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Brief paper

Event-triggered control of systems with sector-bounded nonlinearities and intermittent packet transmissions[☆]

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ABSTRACT

The problem of event-triggered sampled-data control of nonlinear systems with sector-bounded nonlinearities is considered. We assume that sensors transmit their measurements to the controller over a communication channel, where the success of transmissions is defined by an i.i.d. Bernoulli process. For the analysis of the closed-loop system stability, we use the Lyapunov–Krasovskii technique. As a result, we obtain stability conditions in terms of linear matrix inequalities (LMIs), which can be used to design the appropriate triggering parameters. A global strictly positive minimum inter-event time is guaranteed to exist by design with the proposed triggering condition. A numerical example demonstrates the efficiency of the event-triggered approach in reducing the number of transmissions compared to periodic sampling, where the period is the enforced minimum time in the event-triggering condition.

1. Introduction

Nowadays, the problem of digital control of continuous processes has received considerable attention due to increased interest in wireless control systems. One of the advantages of such systems is that the communication between the components can be realized remotely. In addition, eliminating unnecessary wiring reduces network complexity and allows network modifications without complex architectural changes. Moreover, wireless sensors are often equipped with rechargeable batteries and energy harvesting elements to extract energy from the environment and store it for future use (Ahlén et al., 2019; Knorn, Dey, Ahlén, & Quevedo, 2019). This can significantly reduce the overall cost of implementing and maintaining the network. On the other hand, the use of wireless sensors and fading channels may lead to constraints and limitations that can significantly degrade the performance of the control loop (Heemels, Teel, van de Wouw, & Nesic, 2010).

Since controllers, sensors, and communication channels are most often digital, a controlled system receives sampled-data, i.e., it involves both continuous-time dynamics and a discrete-time controller. In the intervals between transmissions, the system becomes open-loop, and therefore an appropriate choice of sampling instants plays an essential role for the system stability. A standard approach to reduce the amount of transmitted information without significant degrading the system performance is an event-trigger (ET), which avoids unnecessary transmissions when the actual sensor signal does not differ significantly from the one available to the controller. The preliminary ideas of event-triggered control (ETC) were given in Årzén (1999), while a more systematic design of event-based controllers was proposed in Tabuada (2007). Since then, various ETC schemes have been extensively studied and developed in many works, see, e.g., Heemels, Johansson, and Tabuada (2012) and Peng and Li (2018) and references therein. For instance, continuous ETs were studied in, e.g., Lunze and Lehmann (2010) and Tallapragada and Chopra (2012), and discrete periodic ETC schemes were considered in Heemels, Donkers, and Teel (2013) and Wang, Postoyan, Nesic, and Heemels (2020). In, e.g., Dimarogonas, Frazzoli, and Johansson (2012) and Mazo and Tabuada (2011), distributed ET strategies were proposed for networked systems. For stochastic delay systems, an event-triggered feedback control problem was firstly solved in Zhu (2019). At the same time, the event-triggered strategy assumes that after the sampling instant is generated, the measured signal must be sent to the controller. However, it may not always be delivered successfully, i.e., packet dropouts may occur. Most works in this area concentrate on estimation problems, see, e.g., Leong, Dey, and Quevedo (2017), or rely on

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the assumption that the number of consecutive losses is bounded, see Dolk and Heemels (2017) and Lehmann and Lunze (2012). At the same time, when intermittent transmissions are caused by, e.g., sensor energy shortage, it does not allow a bound on the number of consecutive dropouts. Instead, transmissions can be described in terms of a probability distribution that can be derived from stochastic models of sensor energy and communication channel gain (Olofsson, Ahlén, & Gidlund, 2016; Seifullaev, Knorn, & Ahlén, 2019, 2021). Therefore, the use of ETC systems with intermittent packet transmissions requires appropriate tools for design and stability analysis.

In this paper, we consider ETC of nonlinear systems with multiple sector-bounded nonlinearities, like in Moreira, Gomes da Silva, Tarbouriech, and Seuret (2020) and Moreira, Tarbouriech, Seuret, and Gomes da Silva (2019). However, contrary to the above works, we assume that the measured signal is delivered to the controller successfully only with a certain probability, i.e., we assume that whether the transmission was successful or not is described by an i.i.d. sequence of Bernoulli random variables. The resulting closed-loop system is then considered as a stochastic impulsive system as proposed in Liu, Fridman, and Johansson (2015), where the continuous part is described by a switching system between two alternative continuous dynamics, see Selivanov and Fridman (2016). However, the analysis of mean-square exponential stability becomes nontrivial, since a discontinuous (in time) Lyapunov–Krasovskii (LK) functional, which decreases in the intervals of continuity, can grow in jumps at discrete time instants. This requires certain changes in the event-trigger structure and a special analysis of the LK functional. The main contribution of the paper lies in the stability conditions formulated in terms of linear matrix inequalities.

The rest of the paper is organized as follows. In Section 2, the problem description and necessary definitions are provided. The main result on the stability analysis is presented in Section 3. Section 4 presents a numerical example demonstrating how the event-triggered control may reduce the number of sent measurements.

2. Problem formulation

Consider the following nonlinear system

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} q_i\xi_i(t) + Bu(t),\ y(t) = Cx(t), \qquad (1)$$
$$\sigma_i(t) = r_i^{\mathrm{T}}x(t),\ \xi_i(t) = \varphi_i(\sigma_i(t), t),\ i = 1, \ldots, N,$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $y(t) \in \mathbb{R}^{n_y}$ and $\sigma_i(t) \in \mathbb{R}$ are the outputs, $u(t) \in \mathbb{R}^{n_u}$ is the control input, $A \in \mathbb{R}^{n_x\times n_x}$, $B \in \mathbb{R}^{n_x\times n_u}$, $C \in \mathbb{R}^{n_y\times n_x}$ are constant matrices, $q_i \in \mathbb{R}^{n_x}$, $r_i \in \mathbb{R}^{n_x}$ are constant vectors. We assume that $\varphi_i(\sigma_i(t), t)$ satisfies the next sector inequalities for all $\sigma_i \in \mathbb{R}$ and $t \geq 0$

$$\mu_i^- \sigma_i^2 \leq \sigma_i\, \varphi_i(\sigma_i, t) \leq \mu_i^+ \sigma_i^2, \qquad (2)$$

where $\mu_i^- \leq \mu_i^+$ are real numbers.

The output $y(t)$ is measured by $\bar{m}$ wireless sensors, where each sensor m, $m \in \{1, 2, \ldots, \bar{m}\}$, can measure a signal $y_m(t) \in \mathbb{R}^{n_m}$, i.e., $y(t) = \left[y_1^{\mathrm{T}}(t), \ldots, y_{\bar{m}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, where $y_m(t) = C_m x(t)$, $C = \left[C_1^{\mathrm{T}}, \ldots, C_{\bar{m}}^{\mathrm{T}}\right]^{\mathrm{T}}$ and $\sum_{m=1}^{\bar{m}} n_m = n_y$. Assume that $\{t_k\}$ is a sequence of sampling times, when the sensors should transmit measured data to the controller. However, each sensor m transmits data successfully only with a certain probability, β_m, otherwise the packet is not delivered. We assume that $\pi_m(k) \in \{0, 1\}$ indicates whether the transmission has occurred or not and consider it as an i.i.d. Bernoulli process with the probability of success β_m, i.e., $\begin{cases}\Pr\left\{\hat{y}_m(t_k) = y_m(t_k)\right\} = \Pr\{\pi_m(k) = 1\} = \beta_m,\\ \Pr\left\{\hat{y}_m(t_k) = \hat{y}_m(t_{k-1})\right\} = \Pr\{\pi_m(k) = 0\} = 1 - \beta_m,\end{cases}$ where $\hat{y}(t_k) = \left[\hat{y}_1^{\mathrm{T}}(t_k), \ldots, \hat{y}_{\bar{m}}^{\mathrm{T}}(t_k)\right]^{\mathrm{T}}$ is the most recently received output information on the controller side, and $\hat{y}(t_{-1}) \equiv \mathbf{0}$. We consider a static output feedback law implemented using zero-order-hold devices

$$u(t) = K\hat{y}(t_k) = \sum_{m=1}^{\bar{m}} K_m\hat{y}_m(t_k),\ \ t \in [t_k, t_{k+1}), \qquad (3)$$

where $K = [K_1 \ldots K_{\bar{m}}] \in \mathbb{R}^{n_u\times n_y}$, $K_m \in \mathbb{R}^{n_u\times n_m}$. Consider the differences between the outputs $y_m(t)$ and the last available measurements $\hat{y}_m(t_k)$ on the controller side:

$$\omega_m(t) = \hat{y}_m(t_k) - y_m(t), \quad t \in [t_k, t_{k+1}). \qquad (4)$$

