Published for SISSA by Springer

Received: *January 27, 2023*
Accepted: *February 21, 2023*
Published: *March 20, 2023*

Taming the 11D pure spinor b-ghost

Max Guillen

Department of Physics and Astronomy, Uppsala University,
75108 Uppsala, Sweden

E-mail: max.guillen@physics.uu.se

Abstract: We provide an alternative compact expression for the 11D pure spinor b-ghost by introducing a new set of negative ghost number operators made out of non-minimal pure spinor variables. Using the algebraic properties satisfied by these operators, it will be straightforwardly shown that $\{Q, b\} = \frac{P^2}{2}$, as well as $\{b, b\} = Q\Omega$. As an application of this novel formulation, the ghost number two vertex operator will easily be obtained in a completely covariant manner from a standard descent relation, the ghost number three vertex operator will be shown to satisfy the generalized Siegel gauge condition, and the 11D supergravity two-particle superfield will be constructed in a quite simple way.

Keywords: BRST Quantization, M-Theory, Supergravity Models

ArXiv ePrint: 2212.13653

Dedicated to the memory of Pedro Quiroz Santillan

https://doi.org/10.1007/JHEP03(2023)135

JHEP03(2023)135

Contents

1 Introduction **1**

2 11D Non-minimal pure spinor superparticle **3**
2.1 The b-ghost 4

3 11D Physical operators **5**
3.1 Physical operators 5
3.2 A simple expression for the b-ghost 8

4 Some applications **9**
4.1 The ghost number two vertex operator 9
4.2 Generalized Siegel gauge 9
4.3 The two-particle superfield 10

5 Discussions **12**

A Linearized 11D supergravity **13**
A.1 Equations of motion 13

B 11D Pure spinor projector **14**
B.1 Equivalence of eqs. (3.9) and (3.14) 15

1 Introduction

10D super-Yang-Mills and 11D supergravity at linearized level have been shown to be elegantly described in a manifestly super-Poincaré covariant manner by the quantization of the 10D and 11D superparticles, respectively, using pure spinor variables [1–3]. These objects were introduced for the first time in the context of the superstring in [4], and then generalized to the study of supermembranes in [5]. The full descriptions of maximally supersymmetric gauge theories, including the aforementioned theories, on pure spinor superspace were later discovered by Cederwall in a series of papers [6–10, 35], by making use of the pure spinor superfield formalism. In this framework, the pure spinor actions take strikingly simple polynomial forms in a fundamental pure spinor superfield Ψ, and contain all the Batalin-Vilkovisky fields of the theories in study.

The kinetic term of the pure spinor field theories presents the standard form "$\Psi Q \Psi$", where Q is the ordinary non-minimal pure spinor BRST operator [29]. Consequently, the propagator of these theories is proportional to the so-called b-ghost, a negative ghost number composite operator satisfying the property $\{Q, b\} = \frac{P^2}{2}$. This operator was first constructed in the pure spinor superstring, and shown to play a crucial role for computing several multiloop scattering amplitudes [11–13, 29]. Likewise, their properties have been shown to be substantial to design a covariant map between the pure spinor formalism and the conventional RNS setting [14, 15].

In a recent work [16], it has been shown that the pure spinor master action of 10D super-Yang-Mills in the gauge $b\Psi = Q\Omega$, for some Ω, referred to as the generalized Siegel gauge, reproduces the same scattering amplitudes as those obtained from the open pure spinor superstring in the field-theory limit [17]. More interestingly, the kinematic numerators at any multiplicity were found to be proportional to nested b-ghost expressions, and to match the multiparticle superfields constructed in [18] up to generalized gauge transformations and BRST-exact terms. These computations were possible to be methodically carried out due to the existence of simpler alternative expressions for the 10D b-ghost [19–21]. Such expressions make use of negative ghost number operators, referred to as physical operators, satisfying a set of defining relations resembling the 10D super-Yang-Mills equations of motion at linearized order. Remarkably, these very same operators were ingeniously used to show that the Siegel gauge condition $b\Psi = 0$, implies a Poisson algebra structure for kinematic numerators, elegantly thus realizing the kinematic algebra of the Bern-Carrasco-Johansson (BCJ) duality [22] from an action principle viewpoint.[1]

In this work, we introduce the 11D analogues of the physical operators above mentioned, and provide a novel compact formula for the 11D b-ghost, introduced for the first time in [25], which will make computations involving the b-ghost more tractable and efficient. To illustrate this, we show that $\{Q, b\} = \frac{P^2}{2}$ and $\{b, b\} = Q\Omega$, in a straightforward and elegant way, as a consequence of the simple properties satisfied by the physical operators. In addition, we use our new formula to construct a ghost number two vertex operator via a standard descent relation involving the ghost number three vertex operator. Up to BRST-exact terms, the operator thus obtained is shown to match that introduced in [26] using the Y-formalism [27] in 11D. Furthermore, we find that the ghost number three operator satisfies the generalized Siegel gauge condition after letting the b-ghost act on it as a second-order differential operator. Finally, we apply the perturbiner method [28] to the pure spinor description of 11D supergravity, and by making use of our new formula for the b-ghost, we readily solve the two-particle superfield equation of motion.

This paper is organized as follows. In section 2, we review the non-minimal pure spinor construction of the 11D superparticle, and discuss the formulae found for the b-ghost in [25] and [30]. In section 3, we introduce the 11D physical operators, and compute their actions on the ghost number three vertex operator. We then write down a compact formula for the 11D b-ghost in terms of the physical operators and, after full expansion, it is shown to coincide with the original proposal in [25]. In section 4, we give some applications showing how our new formula for the b-ghost considerably simplifies computations relevant to scattering processes in 11D supergravity. We close with discussions and future directions in section 5. Appendix A is devoted to a short review of the superspace equations of motion

[1] As discussed in [16], this statement is sensitive to the actual ability of computing pure spinor correlators in a certain regularization scheme. Hence, higher-loop generalizations of this kinematic algebra might be subtle due to the highly non-local behavior of pure spinor kinematic numerators.

of linearized 11D supergravity, and appendix B spells out the 11D pure spinor projector used in this work.

2 11D Non-minimal pure spinor superparticle

The 11D pure spinor superparticle action in flat space is defined by [3, 5]

$$S = \int d\tau[P^a\partial_\tau X_a + p_\alpha\partial_\tau\theta^\alpha + w_\alpha\partial_\tau\lambda^\alpha - \frac{1}{2}P^2] \tag{2.1}$$

We will use letters from the beginning of the Greek/Latin alphabet to denote spinor/vector $SO(1,10)$ indices. The variables (X^a, θ^α) are the usual 11D superspace coordinates, and (P_a, p_α) are their respective conjugate momenta. The bosonic spinor λ^α satisfies the 11D pure spinor constraint, i.e. $\lambda\gamma^a\lambda = 0$. Its respective conjugate momentum w_α is thus only defined up to the gauge transformation $\delta w_\alpha = (\gamma^a\lambda)_\alpha\sigma_a$, for any vector σ_a. Due to their wrong statistics, they will be called ghosts and assigned to carry ghost numbers 1 and -1, respectively. The 11D gamma matrices will be represented by $(\gamma^a)_{\alpha\beta}$, $(\gamma^a)^{\alpha\beta}$, and they satisfy the Clifford algebra: $(\gamma^a)_{\alpha\beta}(\gamma^b)^{\beta\delta} + (\gamma^b)_{\alpha\beta}(\gamma^a)^{\beta\delta} = 2\eta^{ab}\delta^\delta_\alpha$. We will raise and lower spinor indices by using the antisymmetric charge conjugation matrix $C_{\alpha\beta}$ and its inverse $C^{\alpha\beta}$, which obey the relation $C_{\alpha\beta}C^{\beta\delta} = \delta^\delta_\alpha$, so that $(\gamma^a)^{\alpha\beta} = C^{\alpha\epsilon}C^{\beta\delta}(\gamma^a)_{\epsilon\delta}$, etc.

The Hilbert space is described by the BRST-cohomology of the operator $Q_0 = \lambda^\alpha d_\alpha$, where d_α is the Brink-Schwarz fermionic constraint [31] defined as

$$d_\alpha = p_\alpha - \frac{1}{2}(\gamma^a\theta)_\alpha P_a \tag{2.2}$$

Such a cohomology can be shown to be non-trivial up to ghost number 7, describing the 11D supergravity states in its Batalin-Vilkovisky formulation. Concretely, the ghost number 0, 1, 2 and 3 sectors respectively host the gauge symmetry ghost-for-ghost-for-ghost; the gauge symmetry ghost-for-ghost; the supersymmetry, diffeomorphism and gauge symmetry ghosts; and the 11D supergravity physical fields. The higher ghost number sectors form a mirror cohomology of those above described, and reproduce the 11D supergravity antifields. In order to illustrate this, let us analyze the cohomology at ghost number three, $U^{(3)} = \Psi = \lambda^\alpha\lambda^\beta\lambda^\delta A_{\alpha\beta\delta}$. The physical state conditions then imply that

$$Q_0\Psi = 0 \to D_{(\alpha}A_{\beta\delta\epsilon)} = (\gamma^a)_{(\alpha\beta}A_{a\delta\epsilon)} \tag{2.3}$$

$$\delta\Psi = Q_0\Lambda \to \delta A_{\alpha\beta\delta} = D_{(\alpha}\Lambda_{\beta\delta)} \tag{2.4}$$

where $\Lambda = \lambda^\alpha\lambda^\beta\Lambda_{\alpha\beta}$, and $\Lambda_{\alpha\beta}$ is any superfield. These equations match the linearized equations of motion of 11D supergravity in superspace [32], we thus identify $A_{\alpha\beta\delta} = C_{\alpha\beta\delta}$, where $C_{\alpha\beta\delta}$ is the linearized version of the lowest-dimensional component of the 11D supergravity super-3-form. In a particular gauge, one can show that Ψ has the following θ-expansion:

$$\begin{aligned}\Psi =& (\lambda\gamma^a\theta)(\lambda\gamma^b\theta)(\lambda\gamma^c\theta)C_{abc} + (\lambda\gamma^{ab}\theta)(\lambda\gamma_b\theta)(\lambda\gamma^c\theta)h_{ac} + (\lambda\gamma^a\theta)(\lambda\gamma^b\theta)(\lambda\gamma^c\theta)(\theta\gamma_{bc}\psi_a)\\ &- (\lambda\gamma^a\theta)(\lambda\gamma^{bc}\theta)(\lambda\gamma_b\theta)(\theta\gamma_c\psi_a) + O(\theta^5)\end{aligned} \tag{2.5}$$

JHEP03(2023)135

with C_{abc}, h_{ab}, ψ^a_α being respectively the 3-form, graviton and gravitino of 11D supergravity. Indeed, they can be shown to satify the linearized equations of motion

$$\partial^d \partial_{[d} C_{abc]} = 0\,, \qquad \Box h_{bc} - 2\partial^a \partial_{(b} h_{c)a} + \partial_b \partial_c (\eta^{ad} h_{ad}) = 0\,, \qquad (\gamma^{abc})_{\alpha\beta} \partial_b \psi^\beta_c = 0 \tag{2.6}$$

and gauge transformations

$$\delta C_{abc} = \partial_a B_{bc}\,, \qquad \delta h_{ab} = \partial_{(a} t_{b)}\,, \qquad \delta \psi^\alpha_a = \partial_a \kappa^\beta \tag{2.7}$$

where B_{ab}, t_b and κ^β are arbitrary gauge parameters.

