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Round and round in circles—shifting relevance structures as students discuss acceleration and forces during circular motion in a vertical plane

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Abstract

Working out the relations between the forces involved in circular motion in a vertical plane can be challenging for first-year students, as illustrated in this analysis of a 30 min group discussion of a textbook problem where a remote-control model car moves with constant speed inside a cylinder. The analysis includes timelines of semiotic resources used, as well as of topics brought up by individual students. Questions from the students include: what is that force you drew on the paper? Does it act on the car or on the wall? What keeps the car from falling down? The normal force and the 'centripetal force' both point to the center—does it mean they are the same? Is it only a gravitational force at the top? Does the normal force at the bottom just cancel gravity or does it need to be larger? What is 'normal' about the normal force? Arriving at the correct numerical result is insufficient evidence for student understanding of forces in circular motion! Can students with fragmentary understanding bring their pieces together to solve the puzzle? From the timelines, we can identify a few critical moments where the discussion changes focus. This happens when one of the students in the group introduces a new dimension of variation, e.g. a

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reminder about the force of gravity, a free-body diagram drawn, as well as diagrams drawn in other parts of the circle than the top or bottom, where the centripetal and normal forces are no longer in the same direction. Embodied experiences are invoked, but only at a very late stage in the discussion. For teachers, an awareness of the different ways students use terms and think about the forces can be a guide to offering a larger variation in the interventions, as well as in problems assigned.

Supplementary material for this article is available online

Keywords: circular motion, relevance structure, semiotic resources, variation theory, affordances, embodiment, group discussion

(Some figures may appear in colour only in the online journal)

1. Introduction

Anybody teaching introductory physics knows that the dynamics of circular motion is challenging (see e.g. [1]), not least when the motion is in a vertical plane. If your teaching has included discussions in smaller groups, you have probably heard many statements revealing incomplete understanding of how to apply Newton's second law. In this paper, we present an analysis of a video-recorded discussion between 4 first-year students trying to make sense of the forces acting on a remote-control model car in the highest and lowest points of circular motion in a vertical plane. Over many years of teaching we have found that for these points most new students can obtain correct numerical results relatively quickly, while unable to apply consistent strategies for other points [2]. That formula manipulation is not in itself evidence of conceptual understanding is well known [3–5]. The discussion analyzed in this work shows students' uncertainty about what forces should be included in the free-body diagrams, and what these forces represent conceptually.

The students' struggle to recognize and understand the relevant concepts is the focus of this paper. The analysis shows that the participants in the discussion ascribe different meaning to the terms used. It also hints to different approaches to circular motion problems. (A few possible strategies were discussed in [2, 6].) Can the students with their fragmentary understanding of Newton's laws bring their pieces of knowledge together to solve the puzzle of forces in circular motion?

The forces in the highest and lowest points are the easiest to calculate. Recalling embodied experiences of feeling heavier or lighter are often found to be sufficient for students to select the right sign. However, the simplicity related to this choice also hides qualitative aspects of motion in a vertical plane. In the discussion analyzed in this work, the students were found to add a dimension of variation, by considering forces in other parts of the circle.

2. Theoretical framework

In this paper, we are using a combination of the Variation Theory of Learning [7, 8] and Social Semiotics in physics education to analyze student learning [9].

2.1. Social semiotics

An essential aspect of social semiotics is that meanings get made, shared, interpreted and remade through 'modes' of representational and communicational resources, including language, equations, graphs and gestures. A key concept is disciplinary-relevant aspects (DRAs) of a phenomenon, i.e. those aspects that have particular relevance for carrying out a specific task. Thus, disciplinary-relevant aspects in physics are those aspects that physicists would draw on in order to solve a particular problem or explain a given phenomenon [10]. Physics learning thus involves coming to appreciate the disciplinary affordances of semiotic resources [11]. The earlier work analyzing this student discussion about a car moving in a circle in a vertical plane [9] identified radius, mass, normal force, gravitational force, centripetal force, centripetal acceleration, instantaneous velocity, and the system, as DRAs recognised by the students.

Section 5 presents timelines of the semiotic resources used by the students and of DRAs and questions brought up by the individual students during the discussions. The *affordances* of the different semiotic resources used by the students, as well as resources resulting from the analysis, are discussed in section 6.

2.2. Variation theory of learning

In the Variation Theory of Learning, discernment is seen as a key condition for learning. However

> 'no discernment can happen without the experience of difference. But no difference can be experienced without the simultaneous experience of the things that differ. And two things cannot be experienced simultaneously—as two things—without being discerned [7, p 66].'

Marton [8, p 263] suggests a sequence of variations 'to bring learning about.' Students first need to encounter an 'undivided object of learning', e.g. as a problem to be tinkered with, providing a 'perspective from which the learning situation might be seen (relevance structure).' In the textbook problem discussed here, the students quickly identified that speed and radius were needed for the calculation of the centripetal acceleration and that the mass must be taken into account to obtain a numerical value for the 'centripetal force.'

The next step is to provide *contrast* in one critical aspect at a time. The textbook problem only contrasted forces in the highest and lowest points of the circular motion. As seen from the analysis, the student discussions introduced additional variation and contrasts by considering other points.

To solve the physics problem under consideration, the students need to discern the critical aspects discussed in section 3.1.1. E.g. they need to experience or be aware of the possibilities, e.g. that the normal force need not be exactly opposing gravity, that forces may act on different 'bodies' or 'systems' and that force arrows may have different lengths. In the analysis, we identified a few critical events where a student added new dimensions of variations, thereby changing the path of the discussion and the relevance structure perceived by the students.

The next step is to guide students to *generalization*, by keeping critical dimensions invariant, while varying other aspects. The final step is *fusion* where all critical dimensions are varied [8, p 263].

The variation theory of learning suggests a pathway for teachers to guide the students to *generalization* and *fusion* in lessons following the group discussion, by increasing the variation. Section 3 gives a range of examples of variations within the theme of circular motion.

2.3. Combining social semiotics and variation theory of learning

The frameworks of social semiotics and variation theory of learning were combined in earlier work [9]. The analysis in this work presents several examples of how variation in the examples discussed by students helps them discern additional aspects (DRAs) of the problem and sometimes changes the direction of the group discussion.

3. Educational context of the group discussion on circular motion

The group discussion analyzed in this paper was part of a weekly tutorial session complementing lectures. It took place during one of the first weeks of the course. The students were seated in groups of 4–5 students around separate small tables. They were informed that the discussion at some of the tables were going to be recorded and at those tables all students signed an 'informed consent' form. The discussion analyzed in this paper was selected because it exhibited many of the common learning challenges faced by first-year students related to circular motion.

