



Balancing and scheduling human-robot collaborated assembly lines with layout and objective consideration

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ABSTRACT

The recent Industry 4.0 trend, followed by the technological advancement of collaborative robots, has urged many industries to shift towards new types of assembly lines with human-robot collaboration (HRC). This type of manufacturing line, in which human skill is supported by robot agility, demands an integrated balancing and scheduling of tasks and operators among the stations. This study attempts to deal with these joint problems in the straight and U-shaped assembly lines while considering different objectives, namely, the number of stations (Type-1), the cycle time (Type-2), and the cost of stations, operators, and robot energy consumption (Type-rw). The latter type often arises in the real world, where multiple types of humans and robots with different skills and energy levels can perform the assembly tasks collaboratively or in parallel at stations. Additionally, practical constraints, namely robot tool changes, zoning, and technological requirements, are considered in Type-rw. Accordingly, different mixed-integer linear programming (MILP) models for straight and U-shaped layouts are proposed with efficient lower and upper bounds for each objective. The computational results validate the efficiency of the proposed MILP model with bounded objectives while addressing an application case and different test problem sizes. In addition, the analysis of results shows that the U-shaped layout offers greater flexibility than the straight line, leading to more efficient solutions for JIT production, particularly in objective Type-2 followed by Type-rw and Type-1. Moreover, the U-shaped lines featuring a high HRC level can further enhance the achievement of desired objectives compared to the straight lines with no or limited HRC.

1. Introduction

From the times of Henry Ford up to today's industry 5.0 era, assembly lines, as a flow-oriented manufacturing system, have been widely used in various industries such as automotive and electronics for mass-customized production. An assembly line comprises a set of tasks performed on workpieces moving from one station to another through a material handling device until a complete product is assembled. Each assembly line's most elementary optimization problem is known as the assembly line balancing problem (ALBP) (Fathi et al., 2018, 2019). The ALBP decides on the assignment of tasks among the stations to optimize some objectives while taking precedence relations into account. According to the objective function, ALBP is classified into four types: (1) Type-1 minimizes the number of stations (NS) for a given cycle time (CT); (2) Type-2 minimizes the CT for a given NS; (3) Type-E optimizes the CT and the NS simultaneously; and (4) Type-F aims to find a feasible solution given the CT and the NS (Boysen et al., 2022).

Considering the layout, the assembly lines can be mainly divided into straight and U-shaped lines (Mukund Nilakantan and Ponnambalam, 2016). From the ALBP perspective, in straight lines, the task assignment depends on all its predecessors, while in U-shaped lines, the assignment of a task is subjected to either all its predecessors or successors (Fathi et al., 2016). In the latter, the tasks dependent on their predecessors are assigned to the line's entrance, while the tasks dependent on their successors are allocated to the line's exit. Accordingly, U-shaped assembly lines are essential for just-in-time (JIT) production (Nourmohammadi et al., 2019).

On the other hand, since the advent of industrial robots, many manufacturers have used them as the technology enabler for more agile manufacturing (Weckenborg and Spengler, 2019). The first generation of industrial robots was insulated in cages to avoid the risks of intervening in human workplaces. Recently, as a consequence of Industry 4.0, the new generation of robots known as collaborative robots (cobots) have been acquainted with the capability of operating like humans

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thanks to their embedded software and hardware with advanced sensing and interacting systems (Nourmohammadi et al., 2022a). With such technologies, cobots can be deemed resilient operators sharing the same workplace with humans. Moreover, the design of human-centered workplaces motivated by Industry 5.0 has recently attracted many manufacturers' attention to support humans with cobots, where humans play a central role (De Nul et al., 2021). These automation trends have convinced manufacturers to shift their manual or robotic lines toward hybrid assembly lines with human-robot collaboration (HRC), where human skill and cobot agility can be utilized in the same station.

Several studies have underlined the advantages of assembly line balancing with HRC. Dalle Mura and Dini (2019) and Weckenborg and Spengler (2019) discussed that HRC is a good option for efficient assembly line balancing. Weckenborg et al. (2020), Koltai et al. (2021), Nourmohammadi et al. (2022a), and Nourmohammadi et al. (2022b) studied how cobots can significantly enhance the productivity of manual assembly lines. Dalle Mura and Dini (2022), Stecke and Mokhtarzadeh (2022), and Weckenborg et al. (2022) have emphasized the advantages of HRC for the design of both economic and ergonomic assembly lines. From the layout perspective, Li et al. (2023), Mao et al. (2023), and Rahman et al. (2023) have dealt with HRC in the U-shaped assembly lines. To the authors' knowledge, the assumptions were that only one human and robot were allowed at each station, or the human and robot could not perform different tasks in parallel. However, this assumption (i.e., limiting the HRC level) tends to inadvertently limit the full HRC potential for a flexible and efficient utilization of resources. Moreover, from the objective point of view, in line with energy sustainability in Industry 4.0 (Ghobakhloo and Fathi, 2021), the costs of robot energy consumption are not incorporated into the cost of stations and operators in the literature. At the same time, different objectives, specifically in the U-shaped settings, are yet to be explored.

To bridge the above gaps between research and practical implementation, this study aims to deal with the joint problem of balancing and scheduling assembly lines with HRC in straight and U-shaped layouts, considering different objectives. The considered objectives are minimizing the *NS* (Type-1), the *CT* (Type-2), the cost of stations, operators, and energy consumption of robots (Type-rw). In the latter type, the HRC can benefit from incorporating multiple humans and robots with different skills and capabilities while performing the tasks at stations collaboratively and in parallel. Moreover, other real-world aspects will be considered, namely robot energy cost, robot tool changes, zoning, and technological requirements. The optimization problems will be formulated as mixed-integer linear programming (MILP) models.

Moreover, efficient lower and upper bounds for each objective type are proposed to decrease the proposed MILPs' computational dimensionality and find promising solutions. The validity of the proposed models is shown in an application case study and different test problems taken from the literature. The computational experiments provide analyses of results in terms of layout, objectives, and HRC. Overall, the main contributions of this study can be summarized as follows:

- Considering straight and U-shaped assembly lines with HRC and different types of objectives, namely Type-1, Type-2, and Type-rw, the latter includes the cost of stations, operators, and robot energy consumption.
- Incorporating real-world constraints, namely multiple humans and robots with different skills per station, robot tool changes, technological requirements, and zoning of tasks.
- Proposing mixed-integer linear programming models with efficient lower and upper bounds for each objective and layout while testing them on a real case and a set of test problems.
- Analyzing the effect of objective, layout, and HRC scenarios.

The remainder of this paper is organized as follows. In Section 2, the related literature is reviewed. In Section 3, the description of the problem is presented. The mathematical formulations for balancing and

scheduling the U-shaped assembly lines with HRC and the efficient lower and upper bounds for each objective are presented in Section 4. Section 5 presents the computational study of the proposed models while solving a real-world case and test problems with further analysis of results in terms of objective, layout, and HRC. Finally, Section 6 outlines the conclusions and future research directions.

2. Literature review

This section reviews the relevant studies focusing only on balancing the assembly line with HRC (AL-HRC). The interested readers are referred to Boysen et al. (2021) and Battaia and Dolgui (2022) for a recent survey on ALBP literature.

In the study performed by Dalle Mura and Dini (2019), given a *CT*, the cost of the assembly line in terms of the number of stations, workers, robots, and workers' skills and energy load variance, were minimized. The authors developed a genetic algorithm (GA) to assign the tasks to stations with HRC capability in a scooter assembly line. Weckenborg and Spengler (2019) minimized the costs measured by the number of stations, humans, and robots while limiting the ergonomic load of stations formulated as a mathematical optimization model. Weckenborg et al. (2020) extended their previous study by minimizing the *CT* in which a hybrid GA and mathematical model were proposed with customized crossover, mutation, and repair operators to deal with larger test problems. Çil et al. (2020) dealt with the minimization of *CT* given the *NS* where only one robot was allowed, and no collaboration between humans and robots was permissible. Given the *CT*, Yaphiar et al. (2020) minimized the total cost of resources, including humans, robots, machinery, and equipment, by proposing a mathematical model for the mixed-model assembly line. Raatz et al. (2020) dealt with the balancing and scheduling assembly tasks in a real-case assembly line by proposing a GA-based framework to optimize the *CT* and the ergonomic load as the primary and secondary objectives, respectively.

In another study, Li et al. (2021) used a cost-oriented approach to optimize the *CT* and the costs of robot purchase and human salary. A mixed-integer mathematical programming model and multi-objective migration bird optimization algorithm were proposed to solve the problem. Gualtieri et al. (2021) proposed a systematic method for designing a human-robot collaborative assembly system by optimizing the *CT* in an industrial case study. Koltai et al. (2021) addressed different scenarios while shifting from the manual to HRC production systems by proposing mathematical models for optimizing the *NS* and *CT* in a real case. Dalle Mura and Dini (2021) considered job rotations and HRC to optimize costs in terms of the number of stations and equipment, including the cobots and ergonomic factors regarding the energy load variance among workers. The authors developed a GA to solve the assembly line balancing in an industrial case. Vieira et al. (2021) dealt with the planning and scheduling a human-robot assembly line in an industrial case study by developing a hierarchical optimization-simulation approach. The optimization method sequentially dealt with the planning and scheduling by minimizing the total costs and make-span. The simulation module dealt with the feasibility of the generated solutions by considering the dynamic operating situation in line layouts and the dispatching rules.

Stecke and Mokhtarzadeh (2022) studied the design of HRC assembly lines to optimize efficiency and ergonomic risks by considering mobile and immobile robots and zoning constraints. Three mathematical models were proposed to solve real-case and test problems, including mixed-integer programming, constraint programming, and Benders decomposition. Nourmohammadi et al. (2022) dealt with the balancing and scheduling tasks in the HRC assembly lines to optimize the *CT* and the number of operators while considering multiple humans and robots at the stations. A mathematical model and simulated annealing algorithm were proposed to solve case studies and test problems. Weckenborg et al. (2022) studied the economic and ergonomic aspects of AL-HRC when cobots and exoskeletons are utilized to

(a) A semi-assembled MBS and its components



(b) AL-HRC layout

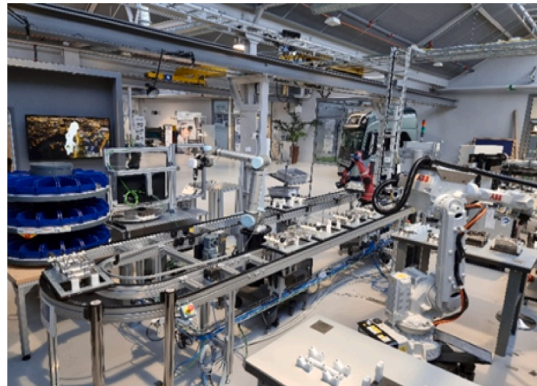


Fig. 1. The case study: (a) a semi-assembled MBS and (b) the AL-HRC layout.

optimize the cost and ergonomic risk measures while parallelizing tasks was not allowed in the presence of multiple operators. Kinast et al. (2022) considered the assembly line balancing with cobot assignment to stations in a real case. They proposed a CP formulation of the problem while assuming that only one cobot can be assigned to each station. They also presented a GA hybridized with variable neighborhood search to solve the problem in an extensive computational study. Sikora and Weckenborg (2022) studied the balancing of the AL-HRC using Benders decomposition. The authors developed some variants of Benders decomposition algorithms and tested them on a comprehensive range of test problems from the literature. The balancing U-shaped assembly line with HRC has been recently addressed by Li et al. (2023) to minimize the CT by assuming that one robot can be assigned to each station and parallel tasks by human and robot are not allowed. Three variants of mathematical models and two meta-heuristic algorithms were proposed to solve the problem. Mao et al. (2023) studied the U-shaped assembly line with HRC by assuming that each station can be equipped with one human and one robot. The problem was formulated as mixed-integer programming and a simulated annealing algorithm with enhanced search procedure was proposed to minimize the CT. Rahman et al. (2023) dealt with semi-automated ALBP with sustainability-based objectives with limited agents per station and uncertain processing times, while a memetic algorithm was proposed to solve the problem.

Based on the above literature, considering different objectives, this study is among the first attempts to balance and schedule assembly lines with HRC in straight and U-shaped layouts. This study extends the HRC level to benefit from incorporating multiple humans and robots with different skills and capabilities while performing the tasks at stations collaboratively and in parallel. Moreover, other real-world aspects, namely robot energy cost, robot tool changes, zoning, and technological requirements, are considered while addressing a case study and test

problems. The effects of layout, objectives, and HRC on the optimization results are also studied.

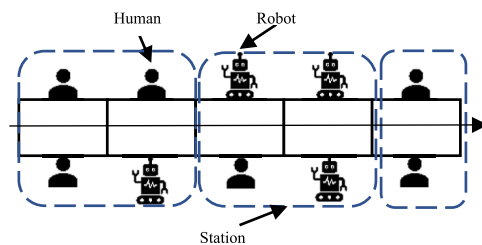
3. Problems description

This study originates from an application case in the pilot assembly line of the engine module of a well-known automotive manufacturer. The assembly line produces a mass balancing system (MBS) unit used by different engine models. It contains two machined aluminum halves with two axles assembled between them, as shown in Fig. 1 (a). Moreover, Fig. 1 (b) shows the assembly line layout of the application case with the cobots. The assembly line consists of stations with embedded sensing and interacting software and hardware that perform the tasks separately, in parallel, or collaboratively by multiple humans and robots with different skills and capabilities.

