

The Collapse of Decoherence

Can Decoherence Theory Solve The Problems of Measurement?

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Abstract

In this review study, we ask ourselves if decoherence theory can solve the problems of measurement in quantum mechanics. After an introduction to decoherence theory, we present *the problem of preferred basis*, *the problem of non-observability of interference* and *the problem of definite outcomes*. We present Zurek's theory of *environment induced superselection rules* and find that the problem of preferred basis and the problem of non-observability of interference can be solved through decoherence theory, but not the problem of outcomes, *if* we accept the eigenstate-eigenvalue link and the Born statistical interpretation. We reveal that these two concepts are essential in the *Copenhagen interpretations* of quantum mechanics, and give an account for von Neumann's and Wigner's conscious collapse interpretation as well as a detailed description of Bohr's and Heisenberg's interpretation. We discuss how Bohr's and Heisenberg's interpretation relates to decoherence with a special emphasis on the irreducibility of classical concepts as interpreted by Don Howard. During the discussion, we critique Wigner's use of the word "consciousness" as opposed to von Neumann's use, as well as Howard's decisively ontological approach to Bohr through an antithetical Kantian approach. We conclude by stating that decoherence theory cannot decisively solve the problem of definite outcomes of quantum mechanics, even when considering it in relation to the Copenhagen interpretation.

Populärvetenskaplig Sammanfattning

Den moderna fysikens beskrivning av mikrokosmos, kvantfysiken, strider emot grundläggande intuitioner om den fysiska världen. Här finns kvantobjekt vilka matematiskt bör beskrivas som superpositioner av tillstånd med flera till synes motsägande egenskaper samtidigt, exempelvis flera energinivåer eller positioner. Här finns också osäkerhetsprinciper som till exempelvis innebär att kvantobjekt inte kan ha en bestämd hastighet *och* position samtidigt. Dessa egenskaper lyser dock med sin frånvaro i den makroskopiska värld som vi upplever runt omkring oss, den som förklaras av den klassiska fysiken. Men, om det nu är så att allt runt omkring oss är uppbyggt av atomer som fysiker säger, alltså kvantmekaniska objekt, hur kommer det sig att dessa kvantlagar försvinner när man går från det mikroskopiska till det makroskopiska? Detta är en kort beskrivning av kvantfysikens *mätproblem*, som idag ofta kallas den *kvant-klassiska övergången*. Anledningen till att frågan just kallas för ett mätproblem är att det största problemet med detta berör hur vi kan gå från en superposition av tillstånd till ett definitivt tillstånd när vi väl genomför en mätning, en fråga som inte har blivit löst på snart hundra år.

Under 1980-talet började fysiker intressera sig för egenskaper hos kvantobjekt som *sammanfläter* med sin omgivning, då denna omgivning också är beskriven av kvantfysiken. Sammanflätning är en exklusivt kvantmekanisk effekt och innebär att man under vissa experimentella villkor inte längre kan beskriva partiklar som individuella utan måste beskriva dem som *ett* enat system. Denna forskning visade att partiklar som sammanflätade med sin omgivning förlorade kvantmekaniska egenskaper och närmade sig en beskrivning som liknar den klassiska fysiken genom en effekt som kallas för *dekoherens*. Skulle denna dekoherenseffekt kunna ge oss nyckeln till att förstå hur kvantfysiken ger upphov till den klassiska verklighet som vi ser runt omkring oss? Min uppsats diskuterar just denna fråga. Uppsatsen börjar med en teknisk genomgång av dekoherenseffekten, för att sedan gå igenom kvantfysikens mätproblem. Efter detta diskuteras mätproblemen i relation till denna dekoherenseffekt. Vi slår fast att fysiker har, med vissa antaganden, löst ett antal frågor som berör mätproblemen, men inte hur en superposition av tillstånd kan realiseras i ett definitivt tillstånd vid en mätning.

En viktig aspekt av denna diskussion är inte bara fysiken i sig, utan också tolkningar av den. Alla fysiker håller med om att kvantfysiken är en av de mest generella och precisa teorierna som människan någonsin har skapat, men inga kan komma överens om vad den faktisk betyder. Refererar kvantfysikens matematiska objekt till metafysiska realiteter? Är det verkligen så att vi måste beskriva kvantfysiken på ett så världsfrånvänt sätt? Vad är egentligen en mätning? Alla dessa frågor har diskuterats sedan kvantfysikens begynnelse. Det förhåller sig så att ställningstaganden till dessa typer av frågor implicit antas vid en viss presentation av kvantfysiken, till exempel vid användandet av vissa matematiska operationer. Jag har i denna uppsats valt att presentera kvantfysiken

på det mest vedertagna sättet, vilket i mångt och mycket bygger på en tolkning av kvantfysiken som kallas *Köpenhamstolkningen*. Köpenhamstolkningen har flera olika skolor, två av vilka hade Niels Bohr respektive John von Neumann som frontfigurer. Den sista delen av min uppsats diskuterar dessa två köpenhamstolkningar, samt dess relationer till dekoherenseffekten.

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*"[Wittgenstein] once greeted me with the question: 'Why do people say that it was natural to think that the sun went round the earth rather than that the earth turned on its axis?' I replied: 'I suppose, because it looked as if the sun went round the earth.' 'Well,' he asked, 'what would it have looked like if it had looked as if the earth turned on its axis?'"*¹

—G.E.M Anscombe, in conversation with Ludwig Wittgenstein.

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1 Introduction

How is it possible that the unintuitive nature of the micro cosmos gives rise to the rules dictating the classical world of our everyday experience? It is a peculiar experience that something as precise and general as the description of nature through quantum theory can still feel so shaky and wobbly, like there is no firm ground to stand on. I *don't* find myself in a superposition of states, constantly diffracting and interfering with everyday objects, which indeed seems very rigid and defined to me. Yet, looking deep down, it seems that it is all described by quantum theory. How could this be?

¹Anscombe, G.E.M; *"An Introduction To Wittgenstein's Tractatus"*; Harper and Row, publishers, Inc, p. 151, (1959).

This, in essence, is the fundamental question behind *the measurement problems of quantum mechanics*. It is nothing less than a mystery, quite simple to understand yet notoriously difficult to solve, if indeed a solution is possible or even needed. Yet, far away from the long nights at the Solvey conference of 1927, as tools have gotten more advanced and physicists more used to working with them, cracks have slowly started to form. One of these tools is *decoherence theory*, starting from the unintuitive point of introducing entanglement dynamics to try and explain the classical world that we perceive in our everyday lives.

In this essay, we will try to answer one question: "*Can Decoherence Theory Solve the Problems of Measurement?*". It is made up of six main parts. Firstly, an introduction to decoherence theory will be provided. Then, we will explain the *problem of preferred basis*, the *problem of non-observability of interference* and the *problem of definite outcomes*. After providing an account for environment induced superselection rules, we investigate whether decoherence can solve the problems of measurement. After this, an account for the Copenhagen interpretations is given, and a discussion regarding their relations to decoherence is provided. We end the essay with a discussion of our findings, a conclusion and a summary, as is customary.

2 Decoherence Theory

2.1 Preliminaries

Before moving on to Decoherence theory, it is important to recall some basic concepts of quantum theory.

2.1.1 Density Operators

Definition: An operator $\hat{\rho}$ is a density operator² if and only if

$$\hat{\rho}^\dagger = \hat{\rho}, \quad (1)$$

$$\hat{\rho}|\psi\rangle = \lambda|\psi\rangle \iff \lambda \geq 0, \quad (2)$$

$$\text{Tr}\{\hat{\rho}\} = 1. \quad (3)$$

Through this definition, we can easily see that

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle \langle \psi_i|, \quad (4)$$

where $\sum_i p_i = 1$, $0 \leq p_i \leq 1$, is the most general form of a density operator, as

$$\left(\sum_i p_i |\psi_i\rangle \langle \psi_i| \right)^\dagger = \sum_i p_i \langle \psi_i|^\dagger |\psi_i\rangle^\dagger = \sum_i p_i |\psi_i\rangle \langle \psi_i|, \quad (5)$$

²LaRose, R.; <https://www.ryanlarose.com/uploads/1/1/5/8/115879647/quic06-states-trace.pdf>

$$\sum_i p_i |\psi_i\rangle \langle \psi_i | \psi \rangle = \sum_i p_i \langle \psi_i | \psi \rangle |\psi_i\rangle = p |\psi\rangle, \quad (6)$$

where $p \geq 0$, and finally

$$\text{Tr}\{\hat{\rho}\} = \sum_{i,k} p_i \langle k | \psi_i \rangle \langle \psi_i | k \rangle = \sum_i p_i = 1. \quad (7)$$

A special case of the density operator is when there is only one element $p_i = 1$, as

$$\hat{\rho} = \sum_i p_i |\psi_i\rangle \langle \psi_i| = |\psi\rangle \langle \psi|. \quad (8)$$

This is called a *pure state*, which signifies a state in which the wave function is known with certainty. A *mixed state*, then, is represented by equation (4) when $p_i \neq 1$ and is made up by an *ensemble* of pure states, where the wave function is not known with certainty.

Theorem:

$$\hat{\rho}_A \otimes \hat{\rho}_B, \quad (9)$$

$$\hat{U} \hat{\rho} \hat{U}^\dagger, \quad (10)$$

$$\frac{\hat{A} \hat{A}^\dagger}{\text{Tr}[\hat{A} \hat{A}^\dagger]} \quad (11)$$

are density operators, where \hat{U} is a unitary operator and \hat{A} a non-zero operator.

Proof: We have that

$$(\hat{\rho}_A \otimes \hat{\rho}_B)^\dagger = \hat{\rho}_A^\dagger \otimes \hat{\rho}_B^\dagger = \hat{\rho}_A \otimes \hat{\rho}_B, \quad (12)$$

$$(\hat{\rho}_A \otimes \hat{\rho}_B)(|\psi_A\rangle \otimes |\psi_B\rangle) = \hat{\rho}_A |\psi_A\rangle \otimes \hat{\rho}_B |\psi_B\rangle = \lambda_A |\psi_A\rangle \otimes \lambda_B |\psi_B\rangle = \hat{\rho}_A \otimes \hat{\rho}_B, \quad (13)$$

$$\text{Tr}[\hat{\rho}_A \otimes \hat{\rho}_B] = \text{Tr}[\hat{\rho}_A] \otimes \text{Tr}[\hat{\rho}_B] = 1, \quad (14)$$

where we have used the fact that $\hat{\rho}_A$ and $\hat{\rho}_B$ are density operators. Thus, $\hat{\rho}_A \otimes \hat{\rho}_B$ is a density operator. Also, we have that

$$(\hat{U} \hat{\rho} \hat{U}^\dagger)^\dagger = \hat{U} \hat{\rho}^\dagger \hat{U}^\dagger = \hat{U} \hat{\rho} \hat{U}^\dagger, \quad (15)$$

$$\hat{U} \hat{\rho} \hat{U}^\dagger |\psi\rangle = \lambda |\psi\rangle \implies \hat{\rho}(\hat{U}^\dagger |\psi\rangle) = \lambda(\hat{U}^\dagger |\psi\rangle), \quad (16)$$

$$\text{Tr}[\hat{U} \hat{\rho} \hat{U}^\dagger] = \text{Tr}[\hat{\rho} \hat{U}^\dagger \hat{U}] = \text{Tr}[\hat{\rho}] = 1 \quad (17)$$

where we have used the fact that $\hat{\rho}$ is a density operator. Thus, $\hat{U} \hat{\rho} \hat{U}^\dagger$ is a density operator. Finally, we have that

$$\left(\frac{\hat{A}\hat{A}^\dagger}{\text{Tr}[\hat{A}\hat{A}^\dagger]} \right)^\dagger = \frac{\hat{A}\hat{A}^\dagger}{\text{Tr}[\hat{A}\hat{A}^\dagger]}, \quad (18)$$

$$\frac{\hat{A}\hat{A}^\dagger}{\text{Tr}[\hat{A}\hat{A}^\dagger]} |\psi\rangle = \lambda |\psi\rangle \implies \frac{1}{\text{Tr}[\hat{A}\hat{A}^\dagger]} (\hat{A}^\dagger |\psi\rangle) = \lambda (\hat{A}^\dagger |\psi\rangle), \quad (19)$$

$$\text{Tr} \left[\frac{\hat{A}\hat{A}^\dagger}{\text{Tr}[\hat{A}\hat{A}^\dagger]} \right] = \frac{\text{Tr}[\hat{A}\hat{A}^\dagger]}{\text{Tr}[\hat{A}\hat{A}^\dagger]} = 1, \quad (20)$$

where we have used the fact that $\frac{1}{\text{Tr}[\hat{A}\hat{A}^\dagger]} \geq 0 \implies \lambda \geq 0$. Thus, $\frac{\hat{A}\hat{A}^\dagger}{\text{Tr}[\hat{A}\hat{A}^\dagger]}$ is a density matrix. ■

Theorem: If $\hat{\rho}$ is in a pure state, then

$$\hat{\rho}^2 = \hat{\rho} \quad (21)$$

$$\text{Tr}[\hat{\rho}^2] = 1 \quad (22)$$

Proof:

$$\hat{\rho}^2 = |\psi\rangle \langle \psi| \psi\rangle \langle \psi| = |\psi\rangle \langle \psi| = \hat{\rho}, \quad (23)$$

where we have used the normalization condition $\langle \psi| \psi\rangle = 1$. Then,

$$\text{Tr}[\hat{\rho}^2] = \text{Tr}[\hat{\rho}] = 1 \quad (24)$$

per definition. ■

Theorem: If $\hat{\rho}$ is in a mixed state, then

$$\hat{\rho}^2 \neq \hat{\rho} \quad (25)$$

$$\text{Tr}[\hat{\rho}^2] < 1. \quad (26)$$

Proof:

We have that

$$\left(\sum_i p_i |\psi_i\rangle \langle \psi_i| \right)^2 = \sum_{i,j} p_i p_j |\psi_i\rangle \langle \psi_i| \psi_j\rangle \langle \psi_j| = \sum_i p_i^2 |\psi_i\rangle \langle \psi_i| \neq \sum_i p_i |\psi_i\rangle \langle \psi_i|, \quad (27)$$

and

$$\text{Tr}[\hat{\rho}^2] = \text{Tr} \left[\sum_i p_i^2 |\psi_i\rangle \langle \psi_i| \right] = \sum_i p_i^2 < \sum_i p_i = 1. \quad (28)$$

We note that if $p_i = 1$, these conditions may no longer be satisfied. This is because pure states are a special case of mixed states. The assumption above is therefore that $p_i \neq 1$. ■

Theorem: Let $\hat{\rho}$ be a pure state and \hat{O} any hermitian operator, then

$$\text{Tr}[\hat{\rho}\hat{O}] = \langle \psi | \hat{O} | \psi \rangle \quad (29)$$

Proof:

We have that

$$\begin{aligned} \text{Tr}[\hat{\rho}\hat{O}] &= \sum_i \langle i | \hat{\rho}\hat{O} | i \rangle \\ &= \sum_i \langle i | \psi \rangle \langle \psi | \hat{O} | i \rangle \\ &= \sum_i \langle \psi | \hat{O} | i \rangle \langle i | \psi \rangle \\ &= \langle \psi | \hat{O} | \psi \rangle. \blacksquare \end{aligned} \quad (30)$$

Theorem: Let $\hat{\rho}$ be a mixed state and \hat{O} any hermitian operator, then

$$\text{Tr}[\hat{\rho}\hat{O}] = \sum_j p_j \langle \psi_j | \hat{O} | \psi_j \rangle. \quad (31)$$

Proof:

We have that

$$\begin{aligned} \text{Tr}[\hat{\rho}\hat{O}] &= \sum_i \langle i | \hat{\rho}\hat{O} | i \rangle \\ &= \sum_i \langle i | \left[\sum_j p_j |\psi_j\rangle \langle \psi_j| \right] \hat{O} | i \rangle \\ &= \sum_{i,j} p_j \langle i | \psi_j \rangle \langle \psi_j | \hat{O} | i \rangle \\ &= \sum_{i,j} p_j \langle \psi_j | \hat{O} | i \rangle \langle i | \psi_j \rangle \\ &= \sum_j p_j \langle \psi_j | \hat{O} | \psi_j \rangle. \blacksquare \end{aligned} \quad (32)$$

Assuming that \hat{O} has the eigenstates $|o_i\rangle$ and eigenvalues o_i we realize that, for a pure state $\hat{\rho}$,

$$\text{Tr}[\hat{\rho}\hat{O}] = \sum_i \langle o_i|\psi\rangle \langle\psi|\hat{O}|o_i\rangle = \sum_i o_i |\langle o_i|\psi\rangle|^2. \quad (33)$$

$|\langle o_i|\psi\rangle|^2$ is the *Born probability* of the outcome o_i ³. $\text{Tr}[\hat{\rho}\hat{O}]$ thus represents the average of eigenvalues o_i obtained from measurements of \hat{O} , which is just the expectation value $\langle\hat{O}\rangle$,

$$\langle\hat{O}\rangle = \text{Tr}[\hat{\rho}\hat{O}]. \quad (34)$$

We note the importance of the fact that this equality is based on the physical concept of the Born rule and thusly the *statistical interpretation*, a fact important for later subject matter (see section 4.5).

2.1.2 Entangled and Separable States

Definition: Given two pure quantum states $|\psi\rangle_A = \sum_i \alpha_i |\psi_i\rangle_A$ and $|\psi\rangle_B = \sum_j \beta_j |\psi_j\rangle_B$, where $|\psi\rangle_A \in \mathcal{H}_A$ and $|\psi\rangle_B \in \mathcal{H}_B$, the composite quantum state

$$|\psi\rangle_{AB} = \sum_{i,j} \gamma_{ij} |\psi_i\rangle_A \otimes |\psi_j\rangle_B \quad (35)$$

where $|\psi\rangle_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ is said to be **separable** if and only if

$$\gamma_{ij} = \alpha_i \beta_j, \quad \forall i, j \quad (36)$$

and **entangled** if and only if

$$\exists i, j \text{ s.t. } \gamma_{ij} \neq \alpha_i \beta_j. \quad (37)$$

From this definition, we see that separable states can be written as

$$\begin{aligned} |\psi\rangle_{AB} &= \sum_{i,j} \gamma_{ij} |\psi_i\rangle_A \otimes |\psi_j\rangle_B = \sum_{i,j} \alpha_i \beta_j |\psi_i\rangle_A \otimes |\psi_j\rangle_B \\ &= \left(\sum_i \alpha_i |\psi_i\rangle_A \right) \otimes \left(\sum_j \beta_j |\psi_j\rangle_B \right) = |\psi\rangle_A \otimes |\psi\rangle_B. \end{aligned} \quad (38)$$

Similarly, a mixed state $\hat{\rho}_{AB} \in \mathcal{H}_A \otimes \mathcal{H}_B$ is separable if and only if there exists a decomposition of $\hat{\rho}_{AB}$ such that

$$\hat{\rho}_{AB} = \sum_i p_i \hat{\rho}_A^i \otimes \hat{\rho}_B^i. \quad (39)$$

³Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, p. 35-36, (2008).

2.1.3 The Partial Trace

Moving forward, we must introduce the method of taking a *partial trace*. As is customary, the trace of an operator \hat{A} is defined as

$$\text{Tr}[\hat{A}] = \sum_i \langle i | \hat{A} | i \rangle, \quad (40)$$

where $\{|i\rangle\}$ is any orthonormal basis. However, when dealing with decoherence theory this trace may be unsatisfactory for studying certain systems. Consider for example the composite system $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$, $\hat{\rho}_A \in \mathcal{H}_A$ and $\hat{\rho}_B \in \mathcal{H}_B$. When performing the trace operation, we obtain the diagonal elements of the entire composite system, with no regard for either of them in isolation. If we wish to analyse the *measurement statistics* of $\hat{\rho}_A$ or $\hat{\rho}_B$ in isolation, at any time during their respective evolution, we use the *partial trace*:

Definition: Let $\hat{\rho}_{AB} = \hat{\rho}_A \otimes \hat{\rho}_B$, where $\hat{\rho}_A \in \mathcal{H}_A$ and $\hat{\rho}_B \in \mathcal{H}_B$. Then **The partial trace over B** is given by

$$\hat{\rho}_A \equiv \text{Tr}_B[\hat{\rho}_{AB}] = \sum_j (I_A \otimes \langle j |_B) \hat{\rho}_{AB} (I_A \otimes |j \rangle_B), \quad (41)$$

where $\{|j\rangle_B\} \in \mathcal{H}_B$ is any orthonormal basis. **The partial trace over A** is similarly defined as

$$\hat{\rho}_B \equiv \text{Tr}_A[\hat{\rho}_{AB}] = \sum_i (\langle i |_A \otimes I_B) \hat{\rho}_{AB} (|i \rangle_A \otimes I_B), \quad (42)$$

where $\{|i\rangle_A\} \in \mathcal{H}_A$ is an orthonormal basis.