The problem discussed by the students in this work is taken from the chapter 5 of their textbook [12, 13], *Applying Newton's Llaws*, where the topics from earlier chapters are brought together. Chapter 3 had introduced students to motion in two or three dimensions, and included several examples asking the students to calculate the centripetal acceleration in different situations where the radius and speed were given. Chapter 4 introduced Newton's laws. In subchapter 5.4 students encounter *Dynamics of circular motion*.

3.1. The textbook problem discussed by the students

The problem discussed by the students involves a small remote-control car with mass $m = 1.60$ kg moving with constant speed, $v = 12\ \mathrm{m\,s^{-1}}$ inside a cylinder with radius $R = 5$ m. The question given was 'What is the magnitude of the normal force exerted on the car by the walls of the cylinder at

(a) point A (the bottom of the track)
(b) point B (top the track)?

The students need to figure out how the force from the cylinder wall acting on the car combines with the force of gravity to give rise to centripetal acceleration.

Since the problem states that speed is constant, the circular motion is uniform and involves a centripetal acceleration with constant magnitude, $a_c = v^2/R \approx 28.8\ \mathrm{m\,s^{-2}}$, as obtained by the group after a couple of minutes (4:20). Multiplying by the mass leads to the numerical value 46.08 (in N) for the force required for the centripetal acceleration.

The normal force exerted by the wall on the car, $\boldsymbol{F}_{\mathrm{wall-on-car}}$, added as a vector to the force of gravity should provide the resultant force $ma_c \approx 2.9\,mg$, pointing in from the walls, as required by Newton's second law. Figure 1 shows a graphic representation of how the forces combine for the points at the top and bottom of the cylinder, as well as on the sides and between. Working with vectors, there is no need to define a coordinate system to see how the addition or subtraction works out.

Note that since the speed was defined (rather unrealistically) to be constant, it is only at the top and bottom that the force from the wall on the car is a purely 'normal' force.

To calculate the size of the force, you just need to add the force of gravity, mg, to ma_c—or subtract it. Students usually manage to calculate the numerical value at the top or bottom, give or take a sign, by procedural mimicking. Sometimes referring to the experiences of their body

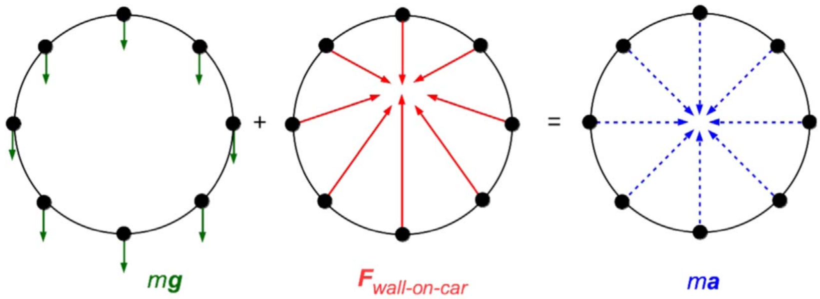

Figure 1. A visual representation of Newton's second law, $\sum\mathbf{F} = m\mathbf{g} + \mathbf{F}_{\text{wall-on-car}} = m\mathbf{a}$, for the problem discussed by the students, where the acceleration is purely centripetal with a value $a_c \approx 2.9g$.

in similar situations can help them get the sign correct, without consciously applying Newton's second law.

3.1.1. Critical aspects of the problem discussed by the students. The textbook chapter title 'Applying Newton's Laws' indicates learning objectives of the chapter, where the problem was found. To apply Newton's third law, students need to understand that the forces involved act on different bodies/systems, and that all forces involve *interactions*. The 'centripetal force' needed to provide a centripetal acceleration is not in itself a force—there is no such thing as a centripetal interaction.

The textbook problem is an application of Newton's second law, $\sum\mathbf{F} = m\mathbf{a}$, and students need to discern that the different force arrows may have different lengths, as well as directions: Newton's second law is a vector relation! From the discussion, an additional critical aspect becomes visible: the properties of a 'normal' force, i.e. being orthogonal to a surface (but not necessarily canceling the force of gravity).

The students may recognize only a subset of the critical aspects as relevant for them to solve the problem. Within variation theory of learning what they see as relevant is referred to as the students' *Relevance structure*.

3.2. Circular motion in the students' textbook

In the textbook used in the course, circular motion is treated as an example of applying Newton's laws, and about a third of a page telling students not to use 'centrifugal forces' ([12, p 175], [13, p 181]). After three pages discussing uniform circular motion in a horizontal plane, including the examples conical pendulum, rounding a banked curve and the flight of airplanes, the chapter continues with motion in a vertical plane, noting that

> 'Motion in a vertical circle is no different in principle, but the weight of the object has to be treated carefully.'

This observation is followed by an example of a rider at the top and bottom of a Ferris wheel. For these positions all forces are vertical and the equations for the forces at the top and bottom differ only by the sign for the centripetal acceleration. The resulting forces at the top and bottom are presented as $n_T = mg(1 - v^2/gR)$ and $n_B = mg(1 + v^2/gR)$.

The example problems in the book discusses the steps: identify and set up, execute, and evaluate. The evaluation discussion of circular motion in a vertical plane in the book brings up the qualitative differences between situations where $v^2/R = g$ and $v^2/R > g$. Towards the end of the group discussions, the group seems to make references to these distinctions.

3.3. Lecture before the group discussion

The day before the group discussion analyzed in this paper, the students listened to a flipped classroom lecture. As preparation, the students were expected to have studied the chapter on acceleration in curvilinear motion, including the special case of uniform circular motion. During the lecture, the students answered clicker questions about the direction of velocity and acceleration for different situations. The rate of correct responses was typically 70%–85%.

The chapter on motion beyond one dimension preceded the introduction of Newton's laws, and the relation between net force and acceleration was not discussed in the lecture. Nor was Newton's second law prominent in the group discussion, although the students have certainly learned it before entering university.

3.4. A special case of two-dimensional motion

Circular motion is a special case of curvilinear motion, where the radius of curvature is constant. In a pendulum, the motion is along a circular arc, where the mechanical energy is nearly constant, whereas speed and centripetal acceleration are not. This applies also to parts of the motion along a roller coaster track, although the radius of curvature may be changing as the train moves along.

Even the special case of uniform circular motion in a vertical plane includes a number of qualitatively different types of motion: Amusement-ride examples include large wheels where $a < g$ and the motion is primarily a translation along a circle. As well as faster rides where $a \approx g$ or $a > g$ where riders may be upside down on the top. For large Ferris wheels, the acceleration is often so small that it is hardly perceptible by a person in the ride.

