



Energy Conversion through a Fluctuation–Dissipation Relation at Kinetic Scales in the Earth’s Magnetosheath

Federica Chiappetta¹ , Emiliya Yordanova² , Zoltán Vörös^{3,4} , Fabio Lepreti^{1,5} , and Vincenzo Carbone^{1,5}

¹ Dipartimento di Fisica, Università della Calabria, Ponte P. Bucci Cubo 31C, Rende (CS), Italy; federica.chiappetta@unical.it

² Swedish Institute of Space Physics, Box 537, SE-751 21 Uppsala, Sweden

³ Space Research Institute, Schmiedlstraße 6 Graz, Austria

⁴ Institute of Earth Physics and Space Science, HUN-REN, Sopron, Hungary

⁵ Istituto Nazionale di Astrofisica (INAF), Direzione Scientifica, Roma, Italy

Received 2023 April 3; revised 2023 September 21; accepted 2023 September 21; published 2023 November 3

Abstract

Low-frequency fluctuations in the interplanetary medium represent a turbulent environment where universal scaling behavior, generated by an energy cascade, has been investigated. On the contrary, in some regions, for example, the magnetosheath, universality of statistics of fluctuations is lost. However, at kinetic scales where energy must be dissipated, the energy conversion seems to be realized through a mechanism similar to the free solar wind. Here we propose a Langevin model for magnetic fluctuations at kinetic scales, showing that the resulting fluctuation–dissipation relation is capable of describing the gross features of the spectral observations at kinetic scales in the magnetosheath. The fluctuation–dissipation relation regulates the energy conversion by imposing a relationship between fluctuations and dissipation, which at high frequencies are active at the same time in the same range of scales and represent two ingredients of the same physical process.

Unified Astronomy Thesaurus concepts: Interplanetary medium (825); Heliosphere (711); Space plasmas (1544); Interplanetary turbulence (830)

1. Introduction

Magnetic fluctuations in the solar wind are described by classical magnetohydrodynamic (MHD) turbulence with a Kolmogorov-like energy spectrum $E(f) \sim f^{-5/3}$ within the inertial range, namely, for frequencies below the ion scale $f \leq f_i \simeq 1$ Hz (Bruno & Carbone 2016). As far as small scales are concerned, i.e., over the frequencies higher than the ion frequency, the situation is rather different (Leamon et al. 1998). At these scales the statistics of solar wind fluctuations depend on the sample at hand, namely, magnetic fluctuations lose the characteristic of large-scale universality (Alexandrova et al. 2008; Kiyani et al. 2009; Chen et al. 2014; Sorriso-Valvo et al. 2017; Carbone et al. 2018). In particular, at small kinetic scales fluctuations have been described by a further power law with a steeper power spectrum $E(f) \sim f^{-\alpha_i}$, the slope being strongly dependent on the analyzed sample. A statistical analysis of spectral slopes shows that roughly $\alpha_i \in [2.0; 3.1]$, with a peak at about $\alpha_i \simeq 8/3$ (Alexandrova et al. 2009; Sahraoui et al. 2009; Goldstein et al. 2015). Using data from Cluster spacecraft a further breakpoint in the magnetic energy power spectrum has been observed in some samples, at frequencies of the order of the electron gyrofrequency f_e , roughly corresponding to few tens of Hz. Interestingly enough, the whole power spectrum, for $f > f_i$, has been interpreted either as an ad hoc function made by a combination of a power law and an exponential decay (Alexandrova et al. 2012), namely, $E(f) \sim f^{-8/3} \exp(-f/f_d)$, or by a combination of two power laws (Sahraoui et al. 2009). Both interpretations have been claimed compatible with electron Landau damping. The slopes of the secondary power law $E(f) \sim f^{-\alpha_e}$, for $f > f_e$, have been

found to lie in the range $\alpha_e \in [3.5; 5.5]$ with a peak at about $\alpha_e \simeq 4$.

The presence of these fluctuations beyond f_i , with a well-defined structure of the energy spectra, has been attributed to dispersive phenomena generated by velocity–space effects and electron dynamics, perhaps due to a further turbulent energy cascade driven by wave–wave coupling, as for example a quasi two-dimensional cascade of kinetic Alfvén waves (Leamon et al. 1998; Bale et al. 2005; Sahraoui et al. 2009, 2010; Salem et al. 2012; Chen et al. 2013; Kiyani et al. 2013; Podesta 2013; Roberts et al. 2013), magnetosonic-whistler coupling (Gary & Smith 2009; Narita et al. 2011, 2016), kinetic slow modes (Yao et al. 2011; Howes et al. 2012) and ion Bernstein modes (Perschke et al. 2013). According to this point of view, the subionic range of scales is followed by a dissipative region starting near f_e , compatible with electron Landau damping. However, searching for single-wave modes through a frequency–wavenumber diagram does not show clear results (Narita et al. 2011), due to the presence of large scattering, sideband modes, sporadic wave trains as envelope solitons, and zero-frequency modes. The situation is made rather complicated also by the failure of the Taylor hypothesis, implying that measurements in the time domain cannot be simply translated into the wavevector domain (Narita 2018).

