify a larger set of phenomena in nature.

Such theory can then be used to make predictions about previously uninvestigated phenomena and situations in nature. Part of the ultimate test of a theory involves checking the predictions of the theory against nature itself: Does nature behave as the theory predicts? If this is not sufficiently accurately the case, we might have to modify the theory or come up with a new theory. It is important to stress that we can never prove a theory to be true, we can only, as Popper emphasized, prove a theory to be wrong by identifying a mismatch between our theory and nature. A theory should always be viewed as the current best description available of the subset of nature we are studying.

So far I have only described a physical theory in very general terms – as a condensed set of identified patterns that can describe a set of phenomena and observations in nature. It is now time to be a little more detailed and to discuss the building blocks of a physical theory and their relation to nature itself.

A physical theory always involves concepts and constructs – well-defined properties of the system we are studying. If we, for instance, are studying the motion of objects, we have concepts such as force, mass, velocity, acceleration. It is important to remember that these concepts do not have a life of their own within the theory – they always represent various features of the actual system we are studying.

Apart from the concepts themselves. a physical theory also consists of relations between these concepts – these are the patterns and regularities that we discussed earlier. Even though it is far from trivial why – as discussed in the previous section – these relations can be captured very elegantly and compactly using the language of mathematics. Going back to the study of the motion of objects I mentioned in the paragraph above, I could argue that we have identified relations between the concepts force, mass and acceleration that say that the acceleration of the centre of mass of an object is proportional to the resultant external force acting on the object and directed in the same direction as that resultant force. Moreover, the acceleration is inversely proportional to the mass of the object – the heavier object the less acceleration for the same resultant force. All these relations can be neatly summarized with a single mathematical formula – known as Newton's second law:

$$\sum \vec{F}_{ext} = m\vec{a}_{cm}$$

For physicists familiar with the concepts and the mathematics involved, this single expression contains all the information about the relations between the concepts that we earlier described in words. So, in a physical theory, we define concepts and then describe the found relationships between these concepts using the language of mathematics.

Some might ask whether the formulation of these relations in mathematical terms is just a matter of aesthetics and compactness. Do we gain anything beyond having to write less text, by formulating these relations as mathematical formulas? In fact we do! Once we have established these relations as mathematical expressions, we can start to use the entire machinery of mathematics to manipulate these expressions. It is by this mathematical manipulation that we can get to well-defined and testable predictions from our theory. So, the mathematical expressions are, along with the concepts themselves, crucial ingredients in any physical theory and could be seen as the condensed products of the process of constructing a physics theory. This means that if we want to explore students' learning of physics it is important to investigate how students experience and deal with physics equations.

6.4 Exploring students experience of equations

As discussed in Section 3.4, a lot of the physics education research as far as equations are concerned had been focusing on the use of equations or on the structural relationships of equations. We felt that many aspects of students' experience of physics equations remained unexplored – such as what students focus on when presented with equations and students' epistemological views of what it means to understand equations.

The study presented in Paper III attempts to map out the variation in what students focus on when presented with physics equations and to discuss the nature of this variation. We believe this to be of importance for the teaching and learning of physics since what students focus on generically reflects how they think about, view and use equations. This is important for a physics teacher to be aware of and should inform the teaching of physics in terms of how we structure the way we teach physics and come to know our students as learners.

Furthermore, apart from problem solving and despite the many research-driven changes that have taken place in physics education teaching and learning environments (e.g. Redish, 2003), the most common form of teaching physics remains teacher presentation and/or students reading textbooks. It is therefore of great interest to explore what students focus on when presented with equations. What aspects of physics equations are brought to the fore?

After investigating what students focus on, we became interested in exploring whether these focuses could have epistemological underpinnings. Epistemologically based research such as that done by Hammer (1994), Roth and Roychoudhury (1994), Redish, Saul and Steinberg (1998), May and Etkina (2002), Adams et al. (2006), and Lising and Elby (2004) have indicated that physics students' learning is significantly related to their perceptions about the nature of physics and about physics learning and knowledge. As stated earlier, a critical ingredient of university physics teaching and learning is the mathematical representations, i.e., *physics equations*. The work reported in Paper IV contributes to the epistemological research as well as to the research on students' experiences of physics equations by exploring students' epistemological mindsets (described and defined in Section 6.6.1) towards what it means to understand physics equations.

6.5 Exploring what students focus on when presented with equations

6.5.1 Research questions

The main research question that we were interested in finding an answer to was:

- What do university physics students focus on when presented with an equation?

To answer this question we used a phenomenographic approach (described in detail in Section 4.4) and in next two sections I will describe and discuss the results of our exploration of this research question.

6.5.2 What do students focus on? - Results of paper III

From our analysis we came up with five categories of what a teacher could anticipate students to focus on when presented with equations (these categories are summarized in Table 2 below). Below we will describe the different categories and provide illustrative examples for some of the categories. For more illustrative examples from the interview data, the reader is referred to Paper III. It should be stressed that these categories are collective constructs and that individual students can focus on several of these facets when presented with equations.

Category A – Name/label

In Category A, students focus on attributing a name (if such a name exists) to the equation. We suggest that the student's experience of the formula has moved from unknown to association with a name, but that there is no focus on the in-depth meaning of the equation.

Category B – Mathematical aspects

In Category B students focus on the mathematical aspects of an equation, i.e., what type of object the equation is mathematically (such as an differential equation) and what mathematical manipulations are necessary to be able to use the equation to attain the desired quantity, i.e., to solve problems.