Looking at the definition for the partial trace, we see how we only perform the trace over one of the subspaces, leaving the other unchanged. In particular, we note that the partial trace is a *non-unitary* operation that is used to investigate the measurement statistics of either part of the composite system⁴. We realise this by setting $\hat{O} = \hat{O}_A \otimes \hat{I}_B$ and finding $\langle \hat{O} \rangle$ as

⁴Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, ch. 2.4, (2008).

$$\begin{aligned}
\langle \hat{O} \rangle &= \\
&= \text{Tr}[\hat{\rho}_{AB} \hat{O}] = \\
&= \sum_{ij} \langle i|_A \langle j|_B \left[\rho_{AB}(\hat{O}_A \otimes \hat{I}_B) \right] |i\rangle_A |j\rangle_B = \\
&= \sum_i \langle i|_A \left[\sum_j \langle j|_B \hat{\rho}_{AB} |j\rangle_B \right] \hat{O}_A |i\rangle_A = \\
&= \sum_i \langle i|_A \text{Tr}_B[\hat{\rho}_{AB}] \hat{O}_A |i\rangle_A = \\
&= \sum_i \langle i|_A \hat{\rho}_A \hat{O}_A |i\rangle_A = \\
&= \text{Tr}_A[\hat{\rho}_A \hat{O}_A],
\end{aligned} \tag{43}$$

showing that

$$\langle \hat{O} \rangle = \langle \hat{O}_A \otimes \hat{I}_B \rangle = \text{Tr}_A[\hat{\rho}_A \hat{O}_A]. \tag{44}$$

Thus, the partial trace operation has endowed us with a means to analyse all relevant measurement statistics of the system $\hat{\rho}_A$. This carries over trivially to the measurement statistics of $\hat{\rho}_B$.

2.2 What is Decoherence?

As a precursor for subjects to come, it is important to give a simple but quite general example on the process of decoherence. In this section such examples are given, as well as a discussion of what decoherence is and is not.

Let a general quantum state $\hat{\rho}_S$, a density operator where S stands for *System*, be defined on the Hilbert space \mathcal{H}_S ⁵. We are not concerned with the structure of $\hat{\rho}_S$, and it is therefore assumed to be a general density operator. Furthermore, we imagine a particle $\hat{\rho}_E$, where E stands for *Environment*, entangling with $\hat{\rho}_S$, assumed here to be in a pure state $\hat{\rho}_E = |\psi_{in}\rangle \langle \psi_{in}|_E$ on the Hilbert space \mathcal{H}_E . To model this entanglement, we must introduce the *scattering operator* \hat{S}_{tot} , satisfying

$$|\psi_{out}\rangle = \hat{S}_{tot} |\psi_{in}\rangle \tag{45}$$

which for sufficiently small interaction times maps a state from before to after interaction sufficiently long after it occurred, in the total System-Environment Hilbert space $\mathcal{H}_{tot} = \mathcal{H}_S \otimes \mathcal{H}_E$. As the system and environment entangle with each other through the scattering operator, they will move from their initial uncorrelated states to a correlated state, a process described as

⁵Hornberger, K.; "Introduction to Decoherence Theory"; arXiv:quant-ph/0612118v3, (2008).

$$\hat{\rho}_{tot} = \hat{\rho}_S \otimes \hat{\rho}_E \longrightarrow \hat{\rho}'_{tot} = \hat{S}_{tot}[\hat{\rho}_S \otimes \hat{\rho}_E]\hat{S}_{tot}^\dagger. \quad (46)$$

As this is a quite general form of the interaction process, some assumptions about the scattering operator are in order. We will assume that the interaction taking place is non-invasive to a certain property of the system. This means, in essence, that the system property is undisturbed by the environmental interaction taking place. Specifically it means that if these properties are represented by $|n\rangle \in \mathcal{H}_S$, they should commute with \hat{S}_{tot} , suggesting the new form

$$\hat{S}_{tot} = \sum_n |n\rangle \langle n| \otimes \hat{S}_n \quad (47)$$

where indeed $\sum_n |n\rangle \langle n|$ only act in \mathcal{H}_S and \hat{S}_n only in \mathcal{H}_E in equation (46). Inserting this expression into equation (46), we find that

$$\begin{aligned} \hat{\rho}'_{tot} &= \hat{S}_{tot}[\hat{\rho}_S \otimes \hat{\rho}_E]\hat{S}_{tot}^\dagger \\ &= \left[\sum_n |n\rangle \langle n| \otimes \hat{S}_n \right] \left[\hat{\rho}_S \otimes |\psi_{in}\rangle \langle \psi_{in}|_E \right] \left[\sum_m |m\rangle \langle m| \otimes \hat{S}_m^\dagger \right] \\ &= \sum_{n,m} |n\rangle \langle n| \hat{\rho}_S |m\rangle \langle m| \otimes \hat{S}_n |\psi_{in}\rangle \langle \psi_{in}|_E \hat{S}_m^\dagger \\ &= \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \otimes \left| \psi_{out}^{(n)} \right\rangle \left\langle \psi_{out}^{(m)} \right|_E. \end{aligned} \quad (48)$$

In taking the *partial trace* (sec. 2.1.3), we can in essence pull out only the quantum state of the system from $\hat{\rho}_S \otimes \hat{\rho}_E$, giving us

$$\begin{aligned} \hat{\rho}'_S &= Tr_E[\hat{\rho}'_{tot}] \\ &= \sum_j \left(\hat{I}_S \otimes \langle j|_E \right) \left[\sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \otimes \left| \psi_{out}^{(n)} \right\rangle \left\langle \psi_{out}^{(m)} \right|_E \right] \left(\hat{I}_S \otimes |j\rangle_E \right) \\ &= \sum_j \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \left\langle j \left| \psi_{out}^{(n)} \right\rangle_E \left\langle \psi_{out}^{(m)} \right| j \right\rangle_E \\ &= \sum_j \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \left\langle \psi_{out}^{(m)} \right| j \right\rangle_E \left\langle j \left| \psi_{out}^{(n)} \right\rangle_E \right. \\ &= \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \left\langle \psi_{out}^{(m)} \right| \psi_{out}^{(n)} \right\rangle_E \\ &= \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \langle \psi_{in} | \hat{S}_m^\dagger \hat{S}_n | \psi_{in} \rangle_E, \end{aligned} \quad (49)$$

which is the quantum state of the system after entanglement. We shall now investigate how the systems diagonal elements, or *populations*, and *coherences*, the systems off-diagonal elements, has developed after the entanglement. We obtain the systems populations by setting $n = m$ in eq. (49), as

$$\begin{aligned}
\hat{\rho}'_{Snn} &= \sum_n \hat{\rho}_{Snn} |n\rangle \langle n| \langle \psi_{in} | \hat{S}_n^\dagger \hat{S}_n | \psi_{in} \rangle_E \\
&= \hat{\rho}_{Snn} \langle \psi_{in} | \psi_{in} \rangle_E = \hat{\rho}_{Snn},
\end{aligned} \tag{50}$$

where we have used the unitarity of the scattering operators in a shared basis, $\hat{S}_n^\dagger \hat{S}_n = \hat{I}$. Thus, we have that

$$\hat{\rho}'_{Snn} = \hat{\rho}_{Snn} \tag{51}$$

meaning that the populations of the system remains unchanged after entanglement with $\hat{\rho}_E$, as is to be expected from the property of non-invasiveness introduced in the scattering operator. However, concerning the coherences of the system, we set $n \neq m$ in equation (49), and see that

$$\begin{aligned}
\hat{\rho}'_{Snm} &= \sum_{n,m} \hat{\rho}_{Snm} |n\rangle \langle m| \langle \psi_{in} | \hat{S}_m^\dagger \hat{S}_n | \psi_{in} \rangle_E \\
&= \sum_{n,m} \hat{\rho}_{Snm} \langle \psi_{in} | \hat{S}_m^\dagger | n \rangle_E \langle m | \hat{S}_n | \psi_{in} \rangle_E \\
&= \sum_{n,m} \hat{\rho}_{Snm} \langle \psi_{out}^{(m)} | n \rangle_E \langle m | \psi_{out}^{(n)} \rangle_E \\
&= \hat{\rho}_{Snm} \langle \psi_{out}^{(m)} | \psi_{out}^{(n)} \rangle
\end{aligned} \tag{52}$$

where the environmental prefix has been removed because $|n/m\rangle$ acting on $|\psi_{out}^{(n/m)}\rangle$ only preserves joint states. Thus, we see that

$$\hat{\rho}'_{Snm} = \hat{\rho}_{Snm} \langle \psi_{out}^{(m)} | \psi_{out}^{(n)} \rangle. \tag{53}$$

Meaning that the coherences of the system has changed by a factor of $\langle \psi_{out}^{(m)} | \psi_{out}^{(n)} \rangle$ after entanglement.

Before discussing the meaning of these results, let us discuss a more concrete example of the same process outlined above. Imagine the classic⁶ double slit experiment set up normally, with two slits in front of a detector screen⁷. As is customary, we imagine a particle, our system, racing towards the double slit and model it as

$$|\psi\rangle = \alpha |s_L\rangle + \beta |s_R\rangle \tag{54}$$

after it has passed the double slit, with s standing for "system" and the prefixes L and R signifying passage through the left and right slit, respectively. Now

⁶ *Quantum?*

⁷ Schlosshauer, M.; "The quantum-to-classical transition and decoherence"; arXiv:1404.2635v2 [quant-ph] (2019).

imagine that instead of a vacuum we fill the area between the detector screen and double slit with some kind of environment. This could be a gas of low concentration, for example. Now, just after the particle has passed the double slit, it will entangle with this environment, which we model as

$$|\psi\rangle|E_0\rangle = (\alpha|s_L\rangle + \beta|s_R\rangle)|E_0\rangle \longrightarrow |\psi'\rangle = \alpha|s_L\rangle|E_L\rangle + \beta|s_R\rangle|E_R\rangle. \quad (55)$$

In essence, this is the same step as above where we instead formally introduced the scattering operator. We see that just as the particle has left the double slit, the system and environment have not yet had time to entangle with one another. However, as time passes the environment will become entangled with our system. We can carefully say that the environment is affected by the probabilities of which slit the particle went through, which affects its evolution accordingly and which justifies our choice of prefixes. In computing the density matrix for the pure, entangled system, we find that

$$\begin{aligned} \hat{\rho}_{SE} &= |\psi'\rangle\langle\psi'| \\ &= (\alpha|s_L\rangle|E_L\rangle + \beta|s_R\rangle|E_R\rangle)(\alpha^*\langle s_L| \langle E_L| + \beta^*\langle s_R| \langle E_R|) \\ &= |\alpha|^2|E_L\rangle|s_L\rangle\langle s_L| \langle E_L| + |\beta|^2|E_R\rangle|s_R\rangle\langle s_R| \langle E_R| \\ &\quad + \alpha\beta^*|E_L\rangle|s_L\rangle\langle s_R| \langle E_R| + \alpha^*\beta|E_R\rangle|s_R\rangle\langle s_L| \langle E_L|. \end{aligned} \quad (56)$$

Again, as before, we are interested in the evolution of the system and not the environment. We therefore perform a partial trace over the environment, as

$$\begin{aligned} \hat{\rho}_S &= \text{Tr}_E[\hat{\rho}_{SE}] = \sum_j (\hat{I}_S \otimes \langle j|_E) \hat{\rho}_{SE} (\hat{I}_S \otimes |j\rangle_E) \\ &= (\hat{I}_S \otimes \langle E_L|) \hat{\rho}_{SE} (\hat{I}_S \otimes |E_L\rangle) + (\hat{I}_S \otimes \langle E_R|) \hat{\rho}_{SE} (\hat{I}_S \otimes |E_R\rangle) \\ &= |\alpha|^2|s_L\rangle\langle s_L| \left(|\langle E_L|E_L\rangle|^2 + |\langle E_L|E_R\rangle|^2 \right) \\ &\quad + |\beta|^2|s_R\rangle\langle s_R| \left(|\langle E_R|E_R\rangle|^2 + |\langle E_L|E_R\rangle|^2 \right) \\ &\quad + \alpha\beta^*|s_L\rangle\langle s_R| \left(\langle E_L|E_L\rangle \langle E_R|E_L\rangle + \langle E_R|E_L\rangle \langle E_R|E_R\rangle \right) \\ &\quad + \alpha^*\beta|s_R\rangle\langle s_L| \left(\langle E_L|E_R\rangle \langle E_L|E_L\rangle + \langle E_R|E_R\rangle \langle E_L|E_R\rangle \right). \end{aligned} \quad (57)$$

Seeing that

$$\begin{aligned} &|\langle E_L|E_L\rangle|^2 + |\langle E_L|E_R\rangle|^2 \\ &= \langle E_L|E_L\rangle \langle E_L|E_L\rangle + \langle E_L|E_R\rangle \langle E_R|E_L\rangle \\ &= \sum_{i=L,R} \langle E_L|E_i\rangle \langle E_i|E_L\rangle \\ &= \langle E_L|E_L\rangle = 1 \end{aligned} \quad (58)$$

where we have used the normalization condition, and

$$\begin{aligned}
& \langle E_L|E_L\rangle \langle E_R|E_L\rangle + \langle E_R|E_L\rangle \langle E_R|E_R\rangle \\
&= \langle E_R|E_L\rangle \langle E_L|E_L\rangle + \langle E_R|E_R\rangle \langle E_R|E_L\rangle \\
&= \sum_{i=L,R} \langle E_R|E_i\rangle \langle E_i|E_L\rangle \\
&= \langle E_R|E_L\rangle,
\end{aligned} \tag{59}$$

and similarly for the others, we finally obtain the reduced density matrix,

$$\begin{aligned}
\hat{\rho}_S &= |\alpha|^2 |s_L\rangle \langle s_L| \\
&\quad + |\beta|^2 |s_R\rangle \langle s_R| \\
&\quad + \alpha\beta^* |s_L\rangle \langle s_R| \langle E_R|E_L\rangle \\
&\quad + \beta\alpha^* |s_R\rangle \langle s_L| \langle E_L|E_R\rangle.
\end{aligned} \tag{60}$$

As we are dealing with the double slit experiment, we are interested in how this state acts under the position operator \hat{x} . Referencing section 2.1.1, we know that

$$\text{Tr}[\hat{\rho}\hat{O}] = \langle \psi | \hat{O} | \psi \rangle \tag{61}$$

where $\hat{\rho}$ is a pure state density matrix and \hat{O} any Hermitian operator. With

$$|\psi\rangle = \alpha |s_L\rangle + \beta |s_R\rangle + \alpha |s_L\rangle |E_L\rangle + \beta |s_R\rangle |E_R\rangle, \tag{62}$$

we have that

$$\begin{aligned}
\text{Tr}[\hat{\rho}_S \hat{x}] &= \langle \psi | x | \psi \rangle = \langle x | \psi \rangle \langle \psi | x \rangle = \langle x | \rho_S | x \rangle \\
&= |\alpha|^2 |\psi_L(x)|^2 + |\beta|^2 |\psi_R(x)|^2 + 2 \text{Re}(\alpha\beta^* \psi_L(x) \psi_R^*(x) \langle E_R|E_L\rangle),
\end{aligned} \tag{63}$$

where we have defined $\psi_i \equiv \langle x | s_i \rangle$.

Let us discuss these results, starting with the latter. Anyone who has dealt with the double slit experiment knows of the characteristic *interference pattern* result of it and the general rule concerning it. As Gasiorowicz put it⁸:

if the paths are not determined, add the wave function and square; if the paths are determined, square the wave function and add.

Looking at equation (63), we wish to analyse the overlap of $\langle E_R|E_L\rangle$. We realize that this term resides as an added factor in the interference term of the double slit experiment model without an environment. As can easily be seen, we have that

$$\begin{aligned}
& \langle E_R|E_L\rangle = 1 \\
\implies \text{Tr}[\hat{\rho}_S \hat{x}] &= |\alpha|^2 |\psi_L(x)|^2 + |\beta|^2 |\psi_R(x)|^2 + 2 \text{Re}(\alpha\beta^* \psi_L(x) \psi_R^*(x)) \\
&= |\alpha\psi_L(x) + \beta\psi_R(x)|^2,
\end{aligned} \tag{64}$$

⁸Gasiorowics, S.; "Quantum Physics"; John Wiley Sons, Inc., third edition, p. 30, (2003).

and

$$\langle E_R|E_L\rangle = 0 \implies \text{Tr}[\hat{\rho}_S \hat{x}] = |\alpha|^2 |\psi_L(x)|^2 + |\beta|^2 |\psi_R(x)|^2 = |\alpha \psi_L(x)|^2 + |\beta \psi_R(x)|^2. \quad (65)$$

We can conclude that the overlap of $\langle E_R|E_L\rangle$ is vital for the appearance of interference, as it regulates the appearance of the interference term. When $|E_L\rangle$ and $|E_R\rangle$ are indistinguishable, we obtain a result which is in line with the result we obtain if the experiment was run in a perfect vacuum. If, however, $|E_L\rangle$ and $|E_R\rangle$ are fully distinguishable, no interference pattern is measured. Of course, $\langle E_R|E_L\rangle$ is not restrained to only being equal to 0 or 1, and could be realised as $0 \leq \langle E_R|E_L\rangle \leq 1$. In this case, we measure an *interference pattern of reduced visibility*, meaning that there is a probability of $p = 1 - |\langle E_R|E_L\rangle|^2$ to observe a particle influenced by interference.

What are we to make of this? Given the structure of $\langle E_R|E_L\rangle$, we realise that the environment could give us *which-way information* about system in question, if we were to make a measurement of the environment. The determination of paths that Gasiorowicz speaks of can only be realized through some form of interaction with the double slit experiment inside of a perfect vacuum during measurements, and it is this that the introduced environment has succeeded in doing, if only $\langle E_R|E_L\rangle \neq 1$. It is important to recall that we are measuring the state $\hat{\rho}_S$, and not $\hat{\rho}_{SE}$. Indeed, measurements on $\hat{\rho}_S$ with full distinguishability between $|E_L\rangle$ and $|E_R\rangle$ reveals that interference has disappeared, but it has not disappeared from the $\hat{\rho}_{SE}$ state as a whole. Interference is again introduced if we were to measure the complete composite system, $|s_L\rangle |E_L\rangle$ and $|s_R\rangle |E_R\rangle$.

I have deliberately skipped the conversation of time evolution in this preliminary discussion and have settled with semi-evolutionary terms such as "before" and "after" entanglement. However, a preliminary discussion must include some discussion of it. As might have become apparent, a more general formulation of equation (55) is given by the form

$$\left(\sum_i c_i |s_i\rangle \right) |E(t=0)\rangle \longrightarrow \sum_i c_i |s_i\rangle |E_i(t)\rangle \quad (66)$$

where as the system evolves, the environment $|E_i(t)\rangle$ evolves in unison with the system state $|s_i\rangle$. As a concrete example, this could model a slit experiment with $i = 1, 2, \dots, n$ number of slits. It is of central importance to determine in general the overlap $\langle E_j(t)|E_i(t)\rangle$ over time to model all decoherence processes. Many decoherence models finds this overlap to evolve as⁹

$$\langle E_j(t)|E_i(t)\rangle \propto e^{-\frac{t}{\tau}} \quad (67)$$

⁹For examples: Schlosshauer, M.; "The quantum-to-classical transition and decoherence", arXiv:1404.2635v2 [quant-ph], ch. 5 (prerequisite at least ch. 4 to understand *Master equations*), (2019).

for $i \neq j$, where τ is a constant. As we have seen in our two examples discussed above, these factors are found in the interference or coherence terms, and through this proportionality we realise that these terms asymptotically decay away in a duration dictated by τ . We can also deduce the chronology of events in our double slit experiment model, as our system just before passing the double slit is modeled by equation (64), to directly realise an interference pattern of reduced visibility which asymptotically evolves into the classical prediction of equation (65), as can be seen from equation (67). This process should be understood as the rapid increase of entanglement between system and environment, where the potentially very numerous *degrees of freedom* of the environment (velocities, rotational and vibrational states, etc.) continuously change in accordance with the superposition of states the system introduces to the environment. Hence, we should expect a very large overlap between $\langle \psi_{out}^{(m)} | \psi_{out}^{(n)} \rangle$ in our first example because we only modeled one particle entanglement, while in our second example the rapidity of decoherence should be determined by the concentration of the gas. In essence, the processes outlined here is the pure quantum mechanical effect whereby coherence terms asymptotically disappear when measuring the system after system-environment entanglement; *de*-coherence.