At variance with free solar wind, magnetic fluctuations within the Earth’s magnetosheath, at large scale, seem to be characterized by the absence of a well-defined Kolmogorov’s spectrum, mainly near the subsolar point (Huang et al. 2017; Rakhmanova et al. 2021). Rather, magnetic fluctuations show in general a less steep slope (Huang et al. 2017); the spectral energy show a spectral slope, which lies within $\alpha_i \in [0.3; 2.2]$, with a peak at about $\alpha_i \simeq 1.2$, strongly depending on the region. Interestingly enough, at variance with large scales, at small kinetic scales fluctuations in the magnetosheath were found to be weakly dependent on the location. Magnetic



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](https://creativecommons.org/licenses/by/4.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

fluctuations seem to have spectral shape similar to what is observed in the free solar wind (Huang et al. 2014; Hadid et al. 2015) as we described before. This is quite interesting, meaning that even if in the magnetosheath the magnetic energy is transferred to small scales perhaps with a mechanism different from a classical “turbulent cascade,” the energy conversion at small scales shares some kinds of universal features with the free solar wind, thus showing similar spectral properties with universal-like scaling distribution (Huang et al. 2017). Even for the magnetosheath, fluctuations at frequencies higher than the electron scale $f > f_e$, have been interpreted as a further power law, as in solar wind, with a distribution of slopes in the range $\alpha_e \in [4; 7.2]$ with a peak of about $\alpha_e \simeq 5.2$, rather steeper and broader than the solar wind counterpart (Huang et al. 2014).

As a rather general note, a power law is characterized uniquely by the scaling index; that is, the physics of the problem determines only the scaling index, not the amplitudes. A variable scaling index perhaps means that we cannot describe the observations with a true power law (Rakhmanova et al. 2021). From a physical point of view, at small scales, say, at about the ion or electron gyroradii or inertial lengths, the linear mode waves become kinetic, exhibiting both and at the same time, a dispersive and dissipative character due to various kinds of wave–particle interactions such as coherent scattering processes or incoherent processes (like pitch-angle scattering). The collisionless damping mechanisms include cyclotron damping, Landau damping, and the pressure-strain term (Matthaeus et al. 2020), but also energization of particles at current sheets (Chasapis et al. 2015), perhaps spontaneously generated by a large-scale intermittent turbulent cascade and stochastic heating. What is generally agreed is that the nonlinear energy cascade, which is surely active at the largest scales, transfers energy beyond the ion cyclotron frequency. The free energy mainly excites electric fluctuations (Bale et al. 2005; Perri et al. 2021), while the energy content in the magnetic fluctuations is lower, and fluctuations must be damped by plasma kinetic effects, thus providing a mechanism for heating in the collisionless plasma. However, note that the presence of collisionless dissipation represents an input for fluctuations, because the wave–particle mechanism involved in the dissipation generates particle beams (Sorriso-Valvo et al. 2019), which, in turn, are able to excite further fluctuations.

The complexity of describing fluctuations at small scales in space environments, and the absence of universality, registered even using recent spacecraft (Carbone et al. 2021), have given rise in recent years to a number of attempts that address the problem of the origin of fluctuations at kinetic scales also from a viewpoint different from a secondary turbulent cascade (Alberti et al. 2022; Benella et al. 2022; Carbone et al. 2022). In the present paper we would like to approach the problem of energy conversion at small scales in the magnetosheath. We use a different framework, recently introduced to describe fluctuations at kinetic scales in the free solar wind (Carbone et al. 2022), which, even if compatible, is rather different from a nonlinear energy cascade framework. The model is based on a Langevin process to describe fluctuations; the present paper is, in some way, a continuation of the previous one (Carbone et al. 2022). From a physical point of view the situation is not exactly the same. In the free solar wind large scales are universal and described by a turbulent cascade. As we said before, in the magnetosheath this is not true; large scales are

not universal, but rather they are described by a power spectrum with a strong variability in the slopes. This means that large-scale fluctuations in the magnetosheath cannot be described by a classical turbulent cascade. Here we would like to test to what extent a Langevin-like model is able to describe small scales in both free solar wind and magnetosheath, thus representing a class of universality.

2. A Langevin Model for Magnetic Fluctuations

Looking at the complex plasma activity at small scales, well documented in literature and far from being fixed in a unique framework, we are dealing with a medium where random fluctuations and dissipation compete in generating magnetic fluctuations. In a region where collisionless dissipation and plasma heating could occur and the presence of characteristic frequencies breaks the scale-free behavior, the role of dispersion and dissipation is still poorly understood, and the origin of fluctuations is far from being clearly established. This is rather different from classical turbulence methodology, where the nonlinear cascade is active within a scale-free region, which is well separated from the smallest scales where dissipation is confined.

Let us consider a simple framework where magnetic fluctuations $\mathbf{b}(t)$ at small scales can be roughly described by a Langevin stochastic differential equation

$$d\mathbf{b}(t) = \Gamma[\mathbf{b}(t), t]\mathbf{e}_b dt + \Psi[\mathbf{b}(t), t] dW(t)\mathbf{e}_b \quad (1)$$

(where \mathbf{e}_b is the direction of the magnetic fluctuations); that is, we assume that the dynamics of fluctuations is due to two different contributions. A first contribution, described by the stochastic process $dW(t)$, which mimics all the complex wave dynamics within the plasma. Here for mathematical simplicity $\Psi[\mathbf{b}(t), t] \sim F_0$ is assumed constant, proportional to the rms of fluctuations $F_0 = \sqrt{\langle b^2 \rangle}$. We can interpret the random forcing as $dW(t)\mathbf{e}_b = \xi(t)dt$, which is the natural physically acceptable choice for an interpretation which assumes $\xi(t)$ as a real noise, possibly different from a white noise, with finite correlation times (Gardiner 2009; Stawarz et al. 2022). Moreover, we assume that $\xi(t)$ is uncorrelated with the initial values of magnetic fluctuations $\mathbf{b}(0)$, say, $\langle \xi(t) \cdot \mathbf{b}(0) \rangle = 0$. The second contribution is due to the collisionless dissipative processes, which we parameterize with a linear damping term, proportional to a constant damping rate γ , say, $\Gamma[\mathbf{b}(t), t]\mathbf{e}_b \simeq -\gamma\mathbf{b}(t)$.

