A search for a long-lived, heavy neutral lepton (N) in 139 fb$^{-1}$ of √s = 13 TeV pp collision data collected by the ATLAS detector at the Large Hadron Collider is reported. The N is produced via W → Nμ or W → Ne and decays into two charged leptons and a neutrino, forming a displaced vertex. The N mass is used to discriminate between signal and background. No signal is observed, and limits are set on the squared mixing parameters of the N with the left-handed neutrino states for the N mass range 3 GeV < m_N < 15 GeV. For the first time, limits are given for both single-flavor and multiflavor mixing scenarios motivated by neutrino flavor oscillation results for both the normal and inverted neutrino-mass hierarchies.

The observations of neutrino flavor oscillations [1,2] can be explained by postulating the existence of right-handed neutrino states that carry no standard model (SM) gauge charges, allowing them to have Majorana masses. The resulting “type-I seesaw” model [3–9] explains the light neutrino masses and predicts heavy mass eigenstates, referred to as “heavy neutral leptons” (HNLs) and denoted by N henceforth. The existence of HNLs can also explain the baryon asymmetry of the Universe via leptogenesis [10–12], which is efficient for HNL masses down to the sub-GeV range [13–17]. Moreover, a model with three HNLs can incorporate a dark matter candidate [14,18–21].

Each HNL state carries a small admixture of the left-handed neutrino of flavor α = {e,μ,τ}. It can therefore participate in weak interactions, controlled by dimensionless mixing coefficients U_α, where |U_α| ≪ 1. Previous searches were interpreted only in terms of a one-HNL model with single-flavor mixing (1SFH) [22–32]. This model is a useful benchmark but is not phenomenologically viable as it predicts neutrino masses that are too large and does not account for two neutrino mass splittings or neutrino flavor oscillations [33–35]. The simplest viable model is that of two quasidegenerate HNLs (2QDH), with close masses and couplings, where all U_α are nonzero. A reinterpretation of ATLAS HNL searches in such HNL scenarios has been performed [35]. However, no experiment has directly explored 2QDH models yet.

The search reported here considers the production of HNLs via W → Nαμ, where α = {e, μ} indicates the flavor of the “prompt” lepton αμ. The HNL decays into two oppositely charged leptons and a neutrino: N → ℓα_μγνγ via an intermediate W^+ boson, or N → ν_βℓγ via a Z^* boson, where β, γ = e or μ, as shown in Fig. 1 (the lepton-number-violating processes are shown in Fig. 1 of the Supplemental Material [36]).

The search focuses on the mixing and mass range (up to 20 GeV) in which the HNL is long-lived. The resulting HNL lifetime can be approximated by τ_N ≈ (4.3 × 10^{-12} s)|U_α|^2m_N/1 GeV)^{-5} [37], where |U_α|^2 = \sum_β |U_β|^2 is taken from Ref. [38]. The HNL decay occurs at a significantly displaced position from the proton-proton (pp) collision point, forming a displaced vertex (DV) of two charged leptons, ℓ_βγ or ℓ_γ. The measured final states are labeled according to the prompt and displaced

![Diagram](image-url)

**FIG. 1.** Feynman diagrams for the HNL production and decay modes targeted in this analysis. Only lepton-number-conserving processes are shown. The flavors of the leptons in the diagrams, labeled by α, β, and γ, are either muons or electrons. If the charged leptons in the HNL decay have the same flavor, then both the diagrams with the virtual W (a) and virtual Z (b) contribute to the process. Equivalent processes are also valid for an initial state W^- boson.
charged leptons therein, denoted by "$\ell_\alpha - \ell_\beta e_\gamma$" (explicitly listed in Table 1). Decays of the W or $\bar{N}$ to $\tau$ leptons were determined to have negligible contribution to the analysis, since the leptonic branching fractions of the $\tau$ and the soft lepton spectrum make their selection highly inefficient. In ISFH scenarios, the analysis is sensitive to the squared mixing parameter $|U_{\mu \ell}|^2$ via the final states $\mu - \mu$, $\mu - e$, and $\mu - e$, while $|U_{e \ell}|^2$ is accessible via $e - e$, $e - e\mu$, and $e - \mu$. In 2QDH scenarios, the combination of the six final states provides sensitivity to $|U_{\mu \ell}|^2$, $|U_{e \ell}|^2$, and $|U_{e \ell}|^2$. For both scenarios, bounds on the mixing parameters are extracted in the “Dirac limit” of lepton-number-conserving (LNC) HNL interactions, where the $W$-mediated final state is $\ell_\alpha - \ell_\beta e_\gamma$, and in the “Majorana limit” of equal branching fractions for LNC and lepton-number-violating (LNV, $\ell_\alpha - \ell_\beta \ell_\gamma$) decays [36]. The analysis can separate LNC and LNV decays only by using an explicit charge requirement for the ISFH model in the $\mu - \mu$ and $e - e\mu$ channels, where the displaced leptons are experimentally distinguishable by their different flavors. The bounds are tighter than and supersede those of Ref. [22], where only the final states $\mu - \mu$ and $\mu - \mu$ were studied.