We assume that each sensor m has access to the recently received value $\hat{y}_m(t_k)$ and implements a continuous ET mechanism generating an event if $t \geq t_k + h$ and at least one of the following conditions is satisfied

$$\omega_m^{\mathrm{T}}(t)\, \Omega_m\, \omega_m(t) \geq \varepsilon_m\, y_m^{\mathrm{T}}(t)\, \Omega_m\, y_m(t), \qquad (5)$$
$$(y_m(t) - y_m(t_k))^{\mathrm{T}} G_m (y_m(t) - y_m(t_k)) \geq \epsilon_m c_{k,m}(y_m), \qquad (6)$$

where the constant $h > 0$ is the minimal distance between two consecutive sampling times, $\varepsilon_m \geq 0$ and $\epsilon_m \geq 1$ are scalar threshold parameters, $\Omega_m \in \mathbb{R}^{n_m\times n_m}$ and $G_m \in \mathbb{R}^{n_m\times n_m}$ are constant positive semi-definite weighting matrices, and $c_{k,m}(y_m) = (y_m(t_k+h)-y_m(t_k))^{\mathrm{T}} G_m(y_m(t_k+h)-y_m(t_k))$. Note that the matrices Ω_m and G_m are used to assign weights to the components of the vector y_m and can be considered as free parameters. The next sampling instant is generated if at least one sensor generates an event, i.e.,

$$t_{k+1} = \min\left\{ t \geq t_k + h \,\middle|\, (5) \text{ or } (6) \text{ is satisfied for some } m \in \{1, 2, \ldots, \bar{m}\} \right\}. \qquad (7)$$

Note that the ET rule (7) is based on time-regularization (see, e.g., Heemels et al. (2012), Selivanov and Fridman (2016) and Tallapragada and Chopra (2012)), which leads to $t_{k+1} - t_k \geq h$, meaning that the so-called Zeno phenomenon is avoided. Thus, the time instant t_{k+1} is generated if one or several sensors generate an event. Then a synchronizer signals to all sensors that it is time to transmit and each sensor should transmit its current value. The receiver at the controller side forms $\hat{y}_m(t_{k+1})$, $m = 1\ldots\bar{m}$, which can be either $y_m(t_{k+1})$ or $\hat{y}_m(t_k)$, depending on what was received from each particular sensor. If nothing was received from a particular sensor, due to either a drop-out or energy scarcity, then the previously received value, $\hat{y}_m(t_k)$, is used. The resulting closed-loop system is schematically illustrated in Fig. 1. Note that ETs usually have a single triggering rule for all sensors requiring a continuous exchange between sensors. However, using (7), we have that although all transmissions are synchronized, each sensor node is equipped with a local triggering rule, meaning that the sensors do not exchange their measurements. With this policy, the sensors consume only a small amount of energy required to send a message that an event has occurred, while much more energy would be required to transfer the whole measurement vectors between the sensors.

According to the approach, proposed in Liu et al. (2015), the control law (3) can be rewritten as follows:

$$u(t) = Ky(t_k) + \sum_{m=1}^{\bar{m}}(1 - \pi_m(k))K_m e_m(t), \qquad (8)$$
$$e_m(t) = \hat{y}_m(t_{k-1}) - y_m(t_k), \quad t \in [t_k, t_{k+1}). \qquad (9)$$

Based on the input-delay approach (Fridman, Seuret, & Richard, 2004), the output $y(t_k)$ can be rewritten as an output with variable delay

$$y(t_k) = y(t - \tau(t)) = Cx(t - \tau(t)), \qquad (10)$$

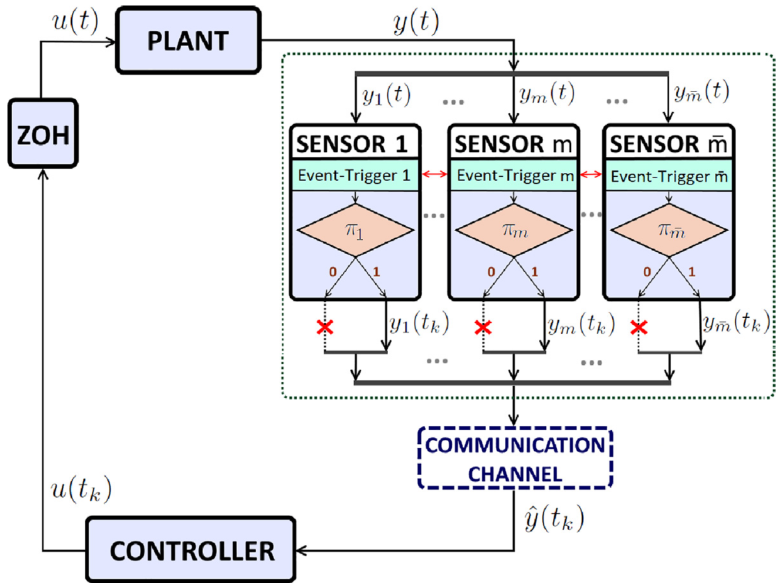

Fig. 1. Event-triggered sample-data control system.

where $\tau(t) = t - t_k$, $t \in [t_k, t_{k+1})$. Then the closed-loop system (1), (3) can be considered as a time-delay system (1), (8)–(10). Since the delay $\tau(t)$ is bounded only on the intervals $t \in [t_k, t_k + h)$, i.e., $0 \le \tau(t) < h$, we will use the representation (1), (8)–(10) only for $t \in [t_k, t_k + h)$. On $[t_k + h, t_{k+1})$, we will use the original form (1), (3), but represent the output $\hat{y}$ at time t_k as the output y at current time t with additional input error $\omega(t) = \left[\omega_1^{\mathrm{T}}(t), \dots, \omega_{\bar{m}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, i.e., $\hat{y}(t_k) = y(t) + \hat{y}(t_k) - y(t) = y(t) + \omega(t)$. Since the triggering condition (5) is not satisfied on the interval $[t_k + h, t_{k+1})$, we know that $\omega^{\mathrm{T}}(t)\,\Omega\,\omega(t) < \varepsilon\, y^{\mathrm{T}}(t)\,\Omega\, y(t)$, where $\Omega = \operatorname{diag}\{\Omega_1, \dots, \Omega_{\bar{m}}\}$ and $\varepsilon = \max\{\varepsilon_1, \dots, \varepsilon_{\bar{m}}\}$. This is a switching approach proposed in Selivanov and Fridman (2016). Also note that the functions $e_m(t)$ have a discontinuity at t_{k+1}, i.e., $e_m(t_{k+1}) = (1 - \pi_m(k))e_m(t_{k+1}^-) + C_m(x(t_k) - x(t_{k+1}))$. Here the superscript "-" denotes the left-side limit. Thus, the closed-loop system (1)–(3) should be considered as the stochastic impulsive system. The continuous dynamic is described by the following switching system

$$
\begin{cases}
\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} q_i \xi_i(t) + BKCx(t - \tau(t)) \\
\quad + \sum_{m=1}^{\bar{m}} (1 - \pi_m(k))BK_m e_m(t),\ t \in [t_k, t_k + h), \\
\dot{x}(t) = (A + BKC)x(t) + \sum_{i=1}^{N} q_i \xi_i(t) + BK\omega(t), \\
\qquad\qquad t \in [t_k + h, t_{k+1}). \\
\dot{e}(t) = 0, \quad t \in [t_k, t_{k+1}),
\end{cases}
\tag{11}
$$

where $\tau(t) = t - t_k$ and $e(t) = \left[e_1^{\mathrm{T}}(t), \dots, e_{\bar{m}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. The impulsive part is given by

$$
\begin{cases}
x(t_{k+1}) = x(t_{k+1}^-), \\
e_m(t_{k+1}) = (1 - \pi_m(k))e_m(t_{k+1}^-) \\
\qquad + C_m(x(t_k) - x(t_{k+1})),\ m = 1, \dots, \bar{m}.
\end{cases}
\tag{12}
$$

Note that the first system in (11) is described by a differential equation with time-delay, and hence, infinite-dimensional, i.e., its solution $x(t)$ at every point t belongs to the following infinite-dimensional state space.

Definition 1. The space of functions $f : [-h, 0] \to \mathbb{R}^n$ with the norm $\|f\|_{\mathbb{L}_2}^2 = \int_{-h}^{0} \|f(s)\|_2^2 ds$ is denoted by $\mathbb{L}_2$, where $\|\cdot\|_2$ is the Euclidean norm. The space of absolutely continuous functions $f : [-h, 0] \to \mathbb{R}^n$, which have square integrable first-order derivatives is denoted by $\mathbb{W}$ with the norm $\|f\|_{\mathbb{W}} = \max_{\theta \in [-h,0]} \|f(\theta)\|_2 + \|\frac{df}{d\theta}\|_{\mathbb{L}_2}$, see Fridman (2010).