In order to define negative ghost number pure spinor operators, one needs to introduce the so-called non-minimal pure spinor variables [29]. These ones consist of two pairs of conjugate variables $(\bar\lambda_\alpha, \bar w^\beta)$, (r_α, s^β), where $\bar\lambda_\alpha$ is a ghost number -1 pure spinor variable satisfying $\bar\lambda\gamma^a\bar\lambda = 0$, and r_α is a ghost number 0 fermionic spinor constrained via $\bar\lambda\gamma^a r = 0$. The 11D non-minimal pure spinor superparticle is then defined by the action [30]

$$S = \int d\tau [P^a \partial_\tau X_a + p_\alpha \partial_\tau \theta^\alpha + w_\alpha \partial_\tau \lambda^\alpha + \bar w^\alpha \partial_\tau \bar\lambda_\alpha + s^\alpha \partial_\tau r_\alpha - \frac{1}{2} P^2] \tag{2.8}$$

together with the BRST operator

$$Q = Q_0 + s \tag{2.9}$$

where $s = r_\alpha \bar w^\alpha$. Using the quartet mechanism [33], one can show that the cohomology of Q will be independent of the non-minimal variables, therefore matching that of Q_0.

2.1 The b-ghost

As in 10D, it is possible to construct the so-called b-ghost, a ghost number -1 operator, obeying $\{Q, b\} = \frac{1}{2} P^2$. This object was originally constructed in [25], and shown to take the complicated form

$$\begin{aligned} b = {} & \frac{1}{2\eta} (\bar\lambda\gamma^{ab}\bar\lambda)(\lambda\gamma^{ab}\gamma^c d) P_c + \frac{2}{\eta^2} L^{(1)}_{ab,cd} \Big[(\lambda\gamma^a d)(\lambda\gamma^{bcd} d) + 2(\lambda\gamma^{abcef}\lambda) N^d{}_e P_f && (2.10) \\ & + \frac{2}{3} (\delta^b_e \delta^d_f - \eta^{bd}\eta_{ef})(\lambda\gamma^{aecgh}\lambda) N_{gh} P^f \Big] - \frac{4}{3\eta^3} L^{(2)}_{ab,cd,ef} \Big[(\lambda\gamma^{abcgh}\lambda)(\lambda\gamma^{def} d) N_{gh} && (2.11) \\ & - 12 [(\lambda\gamma^{abceg}\lambda)\eta^{fh} - \frac{2}{3} \eta^{f[a} (\lambda\gamma^{bce]gh}\lambda)](\lambda\gamma^d d) N_{gh} \Big] && (2.12) \\ & + \frac{8}{3\eta^4} L^{(3)}_{ab,cd,ef,gh} (\lambda\gamma^{abcij}\lambda) \Big[(\lambda\gamma^{defgk}\lambda)\eta^{hl} - \frac{8}{3} \eta^{h[d} (\lambda\gamma^{efgk]l}\lambda) \Big] \{N_{ij}, N_{kl}\} && (2.13) \end{aligned}$$

where $\eta = (\bar\lambda\gamma^{ab}\bar\lambda)(\lambda\gamma_{ab}\lambda)$, $N_{ab} = \frac{1}{2}(\lambda\gamma^{ab} w)$ is the usual ghost Lorentz current, and $L^{(n)}_{a_0 b_0, a_1 b_1, \ldots, a_1 b_1} = (\bar\lambda\gamma_{[[a_0 b_0}\bar\lambda)(\bar\lambda\gamma_{a_1 b_1} r) \ldots (\bar\lambda\gamma_{a_n b_n]]} r)$, with [[ ]] denoting antisymmetrization between each pair of indices. Remarkably, this operator was simplified in [30] to the simpler expression

$$b = P^a \bar\Sigma_a - \frac{4}{\eta} (\bar\lambda\gamma^{ab} r)(\lambda\gamma_a{}^c\lambda)\bar\Sigma_c \bar\Sigma_b - \frac{2}{\eta} (\bar\lambda r)(\lambda\gamma^{ab}\lambda)\bar\Sigma_a \bar\Sigma_b \tag{2.14}$$

where the fermionic vector $\bar{\Sigma}^i$, defined by

$$\bar{\Sigma}^i = \frac{1}{2\eta}(\bar{\lambda}\gamma^{ab}\bar{\lambda})(\lambda\gamma^{ab}\gamma^i d) + \frac{4}{\eta^2}L^{(1)}_{ab,cd}(\lambda\gamma^{abcei}\lambda)N^d{}_e + \frac{4}{3\eta^2}L^{(1)}_{ab,c}{}^i(\lambda\gamma^{abcde}\lambda)N_{de} \\ - \frac{4}{3\eta^2}L^{(1)}_{ad,c}{}^d(\lambda\gamma^{aicde}\lambda)N_{de} \tag{2.15}$$

obeys $(\bar{\lambda}\gamma^{ab}\bar{\lambda})\bar{\Sigma}_b = 0$, and

$$\{Q,\bar{\Sigma}^a\} = \frac{P^a}{2} + \frac{1}{\eta}[(\bar{\lambda}\gamma^{cb}\bar{\lambda})(\lambda\gamma_{ba}\lambda) - (\bar{\lambda}\gamma^{ab}\bar{\lambda})(\lambda\gamma_{bc}\lambda)]P^c - \frac{2}{\eta}(\bar{\lambda}\gamma^{ba}r)(\lambda\gamma_b{}^c\lambda)\bar{\Sigma}_c \\ - \frac{4}{\eta}(\bar{\lambda}\gamma^{bc}r)(\lambda\gamma_b{}^a\lambda)\bar{\Sigma}_c + \frac{2}{\eta}(\bar{\lambda}r)(\lambda\gamma^{ab}\lambda)\bar{\Sigma}_b - \frac{2}{\eta^2}(\bar{\lambda}\gamma^{cd}r)(\lambda\gamma_{cd}\lambda)(\bar{\lambda}\gamma^{ab}\bar{\lambda})(\lambda\gamma_{be}\lambda)\bar{\Sigma}^e \tag{2.16}$$

Using the identity (2.16), it was shown in [30], the simplified expression (2.14) indeed satisfies the property $\{Q,b\} = \frac{1}{2}P^2$, and it is nilpotent up to BRST-exact terms.

3 11D Physical operators

In this section we introduce the 11D analogues of the operators studied in [10] in the 10D case. These will be proven to be essential for a new formulation of the b-ghost exhibiting its close relation to 11D supergravity.

3.1 Physical operators

The 11D physical operators will be defined as follows

$$[Q,\mathbf{C}_\alpha] = -\frac{1}{3}d_\alpha - (\gamma^a\lambda)_\alpha\mathbf{C}_a \tag{3.1}$$
$$\{Q,\mathbf{C}_a\} = \frac{1}{3}P_a + (\lambda\gamma^{ab}\lambda)\mathbf{\Phi}_b \tag{3.2}$$
$$[Q,\mathbf{\Phi}^a] = (\lambda\gamma^a\mathbf{\Phi}) \tag{3.3}$$
$$[Q,\mathbf{\Phi}^\alpha] = \frac{1}{4}(\lambda\gamma^{ab})^\alpha\mathbf{\Omega}_{ab} \tag{3.4}$$
$$\vdots$$

These relations follow immediately from the linearized 11D supergravity equations of motion (see appendix A for a short review). The elipsis below (3.4) represent additional equations which will not be relevant for our purposes. The system of equations above displayed is solved by

$$\mathbf{C}_\alpha = \frac{1}{3}K_\alpha{}^\beta w_\beta \tag{3.5}$$
$$\mathbf{C}^a = \frac{1}{\eta}(\lambda\gamma^{abc})^\alpha(\bar{\lambda}\gamma_{bc}\bar{\lambda})\left[\frac{1}{3}d_\alpha + [Q,\mathbf{C}_\alpha]\right] \tag{3.6}$$
$$\mathbf{\Phi}^a = \frac{2}{\eta}(\bar{\lambda}\gamma^{ab}\bar{\lambda})\left[\frac{1}{3}P_b - \{Q,\mathbf{C}_b\}\right] \tag{3.7}$$
$$\mathbf{\Phi}^\alpha = -\frac{2}{\eta}(\gamma^{abc}\lambda)^\alpha(\bar{\lambda}\gamma_{bc}r)\mathbf{\Phi}_a \tag{3.8}$$

where $K_\alpha{}^\beta$ is an 11D pure spinor projector defined as

$$K_\alpha{}^\beta = -\frac{1}{6\eta}(\lambda\gamma^{ab})^\beta(\bar\lambda\gamma^{cd}\bar\lambda)(\lambda\gamma_{abcd})_\alpha - \frac{4}{3\eta}(\lambda\gamma^{ab})^\beta(\lambda\gamma_b{}^d)_\alpha(\bar\lambda\gamma_{ad}\bar\lambda) - \frac{2}{3\eta}(\lambda\gamma^{cd})^\beta\lambda_\alpha(\bar\lambda\gamma_{cd}\bar\lambda)$$
$$+\frac{1}{3\eta}\lambda^\beta(\lambda\gamma^{cd})_\alpha(\bar\lambda\gamma_{cd}\bar\lambda) \tag{3.9}$$

and the operators are constrained to satisfy

$$\xi^\alpha_a\mathbf{C}_\alpha = 0, \qquad (\bar\lambda\gamma^{ab}\bar\lambda)\mathbf{C}_a = 0, \qquad (\bar\lambda\gamma^a)_\alpha\mathbf{\Phi}_a = 0, \qquad R_\alpha{}^\beta\mathbf{\Phi}^\alpha = 0 \tag{3.10}$$

with ξ^β_a and $R_\alpha{}^\beta$ taking the explicit forms

$$\xi^\beta_a = \frac{1}{2}(\gamma_{abc})^{\beta\delta}\lambda_\delta(\bar\lambda\gamma^{bc}\bar\lambda) \tag{3.11}$$

$$R_\alpha{}^\beta = \Big[-\frac{1}{2}(\lambda\gamma^b)_\alpha(\lambda\gamma^c)^\beta - \frac{1}{4}(\lambda\gamma^{bk}\lambda)(\gamma^c\gamma^k)_\alpha{}^\beta + \frac{1}{2}(\lambda\gamma^{bk})_\alpha(\lambda\gamma^{ck})^\beta - \frac{1}{2}(\lambda\gamma^{bc})_\alpha\lambda^\beta\Big](\bar\lambda\gamma_{bc}\bar\lambda) \tag{3.12}$$