Category C – Linguistic reading of the equation

In Category C students focus on being able to "read" the equation, i.e., substituting terms for symbols such as "velocity equals frequency times wavelength". An example of this from our interview data is seen when one of the students is asked about the meaning of the electromagnetic relation $\nabla \times \mathrm{E} = 0$:

Interviewer: *Can you tell me what this means to you?*
Student: *Um, I think the E is the intensity of an electric field. And the the curl of E…*[quietly to herself] *equals zero… I think this is a conservative vector field – and I know how to calculate it but I don't know what it means.*

Category D – Understanding of the parts

In Category D, as in Category C, students focus on the parts of the equation. However, in Category D it is not enough to know the names of the terms that the symbols represent. Here students focus on *understanding* what e.g. velocity, frequency and wavelength *mean* in physics or what a partial derivative or multivariable function *means* in mathematics. We differentiate this from level C since it is far from clear whether saying 'frequency', for example, automatically means that the students understand what "frequency" means[4]. An example from our data is the excerpt above, where the student directly associates the fact that the curl of the electric field is zero to the field being a 'conservative vector field' without knowing what such type of field means.

Category E – Understanding of the whole

In Category E, students move beyond the parts of the equations and focus on the meaning of the equation *as a whole*. Here the students try to relate or link the equation as a whole to a meaning and try to situate the equation in an appropriate context. In the short excerpt below which is translated from Swedish, the student is commenting on the Schrödinger equation:

Interviewer: *What do you see when you see this equation?*
Student: *[...] I wasn't sure about this, this… psi… or what ever it's called, like what does it do and what do I need it for? I found it hard to link it to anything in reality.*

This student is focusing on the meaning of the equation itself in "reality" and feels that something is missing as far as equations is concerned unless there is a relation to "reality" in one way or another.

These categories are summarised in Table 1.

[4] As diSessa (1993) and others have pointed out, a student can easily learn to express an equation in linguistic terms, repeating it as a *slogan* without actually understanding *what* the slogan means.

	Category	Focus
A	Name/label	The name of the equation
B	Mathematical aspects	The mathematical nature of the equation
C	Linguistic reading of the equation	Reading out the symbols in physics/mathematics terms.
D	Understanding of the parts	Understanding the physics or real world meaning of the *symbols* in the equation
E	Understanding of the whole	Relating the equation as *a whole* to appropriate physics/everyday-world situations

Table 1: Students' focus when presented by a physics equation.

6.5.3 Discussion of the results of Paper III

Looking at the categories summarized in Table 1, it is clear that there is a range in what students focus on when presented with equations. Some students focus on relating the equation as a whole to appropriate physics or everyday situations, while others simply focus on being able to read the equation or understand its mathematical structure.

What struck us as interesting was that, although there is a spread in students' focuses, many of the students focused on the mathematical aspects of physics equations, especially in their initial encounter with physics equations. When asked to describe what they saw when presented with an equation or what the equations meant, a number of students plunged straight into the mathematics. An example is provided in the excerpt below, where the equation $P = \Psi\ \Psi^* = |\Psi|^2$ is being discussed. To clarify to the reader, the student talks about the wave function, but is erroneously referring to it as the "Schrödinger equation".

Interviewer: *What does this tell you? What does this mean to you when you see it?*

Student: *Ehh.. this is the big letter psi, right? [Yes]. So it's… I think it is the Schrödinger equation and.. the complex conjugate…so it's squared and... like that it means that it is always positive, so… one thing I've learned is that it's always positive when it's a square… that's one of the basic things they print in your mind, that's just stuck.*

Interviewer: *It's just stuck there? Square means positive?*

Student: *Yeah (laugh), so… it's like… the probability function will always be positive. It will be the, yeah, you know, square of the Schrödinger equation.*

Interviewer: *Can you… Is there something else this equation tells you?*

Student: *You see like x and t, it's a two-variable… eh… equation or… yeah. And… it's just one-dimensional if you could call it… well, it's two-dimensional with the time, but we usually work with it in one dimension, only x.*

This example serves as a nice illustration of the fact that many students focus on the mathematical structure of an equation. There is much that could be said about the meaning of this equation, yet the student immediately turns to describe the mathematics involved. Another example is provided below where a student is asked about what she was thinking when presented with the same equation:

Interviewer: *What were you thinking when the lecturer presented this equation?*

Student: *I was thinking about psi and the wave function. She said it's a complex function so you can't just write it as a square, but… I still don't quite understand what psi is, how you use it.*

Once again, the conceptual understanding of what the equation means is suppressed and the mathematical details of the equation are the focus of the student.

Others simply looked at the mathematical level of complexity of the equations, thereby dubbing the equations as complex or not, supposedly dependent on whether they could manipulate the equations mathematically or not. An example is the excerpt below where the student is asked what she believes is difficult to understand about the Schrödinger equation and what she was thinking of when presented with this equation.

Student: *Eh… I don't know, what one was thinking was that… it's a second degree equation and it is a differential equation and there are partial… derivatives… which makes it more complicated…it feels very advanced.*

Again we have an example were the mathematical aspects tend to dominate what students are focusing on when presented with an equation.

As another example, for a number of students there was no difference between the equation $k=2\pi/\lambda$ (which is a simple definition of the wave number k in terms of the wavelength λ) and the de Broglie relation $p = h/\lambda$, which says something fundamental about nature by relating wave and particle properties. They both had the same simple mathematical structure of an algebraic relation and were therefore seen as understood by the students, even though a closer investigation of this understanding revealed that in some cases the students were not even sure what the symbols in the equations represented.

So even though there is a spread in what students focus on, the mathematical aspects of equations are considered among the most important and seem to be the determining factor when ruling whether an equation is understood or not. If the mathematical structure is understood and the students know how to manipulate the equation mathematically to obtain the desired quantity, the equation is considered understood.