This search is performed with 139 fb$^{-1}$ of 13 TeV $pp$ collision data collected by the ATLAS experiment at the LHC from 2015 to 2018. To study the signal sensitivity, Monte Carlo (MC) signal samples were generated using PYTHIA8 [39] with the A14 set of tuned parameters [40] and the NNPDF2.3LO PDF set [41]. The impact of multiple $pp$ interactions per bunch crossing was modeled by adding simulated minimum-bias events generated with MADGRAPH5_AMC@NLO2.9.3 [47] using the NNPDF2.3LO PDF set. Particles were propagated through a detector simulation [43] based on GEANT4 [44]. To properly simulate spin correlations between W-boson decay products [35,45,46], which are not accounted for in PYTHIA8, events are weighted to reproduce the angular distributions obtained with MADGRAPH5_AMC@NLO2.9.3 [47] using the HEAVY model [48,49]. The weighting procedure is validated by comparing the momentum spectra of each of the charged-lepton flavors and the neutrino between the weighted PYTHIA8 and MADGRAPH5_AMC@NLO samples. For each $\ell_\alpha - \ell_\beta e_\gamma$ final state, signal samples were generated with HNL masses in the range $3\text{ GeV} < m_N < 20\text{ GeV}$ and proper decay lengths $\tau_N = 1, 10, 100, 1000$ mm.

The ATLAS detector [50–52] is a cylindrical detector with forward-backward symmetry and nearly $4\pi$ solid-angle coverage [53]. It is composed of three major subsystems: the inner detector (ID) closest to the $pp$ interaction point (IP), the electromagnetic and hadronic calorimeters, and the muon spectrometer farthest from the IP. The ID is used to reconstruct the trajectories of charged particles (tracks) in an almost uniform 2 T magnetic field, and comprises three subsystems: pixel, silicon microstrip tracker (SCT), and transition radiation tracker. An extensive software suite [54] is used in the reconstruction and analysis of data and MC events, in detector operations, and in the trigger and data acquisition systems of the experiment.

Events in the signal region (SR) of this analysis were selected with triggers [55] that require a single isolated electron [56] or muon [57] with a minimum transverse momentum ($p_T$) of $20$–$26$ GeV, depending on the lepton flavor and year. Events passing the trigger are required by a filter algorithm to contain at least one lepton [58,59] with $p_T > 28$ GeV and $|\eta| < 2.5$.

To ensure isolation of this lepton from hadronic activity, the scalar sum of the $p_T$ of other tracks within a cone of size $\Delta R = 0.3$ around the lepton momentum ($\Sigma p_T^{(0.3)}$) is required to be less than 5% of the lepton $p_T$. The filter also requires at least one additional lepton with $p_T > 5$ GeV, $|\eta| < 2.5$, and $\Sigma p_T^{(0.3)} / p_T < 1.0$. To reduce the number of events with prompt decays while maintaining efficiency for displaced leptons, the second lepton must have a transverse impact parameter ($d_0$) with respect to the IP of $|d_0| < 0.1$ mm ($|d_0| > 1$ mm) for muons (electrons). Events that pass the filter are then processed with a large-radius tracking (LRT) algorithm [60], that is efficient for tracks with $|d_0| < 300$ mm. The LRT is run using hits leftover after the standard tracking [61], which is efficient only for $|d_0| < 10$ mm. Standard and large-radius tracks are combined with muon-spectrometer tracks (electromagnetic energy clusters) to reconstruct muons (electrons). Events are required to contain a reconstructed primary vertex (PV) with at least two tracks, each having $p_T > 500$ GeV. When more than one PV is reconstructed, the one with the highest $\Sigma p_T^2$ is used, where the sum is over the tracks associated with the PV.