These objects were previously defined in [26] where they were shown to play an important role in the construction of a ghost number -2 operator mapping the cohomology of the ghost number three vertex operator into that of the ghost number one vertex operator. They obey the useful relation $(\lambda\gamma^a)_\alpha\xi^\beta_a = \frac{1}{2}\delta^\beta_\alpha\eta + R_\alpha{}^\beta$.

The projector of eq. (3.9) satisfies the desired properties

$$\lambda^\alpha K_\alpha{}^\beta = \lambda^\beta, \qquad (\lambda\gamma^{ab})^\alpha K_\alpha{}^\beta = (\lambda\gamma^{ab})^\beta, (\lambda\gamma^a)_\beta K_\alpha{}^\beta = 0, \qquad K_\alpha{}^\beta K_\beta{}^\delta = K_\alpha{}^\delta \tag{3.13}$$

and its trace can be shown to match the dimension of the 11D pure spinor space, that is $K_\alpha{}^\alpha = 23$. This statement can easily be proven by rewriting $K_\alpha{}^\beta$ in the more compact form

$$K_\alpha{}^\beta = \delta^\beta_\alpha + \frac{1}{\eta}(\lambda\gamma^{abc})^\beta(\bar\lambda\gamma_{bc}\bar\lambda)(\lambda\gamma_a)_\alpha \tag{3.14}$$

A demonstration of the equivalence between eqs. (3.9) and (3.14) is provided in appendix B.

Explicitly, the physical operators read

$$\mathbf{C}_\alpha = \frac{w_\alpha}{3} + \frac{1}{3\eta}(\lambda\gamma^{abc}w)(\bar\lambda\gamma_{bc}\bar\lambda)(\lambda\gamma_a)_\alpha \tag{3.15}$$

$$\mathbf{C}_a = \frac{1}{3\eta}(\bar\lambda\gamma^{bc}\bar\lambda)(\lambda\gamma_{abc}d) - \frac{2}{3\eta}(\bar\lambda\gamma^{bc}r)(\lambda\gamma_{abc}w) + \frac{2}{3\eta^2}\phi(\bar\lambda\gamma^{bc}\bar\lambda)(\lambda\gamma_{abc}w)$$
$$+\frac{4}{3\eta^2}(\lambda\gamma_{ac}\lambda)(\bar\lambda\gamma^{bc}\bar\lambda)(\bar\lambda\gamma^{de}r)(\lambda\gamma_{bde}w) \tag{3.16}$$

$$\mathbf{\Phi}^a = \frac{2}{3}\Big[\frac{1}{\eta}(\bar\lambda\gamma^{ab}\bar\lambda)P_b - \frac{2}{\eta^2}(\bar\lambda\gamma^{ab}\bar\lambda)(\bar\lambda\gamma^{cd}r)(\lambda\gamma_{bcd}d) + \{s, \frac{2}{\eta^2}(\bar\lambda\gamma^{ab}\bar\lambda)(\bar\lambda\gamma^{cd}r)\}(\lambda\gamma_{bcd}w)$$
$$-\frac{8}{\eta^3}(\lambda\gamma^a\xi_b)(\bar\lambda\gamma^{cb}r)(\bar\lambda\gamma^{de}r)(\lambda\gamma_{cde}w)\Big] \tag{3.17}$$

$$\mathbf{\Phi}^\alpha = \frac{8}{3}\xi^\alpha_a\Big[\frac{1}{\eta^2}(\bar\lambda\gamma^{ab}r)P_b - \frac{4}{\eta^4}(\bar\lambda\gamma^{ab}r)(\lambda\gamma_{cb}\lambda)(\bar\lambda\gamma^{cd}\bar\lambda)(\bar\lambda\gamma^{ef}r)(\lambda\gamma_{def}d)$$
$$-\Big(\frac{8}{\eta^4}(\bar\lambda\gamma^{ab}r)(\lambda\gamma_{cb}\lambda)(\bar\lambda\gamma^{cd}r)(\bar\lambda\gamma^{ef}r) - \frac{16}{\eta^5}(\bar\lambda\gamma^{ab}r)\phi(\lambda\gamma_{cb}\lambda)(\bar\lambda\gamma^{cd}\bar\lambda)(\bar\lambda\gamma^{ef}r)\Big)(\lambda\gamma_{def}w)\Big] \tag{3.18}$$

where $\phi = (\lambda\gamma^{ab}\lambda)(\bar{\lambda}\gamma_{ab}r)$. These relations can be rewritten in a manifestly gauge invariant form, as follows

$$\begin{aligned}\mathbf{C}_\alpha =& -\frac{1}{9\eta}N^{ab}(\bar{\lambda}\gamma^{cd}\bar{\lambda})(\lambda\gamma_{abcd})_\alpha - \frac{8}{9\eta}N^{ab}(\lambda\gamma_b{}^d)_\alpha(\bar{\lambda}\gamma_{ad}\bar{\lambda}) - \frac{4}{9\eta}N^{cd}\lambda_\alpha(\bar{\lambda}\gamma_{cd}\bar{\lambda}) \\ &+\frac{2}{9\eta}J(\lambda\gamma^{cd})_\alpha(\bar{\lambda}\gamma_{cd}\bar{\lambda})\end{aligned} \tag{3.19}$$

$$\begin{aligned}\mathbf{C}_a =& \frac{1}{3\eta}(\lambda\gamma_{abc}d)(\bar{\lambda}\gamma^{bc}\bar{\lambda}) + \frac{8}{3\eta^2}(\bar{\lambda}\gamma^{bc}\bar{\lambda})(\bar{\lambda}\gamma^{de}r)(\lambda\gamma_{bcdfa}\lambda)N_e{}^f + \frac{8}{9\eta^2}(\bar{\lambda}\gamma^{bc}\bar{\lambda})(\bar{\lambda}r)(\lambda\gamma_{abcde}\lambda)N^{de} \\ &+\frac{4}{9\eta^2}(\bar{\lambda}\gamma_{bc}\bar{\lambda})(\bar{\lambda}\gamma_{da}r)(\lambda\gamma^{bcdef}\lambda)N_{ef}\end{aligned} \tag{3.20}$$

$$\begin{aligned}\mathbf{\Phi}^a =& \frac{2}{3}\left[\frac{1}{\eta}(\bar{\lambda}\gamma^{ab}\bar{\lambda})P_b - \frac{2}{\eta^2}(\bar{\lambda}\gamma^{ab}\bar{\lambda})(\bar{\lambda}\gamma^{cd}r)(\lambda\gamma_{bcd}d) - \frac{16}{\eta^3}(\bar{\lambda}\gamma^{ab}\bar{\lambda})(\bar{\lambda}\gamma^{cd}r)(\bar{\lambda}\gamma^{ef}r)(\lambda\gamma_{bcdeg}\lambda)N_f{}^g\right. \\ &\left.-\frac{8}{\eta^3}(\bar{\lambda}\gamma^{ab}\bar{\lambda})(\bar{\lambda}\gamma^{cd}r)(\bar{\lambda}r)(\lambda\gamma_{bcdef}\lambda)N^{ef}\right]\end{aligned} \tag{3.21}$$

$$\begin{aligned}\mathbf{\Phi}^\alpha =& \frac{8}{3}\xi^\alpha_a\left[\frac{1}{\eta^2}(\bar{\lambda}\gamma^{ab}r)P_b - \frac{2}{\eta^3}(\bar{\lambda}\gamma^{ab}r)(\bar{\lambda}\gamma^{cd}r)(\lambda\gamma_{bcd}d) - \frac{8}{\eta^4}(\bar{\lambda}\gamma^{ab}r)(\bar{\lambda}\gamma^{cd}r)(\bar{\lambda}r)(\lambda\gamma_{bcdef}\lambda)N^{ef}\right. \\ &\left.-\frac{16}{\eta^4}(\bar{\lambda}\gamma^{ab}r)(\bar{\lambda}\gamma^{cd}r)(\bar{\lambda}\gamma^{ef}r)(\lambda\gamma_{bcdeg}\lambda)N_f{}^g\right]\end{aligned}$$

where $J = \lambda^\alpha w_\alpha$.