Note that, in our approach, there is no scale separation between dissipation and generation of fluctuations, as in a turbulent environment. In a turbulent medium, according to the Kolmogorov picture, dissipation is confined to small scales (large wavevectors), because the dissipation in fluid (or even in MHD) equations is due to a viscous term that is proportional to the inverse of the Reynolds number, multiplied by the second-order derivative in space of the velocity (or magnetic) field. Since the Reynolds number is in general very high, dissipation is effective only at very small scales, where the derivative term is large enough to compensate the inverse of the Reynolds number. In the collisionless solar wind, when describing the turbulent cascade, which is responsible for the generation of fluctuations at large scales, we must always assume that dissipation is confined to very small scales, but we know that, in the solar wind, it is due to kinetic effects rather than a viscous term. However, beyond the ion scale, it is hard to

assume that dissipation is further separated from the mechanism that is responsible for the birth of fluctuations as, for example, is evident from the pressure-strain term in numerical simulations (Matthaeus et al. 2020).

Under the above approximation, the Langevin equation can be solved by Fourier transforms. This gives an obvious relation between the correlation of the Fourier modes of the forcing ξ_ω and the power spectrum of magnetic energy modes b_ω , in such a way that, in a stationary situation

$$E(\omega) = F_0^2 G(\omega) (\omega^2 + \gamma^2)^{-1} \quad (2)$$

where $E(\omega) = \langle b_\omega \cdot b_\omega^* \rangle$ (brackets means time averaging and $*$ stands for complex conjugate) and $\langle \xi_\omega \cdot \xi_{\omega'} \rangle = 2\pi G(\omega) \delta(\omega + \omega')$. It is worth to remark that “stationary” here means a physical situation where a dynamical system covers all the available phase space. As a simple example let us suppose that magnetic fluctuations are generated by the action of completely uncorrelated stochastic wave trains, so that $\langle \xi_\omega \cdot \xi_{\omega'} \rangle = 2\pi \delta(\omega + \omega')$. In this case the magnetic energy spectrum is given by a Lorentzian function $E(\omega) \simeq F_0^2 / (\omega^2 + \gamma^2)$, which does not describe the broad variety of spectral shapes of magnetic energy density spectrum as observed in the high-frequency solar wind plasma.

Since purely stochastic fluctuations do not describe the high-frequency range; as a realistic example (Carbone et al. 2022) let us consider the case where, as the energy of fluctuations cascading from large scales reaches the ion breakpoint, a variety of waves takes part in the complex process of energy conversion (Narita et al. 2011), so that we expect that wave-wave couplings, wave-particles interactions, and dispersive effects can play a role. In this situation we can expect that $\xi(t)$ can be considered, as a rough approximation, as a colored noise rather than a white noise. This means that the two-point correlations of the stochastic forcing term are different from zero even for relatively large separation times, so that, as usual (Gardiner 2009), we can expect that the two-point correlations decays exponentially in time, so that

$$\langle \xi(t') \cdot \xi(t) \rangle = \exp[-\lambda(t' - t)]. \quad (3)$$

By using the inverse Fourier spectrum we can easily obtain $G(\omega) = 1/(\omega^2 + \lambda^2)$. However, since a classical dispersion relation is absent (Narita et al. 2011), all kinetic modes are present, so that, to be more realistic, we must assume that the decay of correlations does not happen by a single decay rate. Let us consider a situation where there exists a continuous distribution of relaxation rates λ described by a probability of occurrence $P(\lambda)$, and the power spectrum of the external forcing can be calculated from the superposition of all λ 's according to their probability of occurrence. We expect that the decorrelation rates depend on a characteristic value, say, λ_0 , depending, for example, on the wave modes involved in the decorrelation process, and decay for values far from λ_0 . Among the possible functional shape, we choose a convenient integrable form, namely, for simplicity a power law $dP(\lambda) \simeq (\lambda/\lambda_0)^{-\mu} d\lambda$, where $\mu > 1$. This is physically acceptable, even if arbitrary, our choice being due to the relative simplicity of the calculations. In fact, in this case we get

$$G(\omega) \sim \int \frac{dP(\lambda)}{\omega^2 + \lambda^2} = \omega^{-(\mu+1)} \lambda_0^{-\mu} \int \frac{y^{-\mu}}{1 + y^2} dy. \quad (4)$$

Inserting this relation in Equation (2), we obtain an expression for the power spectrum at frequencies f , which can be used to describe the data

$$E(f) = Ag(\mu) f^{-(\mu+1)} (f^2 + \gamma^2)^{-1} \quad (5)$$

where A is a constant and $g(\mu)$ is the integral in Equation (4), namely, a smooth function of μ . Equation (5), already used either as an ad hoc fitting function Sahraoui et al. (2009), or in the framework of a Langevin approach (Carbone et al. 2022), successfully reproduces the overall shape of the free solar wind spectra with different pairs (μ, γ) . The observed spectrum in Equation (5) can then be interpreted as the result of a whole class of flicker noises $\xi(t)$, compatible with the presence of random fluctuations generated by sporadic wave train interactions, with the two-point correlations decaying according to a generic exponential function with variable rate. Note that (5) is compatible with a double power law, because the ion and electron breakpoints are well separated (Sahraoui et al. 2009; Carbone et al. 2021). Of course, Equation (5) is compatible also with the expression introduced ad hoc in Alexandrova et al. (2012), provided that the scaling exponent is fixed $(1 + \mu) = 8/3$ and the free parameters are related by $\exp(-f/f_d) = (f^2 + \gamma^2)$ for each observed frequency. The parameter $1 + \mu$ then represents the scaling exponent of the energy spectra as observed beyond the ion break in solar wind data.