Event selection relies on the reconstruction of two physics objects: a prompt lepton and a DV. The prompt-lepton candidate, $\ell_\alpha$, is taken to be the highest-$p_T$ muon (electron) that satisfies $p_T > 3(4.5)$ GeV, $|d_0| < 3$ mm, and $|z_0 - z_{PV}| \sin \theta < 0.5$ mm, where $z_0$ is the track’s longitudinal impact parameter and $z_{PV}$ is the z coordinate of the PV. If a prompt muon and a prompt electron have an angular separation $\Delta R < 0.05$, the event is rejected. Reconstruction of DVs is performed with an optimized version of the secondary vertexing algorithm described in Ref. [62]. First, “seed” DVs are formed from pairs of tracks from both the standard tracking and LRT algorithms. Subsequently, tracks are added to the DVs, and closely spaced DVs are merged. The secondary vertexing is run with the following configuration changes relative to Ref. [62]: seed DVs are formed from leptons only, with at least one lepton satisfying $|d_0| > 1$ mm, and each having at least eight pixel plus SCT hits; leptonic and hadronic tracks are subsequently attached to the DVs, but selected DVs must have exactly two leptons and no additional tracks.

Events must contain a prompt lepton and a DV comprising a pair of leptons with opposite-sign (OS) electric charge, although same-sign (SS) DVs are retained and
used for background studies. If a displaced track is identified as both a muon and an electron, the track is taken to be a muon (electron) if the muon- (electron-) identification quality is stricter. The displaced muons (electrons) must have $p_T > 3$ (4.5) GeV. If a displaced track in the DV is within $\Delta R = 0.05$ of the prompt lepton, the event is rejected. The DV radial position ($r_{DV}$) must satisfy $4 \text{ mm} < r_{DV} < 300$ mm. The invariant mass of the DV and the prompt lepton, which is generally smaller than the $W$ mass, must satisfy $40$ GeV $< m_{DV+\ell} < 90$ GeV.

Background arises from five sources: DVs from particle interactions with detector material; decays of metastable SM particles; $Z \rightarrow \ell\ell$ decays; cosmic-ray muons; and DVs from random crossings of lepton tracks. The following SR selection is designed to retain high signal efficiency and suppress the first four types of background to negligible levels, with random-crossing remaining the dominant background. Cosmic-ray muons, which can be reconstructed as two back-to-back muons in a DV, are rejected by requiring the two displaced tracks to satisfy $\sqrt{(\Sigma p)^2 + (\pi - \Delta \phi)^2} > 0.05$ [63]. Dielectron ($ee$) DVs have the most background from particle interactions with detector material, so those selected must be in regions without detector material, determined from a three-dimensional map of the ID [64].

The displaced dilepton's invariant mass ($m_{DV}$), which is generally smaller than $m_N$, due to the unobserved final-state neutrino, is used to suppress background from $J/\psi$ and other heavy-flavor decays. For $\mu\mu$ DVs, $m_{DV} > 5.5$ GeV is required. For $e\mu$ and $ee$ DVs, the selection efficiency is smaller, motivating looser requirements that exploit correlations between $r_{DV}$ and $m_{DV}$. These requirements are: $m_{DV} > 5.5$ GeV for $r_{DV} < (225/7)$ mm; $m_{DV} > 2$ GeV for $r_{DV} > (750/7)$ mm; and $m_{DV} > 7$ GeV $\times [1 - r_{DV}/(150 \text{ mm})]$ between these $r_{DV}$ regions [36].

Background from $Z \rightarrow \ell\ell$ decays, in which one of the leptons forms a DV with a third lepton, is suppressed by vetoing events where the invariant mass of the prompt lepton and the displaced lepton with the same flavor (i.e., $\alpha = \beta$) and opposite charge satisfies $80$ GeV $< m(\ell^+\ell^-) < 100$ GeV. In channels with $e\mu$ DVs, the random crossing background is reduced by roughly 50% for 1SFH, LNC interpretations, by requiring the prompt and displaced lepton with the same flavor to have opposite charges: $\mu^\pm - e^\pm e^\pm$ or $e^\pm - e^\pm \mu^\pm$.

The four-momentum of the HNL is obtained by applying four-momentum conservation in the $W$ and $N$ decays, using the kinematics of the charged leptons, the known $W$ mass, an approximation where the leptons and neutrino are massless, and the flight direction of the $N$, given by the vector connecting the PV and DV (see the Appendix). This calculation yields a quadratic equation with two solutions. The positive-radical solution is used to define the invariant mass ($m_{HNL}$) of the HNL candidate. In MC signal events, the distribution of $m_{HNL}$ peaks at the generated value $m_N$, as shown in Fig. 2(a).

The final SR selection is $m_{HNL} < 20$ GeV. The maximum signal selection efficiency is approximately 4%. A control region (CR) is defined as events with $20$ GeV $< m_{HNL} < 50$ GeV. Since HNLs with $|U_{\alpha}|^2 > 20$ GeV and $|U_{\alpha}|^2$ values that the search is sensitive to are short-lived, they fail the $r_{DV}$ requirements, resulting in negligible signal contamination in the CR.