Now it is easy to calculate the action of the 11D physical operators on Ψ. Let us start with $\mathbf{C}_\alpha$. The formula (3.14) immediately implies that

$$\mathbf{C}_\alpha\Psi = C_\alpha + (\lambda\gamma^a)_\alpha\rho_a \tag{3.22}$$

where $C_\alpha = \lambda^\beta\lambda^\delta C_{\alpha\beta\delta}$, and $\rho^a = \frac{1}{\eta}(\lambda\gamma^{abc})^\alpha C_\alpha(\bar{\lambda}\gamma_{bc}\bar{\lambda})$. Using eq. (3.22), one can compute the action of $\mathbf{C}^a$ on Ψ. Indeed, one finds that

$$\mathbf{C}_a\Psi = C_a + (\lambda\gamma_{ac}\lambda)s^c - Q\rho_a \tag{3.23}$$

where $C_a = \lambda^\beta\lambda^\delta C_{a\beta\delta}$, and $s^b = -\frac{2}{\eta}(\bar{\lambda}\gamma^{bc}\bar{\lambda})(C_c - Q\rho_c)$. Similarly, the use of eq. (3.23) allows one to show that

$$\mathbf{\Phi}^a\Psi = \Phi^a + (\lambda\gamma^a\kappa) + Qs^a \tag{3.24}$$

where $\Phi^a = \lambda^\alpha h_\alpha{}^a$, and $\kappa^\alpha = -2\xi^\alpha_a(\Phi^a + Qs^a)$. Finally, eq. (3.24) implies that the action of $\mathbf{\Phi}^\alpha$ on Ψ is given by

$$\mathbf{\Phi}^\alpha\Psi = \Phi^\alpha + (\lambda\gamma^{ab})^\alpha f_{ab} + \lambda^\alpha f + Q\kappa^\beta \tag{3.25}$$

where $\Phi^\alpha = \lambda^\beta h_\beta{}^\alpha$, $f_{ab} = \frac{2}{3\eta}(\bar{\lambda}\gamma_{ab}\bar{\lambda})\lambda_\delta\tau^\delta + \frac{4}{3\eta}(\bar{\lambda}\gamma_{k[a}\bar{\lambda})(\lambda\gamma^k{}_{b]})_\alpha\tau^\alpha + \frac{1}{6\eta}(\bar{\lambda}\gamma^{cd}\bar{\lambda})(\lambda\gamma_{cdab})_\alpha\tau^\alpha$, $f = -\frac{1}{3\eta}(\bar{\lambda}\gamma_{ab}\bar{\lambda})(\lambda\gamma^{ab})_\delta\tau^\delta$, $\tau^\alpha = \Phi^\alpha + Q\kappa^\alpha$, and we used the alternative expression for $R_\alpha{}^\beta$

$$R_\alpha{}^\beta = \left[\frac{1}{12}(\lambda\gamma^{abcd})_\alpha(\lambda\gamma_{ab})^\beta + \frac{2}{3}(\lambda\gamma^{kd})_\alpha(\lambda\gamma^c{}_k)^\beta + \frac{1}{3}\lambda_\alpha(\lambda\gamma^{cd})^\beta - \frac{1}{6}(\lambda\gamma^{cd})_\alpha\lambda^\beta\right](\bar{\lambda}\gamma_{cd}\bar{\lambda}) \tag{3.26}$$

3.2 A simple expression for the b-ghost

The physical operators recently studied allow us to write the following alternative expression for the 11D b-ghost:

$$b = \frac{3}{2}P^a \mathbf{C}_a + \frac{3}{2}(\lambda\gamma^a d)\mathbf{\Phi}_a - \frac{3}{2}(\lambda\gamma^a w)(\lambda\gamma_a \mathbf{\Phi}) \tag{3.27}$$

or in a gauge invariant form

$$b = \frac{3}{2}P^a \mathbf{C}_a + \frac{3}{2}(\lambda\gamma^a d)\mathbf{\Phi}_a - \frac{1}{2}N^{ab}(\lambda\gamma_{ab}\mathbf{\Phi}) \tag{3.28}$$

It is easy to show that $\{Q, b\} = \frac{P^2}{2}$. Indeed, the use of the defining properties (3.1)–(3.4) imply that

$$\begin{aligned}\{Q, b\} &= \frac{1}{2}P^2 + \frac{3}{2}(\lambda\gamma^{ab}\lambda)P_a\mathbf{\Phi}_b + \frac{3}{2}(\lambda\gamma^{ab}\lambda)P_b\mathbf{\Phi}_a - \frac{3}{2}(\lambda\gamma^a d)(\lambda\gamma_a\mathbf{\Phi}) + \frac{3}{2}(\lambda\gamma^a d)(\lambda\gamma_a\mathbf{\Phi})\\ &= \frac{1}{2}P^2\end{aligned} \tag{3.29}$$

One can also check that b is nilpotent up to BRST-exact terms. To see this, it is enough to show that $\{b, b\}$ does not contain any term independent of r_α [30]. This easily follows from the explicit relations (3.15)–(3.18), and the convenient rewriting

$$\begin{aligned}b &= -\frac{1}{\eta}(\bar\lambda\gamma^{ab}\bar\lambda)P_a(\lambda\gamma_b d) + \frac{1}{2\eta}(\bar\lambda\gamma^{bc}\bar\lambda)P^a(\lambda\gamma_{abc}d) + O(r)\\ &= \frac{1}{2\eta}(\bar\lambda\gamma_{bc}\bar\lambda)(\lambda\gamma^{bc}\gamma^a d)P_a + O(r)\end{aligned} \tag{3.30}$$

The constraint algebra $\{d_\alpha, d_\beta\} = -(\gamma^a)_{\alpha\beta}P_a$, then shows our claim.

Finally, after expanding eq. (3.28), and do some algebraic manipulations, one finds that

$$\begin{aligned}b =& \frac{1}{2\eta}(\bar\lambda\gamma_{bc}\bar\lambda)(\lambda\gamma^{bc}\gamma^a d)P_a + \frac{4}{\eta^2}L^{(1)}_{bc,de}(\lambda\gamma^{bcdfa}\lambda)P_a N^e{}_f + \frac{4}{3\eta^2}L^{(1)}_{bf,cf}(\lambda\gamma^{abcde}\lambda)P_a N_{de}\\ &+ \frac{4}{3\eta^2}L^{(1)}_{bc,da}(\lambda\gamma^{bcdef}\lambda)P^a N_{ef} + \frac{2}{\eta^2}L^{(1)}_{ab,cd}(\lambda\gamma^a d)(\lambda\gamma^{bcd}d) - \frac{16}{\eta^3}L^{(2)}_{ab,cd,ef}(\lambda\gamma^{bcdeg}\lambda)N^f{}_g(\lambda\gamma^a d)\\ &- \frac{8}{\eta^3}L^{(2)}_{ag,cd,bg}(\lambda\gamma^{bcdef}\lambda)N_{ef}(\lambda\gamma^a d) - \frac{4}{3\eta^3}L^{(2)}_{ij,ab,cd}(\lambda\gamma^{aijkl}\lambda)(\lambda\gamma^{bcd}d)N_{kl}\\ &- \frac{16}{3\eta^4}L^{(3)}_{ig,ab,cd,jg}(\lambda\gamma^{bcdef}\lambda)(\lambda\gamma^{aijkl}\lambda)N_{kl}N_{ef} - \frac{32}{3\eta^4}L^{(3)}_{ij,ab,cd,ef}(\lambda\gamma^{aijkl}\lambda)(\lambda\gamma^{bcdeg}\lambda)N_{kl}N^f{}_g\end{aligned} \tag{3.31}$$

which, by simple inspection, coincides with the original expression displayed in (2.10).

Next we use the new form for the b-ghost, eq. (3.27), to calculate different quantities relevant to the computation of scattering amplitudes in pure spinor worldline and field theory.

JHEP03(2023)135

4 Some applications

4.1 The ghost number two vertex operator

In [26], a ghost number two vertex operator was defined by letting a non-Lorentz covariant b-ghost act on the ghost number three vertex operator Ψ. The result was remarkably shown to be independent of non-minimal variables up to BRST-exact terms. Here, we define the ghost number two vertex operator following the same prescription of [26]

$$U^{(2)} = \{b, \Psi\} \tag{4.1}$$

Notice that this computation would be pretty complicated to carry out by using the original or simplified expressions for the b-ghost, eqs. (2.10) and (2.14). However, the use of the physical operators discussed in previous section provides a simple and efficient treatment to the problem. Concretely, eqs. (3.23), (3.24), (3.25) yield

$$\begin{aligned} U^{(2)} =& \frac{3}{2}P^a C_a + \frac{3}{2}(\lambda\gamma^a d)\Phi_a - \frac{1}{2}N^{ab}(\lambda\gamma_{ab}\Phi) + Q\left[-\frac{3}{2}P^a\rho_a - \frac{3}{2}(\lambda\gamma^a d)s_a - \frac{3}{2}(\lambda\gamma^a w)(\lambda\gamma_a\kappa)\right] \\ &+ \frac{3}{2}\mathbf{C}^a\partial_a\Psi + \frac{3}{2}\mathbf{\Phi}^a(\lambda\gamma_a D)\Psi + \frac{9}{2}(\lambda\gamma^a C)(\lambda\gamma_a\mathbf{\Phi}) \end{aligned} \tag{4.2}$$

On the other hand, the use of the 11D supergravity equations of motion allows us to show the following identity

$$\begin{aligned} \frac{3}{2}\mathbf{C}^a\partial_a\Psi + \frac{3}{2}\mathbf{\Phi}^a(\lambda\gamma_a D)\Psi + \frac{9}{2}(\lambda\gamma^a C)(\lambda\gamma_a\mathbf{\Phi}) =& \frac{3}{2}P^a C_a + \frac{3}{2}(\lambda\gamma^a d)\Phi_a - \frac{3}{2}(\lambda\gamma^a w)(\lambda\gamma_a\Phi) \\ &+ Q\left[-\frac{9}{2}\mathbf{C}^a C_a - \frac{9}{2}\mathbf{\Phi}^a(\lambda\gamma_a C) + \frac{9}{2}(\lambda\gamma^a\mathbf{C})\Phi_a\right] \end{aligned} \tag{4.3}$$

In this manner, one learns that

$$\begin{aligned} U^{(2)} =& 3P^a C_a + 3(\lambda\gamma^a d)\Phi_a - 3(\lambda\gamma^a w)(\lambda\gamma_a\Phi) + Q\left[-\frac{3}{2}P^a\rho_a - \frac{3}{2}(\lambda\gamma^a d)s_a - \frac{3}{2}(\lambda\gamma^a w)(\lambda\gamma_a\kappa)\right. \\ &\left. - \frac{9}{2}\mathbf{C}^a C_a - \frac{9}{2}\mathbf{\Phi}^a(\lambda\gamma_a C) + \frac{9}{2}(\lambda\gamma^a\mathbf{C})\Phi_a\right] \end{aligned} \tag{4.4}$$

The vertex (4.4) is manifestly Lorentz covariant and invariant under the pure spinor constraint, and its non-BRST-exact piece is remarkably independent of non-minimal variables. Such a sector matches the vertex found in [26] using the Y-formalism in 11D.