Similar results to those we found in Paper II have been reported by Redish, Saul and Steinberg (1998) in a study of students' expectations in introductory physics. In this study, the authors state that:

> An important component of the calculus-based physics course is the development of students' ability to use abstract and mathematical reasoning in describing and making predictions about the behaviour of real physics systems. Expert scientists use mathematical equations as concise summaries of complex relationships among concepts and/or measurements. They can often use equations as a framework on which to construct qualitative arguments. (p. 11)

However, from their study they conclude that the *actual* treatment of equations as far as students are concerned is far less fruitful and sophisticated:

> Many introductory students, however, fail to see the deeper physical relationships present in an equation and instead use the math in a pure arithmetic sense – as a way to calculate numbers. When students have this expectation about equations there can be a serious gap between what the instructor intends and what the students infer. (p.11)

So it seems that students carry expectations that the mathematical aspects of an equation are important – mainly due to the fact that it allows them to solve problems. However, as shown in the research in Paper III and by Redish, Saul and Steinberg (1998) this leads to too heavy a focus on the mathematical aspects, where other important facets of physics equations never receive an appropriate focus. As described by Redish, Saul and Steinberg this means that:

> …an instructor may go through extensive mathematical derivations in class, expecting the students to use the elements of the derivation to see the structure

> and sources of the relationship in the equation. The students, on the other hand, may not grasp what the instructor is trying to do and reject it as irrelevant "theory". Students who fail to understand the derivation and structure of an equation may be forced to rely on memorization – an especially fallible procedure if they are weak in coherence and have no way to check what they recall.

A logical question at this stage is then: "Where does this heavy focus on the mathematical aspects come from?" It is likely that this situation occurs due to the heavy focus on mathematics and rote problem solving involved in the traditional teaching that still dominates in many physics departments and courses. This claim is supported by the results in Redish, Saul and Steinberg (1998), where no improvement with regard to students' expectations of the role of equations can be found after attending a university physics class. In fact, it is even worse than that since some classes "showed a significant and substantial deterioration" (p.11).

It seems like we need to go beyond the traditional mathematical focus and actively discuss and incorporate other important aspects of equations by exposing the students to a more varied presentation of physics equations. Otherwise, important aspects of equations besides the mathematics (such as range of validity and meaning) could go unnoticed and students would continue to have a too strong focus on the mathematical aspects of physics equations and inappropriate expectations of the role of physics equations.

6.6 Exploring university students' epistemological mindsets towards the understanding of physics equations

6.6.1 Mindsets

An ongoing debate exists over how to model a person's epistemology to better understand and inform student learning. In general, the most common models are beliefs, traits/styles, and resources (Elby & Hammer, 2001; Hofer & Pintrich, 1997). Among other things, these models differ as to the form of the epistemology, whether it is explicit or implicit for the student, and how context-dependent it is. One additional model makes no claim as to form and this is the phenomenographic *relevance structure of the learning situation* model. Marton & Booth (1997, p.143) describe relevance structure as being the "persons' experience of what the situation calls for, what it demands. It is a sense of aim of direction, in relation to which different aspects of the situation appear more or less relevant". At the same time the phenomenographic perspective is firmly anchored in its associated empirical findings that reveal variation in ways of experiencing as being related to context. Thus for our research question we would argue that it is most fruitful to draw on the notion of *relevance structure* and in doing so we are characterizing the notion of relevance structure as a mindset. We then define mindset as *the epistemologically-driven bringing to the fore of perceived critical attributes of a learning, application, or problem solving situation*.

6.6.2 Research questions

In order to begin an investigation of students' epistemological mindsets towards the understanding of equations, we focus on three main research questions in Paper IV:

- When students say that they understand an equation, how do they describe what that means to them?
- How can these descriptions be characterised in terms of epistemological mindsets?
- Are similar epistemological mindsets observable for students at various stages in their academic career?

Using the analytical process described in Section 4.5.2 we arrived at the results presented and discussed in the next two sections.

6.6.3 Results of Paper IV

In our analysis of the data, it became clear to us that students' descriptions of what it means to understand an equation could be seen as epistemological mindsets involving one or more epistemological components. In this section, we begin by presenting and describing our characterisations of the components of students' epistemological mindsets towards the understanding of physics equations. Illustrative examples of the characterisations from the interview data can be found in Paper IV.

Throughout the description of these epistemological components, we use the equation providing the speed of a longitudinal wave in a fluid, $v = \sqrt{B/\rho}$, where v is the speed of the wave, B is the bulk modulus of the fluid through which the wave propagates and ρ is the density of the fluid. The use of this equation is for illustrative and clarification purposes and does not reflect or represent actual excerpts from the interview data.

Epistemological component A – understanding involves being able to recognise the symbols in the equation in terms of the corresponding physics quantities

For this epistemological component, understanding an equation involves being able to recognise what all the symbols in the equation represent in terms of corresponding physics quantities. In the case of $v = \sqrt{B/\rho}$, this would correspond to identifying v as the speed of the wave, B as the bulk modulus and ρ as the density of the fluid through which the wave propagates.

Epistemological component B – understanding an equation involves being able to recognise the underlying physics of the equation

Here, understanding an equation involves recognising the underlying physics of the equation. This involves one or several subcomponents such as knowing what the quantities in the equation mean from a disciplinary physics point of view, what the underlying concepts and principles of the equation are or being able to know the origin of the equation in terms of how it is derived. If we once again look at the equation $v = \sqrt{B/\rho}$, this could correspond to an understanding of what the speed of a wave means, what the bulk modulus of a substance represents and what density is. It could also involve knowledge of waves (longitudinal waves in particular) as well as an idea of how the equation is and can be derived from more fundamental concepts.