When the circular motion is in a vertical plane, the changing angle between the centripetal acceleration and the acceleration of gravity introduces a complication that may be challenging for students. This variation was reduced in the textbook problem discussed in this paper, which only asked for the forces at the top and bottom, where all forces are either parallel or anti-parallel to gravity.

Figure 2 places the example studied in this paper in the context of more general two-dimensional motion, and can be used as inspiration for additional variations that may be introduced in the teaching of circular motion.

4. Methodology

This paper presents a further analysis of the multimodal transcription of a 30 min group discussion among four first-year university students discussing circular motion, as described in [9]. That transcript included not only the spoken language, but was annotated with notes about other semiotic systems including gestures, diagrams and mathematics. Figure 3 presents a timeline of semiotic resources used by the four participants (pseudo-named as Alex, Billie, Charlie and Drew). The timeline was created from a transcript spreadsheet, where numerical values were assigned to the different types of semiotic resources. Additional annotations were then added in using a drawing program.

For the analysis presented in this paper, the transcript was reread with a focus on the physics content, noting the various themes brought up during the discussions, as the students struggled to make sense of what forces act during circular motion. The themes of the

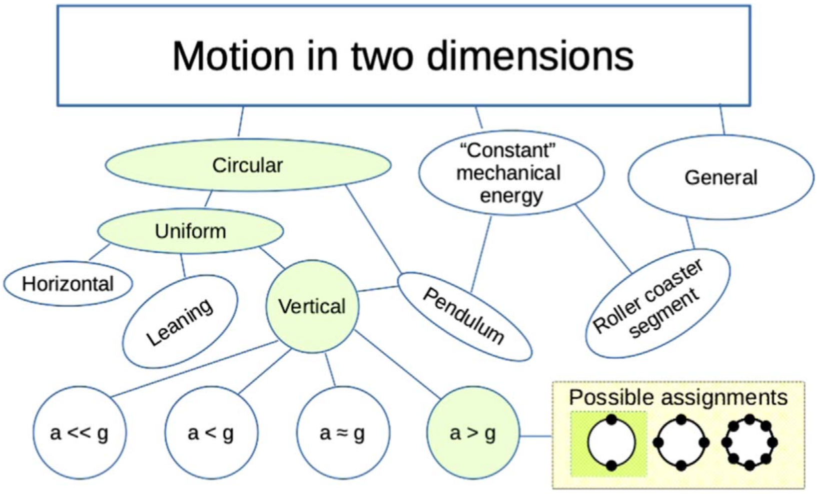

Figure 2. Uniform circular motion in a vertical plane as a special case of motion in two dimensions. The textbook problem discussed by the students asks for the forces on the car in the highest and lowest point of a circular motion where the centripetal acceleration is larger than g.

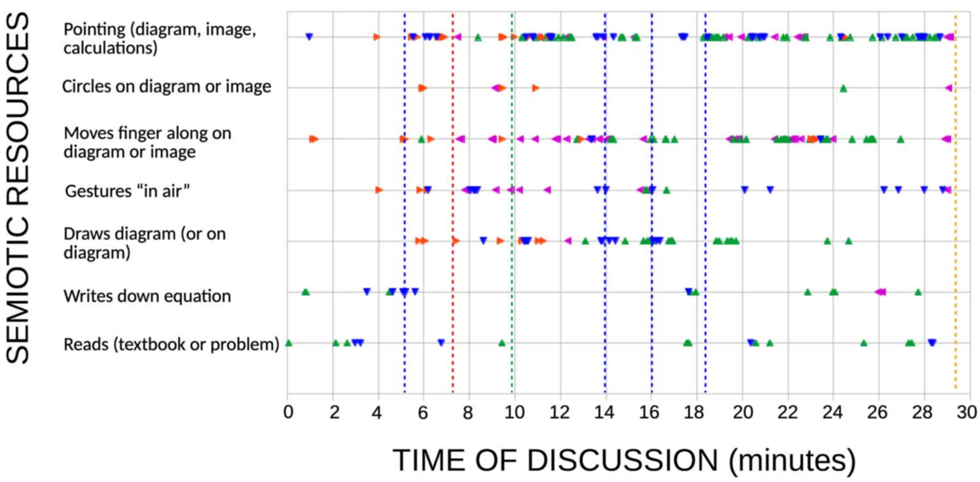

Figure 3. A timeline of semiotic resources used by the students during the discussion which lasted for 30 min. The vertical lines mark a few critical events. Each student is represented by an individual symbol, Alex by a green triangle pointing up, Billie by a red triangle pointing right, Charlie by a blue triangle pointing down and Drew by a purple triangle pointing left. The vertical lines mark critical events discussed in section 6.1.

discussion was noted in an additional column in the spreadsheet. The authors then discussed which categories should be used to summarize the discussion and how different discussion topics might be collected into joint categories.

Separate sheets were then created for each participant to help the identification of individual ways of using terms, as well as patterns of reasoning. In section 5 we present in more

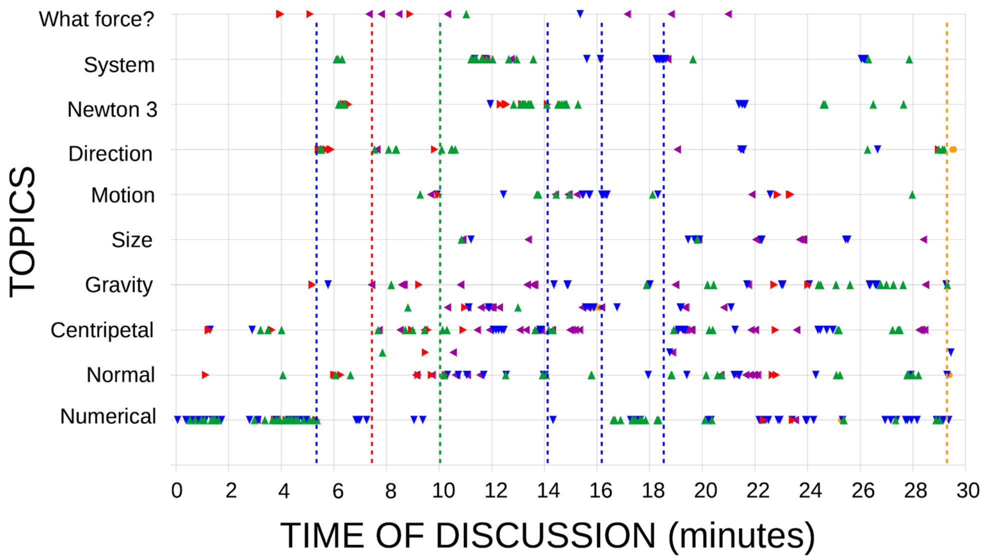

Figure 4. A timeline of the discussions during 30 min. As in figure 3, each student is represented by an individual symbol. The vertical lines mark critical events including equations and drawings added to the shared sheet on the table. The orange line and marks represent another group joining the discussion on their way out.

detail the different themes that were identified, and how the discussions around them developed in the group.