Define $x_t(\theta) \in \mathbb{W}$ as $x_t(\theta) \triangleq x(t + \theta)$, where we assume

$$
x(t_0 + \theta) \equiv 0, \quad \theta \in [-h, 0). \tag{13}
$$

Definition 2. The closed-loop system (11)–(12) will be called h-exponentially mean-square stable with decay rate $\alpha > 0$ if there exists a $\gamma > 0$ such that for any initial condition x_{t_0} the corresponding solution of (11)–(12) satisfies the following inequalities for all $t \ge t_0$

$$
\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le \gamma\, e^{-2\alpha(t - (t_k - hk))} \mathbb{E}\left\{\|x_{t_0}\|_W^2 + \|e(t_0)\|_2^2\right\}, \tag{14}
$$

$$
\mathbb{E}\left\{\|e(t)\|_2^2\right\} \le \gamma\, e^{-2\alpha(t - (t_k - hk))} \mathbb{E}\left\{\|x_{t_0}\|_W^2 + \|e(t_0)\|_2^2\right\}, \tag{15}
$$

where the index k is defined from $t_k \le t < t_{k+1}$.

Remark 1. With the assumption $\hat{y}(t_{-1}) \equiv \mathbf{0}$ the initial value $e(t_0)$ is defined from $x(t_0)$ and (9). Also note that with (13) we have $\|x_{t_0}\|_W = \|x(t_0)\|_2$.

Remark 2. The h-exponential mean-square stability leads to asymptotic mean-square stability, i.e., $\lim_{t\to\infty} \mathbb{E}\left\{\|x(t)\|_2^2\right\} = 0$ and $\lim_{t\to\infty} \mathbb{E}\left\{\|e(t)\|_2^2\right\} = 0$, since $t - (t_k - hk) \to \infty$ as $t \to \infty$. Moreover, without ET, i.e., for $\varepsilon = 0$, the h-exponential mean-square stability coincides with the classical exponential mean-square stability, i.e., since $t_{k+1} - t_k = h$, (14) takes the form

$$
\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le \gamma e^{-2\alpha(t - t_0)}\, \mathbb{E}\left\{\|x_{t_0}\|_W^2 + \|e(t_0)\|_2^2\right\},
$$

for all $t \ge t_0$. The same is true for (15).

Remark 3. Under the assumption that the controller is directly connected to the actuator, we can consider dynamic output feedback instead of the static one. In this case, we obtain augmented system matrices and an augmented state, where the new state will contain the plant and controller states, see Liu et al. (2015) for the details.

3. Stability analysis based on Lyapunov–krasovskii technique

Denote for brevity $t_k' = t_k + h$. The proof of our main result is based on the following lemma.

Lemma 1. *Let there exist positive numbers $\gamma_1, \ldots \gamma_4$, positive definite $n_m \times n_m$ matrices U_m, Q_m satisfying*

$$U_m \le Q_m, \quad \forall m = 1, \ldots, \bar{m}, \tag{16}$$

and functionals $V_1 : \mathbb{R} \times \mathbb{W} \times \mathbb{L}_2 \to \mathbb{R}$ and $V_2 : \mathbb{R}^n \to \mathbb{R}$ such that

$$\gamma_1 \mathbb{E}\left\{\|\phi(0)\|_2^2\right\} \le \mathbb{E}\left\{V_1(t, \phi, \dot{\phi})\right\} \le \gamma_2 \mathbb{E}\left\{\|\phi\|_W^2\right\}, \tag{17}$$

$$\gamma_3\, \mathbb{E}\left\{\|z\|_2^2\right\} \le \mathbb{E}\left\{V_2(z)\right\} \le \gamma_4\, \mathbb{E}\left\{\|z\|_2^2\right\}. \tag{18}$$

Let the function $\bar{V}(t) = \begin{cases} V_1(t, x_t, \dot{x}_t), & t \in [t_k, t_k'), \\ V_2(x(t)), & t \in [t_k', t_{k+1}), \end{cases}$ be continuous from the right along (11)–(12), absolutely continuous for all $t \ne t_k$, $t \ne t_k'$, and satisfying

$$\begin{gathered} \mathbb{E}\left\{V_e(t_{k+1}) - V_e(t_k'^{-}) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_k) U_m e_m(t_k)\right\} \le 0, \\ \mathbb{E}\left\{V_e(t_k') - V_e(t_k'^{-})\right\} \le 0, \end{gathered} \tag{19}$$

where $V_e(t) = \bar{V}(t) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t) Q_m e_m(t)$.
Let for a given $\alpha > 0$

$$\mathbb{E}\left\{\mathcal{L}V_e(t) + 2\alpha V_e(t) - \frac{1}{h}\sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t) U_m e_m(t)\right\} \le 0, \tag{20}$$

along (11)–(12) for all $t \in (t_k, t_k')$, and

$$\mathbb{E}\left\{\mathcal{L}\bar{V}(t) + 2\alpha \bar{V}(t)\right\} \le 0, \tag{21}$$

along (11)–(12) for all $t \in (t_k', t_{k+1})$, $k = 0, 1, 2, \ldots$, where the infinitesimal operator $\mathcal{L}$ is defined as $\mathcal{L}V_e(t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}\left[\mathbb{E}\left\{V_e(t + \Delta) \mid t\right\} - V_e(t)\right]$. Then the closed-loop system (11)–(12) is h-exponentially mean-square stable with decay rate α.

Proof. See Appendix A.

Remark 4. Note that the system is deterministic on $[t_k', t_{k+1})$. Then the operator $\mathcal{L}\bar{V}(t)$ becomes the usual Lyapunov operator $\frac{dV}{dt}$ for $t \in [t_k', t_{k+1})$.

Remark 5. The conditions (20) and (21) guarantee that V_e does not increase on (t_k, t_k') and (t_k', t_{k+1}) along (11)–(12). The second inequality of (19) does not allow $V_e(t)$ to grow at jumps corresponding to t_k'. Note that V_e may grow in jumps at t_{k+1}. However, the first inequality of (19) guarantees that $\mathbb{E}\{V_e(t_{k+1})\} \le \mathbb{E}\left\{V_e(t_k'^{-})\right\}$, and hence, $\mathbb{E}\{V_e(t_{k+1})\} \le \mathbb{E}\{V_e(t_k)\}$, see Fig. 2.

Lemma 1 defines sufficient conditions for h-exponential mean-square stability of the system (11)–(12), where the main difficulty is to design an appropriate LK functional.

On the intervals $[t_k', t_{k+1})$ the closed-loop system is represented by the second model in (11). For its stability analysis consider the Lyapunov function V_2 as a quadratic form $V_2(x(t)) = x^{\mathrm{T}}(t) P x(t)$, where $P > 0$.

Proposition 1. *Let there exist $n_x \times n_x$ matrices $P > 0$, R_2, R_3, and positive real scalars ν_i, $i = 1, \ldots, N$, such that the LMI*

$$\Psi_0 + \Psi_1 \le 0 \tag{22}$$

(see Table 1) is feasible. Then the conditions (18) and (21) of Lemma 1 are satisfied.

Proof. See Appendix B.

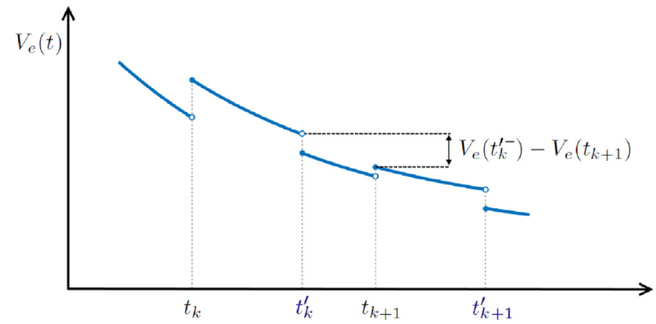

Fig. 2. Discontinuous in time LK functional.

On the intervals $[t_k, t_k')$ the closed-loop system is represented by the time-delay system described by the first equation in (11). For the stability analysis consider the following LK functional $V_1(t, x_t, \dot{x}_t) = V(t, x_t, \dot{x}_t) + V_G(t, \dot{x}_t)$, with $V_G(t, \dot{x}_t) = \sum_{m=1}^{\bar{m}} h \int_{t_k}^{t} e^{2\alpha(s-t)} \dot{x}^{\mathrm{T}}(s) C_m^{\mathrm{T}} G_m C_m \dot{x}(s) ds$ that was proposed in Liu et al. (2015) to cope with the reset conditions (12), and

$$V(t, x_t, \dot{x}_t) = x^{\mathrm{T}}(t) P x(t) + V_Q(t, \dot{x}_t) + V_R(t, x_t), \tag{23}$$

where $V_Q(t, \dot{x}_t) = (h - \tau(t)) \int_{-\tau(t)}^{0} e^{2\alpha s}\, \dot{x}_t^{\mathrm{T}}(s)\, Q\, \dot{x}_t(s) ds$, $V_R(t, x_t) = (h - \tau(t))\, \zeta^{\mathrm{T}}(t, x_t) R\, \zeta(t, x_t)$, $R = \begin{bmatrix} \frac{X + X^{\mathrm{T}}}{2} & -X + X_1 \\ * & -X_1 - X_1^{\mathrm{T}} + \frac{X + X^{\mathrm{T}}}{2} \end{bmatrix}$, and the matrix P is as above, $Q > 0$, X, X_1 are some $n_x \times n_x$ matrices, the vector $\zeta(t, x_t) = \left[x_t^{\mathrm{T}}(0), x_t^{\mathrm{T}}(-\tau(t))\right]^{\mathrm{T}}$. The functional (23) was introduced in Fridman (2010) and applied to nonlinear systems in Seifullaev and Fradkov (2016).