A validation region (VR) is used for data-driven background modeling and evaluation of systematic uncertainties. The VR comprises events that passed a variety of triggers, underwent LRT reconstruction, and do not contain a prompt lepton. The DVs in the VR must satisfy the $r_{DV}$ requirements and pass the cosmic-ray muon veto. For $ee$ DVs, the detector material veto is also applied. The expected signal contamination in the VR is less than two events for a 100% HNL branching fraction into the channel of interest. Since the VR contains more than 100 events in each DV channel, the signal contamination is negligible.

Background from random track crossings is expected to yield equal numbers of OS and SS DVs, given the large number of tracks produced in each event. By contrast, background from $Z \rightarrow \ell\ell$ or cosmic-ray muons yields only OS DVs, and backgrounds from particle interactions with detector material or from decays of metastable hadrons preferentially yield OS DVs. Figure 2(b) shows the $m_{DV}$ distributions for SS and OS DVs in the VR. Good agreement is seen between the yield and shape of the distributions, shown for $e\mu$ DVs. This indicates that the dominant source of background in the SR is random lepton crossings. Therefore, the background model described next focuses on this background type. A systematic uncertainty related to this assumption is described below.

The signal and background yields are obtained with the following fit. The fit uses a data-driven background model obtained from a sample of "shuffled events." This sample is created by combining each OS DV in the VR with each prompt lepton found in a non-VR event that contains an SS DV satisfying loose requirements: $m_{DV} > 1$ GeV, with no lepton identification criteria imposed on its displaced leptons. For each channel, the shuffled sample has at least $2 \times 10^3$ times the number of events in the "unshuffled" data sample, in which the DV and the prompt lepton are from the same event. As with an unshuffled event, a shuffled event may have $m_{HNL} < 20$ GeV (SR) or $20$ GeV $< m_{HNL} < 50$ GeV (CR). The background model in the SR and CR is given by the shuffled events [shown in Fig. 2(a)] with an independent floating normalization factor for each channel. The signal model for the fit is taken from simulation and is assigned a single floating signal strength for all channels. The input to the fit is the OS event yields observed in the SR and CR. Inclusion of the CR in the fit directly constrains the predicted background yield in the SR.

061803-3
The shuffled-event background model relies on the assumption that the absence of correlation between the randomly crossing tracks results in an absence of correlation between the DV and the prompt lepton. The validity of this assumption is checked by comparing the \(m_{HNL}\) distributions and the \(m_{DV+\ell}\) distributions of shuffled events with the distributions of unshuffled events. Only SS DVs are used in this test. In order to have a sufficient number of unshuffled events, the requirements on \(m_{HNL}\), \(m(\ell^+_\ell\ell^-_\ell)\), and \(m_{DV+\ell}\) are removed, and that on \(m_{DV}\) is loosened to \(m_{DV} > 2\) GeV. The unshuffled-event samples have between 36 and 187 events in each channel, and the shuffled-event samples are more than 50 times larger. The comparison based on a Kolmogorov-Smirnov test yields probabilities ranging from 20% to 99% for the different channels, indicating the validity of the no-correlation assumption.

Systematic uncertainties in the background model, taken to be 100% correlated between the CR and the SR, are evaluated for two sources. The first estimates the uncertainty from the assumption that nonrandom backgrounds are negligible and is estimated from differences between the \(m_{HNL}\) distributions of shuffled events created from SS and OS DVs. This uncertainty varies between 5% for the \(e^-\mu^-\) channel and 79% for the \(\mu^-\mu^-\) channel. The second uncertainty accounts for statistical fluctuations in the \(m_{HNL}\) distribution of the shuffled sample due to the finite number of prompt leptons used therein. It is estimated from the differences between the \(m_{HNL}\) distributions for shuffled events of two types: in type 1 (2), the combined DV and prompt lepton originate from events in identical (different) DV channels (ee, \(\mu\mu, e\mu\)). This uncertainty is largest for the \(\mu^-\mu^-\) channel, reaching 5%.

The total systematic uncertainty of the signal efficiency varies between 8% and 42% depending on the channel, \(m_N\), and \(cT_N\). Its largest contribution (up to 28%) arises from the reconstruction of displaced tracks and vertices. This is evaluated by comparing \(K_{S0} \rightarrow \pi^+\pi^-\) event yields in the VR with those in MC samples produced with PYTHIA8.186 in bins of \(p_T\) and \(r_{DV}\), as in Ref. [65]. An additional uncertainty of 3% in the track reconstruction efficiency is calculated by randomly removing tracks from each signal MC event with a \(p_T\)- and \(\eta\)-dependent probability [66].