Epistemological component C – understanding involves recognising the structure of the equation

For this epistemological component, the *structure* of the equation is involved – an understanding of how the different quantities in the equation are related to each other and the equation as a whole in terms of where the quantities are situated in the equation and what this implies. Using $v = \sqrt{B/\rho}$ to clarify, this epistemological component would involve considerations of whether it makes sense to have the bulk modulus B in the numerator and the density ρ in the denominator, i.e., does it makes sense that the speed of the wave increases if we have a larger bulk modulus B and that the speed increases if we decrease the density? It also involves asking questions such as: What happens to the speed if we have a bulk modulus that is four times larger?

Epistemological component D – understanding involves establishing a link between the equation and everyday life

For this epistemological component, understanding involves establishing a link to everyday life. Two main types of links could be identified in the data. The first type involves situating the equation in an everyday context, by identifying examples and situations in everyday life where the equation applies. The second type of link between an equation and everyday life consists of finding analogies from everyday life that help in appreciating the meaning of the equation. For the first type of link, an example would be realising that $v = \sqrt{B/\rho}$ could describe the propagation speed of sound in air. For the second type of everyday linking through analogies, one could compare swimming through water to walking through air in order to appreciate the dependence of the speed on the density.

Epistemological component E – understanding involves knowing <u>how</u> to use the equation to solve physics problems

In this epistemological component, understanding involves being able to know how to use the equation, i.e., solving physics problems by using the mathematical manipulations that are needed to extract the sought information from the equation. This component also involves identifying which information is sought as well as what other information is available or needed. Once again using $v = \sqrt{B/\rho}$ to clarify this component would involve being able to use this equation to calculate the speed of longitudinal waves for a fluid with a given bulk modulus and density, or more generally being able to calculate any of the three quantities from the equation given the other two.

Epistemological component F – understanding involves being able to know <u>when</u> to use the equation

For epistemological component F, an understanding of *when* to apply an equation and when or when not an equation can be applied is explicitly put forward and described to be an important part of understanding an equation. This involves knowledge of the range of validity of the equation, inherent approximations and idealizations and in some cases also what branch of physics the equation is supposed to describe. In the case of $v = \sqrt{B/\rho}$ this would involve knowing for what kind of waves this equation can be used and for what kind of waves it cannot be used. It would also involve acknowledging factors such as the fact that this equation presumes small amplitudes and linear waves and that the fluid is considered to be a continuum.

After identifying the different components of students' epistemological mindsets towards the understanding of an equation which we could identify, the next step was to map the epistemological mindset of individual students in terms of which epistemological components we could identify as being described by the students as an important part of understanding an equation.

	A **Recognise symbols**	**B** **Underlying physics**	**C** **Structure**	**D** **Everyday life**	**E** **How to use**	**F** **When to use**
1	*			*	*	
2	*		*			
3	*				*	
4					*	
5		*			*	
6				*		
7	*	*			*	
8	*		*			*
9				*		
10				*		
11				*		
12	*				*	
13	*	*		*	*	*
14	*					
15					*	*
16				*		*
17	*	*			*	
18		*			*	*
19		*			*	
20		*				
Σ	**9**	**7**	**2**	**7**	**11**	**5**

Table 2. A mapping of the epistemological components for individual students.

We arrived at Table 2, which shows which epistemological components (labelled horizontally from A to F, referring to the different components presented earlier in this section) are present for the individual students in this study (labelled vertically from 1 to 20). Students 1-7 are first year undergraduate students, students 8-16 are second or third year undergraduate students and students 17-20 are PhD students. The last row involves a count of the number of occurrences of the components corresponding to each column.

6.6.4 Discussion of the results of Paper IV

The results presented in the previous section contain several interesting features. Let us look at these results in terms of the original research questions we posed. The first two questions of the research investigation were:

- When students say that they understand an equation, how do they describe what that means to them?
- How can these descriptions be characterised in terms of epistemological mindsets?

First of all, we found that students' descriptions of what it means to understand physics equations could be seen as epistemological mindsets, composed of one or several components. Looking at our characterisations of the components, it can be concluded that to understand an equation could mean a range of things for the students. For some students understanding simply involves being able to recognise the symbols and being able to use the equation to solve physics problems. For others, understanding has to involve an appreciation of the underlying physics as well as an ability to link the equation to an everyday life situation. This range in what it means to understand an equation becomes even more apparent when looking at Table 2. The most frequently occurring components are how to use the equation and being able to recognise the symbols, followed by understanding the underlying physics and relating the equation to everyday life. Thus, the most striking structure of Table 2 is that there is no discernable structure; there are no clear correlations between different components and no obvious pattern can be found. So in conclusion, students' views of what it means to understand an equation could be characterised as mindsets involving one or more components, with a large spread in the composition of these mindsets.

The third question we were interested in answering was:

- Are similar epistemological mindsets observable for students at various stages in their academic career?

Looking again at Table 2, no clear differences could be identified between students at various academic levels. The only thing that could be said, although the low number of students makes such a statement highly tentative, is that all of the higher level students find an understanding of the underlying physics important. So, apart from this, there does not seem to be any temporal convergence to a particular view of what it means to understand an equation or a particular set of components emerging in students' mindsets.