4.2 Generalized Siegel gauge

The maximally supersymmetric theories admitting pure spinor field theory descriptions exhibit a notable symmetry between fields and antifields in a single pure spinor superfield, and thus cannot be quantized by using conventional gauge-fixing techniques. Indeed, it was suggested in [35] that, in analogy with string field theory, the Siegel gauge $b\Psi = 0$ may be used as a consistent gauge-fixing condition in pure spinor master actions. A slightly modified version, referred to as the generalized Siegel gauge, $b\Psi = Q\Omega$ for some Ω, was used in [16] in the context of 10D super-Yang-Mills to show that the scattering amplitudes obtained from the field theory action, match those obtained from CFT techniques in the open superstring [17].

The new expression for the 11D b-ghost (3.27) will now be used to show that the ghost number three vertex operator Ψ satisfies the generalized Siegel gauge. This easily follows from our results (3.23), (3.24), (3.25):

$$\begin{aligned} b\Psi =& \frac{3}{2}\partial^a C_a + \frac{3}{2}(\lambda\gamma^a D)\Phi_a + \frac{3}{2}(\lambda\gamma^a D)(\lambda\gamma_a\kappa) - \frac{3}{2}(\lambda\gamma^a\partial_\lambda)(\lambda\gamma_a\Phi) - \frac{3}{2}(\lambda\gamma^a\partial_\lambda)(\lambda\gamma_a Q\kappa) \\ &+ Q\Big[-\frac{3}{2}\partial^a\rho_a - \frac{3}{2}(\lambda\gamma^a D)s_a\Big] \end{aligned} \tag{4.5}$$

Using the transversality of $C_{a\alpha\beta}$, and the linearized 11D supergravity equations of motion (see appendix A), one then concludes that

$$b\Psi = Q\Big[-\frac{3}{2}(\lambda\gamma_a h^a) - \frac{3}{2}\partial^a\rho_a - \frac{3}{2}(\lambda\gamma^a D)s_a - \frac{3}{2}(\lambda\gamma^a\partial_\lambda)(\lambda\gamma_a\kappa)\Big] \tag{4.6}$$

as stated.

4.3 The two-particle superfield

The pure spinor description of 11D supergravity was introduced by Cederwall in [9]. The action is quartic in the pure spinor superfield Ψ, and produces the following equation of motion

$$Q\Psi + \frac{\kappa}{2}(\lambda\gamma_{ab}\lambda)\mathbf{\Phi}^a\Psi\mathbf{\Phi}^b\Psi + \frac{\kappa}{2}\Psi\{Q,\mathbf{T}\}\Psi - \kappa^2(\lambda\gamma_{ab}\lambda)\mathbf{T}\Psi\mathbf{\Phi}^a\Psi\mathbf{\Phi}^b\Psi = 0 \tag{4.7}$$

where $\mathbf{\Phi}^a$ is a physical operator introduced in section 3, and $\mathbf{T}$ is defined as

$$\mathbf{T} = \frac{32}{9\eta^3}(\bar\lambda\gamma^{ab}\bar\lambda)(\bar\lambda r)(rr)N_{ab} \tag{4.8}$$

The use of the perturbiner method allows one to solve eq. (4.7) in terms of multiparticle superfields. Concretely, the expansion

$$\Psi = \sum_{\mathcal{P}}\Psi_{\mathcal{P}}e^{ik_{\mathcal{P}}\cdot X} \tag{4.9}$$

where $\mathcal{P}$ denotes non-empty words $p_1p_2\ldots p_m$, with $p_1 < p_2 < \ldots < p_m$, and $k_{\mathcal{P}} = k_{p_1} + k_{p_2} + \ldots + k_{p_m}$, yields the following set of relations:

$$Q\Psi_{p_1} = 0 \tag{4.10}$$

$$Q\Psi_{p_1p_2} = -\kappa(\lambda\gamma_{ab}\lambda)\mathbf{\Phi}^a\Psi_{p_1}\mathbf{\Phi}^b\Psi_{p_2} - \frac{\kappa}{2}\Psi_{p_1}\{Q,\mathbf{T}\}\Psi_{p_2} - \frac{\kappa}{2}\Psi_{p_2}\{Q,\mathbf{T}\}\Psi_{p_1} \tag{4.11}$$

$$\begin{aligned} Q\Psi_{p_1p_2p_3} =& -\sum_{\mathcal{P}=\mathcal{Q}\cup\mathcal{R}}\kappa\Big[(\lambda\gamma_{ab}\lambda)\mathbf{\Phi}^a\Psi_{\mathcal{Q}}\mathbf{\Phi}^b\Psi_{\mathcal{R}} + \frac{1}{2}\Psi_{\mathcal{Q}}\{Q,\mathbf{T}\}\Psi_{\mathcal{R}}\Big] \\ &+ \sum_{\mathcal{P}=\mathcal{Q}\cup\mathcal{R}\cup\mathcal{S}}\kappa^2(\lambda\gamma_{ab}\lambda)\mathbf{T}\Psi_{\mathcal{Q}}\mathbf{\Phi}^a\Psi_{\mathcal{R}}\mathbf{\Phi}^b\Psi_{\mathcal{S}} \end{aligned} \tag{4.12}$$

$$\vdots$$

JHEP03(2023)135

where $\mathcal{P} = \mathcal{Q}_1 U \mathcal{Q}_2 U \dots U \mathcal{Q}_s$, indicates a distribution of the words $\mathcal{P}$ into the non-empty ordered words $\mathcal{Q}_1$, $\mathcal{Q}_2$, ..., $\mathcal{Q}_s$. The first equation is nothing but the linearized equation of motion of 11D supergravity obtained from the 11D pure spinor superparticle cohomology. The other equations define the multiparticle superfields of 11D supergravity after removing all BRST-exact terms, as explained in [34]. To illustrate this, let us study the two-particle superfield. Eqs. (3.24), (4.10) imply that

$$Q\tilde{\Psi}_{p_1p_2} = -\kappa(\lambda\gamma_{ab}\lambda)\Phi^a_{p_1}\Phi^b_{p_2} \tag{4.13}$$

where

$$\begin{aligned}\tilde{\Psi}_{p_1p_2} &= \Psi_{p_1p_2} + \kappa(\lambda\gamma_{ab}\lambda)s^a_{p_1}\Phi^b_{p_2} - \kappa(\lambda\gamma_{ab}\lambda)\Phi^a_{p_1}s^b_{p_2} \\ &\quad + \kappa(\lambda\gamma_{ab}\lambda)s^a_{p_1}Qs^b_{p_2} - \frac{\kappa}{2}\Psi_{p_1}\mathbf{T}\Psi_{p_2} - \frac{\kappa}{2}\Psi_{p_2}\mathbf{T}\Psi_{p_1}\end{aligned} \tag{4.14}$$

Using that $\{Q, b\} = \frac{P^2}{2}$, one finds that

$$\tilde{\Psi}_{p_1p_2} = -\frac{2\kappa}{k^2_{p_1p_2}}b\Big[(\lambda\gamma_{ab}\lambda)\Phi^a_{p_1}\Phi^b_{p_2}\Big] \tag{4.15}$$

It is not hard to check that the physical operators studied in section 3.1, shape the solution of (4.15) as

$$b[(\lambda\gamma_{ab}\lambda)\Phi^a_{p_1}\Phi^b_{p_2}] = \tilde{C}_{p_1p_2} + Q\Lambda_{p_1p_2} \tag{4.16}$$

where $\Lambda_{p_1p_2} = -\frac{2}{k^2_{p_1p_2}}b(\tilde{C}_{p_1p_2})$, up to BRST-exact terms, and

$$\begin{aligned}\tilde{C}_{p_1p_2} =& \frac{1}{2}\Big[(\lambda\gamma^{bc}\lambda)h_{p_1,ab}k^a_{p_2}\Phi_{p_2,c} + \Omega_{p_1,ab}k^a_{p_2}C^b_{p_2} - (\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta C^{ab}_{p_2} \\ &+ (\lambda\gamma^{bc}\lambda)h_{p_2,ab}k^a_{p_1}\Phi_{p_1,c} + \Omega_{p_2,ab}k^a_{p_1}C^b_{p_1} - (\lambda\gamma^b)_\delta\lambda^\alpha T_{p_2,\alpha a}{}^\delta C^{ab}_{p_1}\Big]\end{aligned} \tag{4.17}$$

with $T_{\alpha a}{}^\delta = \frac{1}{36}\Big[(\gamma^{bcd})_\alpha{}^\delta H_{abcd} + \frac{1}{8}(\gamma_a{}^{bcde})_\alpha{}^\delta H_{bcde}\Big]$ (see [40, 41] for details). An easy way of checking this is through the use of the equations of motion listed in appendix A. For instance, eqs. (A.6), (A.9), (A.14) lead to

$$\begin{aligned}Q\tilde{C}_{p_1p_2} =& \frac{1}{2}\Big[(\lambda\gamma^{bc}\lambda)(k_{p_1}\cdot k_{p_2})\Phi_{p_1,b}\Phi_{p_2,c} - (\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta k^{[a}_{p_2}C^{b]}_{p_2} \\ &+ Q\Big[-(\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta C^{ab}_{p_2}\Big] + (1\leftrightarrow 2)\Big]\end{aligned} \tag{4.18}$$