3 The Measurement Problems

In what is perhaps the most infamous problem of physics and the philosophy of physics, *The Measurement Problem* remains a hair tearing riddle for physicists and philosophers and has done so for the better part of a century. Although the term "measurement problem" operates as a *lingua franca* among the wider physics community, a more suitable name would be *The Problem of Quantum-to-Classical Transition*. This is so because as the formulation of the problem of measurement matured over the decades, it was realised that indeed the problem can be divided into several distinct problems, all relating to the question of how nature which at its foundation is most generally and accurately described by quantum theory could realise itself as a classical theory on macroscopic scales. Nevertheless, I digress and shall henceforth refer to the problem by its household name.

What are these measurement *problems*, then? A suitable division is *the problem of preferred basis*, *the problem of observability of interference* and *The problem of definite outcomes*, and it is the goal of this chapter to explain them as adequately as possible for later treatment.

3.1 The Preferred Basis Problem

In introducing the preferred basis problem, let us indulge in a thought experiment. Suppose we set up a composition of Stern-Gerlach apparatuses as is

indicated in fig. (1). This setup is an example of a so called *reversible Stern-Gerlach apparatus*, of which the trajectories of states is also represented in fig. (1). To represent the results of the experiment in the formalism of quantum mechanics, suppose we prepared an ensemble of pure $\frac{1}{2}$ -spin states¹⁰

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \quad (68)$$

where $|\uparrow\rangle$ is the spin-up component and $|\downarrow\rangle$ is the spin-down component along the z-axis, as is customary. As this state passes through the first component of our experimental setup, the deviation of the beam into two distinguishable ones consisting of only $|\uparrow\rangle$ - and $|\downarrow\rangle$ -states respectively tells us that we have achieved an entangled state consisting of the spin component and the position of the wave packet, as

$$|\psi\rangle \otimes |\phi_0(r, t)\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\phi_\uparrow(r, t)\rangle + |\downarrow\rangle \otimes |\phi_\downarrow(r, t)\rangle) \quad (69)$$

where $|\phi_x(r, t)\rangle$ is the position of the wave packet, dependant on position in space and of time. As the two beams pass through the last two components, they will recombine and remain so after the experiment has concluded. Hence, we go from the entangled spin-position state to a separable state,

$$\begin{aligned} & \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\phi_\uparrow(r, t)\rangle + |\downarrow\rangle \otimes |\phi_\downarrow(r, t)\rangle) \rightarrow \\ & \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\phi_0(r, t)\rangle + |\downarrow\rangle \otimes |\phi_0(r, t)\rangle) = |\psi\rangle \otimes |\phi_0(r, t)\rangle \end{aligned} \quad (70)$$

which is exactly where we started (besides a change of position in the x-axis which for our purposes are irrelevant).

¹⁰Zurek, W. H.; "Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?"; Physical Review D, Vol. 24, No. 6, p. 1516-1525, (1981).

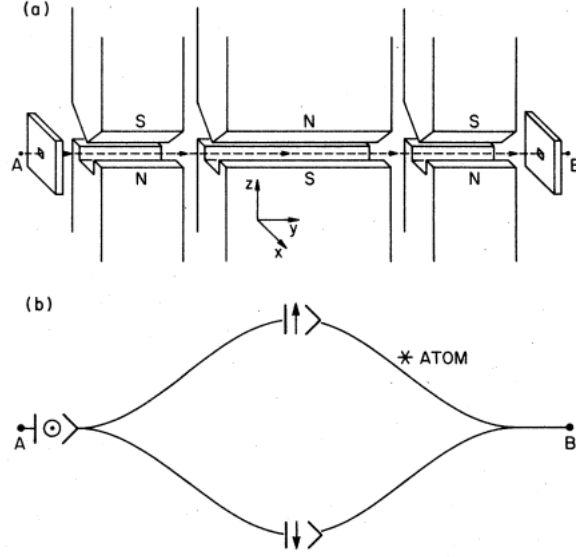


Figure 1: (a) A reversible Stern-Gerlach apparatus.

(b) spin-state trajectories in the reversible Stern-Gerlach apparatus, also including the to-be introduced bi-stable atom.

Picture copied from: Zurek, W. H.; "Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?"; Physical Review D, Vol. 24, No. 6, p. 1516-1525, (1981).

Suppose now that we add to this setup a bi-stable atom along the path of one of the trajectories taken by our $\frac{1}{2}$ -spin states, say $|\uparrow\rangle \otimes |\phi(r_{\uparrow}, t)\rangle$. We further suppose that this bi-stable atom is in the state $|\varphi_g\rangle$ initially, and that the interaction Hamiltonian between the spin state and the atom is given by

$$\hat{H}_{SA} = gv(r - r_A)(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|)(|\varphi_g\rangle\langle\varphi_e| + |\varphi_e\rangle\langle\varphi_g|) \quad (71)$$

where $|\varphi_e\rangle$ is the second stable state of the atom, g is the coupling constant and $v(r - r_A)$ is a short range interaction potential, where r_A is the position of the atom. Hence, expressing again the spin-position state, now including the atom, after the state $|\psi\rangle \otimes |\phi_{00}(r, t)\rangle$ has left the first component, we find that

$$|\Psi\rangle = |\psi\rangle \otimes |\phi_{00}(r, t)\rangle \otimes |\varphi_g\rangle \rightarrow |\Psi'\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\phi_{0\uparrow}(r, t)\rangle + |\downarrow\rangle \otimes |\phi_{0\downarrow}(r, t)\rangle) \otimes |\varphi_g\rangle. \quad (72)$$

Now, as this state becomes an entangled state between the spin-position and the atom, we must use the Schrödinger equation to embark on the further dynamics of $|\Psi'\rangle$, as

$$i\hbar \frac{\partial}{\partial t} |\Psi'\rangle = (\hat{H}_S + \hat{H}_A + \hat{H}_{SA}) |\Psi'\rangle. \quad (73)$$

In considering the evolution of $|\Psi'\rangle$ we can divide it into four orthogonal states: one which in its separate form describes the atom remaining in its ground state after being passed by the spin-up state, one in which the atom is excited after being passed by the spin-up state, one in which the atom remains in its ground state after not being passed at all and one in which the atom is excited after not being passed at all, and solve the Schrödinger equation for each as¹¹

$$i\hbar \frac{\partial}{\partial t} |\uparrow\rangle \otimes |\phi_{g\uparrow}(r, t)\rangle \otimes |\varphi_g\rangle = (\hat{H}_S + \hat{H}_A) |\uparrow\rangle \otimes |\phi_{g\uparrow}(r, t)\rangle \otimes |\varphi_g\rangle + \quad (74)$$

$$gv(r - r_A) |\uparrow\rangle \otimes |\phi_{e\uparrow}(r, t)\rangle \otimes |\varphi_e\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\uparrow\rangle \otimes |\phi_{e\uparrow}(r, t)\rangle \otimes |\varphi_e\rangle = (\hat{H}_S + \hat{H}_A) |\uparrow\rangle \otimes |\phi_{e\uparrow}(r, t)\rangle \otimes |\varphi_e\rangle + \quad (75)$$

$$gv(r - r_A) |\uparrow\rangle \otimes |\phi_{g\uparrow}(r, t)\rangle \otimes |\varphi_g\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\downarrow\rangle \otimes |\phi_{g\downarrow}(r, t)\rangle \otimes |\varphi_g\rangle = (\hat{H}_S + \hat{H}_A) |\downarrow\rangle \otimes |\phi_{g\downarrow}(r, t)\rangle \otimes |\varphi_g\rangle + \quad (76)$$

$$gv(r - r_A) |\downarrow\rangle \otimes |\phi_{e\downarrow}(r, t)\rangle \otimes |\varphi_e\rangle$$

$$i\hbar \frac{\partial}{\partial t} |\downarrow\rangle \otimes |\phi_{e\downarrow}(r, t)\rangle \otimes |\varphi_e\rangle = (\hat{H}_S + \hat{H}_A) |\downarrow\rangle \otimes |\phi_{e\downarrow}(r, t)\rangle \otimes |\varphi_e\rangle + \quad (77)$$

$$gv(r - r_A) |\downarrow\rangle \otimes |\phi_{g\downarrow}(r, t)\rangle \otimes |\varphi_g\rangle$$

where we have simply applied the known \hat{H}_{SA} to all orthogonal states, respectively. Considering only the time-dependant states of these expressions, we make the assumption that

$$|\phi_{g\uparrow}(r, t)\rangle \approx |\phi_{\uparrow}(r)\rangle \otimes |\phi_g(t)\rangle, \quad (78)$$

and similarly for the other expressions. We notice that $|\phi_{\uparrow}(r)\rangle$ is the spin-position state while $|\phi_g(t)\rangle$ is the probability amplitude of finding the bi-stable atom in its ground or excited state. This approximation is justified by assuming firstly that the interaction between atom and spin state does not change the position spin state, and secondly that because of the short interaction potential $v(r - r_A)$ the spin position state can be assumed to remain constant during the period of interaction with the atom. However, we note in passing that this is only true for the short timescale of interaction at $r \approx r_A$, and that the evolution is at all other times given by the left hand side of equation (78).

Our goal is to find the probability amplitudes $|\phi_g(t)\rangle$ and $|\phi_e(t)\rangle$. We shall only calculate $|\phi_g(t)\rangle$ and $|\phi_e(t)\rangle$ as they appear in equation (74) and (75) due to assumptions we are to introduce later. The states $|\phi_{g\uparrow}(r, t)\rangle$ and $|\phi_{e\uparrow}(r, t)\rangle$

¹¹Scully, M. O.; Shea, R.; "State Reduction in Quantum Mechanics: A Computational Example"; Physics Reports (Section C of Physics Letters) Vol. 43, No. 13, p. 485-498 (1978).

are as known one degree of freedom in our system expressed in their own Hilbert space, and we therefore only consider their evolution as

$$i\hbar \frac{\partial}{\partial t} |\phi_{g\uparrow}(r, t)\rangle = (\hat{H}_S + \hat{H}_A) |\phi_{g\uparrow}(r, t)\rangle + gv(r - r_A) |\phi_{e\uparrow}(r, t)\rangle \quad (79)$$

$$i\hbar \frac{\partial}{\partial t} |\phi_{e\uparrow}(r, t)\rangle = (\hat{H}_S + \hat{H}_A) |\phi_{e\uparrow}(r, t)\rangle + gv(r - r_A) |\phi_{g\uparrow}(r, t)\rangle. \quad (80)$$

Making use of equation (78) and letting $\int dr^3 \langle \phi_{\uparrow}(r) |$ act on each side, we obtain

$$i\hbar \frac{\partial}{\partial t} |\phi_g(t)\rangle = (\hat{H}_S + \hat{H}_A) |\phi_g(t)\rangle + g \int dr^3 \langle \phi_{\uparrow}(r) | v(r - r_A) |\phi_{\uparrow}(r)\rangle |\phi_e(t)\rangle \quad (81)$$

and similarly for equation (80). Assuming for simplicity that $\hat{H}_S + \hat{H}_A = 0$, we find the coupled differential equations

$$\frac{\partial}{\partial t} |\phi_g(t)\rangle = \frac{g}{i\hbar} \int dr^3 \langle \phi_{\uparrow}(r) | v(r - r_A) |\phi_{\uparrow}(r)\rangle |\phi_e(t)\rangle \quad (82)$$

$$\frac{\partial}{\partial t} |\phi_e(t)\rangle = \frac{g}{i\hbar} \int dr^3 \langle \phi_{\uparrow}(r) | v(r - r_A) |\phi_{\uparrow}(r)\rangle |\phi_g(t)\rangle, \quad (83)$$

which are easily solved as

$$|\phi_g(t)\rangle = \cos\left(\frac{g}{\hbar} \int \int dt dr^3 \langle \phi_{\uparrow}(r) | v(r - r_A) |\phi_{\uparrow}(r)\rangle\right) \equiv \cos A(t) \quad (84)$$

$$|\phi_e(t)\rangle = -i \sin\left(\frac{g}{\hbar} \int \int dt dr^3 \langle \phi_{\uparrow}(r) | v(r - r_A) |\phi_{\uparrow}(r)\rangle\right) \equiv -i \sin A(t). \quad (85)$$

Thus, we have found the probability amplitudes we were looking for. Assuming that the atom never interacts with the spin down states due to the short range interaction potential $v(r - r_A)$, we set $|\phi_g(t)\rangle = 1$ and $|\phi_e(t)\rangle = 0$ in equations (76) and (77) respectively, which without this assumption leads to an equivalent solution for the probability amplitudes. This assumption leads us to the final solution

$$|\Psi'\rangle = |\uparrow\rangle \otimes [\cos A(t) |\varphi_g\rangle - i \sin A(t) |\varphi_e\rangle] \otimes |\phi_{\uparrow}(r)\rangle + |\downarrow\rangle \otimes |\varphi_g\rangle \otimes |\phi_{\downarrow}(r)\rangle. \quad (86)$$

As evident from the probability amplitude coefficients, the bi-stable atomic state

$$|\Psi'_{g/e}\rangle = \cos A(t) |\varphi_g\rangle - i \sin A(t) |\varphi_e\rangle \quad (87)$$

oscillates in \mathbb{S}^1 due to its time dependence. However, after the spin states have passed the last component, it is easily seen that

$$\lim_{t \rightarrow \infty} A(t) = A < \infty \quad (88)$$

because of the failure of assumption in equation (78) when $r \approx r_A$, where A is a constant. Thus,

$$|\Psi'\rangle = |\uparrow\rangle \otimes [\cos A |\varphi_g\rangle - i \sin A |\varphi_e\rangle] \otimes |\phi_\uparrow(r)\rangle + |\downarrow\rangle \otimes |\varphi_g\rangle \otimes |\phi_\downarrow(r)\rangle. \quad (89)$$

when $t \rightarrow \infty$. To further our discussion away from the discussion of reversible evolution of quantum states and into the irreversible dynamics of measurement, we choose the suitable constant $A = \frac{\pi}{4}$ and obtain

$$|\Psi'\rangle = \frac{1}{\sqrt{2}} [|\downarrow\rangle \otimes |\varphi_g\rangle - i |\uparrow\rangle \otimes |\varphi_e\rangle] \otimes |\phi_0(r)\rangle \quad (90)$$

where as before $|\phi_{\uparrow/\downarrow}(r)\rangle \rightarrow |\phi_0(r)\rangle$. The usefulness of the choice of A should be apparent from the fact that spin up and down states now are 100% correlated with excited and ground state of the atom, respectively.

With equation (90) we see that a reasonably realistic physical pathway for obtaining 100% correlation in the $(|\varphi_g\rangle, |\varphi_e\rangle)$ basis has been achieved. Indeed,

$$|\varphi_g\rangle \langle \varphi_g | \Psi' \rangle = |\downarrow\rangle \otimes |\varphi_g\rangle \otimes |\phi_0(r)\rangle \quad (91)$$

$$|\varphi_e\rangle \langle \varphi_e | \Psi' \rangle = -i |\uparrow\rangle \otimes |\varphi_e\rangle \otimes |\phi_0(r)\rangle. \quad (92)$$

Of course, a standard experimental interpretation regarding the application of the operators in equation (91) and (92) would be that a measurement of the observables $|\varphi_g\rangle \langle \varphi_g|$ or $|\varphi_e\rangle \langle \varphi_e|$ in the $(|\varphi_g\rangle, |\varphi_e\rangle)$ basis has occurred on the system, in this case our bi-stable atom state. However, we shall see that this reasoning is faulty and incomplete. For suppose that we would employ a change of basis, $(|+\rangle, |-\rangle)$ and $(|\rightarrow\rangle, |\leftarrow\rangle)$, where

$$|+\rangle = \frac{1}{\sqrt{2}} (|\varphi_g\rangle + |\varphi_e\rangle) \quad (93)$$

$$|-\rangle = \frac{1}{\sqrt{2}} (|\varphi_g\rangle - |\varphi_e\rangle) \quad (94)$$

and

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle + i |\downarrow\rangle) \quad (95)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle - i |\downarrow\rangle) \quad (96)$$

which is to be interpreted as a system whose basis is aligned with the y-axis instead of the z-axis, under the same experimental setup as presented above. Represented in this new basis, equation (90) becomes

$$|\Psi'\rangle = -\frac{i}{\sqrt{2}}[|\rightarrow\rangle \otimes |+\rangle - |\leftarrow\rangle \otimes |-\rangle] \otimes |\phi_0(r)\rangle \quad (97)$$

which is obtained through solving equations (93) and (94) for $|\varphi_g\rangle$ and $|\varphi_e\rangle$, and (95) and (96) for $|\uparrow\rangle$ and $|\downarrow\rangle$, followed by a simple substitution into equation (90). Applying a new set of observables represented in this basis, we find that

$$|+\rangle \langle +|\Psi'\rangle = -i|\rightarrow\rangle \otimes |+\rangle \otimes |\phi_0(r)\rangle \quad (98)$$

$$|-\rangle \langle -|\Psi'\rangle = -i|\leftarrow\rangle \otimes |-\rangle \otimes |\phi_0(r)\rangle, \quad (99)$$

which is again a 100% correlated system, now represented in the $(|+\rangle, |-\rangle)$ basis. It is important to note that the projection of these new operators upon state (90) gives us the same answer, and a projection of $|\varphi_g\rangle \langle \varphi_g|$ or $|\varphi_e\rangle \langle \varphi_e|$ upon state (97) gives us state (91) and (92), respectively, as is obviously the case due to their equivalence.

It is here that our trouble starts. Indeed, it appears as though our experiment achieves 100% correlation not only in the z-axis, but also the y-axis. This means that as our bi-stable atom interacts with the spin state in the $(|\uparrow\rangle, |\downarrow\rangle)$ basis, it has seemingly also interacted with the spin state in the $(|\rightarrow\rangle, |\leftarrow\rangle)$ basis. Worryingly, we also note that as we apply operators of the bi-stable atom state in our separate bases, we obtain definite spin states in both the x-axis *and* y-axis, seemingly violating the non-commutability of the pauli matrices σ_z and σ_y . This stands in direct contradiction with the laws of quantum mechanics and demands an explanation. Absenting ourselves from the unintuitive nature of quantum mechanics, these results also offer little in light of common sense. It seems as though we have constructed a device which naturally measures several physical quantities in *one* measurement, exhibiting little regard for our experimental setup prepared to measure spin states in the z-axis. Also, as an oriented reader might have appreciated, our experimental setup is an example of a *delayed choice experiment*¹² if we were to set up a measurement apparatus recording the bi-stable atom state after the contraption in figure 1 (a); seemingly, we can inquire the relation between the bi-stable atom and spin state *after* interaction between them has occurred. It is an objectionable conclusion, but we could reasonably (but incorrectly) conclude that the choice of basis is not purely a convention but a defining feature in the isomorphism between theory and outcome.

These are ponderings concerning the *preferred basis problem*. The core of the problem deals with the question of what actually singles out the preferred physical quantities measured in an experiment, e.g. observables in the $(|\varphi_g\rangle, |\varphi_e\rangle)$ or $(|+\rangle, |-\rangle)$ basis¹³. Alternatively, one could grasp the question by asking what

¹²Wheeler, J. A.; In "Mathematical Foundations of Quantum Theory" as "The 'Past' and the 'Delayed-Choice' Double-Slit Experiment"; Elsevier Inc, p. 9-48, (1978).