Taken together, these results indicate that, apart from being able to use equations, students should also be given an opportunity to reflect on what it means to understand physics equations. Taking previous epistemological research into account, it seems plausible that students' views of what it means to understand an equation affects how they approach equations. As an example, take a student with an epistemological mindset where understanding an equation means being able to use it. It is likely that this student's efforts go into being able to manipulate the equation mathematically, thereby being at risk of missing other important attributes of physics equations. So we suggest that epistemological discussions should be a natural and important ingredient in introductory physics courses. Questions such as "What is physics?" and "What does it mean to understand a physics equation?" should be addressed and students should be given an opportunity to reflect on these matters. Due to the importance of epistemology shown by educational research, such an intervention is likely to have a positive effect on students' learning of physics, especially in the light of a successful implementation of such an approach by Linder and Marshall (1998).

7. Conclusions and outlook

7.1 Concluding remarks

Being close to the end of this thesis, I hope that I have been able to provide the reader with an accessible account of my research and associated pedagogical implications. I also hope that at this stage the reader, after being presented the thesis with my research questions and findings originating from these questions, agrees with what I claimed in the introduction – that this thesis can be seen as making research contributions in three main areas:

- understanding and investigating the nature of some potentially puzzling aspects of de Sitter space in relation to inflationary cosmological models;
- a contribution to the exploration of university students' understanding of quantum mechanics in the area of quantum tunnelling; and,
- an exploration of university students' experience of physics equations in terms of what they focus on when presented with equations and how they epistemologically view what it means to understand physics equations.

Looking at this list, a natural follow-up question that emerges is: "Where do we go next?" I attempt to provide the reader with an answer to this question in the next (and last) section of this thesis.

7.2 Future research

Looking at the research presented in this thesis, many ideas comes to my mind as I reflect on possible future research directions inspired by the research reported here.

My research in theoretical physics has involved what is known as the de Sitter space. This has always been a favourite laboratory for many ideas and toy models in theoretical physics. With the discovery that our Universe is not only expanding, but accelerating and approaching a de Sitter space, many questions arise that pose difficulties for theoretical physics and that need to be resolved. One such difficulty is obtaining a consistent string theory in de Sitter space, which turns out to be a highly non-trivial task. Another difficulty associated with the discovery that our Universe evolves towards a de Sitter space is that Einstein's famous cosmological constant has "returned from the dead". An understanding of the origin and nature of the cosmological constant is currently one of the most extensively researched areas in theoretical physics, with a very large potential impact on our view of nature. So aspects of de Sitter space are likely to play an important role in future theoretical physics research.

As far as the conceptual understanding of quantum mechanics is concerned, this is an area where I believe that many things remain to be done. It is not easy to find concepts which are unexplored in for example mechanics, while the situation is very different for quantum mechanics. There still exist many unexplored areas of quantum mechanics where we do not have an understanding of students' ideas, conceptions and understandings. The surveys by Falk (2004) and McKagan and Wieman (2006) are important steps towards a fuller understanding of students' conceptual understanding in quantum mechanics, but further development of such surveys hinges on data acquired from explorations of previously unexplored students' conceptions.

Apart from mapping student conceptions, it would also be interesting to explore the origins of these conceptions. Unlike mechanics, where many conceptions and conceptual hurdles stem from prior everyday life experiences, this is due to the nature of quantum mechanics, not the case for conceptions of quantum phenomena. Thus, many of these conceptions are born at the same time quantum mechanics is first encountered and learnt, and it would be interesting to explore the origin and development of these conceptions and which factors affect their formation and how.

Another area which my research presented in this thesis has explored is students' experience of physics equations. In my research, I have investigated what students focus on when presented with equations as well as students' epistemological views of what it means to understand an equation. I have suggested that the results imply that it would be valuable to incorporate epistemological reflections of what the role of physics equations are and what it means to understand physics equations. It would be interesting to make a coherent implementation of this suggestion and to evaluate the impact as far as student's view of what it means to understand an equation and actions stemming from this view is concerned. "Coherent" refers to the fact that such an implementation would need to involve all aspects of a course. There is no point in making such an intervention if the motivation is unclear to the students or if such things as the assessment or problem solving on that course are not modified in accordance with this introduction of, and focus on, epistemological considerations.

Besides trying out such an implementation, I can see other things related to students' experience and understanding of equations that would be interesting to explore. One such thing would be to make similar explorations for physics teachers and/or researchers and comparing and relating their views to those of the students. Another interesting thing would be to investigate what physicists consider to be the key ingredients of a fruitful and appropriate understanding of physics equations. This could then be followed by empirical studies mapping students' understanding of these key ingredients.

In Paper IV we introduce the notion of epistemological mindsets. From a theoretical point of view it would be interesting to expand the idea of mindsets in terms of the phenomenographic construct of relevance structure in relation to how students frame their learning.

In summary, I see many possible research questions and directions building on the research presented in this thesis, and I hope that future research will be able to explore these areas further, thereby expanding and adding to our understanding of physics as well as the teaching and learning of physics, in the same way as I believe that my research presented in this thesis has made such a contribution.

8. Svensk sammanfattning

8.1 Introduktion

Denna avhandling innefattar forskning inom två skilda områden: teoretisk fysik och fysikens didaktik. Vid en första anblick kan det tyckas att dessa båda områden har väldigt lite gemensamt. Det finns dock en explicit gemensam nämnare mellan min forskning i teoretisk fysik och i fysikens didaktik: i båda fallen rör det sig om ett utforskande av abstrakt fysik och som en följd av detta även av de matematiska representationerna av fysik. I det ena fallet ”från insidan” innefattandes mina egna erfarenheter och i det andra fallet ”från utsidan” via en kartläggning av studenters erfarenheter. Från en inlärningssynvinkel finns många frågeställningar och erfarenheter som är desamma oavsett om du är forskare eller student och jag är övertygad om att mina egna erfarenheter av forskning inom abstrakt fysik har varit ovärderliga när jag har utforskat studenters sätt att erfara abstrakt fysik.