Eq. (A.15) then requires that

$$\begin{aligned}Q\tilde{C}_{p_1p_2} =& \frac{1}{2}\Big[(\lambda\gamma^{bc}\lambda)(k_{p_1}\cdot k_{p_2})\Phi_{p_1,b}\Phi_{p_2,c} + (\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta QC^{ab}_{p_2} \\ &+ (\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta[(\lambda\gamma^{[bc}\lambda)h^{a]}{}_{c,p_2} - (\lambda\gamma^{ab})_\beta\Phi^\beta_{p_2}] \\ &+ Q\Big[-(\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta C^{ab}_{p_2}\Big] + (1\leftrightarrow 2)\Big] \\ =& \frac{1}{2}\Big[(\lambda\gamma^{bc}\lambda)(k_{p_1}\cdot k_{p_2})\Phi_{p_1,b}\Phi_{p_2,c} - (\lambda\gamma_b)_\delta\lambda^\alpha T_{p_1,\alpha a}{}^\delta(\lambda\gamma^{ab})_\beta\Phi^\beta_{p_2} + (1\leftrightarrow 2)\Big]\end{aligned} \tag{4.19}$$

where we used that $(\lambda\gamma_b)_\delta T_{\alpha a}{}^\delta\lambda^\alpha = \frac{1}{12}\Big[(\lambda\gamma^{de}\lambda)H_{abde} + \frac{1}{24}(\lambda\gamma_{ab}{}^{cdef}\lambda)H_{cdef}\Big]$. The Fierz identity $(\lambda\gamma_{ab})_\alpha(\lambda\gamma^{abcdef}\lambda) = -24(\lambda\gamma^{[cd})_\alpha(\lambda\gamma^{ef]}\lambda)$, then states that

$$Q\tilde{C}_{p_1p_2} = (\lambda\gamma^{bc}\lambda)(k_{p_1}\cdot k_{p_2})\Phi_{p_1,b}\Phi_{p_2,c} \tag{4.20}$$

5 Discussions

The main result of this paper is the introduction and construction of the 11D physical operators, and the finding of an alternative formula for the 11D b-ghost, which significantly simplifies algebraic computations in pure spinor superspace. As an exemplification of this statement, we were able to show the defining properties: $\{Q,b\} = \frac{P^2}{2}$, $\{b,b\} = Q\Omega$, in a systematic and quite simple way. Besides, we provided a few useful applications which will be relevant for studying 11D supergravity interactions from the pure spinor perspective. For instance, the two-particle superfield displayed in eq. (4.17) will be substantial for calculating the 4-point amplitude in pure spinor superspace from the perturbiner method applied to the pure spinor 11D supergravity field theory, see eq. (4.12). Higher-order interactions will require a solid understanding of the different properties associated to the physical operators, e.g. (anti)commutation relations, algebraic identities, and so forth. Likewise, this knowledge might potentially be used for studying consistent deformations of 11D supergravity, in analogy with the maximally supersymmetric Born-Infeld action deduced as the only possible deformation of 10D super-Yang-Mills, satisfying the pure spinor master action [10]. We plan to tackle these open questions in the near future.

It is exciting to see that the simplification of the 10D b-ghost gave rise to the unravelling of a kinematic algebra which automatically realizes the color-kinematics duality when external states are described by Siegel gauge operators. It would be interesting to use the formulae presented in this work, and to investigate which kind of underlying algebraic structure rules the 11D scattering amplitudes when vertex operators satisfy the Siegel gauge condition. Furthermore, the fundamental role of the 10D b-ghost in loop-level superstring scattering amplitudes suggests that multi-loop 11D pure spinor correlators will require the use and efficient manipulation of this operator, task which might effectively be carried out with the ideas developed in this paper.

It is also worthy pointing out that the simplified version of the 10D b-ghost has been found to be related to a twistorial formulation of 10D super-Yang-Mills using pure spinor variables [36–38]. This framework was showed to be equivalent to the supertwistor description of ambitwistor strings presented in [39]. It is tempting to use the formulae introduced in this work for the 11D b-ghost, and propose a new twistor description of 11D supergravity using pure spinors, with possible stringy realizations. We leave these problems and related issues for future work.

Acknowledgments

I am grateful to Martin Cederwall for enlightening discussions on topics related to 11D pure spinors. This work was partially funded by the European Research Council under

ERC-STG-804286 UNISCAMP, and by the Knut and Alice Wallenberg Foundation under grant KAW 2018.0162 (Exploring a Web of Gravitational Theories through Gauge-Theory Methods).

A Linearized 11D supergravity

This appendix briefly reviews the geometrical construction in superspace which directly reproduces the 11D supergravity equations of motion at linearized order.

A.1 Equations of motion

Let us first set some notation. We will use capital letters from the beginning/middle of the Latin alphabet to represent tangent/curved superspace indices, and lowercase letters from the beginning (middle) of the Latin/Greek alphabet to denote tangent (curved) space vector/spinor indices. The 11D supergeometry is then defined by the 1-form superfields E^A and $\Omega_B{}^C$, referred to as the vielbein and spin-connection, respectively, and the super-Bianchi identities

$$\mathcal{D}T^A = E^B R_B{}^A, \qquad \mathcal{D}R_A{}^B = 0 \tag{A.1}$$

where $T^A = \mathcal{D}E^A$ is the super-torsion, $R_A{}^B = \mathcal{D}\Omega_A{}^B$ is the super-curvature, and $\mathcal{D} = E^A\nabla_A$ is the super-covariant derivative defined to act on the arbitrary tensor $\mathcal{F}_{A_1\ldots A_m}{}^{B_1\ldots B_n}$ as

$$\mathcal{D}\mathcal{F}_{A_1\ldots A_m}{}^{B_1\ldots B_n} = d\mathcal{F}_{A_1\ldots A_m}{}^{B_1\ldots B_n} - \Omega_{A_1}{}^C\mathcal{F}_{CA_2\ldots A_m}{}^{B_1\ldots B_n} + \ldots + \mathcal{F}_{A_1\ldots A_m}{}^{C\ldots B_n}\Omega_C{}^{B_1} + \ldots \tag{A.2}$$

and d is the ordinary exterior derivative. Eqs. (A.1) imply the familiar relations

$$[\nabla_A, \nabla_B\} = -T_{AB}{}^C\nabla_C - 2\Omega_{[AB\}}{}^C\nabla_C, \tag{A.3}$$

$$R_{AB,C}{}^D = 2\nabla_{[A}\Omega_{B\}C}{}^D + T_{AB}{}^F\Omega_{FC}{}^D + \Omega_{[AB\}}{}^F\Omega_{FC}{}^D \tag{A.4}$$

where $[\,,\,\}$ means graded commutator. The spectrum of 11D supergravity contains a 3-form gauge field which can be promoted to the 3-form superfield $F = E^C E^B E^A F_{ABC}$, satisfying the gauge transformation $\delta F = dL$, for any 2-form superfield L. Its field strength takes the form $G = dF$, and it satisfies the Bianchi identity $dG = 0$. In order to describe linearized 11D supergravity, one first writes the covariant derivative $\nabla_A = E_A{}^M\partial_M$ at linear order as

$$\nabla_A = D_A - h_A{}^B D_B \tag{A.5}$$

where $D_A = \hat{E}_A{}^M\partial_M$, $h_A{}^B = \hat{E}_A{}^M E_M^{(1)B} = -E_A^{(1)M}\hat{E}_M{}^B$, $(\hat{E}_A{}^M,\ \hat{E}_M{}^B)$ are the background values of the vielbeins, and $(E_A^{(1)M},\ E_M^{(1)A})$ are their corresponding first order perturbations. Additionally, one imposes the conventional constraints $T_{\alpha\beta}{}^\delta = T_{a\alpha}{}^c = T_{ab}{}^c = G_{\alpha\beta\delta\epsilon} = G_{a\alpha\beta\delta} = G_{abc\alpha} = 0$, and the dynamical contraints $T_{\alpha\beta}{}^a = (\gamma^a)_{\alpha\beta}$, $G_{\alpha\beta ab} = (\gamma_{ab})_{\alpha\beta}$. After plugging (A.5) into eq. (A.3), one obtains the following set of

JHEP03(2023)135

equations of motion [40, 41]

$$2D_{(\alpha}h_{\beta)}{}^a - 2h_{(\alpha}{}^\delta(\gamma^a)_{\beta)\delta} + h_b{}^a(\gamma^b)_{\alpha\beta} = 0 \tag{A.6}$$

$$2D_{(\alpha}h_{\beta)}{}^\delta - 2\Omega_{(\alpha\beta)}{}^\delta + (\gamma^a)_{\alpha\beta}h_a{}^\delta = 0 \tag{A.7}$$

$$\partial_a h_\alpha{}^\beta - D_\alpha h_a{}^\beta - T_{a\alpha}{}^\beta - \Omega_{a\alpha}{}^\beta = 0 \tag{A.8}$$

$$\partial_a h_\alpha{}^b - D_\alpha h_a{}^b - h_a{}^\beta(\gamma^b)_{\beta\alpha} + \Omega_{\alpha a}{}^b = 0 \tag{A.9}$$

$$\partial_a h_b{}^\alpha - \partial_b h_a{}^\alpha - T_{ab}{}^\alpha = 0 \tag{A.10}$$

$$\partial_a h_b{}^c - \partial_b h_a{}^c - 2\Omega_{ab}{}^c = 0 \tag{A.11}$$

The equations of motion associated to the components of the linearized version of the 3-form superfield F, can directly be deduced from a 4-form superfield H defined from the field strength G as

$$H_{ABCD} = \hat{E}_{[D}{}^Q\hat{E}_C{}^P\hat{E}_B{}^N\hat{E}_{A\}}{}^M G_{MNPQ} \tag{A.12}$$

which can equivalently be written as $H_{ABCD} = 4D_{[A}C_{BCD\}} + 6\hat{T}_{[AB}{}^E C_{ECD\}}$, where $C_{ABC} = \hat{E}_{[C}{}^P\hat{E}_B{}^N\hat{E}_{A\}}{}^M F_{MNP}$, and $\hat{T}^A$ is the flat space-valued torsion. The expansion of (A.12) then yields

$$4D_{(\alpha}C_{\beta\delta\epsilon)} + 6(\gamma^a)_{(\alpha\beta}C_{a\delta\epsilon)} = 0 \tag{A.13}$$

$$\partial_a C_{\alpha\beta\delta} - 3D_{(\alpha}C_{a\beta\delta)} + 3(\gamma^b)_{(\alpha\beta}C_{ba\delta)} = 3(\gamma_{ab})_{(\alpha\beta}h_{\delta)}{}^b \tag{A.14}$$

$$2\partial_{[a}C_{b]\alpha\beta} + 2D_{(\alpha}C_{\beta)ab} + (\gamma^c)_{\alpha\beta}C_{cab} = 2(\gamma_{[b}{}^c)_{\alpha\beta}h_{a]c} + 2(\gamma_{ab})_{(\alpha\delta}h_{\beta)}{}^\delta \tag{A.15}$$

$$3\partial_{[a}C_{bc]\alpha} - D_\alpha C_{abc} = 3(\gamma_{[ab})_{\alpha\beta}h_{c]}{}^\beta \tag{A.16}$$

The defining relations for the physical operators studied in section 3.1 can then be easily found from these equations. For instance, after multiplying by $\lambda^\alpha\lambda^\beta\lambda^\delta$, eq. (A.13) implies that

$$3QC_\epsilon + D_\epsilon\Psi = -3(\lambda\gamma^a)_\epsilon C_a \tag{A.17}$$

where $C_\epsilon = \lambda^\alpha\lambda^\beta C_{\alpha\beta\epsilon}$, $C_a = \lambda^\alpha\lambda^\beta C_{a\alpha\beta}$. Assuming that there exist the linear operators $\mathbf{C}_\epsilon$, $\mathbf{C}_a$ such that their action on the ghost number three vertex operator Ψ are described by the relations: $\mathbf{C}_\epsilon\Psi = C_\epsilon + \ldots$, $\mathbf{C}_a\Psi = C_a + \ldots$, where $\ldots$ denote shift symmetry terms [10], then eq. (A.17) can be written in the operator form

$$[Q, \mathbf{C}_\epsilon] = -\frac{1}{3}d_\epsilon - (\lambda\gamma^a)_\epsilon\mathbf{C}_a \tag{A.18}$$

which is exactly the relation displayed in (3.1). Similar arguments follow for the other operators.