The statistical properties of fluctuations can be related to the global properties of dissipation even without considering a turbulent cascade. To make this feature clearer, let us consider a modified version of the Langevin equation

$$db_j(t) = -\alpha(b_j, \xi_j) b_j dt + F_0 \xi_j dt \quad (6)$$

where α is a stochastic function describing the microscopic properties of dissipation. It is easy to show that Equation (6) has an exact constant of motion $\chi(t) = \sum_j b_j^2 / 2\mu_0$ (μ_0 is the vacuum permittivity), providing

$$\alpha(b_j, \xi_j) = \left(\frac{F_0^2}{2\mu_0} \right) \frac{\sum_j b_j \xi_j}{\sum_j b_j^2}; \quad (7)$$

that is, the system in Equation (6) is time invariant, independently of initial conditions, and we can conjecture that statistical properties of Equations (1) and (6) are equivalent (Gallavotti & Cohen 1995; Gallavotti 2014); that is, $\gamma \simeq \langle \alpha \rangle$ describes the average properties of dissipation. The equivalence between the dissipative system and the conservative one allows us to evidence the physical origin of γ . In fact, from the Langevin Equation (1), we get an equation for the average energy of magnetic fluctuations $\epsilon(t) = \langle b^2 \rangle / 2\mu_0 = \langle \chi(t) \rangle$, which can be solved formally, thus obtaining (Carbone et al. 2022)

$$\begin{aligned} \epsilon(t) &= \left(\frac{F_0^2}{2\mu_0} \right) e^{-2\gamma t} \int_0^t ds e^{2\gamma s} \\ &\quad \times \int_0^s dt' e^{\gamma(t'-s)} \langle \xi(t') \xi(s) \rangle. \end{aligned} \quad (8)$$

Since the statistical equivalence between the Langevin Equation (1) and the modified time-invariant model in Equation (6) can be reasonably conjectured (Gallavotti & Cohen 1995; Gallavotti 2014; Carbone et al. 2022), a finite stationary solution can be found for Equation (8), providing

$\lambda < 3\gamma$. Then in the long time evolution, the statistics of the out-of-equilibrium process can be considered as an equilibrium-like process, and $\epsilon(t)$ tends to a function $\rho(\Omega, F, \Gamma) = \int_{\Omega} F(v) \Gamma(dv)$, where Ω is the phase space covered by the dynamics, $F(v)$ is any function of the observable v , and Γ is the Sinai–Ruelle–Bowen (SRB) probability measure (Bowen 1970; Sinai 1977; Ruelle 1980; Gallavotti & Cohen 1995). After straightforward calculation, using Equation (3), we obtain

$$\rho(\Omega, F, \Gamma) \simeq \left(\frac{B_0^2}{2\mu_0} \right) \left(\frac{\gamma}{\lambda_0} \right)^{-(\mu+1)} \times \int_{\zeta_{\min}}^{\zeta_{\max}} \frac{y^{-\mu}}{1-y} dy \quad (9)$$

where $\zeta_{\min} \leq \lambda/\gamma \leq \zeta_{\max}$ is the range of the possible relaxation rates for the two-point correlations, the maximum possible extension being $\zeta_{\min} \simeq 0$ and $\zeta_{\max} \simeq 1$. Equation (9) represents a kind of fluctuation–dissipation relation (FDR; Gardiner 2009; Kanekar et al. 2015; Carbone et al. 2022). Note that here we consider a statistical equilibrium situation just to evidence some rough consequences of FDR. A statistical equilibrium involving a maximal entropy is not strictly required, because a kind of FDR, based on the response theory (Ruelle 1978), can be found also in a stationary chaotic system with a given statistical distribution of orbits in the phase space, thus obtaining information on the fine structure of the attractor in the finite phase space (Gallavotti & Lucarini 2014; Lucarini 2014; Biferale et al. 2018).

The observable $F(v)$ is arbitrary, so that, if we are, for example, interested in the squared electron plasma velocity fluctuations, we are free to use $F(v) = v^2$, thus roughly identifying $\rho(A, F, \Omega)$ with a stationary nonequilibrium “electron temperature” $k_B T$ by using the kinetic velocity distribution function $f(v)$ in the SRB measure $\Gamma(dv) \sim f(v) dv$. From Equation (9), providing $\lambda < \gamma$, the phenomenological damping rate turns out to be proportional to some power of the electron plasma- β parameter, where the thermal energy is defined through the second-order electron velocities

$$\beta_e = \left(\frac{\gamma}{\lambda_0} \right)^{-(\mu+1)} \int_{\zeta_{\min}}^{\zeta_{\max}} \frac{y^{-\mu}}{1-y} dy. \quad (10)$$

The importance of the plasma- β parameter to fix properties of the high-frequency region of fluctuations, and in particular the ionic break, has been already underlined (Leamon et al. 1999; Chen et al. 2014; Parashar et al. 2018) both from observations and numerical simulations. Our results evidence that the electron plasma- β parameter can regulate the equilibrium condition between the excitation of fluctuations and their dissipation through the FDR in the whole high-frequency region.

3. Data Analysis

In this paper we use Magnetospheric Multiscale (MMS; Burch et al. 2016) observations from the terrestrial magnetosheath. In particular, we use high-resolution magnetic field measurements sampled with frequency 8192 Hz by the Search-coil Magnetometer instrument (Lindqvist et al. 2016) on MMS1. We have selected few trajectories from the

magnetosheath database compiled by Stawarz et al. (2022). For the purpose of this study, the selection is based on intervals representing different plasma conditions in terms of electron plasma beta, varying from low (~ 0.1) to high (~ 39) (see Table 1). To characterize the spectral properties of the magnetic field fluctuations, we calculate the power spectral density (PSD) of the absolute value of the magnetic field for each interval using the Welch algorithm with Hanning window and 75% overlap between the data segments. Note that the Taylor hypothesis allowing the conversion of time to spatial scales has been validated for the intervals under consideration (Stawarz et al. 2022).

