Generation and detection of entangled single-photon pairs

Bachleors Thesis

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Abstract

Quantum information technology is an emerging field with important applications such as quantum cryptography and teleportation, quantum imaging and lithography. These applications make use of single photons and pairs of entangled photons. In this work, we experimentally generate and attempt to detect the entangled photons. The entangled photon pairs are produced using a nonlinear crystal of beta barium borate through a process of spontaneous parametric down-conversion (SPDC). Alignment necessary to detect the entangled photon pairs is implemented using a HeNe laser. The experimental results reveal key signatures of the down-converted photons: (i) energy conservation as the wavelength of generated photons (810 nm) is two times larger than that of the photons used to optically pump SPDC (405 nm), which is shown by using a 10-nm band-pass filter centred around 810 nm; (ii) the angles between the two photons of a pair correspond to the configuration of momentum conservation calculated analytically; (iii) the photons arrived at the detectors within the jitter time of those; and (iv) orthogonal polarisation of down-converted photons (810 nm) with respect to pump photons (405 nm). These findings show the consequences of SPDC.
1 Introduction

Spontaneous parametric down-conversion (SPDC) is a nonlinear phenomenon in quantum optics, which plays an essential role in quantum communication and quantum computing. SPDC occurs when a single photon induces nonlinear polarization in a crystal, splitting into two photons with lower energy than the original photon [1].

Quantum entanglement is a phenomenon that occurs when two particles, such as photons, are generated together such that their properties become intertwined in a way that cannot be explained by classical physics. When a photon pair is generated by SPDC, they are entangled in a way that their quantum states are correlated and dependent on each other, even when separated by a great distance. In quantum formalism, this means that the pair’s quantum state is inseparable and cannot be written in terms which are independent of one another [1].

The properties of the entangled photons are determined by the conservation of energy and momentum during the SPDC process. The total energy and momentum of the two photons must be conserved, so if one photon has a certain energy or momentum, the other photon must have the corresponding value to ensure that the total energy and momentum are conserved [1].

As a result of this entanglement, any measurement made on one photon collapses the pair’s state and both the photons’ properties go from being probabilistic to definitive regardless of the distance between them. This property of entanglement is known as non-locality, and it is a fundamental aspect of quantum mechanics [1].

The discovery of the quantum nature of spontaneous parametric down-conversion dates back to the 1960s when David Klyshko [2] proposed a theoretical framework for the phenomenon. The experimental demonstration of SPDC in ammonium dihydrogen phosphate crystal pumped by a 325-nm He-Cd laser was done by David Burnham and Donald Weinberg [3] in 1970. They showed that when a photon passes through a nonlinear crystal, it could split into two photons, each with half the energy and double the wavelength of the original photon. This phenomenon, called parametric fluorescence at that time, was shown to be spontaneous, meaning that it could occur without any external stimulation. It is interesting to note that already at that early days, the coincidences (simultaneous detection) between two photons were demonstrated and measured.

Despite a well-developed understanding of the quantum nature of SPDC, the property of the entanglement of SPDC photons attracted attention only in the late 80s after the experimental demonstration by Y. Shih and C. Alley [4]. After that, the SPDC process became the main method for the production of entangled photons [5].

Since then, researchers have conducted extensive studies on SPDC and the properties of the photons generated through the process, leading to a deeper understanding of the phenomenon and its potential applications. They have shown that entangled photons can exhibit non-local correlations, which cannot be explained by classical physics. These correlations are a fundamental aspect of quantum mechanics and have been studied extensively in the field of quantum information. These studies as well as studies of the Bell’s inequality with entangled photons led to the Nobel Prize in 2022 to Alain Aspect, John F. Clauser and Anton Zeilinger “for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science [6].”

In recent years, researchers have also explored the potential of SPDC in quantum imaging [7]. The ability to generate entangled photon pairs has allowed for the development of imaging systems that can detect hidden objects and materials with high accuracy. This technology has potential applications in a range of fields, including quantum lithography and medicine through ghost imaging, paving the way for future quantum technologies.

One of the most notable applications of SPDC is in quantum cryptography [8]. In this application, entangled photon pairs are used to transmit cryptographic keys securely between two parties. Because the state of the photons is dependent on each other, any attempt to eavesdrop on the transmission would cause a change in the state of the photons, alerting the parties to the presence of an intruder. This method of encryption is much more secure than traditional methods.

Another application of SPDC is in quantum teleportation [9, 10]. In this process, the quantum state of
one photon is transferred to another photon, without the physical transfer of the photon itself. This process relies on the use of entangled photon pairs and is a crucial component of quantum computing. The ability to teleport quantum states is essential for the development of quantum computers, as it allows quantum information to be transmitted without being corrupted by noise or interference.

SPDC has also been used in the development of quantum sensors. The availability of entangled photon pairs through SPDC has stimulated the development of highly sensitive detectors, which can detect very weak signals with high accuracy [11].

In conclusion, SPDC is a fascinating phenomenon in quantum optics that has led to the generation of entangled photon pairs, which are a critical resource for quantum communication and quantum computing. The discovery of SPDC and its subsequent applications have played a crucial role in the development of quantum technologies, and its continued research will undoubtedly lead to further breakthroughs in the field of quantum physics.

The purpose of this work is to generate and characterise entangled photon pairs generated by means of SPDC in a barium borate crystal. There is a practical interest in developing a compact source of entangled photons for applications.

In the text, the terms ‘quantum light’ and ‘classical light’ are used interchangeably with the terms quantum and classical EM field, correspondingly. By default, the EM field is treated classically or quasi-classically unless another situation is specified explicitly. The SI units with common notation for physical terms are used throughout the text.

2 Classical treatment of spontaneous parametric down-conversion

The electromagnetic (EM) field is a state of excitation established in space and time by the presence of charges and currents. The photon is a quantum of this excitation and a carrier of the corresponding electromagnetic force. The generation of entangled photon pairs through the SPDC process is the result of a nonlinear interaction of an electromagnetic field incident on a crystal with vacuum oscillations of the field in the crystal. Hence, we start this section with a quick review of the classical theory of electromagnetism based on the work of J.C. Maxwell. Then, the discussion focuses on a nonlinear three-wave interaction using the framework of classical electromagnetism. This allows deriving the condition of phase matching – a condition that enables an efficient energy transfer from the incident electromagnetic field (also called “pump wave”) to other waves. There are two possible types of the three-wave nonlinear interaction in a crystal: (1) a pump wave viewed as two identical waves with half of the amplitude of the initial one gives rise to the generation of the second harmonic – a process widely used in lasers to generate higher frequencies but a parasitic and unwanted effect during the SPDC process; (2) a pump wave produces two lower frequency waves often referred to as a “signal” and “idler” wave. Since the two mentioned nonlinear interaction are in generally connected, it is instructive to discuss them both using the framework of classical electromagnetism. This framework allows capturing most of the essential physics with a simple mathematical formalism.

2.1 Electromagnetic theory of light

Through the seminal work of James Clerk Maxwell, the relation between magnetism, electricity as well as charges and currents was summarised in the four Maxwell equations, [12]. This resulted in the fundamental theoretical construct of the electromagnetic theory of light illustrated below:

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},
\]

\[
\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t},
\]

\[
\nabla \cdot \vec{B} = 0,
\]

\[
\nabla \cdot \vec{D} = \rho.
\]

4
Maxwell proposed that light propagates as electric $\vec{E}$ and magnetic fields $\vec{B}$ coupled through the displacement current. In media, these two fields give rise to the electric displacement field, $\vec{D}$ and the magnetic field strength, $\vec{H}$. The magnetic field vector is defined as $\vec{H} = \frac{1}{\mu} \vec{B} - \vec{M}$, where $\vec{M}$ is the average magnetisation in media at a given point. Analogous to the effects of the magnetic field, the electric field also causes polarisation in media. Dielectrics placed in external electric fields tend to become polarised as a result of the displacement of bound charges. The relation between the electric field and the polarisation vector $\vec{P}$, which is material dependent, is summarised in the displacement vector $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$.

For a weak nonlinear dependence of the induced polarisation $\vec{P}$ on an applied field $\vec{E}$, $\vec{P}$ can be expanded into a Taylor series with respect to $\vec{E}$. By retaining only the first and second orders terms, $\vec{P}$ can be written as:

$$P_i = \epsilon_0 \sum_j \chi_{ij} E_j + \sum_{j,k} \chi_{ijk} E_j E_k; \ i, j, k = 1, 2, 3. \quad (5)$$

Here, $P_i$ are the polarisation density components, $E_i$ are the applied electric field components and the coefficients $\chi_{ij}$ and $\chi_{ijk}$ are the components of the $\chi$ tensor that represent the scalar quantities of the electric susceptibility. For dielectrics that have a non-negligible higher-order tensor, $\chi_{ijk}$ or higher are classified as the components of nonlinear susceptibility.

We note that the Lorentz force given by $q(\vec{E} + \vec{v} \times \vec{B})$ – a force acting on a charged particle from an electromagnetic field – contains the fields $\vec{E}$ and $\vec{B}$ and not $\vec{D}$ and $\vec{H}$. Therefore, we consider the $\vec{E}$ and $\vec{B}$ fields as more fundamental and describe the three-wave interaction in terms of the $\vec{E}$ field even inside the crystal [13] [14].

The nonlinearity of a medium gives rise to useful optical properties such as harmonic generation, having numerous technological applications. Some of the most important nonlinear optical effects are the sum frequency generation (SFG) and spontaneous parametric down-conversion (SPDC). SFG occurs when two incoming waves (we will use also the terms “beams” when discuss lasers fields) in a nonlinear medium generate an outgoing wave, whose frequency is the sum of the frequencies of the two incoming ones. One incoming wave can be seen as a degenerate case. SPDC, on the other hand, relies only on one incoming beam and results in two outgoing beams, whose frequencies sum up to that of the incoming one [13].

All these processes can be generated using a second-order nonlinear medium. The polarisation vector of such a material, where higher orders than the second are negligible, is described by the first linear term and the first nonlinear term in Eq. (5). The first nonlinear term is responsible for the frequency increase in SFG, which squares the electric field and, in turn, adds the frequencies of the two incoming beams. A deeper description of SPDC and second harmonic generation (SHG) will be discussed in coming sections. The two processes are summarised in the figure below:
Figure 1: Schematic illustration of phenomena occurring in materials having second-order nonlinearity. In both cases, the efficiency of the process is dependent on the crystal used, the quality and profile of the pump, how well the pump is focused, in addition to other factors. Typically the efficiencies in SHG are above 50%. The conversion efficiencies can be increased further, up to 90% using methods such as cavity enhancement. This means that a portion of the pump beam still passes through the material without undergoing SHG, hence why all the three wavelengths are present after the crystal. This is also true for SPDC but the conversion rates are much lower. Among the highest conversion rates generated to date are four photon pairs per a million incoming pump photons [15] [16].