Uncertainties due to data-MC differences in the trigger efficiency [56,57] range up to 1%, and those due to lepton reconstruction, identification, and impact parameter resolution are between 2% and 17% [59,67] for the different channels. As in Ref. [68], an uncertainty in lepton-identification is estimated as the difference in selection efficiency between large and small \(|d_0|\) tracks. Its maximal

![Graphical representation of the ATLAS experiment's results](https://example.com/atlas-graph.png)

**Table I.** Numbers (yields) of estimated postfit background events and of observed events in the signal and control regions.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Background</th>
<th>Observed</th>
<th>Control region</th>
<th>Background</th>
<th>Observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^-e^-)</td>
<td>0.4 ± 0.3</td>
<td>2</td>
<td>3.6 ± 1.8</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>(\mu^-e^+)</td>
<td>0.2 ± 0.1</td>
<td>1</td>
<td>1.8 ± 1.3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>(\mu^-\mu^-)</td>
<td>0.9 ± 0.4</td>
<td>0</td>
<td>4.1 ± 1.9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>(e^-\mu^-)</td>
<td>2.8 ± 0.8</td>
<td>2</td>
<td>12.2 ± 3.2</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>(\mu^-\mu^-)</td>
<td>1.2 ± 0.9</td>
<td>1</td>
<td>2.8 ± 1.6</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(\mu^-\mu^-)</td>
<td>2.2 ± 1.4</td>
<td>2</td>
<td>8.7 ± 2.9</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>(e^-\mu^+\mu^-)</td>
<td>0.6 ± 0.3</td>
<td>0</td>
<td>2.4 ± 1.4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>(\mu^-\mu^+e^-)</td>
<td>1.9 ± 0.6</td>
<td>0</td>
<td>8.1 ± 2.6</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>
in each of the $e-e\nu$, $\mu-\mu\nu$, and $\mu-\mu\nu$ channels and one OS event in each of the $e-e\nu$ and $e-\mu\nu$ channels. No OS $e-e\nu$ events are observed. These yields are consistent with the estimated backgrounds shown. The observed yields in the CR are consistent with the CR background estimates.

Limits are set at 95% confidence level (CL) on $|U_{\alpha}|^2$ vs. $m_{\nu}$ for each HNL scenario, using the CL$_s$ prescription [73] implemented in TRExFitter [74–76]. All systematic uncertainties are included in the fit by using nuisance parameters, whose postfit values do not show any significant pull or constraint. Each MC signal sample corresponds to specific values of $|U_{\alpha}|^2$ vs $m_{\nu}$, for which the efficiency is evaluated and a hypothesis test is performed with $10^4$ pseudoexperiments.

Figure 3 shows the excluded parameter space in the 1SFH and 2QDH scenarios for both the Dirac limit and the Majorana limit. In the 2QDH scenarios, exclusion limits are shown for the two neutrino-mass hierarchy scenarios. In the inverted-hierarchy case, the relative mixing coefficients are taken to be $x_{\alpha} = |U_{\alpha}|^2/[U|^2] = 1/3$ ($\alpha = e, \mu, \tau$); for the normal-hierarchy case, the values $x_e = 0.06$, $x_{\mu} = 0.48$, and $x_{\tau} = 0.46$ are used [35,77]. These values are at the centers of the regions consistent with the neutrino flavor oscillation data. The observed limits are consistent with the expected limits. The feature visible near $m_{\nu} = 5$ GeV is due to the $r_{DY}$-dependent $m_{DY}$ selection, which limits the sensitivity at low mass.

In conclusion, a search for long-lived heavy neutral leptons is conducted in a 139 fb$^{-1}$ data sample of $\sqrt{s} = 13$ TeV $pp$ collisions collected with the ATLAS detector at the LHC. No excess is observed, and limits are set at 95% CL on the squared mixing coefficient $|U_{\alpha}|^2$ in different HNL masses in the approximate range $3 \text{ GeV} < m_{\nu} < 15 \text{ GeV}$. The observed limits exclude a region with wider ranges of $|U_{\mu}|^2$ and $m_{\nu}$ than previously excluded by ATLAS, and the limits on $|U_{\alpha}|^2$ are novel in ATLAS. For the first time, limits are evaluated for the case of multiflavor mixing scenarios that agree with the neutrino flavor oscillation data, for both the normal and inverted neutrino-mass hierarchies. The strongest limits are observed for multiflavor mixing with the inverted hierarchy.