The thematic timeline in figure 4 shows how the themes of the discussion change during the session. Vertical lines mark a few critical events that have been identified, as discussed more in section 6.1.

5. Timelines of the discussions

Throughout the discussions, all students use a wide range of semiotic resources, as seen from the timeline in figure 3. Semiotic resources and their affordances were in focus of an earlier paper [9], whereas this paper focuses on the physics concepts in relation to the words spoken and what was written on the large sheet on the table during the group discussions. During the reading of the transcript, we also identified a number of topics for the discussion. Figure 4 shows a timeline of the topics, as they were brought up by individual students. The categorization is presented in more details below, together with examples of activities or quotes. These timelines give a visual overview over the student discussions of the circular motion problem, including indications of what were critical events.

The students started by finding the expression for the centripetal acceleration and obtained the numerical value of $a_c = 28.8\,\mathrm{m\,s^{-2}}$, then multiplying by the mass 1.6 kg giving $ma_c = 46.08\,\mathrm{N}$.

Billie points out that gravity needs to be taken into account to evaluate the normal force from the walls acting on the car, as requested by the textbook problem. Alex calculates the value $F_g = mg = 15.7N$ and writes down the correct results for the size of the normal force at the top (A) and the bottom (B) after just over five minutes.

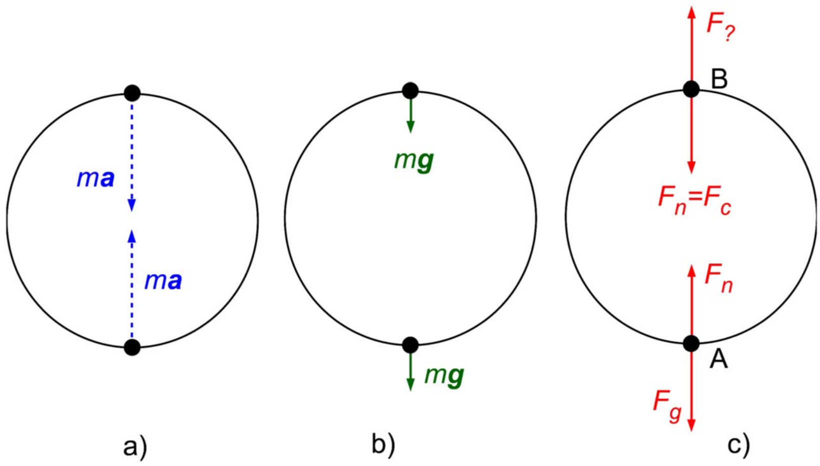

Figure 5. The diagrams (a) and (b) show the force of gravity $m\mathbf{g}$ and the centripetal force, $F_c = ma_c$, in the same scale. The last diagram is a schematic drawing of the first diagram drawn by the student Billie. Note that all force arrows have the same length in their drawing and that F_n is claimed to be equal to F_c at the top (point B). (Note that an upward real force at B would be needed for centripetal accelerations smaller than g.)

The initial discussions thus focus on the *numerical calculations*, which is the first category of the timeline in figure 4. The discussions also mention 'normal' and 'centripetal' which are two additional categories occuring during the first few minutes of the discussion, as indicated in the timeline in figure 4.

During the discussion preceding the equation, Billie asks a couple of times 'Which force is that?.' All particpants asked similar questions or expressed confusion at some stage of the discussion. These occasions are marked in the line at the top of figure 4.

The discussion of the role of the gravitational force intensifies after the equation is put on paper and also about the direction of the forces involved. A few minutes later, Billie draws the first free-body diagram on the paper (analogous to figure 5(c)). The discussion then shifts to the nature of the different forces and the relation between them, in particular the relation between the centripetal force and the normal force: at the top and bottom they are in the same direction—does that mean they are the same thing? Is there really such a thing as a centripetal force—and what keeps the car from falling down at the top? This discussion brings several additional themes to the discussion, included as separate categories in the timeline in figure 4. The different categories are discussed in more detail below.

5.1. Numerical calculations

This category includes searching for equations and inserting numerical values.

During the beginning of the session, the students read the problem and try to remember the relevant equation, $a_c = v^2/R$, and looking for the numbers to insert. There is also some hints of a student looking for the alternative expression $4\pi^2 R/T^2$, which builds on the relation $v = 2\pi R/T$, but is less convenient here, since $v = 12\ \mathrm{m\,s^{-1}}$ was already given in the problem. After 4 min they start to insert numerical values, as indicated in figure 6.

After working out the force of gravity, they discuss whether to add or subtract mg—without using any drawing to support the discussions. After 5:30 Charlie writes down the correct numerical result (figure 7).

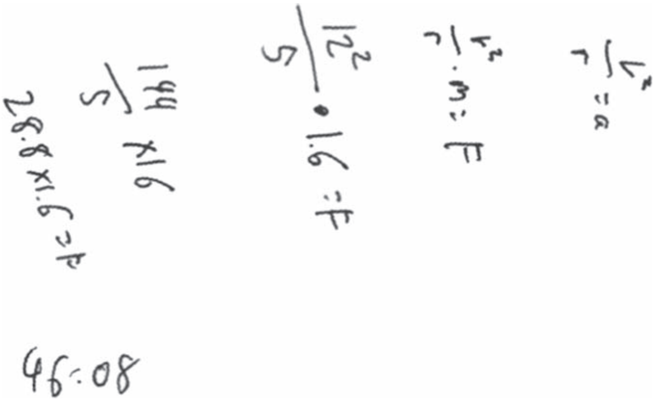

Figure 6. Steps taken by the students in their calculation of the centripetal acceleration and force.

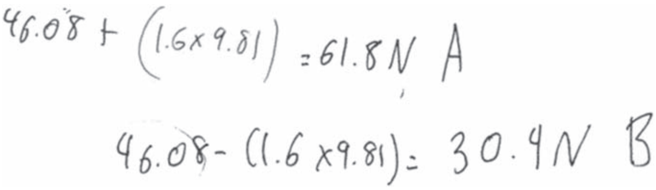

Figure 7. The correct numerical results written down after 5:30.