¹³Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, p. 50, (2008).

observable (and in turn, in what basis) even becomes determinate in the first place, before asking questions regarding which eigenstate of said observable realised itself in a measurement¹⁴. This also makes the preferred basis problem a primary obstacle to overcome before even considering a discussion of the more famous and soon to be discussed problem of outcomes.

It should be noted that these problems are present in a more general framework than the experiment presented here¹⁵. Indeed, given a measurement scheme of the form

$$|\Psi\rangle = |\psi\rangle \otimes |a_0\rangle = \left(\sum_i c_i |s_i\rangle\right) \otimes |a_0\rangle \longrightarrow |\Psi'\rangle = \sum_i c_i |s_i\rangle \otimes |a_i\rangle \quad (100)$$

we can express the right hand side in *any* basis

$$|\Psi'\rangle = \sum_i c_i |s_i\rangle \otimes |a_i\rangle = \sum_i c'_i |s'_i\rangle \otimes |a'_i\rangle, \quad (101)$$

provided that all states $|a_i\rangle$ are mutually orthogonal. Although, we know from the Schmidt decomposition theorem that if $c_i \in \mathbb{R}$, $\sum c_i^2 = 1$ and $c_i \neq c_j \ \forall c_i, c_j$:

$$|\Psi'\rangle = \sum_i c_i |s_i\rangle \otimes |a_i\rangle \quad (102)$$

is unique. If this uniqueness is kept throughout the duration of an experiment, the preferred basis problem does not present itself. But, as we have seen, it is when we allow for $c_i = c_j$ that the problem of preferred basis becomes an obstacle.

3.2 The Problem of Non-Observability of Interference

Superposition, diffraction, interference: the language of wave mechanics is prevalent in quantum theory. Indeed, the Schrödinger equation, as one of the most renowned wave equations, uniquely determines the undisturbed evolution of quantum states. With a theory as general and precise as quantum mechanics, it may seem strange that this language of waves is not prevalent *everywhere*. Sure, by the dock of a lightly breezed bay, in your jostled cup of coffee or in the vibrating strings of an orchestra, wave mechanics are certainly very useful for describing the relevant dynamics of each system. But then, why is it *not* useful concerning for example the motion of macroscopic objects, in light of quantum theory? Why do you as a macroscopic object not diffract when moving through a door, or interfere with yourself when passing by a pillar on the sidewalk? Why is this fundamental language of quantum mechanics so absent in our everyday

¹⁴Barrett, J. A.; "The Preferred-Basis Problem and the Quantum Mechanics of Everything"; Brit. J. Phil. Sci. Vol. 56, No. 2, p. 199-220, (2005).

¹⁵Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, p. 53-55, (2008).

experience?

The Problem of Non-Observability of Interference concerns a very simple question: *Why is it so difficult to observe interference phenomena on macroscopic scales?* Given the apparent ease at which superpositions of states seemingly affects other definite quantum states, as represented in equation (100), why is it that interference effects are only observed for microscopic objects? Certainly, there is nothing stopping us from modeling all objects, being microscopic or macroscopic, as quantum states and using the measurement scheme presented, no matter how complicated these states might be. At the very least, nothing is obviously suggested by the scheme to disallow us from this.

In light of this, let us consider the work of Tonomura *et. al.*¹⁶ who realised a double path (bi-prism) single-electron interference experiment, producing a movie where individual detection events were recorded on a detector screen. With a de Broglie wavelength of approximately $\lambda_e = 0.054 \text{ \AA}$ for the electrons and a double-path spacing of $d = 10 \text{ mm}$ they successfully obtained interference fringes 7000 \AA in size, throughout the entire detector screen with a diameter of 10 mm . A few snapshots from this movie are presented in fig. 1, where the detector screen has been magnified 2000x. We can easily see how the interference pattern builds up over time, and after 70000 electron detections the interference fringes are distinct. Compare these results to Zeilinger *et. al.*¹⁷, who realised a double path (bi-prism) single-*neutron* interference experiment. With a de Broglie wavelength of approximately $\lambda_n = 18.45 \text{ \AA}$ for the neutrons and double path spacing of $d = 150 \text{ }\mu\text{m}$, interference fringes on the order of $100 \text{ }\mu\text{m}$ were observed (10^6 \AA), a result which is provided schematically in fig. 2. In this figure, every dot represents one measurement point on the detector corresponding to 500 s of neutron counting, performed 15 times. The solid curve is the theoretical prediction. When comparing these two results, it should be stressed that the experimenters did not set out to perform the experiments with a main goal of making points regarding the non-observability of interference. Rather, Tonomura *et. al.* set out to clearly show the buildup of electron interference, while Zeilinger *et. al.* set out to as precisely as possible compare experimental results with standard theoretical predictions. Nonetheless, these results give us an excellent starting point for discussing the problem of non-observability of interference.

¹⁶Tonomura, A.; Endo, J.; Matsuda, T.; *et. al.*; "Demonstration of single-electron buildup of an interference pattern"; American Journal of Physics, Vol. 57 No. 2, p. 117-120 (1989).

¹⁷Zeilinger, A.; Gähler, R.; Shull, C. G.; Treimer, W.; Mampe, W.; "Single- and double-slit diffraction of neutrons"; Rev. Mod. Phys. , Vol. 60, No. 4, p. 1067-1073 (1988).

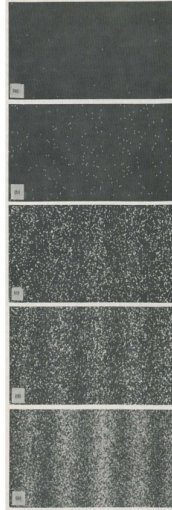


Figure 2: Buildup of interference from the double path single-electron interference experiment performed by A. Tonomura et al. the detector screen has been magnified 2000x. The total number of electrons detected at different snapshot times are, from the top: 10, 100, 3000, 20000, 70000.

Picture copied from: Tonomura, A.; Endo, J.; Matsuda, T.; et. al; "*Demonstration of single-electron buildup of an interference pattern*"; American Journal of Physics, Vol. 57 No. 2, p. 120 (1989).

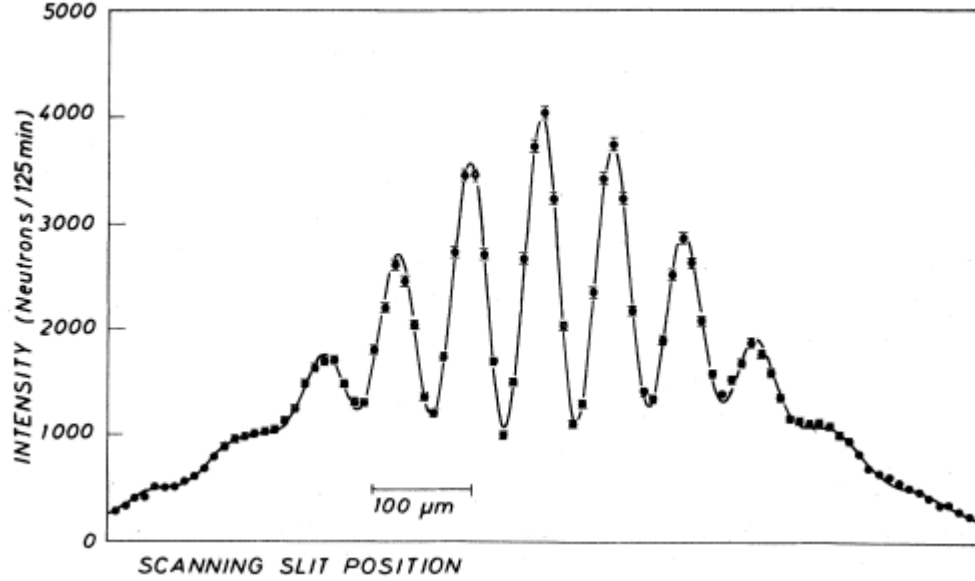


Figure 3: Results and prediction of a double path (bi-prism) single-neutron interference experiment performed by A. Zeilinger *et. al.* Every dot represents one measurement point on the detector corresponding to 500 s of neutron counting, performed 15 times. The solid curve is the theoretical prediction. Picture copied from: Zeilinger, A.; Gähler, R.; Shull, C. G.; Treimer, W.; Mampe, W.; "Single- and double-slit diffraction of neutrons"; Rev. Mod. Phys., Vol. 60, No. 4, p. 1072 (1988).

What do a comparison of these experiments show? For one thing, the interference experiment on neutrons have to be carried out in a more special setting than the electron interference experiment. To obtain a de Broglie wavelength of approximately $\lambda_n = 18.45 \text{ \AA}$, the neutrons have to be extremely slow compared to the electrons ($\frac{v_n}{v_e} \approx 1.6 \cdot 10^{-4}$). What's more, the bi-prism slit spacing has to be much shorter in the neutron experiment ($\frac{d_n}{d_e} = 0.015$). In short, perform A. Tonomura *et. al.* experiment with neutrons, and an interference pattern would be incredibly hard to decipher. Concerning these facts, the reader might think that this adequately explains the problem of non-observability of interference. For, even though the particles are represented by wave packets, the fact that interference phenomena is harder to observe for higher frequency waves due to the difficulty of producing slits with adequately small spacing is fully explained by classical wave mechanics. However, concerning such an objection, I refer the reader to section 2.2 of this essay. Indeed, we saw how the introduction of environmental degrees of freedom in a hypothetical double slit experiment resulted in reduction of interference, a result differing sharply from any classical

prediction. Clearly, the difficulty of observing interference phenomena of non-microscopic objects is not *only* due to a practical problem concerning particle velocities and slit-spacing sizes, and the problem deserves an analysis carried out on pure quantum mechanical grounds.

Actually, concerning section 2.2, we can say that this problem has more or less already been discussed and even solved in this essay already. In discussing the nature of decoherence, we realised that it is nothing more than the local suppression of system coherence terms when letting environmental degrees of freedom entangle with it. Given the large set of degrees of freedoms present in any macroscopic object, this would adequately describe why interference is not observed on this scale. A more thorough discussion about this will be found in section 4.2, as well as references to some demonstrative experiments.

3.3 The Problem of Definite Outcomes

It was von Neumann who first rigorously made an account of the entanglement process of quantum mechanics^{18,19}. *The von Neumann measurement scheme*, which we have used extensively in this text, has been one of the standard means of introducing a formalism of entanglement ever since its introduction in 1932, and it is time for us to more rigorously give an account of it. Despite its name, the key question that von Neumann wanted to answer was how two separate systems could give rise to an entangled composite system under a time evolution of states. As a process arising from the superposition principle and the linearity of the Schrödinger equation, it is quite far from the process of *Measurement* with capital M that will be discussed hereafter. With this in mind, suppose that $|\Psi\rangle \in \mathcal{H}_S$ is a microscopic system with basis states $|\psi_i\rangle$, and that $|A\rangle \in \mathcal{H}_A$ is a macroscopic system with basis states $|a_j\rangle$. $|A\rangle$ is the state of our measuring apparatus, and $|\Psi\rangle$ is the state of whatever object we wish the apparatus to make a measurement on. Further suppose that the state of the apparatus first is given in an initial state $|A\rangle = |a_0\rangle$ and that the state of the object we wish to measure can be expanded as $|\Psi\rangle = \sum_i c_i |\psi_i\rangle$, under the normalization constraint $\sum_i |c_i|^2 = 1$. The entanglement process between object and apparatus is given by

$$|\Psi\rangle \otimes |A\rangle = \left(\sum_i c_i |\psi_i\rangle \right) \otimes |a_0\rangle \longrightarrow \sum_i c_i |\psi_i\rangle \otimes |a_i\rangle, \quad (103)$$

where the arrow signifies a linear time evolution of the separable states. Indeed, returning to section 2.1.2, we see that the object-apparatus system under time evolution goes from a separable system to an entangled system.

¹⁸von Neumann, J.; *Mathematical Foundations of Quantum Mechanics*, Translated from german by Beyer, Robert T.; Princeton University press, Copyright renewed (1983) (original german version published 1932).

¹⁹Johansson, L.-G.; *Interpreting Quantum Mechanics: A Realistic view in Schrödingers Vein*; Ashgate Publishing Company, (2007).

However, a few considerations arise under this formalism. First of all, we said that the apparatus was *macroscopic*. In trying to give an interpretation of the right hand side of equation (103), we can quite clearly see that the apparatus goes from being in the definite position $|a_0\rangle$ to a superposition of states, entangled to the object we wish to measure. Considering a suitable macroscopic measurement apparatus, for example one in which each basis state $|a_j\rangle$ of $|A\rangle$ correspond to a *pointer* position: a literal, visible needle or arrow made to point in different directions corresponding to each possible measurement result on the object, equation (158) seem to imply that this macroscopic pointer is in a superposition of states. Of course, such superpositions of macroscopic objects are never observed. What *is* observed, of course, is the pointer in one *specific* state $|a_i\rangle$, corresponding uniquely to one of the states $|\psi_i\rangle$. Frustratingly, it seems as though the von Neumann measurement scheme skips a step. No less, the scheme seems to skip the very act of measurement²⁰.

How would we explain what is observed? The answer to this question is quite simple. As an entangled state has been prepared in accordance with equation (103), we make a *Measurement* with capital M of the object with our apparatus and obtain a definite pointer state, as is described by the process

$$\sum_i c_i |\psi_i\rangle \otimes |a_i\rangle \longrightarrow |\psi_k\rangle \otimes |a_k\rangle. \quad (104)$$

To put it at its mildest, this transition leaves some questions unanswered.

Firstly, this time evolution is not like any other in quantum mechanics. In fact, as Wigner showed and stated thusly:

*Measurements which leave the system object-plus-apparatus in one of the states with a definite position of the pointer cannot be described by the linear laws of quantum mechanics.*²¹

This is not difficult to show, and the proof goes as follows²². Assuming for simplicity the separation of variables $\Psi(r, t) = \psi(r)\varphi(t)$, we know that any wave function has a time dependence of the form

$$\varphi(t) = Ae^{-i\omega t} = A(\cos \omega t + i \sin \omega t). \quad (105)$$

Of course, any linear combination of states that are themselves solutions to the Schrödinger equation is itself a solution, and therefore

²⁰Here I am only referencing the measurement scheme itself: von Neumann had influential ideas regarding the measurement process, and certainly did not "skip" a step when contemplating the measurement process.

²¹Wigner, E. P.; "The Problem of Measurement"; American Journal of Physics, Vol. 31, No. 6, p. 6-15 (1963).

²²This is not the proof given by Wigner in [21], but it serves the same purpose.

$$\Psi(r, t) = \sum_i \psi(r_i) A_i e^{-i\omega_i t} \quad (106)$$

is a solution. To model a measurement of the form that equation (104) describes, we would have to write

$$\sum_i \psi(r_i) A_i e^{-i\omega_i t} \longrightarrow \psi(r_k) A_k e^{-i\omega_k t_0}, \quad (107)$$

where t_0 is an arbitrary point in time where we wish for a measurement to occur and $A_i = 0 \quad \forall i \neq k$. Of course, the time evolution in equation (107) could be a result of linearity, as there is nothing to stop the evolution from satisfying $A_i = 0 \quad \forall i \neq k$ at any number of times t . However, if this was the case we would expect the evolution to again result in the form

$$\psi(r_k) A_k e^{-i\omega_k t_0} \longrightarrow \sum_i \psi(r_i) A_i e^{-i\omega_i T} \quad (108)$$

for an arbitrary $T > t_0$. This is not what is observed, instead

$$\psi(r_k) A_k e^{-i\omega_k t_0} \longrightarrow \psi(r_k) A_k e^{-i\omega_k T} \quad (109)$$

for all $T > t_0$, if the state is not allowed to develop further. Thus, the measurement process as described by equation (159) is not a linear, or for that matter a reversible or continuous, process. Again, as is excellently explained by Wigner²³, quantum states evolve *linearly, deterministically, continuously and reversibly*, except when we make a measurement of it. During the measurement process, provided that the state of the object we measure is not an eigenstate of the observable we would like to measure, the state evolves *non-linearly, non-deterministically, discontinuously and irreversibly*. As we will discuss later, this statement is somewhat controversial. What is not controversial however, is the fact that this is what we *de facto* observe to happen, which, for example, is demonstrated in the electron interference experiment of section 3.2.

Secondly, what equation (104) has done is to describe what is actually being seen, but it has little explanatory power. Two questions has to be answered here. Firstly, *why do measurements have outcomes at all?* As was discussed earlier, the von Neumann measurement scheme seem to suggest that the apparatus exist in a superposition of states after it has entangled with the object we would like to make a measurement on. *Why* do we even observe states in definite positions given this account? Why do we not observe the apparatus in a superposition of states?²⁴ Secondly, if we are able to describe why measurements have definite outcomes, *why do we observe a particular state over any*

²³Wigner, E. P.; "Interpretation of Quantum Mechanics"; In Wheeler, J. A. and Zurek, W. "Quantum Theory and Measurement", Princeton University press, p. 240-314, (1983).

²⁴This point is similar but subtly different from the problem of non-observability of interference. While the problem of non-observability of interference concerns the fact that interference phenomena born out of the superposition principle are not observed, this problem concerns why superpositions *in general* are never observed.

other realisable one? Of course, given the superposition of states on the right hand side of equation (103), the probability of measuring the state $|\psi_k\rangle \otimes |a_k\rangle$ is $|c_k|^2$, as is dictated by the Born rule. Prepare and perform the same experiment again and we might obtain any other realisable state. The question refers to the *non-deterministic* evolution that Wigner described: what is behind this selection process? Of course, these two questions make up *The problem of outcomes*, one of the most notoriously infamous problems in all of physics.

4 Decoherence and the Measurement Problems

It is in all likelihood a remnant from the familiar dynamics of classical physics that it was only long after the birth of quantum mechanics that environmental dynamics were fully appreciated. As we will discover, there exists a lovely irony in the fact that the uniqueness of the quantum mechanical formalism, especially through entanglement, could give important clues towards a dynamical, physical story of how quantum mechanics gives rise to a strikingly non-quantum world of our everyday experience.

In this section, we will discuss all measurement problems considered in the last section and discuss what decoherence theory has to say about them, specifically by introducing the theory more formally through environment-induced superselection rules. As is not surprising for an essay concerning the measurement problems, we will end this chapter by discussing some interpretational issues as a prerequisite for the next chapter.

4.1 Environment-Induced Superselection Rules

Concerning the problems of measurement described above, a game-changing theoretical framework was found in so called *environment-induced superselection rules*²⁵. Clearly a name referencing common selection rules inhibiting certain transitions set by the Hamiltonian, *superselection rules* put constraints on all conceivable dynamical evolution²⁶. In general, a characterisation of a discrete set of superselection rules can be built by postulating a finite or countably infinite set of mutually orthogonal and exhaustive projection operators \hat{P}_i defined on a Hilbert space \mathcal{H} such that all observables commute with all \hat{P}_i . Specifying this for only two so called *superselection sectors*, we can define a Hilbert space which decomposes as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$, for which vectors $|\psi\rangle_A \in \mathcal{H}_A$ and $|\psi\rangle_B \in \mathcal{H}_B$ under the action of any operator \hat{O}

$$\hat{O} |\psi\rangle_A \in \mathcal{H}_A \quad (110)$$

$$\hat{O} |\psi\rangle_B \in \mathcal{H}_B. \quad (111)$$

²⁵Zurek, W.H.; "*Environment-induced superselection rules*"; Physical Review D Vol. 26 No. 8, p. 1862-1880 (1982).

²⁶Giulini, D.; "*Superselection Rules*"; arXiv:0710.1516 [quant-ph], v.2 (2009).