From the layout perspective, the stations in the considered assembly line can be ordered based on two main types of layouts, namely, (a) the straight line and (b) the U-shaped line, as graphically shown in Fig. 2. The stations are arranged serially in a straight line so that the tasks can only be assigned in one direction from the left to the right. On the other hand, in the U-shaped lines, the tasks are assigned both on the entrance and exit sides of the line, allowing the operators to serve in both directions, increasing flexibility and productivity (Boysen et al., 2022). The configuration of AL-HRC entails solving three joint decisions, namely, (1) assignment of tasks to stations, (2) assignment of operators (humans and robots) to stations, and (3) scheduling of tasks among operators.

The current study considers three different objective types to be optimized depending on the nature of the problem, as discussed below:

(a) Straight line



(b) U-shaped line

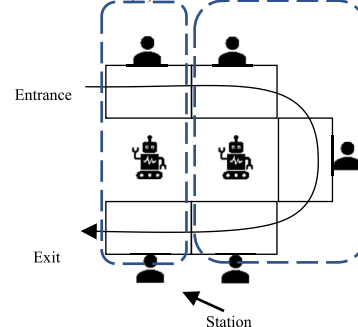


Fig. 2. The two layouts of AL-HRC: (a) Straight line and (b) U-shaped line.

Table 1
The problems and their specifications considered in this study.

Problem	Assembly line (AL)	Layout	Objective NS (Type-1)	CT (Type-2)	Cost (Type-rw)	Constraint Precedence relations	Multiple H&R	Collaborative tasks	Robot tool requirement	Zoning	Technological requirement	Abbreviation of problems
Balancing and scheduling (BS)	AL-HRC	Straight (S)	X	X		X	X					BS-AL-HRC-S-1
						X	X					BS-AL-HRC-S-2
		U-shape (U)	X	X	X	X	X	X	X	X	X	BS-AL-HRC-S-rw
						X	X					BS-AL-HRC-U-1
						X	X	X	X	X	X	BS-AL-HRC-U-2
					X	X	X	X	X	X	X	BS-AL-HRC-U-rw

- *Type-1*: Minimizes the *NS* given a *CT*. Traditionally, this objective arises when a new AL-HRC has to be established. The resulting optimization problem aims to find the lowest *NS* according to which the target *CT* can be satisfied, leading to higher line efficiency.
- *Type-2*: Minimizes the *CT* given an *NS*. Traditionally, this objective arises when an existing AL-HRC attempts to respond to customer demand by minimizing the *CT* to react to the market demand. The resulting optimization problem aims to find the lowest *CT* according to which the target *NS* can be satisfied.
- *Type-rw*: Minimizes the total costs of stations, operators, and robot energy consumption given a *CT*. Unlike the above types that consider the theoretical aspects of AL-HRC, this objective arises when a real-world AL-HRC has to be established in which the total cost (*TC*) of stations, operators, and robot energy consumption is minimized. With a cost-oriented objective, a trade-off is reached between the fixed cost of stations and operators and the operational cost of cobot energy consumption while performing the tasks. The energy consumption of each robot while performing each task is calculated by considering the task time, robot standard energy consumption in a unit of time, and the cost of each unit of energy consumed in the manufacturing company.

Each of these objectives results in new types of optimization problems for balancing and scheduling AL-HRC in which some or all of the following assumptions/constraints are considered:

- *Precedence relations*: The assignment of tasks to the stations and operators and the scheduling of tasks are subjected to the precedence relationships among tasks.
- *Multiple humans and robots*: Each station can be equipped with multiple humans and robots thanks to HRC safety sensors and interacting technologies. A set of available operators (humans and robots) at each station can perform the assembly tasks separately, in parallel, or collaboratively. The task times by each human and robot are determined based on their related skills and capabilities. The traveling times of operators are not considered.
- *Collaborative tasks*: These sets of tasks have to be performed by HRC in which a team consisting of a human and a robot is scheduled to perform them collaboratively.
- *Robot tool requirement*: This constraint considers the robot tool requirement while performing different tasks. Accordingly, each robot's total number of tool changes per station is limited to a fixed number (e.g., three) to avoid a higher number of tool changes.
- *Zoning constraint*: This constraint considers that two linked tasks (e.g., using a screwdriver to tighten two screws) are performed at the same station.
- *Technological requirement*: This attempts to ensure that a task that requires a specific technology available at a particular station cannot be performed at other stations.

Table 1 presents the specifications of the considered optimization problems in this study in terms of layout, objective, and constraint with the related abbreviations referred to hereafter.

4. Mathematical models for balancing and scheduling AL-HRC

This section deals with the mathematical formulation of the optimization problems discussed in the previous section, followed by the lower and upper bounds proposed for the objectives. Table 2 shows the notations used in the mathematical modeling of different optimization problems.

4.1. Straight AL-HRC

Assuming a straight assembly line with serial stations where a set of tasks ($i = 1, \dots, NT$) are assigned to a group of stations ($j = 1, \dots, NS$) to be

Table 2

The notation used in the formulation of BS-AL-HRC-S and BS-AL-HRC-U models.

Notation	Definition
Indices:	
i, h, f	Set of tasks ($i = 1, \dots, NT$)
j, g	Set of stations ($j = 1, \dots, NS$)
k, l	Set of humans ($k = 1, \dots, NH$)
r, q	Set of robots ($r = 1, \dots, NR$)
d	Set of directions ($d = 1, \dots, D$) in U-shaped line; $d = 1$ forward direction and $d = 2$ backward direction
Parameters:	
NT	Number of tasks
NS	Number of stations
NH	Maximum number of humans that can be assigned to each station
NR	Maximum number of robots that can be assigned to each station
th_{ik}	Time of task i if performed by human k
tr_{ir}	Time of task i if performed by robot r
$Pr(i)$	Set of immediate predecessors of task i
$Prall(i)$	Set of all predecessors of task i
$Su(i)$	Set of immediate successors of task i
$Suall(i)$	Set of all successors of task i
$PSall(i)$	Set of all predecessors and successors of task i
cos_j	Cost of station j based on space and equipment costs per year
coh_l	Cost of human l based on the yearly salary
cor_r	Cost of robot r based on the purchase cost per year
En_{ir}	Cost of energy consumption of robot r to perform task i per year
ZC^+_{ih}, ZC^-_{ih}	$\begin{cases} 1; \text{If task } i \text{ and } h \text{ have to be performed } (ZC^+) \text{ or not be performed } (ZC^-) \text{ at the same station} \\ 0; \text{otherwise} \end{cases}$
TER_{ij}	$\begin{cases} 1; \text{If task } i \text{ can not be performed in station } j \\ 0; \text{otherwise} \end{cases}$
JT_{ih}	$\begin{cases} 1; \text{If task } i \text{ and } h \text{ have to be performed by human and robot collaboration} \\ 0; \text{otherwise} \end{cases}$
RTC_{ih}	$\begin{cases} 1; \text{If task } i \text{ and } h \text{ need a robot tool change if performed by a robot} \\ 0; \text{otherwise} \end{cases}$
M	A large number
RTC_{max}	Maximum tool changes allowed for each robot at each station
Decision variables:	
Binary:	
x_{ijk}	$\begin{cases} 1; \text{If task } i \text{ is assigned to human } k \text{ in station } j \text{ in a straight line} \\ 0; \text{otherwise} \end{cases}$
y_{ijr}	$\begin{cases} 1; \text{If task } i \text{ is assigned to robot } r \text{ in station } j \text{ in a straight line} \\ 0; \text{otherwise} \end{cases}$
x_{ijkd}	$\begin{cases} 1; \text{If task } i \text{ is assigned to human } k \text{ in station } j \text{ in direction } d \text{ in a U-shaped line} \\ 0; \text{otherwise} \end{cases}$
y_{ijrd}	$\begin{cases} 1; \text{If task } i \text{ is assigned to robot } r \text{ in station } j \text{ in direction } d \text{ in a U-shaped line} \\ 0; \text{otherwise} \end{cases}$
u_{ih}	$\begin{cases} 1; \text{If task } i \text{ is executed before task } h \text{ in the sequence of tasks assigned to the same human} \\ 0; \text{otherwise} \end{cases}$
v_{ih}	$\begin{cases} 1; \text{If task } i \text{ is executed before task } h \text{ in the sequence of tasks assigned to the same robot} \\ 0; \text{otherwise} \end{cases}$
wk_{jk}	$\begin{cases} 1; \text{If human } k \text{ is used in station } j \\ 0; \text{otherwise} \end{cases}$
rk_{jr}	$\begin{cases} 1; \text{If robot } r \text{ is used in station } j \\ 0; \text{otherwise} \end{cases}$
ws_j	$\begin{cases} 1; \text{If station } j \text{ is opened} \\ 0; \text{otherwise} \end{cases}$
YY_{ihjr}	A binary-auxiliary variable for linearization in a straight line
YY_{ihjrd}	A binary-auxiliary variable for linearization in a U-shaped line
Real positive (\mathbb{R}^+):	
CT	Cycle time
t_i	Processing time of task i after being assigned to either a human or a robot
ST_i	Start time of task i
C_i	Completion time of task i
CW_j	Completion time of all tasks assigned to station j

performed by a set of humans ($l = 1, \dots, NH$) and robots ($r = 1, \dots, NR$). The processing time of task i when assigned to either human l or robot r is determined based on their skills and capabilities, which are presented by th_{il} or tr_{ir} , respectively. The following subsections describe the mathematical formulations of BS-AL-HRC-S-1, BS-AL-HRC-S-2, and BS-AL-HRC-S-rw problems in detail.

4.1.1. BS-AL-HRC-S-1

The mathematical formulation for BS-AL-HRC-S-1 as a mixed-integer linear programming (MILP) is presented below.

$$\text{Minimize } z_1 = \sum_{j=1}^{NS} ws_j \quad (1)$$

$$\sum_{j=1}^{NS} \sum_{l=1}^{NH} x_{ijl} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} y_{ijr} = 1; \forall i \quad (2)$$

$$\begin{aligned} & \sum_{g=1}^{NS} \sum_{l=1}^{NH} g \times x_{hgl} + \sum_{g=1}^{NS} \sum_{r=1}^{NR} g \times y_{hgr} \\ & \leq \sum_{j=1}^{NS} \sum_{l=1}^{NH} j \times x_{ijl} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} j \times y_{ijr}; \forall i, \forall h \in Pr(i) \end{aligned} \quad (3)$$

$$t_i = \sum_{j=1}^{NS} \sum_{l=1}^{NH} th_{il} \times x_{ijl} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} tr_{ir} \times y_{ijr}; \forall i \quad (4)$$

$$\begin{aligned} ST_i - ST_h + M \times (1 - \sum_{l=1}^{NH} x_{hil} - \sum_{r=1}^{NR} y_{hjr}) + M \times (1 - \sum_{l=1}^{NH} x_{ijl} \\ - \sum_{r=1}^{NR} y_{ijr}) \\ \geq t_h; \forall i, \forall j, \forall h \in Pr(i) \end{aligned} \quad (5)$$

$$\begin{aligned} ST_f - ST_i + M \times (1 - x_{fil}) + M \times (1 - x_{ijl}) + M \times (1 - u_{if}) \geq t_i; \forall i, \forall j, \forall l, \forall f \\ \notin \{PSall(i), i < f\} \end{aligned} \quad (6)$$

$$\begin{aligned} ST_i - ST_f + M \times (1 - x_{fjl}) + M \times (1 - x_{ijl}) + M \times u_{if} \geq t_f; \forall i, \forall j, \forall l, \forall f \\ \notin \{PSall(i), i < f\} \end{aligned} \quad (7)$$

$$\begin{aligned} ST_f - ST_i + M \times (1 - y_{fir}) + M \times (1 - y_{ijr}) + M \times (1 - v_{if}) \geq t_i; \forall i, \forall j, \forall r, \forall f \\ \notin \{PSall(i), i < f\} \end{aligned} \quad (8)$$

$$\begin{aligned} ST_i - ST_f + M \times (1 - y_{fir}) + M \times (1 - y_{ijr}) + M \times v_{if} \geq t_f; \forall i, \forall j, \forall r, \forall f \\ \notin \{PSall(i), i < f\} \end{aligned} \quad (9)$$

$$\sum_{i=1}^{NT} x_{ijl} - NT \times wk_{jl} \leq 0; \forall j, \forall l \quad (10)$$

$$\sum_{i=1}^{NT} y_{ijr} - NT \times rk_{jr} \leq 0; \forall j, \forall r \quad (11)$$

$$\sum_{l=1}^{NH} wk_{jl} - NH \times ws_j \leq 0; \forall j \quad (12)$$

$$\sum_{r=1}^{NR} rk_{jr} - NR \times ws_j \leq 0; \forall j \quad (13)$$

$$ws_j \geq ws_{j+1}; \forall j \quad (14)$$

$$ST_i + t_i \leq CT; \forall i \quad (15)$$

The objective function (1) minimizes the total number of stations. Equation (2) ensures that each task is assigned to only one operator, i.e., a human or a robot. Equation (3) imposes that all precedence relationships among tasks are satisfied. Equation (4) determines the processing time of each task after its assignment to either a human or a robot. The scheduling constraints are modeled with Equations (5) to (9). For every pair of tasks i and h , if task h is an immediate predecessor of task i , Equation (5) ensures that task i is started after finishing task h , in order to satisfy the precedence constraints. If two tasks i and f have no precedence relation but they are assigned to the same human or robot, either Equations (6) or (7) for the human or Equations (8) or (9) for the robot become active. For a human, if i is assigned earlier than f ($u_{if} = 1$), then Equation (6) become $ST_f - ST_i \geq t_i$. Otherwise, if f is assigned earlier than i , then Equation (7) becomes $ST_i - ST_f \geq t_f$. For a robot, if i is assigned earlier than f ($v_{if} = 1$), then Equation (8) becomes $ST_f - ST_i \geq t_i$. Otherwise, if f is assigned earlier than i , then Equation (9) becomes $ST_i - ST_f \geq t_f$. Using a sufficiently large number M , if two tasks are assigned to different stations, they are not considered in Equation (5), and if two tasks are assigned to different workers, they are not considered in Equations (6) and (7) and similarly if two tasks are assigned to different robots, they are not considered in Equations (8) and (9). Equations (10) and (11) ensure that if a task is assigned to the l -th worker or r -th robot, and that worker or robot is assigned to the j -th station, then wk_{jl} and rk_{jr} must be equal to 1, respectively. Equation (12) verifies the use of stations; if at least one worker is used in the j -th station, ws_j will be equal to 1. Besides, such constraint ensures that the maximum number of workers that can be assigned to each station is NH . Similarly, Equation (13) satisfies the above condition for the robot while ensuring that the maximum number of robots assigned to each station is

NR . Equation (14) deals with the index of stations and ensures that stations are loaded in increasing order of their indexes. Equation (15) is the cycle time constraint, which ensures that each task has to be completed within the given cycle time.