Proposition 2. *Let there exist $n_x \times n_x$ matrices $P > 0$, X, and X_1, such that the LMI*

$$\Theta = \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + hR > 0 \tag{24}$$

is feasible. Then the condition (17) of Lemma 1 is satisfied.

Proof. See Appendix C.

Proposition 3. *Let there exist $n_m \times n_m$ matrices $Q_m > 0$, $U_m > 0$, and $G_m > 0$, such that the LMIs*

$$\Theta_m \le 0, \quad m = 1, \ldots, \bar{m}, \tag{25}$$

where $\Theta_m = \begin{bmatrix} -\beta_m Q_m + U_m & (1 - \beta_m) Q_m \\ * & Q_m - \frac{1}{\epsilon} e^{-2\alpha h} G_m \end{bmatrix}$, $\epsilon = \max\{\epsilon_1, \ldots, \epsilon_{\bar{m}}\}$, are feasible. Then the conditions (16) and (19) of Lemma 1 are satisfied.

Proof. See Appendix D.

Proposition 4. *Let there exist $n_x \times n_x$ matrices $P > 0$, $Q > 0$, P_2, P_3, X, X_1, Z, Y_1, Y_2, $Y_3^{(i)}$, $n_m \times n_m$ matrices $Q_m > 0$, $U_m > 0$, $G_m > 0$, and positive real scalars κ_i^- and κ_i^+, $i = 1, \ldots, N$, $m = 1, \ldots, \bar{m}$, such that the LMIs*

$$\Phi_0' + \Phi_0'' \le 0, \quad \Phi_1' + \Phi_1'' \le 0, \tag{26}$$

where $\Phi_1' = \Phi(\tau)_{|\tau=h}$, and $\Phi_0' = \Phi(\tau)_{|\tau=0}$ (with elimination the last zero raw and column), and Φ_0'' is obtained from Φ_1'' by replacing κ_i^+ to κ_i^- and eliminating the last zero row and column (see Table 1), are feasible. Then the condition (20) of Lemma 1 is satisfied.

Proof. See Appendix E.

Table 1
Matrices in Propositions 1 and 4.

$$\Psi_0 = \begin{bmatrix} \Psi_{11} & \Psi_{12} & R_2^{\mathrm{T}} q_1 & \dots & R_2^{\mathrm{T}} q_N & R_2^{\mathrm{T}} BK \\ * & -R_3 - R_3^{\mathrm{T}} & R_3^{\mathrm{T}} q_1 & \dots & R_3^{\mathrm{T}} q_N & R_3^{\mathrm{T}} BK \\ * & * & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & 0 & \dots & 0 & 0 \\ * & * & * & \dots & * & -\Omega \end{bmatrix},$$

$$\Psi_{11} = A_{\mathrm{cl}}^{\mathrm{T}} R_2 + R_2^{\mathrm{T}} A_{\mathrm{cl}} + 2\alpha P + \varepsilon\, C^{\mathrm{T}} \Omega\, C,$$

$$\Psi_1 = \begin{bmatrix} \bar{\Psi}_{11} & 0 & \bar{\Psi}_{13}^{(1)} & \dots & \bar{\Psi}_{13}^{(N)} & 0 \\ * & 0 & 0 & \dots & 0 & 0 \\ * & * & \bar{\Psi}_{33}^{(1)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & 0 & \dots & \bar{\Psi}_{33}^{(N)} & 0 \\ * & * & * & \dots & * & 0 \end{bmatrix}, \quad \begin{aligned} &\Psi_{12} = P - R_2^{\mathrm{T}} + A_{\mathrm{cl}}^{\mathrm{T}} R_3, \\ &A_{\mathrm{cl}} = A + BKC, \\ &\bar{\Psi}_{11} = -\sum_{i=1}^{N} \nu_i \mu_i^- \mu_i^+ r_i r_i^T, \\ &\bar{\Psi}_{13}^{(i)} = \tfrac{1}{2} \nu_i (\mu_i^- + \mu_i^+) r_i, \\ &\bar{\Psi}_{33}^{(i)} = -\nu_i, \end{aligned}$$

$$\Phi(\tau) = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14}^{(1)} & \dots & \Phi_{14}^{(N)} & \Phi_{15}^{(1)} & \dots & \Phi_{15}^{(\bar{m})} & \Phi_{16} \\ * & \Phi_{22} & \Phi_{23} & \Phi_{24}^{(1)} & \dots & \Phi_{24}^{(N)} & \Phi_{25}^{(1)} & \dots & \Phi_{25}^{(\bar{m})} & \Phi_{26} \\ * & * & \Phi_{33} & \Phi_{34}^{(1)} & \dots & \Phi_{34}^{(N)} & 0 & \dots & 0 & \Phi_{36} \\ * & * & * & 0 & \dots & 0 & 0 & \dots & 0 & \Phi_{46}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \dots & 0 & 0 & \dots & 0 & \Phi_{46}^{(N)} \\ * & * & * & * & \dots & * & \Phi_{55}^{(1)} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & * & \dots & * & * & \dots & \Phi_{55}^{(\bar{m})} & 0 \\ * & * & * & * & \dots & * & * & \dots & * & \Phi_{66} \end{bmatrix},$$

$$\Phi_{11}(\tau) = A^{\mathrm{T}} P_2 + P_2^{\mathrm{T}} A + 2\alpha P - Y_1 - Y_1^{\mathrm{T}} - (1 - 2\alpha(h-\tau)) \frac{X + X^{\mathrm{T}}}{2},$$

$$\Phi_{12}(\tau) = P - P_2^{\mathrm{T}} + A^{\mathrm{T}} P_3 - Y_2 + (h-\tau) \frac{X + X^{\mathrm{T}}}{2},$$

$$\Phi_{13}(\tau) = Y_1^{\mathrm{T}} + P_2^{\mathrm{T}} BKC^{\mathrm{T}} - Z + (1 - 2\alpha(h-\tau))(X - X_1),$$

$$\Phi_{22}(\tau) = -P_3 - P_3^{\mathrm{T}} + (h-\tau) Q + h \sum_{m=1}^{\bar{m}} C_m^{\mathrm{T}} G_m C_m,$$

$$\Phi_{23}(\tau) = Y_2^{\mathrm{T}} + P_3^{\mathrm{T}} BKC^{\mathrm{T}} - (h-\tau)(X - X_1),$$

$$\Phi_{33}(\tau) = Z + Z^{\mathrm{T}} - (1 - 2\alpha(h-\tau)) \frac{X + X^{\mathrm{T}} - 2X_1 - 2X_1^{\mathrm{T}}}{2},$$

$$\Phi_{14}^{(i)} = P_2^{\mathrm{T}} q_i - Y_3^{(i)} q_i, \quad \Phi_{24}^{(i)} = P_3^{\mathrm{T}} q_i, \quad \Phi_{34}^{(i)} = Y_3^{(i)} q_i,$$

$$\Phi_{15}^{(m)} = (1 - \beta_m) P_2^{\mathrm{T}} BK_m, \quad \Phi_{25}^{(m)} = (1 - \beta_m) P_3^{\mathrm{T}} BK_m,$$

$$\Phi_{55}^{(m)} = 2\alpha Q_m - \frac{1}{h} U_m, \quad \Phi_{16}(\tau) = \tau Y_1^{\mathrm{T}}, \quad \Phi_{26}(\tau) = \tau Y_2^{\mathrm{T}},$$

$$\Phi_{36}(\tau) = \tau Z^{\mathrm{T}}, \quad \Phi_{46}^{(i)}(\tau) = \tau q_i^{\mathrm{T}} Y_3^{(i)^{\mathrm{T}}}, \quad \Phi_{66}(\tau) = -\tau Q e^{-2\alpha h},$$

$$\Phi_1'' = \begin{bmatrix} \bar{\Phi}_{11} & 0 & 0 & \bar{\Phi}_{14}^{(1)} & \dots & \bar{\Phi}_{14}^{(N)} & 0 & \dots & 0 & 0 \\ * & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ * & * & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ * & * & * & \bar{\Phi}_{44}^{(1)} & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & 0 \\ * & * & * & 0 & \dots & \bar{\Phi}_{44}^{(N)} & 0 & \dots & 0 & 0 \\ * & * & * & * & \dots & * & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & 0 \\ * & * & * & * & \dots & * & * & \dots & 0 & 0 \\ * & * & * & * & \dots & * & * & \dots & * & 0 \end{bmatrix}, \quad \begin{aligned} &\bar{\Phi}_{11} = -\sum_{i=1}^{N} \kappa_i^+ \mu_i^- \mu_i^+ r_i r_i^T, \\ &\bar{\Phi}_{14}^{(i)} = \tfrac{1}{2} \kappa_i^+ (\mu_i^- + \mu_i^+) r_i, \\ &\bar{\Phi}_{44}^{(i)} = -\kappa_i^+, \end{aligned}$$

Thus, Propositions 1–4 determine the sufficient conditions for the fulfillment of Lemma 1. Therefore, we immediately arrive to our main result, which provides sufficient conditions for h-exponential mean-square stability.