Hence, as selection rules are set by the condition $\langle \psi |_A \hat{H} |\psi \rangle_B = 0$ in this framework, superselection rules are set by the condition

$$\langle \psi |_A \hat{O} |\psi \rangle_B = 0 \quad (112)$$

for all operators \hat{O} . Crucially, this condition makes it impossible for any given operator to produce coherent superpositions between the states $|\psi \rangle_A$ and $|\psi \rangle_B$. For, given a state $|\psi \rangle_{\pm} = \frac{1}{\sqrt{2}}(|\psi \rangle_A \pm |\psi \rangle_B)$, we realise that

$$\langle \psi |_{\pm} \hat{O} |\psi \rangle_{\pm} = \frac{1}{2} (\langle \psi |_A \hat{O} |\psi \rangle_A \pm \langle \psi |_B \hat{O} |\psi \rangle_B) = \text{Tr}[\hat{\rho} \hat{O}], \quad (113)$$

where

$$\hat{\rho} = \frac{1}{2} (|\psi \rangle_A \langle \psi |_A \pm |\psi \rangle_B \langle \psi |_B). \quad (114)$$

This density operator is obviously mixed, as it is given by a convex combination of pure states, see sec. 2.1.1. Hence, we realise that no coherent superpositions of states between \mathcal{H}_A and \mathcal{H}_B can be measured.

Following its conception in 1952²⁷, superselection rules functioned as an offensive line towards the problem of measurement well into the 1970:s. Their role in this regard is clear: by introducing superselection rules on pointer states of a given apparatus measuring quantum states, we could get rid of the apparent superposition of pointer states that the von Neumann measurement scheme seems to imply by imposing restrictions made on purely physical grounds. However pragmatic an approach, the reader is not unjustified in thinking its use in this regard is *ad hoc*. For, while imposing superselection rules on macroscopic objects where coherence terms are not observed certainly predicts what is measured, there was indeed no known physical mechanism for why such a divide would exist.

This changed in 1981-1982 when Zurek published two papers concerning the use of environment-induced superselection rules²⁸ in order to shed light upon the problem of preferred basis. Indeed, as the name implies, environment-induced superselection rules give physical justification for inducing selection rules based on entanglement dynamics between an environment and a given system. We shall spend some time understanding how this comes to be before tackling the problem of measurement, and specifically the problem of preferred basis, as it essentially equates to a general solution for the latter which needs to be specified for clarity.

Consider now a combined Hilbert space of a system S and an environment E ,

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_E, \quad (115)$$

²⁷Wick, G. C.; Wightman, A. S.; Wigner, E. P.; "The Intrinsic Parity of Elementary Particles"; Phys. Rev., Vol. 88, No. 101, p. 102-106, (1952).

²⁸[12][28]

for which we define a Hamiltonian \hat{H} that acts in this composite system-environment Hilbert space. Further suppose that this Hamiltonian can be divided into three parts: a self-Hamiltonian \hat{H}_S of the system, a self-Hamiltonian \hat{H}_E of the environment and an interaction-Hamiltonian \hat{H}_{SE} between the two, which is most generally defined as

$$\hat{H}_S = \sum_i \delta_i |\psi_i\rangle_S \langle \psi_i|_S \quad (116)$$

$$\hat{H}_E = \sum_j \epsilon_j |\psi_j\rangle_E \langle \psi_j|_E \quad (117)$$

$$\hat{H}_{SE} = \sum_{ij} \gamma_{ij} |\psi_i\rangle_S \langle \psi_i|_S \otimes |\psi_j\rangle_E \langle \psi_j|_E + \lambda \sum_{ii'jj'} \sigma_{ii'jj'} |\psi_i\rangle_S \langle \psi_{i'}|_S \otimes |\psi_j\rangle_E \langle \psi_{j'}|_E. \quad (118)$$

In the further, we shall set $\lambda = 0$ in (118), as

$$\hat{H}_{SE}^0 = \sum_{ij} \gamma_{ij} |\psi_i\rangle_S \langle \psi_i|_S \otimes |\psi_j\rangle_E \langle \psi_j|_E. \quad (119)$$

Physically, this idealization can be understood as assuming that phase coherences of the system-environment are destroyed much faster than thermal equilibrium is achieved between them, and thus their consideration does not matter in the subsequent discussion. Representing the system-environment state vector at $t = 0$ as

$$|\Psi(t=0)\rangle = \left(\sum_i \alpha_i |\psi_i\rangle_S \right) \otimes \left(\sum_j \beta_j |\psi_j\rangle_E \right), \quad (120)$$

we know its state at any later times, as

$$|\Psi(t)\rangle = \sum_{ij} \alpha_i \beta_j e^{-i(\delta_i + \epsilon_j + \gamma_{ij})t} |\psi_i\rangle_S \otimes |\psi_j\rangle_E. \quad (121)$$

With this in hand, we would like to write out the density operator of the system as

$$\hat{\rho}_S(t) = \text{Tr}_E [|\Psi(t)\rangle \langle \Psi(t)|]. \quad (122)$$

We have that

$$\begin{aligned} |\Psi(t)\rangle \langle \Psi(t)| &= \\ & \left(\sum_{ik} \alpha_i \beta_k e^{-i(\delta_i + \epsilon_k + \gamma_{ik})t} |\psi_i\rangle_S \otimes |\psi_k\rangle_E \right) \times \\ & \left(\sum_{jk} \alpha_j^* \beta_k^* e^{+i(\delta_j + \epsilon_k + \gamma_{jk})t} \langle \psi_j|_S \otimes \langle \psi_k|_E \right) = \\ & \sum_{ijk} \alpha_i \alpha_j^* |\beta_k|^2 e^{-i[\delta_i - \delta_j]t - i[\gamma_{ik} - \gamma_{jk}]t} \left(|\psi_i\rangle_S \langle \psi_j|_S \otimes |\psi_k\rangle_E \langle \psi_k|_E \right). \end{aligned} \quad (123)$$

Then, performing the partial trace, we obtain

$$\hat{\rho}_S(t) = \sum_{ijk} \alpha_i \alpha_j^* |\beta_k|^2 e^{-i[\delta_i - \delta_j]t - i[\gamma_{ik} - \gamma_{jk}]t} |\psi_i\rangle_S \langle \psi_j|_S. \quad (124)$$

Investigating the populations and coherence terms, we find that

$$\begin{aligned} i = j &\implies \\ \hat{\rho}_S(t) &= \sum_{ik} |\alpha_i|^2 |\beta_k|^2 e^{-i[\delta_i - \delta_i]t - i[\gamma_{ik} - \gamma_{ik}]t} |\psi_i\rangle_S \langle \psi_i|_S = \\ &= \sum_i |\alpha_i|^2 \left(\sum_k |\beta_k|^2 \right) |\psi_i\rangle_S \langle \psi_i|_S = \\ &= \sum_i |\alpha_i|^2 |\psi_i\rangle_S \langle \psi_i|_S, \end{aligned} \quad (125)$$

and

$$\begin{aligned} i \neq j &\implies \\ \hat{\rho}_S(t) &= \sum_{ijk} \alpha_i \alpha_j^* |\beta_k|^2 e^{-i[\delta_i - \delta_j]t - i[\gamma_{ik} - \gamma_{jk}]t} |\psi_i\rangle_S \langle \psi_j|_S = \\ &= \sum_{ij} \alpha_i \alpha_j^* e^{-i[\delta_i - \delta_j]t} \left(\sum_k |\beta_k|^2 e^{-i[\gamma_{ik} - \gamma_{jk}]t} \right) |\psi_i\rangle_S \langle \psi_j|_S. \end{aligned} \quad (126)$$

We directly see that the population terms are time-independent, which is a consequence of the fact that we set $\lambda = 0$. The coherence terms, however, are time dependant. In analysing one part of the sum, that is, for given i and j , $i \neq j$, we first notice the trivial rotation $e^{-i[\delta_i - \delta_j]t}$ in the complex plane of a vector with length $\alpha_i \alpha_j^*$ from \hat{H}_S . What is much more interesting is the factor

$$z_{ij}(t) = \sum_k |\beta_k|^2 e^{-i[\gamma_{ik} - \gamma_{jk}]t}, \quad (127)$$

contributed by the system-environment interaction Hamiltonian \hat{H}_{SE} . Visually, this factor can be interpreted as a set of vectors of length $|\beta_k|^2$ and direction $e^{-i[\gamma_{ik} - \gamma_{jk}]t}$ in the complex plane. Given that the frequencies $\gamma_{ik} - \gamma_{jk}$ are sufficiently different for each contribution to the sum, $z_{ij}(t = 0)$ starts out as the vector $\sum_k |\beta_k|^2 = 1$, which at $t > 0$ starts to rotate in a clockwise direction. Crucially, because of the difference in frequency, all vectors in the sum will start to move out of phase at $t > 0$, decreasing the absolute value from $|z_{ij}(t = 0)| = 1$ to $|z_{ij}(t = T)| \ll 1$ for $T \gg 0$.

The acute reader may take issue with this statement, given the fact that for a given time $T > 0$, the sum will again arbitrarily closely satisfy $|z_{ij}(t = 0)| = 1$ due to the fact that z_{ij} is an almost periodic function. However, given a sufficiently large environment, or equivalently, many environmental degrees of freedom $|\psi_i\rangle_E$, even for practically small environments the translation time T_ϵ ,

defined by requesting that $1 - |z_{ij}(t)| < \epsilon$ for any given $\epsilon > 0$ at $t = 0$ and $t = T_\epsilon$ but never inside the interval $(0, T_\epsilon)$, is *very* long, and can for macroscopic environments be shown to be on an order much longer than the age of the universe²⁹. Therefore, it is safe to assume that phase coherence for relevant $\epsilon > 0$ is never achieved at $t > 0$ in any practical sense.

Remembering this, we can safely and soundly say that the average of $z_{ij}(t)$ under an infinite time will approach zero,

$$\langle z_{ij}(t) \rangle_{av} = \lim_{t \rightarrow \infty} \langle z_{ij}(t) \rangle = 0, \quad (128)$$

unless, of course, $\gamma_{ik} - \gamma_{jk} = 0$, $\forall i, j, k$. We can also calculate the average absolute value of $z_{ij}(t)$, as

$$\langle |z_{ij}(t)|^2 \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_t^{t+T} |z_{ij}(t)|^2 dt = \sum_{k,k'} p_k p_{k'} \delta(\gamma_{ik} - \gamma_{jk} + \gamma_{ik'} - \gamma_{jk'}), \quad (129)$$

where $\delta(\gamma_{ik} - \gamma_{jk} + \gamma_{ik'} - \gamma_{jk'})$ is the Kronecker delta distribution. Using these results to obtain the standard deviation Δ around the average value $\langle z_{ij}(t) \rangle_{av} = 0$, we have that

$$\Delta = \sqrt{\langle |z_{ij}(t)|^2 \rangle - \langle z_{ij}(t) \rangle_{av}^2} = \sum_{k=1}^N p_k^2 \quad (130)$$

meaning that because we have a set of discrete random variables,

$$\Delta \sim \frac{1}{\sqrt{N}}, \quad (131)$$

where N is the total number of environmental degrees of freedom. Thus, we have started from a state at $t = 0$ where all phases are coherent to at large t discover a situation in which coherence between phases progressively disappears. In equation (131), we also see that this damping of coherence terms is very effective even for a very small environment. As should already be familiar, what has been outlined here is nothing more than the process of decoherence.

So what does all this have to do with environment-induced superselection rules? Well, effectively, we have shown above that environmental entanglement with our system destroys any correlation of states corresponding to eigenvalues of \hat{H}_{SE}^0 . Given the fact that a set of system states can be degenerate in γ_{in} , they would span a subspace $\mathcal{H}_n \subset \mathcal{H}_S$, and a complete set of such subspaces would span the entire system Hilbert space,

$$\mathcal{H}_S = \bigoplus_n \mathcal{H}_n. \quad (132)$$

²⁹Tipler, F. J.; "General relativity, thermodynamics, and the Poincare cycle"; Nature, Vol. 208, No. 5719, p. 203-205, (1979).

The crux is this: under the practicalities we have discussed, an arbitrary subspace \mathcal{H}_n generally only allows for *one* pure state at the same time, which correspond to one of the time in-dependent diagonal elements of the density matrix that we have obtained through system-environment entanglement, or equivalently one term of the density matrix of similar form as equation (114). Building on this further, assuming that the coupling constant of some apparatus is much smaller than the values γ_{ij} , it is practically impossible to measure coherence terms of a system which has experienced decoherence, as we have established. It is only the population terms of the density operator, invariant under environmental entanglement, that can be. Thus, for an operator \hat{O} acting in $\mathcal{H}_n \subset \mathcal{H}_S$, we have discovered the constraint

$$|\psi_n\rangle_S \in \mathcal{H}_n \implies \hat{O} |\psi_n\rangle_S \in \mathcal{H}_n, \quad (133)$$

which of course is a generalisation of equations (110) and (111). Given the results of (132) and (133), we appreciate that environmental decoherence allows us to induce superselection rules on the system state.

These results are everything we need in order to define *the pointer observable*, an operator allowing us to find out which subspace \mathcal{H}_n a given system state $|\psi_n\rangle_S$ is found in after measurement. As can be anticipated, the pointer observable should consist of a finite or countably infinite set of mutually orthogonal and exhaustive projection operators \hat{P}_n projecting on each respective subspace, or superselection sector, \mathcal{H}_n , as

$$\hat{\Lambda} = \sum_n \lambda_n \hat{P}_n \quad (134)$$

under the constraint that $\lambda_n \in \mathbb{R}$ and $\lambda_n = \lambda_{n'} \implies n = n'$. Of course, such an operator can without loss of generality be constructed such that all projection operators are diagonal in the basis $\{|\psi_i\rangle_S\}$, and hence satisfying the relation

$$[\hat{\Lambda}, H_{SE}^0] = 0. \quad (135)$$

It should be stressed that even though these results are obtained through analysing environmental entanglement with a given system, the same relations are indeed obtained through analysing the entanglement between a measuring apparatus and a higher environment entangling with the apparatus. That is, even though our analysis above has concerned only a generic *system-environment* entanglement, we can call a part of this environment *apparatus*, dividing up the environment into two parts; the apparatus that directly entangles with the system and the higher environment that does not, given that this higher environment does not entangle with the system. Given the fact that it is this new environment, separated initially from the apparatus, that contains the enormous amounts of degrees of freedom, decoherence between them becomes incredibly strong. This in turn defines the pointer observable that was discussed above.

This is an important point. Indeed, environment-induced superselection rules laid the groundwork for tackling the problem of measurement through the decoherence process. It is therefore important to set these results in relation to the measurement problem, or *problems*, discussed in chapter 3. Let us start with the problem of preferred basis.

4.2 Decoherence and the Problem of Preferred Basis

In light of the above discussion, we return to the problem formulation of section 3.1. To reiterate, recall that we were seemingly able to obtain 100% correlations in multiple bases under some conditions, see equations (100)-(102). It is now time to investigate what our recently outlined theory of environment-induced superselection rules can teach us about this problem.

We start by considering a Hilbert space \mathcal{H}_S of a system, a Hilbert space \mathcal{H}_A of an apparatus and a Hilbert space \mathcal{H}_E of an environment, related as

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_A \otimes \mathcal{H}_E. \quad (136)$$

With these Hilbert spaces we relate each with a potential quantum state representative of different parts of our experimental setup of section 3.1. The state of a particle moving through the reversible Stern-Gerlach apparatus is our system, the bi-stable atom in its path our apparatus, and every other particle in their vicinity is our environment. Assume that the environment is itself made up by N bi-stable atoms identical in structure to the apparatus atom, with basis states $\{|\sigma_g\rangle_k, |\sigma_e\rangle_k\}$, $k = 1, 2, 3 \dots N$. Assume further that their respective self-Hamiltonians as well as the interaction-Hamiltonians between them is zero. The apparatus-environment interaction Hamiltonian, with structure assumed as

$$\hat{H}_{AE}^k = g_k (|\varphi_g\rangle\langle\varphi_g| - |\varphi_e\rangle\langle\varphi_e|) \otimes (|\sigma_g\rangle_k\langle\sigma_g|_k - |\sigma_e\rangle_k\langle\sigma_e|_k) \prod_{j \neq k} \otimes 1_j \quad (137)$$

for the environmental bi-stable atom k is however not zero, and together with every other environmental bi-stable atom make up the total interaction-Hamiltonian between apparatus and environment as

$$\hat{H}_{AE} = \sum_k^N \hat{H}_{AE}^k. \quad (138)$$

Of course, the system-environment interaction Hamiltonian is assumed to be zero. First, let us accentuate that the pointer observable is defined from the moment that the apparatus-environment interaction Hamiltonian is defined, as the pointer observable is made up of the apparatus basis states which are not perturbed by decoherence, see eq. (135). In our case, it is given by

$$\hat{\Lambda} = \lambda_1 |\varphi_g\rangle \langle \varphi_g| + \lambda_2 |\varphi_e\rangle \langle \varphi_e|, \quad (139)$$

where $\lambda_1, \lambda_2 \in \mathbb{R}$, $\lambda_1 \neq \lambda_2$. It is thus the case that the entirety of the state at $t = 0$ is given by

$$\begin{aligned} |\Phi(t=0)\rangle &= |\Psi'\rangle \prod_{k=1}^N (a_k |\sigma_g\rangle_k + b_k |\sigma_e\rangle_k) = \\ \frac{1}{\sqrt{2}} (|\downarrow\rangle \otimes |\varphi_g\rangle - i |\uparrow\rangle \otimes |\varphi_e\rangle) &\prod_{k=1}^N (a_k |\sigma_g\rangle_k + b_k |\sigma_e\rangle_k) \end{aligned} \quad (140)$$

where we have omitted the wave packet position degree of freedom which is not relevant in the further. To model apparatus-environment entanglement, we simply apply the Hamiltonian \hat{H}_{SA} to the entirety of the state, obtaining

$$\begin{aligned} |\Phi(t)\rangle &= e^{\frac{i}{\hbar} \hat{H}_{SA} t} |\Phi(t=0)\rangle = \\ \frac{1}{\sqrt{2}} |\downarrow\rangle \otimes |\varphi_g\rangle \prod_{k=1}^N (a_k e^{\frac{i}{\hbar} g_k t} |\sigma_g\rangle_k + b_k e^{-\frac{i}{\hbar} g_k t} |\sigma_e\rangle_k) - \\ \frac{i}{\sqrt{2}} |\uparrow\rangle \otimes |\varphi_e\rangle \prod_{k=1}^N (a_k e^{-\frac{i}{\hbar} g_k t} |\sigma_g\rangle_k + b_k e^{\frac{i}{\hbar} g_k t} |\sigma_e\rangle_k). \end{aligned} \quad (141)$$

Continuing to follow our route from the last section, we wish to calculate the reduced density matrix of the system-apparatus state, as

$$\begin{aligned} \hat{\rho}_{SA} &= \text{Tr}_E [|\Phi(t)\rangle \langle \Phi(t)|] = \\ \frac{1}{2} |\downarrow\rangle \langle \downarrow| \otimes |\varphi_g\rangle \langle \varphi_g| &+ \\ \frac{i}{2} z(t) |\downarrow\rangle \langle \uparrow| \otimes |\varphi_g\rangle \langle \varphi_e| - \\ \frac{i}{2} z^*(t) |\uparrow\rangle \langle \downarrow| \otimes |\varphi_e\rangle \langle \varphi_g| &- \\ \frac{1}{2} |\uparrow\rangle \langle \uparrow| \otimes |\varphi_e\rangle \langle \varphi_e| \end{aligned} \quad (142)$$

where

$$z(t) = \prod_{k=1}^N \left[\cos\left(\frac{2}{\hbar} g_k t\right) + i(|a_k|^2 - |b_k|^2) \sin\left(\frac{2}{\hbar} g_k t\right) \right]. \quad (143)$$

As it is a rather cumbersome calculation which for very similar problems have been done above, we have omitted the complete route to obtaining factors from the environmental degrees of freedom. Alas, for the population terms we obtain, after application of the partial trace,

$$\prod_{k=1}^N |a_k|^2 + |b_k|^2 = 1, \quad (144)$$

and to obtain $z(t)$ from the coherence terms,

$$\prod_{k=1}^N |a_k|^2 e^{\frac{2i}{\hbar} g_k t} + |b_k|^2 e^{-\frac{2i}{\hbar} g_k t} = \prod_{k=1}^N \left[\cos\left(\frac{2}{\hbar} g_k t\right) + i(|a_k|^2 - |b_k|^2) \sin\left(\frac{2}{\hbar} g_k t\right) \right], \quad (145)$$

where we have just used Eulers identity, Nonetheless, this is all we need for our analysis of $z(t)$, corresponding indeed to $z_{ij}(t)$ from the last section.

what concerns us now is to show that the apparatus-environment interaction has resulted in decoherence of the system-apparatus state in a particular basis. By the same token as for $z_{ij}(t)$, one can show that

$$\langle z(t) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T z(t) dt = 0 \quad (146)$$

and that

$$\langle |z(t)|^2 \rangle = \frac{1}{2^N} \prod_{k=1}^N [1 + (|a_k|^2 - |b_k|^2)^2]. \quad (147)$$

It is from this easy to see that

$$\Delta = \sqrt{\langle |z(t)|^2 \rangle - \langle z(t) \rangle^2} \implies \Delta \sim 2^{-\frac{1}{2}N}, \quad (148)$$

which, following the reasoning above, leads us to conclude that decoherence has indeed occurred in the system-apparatus state. As per the last chapter, we learnt that environmental decoherence allows us to induce superselection rules on our system. Certainly, the decoherence observed here has done the same on our system-apparatus state. Let us conceptually go through this process and discuss how it can be used to tackle the problem of preferred basis.