The group returns to discussion about the numerical values a few times during the discussion, and also at the end, when they compare their results with the results from another group.

5.2. Normal force

At the top and bottom of the circle, the force from the wall on the car points inward and is a purely *normal force*, orthogonal to the wall. According to Newton's third law, the car exerts an opposite, equally large, normal force on the wall.

In the student discussions the term 'normal' takes on many different meanings. This category includes occasions when the students used the whole phrase *normal force*, or just *normal*, but clearly referring to a normal force, which could be either on the wall or on the car. Sometimes term 'normal' was also used in the context of 'normal to gravity', which was classified as an example of 'Newton's third law.' Sometimes 'normal' and 'centrifugal' were mentioned together. These sentences are categorized as falling between 'normal' and 'centripetal' in the graph. An exception is the context 'normal to the centripetal force', which was considered to imply 'centri*fugal*', which is placed above centripetal in the diagram.

5.2.1. How do the students use the word? Alex uses the term 'normal force' to refer to the contact force between the car and the wall, and it could be either the upward force the car 'emits' or the downward force from the wall on the car, both equal due to Newton's third law.

Knowing that they have the same size, it may not be perceived as important to be very careful to distinguish between them in the discussions. To Alex, the arrow drawn out from the circle is the force exerted (or 'emitted') by the car.

Billie states very early on that the 'The normal force is the force that points to the middle the entire time. That makes the car go in a circle.' This description could also apply to the 'centripetal' force. To Billie 'they are the same thing.' At the lowest point, Billie instead expects that the 'normal force and the gravitation should be equal.' (When normal forces are introduced in textbooks in school, the situation is typically static, with an object resting on a horizontal surface, where the normal force is required to counteract the force of gravity.)

Charlie talks about normal force as a reaction force: 'The normal force to the centripetal force should act here' (pointing to the center of the circle), and concluding that 'There is no centripetal force [...] Yes it is spinning, but there is no force pulling it like this' (pointing up from the lowest point, 12:23-12:28). Later, he states that 'the normal force has to be bigger than the gravity force' (18:24). On the other hand Charlie notes (19:28) that 'The normal force usually cancels with gravity, right?.' In some cases Charlie claims that 'The centripetal and the normal are the same' (18:46), which is also iterated (29:27) at the end of the session, after asking whether the calculated value was the 'net force or did you compute the normal force', but he then directly exclaims 'No', and seems to have noted the contradiction between his different points of view.

Drew expresses that the normal force 'is a reaction force to gravity' and also talks (12:00) about the outward arrow as a 'normal force in reaction to the centripetal force', again an implicit reference to Newton's third law, and also to a centrifugal force. After the teacher had visited the group, Drew notes again: 'I am confused about normal force. That's centripetal force to me.'

5.3. Centripetal

The discussion about normal forces slowly moves over to quite a confused discussion about the centripetal force. The term centripetal may refer both to the centripetal acceleration and to a centripetal force in the group's discussions. The students are uncertain about whether the centripetal force is a real force, or possibly the same as a normal force. These cases are placed between centripetal and normal in the timeline graph.

E.g. at one stage, Billie asks explicitly 'Which force is this?', getting the answer 'centripetal', but when the question is repeated, the answer is 'the normal force exerted by the car.'

Billie also asserts that 'there is not somewhere out of nothing a centripetal force' (9:35). Drew on the other hand claims (19:33) 'But this is the centripetal force, it is not given by the wall. This force is not given by the wall, it is given by the rotation.'

Sometimes during the discussions, Billie seems to think that the force pointing out from the top of the circle is a centri*fugal* force. Cases when we conclude from the context that the student probably means 'centrifugal' are placed above the centripetal category in the timeline. A few more cases are presented below.

5.3.1. Centripetal, centrifugal or a question of what system? The term centrifugal is never mentioned during the discussion—the students seem to be well trained not to use the word. Still there are a cases where they probably think in terms of a centrifugal force.

Figure 8. After 10 min, Billie draws a car to the side (a) and Alex adds the three force arrows. Charlie notes that there should be a third and even a fourth force when the car is on the side. After 14 min, Charlie also draws the diagram of forces (b) when the car has passed the top.

At 8:34, Alex describes the outward force: 'Because of the force of the car creating a centripetal force, I think? I know there's another way of describing it, but I do not remember it.' Later he states 'I do not know which one is centripetal. One of them is.'

After 10 min, Billie draws a car to the side of of the circle (figure 8(a)) and Alex adds the three force arrows. Drew notes (10:13) *Ah, so there is a normal force to the centripetal thing?*.

At 11:48 Drew expresses that the outward force must act on the car 'Otherwise the car would not stay up.'

These outward force examples have been classified as 'centrifugal', together with a few similar remarks, and placed between centripetal and gravity in the diagram. Statements about outward forces not acting on the car but on the wall have instead been classified as 'System' in the timeline.

After 12 min, Alex is getting confused about what causes the centripetal force, not finding anything that would *pull* it to the center. After half a minute, Charlie mentions the normal force again, and Billie refers to 'the Newton.'

Alex and Drew recognize their confusion. In the discussion, at least Alex seems to search for the term 'centrifugal', but noone mentions anything but centripetal forces that the car exerts on the wall. Drew is concerned about all forces pointing down from the top—what keeps it from falling? After 14 min Charlie draws the diagram of the forces just to the side of the top, where the force of gravity is not aligned with the normal force (figure 8(b))

At 14:54-15:20, Drew summarises:

> 'Exactly, so it has an acceleration, and that's our centripetal acceleration. So, it is on the car, so the centripetal force is on the car? It affects the car because it is the car that is affected, since it is [moving finger in a circle] changing its velocity. The wall is the one changing its velocity. So, so the centripetal force is on the car.'

During Drew's monologue Alex looks in the textbook and exclaims that 'I have an idea' and goes on to explaining that the motion is accelerating inwards. He crosses out the outward force for the diagram slightly to the side of the top in figure 8(b) and goes on to show why the car is accelerating.

5.4. Gravitational force

The gravitational force is first brought into the discussion by Billie at 5:11 and then immediately calculated by Charlie and included in the equations giving correct numerical results at 5:30. The group discovers that they need to distinguish between 'normal', 'centripetal' and 'gravity.' For the situation at the top and bottom, the acceleration and the force from the wall have been either parallel or anti-parallel to gravity, and the discussion is not leading to a consensus.

At 10 min, Billie draws a diagram of a car to the side (figure 8(a)). This opens a new dimension of variation, since the gravity is no longer along the same axis as the normal force (or the acceleration).