Using Maxwell’s equations, one can derive the wave equation, which governs the propagation of light in medium, including nonlinear media. The first step as seen below is taking the curl of eq. (1) on both sides and substituting the curl of curl identity $\nabla \times (\nabla \times \vec{E}) = \nabla \cdot (\nabla \cdot \vec{E} - \nabla^2 \vec{E})$ in Eq. (2). By assuming the material to be non-conductive, i.e. $\vec{J} = 0$ in Eq. (2), Eq. (4) can be used to yield

\[
\nabla \times (\nabla \times \vec{E}) = \frac{\partial}{\partial t} (\nabla \times \vec{B}), 
\]

(6)

\[
\nabla \cdot (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial \vec{D}}{\partial t}, 
\]

(7)

\[
\nabla^2 \vec{E} = \frac{\partial^2}{\partial t^2} (\mu_0 \epsilon_0 \vec{E} + \mu_0 \vec{B}), 
\]

(8)

\[
\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{P} = \frac{1}{\epsilon_0 \mu_0} \frac{\partial^2}{\partial t^2} \vec{P}. 
\]

(9)

For linear relationships between the fields in vacuum and in the materials which are modelled by the polarisation vector with only the first order term in equation 5, two simple solutions to the wave equation are a plane and a spherical wave. For a monochromatic plane wave, the electric field vector component reads $\vec{E}(\vec{r}, t) = E_0 e^{-i\omega t + i\vec{k}\vec{r}}$ and its magnetic counterpart is $\vec{B}(\vec{r}, t) = B_0 e^{-i\omega t + i\vec{k}\vec{r}}$, where $\vec{k}$ is the propagation wave vector and its magnitude reads as $k = \omega/c = 2\pi/\lambda$ with $\omega$ and $\lambda$ being the angular frequency and wavelength, respectively.

The homogeneity and linearity of the wave equation implicate that the superposition principle is appli-
cable to electromagnetic waves, i.e.

\[
\vec{E}(\vec{r}, t) = \sum_n \vec{E}_n(\vec{r}, t), \quad (10)
\]

\[
\vec{P}(\vec{r}, t) = \sum_n \vec{P}_n(\vec{r}, t), \quad (11)
\]

\[
\vec{B}(\vec{r}, t) = \sum_n \vec{B}_n(\vec{r}, t). \quad (12)
\]

Harmonic generation is a phenomena that occurs as a result of photon interaction with nonlinear materials possessing second-order nonlinearity. In reality, all materials exhibit a nonlinear behaviour when illuminated by strong-field light but the nonlinear response of some materials is much stronger because of an internal asymmetry of the material. Viewed classically, when an electromagnetic wave with a frequency \( \omega \) penetrates into a material, the electrons of the material start to oscillate and these oscillations can be described by a simple harmonic oscillator model. An increase in the intensity of light results in nonlinear motion of the electrons analogous to a ball on a strongly deformed spring beyond the linear Hooke’s law. This means that the electrons polarise the material in a nonlinear manner. In other words, the polarisation density vector includes terms of higher orders, that is \( \vec{P} = \vec{P}_L + \vec{P}_{NL} \). [1]. Then, the nonlinear harmonic generation is described by [13]:

\[
\nabla^2 \vec{E} - \frac{1}{\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{\epsilon_0 c^2} \frac{\partial^2}{\partial t^2} \vec{B}_{NL}, \quad (13)
\]

where \( \epsilon_L = n^2 \) is the linear dielectric permittivity of the medium and \( n \) is the refractive index.

### 2.2 Sum Frequency Generation (SFG)

The wave equation found in the previous subsection can be used to describe SFG, which is a nonlinear optical phenomena generating new waves with frequencies that are the sum of the input frequencies. The process is illustrated in Fig. 1a. The following derivation is adapted from references [1] [13].

We seek a solution to the wave equation (13) in the form of plane waves propagating in the \( z \)-direction

\[
E_1 = A_1 e^{i(k_1 z - \omega_1 t)} + c.c, \quad (14)
\]

\[
E_2 = A_2 e^{i(k_2 z - \omega_2 t)} + c.c, \quad (15)
\]

\[
E_3 = A_3 e^{i(k_3 z - \omega_3 t)} + c.c., \quad (16)
\]

where ‘c.c’ denotes the complex conjugate term and the following relations hold:

\[
k_i = \frac{n_i \omega_i}{c}, \quad n_i = \epsilon_L(\omega_i), \quad i = 1, 2, 3. \quad (17)
\]

According to Saleh and Teichs argumentation in Ref. [14], to simplify the analysis we consider \( s \)–polarised light so that the electric field contains only one scalar component, which is orthogonal to the direction of propagation. This is a good approximation of laser beams for mild focusing of laser beams, which is usually the case. For strongly focused laser beams, the longitudinal component of the electric field becomes noticeable and is of the order of \( E_\parallel \sim (\lambda/w)E_\perp \), where \( E_\parallel \) and \( E_\perp \) are the longitudinal and transverse components of the laser field. So, all analysis below is valid under the approximation that \( \lambda \) is much smaller than the laser beam size \( w \).

Consider the polarisation term \( P_{NL} \), see (5):

\[
P_{NL} = \chi_{312} E_1 E_2. \quad (18)
\]

It contains two types of terms: (1) \( A_1 A_2 \exp[i(k_1 + k_2) - i(\omega_1 + \omega_2)] \) and (2) \( A_1 A_2^* \exp[i(k_1 - k_2) - i(\omega_1 - \omega_2)] \).

The first term describes second harmonic generation whereas the second one – difference frequency generation.
It is instructive to consider SHG and derive the conditions of phase matching. To this end, we insert model solutions (14) and (15) into $P^{NL}$ and keep only the terms containing $\exp(ik_1 + k_2 - i(\omega_1 + \omega_2))$:

$$P^{NL} = 2\epsilon_0 \chi^{(2)} A_1 A_2 e^{i(k_1 + k_2) - i(\omega_1 + \omega_2)} + c.c.,$$

(19)

where $\chi_{312}$ is replaced by the common notation $2\epsilon_0 \chi^{(2)} [14]$.

By inserting the polarisation term $P^{NL}$ in the form Eq. (19) into the wave equation (13) and replacing the nabla operator with the second derivative with respect to the coordinate $z$, the governing equation for $A_3$ is derived:

$$\left( \frac{d^2 A_3}{dz^2} + 2i k_3 \frac{dA_3}{dz} \right) e^{ik_3z - i\omega_3 t} = \frac{-2\chi^{(2)}}{c^2} (\omega_1 + \omega_2)^2 A_1 A_2 e^{i(k_1 + k_2) - i(\omega_1 + \omega_2)t}.$$  

(20)

In order for this equation to be valid for any time moment, we choose $\omega_3$ to satisfy the relation $\omega_3 = \omega_1 + \omega_2$ – a condition of the sum frequency generation. If $\omega_1 = \omega_2$, then $\omega_3 = 2\omega_1$ and second harmonic generation (SHG) occurs.

Usually, the nonlinearity of a medium is weak and the energy transfer occurs on a scale much larger than the wavelength, i.e. the wave amplitude is a slow varying function. This is the slowly varying amplitude approximation. Then, the second derivative in Eq. (20) can be neglected and Eq.(20) reads:

$$\frac{dA_3}{dz} = i\frac{\omega_3}{cn_3} \chi^{(2)} A_1 A_2 e^{i(k_1 + k_2 - k_3)} + c.c.$$  

(21)

This means that one needs merely to solve a first order ODE. The case is similar for $A_1$ and $A_2$:

$$\frac{dA_2}{dz} = i\frac{\omega_2}{cn_2} \chi^{(2)} A_1^* A_3 e^{-i\Delta k z} + c.c.,$$

(22)

$$\frac{dA_1}{dz} = i\frac{\omega_1}{cn_1} \chi^{(2)} A_2^* A_3 e^{-i\Delta k z} + c.c.$$  

(23)

Here, $\Delta k = k_1 + k_2 - k_3$ is the wave number mismatch parameter playing a crucial role for efficient generation as discussed in detail below.

Let us define the intensities as $I_i = 2n_i \epsilon_0 |A_i|^2$. Then, Eqs. (20), (22) and (23) can be re-written in the form of the Manley–Rowe relations [13] [14]:

$$\frac{d}{dz} \left( \frac{I_1}{\omega_1} \right) = \frac{d}{dz} \left( \frac{I_2}{\omega_2} \right) = - \frac{d}{dz} \left( \frac{I_3}{\omega_3} \right).$$  

(24)

They express the photon-number conservation principle: the number of photons generated at $\omega_1$ or $\omega_2$ per unit length is half the number of photons decaying at $\omega_3$ per unit length. This result indicates that the model is reasonable and gives a correct answer.

### 2.3 Phase matching

Phase matching – the limiting case of $\Delta k = 0$ – directly affects the efficiency of second harmonic generation. Let us first illustrate this mathematically assuming $A_1$ and $A_2$ to be constant in the right hand side of (20). Then, after direct integration we see that $A_3 \propto \exp(i\Delta k z)/\Delta k$ and $A_3$ increases as $\Delta k$ decreases, see [13],

$$I_3 = I_3^{max} \text{sinc}^2 \left( \frac{\Delta k L}{2} \right).$$  

(25)

If the phase relation between the generated waves changes, then the solutions for $A_i$ describe a periodic energy exchange along the crystal between the pump waves $A_1$ and $A_2$ and the generated wave $A_3$. In other words, the phase matching condition can be treated as the situation when the induced atomic dipoles in the matter are phased in such a manner that the field generated from each dipole adds up coherently [13].
In the limiting case $\Delta k = 0$, $A_3$ grows linearly along the crystal and higher output power levels can be achieved. If, on the other hand, the length of the crystal is larger than the reciprocal of the phase mismatching parameter $1/\Delta k$, then the amplitude of generated wave $A_3$ starts decreasing back with the interaction length because the frequency-converted wave $A_3$ becomes out of phase related to the exciting laser field $A_{1,2}$. This leads to the power flowing from the generated wave back to the input waves. The coherence length is therefore defined as:

$$L_c = \frac{2}{\Delta k}.$$  \hfill (26)

Most materials have normal dispersion: the refractive index decreases as a function of wavelength. This makes attaining the matching condition

$$\frac{n_3\omega_3}{c} = \frac{n_2\omega_2}{c} + \frac{n_1\omega_1}{c}$$  \hfill (27)

possible only for the class of materials violating the law of normal dispersion. The class of such materials are crystals that exhibit birefringence. Birefringence is a property that is dependent on the material’s crystal structure. Crystals that, for example, have a cubic crystalline structure cannot be birefringent as they are isotropic. Anisotropic crystals, on the other hand, do not have equivalent crystallographic axes. This means that the material’s interaction with light is highly dependent upon the axis in which it enters [1].

The birefringent crystals can be classified optically as being biaxial or uniaxial. The former having three refractive indices and the latter having two. In the experiments conducted in this project, a negative uniaxial crystal was used. For such crystals, incoming light with perpendicular polarisation with respect to the optical axis and the plane with the vector $\vec{k}$ refract in accordance with the ordinary refractive index, $n^0$. Light orthogonally polarised to the above case, experiences instead the extraordinary refractive index, $n^e$. Birefringence can be negative or positive. Negative birefringence is the case where, $\Delta n = n^e - n^0 < 0$, indicating that the fast axis is situated along the axis with a lower refractive index, $n^e$. For positive crystal $\Delta n > 0$ [1].

The phase matching is also categorised based on the polarisation of the outgoing waves generated. The case, where the waves corresponding to $\omega_1$ and $\omega_2$ have the same polarisation, is called type I phase matching that is $e \rightarrow o + o$. When the polarisation of the waves corresponding to $\omega_1$ and $\omega_2$ are orthogonal to one another is known as type II phase matching, that is $e \rightarrow o + e$.