Jag har dock ännu inte definierat vad jag avser när jag säger abstrakt fysik. Med abstrakt fysik syftar jag på fysik som inte kan relateras till vårt vardagliga liv eller till tidigare erfarenheter. Detta är självfallet något subjektivt och många grenar av fysik innehåller begrepp eller fenomen som skulle kunna klassificeras som exempel på abstrakt fysik. Min egen forskning har innefattat såväl strängteori som kosmologi och kvantmekanik och dessa områden innehåller många aspekter som tveklöst kvalificerar sig som abstrakta och som innefattar begreppsmässiga utmaningar när det gäller inlärandet av dessa aspekter.

Jag hoppas att jag i avhandlingen lyckas övertyga läsaren om att min forskning bidrar till förståelsen av abstrakt fysik samt till förståelsen av studenters erfarenheter av densamma. Mer specifikt bidrar denna avhandling med relevant forskning inom tre huvudområden:

- Förståelse och utforskning av några potentiellt problematiska aspekter hos de Sitter-rummet i relation till kosmologiska modeller som innefattar inflation.

- En undersökning av fysikstudenters begreppsmässiga förståelse av kvantmekanisk tunnling och i synnerhet av sannolikhetens roll för detta fenomen.
- En utforskning av studenters erfarenhet av ekvationer inom fysik i termer av vad de fokuserar på när de möter ekvationer och hur de från en epistemologisk synvinkel betraktar vad det innebär att förstå ekvationer.

8.2 Forskningsfrågor

Den första delen av denna avhandling innefattar forskning inom teoretisk fysik relaterad till kosmologi och förståelse av de Sitter-rummet. De flesta moderna kosmologiska modeller involverar en epok som går under benämningen inflation. Under denna era genomgår vårt universum en hastig, accelererande expansion. Inflationsmodeller förekommer i många varianter, men gemensamt är att de uppvisar lovande resultat när det gäller att förstå och komma till rätta med en del problematiska aspekter av kosmologier som saknar en inflationsfas.

I Artikel I i denna avhandling konstruerar vi ett kosmologiskt scenario som kan ses som en leksaksmodell för inflation, och i detta scenario formulerade vi och utforskade en intressant fråga:

- I ett kosmologiskt scenario där en de Sitter-fas följs av en fas med en skalningsfaktor som utvecklas som t^q, där $1/3<q<1$, finns det en möjlighet att information kan dupliceras i strid med kvantmekanikens förutsägelser?

Den andra delen av denna avhandling omfattar forskning inom fysikens didaktik. Med ett genuint intresse i teoretisk fysik, var det naturligt att från ett lärandeperspektiv studera ett område där teoretisk abstraktion spelar en viktig roll: kvantmekanik. Artikel II undersöker studenters förståelse av kvantmekanisk tunnling. Detta är ett utmärkt didaktiskt laboratorium eftersom tunnling är ett fenomen som dels innefattar aspekter där vi inte har några vardagliga erfarenheter att luta oss mot och som dels kräver en förståelse av ett flertal centrala kvantmekaniska begrepp. Den huvudsakliga forskningsfrågan för denna studie var:

- Hur ser fysikstudenters förståelse av sannolikhet för kvantmekanisk tunnling ut, och vilken variation i denna förståelse är möjlig att påvisa?

Från denna studie blev det uppenbart att det kan vara besvärligt att förstå abstrakt fysik och att matematiska representationer (ekvationer) tenderar att spela en väldigt viktig och dominerande roll för studenter när det gäller förståelsen av denna typ av fysik. Detta ledde till ett intresse av att försöka kartlägga studenters erfarenhet och förståelse av fysikaliska ekvationer. Artikel III och IV är båda produkter av detta intresse. I Artikel III var vi intresserade av att utforska studenters fokus när det gäller ekvationer inom fysik och den centrala frågan var:

- Vad fokuserar fysikstudenter på när fysikaliska ekvationer presenteras för dem?

De svar på denna fråga som vår undersökning fann, fick oss att börja spekulera i om studenters fokus kan vara relaterat till deras syn på vad det innebär att förstå fysikaliska ekvationer. Detta ledde till studien i Artikel IV där vi var intresserade av att utforska studenters epistemologiska syn på vad det innebär att förstå fysikaliska ekvationer. För denna studie formulerade vi tre forskningsfrågor som vi var intresserade av att besvara:

- När studenter säger att de förstår en fysikalisk ekvation, hur beskriver de vad detta innebär för dem?
- Hur kan dessa beskrivningar kategoriseras i termer av epistemologiska attityder?
- Ser dessa epistemologiska attityder likadana ut för studenter på olika nivåer i deras utbildning?

8.3 Metod och metodologi

Denna rubrik är i första hand relevant för Artikel II, III och IV som utgör forskning inom fysikens didaktik. Som i all forskning är inom fysikens didaktik av största vikt att välja ett lämplig sätt att besvara de uppställda forskningsfrågorna. För alla dessa artiklar har det huvudsakliga datamaterialet insamlats genom intervjuer avsedda att utforska studenters erfarenheter och tankar kring det ämne som forskningsfrågan varit avsedd att undersöka. Dessa intervjuer har utformats på ett sätt som tillät studenterna att reflektera över frågeställningar på ett så mångfacetterat sätt som möjligt för att öka sannolikheten att få en adekvat representation av mångfalden i studenternas erfarenheter. Genomgående har intervjuerna även genomsyrats av en ambition att få till stånd ett intervjuklimat där studenterna känner att de kan uttrycka sina erfarenheter på ett tryggt och öppet sätt.