B 11D Pure spinor projector

The 11D pure spinor projector $M_\alpha{}^\beta$ was originally introduced in [25], and shown to be given by

$$\begin{aligned}M_\alpha{}^\beta = \delta_\alpha^\beta &- \frac{1}{4\alpha}(\bar\lambda\gamma_c)^\beta(\lambda\gamma^c)_\alpha - \frac{1}{2\eta\alpha}(\bar\lambda\gamma_a)^\beta(\lambda\gamma^{ab}\lambda)(\bar\lambda\gamma_{cb}\bar\lambda)(\lambda\gamma^c)_\alpha + \frac{1}{8\alpha}(\bar\lambda\gamma_{cd})^\beta(\lambda\gamma^{cd})_\alpha \\ &+ \frac{1}{8\eta\alpha}(\bar\lambda\gamma_{ab})^\beta(\bar\lambda\gamma_{cd}\bar\lambda)(\lambda\gamma^{ab}\lambda)(\lambda\gamma^{cd})_\alpha - \frac{1}{2\eta\alpha}(\bar\lambda\gamma_{ac})^\beta(\bar\lambda\gamma_{bd}\bar\lambda)(\lambda\gamma^{ab}\lambda)(\lambda\gamma^{cd})_\alpha\end{aligned} \tag{B.1}$$

where $\alpha = \lambda\bar{\lambda}$. This expression can be rewritten in the more convenient way

$$\begin{aligned} M_\alpha{}^\beta = \delta_\alpha^\beta - \frac{1}{4\alpha}(\bar{\lambda}\gamma_c)^\beta(\lambda\gamma^c)_\alpha - \frac{1}{2\eta\alpha}(\bar{\lambda}\gamma_a)^\beta(\lambda\gamma^{ab}\lambda)(\bar{\lambda}\gamma_{cb}\bar{\lambda})(\lambda\gamma^c)_\alpha \\ + \frac{1}{8\eta\alpha}(\bar{\lambda}\gamma_{ab})^\beta(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma^{abcde}\lambda)(\lambda\gamma_e)_\alpha \end{aligned} \tag{B.2}$$

where we used the Fierz identity $(\gamma^{[ab})_{(\delta\epsilon}(\gamma^{cd]})_{\mu)\alpha} = -\frac{1}{6}(\gamma_k)_{(\delta\epsilon}(\gamma^{abcdk})_{\mu)\alpha} - \frac{1}{6}(\gamma^{abcdk})_{(\delta\epsilon}(\gamma_k)_{\mu)\alpha}$. The application of the familiar 11D identity $(\gamma^{ab})_{(\alpha\beta}(\gamma_b)_{\delta\epsilon)} = 0$, and [26]

$$(\gamma^{ab})_\alpha{}^\beta(\gamma_{ab})_\delta{}^\epsilon = 2(\gamma^a)_\alpha{}^\beta(\gamma_a)_\delta{}^\epsilon + 4(\gamma^a)_\alpha{}^\epsilon(\gamma_a)_\delta{}^\beta + 4(\gamma^a)_{\alpha\delta}(\gamma_a)^{\epsilon\beta} - 4\delta_\alpha^\epsilon\delta_\delta^\beta + 4C_{\alpha\delta}C^{\epsilon\beta} \tag{B.3}$$

imply the following useful relations

$$\begin{aligned} &\frac{1}{8\eta\alpha}(\bar{\lambda}\gamma_{ab}w)(\lambda\gamma^{abcde}\lambda)(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha \\ &\quad = \frac{1}{8\eta\alpha}(\lambda\gamma_{ab}w)(\bar{\lambda}\gamma^{ab}\gamma^{cde}\lambda)(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha + \frac{1}{4\alpha}(\bar{\lambda}\gamma_a w)(\lambda\gamma^a)_\alpha \\ &\quad\quad - \frac{1}{\eta}(\lambda\gamma_a w)(\bar{\lambda}\gamma^{ac}\bar{\lambda})(\lambda\gamma_c)_\alpha - \frac{1}{\eta}(w\gamma^{cde}\lambda)(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha \\ &- \frac{1}{2\eta\alpha}(\bar{\lambda}\gamma_a w)(\lambda\gamma^{ab}\lambda)(\bar{\lambda}\gamma_{cb}\bar{\lambda})(\lambda\gamma^c)_\alpha \\ &\quad = \frac{1}{\eta\alpha}(\lambda\gamma_a\bar{\lambda})(\lambda\gamma^{ab}w)(\bar{\lambda}\gamma_{cb}\bar{\lambda})(\lambda\gamma^c)_\alpha + \frac{1}{\eta}(\lambda\gamma_a w)(\bar{\lambda}\gamma^{ac}\bar{\lambda})(\lambda\gamma_c)_\alpha \end{aligned} \tag{B.4}$$

Therefore, eq. (B.2) takes the equivalent form

$$\begin{aligned} M_\alpha{}^\beta = \delta_\alpha^\beta + \frac{1}{\eta}(\lambda\gamma^{cde})^\beta(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha + \frac{1}{\eta\alpha}(\lambda\gamma_a\bar{\lambda})(\lambda\gamma^{ab})^\beta(\bar{\lambda}\gamma_{cb}\bar{\lambda})(\lambda\gamma^c)_\alpha \\ + \frac{1}{8\eta\alpha}(\lambda\gamma_{ab})^\beta(\bar{\lambda}\gamma^{ab}\gamma^{cde}\lambda)(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha \end{aligned} \tag{B.5}$$

This equation differs from the 11D projector used in this paper $K_\alpha{}^\beta$, eq. (3.9), in the presence of the last two terms. However, these extra terms trivially satisfy the defining properties of a generic projector, and their traces can readily be shown to vanish, meaning they do not contribute to the dimension of pure spinor space. Indeed, if one defines $M_{1,\alpha}{}^\beta = \frac{1}{\eta\alpha}(\lambda\gamma_a\bar{\lambda})(\lambda\gamma^{ab})^\beta(\bar{\lambda}\gamma_{cb}\bar{\lambda})(\lambda\gamma^c)_\alpha$, $M_{2,\alpha}{}^\beta = \frac{1}{8\eta\alpha}(\lambda\gamma_{ab})^\beta(\bar{\lambda}\gamma^{ab}\gamma^{cde}\lambda)(\bar{\lambda}\gamma_{cd}\bar{\lambda})(\lambda\gamma_e)_\alpha$, it is not hard to convince oneself that

$$\begin{aligned} (\lambda\gamma^a)_\beta M_{1,\alpha}{}^\beta &= 0, & (\lambda\gamma^a)_\beta M_{2,\alpha}{}^\beta &= 0 \\ M_{1,\alpha}{}^\alpha &= 0, & M_{2,\alpha}{}^\alpha &= 0 \end{aligned} \tag{B.6}$$

Thus, the only meaningful information is carried by the first two terms of $M_\alpha{}^\beta$, namely $K_\alpha{}^\beta$, which satisfies the properties of an actual projector, as discussed in (3.13).

B.1 Equivalence of eqs. (3.9) and (3.14)

Now we will show that eq. (3.14) is identical to (3.9). Indeed, the use of the Fierz identity $(\gamma_a)_{(\epsilon\alpha}(\gamma^{abc})_{\delta)\rho} = -(\gamma^{[b})_{(\epsilon\alpha}(\gamma^{c]})_{\delta)\rho} + (\gamma^{[b}{}_k)_{(\epsilon\alpha}(\gamma^{c]k})_{\delta)\rho} + (\gamma^{bc})_{(\epsilon\alpha}C_{\delta)\rho}$, allows one to state

$$\begin{aligned} (\lambda\gamma^a)_\alpha(\lambda\gamma^{abc})_\rho = &-(\lambda\gamma^{[b})_\alpha(\lambda\gamma^{c]})_\rho - \frac{1}{2}(\lambda\gamma^{[b}{}_k\lambda)(\gamma^{c]k})_{\alpha\rho} + (\lambda\gamma^{[b}{}_k)_\alpha(\lambda\gamma^{c]k})_\rho \\ &- \frac{1}{2}(\lambda\gamma^{bc}\lambda)C_{\alpha\rho} - (\lambda\gamma^{bc})_\alpha\lambda_\rho \end{aligned} \tag{B.7}$$

This can be rewritten in the convenient form

$$
\begin{aligned}
(\lambda\gamma_a)_\alpha(\lambda\gamma^{abc})^\beta = & -\delta_\alpha^\beta(\lambda\gamma^{bc}\lambda) + (\lambda\gamma_a)^\beta(\lambda\gamma^a\gamma^{bc})_\alpha - (\lambda\gamma^{[b}{}_k)_\alpha(\lambda\gamma^{c]k})^\beta \\
& -(\lambda\gamma_k\gamma^{[c})_\alpha(\lambda\gamma^{b]k})^\beta + (\lambda\gamma^{bc})_\alpha\lambda^\beta
\end{aligned} \tag{B.8}
$$