In the general case, the matching condition must be applied to wave vectors: $\vec{k}_1 + \vec{k}_2 = \vec{k}_3$. For the type I phase, the matching condition for a negative uniaxial crystal projected onto the direction of the wave vector $\vec{k}_3$ takes the form:

$$n_3^0\omega_2\cos \theta_2 + n_1^0\omega_1\cos \theta_1 = n_3^e\omega_3.$$  \hfill (28)

Up until now the process described corresponds to the classical sum frequency generation. SPDC is the reverse of SFG: a single incoming photon (pump) with pump frequency $\omega_3$ is converted into two photons with frequencies $\omega_1$ and $\omega_2$. The two output photons have historically been named signal and idler photons [17]. In SPDC there is no initial field present at the frequency of the signal and idler photons, that is $A_1(0) = 0$ and $A_2(0) = 0$ in equation (23) [18]. This means that equations (23) and (22) would both give zero result for the amplitude of the signal and idler waves. This, of course, means that neither the signal nor idler would exist. Thus, leading to the conclusion that the classical model of electromagnetism is not sufficient to explain the phenomena of SPDC. The quantum description is necessary [1].

### 2.4 SPDC mechanism

In the experimental setup, described further in section 5, we have type I spontaneous parametric down-conversion. This means that the pump photons with a frequency $\omega_3$ are down-converted into signal and idler photons with frequencies, $\omega_1$ and $\omega_2$ respectively. The down-conversion process generates a cone of photons having a range of different energies. The position in the cone determines the energy of the photon in such a way that energy and momentum are conserved, Eq. (28).
In the degenerate case we are interested in the scenario of signal and idler photons having the same energy. To find where in the cone this is the case, one must look at the equation of conservation of momentum previously discussed in detail in section (2.3):

\[ n_2 \omega_2 \cos \theta_2 + n_1 \omega_1 \cos \theta_1 = n_3 \omega_3 \]  

(29)

For the degenerate case \( \omega_3 / 2 = \omega_2 = \omega_1 \), \( \theta_1 = \theta_2 \) and \( n_1 = n_2 \), the equation (29) simplifies to

\[ n_3 = n_2 \cos \theta_2. \]  

(30)

Equation (30) can be satisfied in a birefringent material that is an anisotropic material having different refractive indices dependent upon material’s orientation. The Beta Barium Borate (BBO) crystal used in this experiment is such a birefringent material [1].

In more detail, in birefringent crystals optical axes are crystallographically distinct unlike in isotropic crystal, where all axes are the same. In such anisotropic crystals when light enters an axis that is not the optical axis, it is refracted in two different rays that emerge having orthogonal polarisation with respect to one another. The rays are known as the extraordinary and ordinary rays. Light that is polarised perpendicularly to the optical axis is the ordinary ray. Light polarised parallel to the optical axis is the extraordinary ray. The extraordinary and ordinary rays each experience the extraordinary and ordinary refractive indices, in particular \( n^e \) and \( n^o \) respectively. The ordinary refractive index, \( n^o \), remains constant regardless of the propagation direction of the pump laser with respect to the optical axis. This is not the case for the extraordinary refractive index: \( n^e \) is dependent upon the phase matching angle \( \theta_m \) that is the angle between the optical axis and the propagation vector of the pump laser. Only the extraordinary ray which experiences \( n^e \) will undergo SPDC and generate down-converted photons. The reason for this is that only with the extraordinary refractive index, \( n^e \), is the phase matching condition in equation (29) satisfied. The down-converted photons that are generated in the setup then all emerge with an ordinary polarisation characteristic of type I phase matching, meaning that they experience the ordinary refractive index and are polarised orthogonally with respect to the pump beam. Further details of why the polarisation changes from extraordinary to ordinary when undergoing SPDC requires a microscopic understanding of the crystal which is beyond the scope of this thesis.

The pump laser as mentioned in the type I phase matching condition has an extraordinary polarisation. The orientation of the crystal along its optical axis with respect to the propagation of the laser affects the refractive index in the following way [13]:

\[ n^e(\theta_m) = \frac{n^o \tilde{n}^e}{\sqrt{(n^o)^2 - ((n^o)^2 - (\tilde{n}^e)^2) \cos^2 \theta_m}}, \]  

(31)

where \( \tilde{n}^e \) is the principal value of the extraordinary refractive index.

When the phase matching angle, \( \theta_m \), equals 29.42°, this satisfies the degenerate case in Eq. (30) and the down-converted photons exit the crystal at an angle \( \theta = \pm 3 \). The crystal used in the setup built was cut to the phase matching angle. In other words, the crystal used is cut in such a way that by placing the crystal at normal incidence to the beam, the phase matching angle is fulfilled and the photons exit at a three degree angle with respect to the propagation direction. This is illustrated in Fig. 2
3 Quantum description of SPDC

In this section, SPDC is described as a quantum event. This is done by first quantising the electromagnetic field and introducing the Hamiltonian of the electromagnetic field in terms of the creation and annihilation operators. The Hamiltonian permits vacuum oscillations of the electromagnetic field and allows for the spontaneous generation of a pair of down-converted photons.

Viewed as a quantum process, the incoming pump photons interact with the quantum vacuum field through the second order nonlinear material and down-convert into the pairs of entangled photons. The process conserves both energy and momentum and does not exchange energy with the crystal due to SPDC. This means that the SPDC process can be described as a quantum evolution of the electromagnetic field. The following derivation is adapted from reference [18].

3.1 Field quantisation

The following derivation is the quantisation of a free electromagnetic field meaning that there are no external charges or currents present, i.e. in the Maxwell equation (4), \( \rho = 0 \) and in equation (2), \( J = 0 \).

The electromagnetic field is assumed to be finite and enclosed in a cube with side \( L \) and volume \( V \). The electric field (and analogously the magnetic counterpart) can be decomposed into plane waves with amplitudes \( \vec{E}_n(t) \) and wave vectors \( \vec{k}_n \). This decomposition is equivalent to a Fourier expansion and reads:

\[
\vec{E}(\vec{r}, t) = \sum_n \vec{E}_n(t) e^{i\vec{k}_n \cdot \vec{r}}
\]

and the components of \( \vec{k}_n \) are defined as

\[
k_{nj} = n_j \frac{2\pi}{L}
\]

where the subscript \( j \) denotes one of the directions along the Cartesian coordinates \( x, y \) or \( z \) and \( n_j \) are the summation indexes along these directions.

In the reciprocal space, which is the space of wave vectors, Maxwell’s equations can be rewritten as

<table>
<thead>
<tr>
<th>Real Space</th>
<th>Reciprocal space</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \vec{\nabla} \cdot \vec{B} = 0 )</td>
<td>( k_n^2 B_n = 0 )</td>
</tr>
<tr>
<td>( \vec{\nabla} \cdot \vec{E} = 0 )</td>
<td>( k_n^2 E_n = 0 )</td>
</tr>
</tbody>
</table>

Table 1: Maxwell’s relations in vacuum in physical and reciprocal space.
The following relations are true for the electric and magnetic fields and will be useful:

$$\vec{E}_{-n}(t) = \vec{E}^*_n(t)$$  \hspace{1cm} (34)

$$\vec{k}_n = -\vec{k}_{-n}$$  \hspace{1cm} (35)

The equations in table 1 tell us that both the electric and magnetic fields are transverse. Additionally, they imply that the magnetic field may be written as

$$\vec{B}_n = \vec{B}_n \vec{e}_{n,1} + \vec{B}_n \vec{e}_{n,2},$$  \hspace{1cm} (36)

where $\vec{e}_{n,1,2}$ are unit vectors, which are orthogonal to one another, and $\vec{B}_n$ are the field amplitudes. The vector $\vec{n} = [n_x, n_y, n_z]$ is the refractive index along the wave vector. The subscripts 1 and 2 are the basis of the polarisations. This means that the equation (32) can be rewritten in terms of the basis $g = [n_x, n_y, n_z, l]$ as

$$\vec{B}(\vec{r}, t) = \sum_g \vec{e}_g \vec{B}_g(t) e^{i\vec{k}_g \cdot \vec{r}}.$$  \hspace{1cm} (37)

In a similar fashion, the electric field can be defined as

$$\vec{E}(\vec{r}, t) = \sum_g \vec{e}_g \vec{E}_g(t) e^{i\vec{k}_g \cdot \vec{r}}.$$  \hspace{1cm} (38)

The electric and magnetic fields constitute 6 scalar components, which are related through Maxwell’s equations. To reduce the number of scalar components and facilitate the derivation of the quantised Hamiltonian for the electromagnetic field, it is advantageous to introduce an additional vector field – the vector potential $\vec{A}(\vec{r}, t)$. Both the electric and magnetic field can be calculated from $\vec{A}(\vec{r}, t)$ as follows:

$$\vec{E}(\vec{r}, t) = -\frac{\partial \vec{A}}{\partial t},$$  \hspace{1cm} (39)

$$\vec{B}(\vec{r}, t) = \vec{\nabla} \times \vec{A}.$$  \hspace{1cm} (40)

Recall that we are working in a source-free region so that the scalar potential is zero and does not appear in the analysis.

Using the Coulomb gauge ($\vec{\nabla} \cdot \vec{A}$), the equation of the harmonic oscillator can be derived for $\vec{A}$. Then, a plane-wave expansion (Fourier expansion) for $\vec{A}$ can be introduced similarly to that of $\vec{E}$

$$\vec{A}(\vec{r}, t) = \sum_g \vec{e}_g A_g(t) e^{i\vec{k}_g \cdot \vec{r}}.$$  \hspace{1cm} (41)

Equations (39) and (40) ensure that the terms $\vec{E}_g(t)$ and $\vec{A}_g(t)$ or equivalently $\vec{B}_g(t)$ are coupled. In order to obtain the solutions to these terms, one can decouple their dynamical equations by introducing the following terms:

$$\beta_g = \frac{1}{2\sigma_g^{(1)}}(\omega_g \vec{A}_g - i\vec{E}_g),$$  \hspace{1cm} (42)

$$\gamma_g = \frac{1}{2\sigma_g^{(1)}}(\omega_g \vec{A}_g + i\vec{E}_g),$$  \hspace{1cm} (43)

where $\sigma_g^{(1)}$ is a constant term. Using these terms, the solutions to $\vec{A}_g(t)$ and $\vec{E}_g(t)$ read

$$\vec{A}_g(t) = \frac{\sigma_g^{(1)}}{\omega_g}(\beta_g(t) + \gamma_g(t)),$$  \hspace{1cm} (44)

$$\vec{E}_g(t) = \frac{\sigma_g^{(1)}}{\omega_g}(i\beta_g(t) - l\gamma_g(t)).$$  \hspace{1cm} (45)

12
The equations (34) translate to the following equivalent relations:

\[
\tilde{A}_g = A_g^* \\
\tilde{E}_g = E_g^* 
\]  

(46)  
(47)

By inserting the equations (46) and (47) into the equations (42) and (43), the relation \( \beta_g^* (t) = \gamma_{-g}(t) \) is obtained. Another important relation \( k_{-1} = -k_1 \) is deduced from equation (35). These two relations can be substituted in the equations for \( \tilde{A}_g(t) \) and \( \tilde{E}_g(t) \). The steps are summarised below

\[
\vec{A}(\vec{r}, t) = \sum_g \mathcal{E}_g \vec{A}(t)e^{i\vec{k}_g \cdot \vec{r}},
\]

(48)

\[
\vec{A}(\vec{r}, t) = \sum_g \mathcal{E}_g \sigma^{(1)}_g (\beta_g(t) + \gamma_g(t))e^{i\vec{k}_g \cdot \vec{r}},
\]

(49)

\[
\vec{A}(\vec{r}, t) = \sum_g \mathcal{E}_g \sigma^{(1)}_g (\beta_g(t) + \beta^*_{-g}(t))e^{i\vec{k}_g \cdot \vec{r}},
\]

(50)

\[
\vec{A}(\vec{r}, t) = \sum_g \mathcal{E}_g \sigma^{(1)}_g (\beta_g(t)e^{i\vec{k}_g \cdot \vec{r}} + \beta^*_{-g}(t)e^{-i\vec{k}_g \cdot \vec{r}}).
\]

(51)

In a similar fashion, the electric and magnetic fields can be derived. The derived normal modes of the fields can be inserted into the Hamiltonian of the electromagnetic field and finally the Hamiltonian of the quantised electromagnetic field emerges.