4.1.2. BS-AL-HRC-S-2

The MILP formulation of BS-AL-HRC-S-2 is given as follows.

$$\text{Minimize } z_2 = CT \quad (16)$$

Equations (2)-(11)

$$\sum_{l=1}^{NH} wk_{jl} - NH \leq 0; \forall j \quad (17)$$

$$\sum_{r=1}^{NR} rk_{jr} - NR \leq 0; \forall j \quad (18)$$

$$ST_i + t_i \leq C_i; \forall i \quad (19)$$

$$C_i \leq CW_j + M \times \left(1 - \sum_{l=1}^{NH} x_{ijl} - \sum_{r=1}^{NR} y_{ijr}\right); \forall i, \forall j \quad (20)$$

$$CW_j \leq CT; \forall j \quad (21)$$

The objective function in (16) minimizes the CT . Constraints (17) and (18) guarantee that the numbers of humans and robots assigned to each station do not exceed the maximum number of humans and the maximum number of robots, respectively. Constraint (19) determines the completion time of each task. Constraint (20) specifies the completion time of the whole set of tasks that are assigned to a station. Constraint (21) is the cycle time constraint.

4.1.3. BS-AL-HRC-S-rw

The mathematical formulation of BS-AL-HRC-S-rw is given as follows.

$$\begin{aligned} \text{Minimize } z_3 = \sum_{j=1}^{NS} cos_j \times ws_j + \sum_{j=1}^{NS} \sum_{l=1}^{NH} coh_l \times wk_{jl} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} cor_r \\ \times rk_{jr} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} \sum_{i=1}^{NT} En_{ir} \times y_{ijr} \end{aligned} \quad (22)$$

Equations (2)-(15)

$$\sum_{k=1}^{NH} \sum_{r=1}^{NR} (x_{ijk} + y_{ijr}) = \sum_{k=1}^{NH} \sum_{r=1}^{NR} (x_{hjk} + y_{hjr}); \forall i, \forall j, \forall h \in (ZC_{ih}^+ = 1) \quad (23)$$

$$\sum_{k=1}^{NH} \sum_{r=1}^{NR} (x_{ijk} + y_{ijr}) + \sum_{k=1}^{NH} \sum_{r=1}^{NR} (x_{hjk} + y_{hjr}) \leq 1; \forall i, \forall j, \forall h \in (ZC_{ih}^- = 1) \quad (24)$$

$$\sum_{k=1}^{NH} \sum_{r=1}^{NR} x_{ijk} + y_{ijr} = 0; \forall i, \forall j \in TER_{ij} = 1 \quad (25)$$

$$ST_i = ST_h; \forall i, \forall h \in JT_{ih} = 1 \quad (26)$$

$$\sum_{k=1}^{NH} x_{ijk} + \sum_{k=1}^{NH} x_{hjk} \leq 1; \forall i, \forall j, \forall h \in JT_{ih} = 1 \quad (27)$$

$$\sum_{r=1}^{NR} y_{ijr} + \sum_{r=1}^{NR} y_{hjr} \leq 1; \forall i, \forall j, \forall h \in JT_{ih} = 1 \quad (28)$$

$$\sum_{k=1}^{NH} x_{ijk} + \sum_{r=1}^{NR} y_{ijr} = \sum_{k=1}^{NH} x_{hjk} + \sum_{r=1}^{NR} y_{hjr}; \forall i, \forall j, \forall h \in JT_{ih} = 1 \quad (29)$$

$$\sum_{i=1}^{NT} \sum_{h=i+1}^{NT} (y_{ijr} \times y_{hjr}) \times RTC_{ih} \leq RTC_{max}; \forall j, \forall r \quad (30)$$

Equation (22) is a cost-oriented objective function regarding the costs of stations, humans, robots, and energy consumption. The positive zoning restrictions among tasks i and h , can be satisfied by Equation (23), while the negative zoning constraints can be ensured by Equation (24). Equation (25) considers the technological requirement between task i and station j given by a TER_{ij} matrix. Assuming that tasks i and h are collaborative tasks to be performed by both human and robot, the Equations (26) to (29) ensure that these tasks are started simultaneously while one human and one robot are specifically assigned to perform them collaboratively. Equation (30) limits the number of tool changes for each robot at each station to RTC_{max} while calculating the total tool switches between tasks i and h according to a given RTC_{ih} matrix.

To avoid the non-linearity term raised by Equation (30), a binary-auxiliary variable (YY_{ihjr}) can be introduced as shown in (31) with (32)-(34) added as its lower and upper bounds. Equation (30) can now be replaced by (35) as a linear Equation. The resulting equations form a MILP for BS-AL-HRC-S-rw.

$$YY_{ihjr} = y_{ijr} \times y_{hjr}; \forall i, \forall h, \forall j, \forall r \quad (31)$$

$$YY_{ihjr} \leq y_{ijr}; \forall i, \forall h, \forall j, \forall r \quad (32)$$

$$YY_{ihjr} \leq y_{hjr}; \forall i, \forall h, \forall j, \forall r \quad (33)$$

$$YY_{ihjr} \geq y_{ijr} + y_{hjr} - 1; \forall i, \forall h, \forall j, \forall r \quad (34)$$

$$\sum_{i=1}^{NT} \sum_{h=i+1}^{NT} YY_{ihjr} \times RTC_{ih} \leq RTC_{max}; \forall j, \forall r \quad (35)$$

4.2. U-shaped AL-HRC

An essential advantage of AL-HRC with a U-shaped layout is that the tasks and operators (humans and robots) can be both assigned to the entrance (forward) and the exit (backward) of the line, as shown in Fig. 2. Assuming an AL-HRC where the stations are arranged in a U-shaped layout, the BS-AL-HRC attempts to assign a set of tasks ($i = 1, \dots, NT$) to a set of stations ($j = 1, \dots, NS$) while scheduling the tasks at each station to be performed by a group of humans ($l = 1, \dots, NH$) and robots ($r = 1, \dots, NR$) with different competency level. The processing time of task i when assigned to either human l or robot r is determined based on their skills and capabilities, which are presented by th_{il} or tr_{ir} , respectively. The notations specific to the formulation of BS-AL-HRC-U are also defined in Table 2. In the following subsections, the mathematical formulations of BS-AL-HRC-U-1, BS-AL-HRC-U-2, and BS-AL-HRC-U-rw problems are described in detail.

4.2.1. BS-AL-HRC-U-1

The proposed MILP for BS-AL-HRC-U-1 is presented below.

$$\text{Minimize } z_1 = \sum_{j=1}^{NS} ws_j \quad (1)$$

$$\sum_{j=1}^{NS} \sum_{l=1}^{NH} \sum_{d=1}^D x_{ijld} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} \sum_{d=1}^D y_{ijrd} = 1; \forall i \quad (36)$$

$$\begin{aligned} & \sum_{g=1}^{NS} (NS - g + 1) \times \left(\sum_{l=1}^{NH} x_{hgld} + \sum_{r=1}^{NR} y_{hgld} \right) \\ & \geq \sum_{g=1}^{NS} (NS - g + 1) \times \left(\sum_{l=1}^{NH} x_{igld} + \sum_{r=1}^{NR} y_{igld} \right); \forall i, \forall h \in Pr(i), d = 1 \end{aligned} \quad (37)$$

$$\begin{aligned} & \sum_{g=1}^{NS} (NS - g + 1) \times \left(\sum_{l=1}^{NH} x_{igld} + \sum_{r=1}^{NR} y_{igld} \right) \\ & \geq \sum_{g=1}^{NS} (NS - g + 1) \times \left(\sum_{l=1}^{NH} x_{hgld} + \sum_{r=1}^{NR} y_{hgld} \right); \forall i, \forall h \in Pr(i), d = 2 \end{aligned} \quad (38)$$

$$t_i = \sum_{j=1}^{NS} \sum_{l=1}^{NH} \sum_{d=1}^D th_{il} \times x_{ijld} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} \sum_{d=1}^D tr_{ir} \times y_{ijrd}; \forall i \quad (39)$$

$$ST_i - ST_h + M \times \left(1 - \sum_{l=1}^{NH} x_{ijld} - \sum_{r=1}^{NR} y_{ijrd} \right) +$$

$$M \times \left(1 - \sum_{l=1}^{NH} x_{hjld} - \sum_{r=1}^{NR} y_{hjrd} \right) \geq t_h; \forall i, \forall h \in Pr(i), \forall j, d = 1 \quad (40)$$

$$ST_h - ST_i + M \times \left(1 - \sum_{l=1}^{NH} x_{hjld} - \sum_{r=1}^{NR} y_{hjrd} \right) +$$

$$M \times \left(1 - \sum_{l=1}^{NH} x_{ijld} - \sum_{r=1}^{NR} y_{ijrd} \right) \geq t_i; \forall i, \forall h \in Pr(i), \forall j, d = 2 \quad (41)$$

$$\begin{aligned} & ST_f - ST_i + M \times \left(1 - \sum_{d=1}^D x_{fjld} \right) + M \times \left(1 - \sum_{d=1}^D x_{ijld} \right) + M \times (1 - u_{if}) \\ & \geq t_i; \forall i, \forall f \notin \{PSall(i), i < f\}, \forall j, \forall l \end{aligned} \quad (42)$$

$$\begin{aligned} & ST_i - ST_f + M \times \left(1 - \sum_{d=1}^D x_{ijld} \right) + M \times \left(1 - \sum_{d=1}^D x_{fjld} \right) + M \times u_{if} \\ & \geq t_f; \forall i, \forall f \notin \{PSall(i), i < f\}, \forall j, \forall l \end{aligned} \quad (43)$$

$$\begin{aligned} & ST_f - ST_i + M \times \left(1 - \sum_{d=1}^D y_{fjrd} \right) + M \times \left(1 - \sum_{d=1}^D y_{ijrd} \right) + M \times (1 - v_{if}) \\ & \geq t_i; \forall i, \forall f \notin \{PSall(i), i < f\}, \forall j, \forall r \end{aligned} \quad (44)$$

$$\begin{aligned} & ST_i - ST_f + M \times \left(1 - \sum_{d=1}^D y_{ijrd} \right) + M \times \left(1 - \sum_{d=1}^D y_{fjrd} \right) + M \times v_{if} \\ & \geq t_f; \forall i, \forall f \notin \{PSall(i), i < f\}, \forall j, \forall r \end{aligned} \quad (45)$$

$$\begin{aligned} & ST_f - ST_i + M \times \left(1 - \sum_{d=1}^D x_{fjld} \right) + M \times \left(1 - \sum_{d=1}^D x_{ijld} \right) + M \times (1 - u_{if}) \\ & \geq t_i; \forall i, \forall f \in \{PSall(i), i < f\}, \forall j, \forall l \end{aligned} \quad (46)$$

$$\begin{aligned} & ST_i - ST_f + M \times \left(1 - \sum_{d=1}^D x_{ijld} \right) + M \times \left(1 - \sum_{d=1}^D x_{fjld} \right) + M \times u_{if} \\ & \geq t_f; \forall i, \forall f \in \{PSall(i), i < f\}, \forall j, \forall l \end{aligned} \quad (47)$$

$$\begin{aligned} & ST_f - ST_i + M \times \left(1 - \sum_{d=1}^D y_{fjrd} \right) + M \times \left(1 - \sum_{d=1}^D y_{ijrd} \right) + M \times (1 - v_{if}) \\ & \geq t_i; \forall i, \forall f \in \{PSall(i), i < f\}, \forall j, \forall r \end{aligned} \quad (48)$$

$$\begin{aligned} & ST_i - ST_f + M \times \left(1 - \sum_{d=1}^D y_{ijrd} \right) + M \times \left(1 - \sum_{d=1}^D y_{fjrd} \right) + M \times v_{if} \\ & \geq t_f; \forall i, \forall f \in \{PSall(i), i < f\}, \forall j, \forall r \end{aligned} \quad (49)$$

Table 3

The lower and upper bounds of the objectives for each model type.