We thank CERN for the very successful operation of the LHC, as well as the support staff from our institutions without whom ATLAS could not be operated efficiently. We acknowledge the support of ANPCyT, Argentina; YerPhI, Armenia; ARC, Australia; BMWF and FWF, Austria; ANAS, Azerbaijan; CNPq and FAPESP, Brazil; NSERC, NRC, and CFI, Canada; CERN; ANID, Chile; CAS, MOST, and NSFC, China; Minciencias, Colombia; MEYS CR, Czech Republic; DNRF and DNSRC, Denmark; IN2P3-CNRS and CEA-DRF/IRFU, France; SRNSFG, Georgia; BMBF, HGF, and MPG, Germany; GSRI, Greece; RGC and Hong Kong SAR, China; ISF and Benoziyo Center, Israel; INFN, Italy; MEXT and JSPS,
Appendix: A HNL mass.—The HNL mass \( (m_{\text{HNL}}) \) can be obtained using energy–momentum conservation in the HNL production \( (W \rightarrow \mathcal{N} \ell_1) \) and decay \( (\mathcal{N} \rightarrow \ell_2 \bar{\ell}_3 \nu) \), where \( \ell_1 \) is the prompt lepton and \( \ell_2 \) and \( \ell_3 \) are the charged leptons in the DV. The problem can be summarized with the following equations. Four-momentum conservation in the \( \mathcal{N} \) decay gives

\[
p_{\mathcal{N}}^\mu = p_2^\mu + p_3^\mu + p_\nu^\mu \equiv p_{23}^\mu + p_\nu^\mu. \tag{A1}
\]

Four-momentum conservation in the \( W \) decay gives

\[
p_W^\mu = p_1^\mu + p_{\mathcal{N}}^\mu = p_1^\mu + p_{23}^\mu + p_\nu^\mu. \tag{A2}
\]

The following are defined:

\[
\begin{align*}
 p_{23}^\mu &= E_{23} - |\vec{p}_{23}|^2 = m_{23}^2, \\
 p_\parallel &\equiv \vec{p}_{23} \cdot \hat{v}, \\
 p_{\perp} &\equiv |\vec{p}_{23} - p_\parallel|, \\
 p_{23}^\ast &\equiv |\vec{p}_{23} - p_\parallel|.
\end{align*}
\]

where \( m, E \), and \( |\vec{p}| \) are the mass, energy, and momentum-vector magnitude of the particles indicated by their subscript and \( \hat{v} \) is the flight direction of the HNL given by the vector connecting the PV and DV.

The solution to the HNL mass is presented in the coordinate system \( k = (x', y', z') \), which is rotated relative to the ATLAS coordinate system, such that the origin of the \( k \) frame is at the PV and the \( z' \) axis points along the flight direction of the HNL. The definition of this coordinate system is

\[
\begin{align*}
 \hat{z} &= \hat{v}, \\
 \hat{x}' &= \frac{\vec{p}_{23} \times \hat{z}}{|\vec{p}_{23} \times \hat{z}|}, \\
 \hat{y}' &= \hat{z} \times \hat{x}'.
\end{align*}
\]

The momenta of \( \ell_2 \) and \( \ell_3 \) constrain the components of the neutrino momentum orthogonal to \( \vec{p}_N \). This means that energy-momentum conservation in the \( W \) and \( \mathcal{N} \) decays can be expressed in terms of one unknown variable \( \alpha \), which is the component of neutrino momentum in the \( z' \) direction. To express Eqs. (A1) and (A2) in terms of \( \alpha \), the following quantities are defined:

\[
\begin{align*}
 \vec{p}_{23}^\ast &\equiv \vec{q}, \tag{A3} \\
 \vec{q} &= (0, |\vec{p}_{23} \times \hat{z}| \equiv q_\perp, \vec{p}_{23} \cdot \hat{z} \equiv q_z). \tag{A4} \\
 \vec{p}_1' &\equiv (0, -q_\perp, \alpha), \tag{A5} \\
 E_1' &= \sqrt{q_\perp^2 + \alpha^2}. \tag{A6}
\end{align*}
\]

Squaring Eq. (A2) gives

\[
p_W^2 = m_W^2 = m_1^2 + m_2^2 + m_{23}^2 + 2p_1' \cdot (p_{23}^\ast + p_1') + 2p_{23}^\ast \cdot p_1'. \tag{A7}
\]

where

\[
\begin{align*}
 p_1' \cdot (p_{23}^\ast + p_1') &= E_1' (E_{23} + E_1') - p_{1,z} (q_z + \alpha), \\
 p_{23}^\ast \cdot p_1' &= E_{23} E_1' + q_z^2 - q_\perpq_z\alpha.
\end{align*}
\]