Theorem 1. *Given $h, \alpha > 0$, $0 < \beta_m \le 1$, $\varepsilon \ge 0$, $\epsilon \ge 1$, and $n_m \times n_m$ matrices $\Omega_m \ge 0$, $G_m \ge 0$. Let there exist $n_x \times n_x$ matrices $P > 0$, $Q > 0$, P_2, P_3, R_2, R_3, X, X_1, Z, Y_1, Y_2, $Y_3^{(i)}$, $n_m \times n_m$ matrices $Q_m > 0$, $U_m > 0$, $G_m > 0$, and positive real scalars ν_i, κ_i^- and κ_i^+, $i = 1, \dots, N$, $m = 1, \dots, \bar{m}$, such that the LMIs (22), (24)–(26) are feasible. Then the closed-loop system (11)–(12) is h-exponentially mean-square stable with decay rate α.*

Remark 6. Since the matrix Φ_0' in (26) is defined as $\Phi(\tau)$ for $\tau = 0$, we obtain that the exponential stability of the system where the sensors communicate at all time instants with the controller is a necessary condition for the LMIs feasibility. Thus, the controller should be designed such that the system with continuous feedback is exponentially stable. Then we can use Theorem 1 to determine the parameters of the event-trigger and the probabilities β_m for which the system remains exponentially (mean-square) stable. In particular, a maximum allowable period $h_{\max}$ can be found by increasing the parameter h until the LMIs become infeasible.

Remark 7. Without packet losses, i.e., for $\beta_m = 1$, the terms $\Phi_{15}^{(m)}$ and $\Phi_{25}^{(m)}$ disappear. Hence, the matrices U_m and Q_m can be chosen sufficiently small such that (25) is feasible for all (even large) $\epsilon > 1$, and the LMIs (26) coincide with those in Seifullaev et al. (2021). Thus, the feasibility of (22), (24), and (26) with $\beta_m = 1$ guarantees exponential stability for the event-triggered control with no packet losses. Moreover, for periodic sampling, i.e., $\varepsilon = 0$, the exponential stability follows from (24) and (26), see Seifullaev and Fradkov (2016). If, in addition, the system is linear, then the LMIs are equivalent to those in Fridman (2010).

4. Numerical example

To illustrate the efficiency of the proposed approach, we consider the model describing rotations of a single-link pendulum

$$\begin{aligned} &\ddot{\varphi}(t) = \frac{g}{l} \sin \varphi(t) - \frac{\varkappa}{l} \dot{\varphi}(t) + \frac{1}{Ml^2} u(t), \\ &u(t) = -K \hat{y}(t_k), \quad y(t) = \varphi(t), \end{aligned} \tag{27}$$

where M and l are the mass and the length of the pendulum, respectively, g is the gravity acceleration, $\varkappa$ is the viscous friction, φ is the deviation angle from the vertical, u is the control torque, and the control objective is to stabilize the pendulum at the upper position, i.e., $\varphi = 0$. The system (27) can be rewritten in the matrix form (1) with $n_x = 2$, $\bar{m} = n_y = n_1 = N = 1$, and

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \varphi(t) \\ \dot{\varphi}(t) \end{bmatrix}, \; A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{\varkappa}{l} \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ \frac{1}{Ml^2} \end{bmatrix},$$

$$C^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; q_1 = \begin{bmatrix} 0 \\ \frac{g}{l} \end{bmatrix}, \; r_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; \xi_1(t) = \sin \sigma_1(t).$$

Since $-0.2173\,\sigma_1^2 \le \sigma_1 \sin \sigma_1 \le \sigma_1^2$ for all σ_1, the nonlinear function $\xi_1(t)$ satisfies the sector-bound inequality (2) for all $t \ge 0$ with $\mu_1^+ = 1$, $\mu_1^- = -0.2173$. Let $M = 1$ kg, $l = 2$ m, $\varkappa = 8$ N/m, $g = 9.8$ m/s^2. Note that the condition $K > 19.6$ is necessary, since otherwise the closed-loop system has more than one equilibrium and, hence, cannot be globally stable. Without packet losses, i.e., $\beta_1 = 1$, by iteratively increasing K and ε and solving the LMIs in Theorem 1, we obtain the maximum allowable periods $h_{\max}$ illustrated in Fig. 3. To reduce the amount of sent measurements (SM), it is reasonable to choose the maximum possible values of h and ε. However, we can see that increasing ε leads to decreasing $h_{\max}$ and vise versa. Numerous simulations have shown that the average amount of SM is minimal when

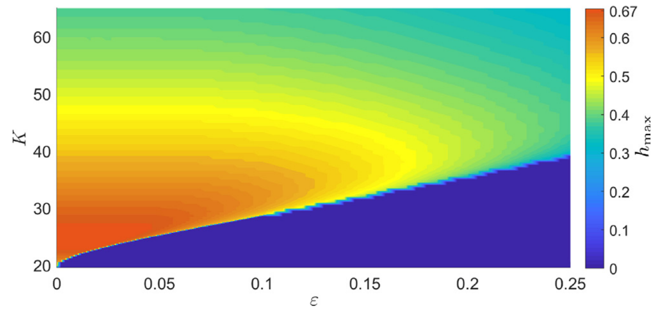

Fig. 3. The dependence of $h_{\max}$ on K and ε. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

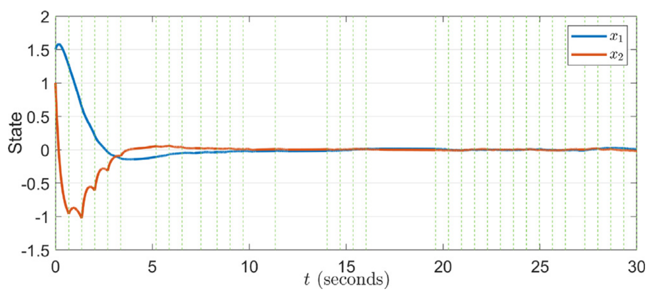

Fig. 4. The solutions of (27) with an additive Gaussian noise (with zero mean and standard deviation 0.1). Vertical green dashed lines denote the time instants t_k generated by the event-trigger (7), $h_{\max} = 0.665$, $\varepsilon = 0.044$, $\beta_1 = 1$.

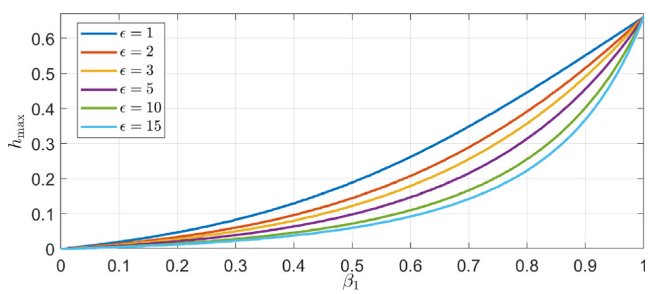

Fig. 5. The dependence of $h_{\max}$ on β_1 and ϵ for $\varepsilon = 0.044$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

priority is given to maximizing $h_{\max}$ rather than ε. Since for each fixed K the dependence of $h_{\max}$ on ε is flat up to a certain value and then $h_{\max}$ rapidly decreases (see the sharp transition between the red and blue areas in Fig. 3), it is reasonable to choose the maximum possible h first, and then increase ε until h drops down. As a result, we choose $K = 26.5$ and $\varepsilon = 0.044$ providing $h_{\max} = 0.665$. Also note that for $\beta_1 = 1$, the feasibility of the LMIs does not depend on ϵ, hence, we can choose it arbitrarily large. Fig. 4 illustrates the solution of the system (27) with the initial conditions $x_0 = [1.5,\ 1]^{\mathrm{T}}$, where measurement and process Gaussian noises were added to the model to demonstrate the robustness of the approach. We can see that the event-trigger resulted in the avoidance of unnecessary transmissions: the sensor sent only 34 measurements, while it would perform 45 transmissions with periodic sampling.