Equation (5) has been used to fit the PSDs, according to the constraint in Equation (10). In other words, once we get the best-fit pair of parameters (γ, μ) for each sample, from Equation (10) we obtain a set of values of λ_0 , which, in general, depends on both ζ_{\min} and ζ_{\max} . Without losing generality we keep $\zeta_{\max} \simeq 1$ fixed, and use ζ_{\min} as variable so that $\lambda_0 = \lambda_0(\zeta_{\min})$. In this way we are able to identify a range of frequencies that characterize the possible decorrelation times for interacting waves that can generate fluctuations in the high-frequency region.

The spectral properties observed in the magnetosheath at high frequencies are nicely reproduced by Equation (5). We used a standard minimization procedure based on a maximum likelihood criterion, thus obtaining the best-fit values for μ and γ for the seven different samples characterized by different values of β_e . Actually the spectra are very smooth and the functional shape of Equation (5) fits very well the data for a wide range of values of λ_0 . In Figure 1 we report the frequency spectra of the seven samples we used, along with the best-fit curve of Equation (5) obtained with the parameters listed in Table 1. In the same table we report the minimum and maximum values of λ_0 , corresponding to $\lambda_{\min} = \lambda_0(\zeta_{\min} \simeq 0)$ and $\lambda_{\max} = \lambda_0(\zeta_{\min} \simeq 1)$, respectively, and the normalized range $\Delta = (\lambda_{\max} - \lambda_{\min})/\gamma$.

As a comment, actually, due to the smoothness of the observed spectra, a simple fit of Equation (5) on the data produces estimates, mainly of the free parameter γ , which are affected by errors that are too large. In particular, results are sensitive to the evaluation of the integral in Equation (5) and each sample is compatible with a relatively large range of possible values of γ . To minimize this bias, we use the FDR Equation (10) in Equation (5), rather than verifying its validity a posteriori (Carbone et al. 2022). This allows us to find reasonable values of the parameters. The goodness of fit, and the statistical reliability of parameters, require a careful analysis of the integral in Equation (9), with different ζ_{\min} , and the determination of one of the parameters.

The minimum value λ_{\min} , which in our model represents the scale where nonlinear effects, say, wave–wave interactions are effective, results to be of the order of few fractions of Hz, thus roughly representing some frequency related to the ion breakpoint observed on the data, and is independent on β . Moreover, $\gamma < 10^2$ Hz, while $\lambda_{\min}/\gamma \simeq 10^2$, a ratio which is independent on β . On the contrary, the maximum value λ_{\max} roughly increases with β , the normalized range of decorrelation rates Δ , increases with β as well. Said differently, in a low- β sample the range of possible values of the two-point correlation for the decay rates λ , is narrow, and is confined to few Hz around the ion breakpoint. On the contrary, when the plasma- β of the sample is high, the range of the

Table 1
Parameters of the Fit

Id	Time	β_e	f_{λ_e}	f_{ρ_e}	f_{\min}	f_{\max}	μ	γ	λ_{\min}	λ_{\max}	Δ
2016 09/30	17:50–18:01	0.08	13.16	46.25	1.88	81.25	1.60	39.26	0.17	4.63	0.11
2018 05/28	22:23–22:30	0.13	20.60	56.45	1.88	62.50	1.17	31.58	0.62	3.05	0.08
2015 11/30	00:21–00:26	1.50	45.90	37.30	1.88	62.50	1.70	56.91	0.48	21.54	0.37
2017 12/09	06:12–06:18	4.00	35.69	17.85	1.88	62.50	2.03	59.47	0.18	34.54	0.58
2016 12/11	15:20–15:32	7.69	27.76	10.05	1.88	62.50	2.03	82.86	0.32	59.71	0.72
2017 01/28	05:29–05:34	11.35	31.41	9.36	1.88	81.25	1.91	66.22	0.46	58.85	0.81
2018 05/23	14:35–14:41	39.31	50.10	8.00	1.88	62.50	1.91	77.52	1.40	77.52	0.98

Note. The date relative to each sample as identification number of the sample itself (id), time interval of each sample, the electron plasma- β parameter, the characteristic frequencies f_{λ_e} and f_{ρ_e} (in Hz), the minimum and maximum frequencies of the data used for the fit of the power spectrum (in Hz), the best-fit parameters μ and γ (in Hz), the minimum and maximum values of λ_0 (in Hz), the normalized interval $\Delta = (\lambda_{\max} - \lambda_{\min})/\gamma$.

two-point correlations for random interacting fluctuations is wide, sometimes ranging from the ion to the electron break-point. When $\beta_e > 1$, the values of γ we found are greater than both the frequencies related to the electron gyroradius f_{ρ_e} and the electron skin depth f_{λ_e} . On the contrary, for $\beta_e < 1$ we find $f_{\lambda_e} < \gamma < f_{\rho_e}$. Specific experimental relationships can be found between the free parameters and β_e , namely, $\lambda_{\max} \sim \beta_e^{1/2}$ and $\gamma \sim \beta_e^{1/7}$, so that their ratio decreases as $(\gamma/\lambda_{\max}) \sim \beta_e^{-5/14}$. Then λ_{\max} increases faster than γ ; thus, for high- β samples, the range of possible decorrelation rates increases, even if high- β samples require decorrelation rates closer to γ . As a reference in Figure 2 we report the dependence of γ and λ_{\max} on β_e . The scaling exponent μ does not show a clear dependence on β ; rather we found that in general the equilibrium is reached in such a way that high values of γ correspond to steeper spectra in the ionic region, namely, to high values of μ .