According to Jackson (1999), the total energy of an electromagnetic field in a volume \( V \) is [12]

\[
E_{tot} = \frac{\varepsilon_0}{2} \int_V (\vec{E}^2 + c^2 \vec{B}^2)
\]

(52)

Assuming the system to be non dissipative and independent of time, the Hamiltonian \( H_R \) corresponds to \( E_{tot} \)

\[
H_R = 2\varepsilon L^3 \sum_g (\sigma^{(1)}_g)^2 |\beta_g|^2.
\]

(53)

According to Grynberg, Aspect and Fabre (2010), the conjugate canonical variables are defined in the following manner

\[
Q_g = \sqrt{\frac{4\varepsilon_0 L^3}{\omega_g}} \sigma^{(1)}_g \Re(\beta_g),
\]

(54)

\[
P_g = \sqrt{\frac{4\varepsilon_0 L^3}{\omega_g}} \sigma^{(1)}_g \Im(\beta_g).
\]

(55)

By manipulating equations (54) and (55), the annihilation, \( \hat{a}_g \), and creation, \( \hat{a}_g^\dagger \), operators being defined as

\[
\hat{a}_g^\dagger = \frac{1}{\sqrt{2\hbar}} (Q_g - i P_g),
\]

(56)

\[
\hat{a}_g = \frac{1}{\sqrt{2\hbar}} (Q_g + i P_g).
\]

(57)

The final step is taking the creation and annihilation operators and inserting it into the Hamiltonian

\[
\hat{H}_R = \sum_g \frac{\omega_g}{2} (\hat{Q}_g^2 + \hat{P}_g^2).
\]

(58)
of the polarisation vector (5) in nonlinear media as just discussed before. The first term in (60) corresponds to the effective Hamiltonian for SPDC, \( \hat{H}_{\text{SPDC}} \), we first recall that the energy of the electromagnetic field in a medium is proportional to \( \vec{E} \cdot \vec{D} = \varepsilon_0 E^2 + \vec{E} \cdot \vec{P} \). It is the second term that is responsible for the nonlinear effects and gives rise to \( \hat{H}_{\text{SPDC}} \). Representing the electric field in the form \( E_0(\hat{a}_g \exp[-i\omega t + i\vec{k}\vec{r}] + \hat{a}_g^\dagger \exp[-i\omega t - i\vec{k}\vec{r}]) \), and inserting this form into the expression for the electromagnetic energy, the effective interaction Hamiltonian for SPDC can be written as

\[
\hat{H}_{\text{SPDC}} = i\hbar \kappa \left( \hat{a}_g \hat{a}_g^\dagger e^{i\Delta k z - i\Delta \omega t} + \hat{a}_s \hat{a}_s^\dagger e^{-i\Delta k z + i\Delta \omega t} \right).
\] (60)

This Hamiltonian contains triple products of creation/annihilation operators reflecting the nonlinear nature of the polarisation vector (5) in nonlinear media as just discussed before. The first term in (60) corresponds to the SFG process or SHG, if \( \omega_1 = \omega_2 \), and describes the creation of a SHG photon at \( \omega_3 \) with the frequency detuning \( \Delta \omega = \omega_3 - \omega_2 - \omega_1 \). The second term in (60) accounts for SPDC, where the pump photon ‘disappears’ to create one signal photon at \( \omega_1 \) and one idler photon at \( \omega_2 \) in the non-degenerate case (\( \omega_2 \neq \omega_1 \)).

The constant \( \kappa \) is given by [1]:

\[
\kappa = \frac{2}{3} \frac{d_{\text{eff}}}{\varepsilon_0 V} \sqrt{\frac{\omega_p \omega_1 \omega_2}{2\varepsilon_0 V}},
\] (61)

where \( d_{\text{eff}} \equiv \chi^{(2)}/2 \) is the effective nonlinear susceptibility and \( V \) is the volume. The smaller the volume, the stronger the coupling \( \kappa \). However, we must keep in mind that the present model is applicable to laser beams only under the condition of mild focusing, the laser spot size is much larger than the wavelength \( w \gg \lambda \).

The Hamiltonian (60) allows us to estimate the number of generated SPDC pairs. Let a pump beam containing \( N_p \) photons interact with a non-linear crystal. We describe the initial state containing 0 photons in the signal and idler states, and \( N_p \) photons in the pump state as \( |0_s, 0_i, N_p \rangle \). From the time-dependent Schrödinger equation, the state function reads

\[
|\Psi(t)\rangle = \exp \left[ \frac{i}{\hbar} \int_0^t \hat{H}_{\text{SPDC}}(t') dt' \right] |0_s, 0_i, N_p \rangle.
\] (62)

Note that the efficiency of SPDC is very low and \( \hbar \kappa \) is a small parameter so that we can Taylor expand the exponential factor up to the first order and write

\[
|\Psi(t)\rangle \approx C_0 |0_s, 0_i, N_p \rangle + \frac{C_1}{i\hbar} \int_0^t \hat{H}_{\text{SPDC}}(t') dt' |0_s, 0_i, N_p \rangle.
\] (63)

Here, \( C_0 \) and \( C_1 \) are the Taylor expansion coefficients present for the normalisation of the wave function. The integrand simplifies for perfect frequency matching, \( \Delta \omega \approx 0 \), and the result of integration is a delta function of time, which implies that \( \hat{a}_s^\dagger \hat{a}_s \hat{a}_p \) is just applied to the initial state

\[
|\Psi(t)\rangle \approx C_0 |0_s, 0_i, N_p \rangle + \kappa C_1 \hat{a}_s^\dagger \hat{a}_s \hat{a}_p e^{-i\Delta k z} |0_s, 0_i, N_p \rangle = C_0 |0_s, 0_i, N_p \rangle + \kappa C_1 e^{-i\Delta k z} |1_s, 1_i, N_{p-1} \rangle.
\] (64)
Note the analogy of (64) with (22) and (23): the exponential (oscillatory) dependence of the mismatch parameter and the direct linear proportionality to the nonlinear susceptibility parameter.

The measured intensity of SPDC is proportional to the square of the wave function (64). Since we are interested only in the situation when both a signal and an idler photon are simultaneously measured, only the second term in (64) is relevant. Hence, the intensity of SPDC is proportional to $\kappa^2$, which gives us an estimate for a typical efficiency of SPDC that is of the order of $10^{-10}$.

In passing, we note that the distribution of SPDC photons follows the Poissonian distribution for the number of photons detected over a fixed observation time interval [1]. We will come back to this fact when we analyse the fluctuations of detected down-converted photons.

4 Experimental setup and hardware

This section presents an optical layout and the parameters of hardware as well as software used for analysing the production SPDC photons with a high temporal resolution. The use of a field programmable gate array allowed seeing the rate of coincidence of entangled photons in real time.

4.1 Main optical layout

The principal scheme of the experimental setup is shown in Fig. 3. The objective of the setup is to create a pair of photons that are entangled via SPDC. The main components of the setup are the pump laser operating at the wavelength of 405 nm, a 5x5x3 mm Beta-Barium Borate crystal (BBO), avalanche photodiode detectors (APD), a high-frequency oscilloscope and a Field Programmable Array (FPGA). Since high temporal resolution was necessary to resolve the signals on the oscilloscope, a single continuous data acquisition was limited to approximately 3 ms. With the FPGA, on the other hand, we were able to conduct continuous measurements that lasted 10 s. Part of the optical path of the pump laser beam can be seen in Fig. 4 as the laser beam passes through the polarizer and the BBO crystal.

![Figure 3: Experimental setup](image)

A He-Ne laser is used to initially align the optics. The laser beam (shown in red) travels the same path as the down-converted photons (yellowish) do after the crystal. Alignment involves adjusting flip mirror M1 and M2 until a maximum count is detected in detectors D1 and D2. Then, the pump laser is set up such that its beam goes through the same point on the crystal as the alignment laser. The polarizer (green) ensures that horizontally polarised pump light enters the BBO crystal and propagates inside the crystals as an extraordinary wave because its polarisation is parallel to the optical axis of the BBO crystal (Chapter 2.4). Type I phase matching generates vertically polarised SPDC photon pairs. Before the pairs’ detection, an analyser controls their polarisation. Bandpass filters (810 nm, 10 nm bandwidth) on the photodetectors reduce noise by enabling only the SPDC photon transmission. An oscilloscope and FPGA measure the number of photons and coincidences arriving to the detector, indicating how well adjusted the setup is.
4.2 Secondary optical layout

The scheme of the subsidiary optical setup intended for single-photon interference is shown in Fig. 5. Although we managed to construct the scheme, time constraints hindered us from optimally aligning the setup. We have therefore not acquired data from this setup. That being said, the following is a description of the construction process. Besides the components named above, this setup also included an Mach-Zehnder interferometer composed of two beam splitters and two mirrors. One of the mirrors was made movable by attaching a piezoelement to it. The objective of this setup was to send one of the photons in the entangled pair to the Mach-Zender interferometer and test single-photon interference. The idea is that when we detect coincidences from both detectors, while taking into account the longer path the photons travel to detector 1, we know that single photons are going through the interferometer. Then, by varying the voltage applied to the piezoelement attached to the mirror, the length of path 2 (red path) in figure 5 varies. When both path 1 and path 2 are equivalent, then the single photon constructively interferes with itself and is detected in detector 1. When the lengths of the paths are different by half of the wavelength, the photon destructively interferes with itself and no detection is made in detector 1. The experimental setup is seen in figure 6 and the ground is prepared for future work.
4.3 Hardware and software

Below, the function and characteristics of hardware and software used in experiments are discussed.

4.3.1 Pump laser

Initially, a 50 mW GaN diode laser was used to optically pump the nonlinear crystal. Unfortunately, the laser exhibited limited power stability that over time led to a drastic decrease in coincidences of the photo-counts. We also saw large fluctuations in the laser output power that made the results obtained less reliable. Therefore, we replaced this model with another laser having a stabilised power supply. Power Technology’s GPD laser with a stabilised power supply was used. Its specifications are as follow [19]:

![Optical scheme for the single photon interference.](image1)

*Figure 5:* Optical scheme for the single photon interference. Description in the text above.