Beroende på frågeställningen har data från dessa intervjuer därefter i iterativa cykler analyserats och tolkats i relation till forskningsfrågorna. I såväl Artikel II som III var vi intresserade i att kartlägga variationen i studenters förståelse av sannolikhet i relation till kvantmekanisk tunnling respektive studenters fokus när det gäller fysikaliska ekvationer. För dessa två studier utnyttjade vi därför fenomenografi, ett teoretiskt ramverk som har för avsikt att kartlägga variationen i personers erfarenhet av ett fenomen, ett begrepp eller en situation. När det gäller Artikel IV var vi inte enbart intresserade av variationen i studenters beskrivningar av vad det innebär att förstå en ekvation, utan även att karaktärisera dessa beskrivningar i termer av epistemologiska attityder samt att jämföra studenter som befinner sig på olika nivå i sin utbildning. Av detta skäl valde vi att utföra en fallstudie av de i projektet ingående studenterna, för att få en så rik beskrivning av studenternas epistemologiska attityder som möjligt, där bredden och den mångfacetterade naturen av studenternas erfarenheter kunde representeras på ett tillfredsställande sätt.

8.4 Resultat

8.4.1 Att undvika en potentiell informationsparadox

I Artikel I presenterar vi ett kosmologiskt scenario där möjligheten till en informationsparadox uppstår. Huvudidén bakom denna potentiella paradox är att ett de Sitter-rum ersätts av en fas med långsammare expansion. I detta scenario uppstår nu möjligheten att ett föremål som inte längre kan kommunicera med en observatör under de Sitter-fasen (det vill säga all eventuell information associerad med föremålet är otillgänglig) plötsligt får möjlighet till kommunikation i den efterföljande fasen, vilket nu gör informationen tillgänglig. Om vi nu antar att informationen från föremålet även når observatören under de Sitter-fasen i form av strålning från den horisont som omger varje observatör har vi ett scenario där informationen kan fördubblas trots att fundamentala kvantmekaniska principer förbjuder detta.

Genom att beräkna den tid det skulle ta för föremålet som försvunnit bortom de Sitter-horisonten att återigen bli tillgängligt för kommunikation visar vi i Artikel I att ett sådant scenario kan undvikas. Det visar sig att denna tid är av samma storleksordning som rekurrenstiden för de Sitter-rummet, vilket innebär att vi betraktar ett experiment som saknar mening eftersom en eventuell detektor hos observatören (med kortare rekurrenstid) bokstavligen tappat minnet efter denna tid, vilket omöjliggör insamlandet av information från strålningen från horisonten i de Sitter-fasen och hotet om duplicerad information är således undanröjt.

8.4.2 Studenters begreppsmässiga förståelse av kvantmekanisk tunnling

I Artikel II undersökte vi hur fysikstudenter förstår kvantmekanisk tunnling av ett vågpaket för olika typer av barriärer. Studenterna intervjuades och i samband med intervjuerna fick de även interagera med datorsimuleringar av kvantmekanisk tunnling. En fenomenografisk analys av de data som intervjuerna genererade visade att det fanns fyra kvalitativt skilda sätt att förstå sannolikhet i samband med kvantmekanisk tunnling och under denna analys fann vi även ett antal exempel på svårigheter i förståelsen av tunnling och kvantmekaniska begrepp hos studenter.

För vissa studenter innefattade förståelsen av sannolikhet i samband med tunnling en syn på sannolikhet som liknar den när man singlar slant. För en slant finns en viss sannolikhet för krona och en viss sannolikhet för klave. På samma sätt finns vid tunnling en viss sannolikhet att passera barriären och en viss sannolikhet att reflekteras.

Andra studenter såg sannolikheten i relation till energi – antingen som en sannolikhet för ett vågpaket att ha en viss energi eller i termer av energifördelningen för de enskilda vågorna som tillsammans bildar vågpaketet. I det senare fallet argumenterade ett flertal studenter att de enskilda vågor som har en tillräckligt hög energi passerar barriären medan de övriga reflekteras.

Ytterligare ett sätt att se på och förstå sannolikhet i samband med kvantmekanisk tunnling som vi kunde finna hos studenterna, var att diskutera sannolikhet i termer av sannolikheten att hitta partikeln som representeras av vågpaketet på en viss plats vid en viss tidpunkt. Slutligen kunde vi även (om än för endast en student) identifiera en förståelse av sannolikhet i termer av upprepade experiment på identiska system.

Förutom vissa av dessa förståelser, som från ett fysikaliskt perspektiv representerar felaktiga uppfattningar om sannolikhet i samband med tunnling, så finns de centrala aspekterna av sannolikhet viktiga för en fullständig förståelse av tunnling med bland kategorierna över studenternas förståelse. Problemet är dock att om vi tittar på enskilda studenter så är det ingen av studenternas förståelse som innefattar alla dessa aspekter. Vi skulle önska att våra studenter förstod och utnyttjade alla dessa komplementära förståelser av sannolikhet för att få en helhetsbild och djupare förståelse av tunnlingsprocessen. I Artikel II presenterar vi ett antal pedagogiska förslag för att öka och förbättra studenters förståelse av tunnling och även andra konceptuella svårigheter vid sidan av sannolikhet som vi kunde identifiera i studien.