In the energy regime of interest, the charged leptons and neutrino can be treated as massless particles, such that \( m_1 = m_2 = 0 \). Rearranging Eq. (A7) to solve for \( E_1' \) gives

\[
E_1' = A + B\alpha, \tag{A8}
\]

where

\[
A = \frac{(m_W^2 - m_{23}^2)/2 - E_1' E_{23} + p_{1,z}q_z - q_z^2}{E_1' + E_{23}}, \quad B = \frac{p_{1,z} + q_z}{E_1' + E_{23}}.
\]

Subtracting Eq. (A8) from Eq. (A7) gives the following quadratic expression in \( \alpha \)
\[(B^2 - 1)\alpha^2 + 2AB\alpha + A^2 - q_\perp^2 = 0.\]

The solution for \(\alpha\) is therefore
\[\alpha = \frac{-AB \pm \sqrt{(B^2 - 1)q_\perp^2 + A^2}}{(B^2 - 1)}. \quad (A9)\]

Both solutions for \(\alpha\) were studied using simulated HNL events and it was noted that the solution that led to a smaller \(|\vec{p}_\nu|\) typically led to a value for \(m_{\text{HNL}}\) that was closer to the simulated \(m_{\chi'}\). This solution often corresponded to forward emission of the neutrino with respect to the HNL decay. Therefore, the definition of \(m_{\text{HNL}}\) in the analysis uses the solution with the positive radical.

The expression for \(\alpha\) in Eq. (A9) depends on \(m_W\). ATLAS has measured the \(W\)-boson pole mass to be \(M_W = 80.370 \pm 0.019\) GeV [79]. This measurement is combined in Ref. [2] with results from other collider experiments to provide a measurement of the \(W\)-boson width, \(\Gamma_W = 2.195 \pm 0.083\) GeV. Since the \(W\) mass has a width, then if \(m_W = M_W\) in Eq. (A9) it is possible that there is no real solution for \(\alpha\). Instead of rejecting these events, \(m_W\) is set equal to the median \(W\) mass in the kinematically allowed region (\(m_W\text{med}\)). This ensures that \(\alpha\) (and correspondingly \(m_{\text{HNL}}\)) always has a real solution.

To define the kinematically allowed region, the minimum \(W\) mass that is consistent with the charged-lepton decay products (\(m_{W,\text{min}}\)) is computed. From Eq. (A7), the mass of the \(W\) boson is given by
\[m_W^2 = m_{\chi'}^2 + 2(E'_1E'_{23} + E'_uE'_1 + E'_{23}) - p'_{1,z}q_z + q_\perp^2 - \alpha(p'_{1,z} + q_z). \quad (A10)\]

and \(m_{W,\text{min}}\) occurs where
\[d(m_W^2/2)/d\alpha = (E'_1 + E'_{23})dE'_u/d\alpha - (p'_{1,z} + q_z) = 0. \quad (A11)\]

Using
\[dE'_u/d\alpha = \frac{d\sqrt{q_\perp^2 + \alpha^2}}{d\alpha} = \frac{\alpha}{E'_u}\]

in Eq. (A11), the chosen value of \(\alpha\) that gives the minimum \(m_W\) is
\[\alpha = \frac{q_\perp B}{\sqrt{1 - B^2}}. \quad (A12)\]

Substituting Eq. (A12) into Eq. (A10), the minimum \(W\) boson mass is
\[m_{W,\text{min}}^2 = m_{\chi'}^2 + 2(E'_1E'_{23} + (E'_1 + E'_{23})\sqrt{q_\perp^2 + \frac{q_\perp^2}{2} B^2}} - p'_{1,z}q_z + q_\perp^2 - \alpha(p'_{1,z} + q_z) - \frac{q_\perp B}{\sqrt{1 - B^2}}.\]

The cumulative probability for the \(W\) boson to have a mass greater than \(m_{W,\text{min}}\) is used to find the median of the remaining distribution. The probability density function \(f\) for \(m_W^2\) satisfies
\[f(m_W^2) \propto \frac{1}{(m_W^2 - M_W^2)^2 + M_W^2\Gamma_W^2}.\]

Therefore, the cumulative distribution function \(F\) is
\[F(m_W^2) = \frac{1}{\pi} \arctan\left(\frac{m_W^2 - M_W^2}{M_W\Gamma_W}\right) + \frac{1}{2}. \quad (A13)\]