Now we assume packet dropouts and start varying β_1. Fig. 5 illustrates the dependence of $h_{\max}$ on β_1 for different ϵ. Let $\beta_1 = 0.8$ and $\epsilon = 2$, which give $h_{\max} = 0.395$. The corresponding solution is illustrated in Fig. 6. We can see that with the ET the

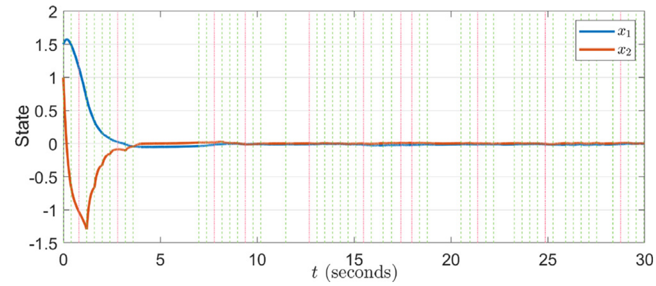

Fig. 6. The solutions of (27). Red vertical dotted lines denote the instants t_k when the packet is not delivered, $h_{\max} = 0.395$, $\varepsilon = 0.044$, $\epsilon = 2$, $\beta_1 = 0.8$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

number of SM (the number of green lines) is 44, while for periodic sampling[1] the average number of SM is 56.

5. Conclusions

In this paper, we have developed appropriate Lyapunov methods and obtained mean-square stability conditions for nonlinear systems with event-triggered sampled-data control and intermittent packet transmissions. The closed-loop system is considered as a stochastic impulsive system where the continuous part is given as a switching system between two continuous dynamics. The main challenge in deriving stability conditions is that the Lyapunov function can grow in jumps at sampling instants requiring a special analysis of its behavior. The numerical example shows how the event-triggered control can reduce the amount of information transmitted over the communication channel. An extension of theoretical results in the case of the presence of process and measurement noise can be a topic of future work.

Appendix A. Proof of Lemma 1

Since $e_m(t) = e_m(t_k)$ for $t \in [t_k, t_{k+1})$, from (20) we have

$$\begin{aligned}
&\mathbb{E}\left\{V_e(t)\right\} \le e^{-2\alpha(t-t_k)}\,\mathbb{E}\left\{V_e(t_k)\right\} \\
&+ \frac{1}{h}\int_{t_k}^{t} e^{-2\alpha(t-s)}ds \sum_{m=1}^{\bar{m}} \mathbb{E}\left\{e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\} \\
&\le e^{-2\alpha(t-t_k)}\,\mathbb{E}\left\{V_e(t_k)\right\} \\
&+ \sum_{m=1}^{\bar{m}} \mathbb{E}\left\{e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\}, \quad t \in [t_k, t_k'),
\end{aligned} \tag{A.1}$$

where the last inequality follows from $\int_{t_k}^{t} e^{-2\alpha(t-s)}ds \le h$, $t \in [t_k, t_k')$. Then taking into account (16) we conclude

$$\mathbb{E}\left\{\bar{V}(t)\right\} \le e^{-2\alpha(t-t_k)}\,\mathbb{E}\left\{V_e(t_k)\right\},\ t \in [t_k, t_k'). \tag{A.2}$$

From (21) we directly get

$$\mathbb{E}\left\{\bar{V}(t)\right\} \le e^{-2\alpha(t-t_k')}\,\mathbb{E}\left\{\bar{V}(t_k')\right\},\ t \in [t_k', t_{k+1}). \tag{A.3}$$

The first inequality of (19) gives $\mathbb{E}\left\{V_e(t_k)\right\} \le \mathbb{E}\left\{V_e(t_{k-1}'^{-})\right\} - \sum_{m=1}^{\bar{m}} \mathbb{E}\left\{e_m^{\mathrm{T}}(t_{k-1})U_m e_m(t_{k-1})\right\}$. Then from (A.1) with $t = t_{k-1}'^{-}$ and $t_{k-1}' - t_{k-1} = h$ we get $\mathbb{E}\left\{V_e(t_k)\right\} \le e^{-2\alpha h}\,\mathbb{E}\left\{V_e(t_{k-1})\right\}$. Continuing by the same way, we obtain

$$\mathbb{E}\left\{V_e(t_k)\right\} \le e^{-2\alpha h k}\,\mathbb{E}\left\{V_e(t_0)\right\}. \tag{A.4}$$

[1] In the case of periodic sampling, we can take $\epsilon = 1$, and then use the blue curve in Fig. 5 instead of the red one. Thus, for $\beta_1 = 0.8$ we obtain $h_{\max} = 0.44$. Note that with the ET and $\epsilon = 2$, we obtain a smaller sampling value, $h_{\max} = 0.395$. However, the number of transmissions is still reduced.

• From the first inequality in (17) and (A.2) it follows that for $t \in [t_k, t_k')$, $\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le \frac{1}{\gamma_1}\mathbb{E}\left\{\bar{V}(t)\right\} \le \frac{1}{\gamma_1}e^{-2\alpha(t-t_k)}\mathbb{E}\left\{V_e(t_k)\right\}$. Then using (A.4) we obtain

$$\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le b_1 e^{-2\alpha(t-(t_k-hk))}\mathbb{E}\left\{V_e(t_0)\right\}, \quad t \in [t_k, t_k'), \tag{A.5}$$

for some $b_1 > 0$. The second inequality of (17) implies

$$\begin{aligned}\mathbb{E}\left\{V_e(t_0)\right\} &= \mathbb{E}\left\{V_1(t_0, x_{t_0}, \dot{x}_{t_0}) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_0)Q_m e_m(t_0)\right\} \\ &\le b_2\mathbb{E}\left\{\|x_{t_0}\|_W^2 + \|e(t_0)\|_2^2\right\}, \; b_2 > 0.\end{aligned} \tag{A.6}$$

• Similarly for $t \in [t_k', t_{k+1})$ from the first inequality in (18) and (A.3) we have $\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le \frac{1}{\gamma_3}\mathbb{E}\left\{\bar{V}(t)\right\} \le \frac{1}{\gamma_3}e^{-2\alpha(t-t_k')}\mathbb{E}\left\{\bar{V}(t_k')\right\}$. Since $e(t_k'^-) = e(t_k') = e(t_k)$, the second inequality in (19) yields $\mathbb{E}\left\{\bar{V}(t_k')\right\} \le \mathbb{E}\left\{\bar{V}(t_k'^-)\right\}$. Hence $\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le \frac{1}{\gamma_3}e^{-2\alpha(t-t_k')}\mathbb{E}\left\{\bar{V}(t_k'^-)\right\}$. Next, applying (A.2) for $t = t_k'^-$ and then (A.4), for some $b_3 > 0$ we obtain

$$\mathbb{E}\left\{\|x(t)\|_2^2\right\} \le b_3 e^{-2\alpha(t-(t_k-hk))}\mathbb{E}\left\{V_e(t_0)\right\}, t \in [t_k', t_{k+1}). \tag{A.7}$$

• Since $e_m(t) = e_m(t_k)$ for $t \in [t_k, t_{k+1})$, from (A.1) we have $\sum_{m=1}^{\bar{m}}\mathbb{E}\left\{e_m^{\mathrm{T}}(t)(Q_m - U_m)e_m(t)\right\} + \mathbb{E}\left\{\bar{V}(t)\right\} \le e^{-2\alpha(t-t_k)}\mathbb{E}\left\{V_e(t_k)\right\}$, $t \in [t_k, t_k')$. Then from the positivity of $\bar{V}(t)$ and (16) it follows that $\mathbb{E}\left\{\|e(t)\|_2^2\right\} \le b_4 e^{-2\alpha(t-t_k)}\mathbb{E}\left\{V_e(t_k)\right\}, b_4 > 0$. Finally, from (A.4) we find that for some $b_5 > 0$

$$\mathbb{E}\left\{\|e(t)\|_2^2\right\} \le b_5 e^{-2\alpha(t-(t_k-hk))}\mathbb{E}\left\{V_e(t_0)\right\}, t \in [t_k, t_{k+1}). \tag{A.8}$$

• Combining (A.5), (A.7), (A.8), and (A.6) we arrive to (14), (15) with $\gamma = \max\{b_1b_2, b_3b_2, b_5b_2\}$, which imply the h-exponential mean-square stability of (11)–(12).