![Top view of the optical layout for the single photon interference.](image2)

*Figure 6:* Top view of the optical layout for the single photon interference.
<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>405 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output power (mW)</td>
<td>1 mW-50 mW</td>
</tr>
<tr>
<td>Power stability, @25 °C</td>
<td>&lt; ±0.2%</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>&lt;1 mrad</td>
</tr>
<tr>
<td>Beam size</td>
<td>1.17 x2.28 mm</td>
</tr>
<tr>
<td>Beam polarisation</td>
<td>random</td>
</tr>
</tbody>
</table>

**Table 2:** Properties of the laser used for pumping.

A spectral bandwidth of this laser was unspecified by the supplier. However, this parameter could severely affect spectral characteristics of the SPDC. More specifically, a broad laser bandwidth leads to low coherence time of the SPDC that can suppress the single photon interference. Effect of the spectral decoherence at the SPDC can be understood intuitively as the slippage in time between the pump and down-converted photons due to the group velocity delay (GVD) in the nonlinear crystal. If such a slippage length gets comparable to or greater than the coherence length $l_c$ of the pump laser, entanglement quality will be reduced or even completely disappeared [20].

The GVD in the BBO crystal was evaluated with the help of SNLO software [21]. SNLO software is a tool used to analyze and simulate nonlinear optical phenomena, enabling the calculations of the GVD characteristics in the BBO crystal. Specifically for type I phase matching, the group velocities for the pump and down-converted photons are equal to $v_{gp} = c/1.740$ and $v_{gc} = c/1.684$, respectively, where $c$ is the light velocity in vacuum. Corresponding GVD slippage length

$$l_{GV D} = l_{cr} c \left( \frac{1}{v_{gp}} - \frac{1}{v_{gc}} \right),$$

equals to 0.168 mm, where $l_{cr} = 3$ mm is the length of the BBO crystal. The coherence length of the pump laser is inversely proportional to its spectral bandwidth $\Delta \lambda$ and reads as [22–24] :

$$l_c = \frac{\lambda^2}{\Delta \lambda}.$$  \hspace{1cm} (66)

This determines a condition on the pump laser bandwidth $\Delta \lambda$. Equation [?] below results from solving Equation [?] with the coherence length, $l_c$ set at the minimum value equivalent to the slippage length.

$$\Delta \lambda < \frac{\lambda^2}{l_{GV D}}$$

which must be less than 1 nm. The requirement on $\Delta \lambda$ means the laser’s coherence time will exceed the crystal-induced slippage length from group velocity delay. This ensures that the SPDC efficiency is maintained. To ensure the laser used has a sufficiently small bandwidth, the coherence length was measured using a Michelson interferometer, how this was built is described in detail in chapter 6 of reference [25]. The laser beam was sent through a beam splitter, whereby each of the split beams met a mirror. One of the mirrors was placed on a translational stage, making it movable. The other mirror was stationary.

Fig. 7 demonstrates the experimentally observed interference pattern with well resolved fringes. The fringe pattern almost disappeared when we translated the movable mirror by a distance longer than $l_{tr} = 0.5$ cm. Thus, the coherence length was estimated to be $l_c = 2 \sqrt{2} l_{tr} = 1.4$ cm. This corresponds to a spectral bandwidth of 0.012 nm, which is well within the necessary limit.
4.3.2 Nonlinear crystal

The photon pairs are generated in a 3 mm long BBO crystal. The crystal was produced by Newlight Photonics, it was 5 x 5 mm in the cross section, and specifically cut for the type-I degenerate SPDC with the 405 nm pump wavelength. Thus, the photons generated in pairs through SPDC have the same polarization, which was perpendicular to the polarization direction of the pump photon. The crystal cut angle was specified by the manufacturer to be $\theta_c = 29.2^\circ$ which is the angle between the crystal optical axis and normal to the crystal. This value was differed from the exact phase matching angle $\theta_m = 29.42^\circ$ for the degenerate SPDC we evaluated earlier in section 2.4. The implications of a deviation of $0.22^\circ$ is a less than 1% change in the refractive index which can be estimated using Eq. 31, meaning that the SPDC photons exit angle from the crystal would also deviate by less than 1%. Characteristics of the crystal are listed in Table 3:

<table>
<thead>
<tr>
<th>Crystal type</th>
<th>BBO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>3 mm</td>
</tr>
<tr>
<td>Cross section</td>
<td>$5 \times 5 \ \text{mm}^2$</td>
</tr>
<tr>
<td>Cut angle, $\theta$</td>
<td>$29.2^\circ$</td>
</tr>
<tr>
<td>Acceptance bandwidth</td>
<td>3.9 nm</td>
</tr>
<tr>
<td>Acceptance angle</td>
<td>2.4 mrad</td>
</tr>
</tbody>
</table>

Table 3: Characteristics of the nonlinear crystal used for the SPDC.

From the table we see that the crystal acceptance bandwidth is more than two orders of the magnitude greater the laser bandwidth. Thus, the SPDC photon rate should be fairly insensitive to possible variations in the pump laser spectrum. Note that if the cut angle $\theta_c$ stated by the manufacturer is exactly equal to $\theta_m$, then the plane of the crystal must be set orthogonal to the laser beam propagation direction. Because this was not our case, it was necessary to perform a precise alignment of the crystal plane orientation by turning the crystal about the $x$ and $y$ axes (see Fig. 8). Next, the crystal should be properly oriented by turning about its axis of rotation $z$. This is necessary to set the crystal optical axis related to the plane of the light polarisation.

Being properly aligned, the crystal must produce pairs of the SPDC photons which are degenerate in energy. Such photons exit the crystal at an angle close to $\pm 3^\circ$ and form a cone of light. According to the conservation of momentum, the entangled photons are detected along a line that intersects the cone, which is explained in detail in section 2.4.
4.3.3 Detectors

As the down-converted photons have a wavelength of approximately 810 nm, a Single Photon Detector Module (SPDM) that has a high efficiency in that range was procured. The model of the detectors used are the Single Photon Counting Module (COUNT) from LaserComponents [26]. The SPDM operates on silicon avalanche photodiodes. It detects photons of 810 nm with an efficiency of 60%. Other important characteristics to consider are the timing jitter and buildup time. The SPDM’s timing jitter is the result of the randomness of the multiplication process in an avalanche photodiode [14]. If the timing jitter were zero, there would be no fluctuations between the moment when a photon arrives at a detector and when an output signal is generated. The buildup time refers to the duration necessary for the detector to reach a steady-state condition [14]. The detectors are coupled into multimode optical fibers (ThorLabs M31L01). The fiber cables have an effective refractive index of 1.482 at 850 nm. To guide the photons effectively into the fiber cables, they were attached to a micro-assembly as seen in Fig. 24. When the micro-lens assembly was connected correctly, one could clearly see laser light passing through the fiber cables as shown in Fig. 9. Regarding the efficiency of the SPDM, it is the absolute efficiency. The 60% efficiency means that 60% of the photons impinging on the crystal result in an electrical signal. This is further corroborated by Table 4, which explicitly states ‘photon detection efficiency’.

Detailed specifications can be found in the manual [26] and a summary of important properties are listed below in Table 4.
<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon detection efficiency at 810 nm</td>
<td>typ. 60% (min 50% )</td>
</tr>
<tr>
<td>Dark count rate variation</td>
<td>5%</td>
</tr>
<tr>
<td>Active area diameter</td>
<td>100 µm</td>
</tr>
<tr>
<td>Buildup time</td>
<td>1000 ps</td>
</tr>
<tr>
<td>Dead time</td>
<td>45 ns</td>
</tr>
<tr>
<td>Pulse length (in 50 ohm)</td>
<td>typ 15 ns (max 17 ns)</td>
</tr>
<tr>
<td>Output amplitude</td>
<td>3 V</td>
</tr>
<tr>
<td>Delay between photon impact and pulse</td>
<td>30 ns</td>
</tr>
<tr>
<td>Maximum count rate</td>
<td>20 MCounts/s</td>
</tr>
<tr>
<td>RMS timing jitter</td>
<td>50 ps</td>
</tr>
</tbody>
</table>

*Table 4: Specifications of LaserComponents SPDM.*

4.3.4 Oscilloscope

For the majority of the experiment, signals were fed out to an oscilloscope through a BNC cable. These coincidences were evaluated using Matlab, the algorithm is found in the appendix. The oscilloscope was capable of a maximum real-time sampling rate of 10 gigasamples per second (GSa/s), allowing measurements to be sampled at an interval of 0.1 ns. However, we predominantly utilised a sampling rate of 2.5 GSa/s. This choice was made to capture the complete ascent of the signal waveform, distinguishing it from potential noise interference. Moreover, this sampling strategy made the Matlab processing easier. By using the signal’s inherent exponential decay—apparent in the oscilloscope’s signal, seen in Figure 16—we could confidently establish a higher threshold in our analysis, effectively discerning between true signal and noise. In other words at the full 10GSa/s rate, we observed the signal in a narrow window which did not encompass the characteristic exponential decay of the signal.
4.3.5  Field Programmable Gate Array (FPGA)

At high sampling rates of GSa/s the oscilloscope cannot take continuous measurements that span several seconds because of memory limitations. The analysis process is also tedious and time consuming. We therefore used an FPGA to automate the process. The Altera DE2 model was used, fig. 11. Further specifications can be found on terasIC website in the reference [27]. In order to feed the signals to the FPGA, we constructed a small box that can input up to three signals via a BNC cable from three separate detectors, seen in Fig. 10. The signals were sent through a total of 50 ohm resistance, which were soldered onto BNC contacts. The signals inputted into the DE2 board had an amplitude of 3 volts. Three volts surpassed the board’s minimum voltage of 1.7 volts needed to be registered as high-level logic and consequently as a signal. The detectors’ signal also exceeds the threshold of 0.8 volts for low level logic, meaning that a signal will not be interpreted as no signal by the board [28]. The BNC contacts were then soldered to a 40 ribbon cable. Two wires were used for each BNC contact, one to ground and the other to a GPIO_0 pin on the FPGA via the ribbon cable. It was connected as in table 5. The cable was then attached to the FPGAs 40 pin expansion header. The program uploaded on to the FPGA was obtained from Mark Beck at Reed College. This program is meant to check if the incoming signals arrive within the coincidence time resolution. Measurements are made and stored using a LabVIEW interface. The LabVIEW code was also obtained from Mark at Reed College. Fig. 12 is a screenshot from the interface. The thermometer icon in LabView labelled B depicts the number of signal counts obtained from one detector and correspondingly the counts from the second detector are labelled B’. The counts of the coincidences are labelled BB’. The switches on the FPGA determined which detectors were being analysed for coincidences. For example, if neither SW0 or SW01 were on, then there would be zero coincidences. If only SW1 were on, then the number of coincidences would equal to the number of signals in detector B. The same is true if only SW01 were on. The truth table 6 illustrates the logic.

![Figure 10: Box built to input signals to the FPGA.](image)
Figure 11: Altera DE2 board.

Figure 12: LabView interface.

<table>
<thead>
<tr>
<th>Detector</th>
<th>GPIO_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>13</td>
</tr>
<tr>
<td>B'</td>
<td>15</td>
</tr>
<tr>
<td>A</td>
<td>19</td>
</tr>
</tbody>
</table>

Table 5: Connections between FPGA and the detectors through the GPIO_0.
<table>
<thead>
<tr>
<th>SW0</th>
<th>SW1</th>
<th>Output Coincidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>None</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>B</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>B'</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>BB'</td>
</tr>
</tbody>
</table>

Table 6: Truth table showing which switches count which coincidences.

The table is also true for switches SW4-SW5, SW8-SW9 and switches SW12-SW13 but only SW0 and SW1 are shown in the truth table above.