Model type	Objective lower bound	Objective upper bound
Type-1	$NS^{LB} = \begin{cases} \frac{\sum_{i=1}^{NT} \min_{k \in NH, r \in NR} (th_{ik}, tr_{ir})}{CT \times (NH + NR)}; & \text{if } NH > 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \min_{r \in NR} (tr_{ir})}{CT \times NR}; & \text{if } NH = 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \min_{k \in NH} (th_{ik})}{CT \times NH}; & \text{if } NH > 0 \text{ and } NR = 0 \end{cases}$	$NS^{UB} = \begin{cases} \frac{\sum_{i=1}^{NT} \max_{k \in NH, r \in NR} (th_{ik}, tr_{ir})}{CT}; & \text{if } NH > 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \max_{r \in NR} (tr_{ir})}{CT}; & \text{if } NH = 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \max_{k \in NH} (th_{ik})}{CT}; & \text{if } NH > 0 \text{ and } NR = 0 \end{cases}$
Type-2	$CT^{LB} = \begin{cases} \frac{\sum_{i=1}^{NT} \min_{k \in NH, r \in NR} (th_{ik}, tr_{ir})}{NS \times (NH + NR)}; & \text{if } NH > 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \min_{r \in NR} (tr_{ir})}{NS \times NR}; & \text{if } NH = 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \min_{k \in NH} (th_{ik})}{NS \times NH}; & \text{if } NH > 0 \text{ and } NR = 0 \end{cases}$	$CT^{UB} = \begin{cases} \frac{\sum_{i=1}^{NT} \max_{k \in NH, r \in NR} (th_{ik}, tr_{ir})}{NS}; & \text{if } NH > 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \max_{r \in NR} (tr_{ir})}{NS}; & \text{if } NH = 0 \text{ and } NR > 0 \\ \frac{\sum_{i=1}^{NT} \max_{k \in NH} (th_{ik})}{NS}; & \text{if } NH > 0 \text{ and } NR = 0 \end{cases}$
Type-rw	$TC^{LB} = \sum_{j=1}^{NS^{LB}} \cos_j + \sum_{j=1}^{NS^{LB}} \min_{k \in NH} (coh_k) + \sum_{j=1}^{NS^{LB}} \min_{r \in NR} (cor_r)$	$TC^{UB} = \sum_{j=1}^{NS^{UB}} \cos_j + \sum_{j=1}^{NS^{UB}} \sum_{l=1}^{NH} coh_l + \sum_{j=1}^{NS^{UB}} \sum_{r=1}^{NR} cor_r + \sum_{i=1}^{NT} \max_{r \in NR} (En_{ir})$

$$\sum_{i=1}^{NT} x_{ijld} \leq NT \times wk_{jl}; \forall j, \forall l, \forall d \quad (50)$$

$$\sum_{i=1}^{NT} y_{ijrd} \leq NT \times rk_{jr}; \forall j, \forall r, \forall d \quad (51)$$

Equations (12)-(15)

Equation (36) ensures that each task can be assigned to only one operator and one direction (either forward or backward). Equations (37) and (38) satisfy the precedence relations among tasks while being assigned to the forward and backward directions, respectively. Constraint (39) determines the processing time of each task after it has been assigned to a specific human or robot. Equations (40) and (41) perform the scheduling of tasks when they have direct (immediate) precedence relationships while being assigned to the forward and backward directions, respectively. Equations (42) and (43) guarantee that if two tasks have no precedence relationships (neither direct nor indirect) and are performed by a human, then their start times are scheduled one after the other. Equations (44) and (45) are similar to (42) and (43), while a robot performs the tasks. When tasks have indirect precedence relationships, Equations (46) and (47) ensure that their start times are scheduled one after the other when performed by a human, while equations (48) and (49) schedule them when performed by a robot. Equation (50) ensures that if a task is assigned to a station to be performed by a human, the related human has been assigned to that station. Similarly, Equation (51) satisfies the above condition for a robot operator.

4.2.2. BS-AL-HRC-U-2

The proposed MILP model for BS-AL-HRC-U-2 is presented as follows:

$$\text{Minimize } z_2 = CT \quad (16)$$

Equations (36)-(51)

Equations (17) to (19), (21)

$$C_i \leq CW_j + M \times \left(1 - \sum_{l=1}^{NH} \sum_{d=1}^D x_{ijld} - \sum_{r=1}^{NR} \sum_{d=1}^D y_{ijrd} \right); \forall i, \forall j \quad (52)$$

The definition of Equation (52) is similar to (20) in Section 4.1.2, in which the completion time of each station in terms of the times of the assigned tasks is calculated.

4.2.3. BS-AL-HRC-U-rw

The proposed MILP model for BS-AL-HRC-U-rw is given below.

$$\begin{aligned} \text{Minimize } z_3 = & \sum_{j=1}^{NS} \cos_j \times ws_j + \sum_{j=1}^{NS} \sum_{l=1}^{NH} coh_l \times wk_{jl} + \sum_{j=1}^{NS} \sum_{r=1}^{NR} cor_r \\ & \times rk_{jr} + \sum_{i=1}^{NT} \sum_{j=1}^{NS} \sum_{r=1}^{NR} \sum_{d=1}^D En_{ir} \times y_{ijrd} \end{aligned} \quad (53)$$

Equations (36)-(51)

Equations (12)-(15)

$$\begin{aligned} \sum_{k=1}^{NH} \sum_{r=1}^{NR} \sum_{d=1}^D x_{ijkd} + y_{ijrd} = & \sum_{k=1}^{NH} \sum_{r=1}^{NR} \sum_{d=1}^D x_{hjkd} + y_{hjrd}; \forall i, \forall j, \forall h \in (ZC_{ih}^{+} \\ & = 1) \end{aligned} \quad (54)$$

$$\begin{aligned} \sum_{k=1}^{NH} \sum_{r=1}^{NR} \sum_{d=1}^D x_{ijkd} + y_{ijrd} + \sum_{k=1}^{NH} \sum_{r=1}^{NR} \sum_{d=1}^D x_{hjkd} + y_{hjrd} \leq & 1; \forall i, \forall j, \forall h \\ \in (ZC_{ih}^{-} = 1) \end{aligned} \quad (55)$$

$$\sum_{k=1}^{NH} \sum_{r=1}^{NR} \sum_{d=1}^D x_{ijkd} + y_{ijrd} = 0; \forall i, \forall j \in TER_{ij} = 1 \quad (56)$$

$$ST_i = ST_h; \forall i, \forall h \in JT_{ih} = 1 \quad (57)$$

$$\sum_{k=1}^{NH} \sum_{d=1}^D x_{ijkd} + \sum_{k=1}^{NH} \sum_{d=1}^D x_{hjkd} \leq 1; \forall i, \forall j, \forall h \in JT_{ih} = 1 \quad (58)$$

$$\sum_{r=1}^{NR} \sum_{d=1}^D y_{ijrd} + \sum_{r=1}^{NR} \sum_{d=1}^D y_{hjrd} \leq 1; \forall i, \forall j, \forall h \in JT_{ih} = 1 \quad (59)$$

$$\begin{aligned} \sum_{k=1}^{NH} \sum_{d=1}^D x_{ijkd} + \sum_{r=1}^{NR} \sum_{d=1}^D y_{ijrd} = & \sum_{k=1}^{NH} \sum_{d=1}^D x_{hjkd} + \sum_{r=1}^{NR} \sum_{d=1}^D y_{hjrd}; \forall i, \forall j, \forall h \in JT_{ih} \\ & = 1 \end{aligned} \quad (60)$$

$$YY_{ihjrd} \leq y_{ijrd}; \forall i, \forall h, \forall j, \forall r, \forall d \quad (61)$$

$$YY_{ihjrd} \leq y_{hjrd}; \forall i, \forall h, \forall j, \forall r, \forall d \quad (62)$$

$$YY_{ihjrd} \geq y_{ijrd} + y_{hjrd} - 1; \forall i, \forall h, \forall j, \forall r, \forall d \quad (63)$$

Table 4

The experimental settings in the case study and test problems.

Size	Problem	NT	CT	NS	NH	NR	Instances	Time limit (s)
Small	Case study	28	43	2	0,1,2	0,1,2	C1,C2,C3,C4,C5,C6,C7,C8	3600
	Bowman	8	25	2	0,1,2	0,1,2	T1,T2,T3,T4,T5,T6,T7,T8	3600
	Jackson	11	8	2	0,1,2	0,1,2	T9,T10,T11,T12,T13,T14,T15,T16	
	Roszieg	25	22	3	0,1,2	0,1,2	T17,T18,T19,T20,T21,T22,T23,T24	
Medium	Buxey	29	55	3	0,1,2	0,1,2	T25,T26,T27,T28,T29,T30,T31,T32	
	Sawyer	30	38	4	0,1,2	0,1,2	T33,T34,T35,T36,T37,T38,T39,T40	7200
	Gunther	35	81	4	0,1,2	0,1,2	T41,T42,T43,T44,T45,T46,T47,T48	
	Kilbridge	45	92	5	0,1,2	0,1,2	T49,T50,T51,T52,T53,T54,T55,T56	
Large	Hahn	53	2806	6	0,1,2	0,1,2	T57,T58,T59,T60,T61,T62,T63,T64	
	Tonge	70	1170	3	0,1,2	0,1,2	T65,T66,T67,T68,T69,T70,T71,T72	10,800
	Arcus1	83	25,236	3	0,1,2	0,1,2	T73,T74,T75,T76,T77,T78,T79,T80	
	Arcus2	111	50,133	3	0,1,2	0,1,2	T81,T82,T83,T84,T85,T86,T87,T88	

$$\sum_{i=1}^{NT} \sum_{h=i+1}^{NT} YY_{ihjrd} \times RTC_{ih} \leq RTC_{max}; \forall j, \forall r, \forall d \quad (64)$$

Equation (53) calculates the cost-oriented objective regarding the costs of stations, humans, robots, and robots' energy consumption. Equations (54) and (55) ensure that the positive and negative zoning constraints among tasks are satisfied, respectively. The technological requirements among tasks are guaranteed by Equation (56). Equations (57) to (60) ensure that HRC schedules the collaborative tasks simultaneously. Equations (61) to (64) satisfy the robot tool changes constraints by introducing a binary-auxiliary variable (YY_{ihjrd}), similar to the

linearization steps taken in [Section 4.1.3](#).

4.3. Proposed lower and upper bounds for objectives

To decrease the computational dimensionality of the proposed MILPs and find promising solutions while solving such complex models and extensions, six bounds, including three lower and three upper bounds, are introduced here to limit the objective ranges at each model type, as shown in [Table 3](#).

In this table, for Type-1, the lower bound and the upper bound of the objective, namely NS^{LB} and NS^{UB} , are calculated based on the minimum

Table 5

The results of MILP-based models for different problems and case instances.

Problem	Case instance	CT / NS	NH	NR	Model MILP	GAP (%)	MILP ^{LB-UB}	GAP (%)
					NS* / CT*/TC*		NS* / CT*/TC*	
BS-AL-HRC-S-1	C1	CT = 43	0	1	6.0	0.50	6.0	0.00
	C2		0	2	4.0	0.25	4.0	0.25
	C3		1	0	3.0	0.00	3.0	0.00
	C4		1	1	2.0	0.00	2.0	0.00
	C5	NS = 2	1	2	2.0	0.00	2.0	0.00
	C6		2	0	2.0	0.00	2.0	0.00
	C7		2	1	2.0	0.00	2.0	0.00
	C8		2	2	2.0	0.50	2.0	0.50
BS-AL-HRC-S-2	C1	NS = 2	0	1	108.5	0.00	108.5	0.00
	C2		0	2	65.6	0.39	65.6	0.17
	C3		1	0	57.4	0.25	57.4	0.00
	C4		1	1	34.0	0.17	34.0	0.15
	C5	CT = 43	1	2	26.6	0.05	26.6	0.05
	C6		2	0	34.0	0.24	34.0	0.15
	C7		2	1	24.6	0.00	24.6	0.00
	C8		2	2	24.6	0.00	24.6	0.00
BS-AL-HRC-S-rw	C4	CT = 43	1	1	4509.9	0.03	4509.9	0.00
	C5		1	2	4509.9	0.03	4509.9	0.03
	C7		2	1	4509.9	0.03	4509.9	0.03
	C8		2	2	4553.8	0.29	4509.9	0.36
BS-AL-HRC-U-1	C1	CT = 43	0	1	6.0	0.66	6.0	0.00
	C2		0	2	4.0	0.50	4.0	0.25
	C3		1	0	3.0	0.33	3.0	0.00
	C4		1	1	2.0	0.50	2.0	0.00
	C5	NS = 2	1	2	2.0	0.50	2.0	0.50
	C6		2	0	2.0	0.50	2.0	0.00
	C7		2	1	2.0	0.50	2.0	0.50
	C8		2	2	1.0	0.00	1.0	0.00
BS-AL-HRC-U-2	C1	NS = 2	0	1	108.5	0.00	108.5	0.00
	C2		0	2	64.4	0.62	64.4	0.15
	C3		1	0	57.4	0.49	57.4	0.00
	C4		1	1	32.4	0.59	32.4	0.11
	C5	CT = 43	1	2	24.0	0.45	24.0	0.20
	C6		2	0	32.4	0.35	32.4	0.11
	C7		2	1	24.0	0.48	24.0	0.20
	C8		2	2	22.0	0.42	22.0	0.34
BS-AL-HRC-U-rw	C4	CT = 43	1	1	4509.8	0.44	4509.6	0.02
	C5		1	2	4509.6	0.47	4707.2	0.49
	C7		2	1	4509.8	0.47	4553.8	0.48
	C8		2	2	3212.1	0.26	3212.0	0.26

Table 6

The results of MILP-based models for different problems and test instances.