Appendix B. Proof of Proposition 1

Since V_2 is a quadratic form, the inequalities in (18) are satisfied with $\gamma_3 = \lambda_{\min}(P)$ and $\gamma_4 = \lambda_{\max}(P)$, the minimal and maximal eigenvalues of a matrix P, respectively. Next, we will derive the conditions implying (21). By direct calculations we have

$$\mathcal{L}V_2(t) + 2\alpha V_2(t) = 2x^{\mathrm{T}}(t)P\dot{x}(t) + 2\alpha x^{\mathrm{T}}(t)Px(t). \tag{B.1}$$

Next, we use a free-weighting matrices technique. Along the solutions of (11) the following equality holds

$$\begin{aligned}&2\left[x^{\mathrm{T}}(t)R_2^{\mathrm{T}} + \dot{x}^{\mathrm{T}}(t)R_3^{\mathrm{T}}\right] \times \Bigg[(A+BKC)x(t) \\ &+ \sum_{i=1}^{N} q_i\xi_i(t) + BK\omega(t) - \dot{x}(t)\Bigg] = 0,\end{aligned} \tag{B.2}$$

on $[t_k', t_{k+1})$, where R_2, R_3 are free-weighting $n_x \times n_x$ matrices. From (5) we have on $[t_k + h, t_{k+1})$

$$\varepsilon x^{\mathrm{T}}(t)C^{\mathrm{T}}\Omega C x(t) - \omega^{\mathrm{T}}(t)\Omega\,\omega(t) \ge 0. \tag{B.3}$$

Then we can add the left-hand sides of (B.2) and (B.3) to the right-hand side of (B.1), and taking expectation on both sides of (B.1) we get $\mathbb{E}\left\{\mathcal{L}V_2(t) + 2\alpha V_2(t)\right\} \le \mathbb{E}\left\{\eta^{\mathrm{T}}(t)\Psi_0\,\eta(t)\right\}$, where $\eta(t) = \left[x^{\mathrm{T}}(t), \dot{x}^{\mathrm{T}}(t), \xi_1(t), \ldots, \xi_N(t), w^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. The inequalities (2) can be rewritten as $F_i(\eta(t)) \triangleq \left(\xi_i(t) - \mu_i^- r_i^{\mathrm{T}}x(t)\right)\left(\mu_i^+ r_i^{\mathrm{T}}x(t) - \xi_i(t)\right) \ge 0$. Thus, for (21) it is sufficient to require $\eta^{\mathrm{T}}(t)\Psi_0\,\eta(t) \le 0$ for $\eta(t)$ satisfying $F_i(\eta(t)) \ge 0$ for all $i = 1, \ldots, N$. This condition can be rewritten with S-procedure: $\eta^{\mathrm{T}}(t)\Psi_0\,\eta(t) + \sum_{i=1}^{N}\nu_i F_i(\eta(t)) \le 0$ for some positive scalars ν_i, $i = 1, \ldots, N$. which corresponds to the LMI $\Psi_0 + \Psi_1 \le 0$ leading to (21).

Appendix C. Proof of Proposition 2

For the first inequality of (17) it is sufficient to require $\Theta > 0$. Indeed, $\mathbb{E}\left\{x_t(0)^{\mathrm{T}}Px_t(0) + V_R(t, x_t)\right\} = \frac{h-\tau(t)}{h} \times \mathbb{E}\left\{\zeta^{\mathrm{T}}(t, x_t)\Theta\zeta(t, x_t)\right\} + \frac{\tau(t)}{h}\mathbb{E}\left\{\zeta^{\mathrm{T}}(t, x_t)\Theta_{|h=0}\zeta(t, x_t)\right\} \ge \gamma_1\mathbb{E}\left\{\|x_t(0)\|^2\right\}$, where $\gamma_1 = \min\left\{\lambda_{\min}(P), \lambda_{\min}(\Theta)\right\}$. Similarly, we obtain that $\mathbb{E}\left\{x_t(0)^{\mathrm{T}}Px_t(0) + V_R(t, x_t)\right\} \le \tilde{\gamma}_2\mathbb{E}\left\{\max_{\theta\in[-h,0]}\|x_t(\theta)\|_2\right\}$, where $\tilde{\gamma}_2 = \lambda_{\max}(P) + \lambda_{\max}(\Theta)$. Moreover, since for any matrix $M > 0$ $\int_{-\tau(t)}^{0} e^{2\alpha s}\dot{x}_t^{\mathrm{T}}(s)M\dot{x}_t(s)ds \le \int_{-h}^{0} e^{2\alpha s}\dot{x}_t^{\mathrm{T}}(s)M\dot{x}_t(s)ds \le \int_{-h}^{0}\dot{x}_t^{\mathrm{T}}(s)M\dot{x}_t(s)ds \le \lambda_{\max}(M)\int_{-h}^{0}\|\dot{x}_t(s)\|_2^2 ds$, then we obtain that $\mathbb{E}\left\{V_G(t, \dot{x}_t) + V_Q(t, \dot{x}_t)\right\} \le \bar{\gamma}_2\mathbb{E}\left\{\int_{-h}^{0}\|\dot{x}_t(s)\|_2^2 ds\right\}$, where $\bar{\gamma}_2 = h\lambda_{\max}(Q) + h\sum_{m=1}^{\bar{m}}\lambda_{\max}(C_m^{\mathrm{T}}G_mC_m)$. Therefore, the second inequality of (17) is fulfilled with $\gamma_2 = \max\{\tilde{\gamma}_2, \bar{\gamma}_2\}$.

Appendix D. Proof of Proposition 3

We will start with the derivation of the conditions for (19). Note that since $\tau(t_k) = 0$ and $\tau(t_k'^-) = h$, we obtain $V_Q(t_k, \dot{x}_{t_k}) = V_Q(t_k', \dot{x}_{t_k'}) = V_R(t_k, x_{t_k}) = V_R(t_k', x_{t_k'}) = 0$. Also note that $V_Q(t_k, \dot{x}_{t_k}) = 0$. Consider two possible cases: $t_{k+1} > t_k'$ and $t_{k+1} = t_k'$, i.e., $t_{k+1} - t_k = h$.

• For $t_{k+1} > t_k'$ the left-hand side of the first inequality (19) takes the following form:
$\mathbb{E}\left\{V_e(t_{k+1}) - V_e(t_k'^-) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\}$
$= \mathbb{E}\left\{x^{\mathrm{T}}(t_{k+1})Px(t_{k+1}) - x^{\mathrm{T}}(t_k')Px(t_k')\right\}$
$+ \sum_{m=1}^{\bar{m}}\mathbb{E}\left\{e_m^{\mathrm{T}}(t_{k+1})(Q_m)e_m(t_{k+1})\right\} - \mathbb{E}\left\{V_{G\,|\,t=t_k'^-}\right\}$
$+ \sum_{m=1}^{\bar{m}}\mathbb{E}\left\{e_m^{\mathrm{T}}(t_k)(U_m - Q_m)e_m(t_k)\right\}$. Since $\bar{V}(t) = V_2(x(t))$ on $t \in [t_k', t_{k+1})$, the condition (21) yields $\mathbb{E}\left\{x^{\mathrm{T}}(t_{k+1})Px(t_{k+1}) - x^{\mathrm{T}}(t_k')Px(t_k')\right\} \le 0$. By direct calculations we obtain $\mathbb{E}\left\{e_m^{\mathrm{T}}(t_{k+1})(Q_m)e_m(t_{k+1})\right\}$
$= \mathbb{E}\Big\{\left[(1-\pi_m(k))e_m(t_k) + C_m(x(t_{k+1}) - x(t_k))\right]^{\mathrm{T}} Q_m$
$\times \left[(1-\pi_m(k))e_m(t_k) + C_m(x(t_{k+1}) - x(t_k))\right]\Big\}$
$= \mathbb{E}\left\{(1-\beta_m)e_m^{\mathrm{T}}(t_k)Q_m e_m(t_k)\right\}$
$+ \mathbb{E}\left\{2(1-\beta_m)e_m^{\mathrm{T}}(t_k)Q_mC_m(x(t_{k+1}) - x(t_k))\right\}$
$+ \mathbb{E}\left\{\left[C_m(x(t_{k+1}) - x(t_k))\right]^{\mathrm{T}} Q_mC_m(x(t_{k+1}) - x(t_k))\right\}$ for $m = 1, \ldots, \bar{m}$, and
$\mathbb{E}\left\{V_{G\,|\,t=t_k'^-}\right\} \ge he^{-2\alpha h}\sum_{m=1}^{\bar{m}}\int_{t_k}^{t_k'}\mathbb{E}\{\dot{x}^{\mathrm{T}}(s)C_m^{\mathrm{T}}G_m$
$\times C_m\dot{x}(s)\}ds \ge e^{-2\alpha h}\sum_{m=1}^{\bar{m}}\mathbb{E}\Big\{\left[C_m(x(t_k') - x(t_k))\right]^{\mathrm{T}} G_m$
$\times C_m(x(t_k') - x(t_k))\Big\}$, where the latter follows from Jensen's inequality, see, e.g., Gu, Kharitonov, and Chen (2003). Since we consider the case $t_{k+1} > t_k'$, from (6) we have $(C_m(x(t) - x(t_k)))^{\mathrm{T}}G_m\,C_m(x(t) - x(t_k)) \le \epsilon_m c_{k,m}$ on $[t_k', t_{k+1})$, and substituting $t = t_{k+1}^-$, we get $-\mathbb{E}\left\{V_{G\,|\,t=t_k'^-}\right\} \le -\frac{1}{\epsilon}e^{-2\alpha h}\sum_{m=1}^{\bar{m}}\mathbb{E}\Big\{(x(t_{k+1}) - x(t_k))^{\mathrm{T}} \times C_m^{\mathrm{T}}G_mC_m(x(t_{k+1}) - x(t_k))\Big\}$.