Billie's diagram after 10 min initiates discussion about the direction of the different forces, as seen from the timeline. The first mention of size relations follows shortly after, as Drew and Alex point to the highest point and noting that the centripetal force has to be larger than the force of gravity. (10:49-10:51).

Charlie continues a few minutes later with another diagram making a distinction between gravity and normal force, as he draws a diagram just to the left of the top of the circle.

Much later (23 min) Charlie notes that 'something has to cancel with gravity' for this situation. Billie believes 'that's something that's controlled by the car I think.' (A common view among new students is that a forward force on a car is due to an 'engine force' from the car rather than a friction force.) However they abandon this discussion of forces on the side, since the problem text only asked for the forces at the top and bottom.

Charlie is not so convinced about the importance of the gravitational force for the result (25:40), whereas Alex argues for the results of the group by resorting to the embodied experiences of feeling light at the top of a lift and heavier as you reach the bottom and note that 'You can go from a ballerina to a sumo wrestler in a second.' (25:49) (This is closer to the truth for a trampoline [14] than for a lift.) Alex also mentions the embodied experience of swinging a mallet.

At 27:16 Alex reminds them about the gravitational force: 'You have not included the weight of the car' and Charlie replies that they included the mass, but Alex insists that they only used it for the acceleration (i.e. to calculate the centripetal force).

5.5. Direction of the forces and relative signs

Everyone who has taught circular motion knows that the direction of the forces can be a stumble block. The main challenge for students in calculating forces at the lowest or highest points is typically to get the sign right. The dialogue studied here is sometimes confusing, with different students possibly talking about different points, different forces or different equations.

Discussions about whether a force acts on the wall or on the car are categorized as 'system.'

Alex describes (10:28) 'Here (top) you minus them because they are in the same direction. And here (bottom) you add them.' It is not so obvious what is added or subtracted, but probably, this can be interpreted this as using the relations $F_N = ma_c - mg$ and $F_N = ma_c + mg$, respectively, which gives the correct numerical result, and agrees with the numerical expressions he wrote down earlier (around 5:20). However, the discussion about direction and nature of the centripetal force continues for several more minutes, as discussed in more detail in section 5.3.

At 10:06 Alex points to a car position on the side, drawn by Billie at 9:50, wondering about the force pointing out and somewhat later notes But there should be a third force—a fourth force' (presumably looking for an upward force to cancel gravity).

After Charlie has drawn the force diagram slightly to the side of the top (figure 8, at 14:22), where the force of gravity is clearly distinct from the force from the wall, the group starts to discuss the size relations between the centripetal force and the force of gravity (figure 10). Charlie then continues to show how the normal force changes direction as the car moves around the circle (figure 9).

At 25 min, Charlie refers to this diagram again and points out the different character of what is 'pulling' the car towards the center as he moves a finger around the circle (figure 9) 'Here [the top] it is the gravity that pulls it ... and here [bottom] the normal pulls it. But they are both centripetal forces here [bottom] but they are not centripetal forces here [another point].'

The diagrams where the centripetal acceleration is not aligned with gravity open 'direction' as a new dimension of variation.

5.6. System

In some occasions, the students are discussing whether a given arrow is intended to describe a force acting on the wall. The discussion often involves pointing to a force arrow or gesturing that the force acts on the wall. Although closely related to the references to Newton's third law, the 'system' category is chosen when the discussion is only about what system a force acts on.

At 10:59 Drew points at an outward arrow on the side of the circle (figure 8) and asks 'Is this centripetal? Because this should be the F of the car.' and the confusion continues. Charlie points to the upward force at the top, asking what force it is, while Alex and Drew both answer 'Normal force' but Drew adds 'to the centripetal force' and Alex seems to agree, but with different words: 'The force of the car as it goes round' Alex protests again that 'This force does not act on the car.' and Charlie exclaims 'No, no, no, that's the force that?s created by the car.'

At 11:44 Charlie asks whether the force acts on the car or on the wall—and gets contradictory responses, with Drew concerned about the car 'staying up.'

Alex expresses again at 13:12 that 'The car is not making any force to the center. It is only focus is on the wall.'

Charlie (21:28) describes the 'centripetal force: 'It is reaction to this one. As a reaction to the normal. But it does not matter for the movement of the car. It is a force—it is a reaction force, that acts on the wall.'

5.7. Newtons third law

The references to Newton's third law are mostly implicit. An examples is Charlie's description (at 12:59) of how 'the car emits a force onto the wall, and the wall creates a force onto the car.' Charlie keeps returning to this description.

Sometimes students just mention 'The Newton' or a 'reaction force', also included in this category, as well as hints of references to Newton's third law from expressions such as 'normal to gravity.' However, phrases like 'normal to the centripetal force' are considered as a reference to a 'centrifugal' force, and placed between the Centripetal and Gravity categories in the timeline.

Figure 9. Charlie draws a set of arrows (after around 19 min) representing the normal force, to show its changing direction.

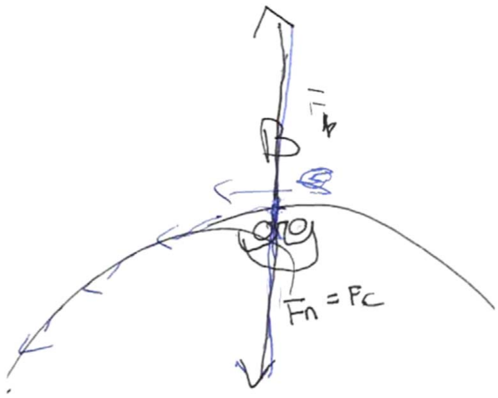

Figure 10. Illustration of the changing direction of the velocity, used by Charlie, after removing the outward arrow in figure 8(b) as discussed in more detail in [9].

5.8. Motion—acceleration and velocity

This category is used when a student is explicitly indicating the motion, through gestures or drawing, or referring to the difference between motion and a stationary situation.

E.g. Alex at 9:17, emphasises that the situation is different from a static situation: 'If it was stationary yeah, but it has got speed and that speed is going round in a circle' and shortly after, Drew describes how 'the centripetal is given by the speed, because you are rotating' (9:41).

At 13:43 Alex refers to 'its changed acceleration as it goes round', which is also placed in this category.

When Charlie is convinced that no outward force acts on the car, the enthusiastically explains to the other students how the velocity is changing its direction, and that this requires an inwards force from the walls which also changes direction (figure 10, 16:11-16:30).