The midpoint of the allowed kinematic region has a value of
\[F_{\text{med}} = \frac{1 + F(m_{W,\text{min}}^2)}{2}. \quad (A14)\]

Rearranging Eq. (A13) for \(m_W^2\) gives
\[m_W^2 = M_W^2 + \Gamma_W M_W \tan \left(\pi \left[1 - \frac{1}{2}\right]\right). \quad (A14)\]

Substituting \(F = F_{\text{med}}\) in Eq. (A14) gives an expression for the median \(W\) mass in the kinematically allowed region
\[m_{W,\text{med}}^2 = M_W^2 + \Gamma_W M_W \tan \left(\pi \left[\frac{1 + F(m_{W,\text{min}}^2)}{2} - \frac{1}{2}\right]\right). \quad (A15)\]

This value of \(m_{W,\text{med}}\) is used in Eq. (A9) to solve for \(\alpha\).

From Eq. (A1) and the definitions in Eq. (A3) to (A6), the expression for the HNL mass in terms of \(\alpha\) is
\[m_{\text{HNL}}^2 = m_{\chi'}^2 + 2p'_{1,z}p'_{23}\]
\[= m_{\chi'}^2 + 2E'_{23}\sqrt{q_\perp^2 + \alpha^2} + 2q_\perp^2 - 2q_z\alpha. \quad (A15)\]

Substituting the expression for \(\alpha\) in Eq. (A9) into Eq. (A15) gives the solution for the HNL mass.


LCb Collaboration, Search for Majorana Neutrinos in $B^+ \rightarrow \pi^+ \mu^+ \mu^- \nu_{\mu}$ decays, Phys. Rev. Lett. 112, 131802 (2014).


See Supplemental Material http://link.aps.org/supplemental/10.1103/PhysRevLett.131.061803 for the Feynman diagrams for the lepton number violating decays, as well as distributions of the opposite-sign and same-sign displaced vertices in the VR.


J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H.-S. Shao, T. Stelzer, P. Torrielli, and M. Zaro,
aDeceased.
bAlso at Department of Physics, King’s College London, London, United Kingdom.
cAlso at Instituto de Fisica Teorica, IFT-UAM/CSIC, Madrid, Spain.
dAlso at Institute of Physics, Azerbaijan Academy of Sciences, Baku, Azerbaijan.
eAlso at TRIUMF, Vancouver BC, Canada.
fAlso at Physics Department, An-Najah National University, Nablus, Palestine.
gAlso at Department of Physics, University of Fribourg, Fribourg, Switzerland.
hAlso at Department of Physics and Astronomy, University of Louisville, Louisville, Kentucky, USA.
iAlso at Departament de Fisica de la Universitat Autonoma de Barcelona, Barcelona, Spain.
jAlso at Affiliated with an institute covered by a cooperation agreement with CERN.
kAlso at The Collaborative Innovation Center of Quantum Matter (CICQM), Beijing, China.
lAlso at Department of Physics, Ben Gurion University of the Negev, Beer Sheva, Israel.
mAlso at Università di Napoli Parthenope, Napoli, Italy.
nAlso at Institute of Particle Physics (IPP), Canada.
oAlso at Bruno Kessler Foundation, Trento, Italy.
pAlso at Borough of Manhattan Community College, City University of New York, New York, New York, USA.
qAlso at Department of Financial and Management Engineering, University of the Aegean, Chios, Greece.
rAlso at Centro Studi e Ricerche Enrico Fermi, Italy.
sAlso at Department of Physics, California State University, East Bay, USA.
tAlso at Instituto Catalana de Recerca i Estudis Avancats, ICREA, Barcelona, Spain.
uAlso at Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Freiburg, Germany.
vAlso at Yeditepe University, Physics Department, Istanbul, Türkiye.
wAlso at Institute of Theoretical Physics, Ilia State University, Tbilisi, Georgia.
xAlso at CERN, Geneva, Switzerland.
yAlso at Hellenic Open University, Patras, Greece.
zAlso at Center for High Energy Physics, Peking University, China.
bAlso at The City College of New York, New York, New York, USA.
cAlso at Department of Physics, California State University, Sacramento, USA.
dAlso at Département de Physique Nucléaire et Corpusculaire, Université de Genève, Genève, Switzerland.
eAlso at Institut für Experimentalphysik, Universität Hamburg, Hamburg, Germany.
fAlso at Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan, USA.
gAlso at Institute for Nuclear Research and Nuclear Energy (INRNE) of the Bulgarian Academy of Sciences, Sofia, Bulgaria.
hAlso at National Institute of Physics, University of the Philippines Diliman (Philippines); Philippines.