Problem	Size	Test instance	CT/NS	NH	NR	Model		MILP ^{LB-UB}	
						NS*/CT*/TC*	GAP (%)	NS*/CT*/TC*	GAP (%)
BS-AL-HRC-S-1	Small	T1	CT = 25	0	1	4.0	0.00	4.0	0.00
		T2		0	2	4.0	0.00	4.0	0.00
		T3		1	0	2.0	0.00	2.0	0.00
		T4		1	1	2.0	0.00	2.0	0.00
		T5		1	2	2.0	0.00	2.0	0.00
		T6		2	0	2.0	0.00	2.0	0.00
		T7		2	1	2.0	0.00	2.0	0.00
		T8		2	2	2.0	0.00	2.0	0.00
		T9	CT = 8	0	1	7.0	0.00	7.0	0.00
		T10		0	2	7.0	0.00	7.0	0.00
		T11		1	0	3.0	0.00	3.0	0.00
		T12		1	1	3.0	0.00	3.0	0.00
		T13		1	2	3.0	0.00	3.0	0.00
		T14		2	0	3.0	0.00	3.0	0.00
		T15		2	1	3.0	0.00	3.0	0.00
		T16		2	2	3.0	0.00	3.0	0.00
		T17	CT = 22	0	1	6.0	0.00	6.0	0.00
		T18		0	2	5.0	0.00	5.0	0.00
		T19		1	0	3.0	0.00	3.0	0.00
		T20		1	1	3.0	0.00	3.0	0.00
		T21		1	2	2.0	0.00	2.0	0.00
		T22		2	0	3.0	0.00	3.0	0.00
		T23		2	1	2.0	0.00	2.0	0.00
		T24		2	2	2.0	0.00	2.0	0.00
		T25	CT = 55	0	1	6.0	0.00	6.0	0.00
		T26		0	2	5.0	0.00	5.0	0.00
		T27		1	0	3.0	0.00	3.0	0.00
		T28		1	1	3.0	0.33	3.0	0.33
		T29		1	2	2.0	0.00	2.0	0.00
		T30		2	0	3.0	0.33	3.0	0.33
	Medium	T31		2	1	2.0	0.00	2.0	0.00
		T32		2	2	2.0	0.00	2.0	0.00
		T33	CT = 38	0	1	9.0	0.00	9.0	0.00
		T34		0	2	7.0	0.00	7.0	0.00
		T35		1	0	5.0	0.20	5.0	0.00
		T36		1	1	3.0	0.00	3.0	0.00
		T37		1	2	3.0	0.00	3.0	0.00
		T38		2	0	3.0	0.00	3.0	0.00
		T39		2	1	3.0	0.00	3.0	0.00
		T40		2	2	3.0	0.00	3.0	0.00
		T41	CT = 81	0	1	7.0	0.00	7.0	0.00
		T42		0	2	6.0	0.00	6.0	0.00
		T43		1	0	3.0	0.00	3.0	0.00
		T44		1	1	3.0	0.00	3.0	0.00
		T45		1	2	3.0	0.33	3.0	0.00
		T46		2	0	3.0	0.00	3.0	0.00
		T47		2	1	2.0	0.00	2.0	0.00
		T48		2	2	2.0	0.00	2.0	0.00
		T49	CT = 92	0	1	7.0	0.42	7.0	0.14
		T50		0	2	5.0	0.40	5.0	0.39
		T51		1	0	3.0	0.00	3.0	0.00
		T52		1	1	3.0	0.33	3.0	0.33
		T53		1	2	2.0	0.00	2.0	0.00
		T54		2	0	3.0	0.33	3.0	0.33
	Large	T55		2	1	2.0	0.00	2.0	0.00
		T56		2	2	2.0	0.00	2.0	0.00
		T57	CT = 2806	0	1	6.0	0.00	6.0	0.16
		T58		0	2	5.0	0.00	5.0	0.00
		T59		1	0	4.0	0.00	3.0	0.00
		T60		1	1	3.0	0.33	3.0	0.00
		T61		1	2	2.0	0.00	2.0	0.00
		T62		2	0	3.0	0.33	3.0	0.33
		T63		2	1	2.0	0.00	2.0	0.00
		T64		2	2	2.0	0.00	2.0	0.00
		T65	CT = 1170	0	1	4.0	0.49	4.0	0.25
		T66		0	2	3.0	0.33	3.0	0.33
		T67		1	0	2.0	0.50	2.0	0.00
		T68		1	1	NA ^o	NA	2.0	0.50
		T69		1	2	1.0	0.00	1.0	0.00
		T70		2	0	2.0	0.50	2.0	0.50
		T71		2	1	1.0	0.00	1.0	0.00

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model			
						MILP NS*/CT*/TC*	GAP (%)	MILP ^{LB-UB} NS*/CT*/TC*	GAP (%)
BS-AL-HRC-S-2	Small	T72	CT = 25236	2	2	NA	NA	1.0	0.00
		T73		0	1	4.0	0.25	4.0	0.25
		T74		0	2	NA	NA	3.0	0.33
		T75		1	0	2.0	0.00	2.0	0.00
		T76		1	1	NA	NA	2.0	0.00
		T77		1	2	NA	NA	2.0	0.00
		T78		2	0	NA	NA	2.0	0.00
		T79		2	1	NA	NA	2.0	0.00
		T80	CT = 50133	2	2	NA	NA	1.0	0.00
		T81		0	1	21.0	0.95	4.0	0.25
		T82		0	2	NA	NA	3.0	0.00
		T83		1	0	NA	NA	2.0	0.00
		T84		1	1	NA	NA	2.0	0.00
		T85		1	2	30.0	0.93	2.0	0.00
		T86		2	0	2.0	0.50	2.0	0.00
		T87		2	1	30.0	0.90	3.0	0.00
		T88		2	2	16.0	0.93	3.0	0.00
		T1	NS = 2	0	1	38.0	0.00	38.0	0.00
		T2		0	2	35.0	0.00	35.0	0.00
		T3		1	0	19.0	0.00	19.0	0.00
		T4		1	1	17.5	0.00	17.5	0.00
		T5		1	2	17.5	0.00	17.5	0.00
		T6		2	0	17.5	0.00	17.5	0.00
		T7		2	1	17.5	0.00	17.5	0.00
		T8		2	2	17.5	0.00	17.4	0.00
		T9	NS = 2	0	1	23.0	0.00	23.0	0.00
		T10		0	2	18.0	0.00	18.0	0.00
		T11		1	0	11.5	0.00	11.5	0.00
		T12		1	1	9.0	0.00	8.9	0.00
		T13		1	2	9.0	0.00	9.0	0.00
		T14		2	0	9.0	0.00	9.0	0.00
		T15		2	1	9.0	0.00	9.0	0.00
		T16		2	2	9.0	0.00	8.9	0.00
		T17	NS = 3	0	1	42.0	0.00	42.0	0.00
		T18		0	2	32.0	0.00	32.0	0.00
		T19		1	0	21.0	0.00	21.0	0.00
		T20		1	1	16.0	0.00	16.0	0.00
		T21		1	2	15.0	0.00	15.0	0.00
		T22		2	0	16.0	0.00	15.9	0.00
		T23		2	1	15.0	0.00	15.0	0.00
		T24		2	2	15.0	0.00	15.0	0.00
		T25	NS = 3	0	1	108.0	0.20	108.0	0.00
		T26		0	2	75.0	0.02	75.0	0.09
		T27		1	0	54.0	0.28	54.0	0.00
		T28		1	1	37.5	0.06	37.5	0.09
		T29		1	2	34.0	0.00	34.0	0.00
		T30		2	0	37.5	0.01	37.4	0.02
		T31		2	1	32.5	0.00	32.5	0.00
		T32		2	2	32.5	0.00	32.5	0.00
	Medium	T33	NS = 4	0	1	82.0	0.30	81.0	0.00
		T34		0	2	55.0	0.09	55.0	0.10
		T35		1	0	41.0	0.26	40.5	0.00
		T36		1	1	27.5	0.09	27.5	0.09
		T37		1	2	25.0	0.00	25.4	0.03
		T38		2	0	27.5	0.09	27.4	0.09
		T39		2	1	23.5	0.00	23.5	0.00
		T40		2	2	23.5	0.00	23.5	0.00
		T41	NS = 4	0	1	121.0	0.09	120.9	0.00
		T42		0	2	89.0	0.00	89.0	0.07
		T43		1	0	60.5	0.14	60.4	0.00
		T44		1	1	44.5	0.00	44.5	0.01
		T45		1	2	44.0	0.00	44.0	0.01
		T46		2	0	44.5	0.03	44.5	0.07
		T47		2	1	43.5	0.05	43.5	0.00
		T48		2	2	43.5	0.00	43.5	0.00
		T49	NS = 5	0	1	111.0	0.49	111.0	0.00
		T50		0	2	77.9	0.29	78.0	0.29
		T51		1	0	55.4	0.43	55.4	0.00
		T52		1	1	39.0	0.29	39.0	0.29
		T53		1	2	34.0	0.19	34.9	0.21
		T54		2	0	38.5	0.28	38.9	0.29
		T55		2	1	30.5	0.09	29.9	0.08
		T56		2	2	29.0	0.05	28.9	0.05

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model			
						MILP NS*/CT*/TC*	GAP (%)	MILP ^{LB-UB} NS*/CT*/TC*	GAP (%)
BS-AL-HRC-S-rw	Large	T57	NS = 6	0	1	2400.0	0.05	2400.0	0.02
		T58		0	2	2242.0	0.00	2242.0	0.00
		T59		1	0	1200.0	0.05	1200.0	0.00
		T60		1	1	1121.0	0.00	1121.0	0.02
		T61		1	2	1096.5	0.00	1096.5	0.00
		T62		2	0	1121.0	0.00	1121.0	0.00
		T63	NS = 3	2	1	1096.5	0.00	1096.5	0.00
		T64		2	2	1096.5	0.00	1096.5	0.00
		T65		0	1	1172.0	0.47	1173.0	0.00
		T66		0	2	797.0	0.38	786.0	0.25
		T67		1	0	585.5	0.50	585.4	0.00
		T68		1	1	393.5	0.46	395.9	0.26
		T69		1	2	363.0	0.31	344.4	0.24
		T70		2	0	398.4	0.38	395.9	0.26
		T71		2	1	318.5	0.24	316.5	0.24
		T72		2	2	304.5	0.19	318.4	0.22
		T73	NS = 3	0	1	25245.0	0.39	25266.9	0.00
		T74		0	2	17935.0	0.50	18062.0	0.30
		T75		1	0	12621.0	0.40	12623.5	0.00
		T76		1	1	9166.0	0.42	9039.4	0.27
		T77		1	2	9313.0	0.35	8288.4	0.17
		T78		2	0	8972.5	0.29	8995.5	0.24
		T79		2	1	7428.5	0.42	7494.0	0.08
		T80		2	2	7284.5	0.10	7044.4	0.00
		T81	NS = 3	0	1	50368.0	0.61	59119.9	0.15
		T82		0	2	38467.0	0.46	34831.9	0.28
		T83		1	0	25092.5	0.55	25086.9	0.00
		T84		1	1	26838.0	0.91	18912.9	0.33
		T85		1	2	23355.5	0.87	16068.5	0.48
		T86		2	0	17930.5	0.41	17340.5	0.27
		T87		2	1	21882.5	0.87	15520.4	0.35
		T88		2	2	19873.0	0.85	13840.9	0.27
	Small	T4	CT = 25	1	1	4351.3	0.00	4351.3	0.00
		T5		1	2	4351.3	0.00	4351.3	0.00
		T7		2	1	4351.3	0.00	4351.5	0.00
		T8		2	2	4351.3	0.00	4351.3	0.00
		T12	CT = 8	1	1	6350.3	0.00	6350.3	0.00
		T13		1	2	6350.3	0.00	6350.3	0.00
		T15		2	1	6350.3	0.00	6350.3	0.00
		T16		2	2	6350.3	0.00	6350.3	0.00
		T20	CT = 22	1	1	6354.6	0.00	6354.4	0.00
		T21		1	2	5360.9	0.00	5360.8	0.00
		T23		2	1	5204.7	0.00	5204.7	0.00
		T24		2	2	5204.7	0.03	5204.7	0.00
		T28	CT = 55	1	1	6351.8	0.21	6351.8	0.21
		T29		1	2	5379.3	0.19	5378.8	0.18
		T31		2	1	5051.8	0.06	5051.8	0.10
		T32		2	2	5051.8	0.06	4863.7	0.07
	Medium	T36	CT = 38	1	1	7526.4	0.08	7526.4	0.08
		T37		1	2	7526.4	0.08	7526.4	0.15
		T39		2	1	7205.5	0.09	7205.7	0.08
		T40		2	2	7205.7	0.09	7207.7	0.11
		T44	CT = 81	1	1	6350.6	0.00	6350.6	0.00
		T45		1	2	6350.6	0.00	6350.6	0.00
		T47		2	1	5212.6	0.00	5212.6	0.00
		T48		2	2	5212.6	0.00	5212.6	0.00
		T52	CT = 92	1	1	23112.7	0.90	6350.9	0.30
		T53		1	2	10060.2	0.76	6350.9	0.40
		T55		2	1	13250.9	0.85	5058.8	0.16
		T56		2	2	6700.9	0.41	5208.2	0.19
		T60	CT = 2806	1	1	7850.4	0.67	6200.2	0.21
		T61		1	2	NA	NA	6553.4	0.28
		T63		2	1	6200.2	0.24	5050.5	0.03
		T64		2	2	NA	NA	14410.8	0.68
	Large	T68	CT = 1170	1	1	4200.4	0.44	4350.7	0.45
		T69		1	2	27251.4	0.96	4200.9	0.44
		T71		2	1	2700.5	0.12	2700.4	0.12
		T72		2	2	2700.7	0.12	2850.3	0.17
		T76	CT = 25236	1	1	NA	NA	10350.1	0.77
		T77		1	2	NA	NA	4200.1	0.44
		T79		2	1	48250.9	0.98	4550.1	0.48
		T80		2	2	47902.2	0.98	2851.3	0.17
		T84	CT = 50133	1	1	NA	NA	11752.2	0.81