8.4.3 Fysikaliska ekvationer och studenters fokus

Vårt huvudsakliga intresse i Artikel III var att kartlägga vad studenter fokusera på i samband med fysikaliska ekvationer. Intervjuer i kombination med en fenomenografisk analys visade att det finns ett flertal olika saker som studenter fokuserar på:

- Huruvida ekvationen har ett namn och i sådana fall vilket
- Att förstå ekvationens matematiska aspekter, det vill säga vilken typ av objekt ekvationen är rent matematiskt och vilka matematiska manipulationer som krävs för att använda ekvationen
- Att kunna ”läsa” ekvationen, det vill säga ersätta symboler med ord
- Att förstå ekvationens olika delar, det vill säga vad de storheter som representeras av symbolerna i ekvationen har för fysikalisk innebörd
- Att förstå ekvationen som helhet i termer av dess innebörd och att länka ekvationen till ett lämpligt sammanhang

Vid sidan av kartläggandet av denna variation, var den mest intressanta upptäckten att en stor andel av studenterna hade ett väldigt starkt fokus på en av dessa kategorier – de matematiska aspekterna. Många studenter besvarade frågorna ”Vad tänker du när du ser den här ekvationen?” eller ”Vad har den här ekvationen för innebörd för dig?” med utläggningar om den matematiska strukturen hos ekvationen. Andra verkade bedöma huruvida de hade förstått en ekvation eller inte utifrån om de kunde förstå och manipulera den matematiskt. Detta starka fokus på matematik är en smula oroväckande eftersom det innebär att andra viktiga aspekter av ekvationer riskerar att aldrig bli föremål för studenternas fokus. Studenterna behöver erfara en mer varierad presentation och diskussion av ekvationer där de matematiska aspekterna utgör *en del* av förståelsen av en ekvation.

8.4.4 Fysikaliska ekvationer och epistemologiska attityder

I Artikel IV undersökte vi studenters epistemologiska beskrivningar av innebörden av att förstå fysikaliska ekvationer. Från vår analys kunde vi konstatera att dessa beskrivningar kan beskrivas i termer av epistemologiska attityder sammansatta av en eller flera komponenter som sågs som viktiga ingredienser i förståelsen en ekvation. De komponenter vi kunde identifiera kan kortfattat summeras av nedanstående lista:

- Att känna igen de symboler som ingår i ekvationen i termer av motsvarande fysikaliska storheter
- Att ha kännedom om ekvationens underliggande fysik
- Att ha en förståelse för strukturen hos en ekvation
- Att etablera en länk mellan ekvationen och det vardagliga livet
- Att förstå hur ekvationen används för att lösa problem
- Att förstå när ekvationen kan (och bör) användas

För de enskilda studenterna bestod deras epistemologiska attityder av en eller flera av dessa komponenter (ingen uttryckte samtliga av dessa) och en av de mest tydliga slutsatserna vi kunde dra var att förståelse av en ekvation kan ha väldigt olika innebörd för olika studenter. För vissa innebar förståelse ett igenkännande av de symboler som ingår i ekvationen samt att kunna använda ekvationen för att lösa problem, medan det för andra krävdes ingredienser såsom att kunna länka ekvationen till en vardagssituation eller att veta när ekvationen kan tillämpas för att en förståelse skulle anses som uppnådd. En relaterad slutsats var att studenternas epistemologiska attityd inte uppvisade någon tydlig struktur. Inga otvetydiga korrelationer mellan olika komponenter eller kunde påvisas. Slutsatsen är således att studenters syn på vad det innebär att förstå en ekvation kan karaktäriseras som epistemologiska attityder innefattande en eller flera komponenter.

Vi var också intresserade av att undersöka om det fanns några märkbara skillnader i studenters syn på vad det innebär att förstå en ekvation, när studenter på olika nivå i deras utbildning jämfördes. Några tydliga sådana skillnader, vid sidan av att alla studenter i ett senare skede av sin utbildning såg förståelse av den underliggande fysiken som en viktig komponent, kunde inte påvisas.

Resultaten från denna studie indikerar att det finns en kraftig spännvidd i studenters syn på vad det innebär att förstå en ekvation och i ljuset av tidigare epistemologisk forskning är det inte orimligt att anta att denna syn på vad det innebär att förstå en ekvation påverkar hur de hanterar ekvationer. Detta föranleder oss att föreslå ett större fokus på epistemologiska frågeställningar när det gäller fysikundervisningen samt en mer varierad presentation och hantering av ekvationer. En sådan presentation bör göra multipla aspekter av vad det innebär att förstå en ekvation tydliga samt föremål för reflektion och diskussion hos studenterna.

8.5 Några avslutande ord

Avslutningsvis vill jag understryka att det dessvärre inte finns några ”mirakelkurer” när det gäller att förbättra fysikundervisningen. Att få en helhetsbild och förståelse av in- och utlärning av fysik är att lägga ett tämligen komplicerat pussel. Dock så har uppkomsten och utvecklingen av fysikens didaktik som ett forskningsfält försett oss med flera viktiga pusselbitar och denna avhandling utgör ytterligare ett bidrag i en strävan efter att få en så komplett bild som möjligt. Ju mer komplett bild vi kan få, desto närmare kan vi komma till vad som rimligtvis måste vara målet - att skapa en kreativ, framgångsrik och stimulerande miljö för våra fysikstudenter.

Vi bör ta chansen att utnyttja de insikter fysikens didaktik har att bidraga med för att vi skall lyckas med att framställa fysik på ett sätt som får våra studenter att inse skönheten och kraftfullheten hos fysik. Min egen resa som doktorand har utan tvekan gett ytterligare näring åt min med största sannolikhet livslånga kärlek till fysik!

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“In my end is my beginning”
Mary – Queen of Scots

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