Thus, $\mathbb{E}\left\{V_e(t_{k+1}) - V_e(t_k'^-) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\} \le \sum_{m=1}^{\bar{m}}\mathbb{E}\left\{\theta_m^{\mathrm{T}}(k)\Theta_m\theta_m(k)\right\}$, where $\theta_m(k) = \left[e_m^{\mathrm{T}}(t_k), \left[C_m(x(t_{k+1}) - x(t_k))\right]^{\mathrm{T}}\right]^{\mathrm{T}}$, $m = 1, \ldots, \bar{m}$.

• Consider now the case $t_{k+1} = t_k'$. Then
$\mathbb{E}\left\{V_e(t_{k+1}) - V_e(t_{k+1}^-) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\}$
$= \sum_{m=1}^{\bar{m}}\mathbb{E}\left\{e_m^{\mathrm{T}}(t_{k+1})(Q_m)e_m(t_{k+1})\right\} - \mathbb{E}\left\{V_{G\,|\,t=t_{k+1}^-}\right\}$
$+ \sum_{m=1}^{\bar{m}}\mathbb{E}\left\{e_m^{\mathrm{T}}(t_k)(U_m - Q_m)e_m(t_k)\right\}$ and, similarly, $-\mathbb{E}\left\{V_{G\,|\,t=t_{k+1}^-}\right\}$
$\le -e^{-2\alpha h} \times \sum_{m=1}^{\bar{m}}\mathbb{E}\Big\{(x(t_{k+1}) - x(t_k))^{\mathrm{T}}C_m^{\mathrm{T}}G_mC_m(x(t_{k+1}) - x(t_k))\Big\}$.

Hence, $\mathbb{E}\left\{V_e(t_{k+1}) - V_e(t_{k+1}^-) + \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t_k)U_m e_m(t_k)\right\} \leq \sum_{m=1}^{\bar{m}} \mathbb{E}\left\{\theta_m^{\mathrm{T}}(k)\tilde{\Theta}_m\theta_m(k)\right\}$, where the matrices $\tilde{\Theta}_m$ are obtained from Θ_m by eliminating the term $\frac{1}{\epsilon}$. Since $\epsilon \geq 1$, the inequalities $Q_m - \frac{1}{\epsilon}e^{-2\alpha h}G_m \leq 0$ lead to $Q_m - e^{-2\alpha h}G_m \leq 0$, $m = 1, \ldots, \bar{m}$. Then to satisfy the first inequality in (19) for both cases, we need $\Theta_m \leq 0$, $m = 1, \ldots, \bar{m}$. For $t_{k+1} = t_k'$, the second inequality in (19) follows from the first one. For $t_{k+1} > t_k'$, it is fulfilled as well, since $V_e(t_k') - V_e(t_k'^-) = \bar{V}(t_k') - \bar{V}(t_k'^-) = V_2(x(t_k)) - x^{\mathrm{T}}(t_k)Px(t_k) - V_{G|t=t_k'^-} = -V_{G|t=t_k'^-} \leq 0$. Therefore, for (19) it is sufficient to require (25). Moreover, (25) guarantees (16).

Appendix E. Proof of Proposition 4

We will derive the conditions for (20) on $[t_k, t_k')$, where $\bar{V}(t) = V_1(t, x_t, \dot{x}_t)$. Since $\frac{d}{dt}x(t-\tau(t)) = (1-\dot{\tau}(t))\dot{x}(t-\tau(t)) = 0$, by direct calculations we obtain

$$
\begin{aligned}
&\mathcal{L}V_e(t) + 2\alpha V_e(t) - \frac{1}{h}\sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t)U_m e_m(t) \leq 2x^{\mathrm{T}}(t)P\dot{x}(t) \\
&+ 2\alpha x^{\mathrm{T}}(t)Px(t) - e^{-2\alpha h}\int_{-\tau(t)}^{0} \dot{x}^{\mathrm{T}}(t+s)Q\dot{x}(t+s)ds \\
&+ \dot{x}^{\mathrm{T}}(t)\left[h\sum_{m=1}^{\bar{m}} C_m^{\mathrm{T}}G_mC_m + (h-\tau(t))Q\right]\dot{x}(t) \\
&+ (2\alpha(h-\tau(t)) - 1)\zeta^T(t, x_t)R\,\zeta(t, x_t) + (h-\tau(t)) \\
&\times \left[\dot{x}^{\mathrm{T}}(t)(X + X^{\mathrm{T}})x(t) + 2\dot{x}^{\mathrm{T}}(t)(-X + X_1)x(t-\tau(t))\right] \\
&+ \sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t)\left(2\alpha Q_m - \frac{1}{h}U_m\right)e_m(t).
\end{aligned}
\tag{E.1}
$$

Denote $v(t) = \frac{1}{\tau(t)}\int_{-\tau(t)}^{0}\dot{x}(t+s)ds$, where $v(t_k)$ is understood as $\lim_{\tau(t)\to 0} v_1 = \dot{x}(t)$. Next, similarly to above, we use the free-weighting matrices technique

$$
\begin{aligned}
0 &= 2\left[x^{\mathrm{T}}(t)Y_1^{\mathrm{T}} + \dot{x}^{\mathrm{T}}(t)Y_2^{\mathrm{T}} + x^{\mathrm{T}}(t-\tau(t))Z^{\mathrm{T}}\right. \\
&\quad \left. + \sum_{i=1}^{N}\xi_i q_i^{\mathrm{T}}Y_3^{(i)\mathrm{T}}\right] \times [\tau(t)v(t) - (x(t) - x(t-\tau(t)))], \\
0 &= 2\left[x^{\mathrm{T}}(t)P_2^{\mathrm{T}} + \dot{x}^{\mathrm{T}}(t)P_3^{\mathrm{T}}\right]\left[-\dot{x}(t) + Ax(t) + \sum_{i=1}^{N}q_i\xi_i(t)\right. \\
&\quad \left. + BKCx(t-\tau(t)) + \sum_{m=1}^{\bar{m}}(1-\pi_m(k))BK_me_m(t)\right],
\end{aligned}
\tag{E.2}
$$

where $P_2, P_3, Y_1, Y_2, Y_3^{(i)}, Z$ are some $n_x \times n_x$ matrices. Using Jensen's inequality $\int_{-\tau(t)}^{0}\dot{x}^{\mathrm{T}}(t+s)\,Q\,\dot{x}(t+s)ds \geq \tau(t)\,v^{\mathrm{T}}(t)Q\,v(t)$, adding (E.2) to the right-hand side of (E.1), and taking expectation on both sides of (E.1) we get

$$
\begin{aligned}
&\mathbb{E}\left\{\mathcal{L}V_e(t) + 2\alpha V_e(t) - \frac{1}{h}\sum_{m=1}^{\bar{m}} e_m^{\mathrm{T}}(t)U_m e_m(t)\right\} \\
&\leq \frac{h-\tau(t)}{h}\mathbb{E}\left\{\eta_0^T(t)\Phi_0'\eta_0(t)\right\} + \frac{\tau(t)}{h}\mathbb{E}\left\{\eta_1^T(t)\Phi_1'\eta_1(t)\right\},
\end{aligned}
\tag{E.3}
$$

where $\eta_0(t) = \left[x^{\mathrm{T}}(t), \dot{x}^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\tau(t)), \xi_1(t), \ldots, \xi_N(t), e_1^{\mathrm{T}}(t), \ldots, e_{\bar{m}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, $\eta_1(t) = \left[\eta_0^{\mathrm{T}}(t), v^{\mathrm{T}}(t)\right]^{\mathrm{T}}$. Thus, taking into account (2), for (20) it is sufficient to require the right hand side of (E.3) to be nonpositive for all $\eta_0(t)$ and $\eta_1(t)$ satisfying (B.3). As above, we can rewrite this condition with S-procedure: $\Phi_0' + \Phi_0'' \leq 0$, $\Phi_1' + \Phi_1'' \leq 0$.

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