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model		MILP ^{LB-UB} NS*/CT*/TC*	GAP (%)
BS-AL-HRC-U-1	Small	T85	CT = 25	1	2	NA	NA	6355.7	0.64
		T87		2	1	NA	NA	9400.5	0.76
		T88		2	2	NA	NA	12403.9	0.82
		T1		0	1	4.0	0.00	4.0	0.00
		T2		0	2	3.0	0.00	3.0	0.00
		T3		1	0	2.0	0.00	2.0	0.00
		T4		1	1	1.0	0.00	1.0	0.00
		T5		1	2	1.0	0.00	1.0	0.00
		T6		2	0	1.0	0.00	1.0	0.00
		T7		2	1	1.0	0.00	1.0	0.00
		T8		2	2	1.0	0.00	1.0	0.00
		T9	CT = 8	0	1	7.0	0.00	7.0	0.00
		T10		0	2	6.0	0.00	6.0	0.00
		T11		1	0	3.0	0.00	3.0	0.00
		T12		1	1	2.0	0.00	2.0	0.00
		T13		1	2	2.0	0.00	2.0	0.00
		T14		2	0	2.0	0.00	2.0	0.00
		T15		2	1	2.0	0.00	2.0	0.00
		T16		2	2	2.0	0.00	2.0	0.00
		T17	CT = 22	0	1	6.0	0.00	6.0	0.00
		T18		0	2	4.0	0.00	4.0	0.00
		T19		1	0	3.0	0.00	3.0	0.00
		T20		1	1	2.0	0.00	2.0	0.00
		T21		1	2	2.0	0.00	2.0	0.00
		T22		2	0	2.0	0.00	2.0	0.00
		T23		2	1	2.0	0.00	2.0	0.00
		T24		2	2	2.0	0.00	2.0	0.00
		T25	CT = 55	0	1	6.0	0.16	6.0	0.00
		T26		0	2	4.0	0.25	4.0	0.00
		T27		1	0	3.0	0.33	3.0	0.00
		T28		1	1	2.0	0.00	2.0	0.00
		T29		1	2	2.0	0.00	2.0	0.00
		T30		2	0	2.0	0.00	2.0	0.00
		T31		2	1	2.0	0.00	2.0	0.00
	Medium	T32	CT = 38	2	2	2.0	0.00	2.0	0.00
		T33		0	1	9.0	0.44	9.0	0.00
		T34		0	2	6.0	0.33	6.0	0.16
		T35		1	0	5.0	0.60	5.0	0.00
		T36		1	1	3.0	0.33	3.0	0.00
		T37		1	2	3.0	0.33	3.0	0.33
		T38		2	0	3.0	0.33	3.0	0.00
		T39		2	1	3.0	0.33	3.0	0.33
		T40		2	2	2.0	0.00	2.0	0.00
		T41	CT = 81	0	1	7.0	0.57	6.0	0.00
		T42		0	2	5.0	0.40	5.0	0.20
		T43		1	0	3.0	0.33	3.0	0.00
		T44		1	1	2.0	0.00	2.0	0.00
		T45		1	2	2.0	0.00	2.0	0.00
		T46		2	0	2.0	0.00	2.0	0.00
		T47		2	1	2.0	0.50	2.0	0.00
		T48		2	2	2.0	0.00	2.0	0.00
		T49	CT = 92	0	1	9.0	0.94	7.0	0.14
		T50		0	2	5.0	0.60	5.0	0.40
		T51		1	0	27.0	0.96	3.0	0.00
		T52		1	1	50.0	0.99	3.0	0.33
		T53		1	2	NA	NA	2.0	0.50
		T54		2	0	2.0	0.50	2.0	0.00
		T55		2	1	NA	NA	2.0	0.50
		T56		2	2	17.0	0.94	2.0	0.50
		T57	CT = 2806	0	1	NA	NA	6.0	0.16
		T58		0	2	NA	NA	4.0	0.25
		T59		1	0	NA	NA	3.0	0.00
		T60		1	1	NA	NA	2.0	0.00
		T61		1	2	NA	NA	2.0	0.00
		T62		2	0	NA	NA	2.0	0.00
		T63		2	1	NA	NA	2.0	0.00
	Large	T64		2	2	NA	NA	2.0	0.00
		T65	CT = 1170	0	1	NA	NA	4.0	0.25
		T66		0	2	NA	NA	3.0	0.33
		T67		1	0	NA	NA	2.0	0.00
		T68		1	1	NA	NA	2.0	0.50
		T69		1	2	20.0	0.95	1.0	0.00
		T70		2	0	2.0	0.50	2.0	0.50

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model			
						MILP NS*/CT*/TC*	GAP (%)	MILP ^{LB-UB} NS*/CT*/TC*	GAP (%)
BS-AL-HRC-U-2	Small	T71	CT = 25236	2	1	NA	NA	1.0	0.00
		T72		2	2	1.0	0.00	1.0	0.00
		T73		0	1	NA	NA	4.0	0.25
		T74		0	2	NA	NA	NA	NA
		T75		1	0	NA	NA	2.0	0.00
		T76		1	1	NA	NA	NA	NA
		T77		1	2	NA	NA	1.0	0.00
		T78		2	0	2.0	0.50	2.0	0.50
		T79	CT = 50133	2	1	NA	NA	1.0	0.00
		T80		2	2	1.0	0.00	1.0	0.00
		T81		0	1	NA	NA	4.0	0.25
		T82		0	2	NA	NA	3.0	0.00
		T83		1	0	NA	NA	2.0	0.00
		T84		1	1	NA	NA	2.0	0.50
		T85		1	2	27.0	0.96	1.0	0.00
		T86		2	0	9.0	0.88	2.0	0.50
		T87	NS = 2	2	1	NA	NA	1.0	0.00
		T88		2	2	30.0	0.96	1.0	0.00
		T1		0	1	38.0	0.00	37.9	0.00
		T2		0	2	26.0	0.00	26.0	0.00
		T3		1	0	19.0	0.00	18.9	0.00
		T4		1	1	13.0	0.00	13.0	0.00
		T5		1	2	13.0	0.00	13.0	0.00
		T6		2	0	13.0	0.00	13.0	0.00
		T7	NS = 2	2	1	13.0	0.00	13.0	0.00
		T8		2	2	13.0	0.00	13.0	0.00
		T9		0	1	23.0	0.00	23.0	0.00
		T10		0	2	16.0	0.00	15.9	0.00
		T11		1	0	11.5	0.00	11.5	0.00
		T12		1	1	8.0	0.00	7.9	0.00
		T13		1	2	7.5	0.00	7.4	0.00
		T14	NS = 3	2	0	8.0	0.00	7.9	0.00
		T15		2	1	7.0	0.00	6.9	0.00
		T16		2	2	6.5	0.00	6.4	0.00
		T17		0	1	42.0	0.39	41.9	0.00
		T18		0	2	28.0	0.21	27.9	0.25
		T19		1	0	21.0	0.40	20.9	0.00
		T20		1	1	14.0	0.25	13.9	0.25
		T21	NS = 3	1	2	12.0	0.08	11.9	0.04
		T22		2	0	14.0	0.24	13.9	0.25
		T23		2	1	11.0	0.09	10.9	0.09
		T24		2	2	10.0	0.00	10.0	0.00
		T25		0	1	108.0	0.49	107.9	0.00
		T26		0	2	72.0	0.41	72.0	0.25
		T27		1	0	54.0	0.52	54.0	0.00
		T28	NS = 4	1	1	36.0	0.38	36.4	0.26
		T29		1	2	31.5	0.30	31.9	0.31
		T30		2	0	36.0	0.42	35.9	0.24
		T31		2	1	28.0	0.25	27.4	0.23
		T32		2	2	25.5	0.15	25.4	0.15
		T33		0	1	81.0	0.52	81.0	0.00
		T34		0	2	55.0	0.38	56.0	0.27
		T35	NS = 4	1	0	40.5	0.53	40.5	0.00
		T36		1	1	27.5	0.09	27.5	0.26
		T37		1	2	24.0	0.29	24.0	0.29
		T38		2	0	27.0	0.37	27.5	0.26
		T39		2	1	21.0	0.19	21.0	0.19
		T40		2	2	20.0	0.15	20.0	0.15
		T41		0	1	121.0	0.65	121.0	0.00
		T42	NS = 5	0	2	82.0	0.45	82.0	0.26
		T43		1	0	60.5	0.56	60.5	0.00
		T44		1	1	41.0	0.39	41.0	0.26
		T45		1	2	38.0	0.36	40.0	0.38
		T46		2	0	41.0	0.36	41.0	0.26
		T47		2	1	34.5	0.33	33.9	0.14
		T48		2	2	32.0	0.09	32.0	0.09
		T49		0	1	111.0	0.50	111.0	0.00
	Medium	T50	NS = 5	0	2	75.0	0.26	76.0	0.27
		T51		1	0	55.5	0.50	55.5	0.00
		T52		1	1	38.0	0.27	37.5	0.26
		T53		1	2	32.5	0.15	32.9	0.16
		T54		2	0	37.5	0.26	37.5	0.26
		T55		2	1	29.0	0.05	29.0	0.05

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model	MILP ^{LB-UB}		
							NS*/CT*/TC*	GAP (%)	NS*/CT*/TC*
BS-AL-HRC-U-rw	Large	T56	NS = 6	2	2	27.5		0.00	27.4
		T57		0	1	2363.0		0.24	2352.0
		T58		0	2	1875.0		0.05	1775.0
		T59		1	0	1200.0		0.26	1200.0
		T60		1	1	1596.5		0.86	913.5
		T61		1	2	902.5		0.01	887.4
		T62	NS = 3	2	0	896.0		0.00	913.5
		T63		2	1	887.5		0.00	971.0
		T64		2	2	971.0		0.55	887.4
		T65		0	1	1181.0		0.87	1170.0
		T66		0	2	788.0		0.80	782.0
		T67		1	0	585.5		0.86	585.0
		T68		1	1	492.0		0.93	392.0
		T69		1	2	350.9		0.77	339.0
		T70	NS = 3	2	0	393.5		0.76	390.4
		T71		2	1	294.5		0.73	299.0
		T72		2	2	290.0		0.73	269.5
		T73		0	1	30537.0		0.87	25245.0
		T74		0	2	17848.9		0.79	17141.9
		T75		1	0	12680.4		0.85	12627.5
	Small	T76	NS = 3	1	1	11528.0		0.86	8944.0
		T77		1	2	51298.0		0.99	7676.9
		T78		2	0	8970.9		0.79	8595.4
		T79		2	1	10348.5		0.92	6667.9
		T80		2	2	8949.0		0.91	6634.0
		T81		0	1	60200.0		0.97	50159.0
		T82	CT = 25	0	2	42372.0		0.86	33520.9
		T83		1	0	35103.0		0.95	25075.9
		T84		1	1	28675.0		0.99	16724.5
		T85		1	2	19709.4		0.94	15924.4
		T86		2	0	23359.0		0.94	18394.0
		T87		2	1	18378.0		0.94	14555.0
	Medium	T88	CT = 8	2	2	24283.0		0.91	12316.0
		T4		1	1	4201.5		0.00	4201.5
		T5		1	2	2857.2		0.00	2857.2
		T7		2	1	2701.5		0.00	2701.5
		T8		2	2	2701.5		0.00	2701.5
		T12	CT = 22	1	1	5003.8		0.00	5003.8
		T13		1	2	5003.8		0.00	5003.8
		T15		2	1	4851.6		0.00	4851.6
		T16		2	2	4851.6		0.00	4851.6
		T20		1	1	5010.1		0.13	5010.1
		T21	CT = 55	1	2	5010.1		0.13	5010.1
		T23		2	1	4854.1		0.10	4854.1
		T24		2	2	4854.1		0.10	4854.1
		T28		1	1	6351.8		0.34	5028.6
		T29		1	2	5028.6		0.22	5028.6
	Large	T31	CT = 38	2	1	5051.8		0.50	4863.7
		T32		2	2	4863.4		0.15	4863.4
		T36		1	1	7526.4		0.41	8505.5
		T37		1	2	7526.4		0.41	7526.4
		T39		2	1	7205.7		0.39	7206.0
		T40	CT = 81	2	2	7205.7		0.39	6427.3
		T44		1	1	6350.6		0.33	6350.6
		T45		1	2	5393.8		0.22	6721.7
		T47		2	1	5050.6		0.16	5050.6
		T48		2	2	5050.6		0.16	5050.6
		T52	CT = 92	1	1	NA		NA	6508.8
		T53		1	2	81038.1		0.99	6875.3
		T55		2	1	NA		NA	5050.9
		T56		2	2	NA		NA	5050.9
		T60		1	1	NA		NA	6350.4
	Large	T61	CT = 2806	1	2	NA		NA	8053.8
		T63		2	1	NA		NA	5050.2
		T64		2	2	NA		NA	4702.2
		T68		1	1	NA		NA	6000.4
		T69		1	2	NA		NA	4200.4
		T71		2	1	NA		NA	10101.5
		T72	CT = 25236	2	2	96004.8		0.99	2700.4
		T76		1	1	NA		NA	NA
		T77		1	2	NA		NA	5202.2
		T79		2	1	NA		NA	6550.3
		T80		2	2	NA		NA	2852.8