Is is most instructive to see how another basis behaves under the same environmental entanglement presented above. In chapter 3.1 we saw how a change of basis of (90) using (93)-(96) led to (97), which exhibited 100% correlations in the $\{|+\rangle, |-\rangle\}$ basis. It can be directly realised that a projection of the operators $|\varphi_g\rangle\langle\varphi_g|$ or $|\varphi_e\rangle\langle\varphi_e|$ on state (141) expectantly shows that it is 100% correlated in the $\{|\varphi_g\rangle, |\varphi_e\rangle\}$ basis, even after environmental entanglement. This is the same situation we saw in chapter 3.1, and we can conclude that the apparatus-environmental entanglement has indeed not changed this correlation. Now, in applying the operators $\{|+\rangle, |-\rangle\}$ upon the same state in order to investigate the correlations in the $\{|\rightarrow\rangle, |\leftarrow\rangle\}$ basis, the situation is quite different. We see that if

$$|\Phi(t)\rangle = \frac{1}{\sqrt{2}} |\downarrow\rangle \otimes |\varphi_g\rangle \otimes |\Sigma^+\rangle - \frac{i}{\sqrt{2}} |\uparrow\rangle \otimes |\varphi_e\rangle \otimes |\Sigma^-\rangle, \quad (149)$$

where

$$|\Sigma^\pm\rangle = \prod_{k=1}^N (a_k e^{\pm \frac{i}{\hbar} g_k t} |\sigma_g\rangle_k + b_k e^{\mp \frac{i}{\hbar} g_k t} |\sigma_e\rangle_k), \quad (150)$$

we have that

$$|+\rangle \langle +| \Phi(t)\rangle = \left(\frac{1}{\sqrt{2}} |\downarrow\rangle \otimes |\Sigma^+\rangle - \frac{i}{\sqrt{2}} |\uparrow\rangle \otimes |\Sigma^-\rangle \right) \otimes |+\rangle \quad (151)$$

$$|-\rangle \langle -| \Phi(t)\rangle = \left(\frac{1}{\sqrt{2}} |\downarrow\rangle \otimes |\Sigma^+\rangle + \frac{i}{\sqrt{2}} |\uparrow\rangle \otimes |\Sigma^-\rangle \right) \otimes |-\rangle. \quad (152)$$

This is an important result. We realise that a set of eigenvectors which previously did show 100% correlation in the $\{|\varphi_g\rangle, |\varphi_e\rangle\}$ basis, which can again be shown to be true by application of its respective operator on state (140), no longer exhibits this property. The states (151) and (152) are still maximally entangled, which is a consequence of apparatus-environment entanglement. Suffice to say, an operator which gave us which-state information of the system now fails completely at this task; information has leaked from the apparatus to the environment due to entanglement.

Why is this? The key to this question lies in the apparatus-environment interaction Hamiltonian. Looking at (137) and (138), we realise that, as correlations between apparatus and environment are established, the only operators which can reliably record which state the system inhabits is the operators in the Hamiltonian making up the pointer observable. The results of (149)-(152) shows us that at least one other operator which before environmental entanglement could also do this no longer can. This is indeed a result which is true in general for all observables not present in the pointer observable, as can easily be realised. What we have seen is the destruction of all correlations between system and apparatus which are unstable under apparatus-environmental entanglement, only and solely because we defined our apparatus-environment interaction Hamiltonian as we did. Alternatively, the states that are not susceptible to environmental entanglement, in our case $|\downarrow\rangle \otimes |\varphi_g\rangle$ and $|\uparrow\rangle \otimes |\varphi_e\rangle$, will keep their respective correlations after entanglement has occurred, remaining undisturbed.

This brings us to a fact that is just as down to earth as it is enlightening. Indeed, for a device to act as a *measuring* device, we have to trust in its ability to give us an accurate representation of what it has measured. In the framework of quantum mechanics, we say that an apparatus is reliable if only if

$$\left(\sum_i \alpha_i |\psi_i\rangle_S \right) \otimes |\psi_0\rangle_A \longrightarrow \sum_i \alpha_i |\psi_i\rangle_S \otimes |\psi_i\rangle_A, \quad (153)$$

absent of states of the form $|\psi_i\rangle_S \otimes |\psi_j\rangle_A$, $i \neq j$ ³⁰. What we have found is indeed the pointer basis $|\psi'_i\rangle_A$ which allows for the process

$$\left(\sum_i \alpha_i |\psi_i\rangle_S \otimes |\psi'_i\rangle_A \right) \otimes |\psi_0\rangle_E \longrightarrow \sum_i \beta_i |\psi_i\rangle_S \otimes |\psi'_i\rangle_A \otimes |\psi_i\rangle_E \quad (154)$$

where $|\alpha_i|^2 = |\beta_i|^2$, in a setup where all other pointer bases $|\psi_i\rangle_A \neq |\psi'_i\rangle_A$ satisfy $|\alpha_i|^2 \neq |\beta_i|^2$. The environmental entanglement has created a situation in which it is any persons guess what correlations actually were established between the system and measuring device, if only considering correlations not involving the pointer basis.

Why would we enquire about the correlations not involving the pointer basis? The question might seem strange at first, but it is here that we ground ourselves. *The only reason that our apparatus-environment interaction Hamiltonian has the structure it has is due to our initially prepared experimental setup.* To understand this point it is helpful to imagine the environment as being part of the immediate surroundings of our apparatus. That is, our "apparatus" is simply one very small part of a complete device which we would more commonly call an apparatus. This larger apparatus has a function, and that function is to measure what state our bi-stable atom is in. In technical terms, this means that it has the ability to select a pointer basis. If our measuring device could not achieve this, *it wouldn't be a measuring device.* We have designed it to reliably select a pointer basis, and what we have done above is essentially to explain why a measuring device made to measure something specific has done just that. Environmental dynamics needs to be considered in explaining this, an estranged fact for our intuition regarding macroscopic measuring devices.

Given the formalism of environment-induced superselection rules in relation to the preferred basis problem, we can explain how entanglement processes often give rise to quasi-classical characteristics, as indeed Zurek did in his seminal papers. Given the fact that radiation and collisions of molecules often constitute the environment in most experiments where the system-environment (alt. apparatus-environment) interaction Hamiltonian is dominating entanglement dynamics, it is the case that the respective force laws of these environment particles need to be considered to construct such a Hamiltonian. Be the potential classical or quantized, it is often dependant on position. The pointer observable, or equivalently the pointer basis for systems such as that considered above, thus has to be dependant on position to ensure that relation (135) is upheld. This explains why eigenstates of position are very sensitive to decoherence, e.g. for our double slit experiment example in section 2.2.

Similar arguments can be made for other quantities such as energy, charge,

³⁰Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, ch. 2.8, (2008).

momentum etc., who also show quasi-classical properties in experiments, considering dynamics differing from the one presented above. Indeed, in the *quantum limit of decoherence*, as opposed to *quantum measurement-limit* studied above where the system-environment Hamiltonian overrules the dynamics, the system Hamiltonian \hat{H}_S overrules the dynamics over the system-environment Hamiltonian³¹. In this case, the environmental monitoring of decoherence as discussed in section 2.3. will only be able to record quantities that are constants of motion, selecting a preferred basis of energy eigenstates. Environment-induced superselection thus reduce any superpositions of energy eigenstates, for example in atoms, whose non-existence we often take for granted but is nonetheless a result of slow environmental monitoring of the system under study.

It is important to mind ones step when considering the "leaking of information content" that was observed above. Decoherence is observed in the $\{|\downarrow\rangle \otimes |\varphi_g\rangle, |\uparrow\rangle \otimes |\varphi_e\rangle\}$ basis after tracing out environmental degrees of freedom as can be seen from the density operator (142). In tracing out environmental degrees of freedom the coherence terms are indeed reduced when considering the system-apparatus state, but not destroyed. In calculating the system-apparatus-environment density operator, dynamically stable coherence terms appear even after environmental entanglement has occurred. "information", or the presence of coherence terms, has "leaked" or "transferred" to the system-apparatus-environment state as a whole, but disappeared when considering only the system-apparatus state. However, because we are only considering the system-apparatus state in our idealized measurement the coherence terms seem to have vanished. Consider the state as a whole, and they reappear.

Has decoherence theory solved the problem of preferred basis? The answer is a resounding yes, provided that the formalism hitherto presented is accepted by the reader. This, I admit, is a rather anticlimactic answer to the question at hand. It is however completely required. It is a necessity in any introduction of any topic in quantum mechanics to choose a certain *interpretational* direction in order to be able to have a fruitful discussion. Such an interpretational direction has indeed also been chosen here. I invite the reader to section 4.5 for a discussion of this direction, functioning as a prerequisite to chapter 5 where this topic will be discussed in full. What is of importance to us, however, is that we indeed have a solution to the preferred basis problem when considering quantum mechanics in its *canon*.

4.3 Decoherence and the Problem of Non-Observability of Interference

Environment-induced superselection rules through decoherence theory is a bow with two strings. Before discussing whether or not its arrows can defeat the

³¹Paz, J. P.; Zurek, W.H.; "Quantum limit of decoherence: Environment induced superselection of energy eigenstate"; Phys. Rev. Lett., Vol. 82, No. 26, p. 5181-5185, (1999).

problem of preferred basis, a treatment requiring some interpretational issues to be properly discussed, let us move on to the problem of non-observability of interference.

As the attentive reader might have realised, the discussion above regarding environment-induced selection rules and the preferred basis problem touches heavily on the problem of non-observability of interference. In fact, as accentuated in section 3.2, the entire process of decoherence as the local suppression of system coherence terms through environmental entanglement *means* that interference terms have been reduced. This reduction of interference through decoherence is indeed what allowed us to introduce the pointer observable in the first place, making it a necessary condition for the entirety of the subsequent discussion. As we have seen in approximations (131) and (148), a larger environment implies a smaller standard deviation around the factor $z_{ij}(t)$ appearing in section 4.1 and 4.2, responsible for the reduction of coherence terms. The very nature of interference experiments as measured in the position basis makes the discussion above regarding collisional decoherence from environmental radiation and molecules highly relevant for the difficulty of observing interference effects of macro- and mesoscopic objects.

In presenting the experimental work associated to the problem of non-observability of interference, this essay either cannot do it justice or move in a direction not connected to the topic at hand. In spite of this, I have decided to try and do it justice to the best of my ability. A great start is with the work of Zeilinger *et. al.*³² who through a near-field Talbot-Lau interferometer³³ observed interference fringes of C₇₀ fullerene molecules. Beautiful in its own right, subsequent experiments by the same group^{34,35} showed how the reduction of interference through collisional environmental decoherence was in remarkable agreement with decoherence models. This was done by performing the C₇₀ interference experiment with different concentrations of gas in the molecule path, in much the same vein as our double slit experiment in section 2.2. In order to understand the difficulty of studying such large systems, we appreciate the fact that molecules of this type are prone to thermal emission through their many internal degrees of freedom (in many ways, a C₇₀ molecule is more like a dust grain than an elementary particle such as electrons studied in 2.2), resulting in the system "spreading out" through emissions of photons during the course of an experiment³⁶, thus inducing decoherence from emissions from the system. Appropriate decoherence models studying these effects show excellent agreement with their experimental

³²Zeilinger, A. ;*et. al.*; "Matter-Wave Interferometer for Large Molecules"; Physical Review Letters, Vol. 88, No. 10, p. 100404-100404 (2002).

³³See [30]

³⁴Zeilinger, A. ;*et. al.*; "Collisional Decoherence Observed in Matter Wave Interferometry"; Phys. Rev. Lett., Vol. 90, No. 16, p. 160401-160401, (2003).

³⁵Hackermuller, L. ;*et. al.*; "Decoherence in a Talbot-Lau interferometer: the influence of molecular scattering"; Appl. Phys. B, Vol. 77, No. 8, p. 781-787, (2003).

³⁶Hackermuller, L. ;*et. al.*; "Decoherence of matter waves by thermal emission of radiation"; Letters to Nature, Vol. 427, No. 6976, p. 711-714, (2004).

results.

Has decoherence theory solved the problem of non-observability of interference? The answer is the same as per the last section. That is, it is certainly solved *if* the reader accepts the formalism thus presented. We refer the reader to section 4.5 and 5 for a deeper discussion on this topic, but as of now we move on to the problem of outcomes.

4.4 Decoherence and the Problem of Definite Outcomes

It has been claimed³⁷ that decoherence solves the problem of outcomes. It is indeed part of the "folklore"³⁸ of decoherence theory that as a local system is decohered, the "classical" probabilities (see ch. 2) are thought as being representative of classical definite states. That is, it is believed that decoherence can, on *purely* physical grounds, describe why *one* measurement results in a definite state. As have been pointed out by many^{39,40,41}, this is a misunderstanding.

In remembering section 3.3, we recall the two questions associated with the problem of outcomes: *why do measurements have outcomes at all*, and *why do we observe a particular state over any other realisable one*? In relation to our recent discussion regarding decoherence and environment-induced superselection rules, we remind ourselves of what we have actually achieved: the local reduction of system coherence terms, contrasting the dynamically stable population terms in the system density operator which makes up the sole ingredient of the pointer observable. Thinking back to the experiment discussed in section 3.1 and 4.2, it is helpful to imagine ourselves actually performing this experiment and discussing what we would see in order to tackle these questions. We would observe, in making a measurement of our apparatus particle, an eigenvalue associated with either the state function $|\varphi_g\rangle\langle\varphi_g|$ or $|\varphi_e\rangle\langle\varphi_e|$ acting on $|\Phi(t)\rangle$, and we would conclude which state function had been applied as such. After sufficiently many such identical experiments, we would see how we have always obtained $|\varphi_g\rangle\langle\varphi_g|$ or $|\varphi_e\rangle\langle\varphi_e|$, and that they have an equal probability of being measured. On this, there are no disputes.

why do we observe a particular state over any other realisable one, then, considering this series of experiments? Decoherence theory has no clear answers to this. Indeed, the density matrix of the system practically only contains population terms after a sufficiently long time, but only *one* of these terms are

³⁷Anderson, P. W. ; "Science: A 'Dappled World' or a 'Seamless Web'?" ; Stud. Hist. Phil. Mod. Phys., Vol. 32, No. 3, p. 487-494, (2001).

³⁸Bacciagaluppi, G.; "The Role of Decoherence in Quantum Mechanics"; Stanford Encyclopedia of Philosophy, (2003), rev. (2020).

³⁹Adler, S. L.; "Why Decoherence has not Solved the Measurement Problem: A Response to P.W. Anderson"; Stud. Hist. Philos. Mod. Phys., Vol. 34, No. 1, p. 135-142, (2003).

⁴⁰Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, p. 60, (2008).

⁴¹[36]

measured. Even when a preferred basis has been "selected" and interference terms of the system asymptotically reduced, there is no agreed upon physical mechanism for deducing *which* population term is observed. We can say that we are back where we started, even after the system has decohered; we have not escaped the selection process of equation (104). We have described why interference terms of the right hand side of equation (103), seen if presented in density operator form, seem to have vanished and why the particular basis states $|a_i\rangle$ remain stable, but the selection process itself remains cloudy. Secondly, *why do measurements have outcomes at all*, then? It is not hard to realise that this question is broader than the first and thus suffers the same problems presented. Even though we understand why interference of different pointer states is not observed, the disappearance of superposition between dynamically stable states is not.

The problem of outcomes remains an especially controversial issue within the physics community with a lively but often healthy debate frequently touching on the topic of decoherence, where it is asked exactly how decoherence fits into interpretational issues. These interpretational issues and their relation to decoherence will be discussed in chapter 5, but it is very important to first illuminate exactly what it is that decoherence has not solved. We presented the problem of outcomes as an unexplained break from the linear evolution of quantum states when a measurement is performed, a process described by equation (104). Decoherence has failed to present a physical mechanism for why the outcome appears as such, akin to a treatment where all population terms except for one, no less the one we measure, is present.

4.5 Concluding Remarks and the Issue of Interpretation

It is time to address the vague answers to the questions of whether or not decoherence once and for all solves the problems of measurement here presented. As was mentioned during the end of section 4.2, an interpretational direction has been chosen when discussing decoherence in this essay, and it is time to reveal what these are⁴². Going all the way back to sections 2.1.1-3, we derived the expressions

$$\text{Tr}[\hat{\rho}\hat{O}] = \langle\psi|\hat{O}|\psi\rangle \quad (155)$$

$$\langle\hat{O}\rangle = \langle\psi|\hat{O}|\psi\rangle \quad (156)$$

using slightly different arguments. Equation (155) is of course nothing more than a mathematical fact, while the equality of equation (156) was argued through realizing that

$$\text{Tr}[\hat{\rho}\hat{O}] = \sum_i \langle o_i|\psi\rangle \langle\psi|\hat{O}|o_i\rangle = \sum_i o_i |\langle o_i|\psi\rangle|^2, \quad (157)$$

⁴²Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, sec. 8.1.1, (2008).

where $|\langle o_i | \psi \rangle|^2$ is the *Born probability* of measuring the eigenvalue o_i after a Measurement with capital "M", akin to the process presented in (104) (see sec. 3.3). Herein lies two important interpretational assumptions if the equality of (157) is to be satisfactory. Firstly, we assume that a quantum state $|\psi\rangle$ possesses a definite value of \hat{O} if and only if $|\psi\rangle$ is in an eigenstate of \hat{O} , such that $\hat{O}|\psi\rangle = o_i|\psi\rangle$, where the definite value is the associated eigenvalue o_i . This property is called the *Eigenstate-Eigenvalue link*⁴³. Secondly, we assume further that the eigenvalues o_i associated with \hat{O} are the only possible outcomes of a Measurement with capital M, and that the probability of obtaining a given eigenvalue o_i is given by the Born probability $|\langle o_i | \psi \rangle|^2$, where $|o_i\rangle$ is an eigenstate of \hat{O} . This property is called the *Born rule*.

These assumptions are implicitly or explicitly stated in most standard textbooks of quantum mechanics, and chances are that the reader does not find them to be controversial in the slightest. What is important to us however are their use in the introduction of the reduced density matrix which has been heavily used throughout the duration of this text. Recall the phrasing used when introducing the partial trace, where it was stated that we were to use the partial trace in order to investigate the measurement statistics of either part of the composite system. The term "measurement statistics" highlights the fact that we are assuming a situation in which an experimenter only has practical access to the local system of interest, thus realising that the local system is entangled to an environment that she is either unable or uninterested to measure. When a measurement is performed on the local system its set of possible outcomes is under all practical purposes approximately equal to a proper ("classical") mixture of pure states, leading the experimenter to the conclusion that coherence terms has vanished from the local system in calculating $\langle \hat{O} \rangle = \langle \hat{O}_S \otimes I_E \rangle = \text{Tr}_S[\rho_S \hat{O}_S]$, where \hat{O}_S is the operator containing all measurement statistics of the system.

Although, recall that the expression $\langle \hat{O} \rangle = \langle \hat{O}_S \otimes I_E \rangle = \text{Tr}_S[\rho_S \hat{O}_S]$ is based on that equality (156) holds true and that measurements already *do* have outcomes prescribed by the Born rule. The trace operation itself, and hence the reduced density matrix, is based in the *Statistical interpretation* of quantum mechanics, meaning that the local system quantum state during a Measurement with capital M is thrown into one of the eigenstates allowed for under the eigenvalue-eigenstate link with probabilities dictated by the Born rule. If these measurement assumptions are dropped we have no justification for using equation (156) and therefore not the formalism of reduced density matrices, leaving us in a situation where the experimenter is forced to only consider the dynamics of the entangled system-environment state. Within this interpretational formalism, the experimenter is therefore unable to say anything at all about the local system and thus not deduce any measurement statistics after a measurement to *describe* the underlying conditions of how she got to a particular result.