Therefore, the projector $K_\alpha{}^\beta$ in eq. (3.14), can be cast as

$$
K_\alpha{}^\beta = \frac{1}{\eta}\Big[(\lambda\gamma_a)^\beta(\lambda\gamma^a\gamma^{bc})_\alpha - \lambda_\alpha(\lambda\gamma^{bc})^\beta + (\lambda\gamma^{bc})_\alpha\lambda^\beta - 2(\lambda\gamma^{[b}{}_k)_\alpha(\lambda\gamma^{c]k})^\beta\Big](\bar\lambda\gamma_{bc}\bar\lambda) \tag{B.9}
$$

Using that $(\lambda\gamma^k)^\beta(\lambda\gamma_k)_\epsilon = -\frac{1}{6}(\lambda\gamma^{ab})^\beta(\lambda\gamma_{ab})_\epsilon - \frac{2}{3}\lambda_\epsilon\lambda^\beta$, one arrives at

$$
\begin{aligned}
K_\alpha{}^\beta = & -\frac{1}{6\eta}(\lambda\gamma^{ab})^\beta(\bar\lambda\gamma^{cd}\bar\lambda)(\lambda\gamma_{abcd})_\alpha - \frac{4}{3\eta}(\lambda\gamma^{ck})_\alpha(\lambda\gamma_k{}^d)^\beta(\bar\lambda\gamma_{cd}\bar\lambda) - \frac{2}{3\eta}(\lambda\gamma^{cd})^\beta\lambda_\alpha(\bar\lambda\gamma_{cd}\bar\lambda) \\
& + \frac{1}{3\eta}\lambda^\beta(\lambda\gamma^{cd})_\alpha(\bar\lambda\gamma_{cd}\bar\lambda)
\end{aligned} \tag{B.10}
$$

which coincides with eq. (3.9).

References

[1] N. Berkovits, *Covariant quantization of the superparticle using pure spinors*, *JHEP* **09** (2001) 016 [hep-th/0105050] [INSPIRE].

[2] N. Berkovits, *ICTP lectures on covariant quantization of the superstring*, *ICTP Lect. Notes Ser.* **13** (2003) 57 [hep-th/0209059] [INSPIRE].

[3] M. Guillen, *Equivalence of the 11D pure spinor and Brink-Schwarz-like superparticle cohomologies*, *Phys. Rev. D* **97** (2018) 066002 [arXiv:1705.06316] [INSPIRE].

[4] N. Berkovits, *Super Poincare covariant quantization of the superstring*, *JHEP* **04** (2000) 018 [hep-th/0001035] [INSPIRE].

[5] N. Berkovits, *Towards covariant quantization of the supermembrane*, *JHEP* **09** (2002) 051 [hep-th/0201151] [INSPIRE].

[6] M. Cederwall, *N=8 superfield formulation of the Bagger-Lambert-Gustavsson model*, *JHEP* **09** (2008) 116 [arXiv:0808.3242] [INSPIRE].

[7] M. Cederwall, *Superfield actions for N=8 and N=6 conformal theories in three dimensions*, *JHEP* **10** (2008) 070 [arXiv:0809.0318] [INSPIRE].

[8] M. Cederwall, *An off-shell superspace reformulation of D=4, N=4 super-Yang-Mills theory*, *Fortsch. Phys.* **66** (2018) 1700082 [arXiv:1707.00554] [INSPIRE].

[9] M. Cederwall, *D=11 supergravity with manifest supersymmetry*, *Mod. Phys. Lett. A* **25** (2010) 3201 [arXiv:1001.0112] [INSPIRE].

JHEP03(2023)135

[10] M. Cederwall and A. Karlsson, *Pure spinor superfields and Born-Infeld theory*, *JHEP* **11** (2011) 134 [arXiv:1109.0809] [INSPIRE].

[11] N. Berkovits and C.R. Mafra, *Some Superstring Amplitude Computations with the Non-Minimal Pure Spinor Formalism*, *JHEP* **11** (2006) 079 [hep-th/0607187] [INSPIRE].

[12] H. Gomez and C.R. Mafra, *The Overall Coefficient of the Two-loop Superstring Amplitude Using Pure Spinors*, *JHEP* **05** (2010) 017 [arXiv:1003.0678] [INSPIRE].

[13] H. Gomez and C.R. Mafra, *The closed-string 3-loop amplitude and S-duality*, *JHEP* **10** (2013) 217 [arXiv:1308.6567] [INSPIRE].

[14] N. Berkovits, *Covariant Map Between Ramond-Neveu-Schwarz and Pure Spinor Formalisms for the Superstring*, *JHEP* **04** (2014) 024 [arXiv:1312.0845] [INSPIRE].

[15] N. Berkovits, *Manifest spacetime supersymmetry and the superstring*, *JHEP* **10** (2021) 162 [arXiv:2106.04448] [INSPIRE].

[16] M. Ben-Shahar and M. Guillen, *10D super-Yang-Mills scattering amplitudes from its pure spinor action*, *JHEP* **12** (2021) 014 [arXiv:2108.11708] [INSPIRE].

[17] C.R. Mafra, O. Schlotterer and S. Stieberger, *Complete N-Point Superstring Disk Amplitude I. Pure Spinor Computation*, *Nucl. Phys. B* **873** (2013) 419 [arXiv:1106.2645] [INSPIRE].

[18] C.R. Mafra and O. Schlotterer, *Multiparticle SYM equations of motion and pure spinor BRST blocks*, *JHEP* **07** (2014) 153 [arXiv:1404.4986] [INSPIRE].

[19] N. Berkovits, *Dynamical twisting and the b ghost in the pure spinor formalism*, *JHEP* **06** (2013) 091 [arXiv:1305.0693] [INSPIRE].

[20] C.-M. Chang, Y.-H. Lin, Y. Wang and X. Yin, *Deformations with Maximal Supersymmetries Part 2: Off-shell Formulation*, *JHEP* **04** (2016) 171 [arXiv:1403.0709] [INSPIRE].

[21] M. Guillen, *Green-Schwarz and pure spinor formulations of chiral strings*, *JHEP* **12** (2021) 029 [arXiv:2108.11724] [INSPIRE].

[22] Z. Bern, J.J.M. Carrasco and H. Johansson, *New Relations for Gauge-Theory Amplitudes*, *Phys. Rev. D* **78** (2008) 085011 [arXiv:0805.3993] [INSPIRE].

[23] Z. Bern, J.J.M. Carrasco and H. Johansson, *Perturbative Quantum Gravity as a Double Copy of Gauge Theory*, *Phys. Rev. Lett.* **105** (2010) 061602 [arXiv:1004.0476] [INSPIRE].

[24] Z. Bern et al., *The Duality Between Color and Kinematics and its Applications*, arXiv:1909.01358 [CERN-TH-2019-135] [INSPIRE].

[25] M. Cederwall and A. Karlsson, *Loop amplitudes in maximal supergravity with manifest supersymmetry*, *JHEP* **03** (2013) 114 [arXiv:1212.5175] [INSPIRE].

[26] M. Guillen, *Notes on the 11D pure spinor wordline vertex operators*, *JHEP* **08** (2020) 122 [arXiv:2006.06022] [INSPIRE].

[27] I. Oda and M. Tonin, *On the Berkovits covariant quantization of GS superstring*, *Phys. Lett. B* **520** (2001) 398 [hep-th/0109051] [INSPIRE].

[28] A.A. Rosly and K.G. Selivanov, *On amplitudes in selfdual sector of Yang-Mills theory*, *Phys. Lett. B* **399** (1997) 135 [hep-th/9611101] [INSPIRE].

[29] N. Berkovits, *Pure spinor formalism as an N=2 topological string*, *JHEP* **10** (2005) 089 [hep-th/0509120] [INSPIRE].

[30] N. Berkovits and M. Guillen, *Simplified $D = 11$ pure spinor b ghost*, *JHEP* **07** (2017) 115 [arXiv:1703.05116] [INSPIRE].

[31] L. Brink and J.H. Schwarz, *Quantum Superspace*, *Phys. Lett. B* **100** (1981) 310 [INSPIRE].

[32] L. Brink and P.S. Howe, *Eleven-Dimensional Supergravity on the Mass-Shell in Superspace*, *Phys. Lett. B* **91** (1980) 384 [INSPIRE].

[33] T. Kugo and I. Ojima, *Local Covariant Operator Formalism of Nonabelian Gauge Theories and Quark Confinement Problem*, *Prog. Theor. Phys. Suppl.* **66** (1979) 1 [INSPIRE].

[34] N. Berkovits and M. Guillen, *Equations of motion from Cederwall's pure spinor superspace actions*, *JHEP* **08** (2018) 033 [arXiv:1804.06979] [INSPIRE].

[35] M. Cederwall, *Pure spinor superfields – an overview*, *Springer Proc. Phys.* **153** (2014) 61 [arXiv:1307.1762] [INSPIRE].

[36] N. Berkovits, *Ten-Dimensional Super-Twistors and Super-Yang-Mills*, *JHEP* **04** (2010) 067 [arXiv:0910.1684] [INSPIRE].

[37] D. García Sepúlveda and M. Guillen, *A pure spinor twistor description of the $D = 10$ superparticle*, *JHEP* **08** (2020) 130 [arXiv:2006.06023] [INSPIRE].

[38] D.G. Sepúlveda and M. Guillen, *A Pure Spinor Twistor Description of Ambitwistor Strings*, arXiv:2006.06025 [INSPIRE].

[39] N. Berkovits, M. Guillen and L. Mason, *Supertwistor description of ambitwistor strings*, *JHEP* **01** (2020) 020 [arXiv:1908.06899] [INSPIRE].

[40] N. Berkovits, E. Casali, M. Guillen and L. Mason, *Notes on the $D = 11$ pure spinor superparticle*, *JHEP* **08** (2019) 178 [arXiv:1905.03737] [INSPIRE].

[41] M. Guillen, *Pure spinors and $D = 11$ supergravity*, Ph.D. thesis, Sao Paulo, IFT, (2019) [arXiv:2006.06014] [INSPIRE].

JHEP03(2023)135