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Table 6 (continued)

Problem	Size	Test instance	CT/NS	NH	NR	Model			
						MILP NS*/CT*/TC*	GAP (%)	MILP ^{LB-UB} NS*/CT*/TC*	GAP (%)
		T84	CT = 50133	1	1	NA	NA	NA	NA
		T85		1	2	NA	NA	NA	NA
		T87		2	1	NA	NA	NA	NA
		T88		2	2	NA	NA	NA	NA

° NA: Not available.

and maximum theoretical number of stations NS , while considering the minimum and maximum processing times of humans and robots (th_{ik} , tr_{ir}), CT , and the maximum number of humans (NH) and robots (NR) per station, respectively. For Type-2, the lower and upper bounds of the objective, namely CT^{LB} and CT^{UB} , can be calculated based on the minimum and maximum theoretical CT , while considering the minimum and maximum processing times of humans and robots (th_{ik} , tr_{ir}), NS , NH , and NR , respectively. Finally, the lower bound and the upper bound of TC for Type-rw, namely TC^{LB} and TC^{UB} , can be calculated based on the associated cost of stations (cos_j), humans (coh_k), robots (cor_r), and robot energy consumption (En_{ir}). The TC^{LB} and TC^{UB} are based on the minimum and the total cost terms while considering the NS^{LB} , NS^{UB} obtained by Type-1, respectively.

5. Computational experiments

This section tests the application of the proposed MILP models by solving different instances of an automotive case study and the standard test problems from the literature. To this end, first, the experimental settings are presented. Then, the numerical results of the proposed MILP-based models for each problem on the case study and test problems are discussed. Third, the effect of the layout and objective on the optimization solutions is investigated. Fourth, the impact of the HRC level on the results is studied. Finally, the research implications and limitations of the study are presented.

5.1. Experimental settings

There are mainly two sets of problems considered in this study: (1) case study and (2) test problem. The case study is based on the assembly line of the MBS unit from the automotive industry, with the main characteristics presented in Table 4. This table shows the problem size, NT (number of tasks), CT (cycle time), NS (number of stations), NH (maximum number of humans), NR (maximum number of robots), instance name, and the time limit allowed to solve the related instances.

Table 7

The comparison of MILP^{LB-UB} vs. MILP results in different sizes.

Size	Status	Objectives (Num)	GAP (Num)	Objectives (%)	GAP (%)
Small	Better	30	22	0.19	0.14
	Equal	127	130	0.79	0.81
	Worse	3	8	0.02	0.05
	Sum	160	160	1.00	1.00
Medium	Better	48	80	0.30	0.50
	Equal	96	60	0.60	0.38
	Worse	16	20	0.10	0.13
	Sum	160	160	1.00	1.00
Large	Better	88	98	0.73	0.82
	Equal	20	19	0.17	0.16
	Worse	12	3	0.10	0.03
	Sum	120	120	1.00	1.00
Total	Better	166	200	0.38	0.45
	Equal	243	209	0.55	0.48
	Worse	31	31	0.07	0.07
	Sum	440	440	1.00	1.00

Further information about the case study is presented in the Appendixes. On the other hand, the test problems are based on the original input data existing online as the standard data set provided by Scholl et al. (1995). The rest of the data for the test problems, specifically for HRC features, were generated randomly and can be accessed in the supplementary material. The experimental settings for the test problem instances include the size of problems (small, medium, and large) and other characteristics similar to the case study, as presented in Table 4. In general, 12 problems, each with eight instances, are considered to be solved by the proposed MILP-based models. The proposed models were solved by the CPLEX solver on a Core i9 PC with a 3.10 GHz processor and 64 GB of RAM. The solver was terminated when the optimal solutions were found or the time limit shown in Table 4 was reached. The number of variables, the number of constraints, and the CPU times of the MILP-based models coded in CPLEX for all problems are reported in the supplementary material. As the problem's dimensionality increases in terms of NT, NS, CT, NH, and NR, the number of variables, constraints, and CPU times of the CPLEX models significantly increase. Over all the considered problems, the complexity of MILP models can be ranked in decreasing order of BS-AL-HRC-U-rw > BS-AL-HRC-U-1 > BS-AL-HRC-S-rw > BS-AL-HRC-U-2 > BS-AL-HRC-S-2 > BS-AL-HRC-S-1, in terms of the number of variables, constraints, and CPU times, simultaneously, where > shows a higher complexity.

5.2. Numerical results

This section deals with the resulting solutions for BS-AL-HRC problems in the considered case study from the automotive industry and the standard test problems from the literature. Two types of models were considered while solving the instances: (1) MILP without bounds for the considered objectives (MILP) and (2) MILP with the proposed lower and upper bounds for objectives (MILP^{LB-UB}), as presented in Section 4.3. The subsections below compare MILP and MILP^{LB-UB} models over the case study and test instances.

5.2.1. Case study instances

This section focuses on the results of the proposed MILP-based models while solving the case instances presented in Table 4. The characteristics of the case instances and the results of the MILP and MILP^{LB-UB} models for each problem type, including the layout (straight and U-shaped) and the objectives (Type-1, Type-2, Type-rw), are shown in Table 5. In this table, the first column shows the problem type, the second column indicates the case instance number and the third column presents the given CT or NS for the related problem type. The fourth and fifth columns illustrate the maximum number of humans (NH) and the maximum number of robots (NR), respectively. The rest of columns compare the results of MILP and MILP^{LB-UB} models in terms of the obtained NS^* , CT^* (second) and TC^* (unit of cost) and the relative GAP(%).

Considering the objectives column under MILP^{LB-UB} in Table 5, the performance of the MILP^{LB-UB}, compared to MILP, was better in 10 instances, similar in 28 instances, and worse in only two instances. On the other hand, considering the GAP column, the performance of the MILP^{LB-UB}, compared to MILP, was better in 19 instances, similar in 18 instances, and worse in three instances. MILP^{LB-UB} could find better,

Table 8

The effects of objective and layout measured by RPD for all instances.

Size	Instance	Type-1 NS^{min}		RPD (%)	Type-2 $CT^{min}(second)$		RPD (%)	Type-rw $TC^{min}(cost\ unit)$		RPD (%)
		Straight	U-shaped		Straight	U-shaped		Straight	U-shaped	
Case	C1	6	6	0.00	108.5	108.5	0.00	—*	—	—
	C2	4	4	0.00	65.6	64.4	0.02	—	—	—
	C3	3	3	0.00	57.4	57.4	0.00	—	—	—
	C4	2	2	0.00	34	32.4	0.05	4509.9	4509.63	0.00
	C5	2	2	0.00	26.6	24	0.10	4509.9	4509.63	0.00
	C6	2	2	0.00	34	32.4	0.05	—	—	—
	C7	2	2	0.00	24.6	24	0.02	4509.9	4509.77	0.00
	C8	2	1	0.50	24.6	22	0.11	4509.9	3211.97	0.29
Small	T1	4	4	0.00	38.0	38.0	0.00	—	—	—
	T2	4	3	0.25	35.0	26.0	0.26	—	—	—
	T3	2	2	0.00	19.0	19.0	0.00	—	—	—
	T4	2	1	0.50	17.5	13.0	0.26	4351.4	4201.6	0.03
	T5	2	1	0.50	17.5	13.0	0.26	4351.4	2857.3	0.34
	T6	2	1	0.50	17.5	13.0	0.26	—	—	—
	T7	2	1	0.50	17.5	13.0	0.26	4351.4	2701.6	0.38
	T8	2	1	0.50	17.5	13.0	0.26	4351.4	2701.6	0.38
	T9	7	7	0.00	23.0	23.0	0.00	—	—	—
	T10	7	6	0.14	18.0	16.0	0.11	—	—	—
	T11	3	3	0.00	11.5	11.5	0.00	—	—	—
	T12	3	2	0.33	9.0	8.0	0.11	6350.3	5003.9	0.21
	T13	3	2	0.33	9.0	7.5	0.17	6350.3	5003.9	0.21
	T14	3	2	0.33	9.0	8.0	0.11	—	—	—
	T15	3	2	0.33	9.0	7.0	0.22	6350.3	4851.7	0.24
	T16	3	2	0.33	9.0	6.5	0.28	6350.3	4851.7	0.24
	T17	6	6	0.00	42.0	42.0	0.00	—	—	—
	T18	5	4	0.20	32.0	28.0	0.13	—	—	—
	T19	3	3	0.00	21.0	21.0	0.00	—	—	—
	T20	3	2	0.33	16.0	14.0	0.13	6354.4	5010.2	0.21
	T21	2	2	0.00	15.0	12.0	0.20	5360.8	5010.2	0.07
	T22	3	2	0.33	16.0	14.0	0.13	—	—	—
	T23	2	2	0.00	15.0	11.0	0.27	5204.8	4854.1	0.07
	T24	2	2	0.00	15.0	10.0	0.33	5204.8	4854.1	0.07
	T25	6	6	0.00	108.0	108.0	0.00	—	—	—
	T26	5	4	0.20	75.0	72.0	0.04	—	—	—
	T27	3	3	0.00	54.0	54.0	0.00	—	—	—
	T28	3	2	0.33	37.5	36.0	0.04	6351.9	5028.6	0.21
	T29	2	2	0.00	34.0	31.5	0.07	5378.9	5028.6	0.07
	T30	3	2	0.33	37.5	36.0	0.04	—	—	—
	T31	2	2	0.00	32.5	27.5	0.15	5051.9	4863.8	0.04
	T32	2	2	0.00	32.5	25.5	0.22	4863.8	4863.5	0.00
Medium	T33	9	9	0.00	81.0	81.0	0.00	—	—	—
	T34	7	6	0.14	55.0	55.0	0.00	—	—	—
	T35	5	5	0.00	40.5	40.5	0.00	—	—	—
	T36	3	3	0.00	27.5	27.5	0.00	7526.4	7526.4	0.00
	T37	3	3	0.00	25.0	24.0	0.04	7526.4	7526.4	0.00
	T38	3	3	0.00	27.5	27.0	0.02	—	—	—
	T39	3	3	0.00	23.5	21.0	0.11	7205.5	7205.8	0.00
	T40	3	2	0.33	23.5	20.0	0.15	7205.8	6427.3	0.11
	T41	7	6	0.14	121.0	121.0	0.00	—	—	—
	T42	6	5	0.17	89.0	82.0	0.08	—	—	—
	T43	3	3	0.00	60.5	60.5	0.00	—	—	—
	T44	3	2	0.33	44.5	41.0	0.08	6350.6	6350.6	0.00
	T45	3	2	0.33	44.0	38.0	0.14	6350.6	5393.8	0.15
	T46	3	2	0.33	44.5	41.0	0.08	—	—	—
	T47	2	2	0.00	43.5	34.0	0.22	5212.7	5050.6	0.03
	T48	2	2	0.00	43.5	32.0	0.26	5212.7	5050.6	0.03
	T49	7	7	0.00	111.0	111.0	0.00	—	—	—
	T50	5	5	0.00	78.0	75.0	0.04	—	—	—
	T51	3	3	0.00	55.5	55.5	0.00	—	—	—
	T52	3	3	0.00	39.0	37.5	0.04	6350.9	6508.8	−0.02
	T53	2	2	0.00	34.0	32.5	0.04	6350.9	6875.3	−0.08
	T54	3	2	0.33	38.5	37.5	0.03	—	—	—
	T55	2	2	0.00	30.0	29.0	0.03	5058.8	5050.9	0.00
	T56	2	2	0.00	29.0	27.5	0.05	5208.3	5050.9	0.03
	T57	6	6	0.00	2400.0	2352.0	0.02	—	—	—
	T58	5	4	0.20	2242.0	1775.0	0.21	—	—	—
	T59	3	3	0.00	1200.0	1200.0	0.00	—	—	—
	T60	3	2	0.33	1121.0	913.5	0.19	6200.2	6350.4	−0.02
	T61	2	2	0.00	1096.5	887.5	0.19	6553.5	8053.8	−0.23
	T62	3	2	0.33	1121.0	896.0	0.20	—	—	—
	T63	2	2	0.00	1096.5	887.5	0.19	5050.5	5050.2	0.00
	T64	2	2	0.00	1096.5	887.5	0.19	14410.8	4702.3	0.67