It is worth reporting that the estimate of the relation γ versus β_e reported here is rather inconsistent with the results of Carbone et al. (2022), which in fact report a scaling relation $\gamma \sim \beta_e^{-3/2}$ for the free solar wind. This can be due to several factors, which should pertain to the different nature of fluctuations between free solar wind and magnetosheath. These multiscale processes can involve multiple conversions between electromagnetic, kinetic, and thermal energies downstream of the bow shock (Vörös et al. 2021). As a technical detail, as said before, here we use the FDR to reduce the dispersion of the possible values of the fitting parameters, different from what has been done in Carbone et al. (2022). Of course, it is always possible, using magnetosheath data, to find values of γ that behave as $\beta_e^{-3/2}$ as in the free solar wind, but the large dispersion of values of γ makes the scaling laws unreliable here. Only using the FDR Equation (10) the dispersion of parameters can be reduced, thus making statistically reliable the estimate and the scaling laws.

4. Summary

To conclude, we showed that a Langevin equation roughly describes the energy conversion at kinetic scales, namely, at the end of the inertial range, generally characterized by a turbulent energy cascade. Beyond the ion break, namely, roughly at frequencies $f > f_i$, the scale-free region ends because some characteristic frequencies are born. In this region, dissipative effects cannot be neglected and a lot of nonlinear phenomena, for example, wave–wave couplings

and wave–particle interactions, are responsible for the birth of magnetic fluctuations. In this region, the FDR obtained from the Langevin equation, relates the magnetic fluctuations, here globally described by a function of the plasma- β parameter, to the dissipative properties of the magnetosheath at small scales, globally described by the parameter γ . The FDR, perhaps more than a classical turbulent cascade, governs this region of the spectrum (Kanekar et al. 2015), thus regulating the energy conversion by imposing a relationship between fluctuations and dissipation, two mechanisms, which, unlike what happens in the inertial range of classical turbulence, are active in the same range of frequencies at the same time. In other words, according to the FDR, fluctuations and dissipation represent two ingredients of the same physical process, namely, the energy conversion. The fact that fluctuations and dissipation must balance locally each other is perhaps the main motivation for the absence of universality of classical statistics of fluctuations at kinetic scales. Rather, the universality is recovered at the level of FDR, say, a well-defined relation with the plasma- β parameter is required. Finally, the values of γ cannot be misinterpreted as the scale where dissipation starts to be active, according to a Richardson-like cascade. Instead, γ represents a characteristic scale required to assure an equilibrium condition between the generation of fluctuations and the dissipation of the fluctuations themselves.

The approach based on a Langevin-like description of small-scale fluctuations is rather different, even if compatible, with the usual turbulent cascade. A turbulent energy cascade is a nonequilibrium process, in the sense that it cannot be described by a maximal entropy approach of usual statistical mechanics. In the dynamical system theory, a chaotic system is far from classical equilibrium. The statistical mechanics of nonequilibrium processes, which does not necessarily require a maximal entropy approach, is rather general and works for both usual statistical systems and chaotic system, including some aspect of turbulence (Gallavotti & Lucarini 2014; Gallavotti 2014; Lucarini 2014; Biferale et al. 2018).

We would like to emphasize once more that our approach does not bring into question the importance of the complexity inherent to the space plasmas and cascade processes. Rather, independently of the specific microphysical plasma dynamics, our approach can account for the gross features of the observations of spectral properties of high-frequency fluctuations in the magnetosheath and, in general, in the