4.3.6 Interferometer

For the second experimental setup, where single photon interference was tested, a Mach Zender type interferometer was setup. Since the down-converted photons have a wider spatial spread than the incoming pump beam, it is important that the lengths be equal to each other within micrometer precision. The interferometer consisted of two beam splitters, a stationary mirror and an additional mirror mounted on an piezoelectric element that could displace the mirror up to $4.6 \mu m$. The displacement occurs at a 45 degree angle from the breadboard direction. This means that the total displacement of path that has the movable mirror is up to $\sqrt{2} \times 4.6 \mu m = 6.5 \mu m$. This corresponds to eight wavelengths of the down-converted photons. The piezoelement is controlled using a tunable voltage source. Its displacement is dependent on the voltage sent through the piezoelement.

![Piezo element glued to the mirror.](image)

Figure 13: Piezo element glued to the mirror.

4.4 Alignment process of the main optical setup

The alignment process and the experimental setup described further down in section 5 closely follow a detailed manual for how to generate correlated photons found in reference [24] in addition to a summarised version by the same author in reference [23].

The main setup is illustrated in Fig. 3. First, we aligned a beam of the GaN diode laser to propagate parallel along a line of the breadboard holes. This was done by placing two iris diaphragms onto opposite sides of the breadboard. Then, the BBO crystal was installed and secured such that the diode laser beam.
passed thought the crystal centre. To determine necessary location of the photodetectors, we used a lower-
power He-Ne laser. A beam of this laser was directed to the BBO crystal by using two metallic mirrors M1 and M2. Mirror M1 was mounted onto a flip holder. Such a configuration enabled the He-Ne laser beams to intersect the diode laser beam under the angle of approximately ±3°, the values which were expected due to SPDC. When the diode laser was off, photodetectors D1 and D2 were installed and positioned to observe the highest rates of the photocounts.

Having had positioned the photodetectors, we turned the diode laser on while the He-Ne laser was turned off. The transmission axis of the polarizer for the diode laser beam was set to be horizontal. To ensure detection of the down-converted photons, which have orthogonal polarisation with respect to the pump photons polarisation, the transmission axis of the analyser before one of the photodetectors was adjusted to be vertical. To set the optical axis of the nonlinear crystal with respect to the pump beam polarisation plane, the crystal was rotated about z axis (see Fig. 8). Additional alignment was performed for the crystal plane orientation (turning the crystal about the x and y axes). The proper crystal orientation was controlled by the photocount rate. To further maximise this rate, we made fine adjustment of positions of the the photodetectors by slowly translating each of them in the horizontal plane across the propagation directions of the SPDC photons (see Fig. 3) until a maximum count rate was obtained. That being said the count rate data as a function of angular position relative to the pump beam was not stored, so the error in the angle is made based on an estimation which follows,

The properly positioned photodetectors composed an angle of about \( \phi = \pm 3.2^\circ \) with respect to the pump beam propagation direction. This angle was somewhat greater than the theoretically expected value of ±3.0°. The difference can be explained by deviation of the crystal cut angle \( \theta_c = 29.2^\circ \) from the exact phase matching angle \( \theta_m = 29.42^\circ \) as well as an experimental error.

To estimate such an error, one needs to determine an accuracy of the alignment. Generally speaking, the alignment accuracy is a function of several variables. More specifically, it depends on the positioning inaccuracies for 1) the iris diaphragms \( \delta x_{ir}, \delta y_{ir} \) which were used to align the diode laser beam, 2) the photodetectors \( \delta x_{det}, \delta y_{det}, \delta z_{det} \), and 3) the nonlinear crystal \( \delta z_{cr} \). Here we consider the pump beam axis as z direction, x and y directions are both orthogonal to z; x is parallel to the plane of the optical table, y orthogonal to it (see Fig. 3). We estimated the inaccuracy along the y direction to be ±0.5 mm which is based on the fact the tool used to measure the position of the detector in the y axis was a standard ruler which has half the graduation value of 0.5 mm.

The x and z inaccuracies were estimated to be much larger, ±3 mm. This is because to find the x and z distances, first, we needed to make vertical projections of the points of interest onto the optical breadboard and, then, to measure distances between the projections. More specifically, we measured distances between the projected positions of the photodetectors and pump beam axis (x coordinates) as well as a distance between the BBO crystal and intersection of an imaginary line connecting two photodetectors with the pump beam axis (z coordinate). We have found that major contribution to the x and z inaccuracies was due to deviation of the projecting lines from exact vertical ones. Such estimates are based on the variation observed in a series of 3-5 measurements for each the projection.

The optical components are aligned within a horizontal plane with a vertical precision of 0.5 mm. The deviations, \( \delta y \) are significantly smaller than \( \delta x \) and \( \delta z \), as such, we can establish a relationship between experimental errors and the angular inaccuracies, \( \delta \phi \), which reads:

\[
\delta \phi = \left| \frac{d}{dx} \arcsin \left( \frac{x}{z} \right) \right| \delta x + \left| \frac{d}{dz} \arcsin \left( \frac{x}{z} \right) \right| \delta z, \tag{68}
\]

where z is the distance between the nonlinear crystal and photo detector and x is the half distance between the detectors. For relatively low \( \phi \) values, that is our case, we can rewrite the above equation to be:

\[
\delta \phi = \left| \frac{d}{dx} \left( \frac{x}{z} \right) \right| \delta x + \left| \frac{d}{dz} \left( \frac{x}{z} \right) \right| \delta z = \frac{1}{z} \delta x + \frac{x}{z^2} \delta z. \tag{69}
\]

Next, we consider \( \delta x_{ir}, \delta x_{det} \) and \( \delta z_{det}, \delta z_{cr} \) as the statistically independent variables. In this case, we can represent \( \delta x \) and \( \delta z \) as follows:

\[
\delta x = \sqrt{\delta x_{ir}^2 + \delta x_{det}^2}, \tag{70}
\]

\[
\delta z = \sqrt{\delta z_{det}^2 + \delta z_{cr}^2}. \tag{71}
\]
\[ \delta z = \sqrt{\delta z_{cr}^2 + \delta z_{det}^2}. \]

By using Eqs. (69 - 71) and taking \( x \) and \( z \) equal to the experimentally measured values 37 and 650 mm, respectively, we obtain \( \delta \phi = \pm 0.39^\circ \).

### 4.5 Alignment of secondary setup

For the secondary setup with the interferometer found in Fig. 6, the same alignment procedure was applied. In addition to that, the interferometer must be aligned correctly. It has a 15 cm long side. As mentioned earlier, the interferometer consists of two beam splitters and two mirrors as illustrated in Fig. 14. The two mirrors are placed on translational stages and also have mounts with which fine adjustments can be made.

The down-converted photons exit the crystal at an angle of \( 3.2\pm0.4^\circ \). At the same time, in a Mach–Zehnder interferometer the optical paths must either be parallel or orthogonal to the direction of incoming photons to make a high-precision alignment possible. This implies that the interferometer would be rotated relative to the holes of the optical table making the installation and alignment challenging. Hence, we decided to reconfigure the SPDC setup such that one of the optical paths of down-converted photons is parallel to the holes of the table. To this end, a mirror is placed in the path of the down-converted photons to redirect the photons such that they travel along the breadboard holes. This is also illustrated in the optical scheme in Fig. 5.

In order to ensure the down-converted photons were travelling in a straight line along the holes, two irises with the smallest possible opening were placed along the breadboard holes. The mirror was then adjusted until the He-Ne laser went through both of them. Then, the first beam splitter was placed. The same process of making sure that the beams travelled along the breadboard holes in both directions was done. Thereafter, both the mirrors are placed at a 15 cm distance from the beam splitter in both directions. Again, we make sure that the laser beam is parallel to the holes of the optical table by adjusting the mirrors using their adjustment degrees of freedom. At this point, the laser beam from both mirrors should intersect in air. Finally, the last beam splitter is placed such that both the beams reflected from the mirrors meet at the same point on the surface of the beam splitter. Then, the beam splitter is tilted until the split beam in both directions travel along the breadboard holes. After the second beam splitter an interference pattern with clear fringes was visible seen in Fig. 15. If the interferometer were well-aligned, the interference pattern would be in the form of concentric circles. The interference pattern obtained is, therefore, indicative of the split beams arriving at the second beamsplitter at different angles.
5 Experiments with the main optical setup

With the main optical setup an experiment was conducted to test that the generated photons are entangled.

5.1 Detection of single photons

The experiment started with testing single-photon detectors in two regimes: (1) The regime of minimum possible illumination of the detector, which was achieved by switching off all light sources in the laboratory except for essential electronics and enclosing the optical setup by a metallic shield. The typical number of pulses observed on the oscilloscope is 1 or 2 for an effective measurement interval of 10 ms (the actual time interval is 1 ms but 10 waveforms are superimposed on top of each other). This is in agreement with the
specification of the dark photon count rate of less than 100 pulses per second stated by the manufacturer. (2) The regime of a high count rate limited by the dead time of the detector. The detector was operated in the presence of daylight laboratory lamps and other operating light sources, and a counting rate of around ten million pulses was observed, being limited by the detector dead time. Thus, the detector operates according to the specifications.

For the study of the SPDC process, all light sources except the pump laser and electronics were switched off and photons of the pump laser at 405 nm were filtered out. Figure 16 shows typical waveforms observed during measurements. Two detectors are connected to two channels of the oscilloscope: channels 2 and 3 with channel signals shown in pink and blue, respectively. The trigger is set on the channel 2: if a detected signal exceed some threshold (in this case around 1V), the signal waveform is measured and depicted in the center. Signals in another channel are also measured simultaneously. The signals from channel 2 and 3 that overlap within the detectors’ time jitter are entangled, given that they also fulfil the other requirements of entanglement. A voltage offset is set between the two channels to make their overlap in time more visually pronounced. The signal rate in the channel 2 is $5 \times 10^6$/s. In the figure the pink signal was triggered 3000 times. Channel 3, the blue channel shows approximately 300 signals, so it has a signal rate of $5 \times 10^5$/s. Approximately 200 signals of the 300 from the blue channel were analysed as coincidences. The data we used in the analysis of the polarisation angle and the arrival time of the pulses did not have a trigger on either of the channels, which gives a higher count rate for the channel that is not triggered and also a more accurate statistical analysis.

For further analysis of the arrival time of the pulses from the single-photon detectors, raw data (without
any internal post-processing in the oscilloscope) were downloaded and analysed with Matlab. An example of pulses is shown in Fig. 17. To compare the arrival time of different pulses, we introduce the notion of the pulse centroid, which is simply the effective centre of mass:

$$t_c = \frac{\sum t_i V_i}{\sum V_i}.$$  \hspace{1cm} (72)

Here, $V_i$ is the voltage at the time moment $t_i$ and the summation is over the points located around the peak of the pulse, which is identified before the calculation of $t_c$ starts. Note that Eq. (72) is nothing but the definition of the mean value of $t$ with $V$ having the meaning of a probability distribution function. Later, we will use this analogy.

The use of $t_c$ greatly increases the accuracy. Specifically, if the time step $\Delta t$ is precisely known, then $t_c$ is known with the accuracy $\Delta t (N/S)$, where $(N/S)$ is the noise-to-signal ratio \cite{29}. We will come back to $(N/S)$ but first let us take a look at the accuracy of the time step $\Delta t$.