5.9. Size relations between different forces (or between a_c and g)

Although the force situation for accelerated motion in a vertical plane has very different character depending on whether the centripetal acceleration is smaller than, equal to, or larger than the acceleration of gravity, the students do not directly compare the sizes. This comparison is also somewhat more obscure if the force of gravity is written as $\mathbf{F}_g$ rather than as $m\mathbf{g}$.

After considering the car at the side of the circle, two of the students note that the centripetal force must be larger than the force of gravity. (10:49).

Towards the end of the discussion, they discuss the relation and what would happen if they were equal or if the 'normal force' at the bottom would be too small (figure 11).

5.10. Which force?

The upper line marks occasions when one of the students expressed confusion or asked about 'Which force is that?', In attempts to bring the participants together in the discussion, clarifying what is discussed.

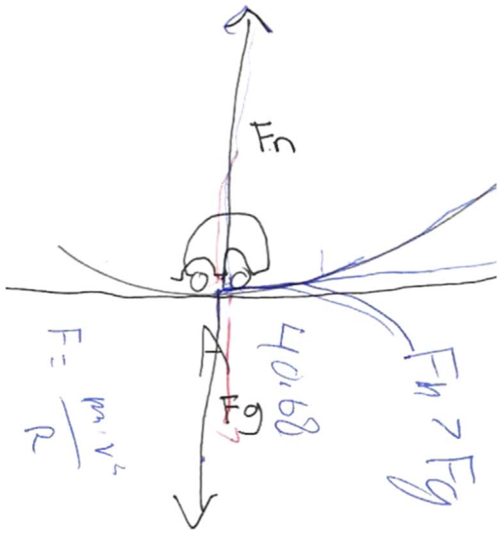

Figure 11. After 18 min, the students discuss the size relation between normal force and gravitational force, and how the car would move depending on the size of the normal force relative to the force of gravity, concluding that $F_n > F_g$, since the car would otherwise move down (or continue in a straight line).

6. Discussion

6.1. Critical events

In the visual timelines in figures 3 and 4 we have noted a few critical events, which change the focus of the discussion.

The first is when Billie reminds the group (at 5:11) that the force of gravity must be taken into account. Soon after, the correct numerical values are written down by Charlie (figure 7). This is followed by a discussion about directions of the forces together with discussions about Newton's third law, and forces acting on the car—or not. Clearly, correct numerical values do not by themselves indicate a good understanding of forces in a circular motion.

At 7:17, Billie draws the first free-body diagram. The discussions about direction continue, with normal, centripetal and gravitational forces mentioned several times with references to the diagram.

At 9:17, Alex emphasizes the difference between stationary situations and the motion around the circle. At 9:50, Billie draws a car on the side of the circle and Alex adds the three arrows shown in figure 8(a). In this situation the gravitational force is no longer parallel or anti-parallel with the normal force and centripetal interaction, which made it possible for the students to focus on the relation between the normal force on the car and the force it exerted on the wall.

The possibility of separating the direction of gravity from the centripetal force is also used by Charlie, drawing a car slightly to the side of the top at 14:22 (figure 8(b)). Drew gets involved and explains how the force from the wall on the car causes the acceleration, followed by Charlie crossing out the outward force, and focusing on the changed direction of motion, as analyzed in more detail in [9]. At 16:06 the teacher visiting the group confirms that there is no outward force action *on the car* and the group goes back to the numerical calculations.

It can be noted that Billie is essentially not making any comment for about 10 min, until she suggests at 22:40 that there is no need for a normal force at the top since there is a downward acceleration.

6.2. Affordances of circular motion problems

The circular motion problem as formulated in the student textbook [13] of a model car moving around with constant speed in a circle lacks the realism that is more likely to invite reference to personal force experiences. Nevertheless, towards the end of the discussion, Alex points out (25:27) that 'the car is heaviest at the bottom.' Charlie suggests that the force of gravity has a negligible effect, but Alex claims that 'It is a massive difference' and refers to the embodied experience where you go from top to bottom in a lift.

We note that in this context, Alex implicitly makes use of the operational definition of weight, which coincides with the intuitive perception of weight [15], although not with the definition chosen by most textbooks, as reviewed by Taibu [16]. The operational definition builds on the equivalence between inertial and gravitational mass, which makes acceleration feel exactly like a force of gravity in the opposite direction [17].

An amusement park can offer many different examples of circular motion in a vertical plane, without rotation, as well as with rotation around different axes of the body, and not only uniform circular motion but also examples such as pendulum motion and roller coaster loops. These allow students to connect the experiences of their bodies with mathematical descriptions. (It can also be noted that a roller coaster track causing a constant centripetal acceleration of around $3g$ in a vertical plane would not be a circle, but a drop-shaped loop [18].)

6.3. Affordances of free-body diagrams

Most of the free-body diagrams drawn during the discussion have the same length for the inward and outward arrows. These can be interpreted in a number of different ways [6]. Only after 18 min do the students start to discuss the size relations between the different forces, noting that the upward normal force must be larger than the force of gravity, or the car would move in a straight line at the bottom, or start to fall downwards. Nowhere do they draw the forces in scale, nor compare the force of gravity.

When free-body diagram is drawn on the side and also slightly to the side of the top (figure 8) this brings in an additional dimension of variation, since the gravitational force is not in the same (or opposite) direction as a normal force. When the teacher confirms that the outward arrow does not exist for the car, this seems to clarify Charlie's understanding of how Newton's third law involves forces on different bodies, and he goes on to discuss how the direction of the velocity changes, requiring an unbalanced force.

Newton's second law is present in the student discussions only as a very general principle. In the group discussion presented in [19], the students seemed surprised to discover that the geometrical vector sum of forces, e.g. at the turning point of a pendulum, gave directly the vector $m\mathbf{a}$. In fact, it is not uncommon for textbooks to omit this geometrical identity, in particular for accelerated motion, and forces are not always drawn to scale [20].

6.4. Inertial forces

The relation $m\mathbf{g} + \mathbf{F}_{\text{wall-on-car}} = m\mathbf{a}_c$ seems to be less clear to the students than the rearranged relation, $m\mathbf{g} - m\mathbf{a}_c + \mathbf{F}_{\text{wall-on-car}} = 0$, moving the acceleration term $m\mathbf{a}_c$ on the right-hand side to the left-hand side of the equation as an 'inertial force', implicitly making use of the equivalence between inertial and gravitational mass. This leads to a mathematically equivalent relation, even if it is conceptually different!