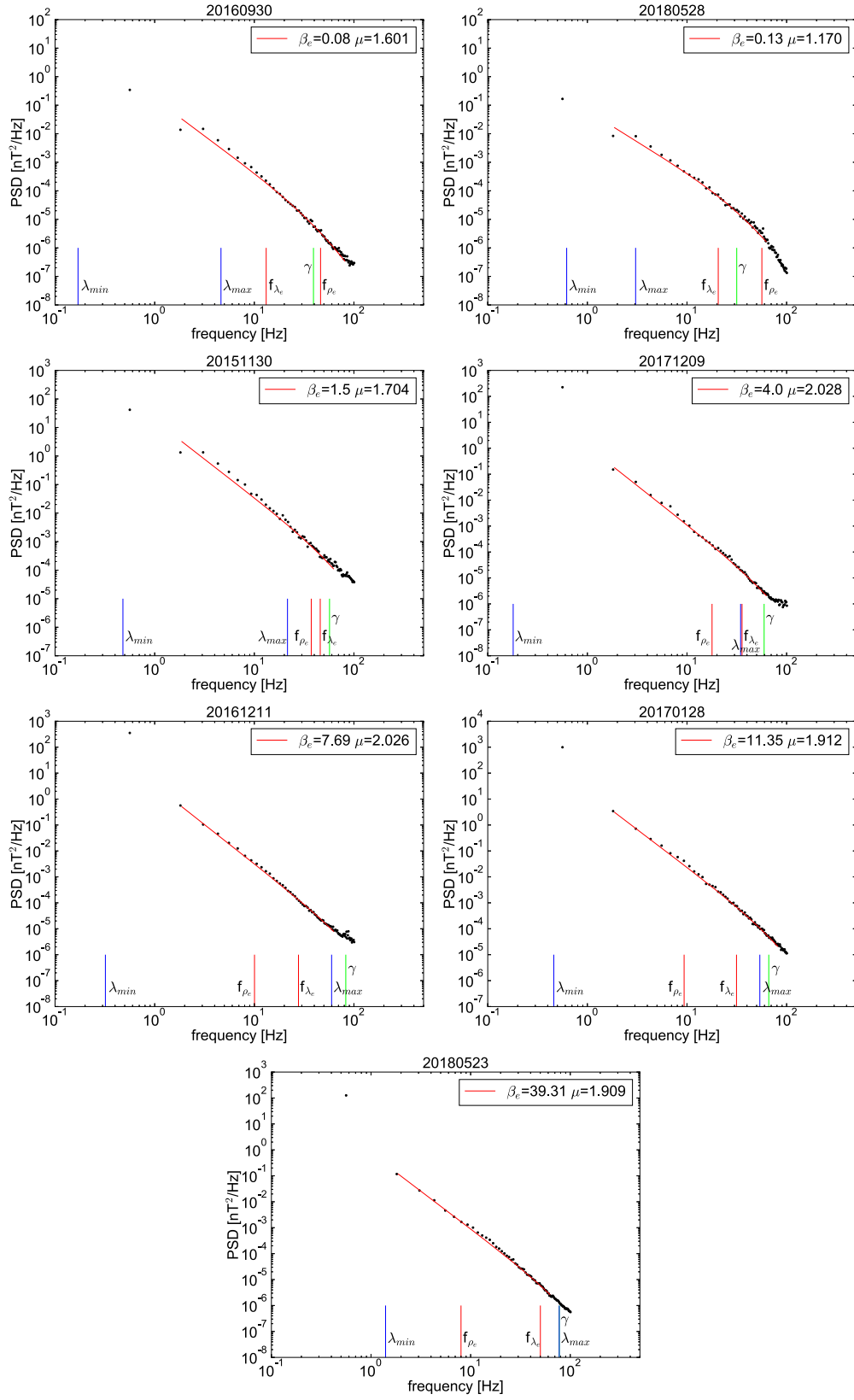


Figure 1. $E(f)$ vs. f as obtained from data (dots) and from Equation (5) (red line) for the seven different samples. The positions of γ , λ_{\min} , λ_{\max} , and of the two characteristic electron plasma frequencies of each sample are indicated on the plots. The values of the electron plasma- β and the scaling exponent μ obtained from the fit are reported in each figure.

interplanetary space (Carbone et al. 2022). Microphysical processes related to specific wave-wave couplings and wave-particle effects could be taken into account, for

example, by specifying the range of values of the decorrelation rate λ . This is out the scope of the paper and will be investigated and reported in a later paper.

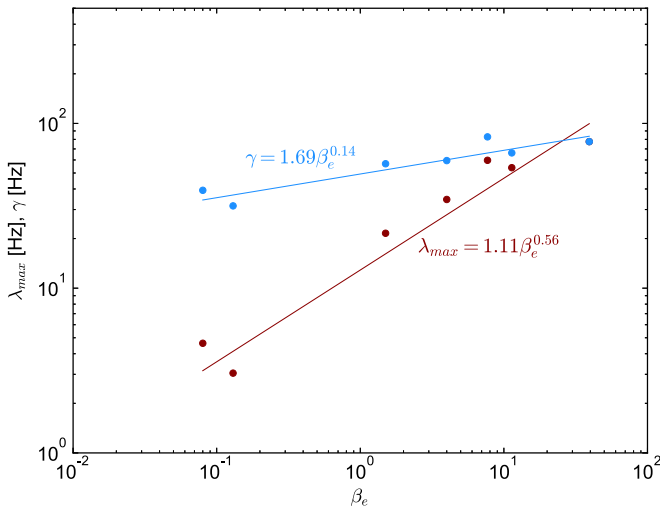


Figure 2. The values of γ and λ_{\max} as a function of the electron plasma- β parameter of the seven samples we used. The full lines correspond to the scaling reported in the text.

Acknowledgments

F.C., V.C., and F.L. were supported by Italian MIUR-PRIN grant 2017APKP7T on Circumterrestrial Environment: Impact of Sun–Earth Interaction and by the Italian Space Agency and the National Institute of Astrophysics, in the framework of the CAESAR (Comprehensive spAce wEather Studies for the ASPIS prototype Realization) project, through the ASI-INAF n. 2020-35-HH.0 agreement for the development of the ASPIS (ASI Space weather InfraStructure) prototype of scientific data center for Space Weather. E.Y.'s research was supported by the Swedish National Space Agency grant 192/20. Z.V. was supported by the Austrian Science Fund (FWF) P33285-N.

ORCID iDs

Federica Chiappetta <https://orcid.org/0000-0001-7221-1382>
 Emiliya Yordanova <https://orcid.org/0000-0002-9707-3147>
 Zoltán Vörös <https://orcid.org/0000-0001-7597-238X>
 Fabio Lepreti <https://orcid.org/0000-0001-5196-2013>
 Vincenzo Carbone <https://orcid.org/0000-0002-3182-6679>

References

Alberti, T., Benella, S., Carbone, V., et al. 2022, *Univ*, **8**, 338
 Alexandrova, O., Carbone, V., Veltri, P., & Sorriso-Valvo, L. 2008, *ApJ*, **674**, 1153
 Alexandrova, O., Lacombe, C., Mangeney, A., Grappin, R., & Maksimovic, M. 2012, *ApJ*, **760**, 121
 Alexandrova, O., Saur, J., Lacombe, C., et al. 2009, *PhRvL*, **103**, 165003
 Bale, S. D., Kellogg, P. J., Mozer, F. S., Horbury, T. S., & Reme, H. 2005, *PhRvL*, **94**, 215002
 Benella, S., Stumpo, M., Consolini, G., et al. 2022, *ApJL*, **928**, L21
 Biferale, L., Cencini, M., De Pietro, M., Gallavotti, G., & Lucarini, V. 2018, *PhRvE*, **98**, 012202
 Bowen, R. 1970, *AmJM*, **92**, 725