In the measurements presented below, the step is large – 0.4 ns, in order to make the acquisition time as long as possible. The timebase accuracy of the oscilloscope is better than 25 parts per million (according to the technical protocol supplied during the delivery). The sampling clock of the oscilloscope operates at 10 GHz. The time interval of 0.4 ns contains 4 clock cycles and thus the uncertainty of $\Delta t$ is $10^{-4}$ cycles, which is 10 fs. Hence, the uncertainty in $t_i$ in Eq. (72) can be safely disregarded.

The second contribution to the uncertainty of $t_c$ comes from the uncertainty in the measured voltage $V_i$. The oscilloscope was set to the maximum bandwidth so that the rms noise floor is 23 mV. This is roughly 0.1% of the peak value of the pulse. Thus, the position of the pulse centroid would be known with an accuracy of around 0.4 ps if the number of measurement points were very large. In our case, the pulse contains around 30 measurement points within the interval corresponding to the FWHM of the pulse. Hence, the statistical analysis and the Central Limit Theorem \cite{30} are strictly speaking not well justified. Therefore, below we perform a “computer experiment” to show that the accuracy of the determination of $t_c$ is in the ps range.

For the “computer experiment”, we start with a Gaussian pulse, apply discretisation and add noise to the data points. Fig. 18 shows a Gaussian pulse with a mean value, $\mu$, equal to $\pi$ and an rms width, $\sigma$, of unity. The number of data points is 30. They are uniformly distributed over the interval corresponding to the FWHM of the pulse. Hence, the statistical analysis and the Central Limit Theorem \cite{30} are strictly speaking not well justified. Therefore, below we perform a “computer experiment” to show that the accuracy of the determination of $t_c$ is in the ps range.

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We define the error as

$$Er = (t_c - \mu)/\mu,$$ \hspace{1cm} (73)

which is a relative deviation of the centroid of the pulse with respect the true mean value of the Gaussian
distribution. The error tends to zero as the artificial noise vanishes. Figure 19 shows the error for different generated realisations of Gaussian pulses with added noise. The peak deviation is around 1%. According to the Central Limit Theorem, fluctuations of the error are connected to the level of noise as

$$\sqrt{E_{r^2}} \approx \frac{\sigma_{\text{noise}}}{\sqrt{N}} \mu,$$  \hspace{1cm} (74)

where $\sigma_{\text{noise}}$ is the rms value of the relative fluctuation of $V$ and $N$ is the number of data points. The bar denotes statistical averaging over different realisations of the Gaussian pulse. Numerical values for statistical parameters are presented in Table 7. The 3rd and 4th columns express the same thing just presented differently – ratio of the standard deviation to the mean, so the numerical values are very close to each other as it should be. Despite a relatively low number of samples – $N = 30$, Eq. (74) is quite accurate.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$E_{r}$</th>
<th>$\sqrt{E_{r^2}}$</th>
<th>$\sigma_{c_e}/\bar{t}_c$</th>
<th>$\sigma_{\text{noise}}/\sqrt{N}\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$1.8 \times 10^{-4}$</td>
<td>0.0035</td>
<td>0.0035</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Table 7: Comparison of statistical parameters. The last digit after the comma is not statistically valid but we choose to write it to highlight the difference between the results.

Using Eq. (74), the uncertainty in the determination of the centroid of the pulses from the single-photon detectors is around 0.5 ps. This uncertainty includes only the fluctuations due to the oscilloscope. There must
also be fluctuations due to the single-photon detector itself but again thanks to the multipoint measurement, it is reasonable to assume that the accuracy will be preserved on the picosecond level. To sum up, the position of the pulse centroid can be determined with a great accuracy. To understand this better, consider a simple example that illustrates the difference between the measurement step and measurement accuracy. Suppose we measure a 1-meter long object with a step of 0.5 meters using a ruler with 1 mm increments. Then, each point is measured with an accuracy of ±0.5 mm. Despite the large step size, the precise increments allow for high measurement accuracy at each point. Similarly, with a large number of measurement points, the centroid of a long object can be obtained with high accuracy. In the table 7 N, the number of measurement points per pulse is taken as 30 to illustrate that despite a relatively low number of samples it is possible to obtain high measurement accuracy. In other words, \( N = 30 \) is sufficient for a detailed reconstruction of the pulse shape, leading to the precise determination of the centroid position despite the longer sampling time.

5.2 Delay in the arrival time of photons

Using the procedure for the determination of the centroid of pulses, a long time series was analysed for two detectors. Each file from the oscilloscope is saved in the csv format with two columns. Each column consists of the voltage level at each detector per given unit time. In most cases, voltage measurements were made every 0.4 ns. The csv files were analysed with a Matlab script that in its entirety is found in the appendix. The script first identifies peaks that exceed the trigger level of 3 volts for each detector. An example of a coincidence is seen in Fig. 16 on the interface screen of the oscilloscope. The pink signal is from one detector and blue from the other. The signals are offset in the y axis to make viewing the signal easier.

The difference in arrival time of the centroids of signals for each detector is plotted as a histogram seen in figure 20. The histogram is normalised such that the sum of all counts gives unity. Such a normalised histogram is known as the probability distribution function.

This pdf is an important indicator of whether we can consider the signals obtained to be coincidences or not. Given that the cables from the detector to the oscilloscope or FPGA are physically identical at first glance, the difference in arrival time should form a Gaussian curve centred at zero ns. This is not the case as the pdf is centred around 1.24 ns. However, the important factor is that the difference in the arrival time between two entangled photons of the same pair must be constant from shot to shot, i.e. constant from pair to pair. And this is the case as the rms width of the probability distribution function in Fig 20 is 0.22 ns. In other words, the spread in the arrival time of two photons of the same pair is comparable or even less than the spontaneous decay time in atoms [31]. This is a strong indication that two photons originate from the same process. At the same time, the systematic deference in the arrival time is strange and in the next section, we describe the measurement of the electrical length of the cables.
Figure 20: The probability distribution function (pdf) of the time difference between the centroids of the signals from two channels of the oscilloscope. A normal distribution with the mean at $\mu = 1.24$ ns and the standard deviation $\sigma = 0.22$ ns is fitted to the pdf.

5.3 Analysis of the signal delay

The above analysis of the arrival time between two pulses measured by the single-photon detectors showed a systematic difference of 1.24 ns. In an attempt to understand this difference, the electrical length of the cables was measured. The measurement setup comprises a vector network analyser (VNA) operating as a time-domain reflectometer, Fig. 21. The VNA is an instrument that primarily operates in the frequency domain with capability of measuring the reflection and transmission characteristics of a device under study. The VNA used in this study measures the amplitude and phase of the reflected signal at different frequencies and then reconstructs the response to a short impulse by means of inverse Fourier transform. Further details can be found in [32]. The result of the reconstruction is the power of the reflected signal as a function of distance. The speed of electrical signals is taken as the speed of light in vacuum during the calculation performed by the software in the VNA. This allows for a simple conversion between the distance of interest and time.

Figure 22 shows an example of measurements. The three marked peaks correspond to different connections along the cables. The peaks can be unambiguously set into the correspondence to connectors along the cable by disassembling the long cable and connecting the end to a matched RF load. The peak marked as (C) corresponds to the open end of the cable. By zooming into this region, Fig. 23, a difference in electrical length of around 75 mm between two cables can be seen. The measurement accuracy is 22 mm. Hence, the difference in propagation time through the two cables is 240 ps with an uncertainty of 73 ps.

We must conclude that the difference in electrical length of the cables cannot explain the difference in the arrival time of the pulses from single-photon detectors. Even if the effect of the cables is taken into account, the remaining difference between the pulses is around 1 ns. Since the difference is a systematic effect, there is no reasons to assume this difference to be a feature of the SPDC process, which is essentially random as it originates from quantum fluctuations.

A possible explanation of this discrepancy is a difference in the response time of two photon detection channels. Such a difference can be due to variation in the response function of the photodetectors. But it also can arise because of the small deviation in the input resistivity of the oscilloscope or FPGA channels. Further studies are needed but no equipment is available to make careful tests of the temporal responses.
Figure 21: Photo of the measurement setup for studying the electrical length of the cables used to transmit signals from single-photon detectors to the oscilloscope. (A): the N-type to BNC-type adapter, (B): the BNC tree, (C): the BNC-to-lemo adapter that goes into a single-photon detector.

Figure 22: Screenshot of the power of the reflected signal (dB) vs distance (m). The peaks marked by (A), (B) and (C) correspond to the reflection from the N-type to BNC-type connector (A), see Fig. 21, reflection from the BNC tree (B) and reflection from the open end of the connector that goes into a single-photon detector (C).
5.4 Polarisation of the down-converted photons

In order to check the polarisation angle of the down converted photons with respect to the pump photons, a polarizer was placed before the crystal as seen in figure 3, while the analyser was simultaneously placed in front of one of the detectors seen in the same figure. The results are then presented as the number of coincidences between the photon arrival at each detector as a function of the difference between the polarisation angle of the polarizer with respect to the analyser. The polarization angle of the analyser in front of the detector was changed in increments of ten degrees, while the polarizer was kept constant. Measurements were taken using the FPGA and oscilloscope. The duration of each measurement using the FPGA was 10 seconds and each such measurement was repeated approximately 10 times.

The data should follow Malus law shifted by \(90^\circ\), since the pump photons should be orthogonally polarized with respect to the down-converted photons. This means that the data should follow a cosine squared distribution, \(\cos^2(\Delta \theta - 90^\circ)\). To this end, the fit of the normalised experimental data is calculated using a nonlinear least square curve fitting function in matlab, where the four parameters being fitted are the amplitude, phase shift, period and vertical shift in the y axis.

The fit shows deviations in the phase shift, period and amplitude from the cosine square distribution given above, possible reasons for this and how it affects the validity of the results are discussed below.

Fig. (25) shows the number of coincidences as a function of the relative angle between the polarizer and analyser as well as the fit of the data. The experimental data is fitted by the equation

\[
0.93 \cos^2(0.97(\Delta \theta - 82) \pi /180)
\]

The deviations in the intensity of the coincidences in Fig. 25 from the theoretical distribution \(\cos^2(\Delta \theta - \pi /2)\) can be explained as follows: The fluctuations of the number of coincidences originate from two effects: (1) the Poissonian nature of SPDC [1] and (2) intensity fluctuations of the pump laser. Consider these two effects in detail. The variance of the Poissonian distribution is simply equal to the mean value, \(\bar{n}\), so that the signal-to-noise ratio is \(1/\sqrt{\bar{n}}\), [31], decreases \(\bar{n}\), i.e. decreases as the measurement interval increases. On the other hand, a large observation time leads to intensity fluctuation of the pump laser. The manufacturer does not provide the information about the coherence time or how the laser is built. We only know that it is a solid-state laser and the coherence time is larger than 100 ps. This large coherence time implies that the laser must contain a microcavity and typically such cavities have a bandwidth of the order of 1 MHz. This implies a coherence time of 1 \(\mu\)s. We do not posses equipment to measure such a narrow signal bandwidth but the measurements of the second-order correlation function indirectly support the assumption of the 1 \(\mu\)s coherence time. The 10 s long measurements of the coincidences greatly exceed the coherence time of the pump laser, and, therefore, the data points in Fig. 25 are affected by the intensity fluctuations of the pump laser, hence the deviations in amplitude.