Do we do students a disservice by not telling them that a proper use of inertial forces leads to the correct result when you calculate the size of the real forces? Completely avoiding the word 'centrifugal' seems to leave the students without terminology to discuss or even perceive different approaches by fellow students. For some situations the use of inertial forces can give simpler descriptions as discussed e.g. in [21]. Eric Rogers' classical book 'Physics for the Inquiring Mind' [22] devotes 10 pages to discussion of forces, changes in motion and experiences, describing centrifugal force as an 'Engineer's headache cure', enabling the conversion to a statics problem, by adding an imaginary force to obtain equilibrium (p 307), but concluding that

> 'Motion in a circle needs a real inward force, provided by real external agents. This view of centri*petal* force will help you to deal with all real problems of circular motion.'

Viennot [23] notes that students' 'intuitive ways of reasoning, although not explicitly enunciated, resists attempts to change them and present a considerable challenge for teaching. Ignoring them makes our teaching surprisingly ineffective.' She also finds that 'There is a general tendency to forget that students' spontaneous reasoning is not necessarily expressed in the same terms as that of physicists.'

6.5. Affordances of embodiment

Can the embodied experience of feeling heavier than normal at the lowest point help students develop an understanding of forces in circular motion? The intuitive focus is probably to think of your body being pressed toward the seat by a 'centrifugal force.' Students often need to be guided to focus instead on the forces acting *on* their own body. (Indeed, distinguishing between the different systems was found to be a challenge for the students in this discussion.) Newton's third law tells us that when our body feels pushed into a seat, the seat pushes back, exerting the additional upward force required for acceleration.

In the discussion analyzed in this manuscript, the group invoked the experience of the body only after 25 min. The textbook car problem chosen probably did not invite thinking of yourself taking part in the motion.

The changing forces on your body may be measured by taking a bathroom scale along in a lift or even in a small Ferris wheel during special events, such as Tivoli Garden's Academic days ('Faglige dage'). The experience of feeling heavier or lighter can also be visualized using a small slinky or other spiral toy taken along, as suggested in [24, 25] or by using smartphone accelerometers [26–28], which measure the vector **a**–**g**.

6.6. Affordances of transcripts, timeline and time stamps of drawings

Just reading the transcript it is not so easy to know who is having the signs and directions right. Being able to insert the development of the drawing, together with their discussion leads to slightly different interpretations in some points. The timeline gives a clearer overview about what concepts are discussed, and how the different students take part (or not) in the exchanges. It may also allow for a connection between the pointing and the physics content of what is being pointed at.

6.7. Affordances for a teacher

For a teacher, the presentation of the student discussion can bring the variation of student backgrounds into the teacher's focal awareness. Possibly, not all students have been taught circular motion in the same way as presented in the textbook. Some may even have used centrifugal forces in their earlier studies.

The group discussion reveals how the students—even individual students—apply different strategies for different parts of the circular motion. A multiple-choice test to expose different strategies [6] was used as part of an exam for first-year students, and showed that most of these students used different strategies for the five situations probed.

The concept of 'Epistemologic commitment' was discussed by Eriksson *et al* [9] to describe how one of the students insisted on using an outward force for the horizontal circular motion in a chain flyer. However, the student may have chosen a different strategy for other situations. Viennot [23] found that is more tempting for students to draw outward forces in cases that can raise concern about not staying up (or 'staying out') than for many other situations. Probably no teacher has encountered a student drawing an upward inertial force arrow for the highest point of a projectile motion!

Many of the concepts needed to deal with circular motion problems in a vertical plane are likely to have been only partially understood by the students. This includes the meaning of 'normal', Newton's third law, as well as the understanding that 'centripetal force' is not in itself the result of an interaction. They may also forget to apply Newton's second law in a consistent way. In addition, students may be confused about the relation between mass and weight and between operational and gravitational definitions of the weight concept.

Viennot [23] notes that 'There are several indications that a centrifugal force is not simply an inertial force transposed to another system, or at least that this is not always the case' and that 'one may tap quite a different reasoning when speaking of force *on* something or force *of* something or force not further specified.'

Awareness of common student difficulties helps teachers perceive the lack of clarity, and ask relevant questions to help students express and clarify their way of thinking, and discover when it deviates from the description in Newtonian physics. Useful questions to use during teacher intervention, discussed in [2] include 'What force is that?' Possibly followed by 'What interaction is involved?' The question 'What if...' can help students be aware of the consequences of their suggestions, possibly with added questions about the relative length of the forces drawn, and how they reflect their calculated values. Many statements by the students hint at incorrect use of the word 'normal': asking 'What does normal mean?' Can bring student attention to the meaning in math and physics.

Clement [3] noted that 'Difficulties at the qualitative level may go undetected because a students' superficial knowledge of formulas and formula manipulation techniques can mask his or her misunderstanding of underlying qualitative concepts.' This problem has been encountered by many teachers over the years. Mazur has described his 'sense of shock when he discovered that his students could not correctly answer seemingly simple qualitative questions, despite being able to work much more difficult quantitative problems' [4, 5]. The group discussion in this work illustrates how students can obtain correct numerical results without having the related conceptual understanding and how a teacher may easily fail to recognize this when answering group questions in a problem-solving tutorial situation.

7. A time for telling

The student discussion in focus of this paper seems to leave the students as confused as when they started. Without intervention, they are unable to combine their pieces of knowledge to solve the puzzle of drawing a diagram of forces in circular motion.

On a positive note, we can hope that their discussion has created a *need to know*, which should precede a 'time for telling' as emphasized by Etkina *et al* [29], in a description of the ISLE method:

> That is why when we tell students something about an issue they have not grappled with on their own, they do not remember or do not care. In our approach, 'time for telling' is a moment when the students can put together their ideas, reflect on them and compare them to what physicists think on the matter.

Marton [8, p 214] also discussed the advantages of finding out first over 'Being told and then finding out' and suggests a sequence of variations to bring learning about (p 263). Section 3 suggests a range of variations of circular motion problems, to assist teachers in the follow-up of frustrated student discussions.

The results presented in this paper are a reminder about useful strategies to help students develop good problem-solving habits, such as encouraging students to define clearly what system is considered, label forces with their origin and on what body they act, and draw force arrows approximately to scale.

The results also show that even if the textbook and teacher(s) in a course avoid using the terms 'centrifugal' and 'centripetal' forces, but only discuss 'centripetal acceleration', students with different educational backgrounds may bring different conventions to group discussions. Brief teacher interventions can help in clarifying the different terms, and explain why some are best avoided. Assigning a wider range of circular motion problems, where students are encouraged to connect a mathematical description of forces with embodied experiences, as well as with sensor data, should leave students better prepared to deal with more general problems involving force and motion.

Data availability statement

No new data were created or analyzed in this study.

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