⁴³Wallace, D.; "What is orthodox quantum mechanics?"; arXiv:1604.05973, (2016).

Although, haven't we throughout this text made claims regarding measurement results concerning the disappearance of coherence terms in a set basis of a local system, which without a doubt have been measured? This is of course true in a sense, and this question is an excellent preamble to the next chapter. We have presented a certain formalism in the study of decoherence phenomena, certainly also the most common one at that, but it is none the less not universally accepted nor the only presentable way. Some formalism of quantum theory rejects the notion of reduced density matrices as it assumes the projection postulate, for example.⁴⁴ Of course, it isn't what *de facto* has been measured that there is disagreement about, but how we *speak* about, *make claims* about, and *interpret* the measurement process itself.

We are more or less left where we left off from section 4.2. With these interpretational cards on the table, we can say that decoherence indeed solves the problem of preferred basis and the problem of the non-observability of interference, but not the problem of outcomes, *if* the formalism of reduced density matrices sprung out from equation (156) is accepted.

5 Decoherence as a Means to Solve the Problem of Measurement Within the Copenhagen Interpretations

Can decoherence theory solve the problem of measurement? We have outlined decoherence theory and the problem of measurement, but the reader is justified in thinking the connection of the two as vague. In the last chapter, one key realization was the dependence on an interpretational direction in order to fully answer if decoherence theory can solve the problem of measurement. This unsatisfactory position boils down to one question which since the birth of quantum mechanics has adamantly haunted physicists and philosophers alike: *What is a measurement in quantum mechanics?* This is of course a condensed version of the measurement problems or the quantum to classical transition presented in chapter 3. So, *what is a measurement?* If we for the moment accept that the formalism of quantum mechanics does not and can not describe the measurement process, which is the *status quo*, how are we to solve the measurement problems? If we are not willing to add new postulates to the formalism (cf. *objective collapse theories*), the only way to discuss a process not included in the formalism and not testable through experimentation is through *interpretations* of the formalism, alternatively changes in the formalism which leaves experimental predictions unchanged (cf. *Bohmian mechanics*), which seeks to argue how the formalism leads to a definite result.

The literature concerning the measurement problems and their related interpre-

⁴⁴Schlosshauer, M.; "DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"; Springer-Verlag Berlin Heidelberg, ch. 8.2, (2008).

tations of quantum mechanics is *vast*. In the interest of time, we will only investigate the relation between decoherence and the *Copenhagen Interpretations* of quantum mechanics. We give two main reasons for investigating the Copenhagen interpretations of quantum mechanics over any other interpretation. Firstly, as outlined in section 4.5, the use of the reduced density matrix presumes an interpretational direction willing to accept the eigenvalue-eigenstate link and the Born rule, two postulates almost universally accepted in any introductory text on not only quantum mechanics itself but also decoherence theory. It is only natural to discuss the interpretation which gives the underlying justifications for these assumptions, as most physicists see it. Second, many of the most famed interpretations which themselves are not Copenhagen interpretations (*Relative state interpretations*, *Bohmian mechanics* and *objective collapse theories* for example) can all be viewed as a collection of footnotes to Copenhagen interpretations, all a critique of certain aspects of Copenhagen interpretations followers of each find uncomfortable. If this is because Copenhagen interpretations were the first serious attempt of an interpretational framework or because most physicists today hold it as the preferred interpretation does not matter; it is again the natural starting point for discussions regarding interpretations of quantum mechanics.

5.1 Copenhagen Interpretations

With a complete account primally sketched out by Niels Bohr for the first time in 1925, the *Copenhagen interpretations* are the first type of interpretations of quantum mechanics.⁴⁵ Held by contemporaries such as Werner Heisenberg, Max Born and many others, it is still the most popular interpretational framework to this day.⁴⁶ Often under the banner "Copenhagen interpretation" in singular form, past and contemporary scholars subscribing to it have disagreements on the specifics (and unfortunately non-specifics), making the plural form more suitable. In fact, the Copenhagen interpretations began as a collection of rather loose ideas shared between Bohr and Heisenberg who amongst themselves had some disagreements on fundamental issues. To guide ourselves through this messy landscape we will stick to a theme corresponding to the views expressed by Niels Bohr and Werner Heisenberg and highlight central disagreements held by prominent persons both during their lifetime and afterwards. Sticking to Bohr or Heisenberg, the hope is then that a coherent and clear interpretation might be revealed to us. Sadly, this is not the case. Even today the debate is fierce about what they actually meant in their writings considering fundamental issues, where there is disagreements on how we should define their rather loose terms and whether there exists any internal inconsistencies in their thought.

⁴⁵Schlosshauer, M.; Camilleri, K.; "The quantum-to-classical transition: Bohr's doctrine of classical concepts, emergent classicality, and decoherence"; arXiv:0804.1609, (2008).

⁴⁶Sivasundaram, S.; Hvidtfelt Nielsen, K.; "Surveying the Attitudes of Physicists Concerning Foundational Issues of Quantum Mechanics"; arXiv:1612.00676 (2016).

Therefore, it is difficult to state what Bohr or Heisenberg *really* meant in their writings and we therefore have to resort to influential interpretations of them.

When providing an account the Copenhagen interpretation, two concepts become indispensable, especially considering the philosophy of Niels Bohr; *The principle of complementarity* and *The necessity of classical concepts*. We will provide an overarching account of these two concepts below, with an emphasis on the necessity of classical concepts as it strongly relates to our discussion about decoherence.

5.1.1 The Principle of Complementarity

Murdoch defined Niels Bohr's concept of complementarity as follows. Introducing two concepts which are said to be complementary, stating that they are complementary implies that they are both necessary for the description of an entity, *but* the conditions necessary for the application of one of the concepts contradicts the application of the other⁴⁷⁴⁸. Realising the necessity to define our terms, a "concept" can be essentially thought of as an observable, be it momentum, position, energy or other. An "entity" is an object which we wish to endow with such concepts. "Conditions" can be thought as experimental conditions, and "application" essentially means the demand of "unambiguous communication" about the entity, to use the typical language of Bohr.

Of course, the most famous example of two complementary concepts are position and momentum. It is easy to see how this might be the case by investigating which experimental conditions are required to measure one or the other, as a position measurement requires a detector fixed in a certain frame of reference relative to for example the lab frame, while a momentum measurement requires a detector able to absorb momentum quanta of the object which is measured. A detector cannot simultaneously be fixed in a lab frame *and* undergo a translation by absorbing momentum quanta, which is why these observables are said to be complementary in accordance with the above definition.

On the face of it the above example could give the impression of revealing to us an unfortunate fact of nature, implying that the micro cosmos behaves in a way familiar to us even though our methods of measurement falls short in accessing this familiarity. On this there is disagreement, mainly between two schools of thought. The first is the *ontic interpretation of complementarity*, whose followers essentially state that objects cannot have definite values of complementary observables simultaneously. The second is the *epistemic in-*

⁴⁷Murdoch, D.; "*Niels Bohr's philosophy of physics*"; Cambridge : Cambridge Univ. Press, (1987).

⁴⁸Johansson, L.-G.; "*Interpreting Quantum Mechanics: A Realistic View in Schrödinger's Vein*"; ASHGATE, (2007).

terpretation of complementarity, stating that objects can have definite values of complementary observables at all times which however cannot be measured with arbitrary accuracy simultaneously. Ontic and epistemic are here obviously referring to the two philosophical fields of *Ontology* (a sub field of *Metaphysics*) and *Epistemology*, respectively. Metaphysics is the field of philosophy asking questions such as "what is reality; what is *there* and what is it *like*?", while Ontology focuses more on the "there" than the "like". Epistemology is the field of philosophy asking questions such as "what is knowledge?". It is obvious from the context that the ontic interpretation of complementarity makes an ontological claim while the epistemic interpretation of complementarity makes an epistemological claim; an object described by complementary variables *cannot have* definite values of them simultaneously, or we are unable to measure, or *know*, whether they do or not.⁴⁹

Interestingly but complicatedly, Bohr himself would most probably have disagreed with both of these schools of thought, even though he adamantly argued for the necessity of complementary concepts. As a prerequisite to his thinking, it is fair to say that he constantly spoke about what we can *know*, alternatively what we can unambiguously communicate, about a system in question.⁵⁰ More to the point, Bohr definitely thought that atoms *existed*, but that quantum theory did not give a pictorial representation of the micro cosmos. It was clear to Bohr that any measurement in quantum mechanics necessarily must result in an inseparable state of that which is measured and the measurement device, and given the difference between necessary experimental conditions required to unambiguously communicate about complementary concepts through the use of measuring devices, no single picture of the entity could be presented. In its most concentrated form, Bohr thought it a necessity that all truth conditions of sentences about kinematics and dynamics in quantum mechanics are dependant on reference to the experimental setup itself as well as the experimental outcome, making all other discussions meaningless. This is most clearly represented in the so called *Bohr-Einstein debates* and his response to the *Einstein-Podolsky-Rosen* paper which we will only mention in passing.

5.1.2 The Necessity of Classical Concepts

Bohr and Heisenberg often spoke about the *necessity* or *irreducibility* of *classical concepts*, a demand given quite clearly by Bohr in the following passage:

It is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all

⁴⁹The attentive reader might wonder about the words "can have" in defining the epistemic interpretation of complementarity, as a "can have" implies a description of the "like". This is of course true, but this largely boils down to that the ontic and epistemic interpretations are not clear cut. Some epistemic interpretations are agnostic while others make ontological claims.

⁵⁰Faye, J.; "*Copenhagen Interpretation of Quantum Mechanics*"; Stanford Encyclopedia of Philosophy (First published Fri May 3, 2002; substantive revision Fri Dec 6, 2019).

evidence must be expressed in classical terms. The argument is simply that by the word ‘experiment’ we refer to a situation where we can tell to others what we have done and what we have learned and that, therefore, the account of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics.⁵¹

Bohr expressed similar views from the very beginning of quantum mechanics to the end of his days. With interpretations of his thought ranging from influences of *Empiricism*, *Experimentalism*, *Pragmatism* or even *Kantianism* and *Darwinism*, it is difficult to give a full description of his philosophy. However, over the years there is relatively large agreement on some of his core tenets, which we here will try to describe.

It is interesting to note that Bohr opposed the von Neumann measurement scheme in describing the infectious nature of inseparability which saw no difference in describing microscopic and macroscopic objects.⁵² In his view, there was certainly no problem in describing nature in way of the scheme as he realised the universal validity of quantum theory, but he asked himself what the use of this would be. Because, according to Bohr, in describing the inseparability of the object under measurement and the measurement apparatus, we have to discard the properties of the measurement apparatus that *makes it* a measurement apparatus in the first place. The properties of a measurement apparatus are nothing less than whatever conditions are required to use *irreducible classical concepts* when communicating measurement results, about such things as position, momentum and energy. In this way, Bohr treated these concepts as *a priori* required in a discussion regarding measurements. In order for the irreducible classical concepts to be obtained from a measurement, the measurement instrument cannot be treated in a quantum mechanical fashion but must be treated classically, even though it would be possible to describe it quantum mechanically. This is what a measurement *is*, according to Bohr. This stance can be called *the quantum-classical divide*, which was clearly an epistemological stance.

There is a deeper insight at play here, realised by Max Born and appreciated by both Heisenberg and Bohr. Considering any measurement in classical physics, the separation between the object we wish to measure and the measurement apparatus is always rather clear and not a point of consideration if not just as a redundant matter of fact. With the introduction of inseparability of states in quantum mechanics this matter of fact can be challenged. For, taking the necessity of inseparability of quantum states during the primary evolution of an

⁵¹Bohr, N.; *"Essays 1932-1957 on Atomic Physics and Human Knowledge"* or *"The Philosophical Writings of Niels Bohr, Vol. II"*; Woodbridge: Ox Bow Press, (1932-1957).

⁵²Schlosshauer, M.; Camilleri, K.; *"The quantum-to-classical transition: Bohr's doctrine of classical concepts, emergent classicality, and decoherence"*; arXiv:0804.1609, (2008).

object we wish to measure and a part of a measurement device at face value, we cannot treat the to be measured object and this part of the measuring device as separate. Indeed, inseparability means that they can no longer be thought of as two entities, but must necessarily be thought as one. If such a situation is carried on *ad infinitum*, for example in accordance with the von Neumann measurement scheme, *we would not be able to distinguish any parts of our experimental setup*. But, Bohr would say, it is an *a priori* requirement of a measurement to be able to distinguish between that which is measured and that which is not measured. Bohr's point here is that we define what a measurement is in the first place through classical terms, in this case the separation of states, and whatever process that does not follow these definitions cannot be considered a measurement, and any device which does not give us the ability to perform such a measurement cannot be considered a measurement device.

5.1.3 Proposed Solutions to the Problem of measurement

It may seem strange that much time has been used to explain complementarity and the irreducibility of classical concepts, but we are yet to even mention *the collapse of the wave function*.⁵³⁵⁴ The postulate of wave function collapse, also known as the projection postulate, states that when an observable o_i associated with the operator $\hat{O} = \sum_i o_i \hat{o}_i$ is measured on a quantum state $|\psi\rangle$, the measurement induces a stochastic transition on the state as

$$|\psi\rangle \longrightarrow |\psi_i\rangle = \frac{\hat{o}_i |\psi\rangle}{|\hat{o}_i |\psi\rangle|^2} \quad (158)$$

where $\hat{O} |\psi_i\rangle = o_i |\psi_i\rangle$ and where the probability of measuring o_i is given by the Born rule. First introduced by Heisenberg and put on mathematical footing by von Neumann in his seminal work of 1932,⁵⁵ wave function collapse is by some seen as almost synonymous with Copenhagen interpretations of quantum mechanics. This view is unfortunately misleading. First of all, this view glosses over important interpretational disagreements on the nature of collapse by its tending to treat the Copenhagen interpretations as *one* interpretation, which it is not. Secondly, its representation through (173) very nearly seems to assume collapse as a physical process, a statement which harbors disagreements within different Copenhagen interpretations. In this respect, there are two schools of thought concerning the measurement and collapse process, pioneers of which are Bohr and Heisenberg, and von Neumann and Wigner, respectively.

Von Neumann and Wigner

⁵³Schlosshauer, M.; *"DECOHERENCE AND THE QUANTUM-TO-CLASSICAL TRANSITION"*; Springer-Verlag Berlin Heidelberg, ch. 8.1, (2008).

⁵⁴Wallace, D.; *"What is orthodox quantum mechanics?"*; arXiv:1604.05973, (2016).

⁵⁵von Neumann, J.; *"Mathematical Foundations of Quantum Mechanics"*, Princeton University Press, ch. VI. (1996, original German version published 1932).

Let us start by accounting for the train of thought belonging to von Neumann and Wigner. As mentioned above, von Neumann described mathematically the collapse postulate put forward by Heisenberg. This mathematical description is nothing more than the von Neumann measurement scheme (118) that was presented in section 3.3 together with the evolution (119) describing the notorious state projection into an eigenstate of the measured observable. The question we wish to answer here is how exactly von Neumann himself justified such a process. For starters, von Neumann realised that (118) and (119) were two distinct processes, as we showed rigorously in section 3.3. To combat this problem, which is indeed the problem of outcomes, he asks us to distinguish between (I) the system under observation, (II) the measurement apparatus and (III) the observer, and consider their entanglement under an experiment described by the von Neumann measurement scheme. He argues that, as entanglement between these three states begin, a collapse process occurs only when the observer sees and becomes aware of the experiment, for example by consulting the then necessarily macroscopic measurement apparatus. He further holds that the dividing line of this collapse process can be drawn arbitrarily depending on the context: (I) can be described by the unitary laws of quantum mechanics while the collapse process could happen at (II) + (III), or (I) + (II) can be described by the unitary laws of quantum mechanics and (III) by the collapse process. His main point is, thusly, that the subjective perception of the observer could never be described by the unitary laws of quantum mechanics. His argument for this lies in the fact that the awareness of the observer, that he takes as non-physical in nature, is fundamentally distinct from anything described by physical processes. Essentially, we are to view everything physical in nature, from micro- to macroscopic, as evolving under the Schrödinger equation *until* an observer becomes conscious of some object within this evolution. The awareness of the observer then induces a collapse of the wave function of the entire system, a proposition that we simply accept as part of the apparently non-physical consciousness' effect on physical systems.

This, admittedly, seems incredibly unfounded. Why would "consciousness", an indeed arduously difficult concept to define, induce a collapse process? What is the context in which we choose where the dividing line for collapse is? This ultimately boils down to a view of the philosophy of science popular in and around the 1930:s when von Neumann wrote his work in which this idea resides. The view is called *instrumentalism*, an idea based on the larger philosophical school of *pragmatism*.⁵⁶ Pragmatism is essentially a philosophy that judges propositions in relation to how useful they are in certain situations, and is therefore not interested in whether the proposition is true in any absolute sense, something that pragmatists deem a meaningless task due to our limited access to the verifiability of truth statements disconnected from our subjective selves. As a

⁵⁶This passage is based on the present writers conversations with Prof. Lars-Göran Johansson

sub-category of pragmatism, instrumentalism then applies this idea to science and states that every model we make within it should only be judged upon how well it is at explaining and predicting natural phenomena.

In light of this, we are now able to see where von Neumann was coming from. The subjective experiences of the observer is inexorably present at all times, which must effect any scientific model. The model we are consulting is, in this case, the von Neumann measurement scheme which (*not including complications regarding decoherence which was not fully understood in the 1930:s*) unequivocally shows that quantum correlations spreads from, for example, microscopic objects to macroscopic ones. This spreading of correlations can seemingly go on indefinitely, but no observer has any subjective experience of this kind of evolution in any direct sense. Therefore, we reason that there must exist something differentiating physical objects and the observer, and that something we call "consciousness". The dividing line for this consciousness inducing collapse should be drawn in whatever way is most practical to us in any situation, as long as it allows us to explain and predict what we measure. This is, in essence, the instrumentalist view of the collapse process that von Neumann propagated.

Perhaps surprisingly, von Neumann himself never explicitly leaped to say that consciousness *causes* collapse. He admitted that the human body could be regarded as a quantum object under the treatment of his scheme, and indeed thusly the brain itself. However, the analysis must stop at the actual mind of the observer, where the cognition process itself is responsible for collapse. Furthermore, as an instrumentalist, he always upheld psycho-physical parallelism, which he saw as a prerequisite for any scientific reasoning. This is a rather peculiar stance, as it would imply that a collapse process only can occur when a conscious observer is present, at the same time as any interaction between the non-physical consciousness and physical objects are disallowed. Indeed, this is an internal inconsistency within Von Neumann's entire interpretation.

Von Neumann gathered a large following which lasted well into the second part of the twentieth century. As perhaps one of his most famous followers, Wigner realised this internal inconsistency in von Neumann's thought and sought to reason out of it. Carrying essentially the same ideas as von Neumann presented above, Wigner took the step to suggest that consciousness actually *causes* a physical collapse process, thus distancing himself from psycho-physical parallelism. However, he never explained how this psycho-physical interaction would work in practice or in theory. His most famous argument for consciousness inducing collapse is the *Wigner's friend*⁵⁷ gedankenexperiment, a complication of the famous *Schrödinger's cat* gedankenexperiment in which we essentially replace Schrödinger's cat with a human observer, an experiment which we here only mention in passing.

⁵⁷Wigner, E.; "Remarks on the Mind-Body Question"; In "Symmetries and Reflections", Indiana University Press, Bloomington, Indiana, p. 171-184, (1967).

In the interest of honesty and clarity, it should be mentioned that this interpretation mostly is one for the history books. As an interpretation obviously conflicting with realism using admittedly large leaps of reasoning, most physicists today regard it as problematic and even on the verge of mysticism.