The phase shift in the fit deviates from the theoretical distribution by \(8^\circ\). The most probable reason
behind this is that, before conducting the experiments, we noticed that the polariser and analyser were not parallel with respect to the hatch markings on the polarisers. We, therefore, rotated the polarisation filters by hand and deemed them to be parallel to one another by eye when they seemed to pass through as much light as possible. This procedure should have been done with the detectors or with a CCD camera.

The deviations in the period are by 3%, which corresponds to an accumulated error of 10.8°. 36 measurements were done in total which corresponds to an error of 0.3° per measurement. This is less than the resolution of the analyser used in the experiments, which is 0.5°. The error in the period can therefore be attributed to a systemic error when changing the analyser’s polarisation angle between measurements.

The results point to the fact that the pump photons are orthogonally polarized with respect to the down-converted photons; since the deviations from the theoretical distribution are small and can be explained by systemic errors, the statistical nature of SPDC and the fact that pump lasers fluctuate in intensity.
The experimental data are the number of photon arrival coincidences between detector 1 and detector 2 in the main optical setup as a function of the difference between the polarization angles of the polarizer in front of the crystal and the analyser in front of detector 1, seen in figure 3. Each point in the figure depicted as an experimental data point was measured ten times. The vertical error bars illustrate the range in number of coincidences from these ten measurements. The resolution of the analyser is $0.5^\circ$.

A nonlinear least squares fit showing that experimental data follows the equation $0.93 \cos^2(0.97(\Delta \theta - 82) * \pi/180)$.

6 Discussion

The entangled photon pairs in this experimental setup are produced using a nonlinear crystal of beta barium borate through a process of spontaneous parametric down-conversion (SPDC). The theory of SPDC predicts that for the generation of entangled photons, the following four requirements must be fulfilled. (i) The sum of the momenta of the photons in each entangled pair must fulfill the requirement of momentum conservation and be emitted along a circle in the cone of the down-converted light. (ii) They also need to fulfill the requirement of energy conservation: sum energy of the outgoing photons is equal to the incoming photon energy. (iii) The down-converted photons should have the orthogonal polarization with respect to the pump photons as the BBO crystal is cut for the type I phase matching. (iv) Finally, the entangled photons should be generated simultaneously, i.e. within the GVD slippage length $L_{GVD}$ (see Chapter 4.3.1) and within the characteristic time interval of the process that generates them. In the course of the project, the above requirements were satisfied.

Both of the detectors were placed at the angle that corresponds to the SPDC phase matching angle, ensuring that the momentum requirement was fulfilled. The energy conservation of the detected pair was ensured by using the band-pass filters before the detectors. The transmission band of the filters was centred at the light frequency which was half the pump laser frequency. The polarizer-analyser test proved orthogonality in polarization of one of the photons in the SPDC photon pair with respect to the pump photons polarization.

Additionally, to test the simultaneous arrival of the SPDC photons pairs generated, the time difference between the centroids of the coincidence signals as seen in figure 16 are plotted as a histogram in figure 20. The histogram follows a Gaussian distribution which is centred at around 1.25 ns, meaning that there is a systematic difference of 1.25 ns in arrival time between the photons in a photon pair. After testing the cables using a vector network analyser to check if the difference can be attributed to the cables not being electrically identical, we found that approximately 1 ns difference could neither be attributed to electrical or physical differences in the cables. The 1 ns discrepancy cannot be attributed to the difference in path travelled between the photons in each pair either, since that would correspond to an approximately 30 cm difference in path.
Although there is a systematic difference of 1.25 ns in the time difference of the centroids, the spread in time arrival is around 0.22 ns. This is approximately an order of magnitude less than the time it takes for the de-excitation of atoms meaning that the coincidences that are detected are likely coming from the same SPDC process which is a requirement for them to be entangled. Additionally the count rate from a single detector, when the SPDC source is switched off, is approximately 100 signals per second, indicating that the coincidences are unlikely to be accidental, as the count rate for SPDC photons is $\sim 10^6$/s.

To test if the polarization requirement (iii) was satisfied, the number of coincidences as a function of the difference in polarisation angle between the polariser in front of the crystal and the analyser in front of one of the detectors was plotted in figure 25. This shows the polarisation of one of the down-converted photons in the generated photon pairs with respect to the polarisation of the pump beam. Since we have type I SPDC, we expect the data to follow the cosine squared distribution, $\cos^2(\Delta \theta - 90^\circ)$. The data points in Fig. 25 slightly deviate (on the per cent level) from the expected cosine squared distribution and can be attributed to the following: the deviations in amplitude can be explained through the Possionian nature of light. The deviations in the period can be attributed to the accumulated systematic errors when changing the polarisation angle of the analyser in the experiment. Finally the deviations in the phase shift can be attributed to not setting the polarisation filters in the polariser and analyser perfectly parallel with respect to each other before conducting the experiment.

7 Conclusions

The purpose of this experiment was to generate and detect the entangled photons using the nonlinear crystal via spontaneous parametric down-conversion. The entangled photon source has been successfully developed. The source comprised the continuous-wave diode laser at 405 nm operation wavelength and BBO crystal. The following results make it possible to conclude that the detected photons were indeed entangled. Firstly, the photons in the pairs were degenerate in energy. This was assured by using a narrow band-pass filters for both the detection channels. The filters transmitted only the photons at a central wavelength of 810 nm that exactly matched the degenerate photon energy. Secondly, the spread in the time difference between the arrival of the photons is around 0.22 ns, which is less than the average time it takes for atoms in a BBO crystal to de-excite and generated entangled photons. This is a strong indication that the photons in the pair were generated from the same process and are therefore entangled. The maximum coincidence rate was detected when the detectors were placed at a $3.2 \pm 0.4^\circ$ angle with respect to the propagation direction of the pump laser beam. The experimentally obtained angle is close to the theoretically expected value of 3 degrees, meaning that the momenta of photons were conserved.

We were not able to confirm that both the photons in the down-converted pair had the same polarisation which should be orthogonal to the pump photons. That being said, it is unlikely that the photons within each generated pair would differ in polarization. This is because the efficient generation of the down-converted photons necessitates that the photon pair are orthogonal in polarization with respect to the pump beam. If the polarisation of the photons in the pair were different, it would violate the phase matching condition that is essential for high generation efficiency.

The presented findings show how one can generate entangled photons through SPDC and confirm theoretical predictions. Entangled photons can be used for further experimental studies such as single-photon and double-photon interferometry.

References


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%Written by Gabriella Habtezion and further upgraded by Vitaliy Goryashko

clearvars

filename = 'RefCurve_2022-07-14_0_083920.Wfm.csv'; % data file ia a two-coloum table ...
    containing a time vector and a voltage vector
delimIn = ';';
headerlinesIn = 1;
data = importdata(filename, delimIn); % 2D data matrix
LL = length(data);

scale = 0.4; % conversion to ns

diff=[];
counts=[];

% initialization of time and voltage data
ch1=data(:,1);
ch2=data(:,2);
t1=1:LL;
t2=1:LL;

% trigger in volt

tgr1_l=3;
tgr2=10;

[pks1,locs1]=findpeaks(ch1,t1,'MinPeakHeight',tgr1_l,'MinPeakdistance',tgr2);
[pks2,locs2]=findpeaks(ch2,t2,'MinPeakHeight',tgr1_l,'MinPeakdistance',tgr2);

tol=10/max(abs([locs1(:);locs2(:)]));
[LIA, LocB] = ismembertol(locs2(1,:),locs1(1,:),tol);

% analysis of the peaks
% t_unif_locs1 = linspace(0,max(locs1),length(locs1)) ;
% plot(t_unif_locs1 - locs1)

% finding the centroids of the pulses
peakT1 = zeros(1,length(pks1));

for i = 1:length(pks1)
    dL_m = 10; % left distance from the peak
    dL_p = 100; % right distance from the peak
    L1_low = locs1(i) - dL_m; % lower limit of integration
    L1_up = locs1(i) + dL_p; % upper limit of integration
    peakT1(i) = (t1(L1_low:L1_up)*ch1(L1_low:L1_up))/sum(ch1(L1_low:L1_up));
end

peakT2 = zeros(1,length(pks2));
for i = 1:length(pks2)
    L2_low = locs2(i) - dL_m;  % lower limit of integration
    L2_up = locs2(i) + dL_p;  % upper limit of integration
    peakT2(i) = (t2(L2_low:L2_up)*ch2(L2_low:L2_up))/sum(ch2(L2_low:L2_up));
end

%% raw data, peaks and centroids
factor = 1000;
figure
plot(t1(1:LL/factor),ch1(1:LL/factor))
xlabel('Time (units)')
ylabel('Signal (V)')
hold on
plot(locs1(1:10),pks1(1:10),'o')
plot(peakT1(1:10),pks1(1:10),'*')
xlim([0 2*10^3])

% difference btw the peaks of the signals
tol=10/max(abs([locs1(:);locs2(:)]));
[LIA_c1, Loc_c1]= ismembertol(locs2(1,:),locs1(1,:),20, 'DataScale', 1);
[LIA_c2, Loc_c2]= ismembertol(locs1(1,:),locs2(1,:),20, 'DataScale', 1);
fprintf('The number of peaks in the c1 is %g 
', length(pks1) );
fprintf('The number of peaks in the c2 is %g 
', length(pks2) );
fprintf('The number of coincedences is %g 
 ', sum(LIA_c1) );
no_co1 = nonzeros(Loc_c1);
no_co2 = nonzeros(Loc_c2);
diff = zeros(length(no_co1),1);
for i = 1:length(no_co1)
    index1 = no_co1(i);
    index2 = no_co2(i);
    diff(i) = scale*(locs1(index1)-locs2(index2));
end
fprintf('Mean time difference btw centroids in [ns] %g 
 ', mean(diff) );
fprintf('rms width of centroids spread in [ns] %g 
 ', rms(diff) );
figure
h=histogram(diff,1000,'EdgeAlpha',0.0);
counts=h.Values;
xlabel('Time (ns)')
title('The time difference between the two coincidence peaks')
xlim([mean(diff)-2*rms(diff) mean(diff)+2*rms(diff)])

% difference btw the centroids of the signals
tol=10/max(abs([peakT1(:);peakT2(:)]));
[LIA_c1, Loc_c1]= ismembertol(peakT2(1,:),peakT1(1,:),20, 'DataScale', 1);
[LIA_c2, Loc_c2]= ismembertol(peakT1(1,:),peakT2(1,:),20, 'DataScale', 1);
fprintf('The number of peaks in the c1 is %g \n ', length(pks1) );
fprintf('The number of peaks in the c2 is %g \n ', length(pks2) );
fprintf('The number of coincidences is %g \n ', sum(LIA_c1) );
no_co1 = nonzeros(Loc_c1);
no_co2 = nonzeros(Loc_c2);
diff = zeros(length(no_co1),1);
for i = 1:length(no_co1)
    index1 = no_co1(i);
    index2 = no_co2(i);
    diff(i) = scale*(peakT1(index1)-peakT2(index2));
end
fprintf(filename)
fprintf('Mean time difference btw centroids in [ns] %g \n ', mean(diff) );
fprintf('rms width of centroids spread in [ns] %g \n ', rms(diff) );
figure
h=histogram(diff,1000,'EdgeAlpha',0.0);
counts=h.Values;
xlabel('Time (ns)');
title('The time difference between the two coincidence centroids')
xlim([mean(diff)-rms(diff) mean(diff)+rms(diff)])