Bruno, R., & Carbone, V. 2016, *Turbulence in the Solar Wind*, 928 (Berlin: Springer)
 Burch, J. L., Moore, T. E., Torbert, R. B., & Giles, B. L. 2016, *SSRv*, **199**, 5
 Carbone, F., Sorriso-Valvo, L., Alberti, T., et al. 2018, *ApJ*, **859**, 27
 Carbone, F., Sorriso-Valvo, L., Khotyaintsev, Y. V., et al. 2021, *A&A*, **656**, A16
 Carbone, V., Lepreti, F., Vecchio, A., Alberti, T., & Chiappetta, F. 2021, *FrP*, **9**, 18
 Carbone, V., Telloni, D., Lepreti, F., & Vecchio, A. 2022, *ApJL*, **924**, L26
 Chasapis, A., Retinó, A., Sahraoui, F., et al. 2015, *ApJL*, **804**, L1
 Chen, C. H. K., Boldyrev, S., Xia, Q., & Perez, J. C. 2013, *PhRvL*, **110**, 225002
 Chen, C. H. K., Leung, L., Boldyrev, S., Maruca, B. A., & Bale, S. D. 2014, *GeoRL*, **41**, 8081
 Chen, C. H. K., Sorriso-Valvo, L., Safránková, J., & Nemecek, Z. 2014, *ApJL*, **789**, L8
 Gallavotti, G. 2014, *Nonequilibrium and Irreversibility* (Berlin: Springer)
 Gallavotti, G., & Cohen, E. G. D. 1995, *PhRvL*, **74**, 2694
 Gallavotti, G., & Lucarini, V. 2014, *JSP*, **156**, 1027
 Gardiner, C. 2009, *Stochastic Methods* (Berlin: Springer)
 Gary, S. P., & Smith, C. W. 2009, *JGRA*, **114**, A12105
 Goldstein, M. L., Wicks, R. T., Perri, S., & Sahraoui, F. 2015, *RSPTA*, **373**, 20140147
 Hadid, L. Z., Sahraoui, F., Kiyani, K. H., et al. 2015, *ApJL*, **813**, L29
 Howes, G. G., Bale, S. D., Klein, K. G., et al. 2012, *ApJL*, **753**, L19
 Huang, S. Y., Hadid, L. Z., Sahraoui, F., Yuan, Z. G., & Deng, X. H. 2017, *ApJL*, **836**, L1
 Huang, S. Y., Sahraoui, F., Deng, X. H., et al. 2014, *ApJL*, **789**, L28
 Kanekar, A., Schekochihin, A. A., Dorland, W., & Loureiro, N. F. 2015, *JPhPh*, **81**, 305810104
 Kiyani, K. H., Chapman, S. C., Khotyaintsev, Y. V., Dunlop, M. W., & Sahraoui, F. 2009, *PhRvL*, **103**, 075006
 Kiyani, K. H., Chapman, S. C., Sahraoui, F., et al. 2013, *ApJ*, **763**, 10
 Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., & Wong, H. K. 1998, *JGR*, **103**, 4775
 Leamon, R. J., Smith, C. W., Ness, N. F., & Wong, H. K. 1999, *JGR*, **104**, 22331
 Lindqvist, P. A., Olsson, G., Torbert, R. B., et al. 2016, *SSRv*, **199**, 137
 Lucarini, V. 2014, *JSP*, **146**, 774
 Matthaeus, W. H., Yang, Y., Wan, M., et al. 2020, *ApJ*, **891**, 101
 Narita, Y. 2018, *LRSF*, **15**, 2
 Narita, Y., Gary, S. P., Saito, S., Glassmeier, K. H., & Motschmann, U. 2011, *GeoRL*, **38**, L05101
 Narita, Y., Nakamura, R., Baumjohann, W., et al. 2016, *ApJL*, **827**, L8
 Parashar, T. N., Matthaeus, W. H., & Shay, A. 2018, *ApJL*, **864**, L21
 Perri, S., Perrone, D., Owen, R., et al. 2021, *ApJ*, **919**, 75
 Perschke, C., Narita, Y., Gary, S. P., Motschmann, U., & Glassmeier, K. H. 2013, *AnGeo*, **31**, 1949
 Podesta, J. J. 2013, *SoPh*, **286**, 529
 Rakhmanova, L., Riazantseva, M., & Zastenker, G. 2021, *FrASS*, **7**, 616635
 Roberts, O. W., Li, X., & Li, B. 2013, *ApJ*, **769**, 58
 Ruelle, D. 1978, *PThPS*, **64**, 339
 Ruelle, D. 1980, *NYASA*, **357**, 1
 Sahraoui, F., Goldstein, M. L., Belmont, G., Canu, P., & Rezeau, L. 2010, *PhRvL*, **105**, 131101
 Sahraoui, F., Goldstein, M. L., Robert, P., & Khotyaintsev, Y. V. 2009, *PhRvL*, **102**, 231102
 Salem, C. S., Howes, G. G., Sundkvist, D., et al. 2012, *ApJL*, **745**, L9
 Sinai, Y. G. 1977, *Lectures in Ergodic Theory*, Lect. Not. Math. (Princeton, NJ: Princeton Univ. Press)
 Sorriso-Valvo, L., Carbone, F., Leonardis, E., et al. 2017, *AdSpR*, **59**, 1642
 Sorriso-Valvo, L., Catapano, F., Retinó, A., et al. 2019, *PhRvL*, **122**, 035102
 Stawarz, J. E., Eastwood, J. P., Phan, T. D., et al. 2022, *PhPi*, **29**, 012302
 Vörös, Z., Roberts, O. W., Yordanova, E., et al. 2021, *FrASS*, **10**, 1163139
 Yao, S., He, J. S., Marsch, E., et al. 2011, *ApJ*, **728**, 146