Bohr and Heisenberg

Certainly, what has been outlined above were not the views of Bohr. As hinted above, there are essentially four pillars of thought that all interpretations of Bohr's thought has to take into account. Firstly, we have the above outlined irreducibility of classical concepts and the principle of complementarity with context dependant measurements. Secondly, the peculiar feature of non-separability between measurement object and part of the measurement device, and lastly the view of the quantum formalism as non-isomorphic to nature *as it is*, disallowing us from a pictorial representation from the formalism and only allowing discussion of how it predicts measurement results in relation to a specific experimental setup. How does these four pillars relate to the question "*what is a measurement in quantum mechanics?*"?

Henrik Zinkernagel stated that "[Bohr's] view constituted a solution, or rather *dissolution*, of the measurement problem"⁵⁸, and it is hard to disagree with his analysis. For Bohr, the essential problem at the heart of interpretational issues of quantum mechanics was the non-separability of states and the fact that the line between measurement device and measured object was completely blurred. Their distinction was nothing less than a prerequisite to obtain objective, empirical knowledge at all, according to Bohr. Their distinction, then, is a pragmatic necessity. It is clear that we certainly *can* discriminate between the measured object and the measurement device when we perform a measurement. This, according to Bohr, is because of an interaction between the measured quantum object and the measurement device that is not, and even *cannot* be, described by the quantum formalism. The formalism, from the very beginning, concerns only the stochastic outcomes of measurements, and should not be seen as a description of nature *as it is*.

To what extent Bohr believed that quantum mechanics was universal is an active debate. However, it is certainly true that at the very least classical concepts were a necessity for describing measurement outcomes according to Bohr, whether he believed that there was an ontological separation between the quantum and classical or that macroscopic objects *could* be described by quantum mechanics, even though this would be meaningless in a context of experimentation. Nonetheless, it is when we necessarily equate a measurement device with classical concepts that we can understand the interaction described above,

⁵⁸Zinkernagel, H.; "*Niels Bohr on the Wave Function and the Quantum/Classical Divide*"; Studies in History and Philosophy of Modern Physics, Vol. 53, p. 9–19, (2016), emphasis added.

indescribable by the formalism. An essential property of classical objects is the fact that they always exist in a certain well defined frame of reference in spacetime, in relation to which we are able to define their position, and consequently their energy and momentum at all times. It is this essential property which makes measurement devices necessarily classical, because such a reference frame is necessary for describing and defining a particular property that we are measuring. The necessarily classical measurement device gives us firm ground to stand on when it comes to defining how a measurement could occur. The classically described measuring device gives the context of a certain classical concept, and it is in relation to this device that the formalism is used to obtain a definite result. Note that, according to Bohr, whatever process exists that does not follow these principles *wouldn't be a measurement*.

It is interesting to note that Bohr would probably object to calling the move from superposition to a definite state a *collapse*. Indeed, using this type of language is exactly what Bohr wanted to move away from in his description. To postulate a physical collapse is to give an isomorphically pictorial description of a natural process that blurs the divide between the quantum and the classical. It would hopelessly miss his main point.

It is reasonable to, at this stage, state that Heisenberg was incredibly influenced by Bohr's views on the subject of measurement, and would have agreed with what has been said above. We will highlight some disagreements between them in the next section.

5.2 Copenhagen Interpretations and Decoherence

Can Decoherence Theory solve the Problem of Measurement? The question might seem flawed from the very start. Indeed, the Copenhagen interpretations are all attempts at an explanation of the measurement process given a formalism which do not include the measurement process. An interpretation is *itself* a proposed solution, to a problem that begins with accepting the formalism as unable to account for the measurement process. However, this does not mean that decoherence is unable to strengthen or weaken an interpretation, or perhaps render the philosophical ramifications of some, or all but one, interpretations superfluous. In this sense, the question "*Can Decoherence Theory solve the Problem of Measurement?*" is not flawed.

In connecting the Copenhagen interpretations with decoherence theory, one central question concerns the choice of observable being measured within the Copenhagen interpretation of Bohr and Heisenberg. Essentially, in Bohr's interpretation, the solution to the preferred basis problem is that the observer has to make a choice of what observable she will measure, ultimately changing the quantum state, a view that has met critique due to its apparent reliance on an observer dependent reality, breaking from a long tradition held in classical physics. Furthermore, this choice is of course reflected in the choice of measure-

ment apparatus, its settings, and ultimately which senses she uses in order to become aware of said measurement result. In pressing the experimentalist on why she chose this particular apparatus with its particular settings it is clear that she would have to give an account of why this particular setup measures the observable that she chose in the first place. This, in turn, undermines her choice as the ultimate reason for why a particular observable was measured, because she is now in a position where a pictorial, physical explanation of the measurement process concerning the preferred basis seems to be required.

This, as has already been technically accounted for in sections 4.1-2, is a problem for decoherence theory. As Zurek, Zeh, and others found, it is clear that the stability criterion created by environmentally induced superselection rules creates a defensive line in favour of the experimentalist. In strictly using the quantum mechanical formalism and introducing an environment, the experimentalist can explain why a preferred basis is dynamically selected without making reference to a measurement inducing an apparent collapse. In this sense, decoherence theory has indeed aided in an explanation for the selection of observables in the Copenhagen interpretation of Bohr and Heisenberg, without strictly making reference to the choice of a subject.

One central disagreement between Bohr and Heisenberg, which has often been overlooked, is the precise nature of the quantum-to-classical divide⁵⁹. The disagreement originates in where to place the *cut* (*Heisenbergs Schnitt*) between the quantum and the classical, and whether or not this cut represents an actual discontinuity between physical laws of nature or should be viewed as a practical tool to make sense of measurements. In Heisenberg's view, the cut between the quantum and classical represented no such discontinuity. According to him, the dividing line could in principle be drawn anywhere provided that it was suitable to the experimental problem at hand, but not so far as to the object under measurement, which should always be described by quantum theory. He argued that, if this was not the case, we would have to accept that nature is described by two sets of physical laws: one for the quantum and one for the classical, and that problems concerning their interaction would arise. Meanwhile, Bohr himself was determined that the quantum-classical divide was not something that could be set arbitrarily by the experimenter, but something that was uniquely defined for each possible experiment.⁶⁰

There has been lots of disagreement among philosophers and physicists about what Bohr exactly meant by this. An especially interesting interpretations of

⁵⁹Schlosshauer, M.; Camilleri, K.; "The quantum-to-classical transition: Bohr's doctrine of classical concepts, emergent classicality, and decoherence"; arXiv:0804.1609, (2008).

⁶⁰It is important to note that Bohr was *not* speaking about an actual, physical discontinuity. Both Bohr and Heisenberg were interested in what can be known about measurement results, and this discussion concerns how to speak about the laws describing them.

Bohr's view on the topic comes from Don Howard⁶¹, who set out to understand this in Bohr's view already defined cut through an especially physical approach, as may be surprising for an interpretation of Bohr. It is the case, according to Howard, that for Heisenberg the classical-quantum divide corresponded exactly to the system-apparatus divide, but that this was not necessarily the case for Bohr. Howard imagines, as we have many times before, A von Neumann chain as

$$|\psi\rangle = \left(\sum_n c_n |s_n\rangle \right) |a_0\rangle \longrightarrow |\psi'\rangle = \sum_n c_n |s_n\rangle |a_n\rangle \quad (159)$$

and suggests that the identification of classical or quantum systems can be made by identifying sub-ensembles appropriate for the measurement context. We can see this by first calculating the density matrix of $|\psi'\rangle$ as

$$\hat{\rho}_{sa} = |\psi'\rangle \langle \psi'| = \sum_{nm} c_n c_m^* |s_n\rangle \langle s_m| \otimes |a_n\rangle \langle a_m|, \quad (160)$$

and state that the measurement context is decided by the apparatus operator

$$\hat{O} = \sum_m o_m |a_m\rangle \langle a_m|. \quad (161)$$

Letting \hat{O} act on $\hat{\rho}_{sa}$, we obtain

$$\hat{O}\hat{\rho}_{sa} = \sum_n |c_n|^2 |s_n\rangle \langle s_n| \otimes |a_n\rangle \langle a_n|, \quad (162)$$

a state that Howard asks us to consider as *ignorance interpretable*. Here, ignorance interpretable means that we are to view the state $\hat{O}\hat{\rho}_{sa}$ as a proper mixture of the possible measurement outcomes $|s_n\rangle |a_n\rangle$, that is, as \hat{O} acts on $\hat{\rho}_{sa}$, the system-apparatus state *really is* in one of the states $|s_n\rangle |a_n\rangle$ and not in any superposition of them, and they are represented as such due to *our* ignorance of which state the system actually is in and not due to any *fundamental* quantum mechanical uncertainty (improper state).

How are we to interpret Bohr from this formalism? In Howard's view, the division between quantum and classical is precisely at the point where we are allowed to obtain a state such as $\hat{O}\hat{\rho}_{sa}$ and make use of the ignorance interpretation. This is achieved through the acting of a specific measurement observable \hat{O} on the state which by its nature is set by the measurement we are performing. One could say that \hat{O} allows us to build proper sub-ensembles of the global system-apparatus state $\hat{\rho}_{sa}$ representing the measurement context by the eigenbase of the final state.

Given this interpretation of Bohr's notion of the Heisenberg cut, it is clear

⁶¹Howard, D.; "What Makes a Classical Concept Classical? Toward a Reconstruction of Niels Bohr's Philosophy of Physics"; Niels Bohr and Contemporary Philosophy, Kluwer Academic Publishers. p. 201–229 (1994).

that the system-apparatus divide does not always correspond to the quantum-classical divide. However, it is plagued by problems similar to the once we saw above. According to Howard's interpretation of Bohr, the question of what proper sub-ensemble is obtained is a question of knowing which observable was measured by an experimental arrangement. However, it is not answered *why* a particular experimental arrangement would result in a certain observable. Furthermore, it is thus also the case that some proper sub-ensembles can be rewritten as a set of many bases, so there is again the question of how to single out a preferred basis. And, as perhaps the most pressing and obvious point, Howard's process can be justifiably deemed as *ad hoc*, as there is no clear justification of using the formalism to remove coherence terms from the system-apparatus density operator except that we simply want to in order to properly define "irreducible classical concepts".

Here, again, decoherence theory can come to the rescue. Of course, as has been technically evaluated in sections 4.1-3, by defining an apparatus-environment interaction Hamiltonian a preferred basis for the system-apparatus state is dynamically singled out which in turn clarifies why a particular experimental arrangement measures a particular observable. Concerning the accusation of *ad hoc*, we remember that decoherence is nothing more than the asymptotic reduction of coherence terms in a particular basis of the system-apparatus state. We can then, at least in principle, obtain a state which for all practical application behaves as $\hat{O}\hat{\rho}_{sa}$, leading to a conclusion that Howard's interpretation is in fact not *ad hoc* but simply a necessity of the formalism under apparatus-environment entanglement. This way, decoherence could give us a dynamical account of why this formalism is suitable.

One aspect, accentuated by Schlosshauer and acknowledged by Howard in his original article, could however problematize this interpretation of Bohr in a modern context, when decoherence is considered. It is indeed the case that, as discussed in 4.5, the reduced density matrix formalism only functions in the framework of the statistical interpretation and as a direct consequence of this the projection postulate. Of course, if we are to equate classical concepts with proper sub-ensembles, a projection in the way described by the projection postulate cannot occur. If decoherence theory is to give physical justification for Howard's interpretation, we would therefore have to state that reduced density matrices are ignorance interpretable, a leap just as major as simply applying the ignorance interpretation on $\hat{O}\hat{\rho}_{sa}$, formally identical to a reduced density matrix. This is a problem which has to be solved if we are to relate Howard's interpretation to a decoherence process.

6 Discussion

Can Decoherence Theory Solve the Problem of Measurement? This question, as we have seen, requires some depth to be answered. We may summarize our

findings in this essay thusly: if accepting the eigenvalue-eigenstate link and the Born probability rule, two essential parts of the Copenhagen interpretations, we can, through the theory of environment induced superselection rules, solve the problem of preferred basis and the problem of non-observability of interference, but not the problem of outcomes. It is the case that through environment induced selection rules a preferred basis is dynamically selected by environmental entanglement, which in itself *implies* that interference phenomena should asymptotically vanish, but we are nonetheless left with a set of population terms, one of which is selected when performing a measurement. The question of *why* this selection process happens or *why that state* is selected has indeed not been answered. The measurement problem, today often synonymous with the problem of outcomes in many ways because of the discovery of decoherence, has therefore *not* been solved.

It should again be stated that everyone does not accept the Born rule or the eigenvalue-eigenstate link. For example, Van Fraasens modal interpretation explicitly starts with a rejection of the eigenvalue-eigenstate link, and the existential interpretation of Zurek rejects the Born interpretation and aims at "deriving" probabilities from definite results. Given this messiness, we nonetheless stuck with our interpretational direction and laid all cards on the table through presenting the Copenhagen interpretations. We should now ask ourselves, could the problem of definite outcomes be solved by one of the Copenhagen interpretations, and if not, could decoherence theory be the final nail in the coffin that once and for all discards it or shows that it is without a doubt the only reasonable interpretation?

Let us first discuss the interpretations of von Neumann and Wigner. As already accounted for, Von Neumann's explanation of the selection process contains the internal consistency of psycho-physical parallelism on the one hand and *some kind* of interaction between "consciousness" and quantum state. Wigner's interpretation does little to account for this interaction, even if he mentions it explicitly. His interpretation is fleeting, since it is heavily inspired by an instrumentalist approach to then seemingly discard it altogether. Given his introduction of an actual, physical collapse process, it is natural to demand and explanation for how this works. Is there an experiment that we could perform to differentiate a conscious collapse process from a non-conscious collapse process, and if not, why not? I would argue that the main problem with Wigner's interpretation, however, is his use of the word "consciousness" or "mind" in relation to Von Neumann's use of it. Given Von Neumann's clear instrumentalist approach, he can introduce the term without a strict definition other than "that which induces a collapse", claiming that it is a necessary mental construct to pragmatically account for the projection of quantum states. Given Wigner's introduction of physical collapse, one must conclude that this "consciousness" is no longer used in this fashion, but an actual, physical "object" that thusly require a physical explanation. So, *what is consciousness?* Without a satisfactory definition, Wigner's interpretation falls flat.

Concerning Wigner's and Von Neumann's interpretation in relation to decoherence, there is quite little to say. Certainly, of course, there exist no inconsistency with the formalism itself. However, the interpretation does not concern anything which we might relate to decoherence, for example the concept of classicality. In fact, decoherence may be seen as an obstacle for conscious collapse, as it acts as a guiding star in explaining phenomena where previously a subject was unsatisfactorily added in other interpretations, for example the one of Bohr and Heisenberg. Decoherence theory seem to have no explanatory power when it comes to conscious collapse.

Bohr's and Heisenberg's interpretations, however, harbors no such internal inconsistencies as Wigner's or Von Neumann's. However, it certainly leaves questions unanswered. What is the suitable definition of an irreducible classical concept? What does the formalism of quantum mechanics actually refer to, if not some kinematics of nature? Why does the formalism seem to require an apparent collapse process to account for measurements if this doesn't happen, or indeed why do we need it at all? Certainly, the critiques and questions regarding Bohr's interpretation are many, and it would be bold to claim that his interpretation solves the measurement problem.

Concerning Howard's interpretation of Bohr, I have a couple of criticisms. Essentially, I would argue that it is a mistake to interpret Bohr through this physical approach. To get my point across, a short comment on *Immanuel Kant* is needed. Kant realised, as we all do sometimes, that the world is *messy* and *chaotic*. Even in the calmest of moments, we are absolutely bombarded with the outside world through different sounds, colors, shapes, smells, all jumbled together and always changing. In Kant's time, right at the very start of the scientific revolution, *empiricism* started to take a hold in scholarly circles, where *David Hume* was one of its most prominent figures, who claimed that all knowledge can only and solely be derived from our senses through experience. Kant, deeply disturbed by this, thought that there must be *some kind* of reason behind reason, so to speak, abling us to categorize, sift through and focus on particular aspects of this messy and chaotic landscape. In order to create concepts through chaos, *a priori* concepts, innate in us, needed to exist. Accounted for in his seminal work *Critique of Pure Reason* are three main ideas concerning these *a priori* concepts, the one of our focus being *The Transcendental Aesthetic*. Essentially, it contains arguments for why space and time are *a priori* concepts, necessary for experience. Summarizing his main conclusion, for something to be an *object*, an *observer*, or perhaps even an *apparatus*, they need to be separated from each other, and without this separation, we cannot meaningfully speak of these concepts at all. Similarly, with time, for an understanding of e.g. experimental results, there need to be an understanding of cause and effect, or a separation in time.

As many have done before us, we realise the similarities of Kant's thoughts

with Bohr's. For example, Bohr explicitly defines classical objects as objects within a well defined frame of reference in space and time as being one of the necessities for empirical knowledge. To argue from the standing point of necessities or irreducibilities, or *a priori* concepts, in relation to obtain empirical knowledge, a theme we surely saw above, is an explicitly Kantian move. It is from this standing point that I would like to critique Howard's view. Admittedly, it fits well with a modern decoherence approach towards Bohr, but in representing classical concepts as synonymous with pure sub-ensembles of the system-apparatus state we make an ontological interpretation of Bohr where it should preferably be more epistemological. I interpret Bohr's approach towards classical concepts as being reachable with our innate sense of *a priori* concepts, meaning that classical concepts is not referencing a particular state of the world but our subjective ability to make categorizations of it. Adding to this point is the fact that Bohr and indeed Heisenberg always spoke about what can be *known*, not what the formalism describe about the kinematics of nature. My view does not challenge what Bohr has said about the Heisenberg cut: the fact that we can not arbitrarily divide nature into classical and non-classical systems in an experiment refers to our common, innate sense of *a priori* concepts, which is indeed according to Kant once and for all defined for all human observers. Given this, we do not have to refer to classical objects as being defined in the formalism. I would also argue against that this interpretation of Bohr harbors a particularly subjective approach, as there is no meaning to speak of knowledge without a subject inhabiting it, and that this knowledge has an objective starting point in *a priori* concepts. It is for these reasons that I would also reject the notion that Howard's interpretation sheds light on Bohr's interpretation because of decoherence theory outlined above. Even though decoherence theory could give an account of showing that it is not *ad hoc*, its starting assumption is problematic.

Given this, we can conclude that it would be bold to claim that any of the Copenhagen interpretation *solves* the problem of outcomes. Decoherence certainly gives a satisfactory explanation to situations which before were treated by unsatisfactorily introducing the choice of subjects, for example concerning the preferred basis problem in Bohr's interpretation. We can say that the relationship between decoherence and the irreducibility of classical concepts certainly draws interesting parallels, but concerning Howard's interpretation I would argue that it is unsatisfactory. Given this, we cannot from what has been presented above in good conscience claim that decoherence theory neither discards the Copenhagen interpretation nor makes it the only reasonable interpretation.

7 Conclusion

If accepting the eigenvalue-eigenstate link and the Born probability rule we can through decoherence theory solve the problem of preferred basis and the problem of non-observability of interference, but not the problem of definite

outcomes. It would be bold to claim that the Copenhagen interpretation could solve the problem of definite outcomes, but decoherence theory can be used to justify some aspects of this interpretation. These justifications are however not so satisfactory as to render all other interpretations superfluous.

8 Summary

After going over some preliminaries, we introduced decoherence as the asymptotic reduction of interference terms in a particular basis through environmental entanglement. We then introduced *the problem of preferred basis*, *the problem of non-observability of interference* and *the problem of definite outcomes*. After outlining the theory behind environment induced superselection rules, we saw that we can solve the problem of preferred basis and the problem of non-observability of interference, but not the problem of definite outcomes, *if* the underlying assumptions concerning the application of reduced density operators are accepted. We saw how these underlying assumptions, the eigenvalue-eigenstate link and the Born rule, are key concepts in the Copenhagen interpretations of quantum mechanics. After accounting for Von Neumann's and Wigner's, and Bohr's and Heisenberg's interpretations, respectively, we discussed how decoherence theory could give justifications for these interpretations, where we found that decoherence is mostly relevant for the interpretation of Bohr and Heisenberg. After discussing these findings, we found that it would be bold to claim that the Copenhagen interpretation could solve the problem of definite outcomes, but that decoherence theory can be used to justify some aspects of this interpretation, where these justifications are however not so satisfactory as to render all other interpretations superfluous.

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