(continued on next page)

Table 8 (continued)

Size	Instance	Type-1 NS^{min}		RPD (%)	Type-2 $CT^{min}(\text{second})$		RPD (%)	Type-rw $TC^{min}(\text{cost unit})$		RPD (%)
		Straight	U-shaped		Straight	U-shaped		Straight	U-shaped	
Large	T65	4	4	0.00	1172.0	1170.0	0.00	–	–	–
	T66	3	3	0.00	786.0	782.0	0.01	–	–	–
	T67	2	2	0.00	585.5	585.0	0.00	–	–	–
	T68	2	2	0.00	393.5	392.0	0.00	4200.4	6000.4	–0.43
	T69	1	1	0.00	344.5	339.0	0.02	4200.9	4200.4	0.00
	T70	2	2	0.00	396.0	390.5	0.01	–	–	–
	T71	1	1	0.00	316.5	294.5	0.07	2700.4	10101.5	–2.74
	T72	1	1	0.00	304.5	269.5	0.11	2700.7	2700.4	0.00
	T73	4	4	0.00	25245.0	25245.0	0.00	–	–	–
	T74	3	NA	NA	17935.0	17142.0	0.04	–	–	–
	T75	2	2	0.00	12621.0	12627.5	0.00	–	–	–
	T76	2	NA	NA	9039.5	8944.0	0.01	10350.1	NA	NA
	T77	2	1	0.50	8288.5	7677.0	0.07	4200.1	5202.3	–0.24
	T78	2	2	0.00	8972.5	8595.5	0.04	–	–	–
	T79	2	1	0.50	7428.5	6668.0	0.10	4550.1	6550.3	–0.44
	T80	1	1	0.00	7044.5	6634.0	0.06	2851.3	2852.8	0.00
	T81	4	4	0.00	50368.0	50159.0	0.00	–	–	–
	T82	3	3	0.00	34832.0	33521.0	0.04	–	–	–
	T83	2	2	0.00	25087.0	25076.0	0.00	–	–	–
	T84	2	2	0.00	18913.0	16724.5	0.12	11752.3	NA	NA
	T85	2	1	0.50	16068.5	15924.5	0.01	6355.8	NA	NA
	T86	2	2	0.00	17340.5	18394.0	–0.06	–	–	–
	T87	3	1	0.67	15520.5	14555.0	0.06	9400.6	NA	NA
	T88	3	1	0.67	13841.0	12316.0	0.11	12403.9	NA	NA

*The Type-rw does not apply to this instance, considering the NH and NR.

equal, and worse objectives in 25 %, 70 %, and 5 % of the case instances, respectively. Moreover, considering the GAP, MILP^{LB-UB} was better, equal, and worse than MILP in 47.5 %, 45 %, and 7.5 % of the case instances.

5.2.2. Test instances

This section concentrates on the results of the proposed MILP-based models while solving the test instances, as presented in Table 4. The characteristics of the test instances and the results of the MILP and MILP^{LB-UB} models for each problem type are shown in Table 6. In this table, the first column shows the problem type, the second column shows the problem size and the third column indicates the test instance number. The fourth column presents the given CT or NS for the related problem type, while the fifth and sixth columns illustrate the NH and NR . The rest of columns compare the results of MILP and MILP^{LB-UB} models in terms of the obtained NS^* , CT^* (second) and TC^* (unit of cost) and the relative $GAP(\%)$.

To better explain the results in Table 6, Table 7 summarizes the performing status (better, equal, worse) of MILP^{LB-UB} versus MILP in different sizes in terms of the objectives and GAP both in numerical (Num) and percentage (%).

According to Table 7, in the small size, considering the objectives (Num), the performance of the MILP^{LB-UB} was better in 30 instances, similar in 127 instances, and worse in only three instances. On the other hand, considering the GAP (Num), MILP^{LB-UB} was better in 22 instances, similar in 130 instances, and worse in 8 instances. In the objectives (%), MILP^{LB-UB} was better in 19 % of instances, similar in 79 % of instances, and worse in only 2 %, while in GAP (%), MILP^{LB-UB} was better in 14 % of instances, similar in 81 % of instances, and worse in only 5 %.

For the medium size, considering the objectives (Num), MILP^{LB-UB} was better in 48 instances, similar in 96 instances, and worse in 16 instances. At the same time, in GAP (Num), MILP^{LB-UB} was better in 80 instances, identical in 60 instances, and worse in 20 instances. Considering the objectives (%), MILP^{LB-UB} was better in 30 % of instances, similar in 60 % of instances, and worse in 10 %, while in the GAP (%), MILP^{LB-UB} was better in 50 % of instances, similar in 38 % of instances, and worse in 13 %.

For the large size, considering the objectives (Num), MILP^{LB-UB} was better in 88 instances, similar in 20 instances, and worse in 12 instances.

At the same time, in the GAP (Num), MILP^{LB-UB} was better in 98 instances, similar in 19 instances, and worse in only three instances. Regarding the objectives (%), MILP^{LB-UB} was better in 73 % of instances, similar in 17 % of instances, and worse in 10 %, while in the GAP (%), MILP^{LB-UB} was better in 82 % of instances, similar in 16 % of instances, and worse in only 3 %.

Overall, MILP^{LB-UB} could find better, similar, and worse objectives than MILP in 166, 243, and 31 instances, respectively, while considering the GAP (Num), MILP^{LB-UB} was better, similar, and worse in 200, 209, and 31 instances, respectively. Considering objective (%), MILP^{LB-UB} was better, equal, and worse than MILP in 38 %, 55 %, and 7 % of all instances, while in the GAP (%), MILP^{LB-UB} was better, similar, and worse than MILP in 45 %, 48 %, and 7 % of all instances, respectively. It is worth mentioning that while moving from the small to the large size problems, the domination of MILP^{LB-UB} over MILP has been increasing for both the objective and GAP measures. Additionally, MILP^{LB-UB} found at least a solution in 56 medium-to-large instances, whereas MILP could not find any solution, as reported by NA (not available) in Table 6.

5.3. Comparison of results by objective and layout

This section investigates the effect of the objective and layout on BS-AL-HRC in the case study and test instances. To measure the effects of the objective (Type-1, Type-2, Type-rw) and the layout (straight, U-shaped) on the results, three relative percent deviation (RPD) measures, namely RPD_{Type-1} , RPD_{Type-2} , and $RPD_{Type-rw}$, are defined by Equations (65) to (67). For each problem type, the related RPD measures the relative percent deviation of the U-shaped results compared to the straight results in terms of the corresponding objective function. In calculating the RPD for each instance, the minimum values of the objectives by the MILP-based models are considered.

$$RPD_{Type-1} = (NS_{BS-AL-HRC-S-1}^{min} - NS_{BS-AL-HRC-U-1}^{min}) / NS_{BS-AL-HRC-S-1}^{min} \quad (65)$$

$$RPD_{Type-2} = (CT_{BS-AL-HRC-S-2}^{min} - CT_{BS-AL-HRC-U-2}^{min}) / CT_{BS-AL-HRC-S-2}^{min} \quad (66)$$

$$RPD_{Type-rw} = (TC_{BS-AL-HRC-S-rw}^{min} - TC_{BS-AL-HRC-U-rw}^{min}) / TC_{BS-AL-HRC-S-rw}^{min} \quad (67)$$

Table 9

The comparison of results by the objective and layout in different sizes.

Size	Status	Type-1 NS^{min}	$NS^{min}(\%)$	Type-2 CT^{min}	$CT^{min}(\%)$	Type-rw TC^{min}	$TC^{min}(\%)$
Case	Better	1	0.13	6	0.75	4	1.00
	Equal	7	0.87	2	0.25	0	0.00
	Worse	0	0.00	0	0.00	0	0.00
	Sum	8	1.00	8	1.00	4	1.00
Small	Better	18	0.56	29	0.91	16	1.00
	Equal	14	0.44	3	0.09	0	0.00
	Worse	0	0.00	0	0.00	0	0.00
	Sum	32	1.00	32	1.00	16	1.00
Medium	Better	11	0.34	23	0.72	9	0.56
	Equal	21	0.66	6	0.19	2	0.13
	Worse	0	0.00	3	0.09	5	0.31
	Sum	32	1.00	32	1.00	16	1.00
Large	Better	5	0.21	21	0.88	2	0.17
	Equal	17	0.71	1	0.04	0	0.00
	Worse	2	0.08	2	0.08	10	0.83
	Sum	24	1.00	24	1.00	12	1.00
Total	Better	34	0.39	73	0.83	27	0.61
	Equal	52	0.59	10	0.11	2	0.05
	Worse	2	0.02	5	0.06	15	0.34
	Sum	88	1.00	88	1.00	44	1.00

The calculated RPD values for each problem type over all instances in the case study and test problems are presented in Table 8. In this table, the first and second columns show the problem size and the instance name, while under columns Type-1, Type-2, and Type-rw, the related minimum objective (NS^{min} , CT^{min} , TC^{min}) by MILP-based models for the straight and U-shaped layouts, and the calculated RPD values are shown, respectively. A positive RPD shows an improvement in the related objective by the U-shaped compared to the straight layout, while a negative RPD shows a decline in that objective.

In Table 8, for instance, in the case instance rows (i.e., C1 to C8), one can observe the related minimum objective values and RPD under columns Type-1, Type-2, and Type-rw. The rest of the rows in Table 8, i.e., T1 to T88, show the minimum objectives of each type and the obtained RPD for the small, medium, and large test instances. To better explain the results in Table 8, Table 9 compares the results by the U-shaped layout versus the straight layout for each problem type and in different sizes. Under columns Type-1, Type-2, and Type-rw, the performing status in terms of the number of better, equal, and worse objectives in terms of NS^{min} , CT^{min} , TC^{min} as well as the related percentage (%), are reported.

According to Table 9, one can observe that in the case study for Type-1, the U-shaped layout outperformed the straight line in 13 % of instances, showed similar performance in 87 % of instances concerning NS^{min} . For Type-2, the U-shaped layout was superior in 75 % of instances while showing equivalent performance in 25 % of instances regarding CT^{min} . In Type-rw case instances, the U-shaped layout demonstrated

better performance in 100 % of instances in terms of TC^{min} . Overall, the results suggest that the U-shaped layout is most effective in Type-rw, followed by Type-2 and Type-1, in the case instances.

In small-size test instances, for Type-1, the U-shaped layout improved performance in 56 % of instances and had similar performance in 44 % of instances concerning NS^{min} . For Type-2, the U-shaped layout showed superior performance in 91 % of instances and had similar performance in 9 % of instances concerning CT^{min} . In Type-rw, the U-shaped layout outperformed the straight line in 100 % of instances concerning TC^{min} . Similar to the case instances, in small-size test problems, the U-shaped layout is most effective in Type-rw, followed by Type-2 and Type-1.

In medium-size test instances, for Type-1, the U-shaped layout improved performance in 34 % of instances and had similar performance in 66 % of instances concerning NS^{min} . For Type-2, the U-shaped layout showed superior performance in 72 % of instances, had similar performance in 19 % of instances, and performed worse in 9 % of instances concerning CT^{min} . In Type-rw, the U-shaped layout outperformed the straight line in 56 % of instances, had a similar performance in 13 % of instances, and performed worse in 31 % of instances concerning TC^{min} . Overall, in the medium-size, the U-shaped layout was most effective in Type-2, followed by Type-rw and Type-1. Notably, the worse performance in Type-rw was due to high GAPs in a few U-shaped results compared to their straight counterparts.

In large-size test problems, for Type-1, the U-shaped layout improved performance in 21 % of instances, had similar performance in 71 % of

Table 10

The comparison of HRC scenarios in terms of the objective and layout in different sizes.

Size	Problem	Type-1 Straight $NS^{min}_{Avg(HRC)}$	U-shaped $NS^{min}_{Avg(HRC)}$	RPD	Type-2 Straight $CT^{min}_{Avg(HRC)}$	U-shaped $CT^{min}_{Avg(HRC)}$	RPD	Type-rw Straight $TC^{min}_{Avg(HRC)}$	U-shaped $TC^{min}_{Avg(HRC)}$	RPD
Small	Case	2.00	1.75	0.13	27.45	25.60	0.07	4509.90	4185.25	0.07
	Bowman	2.00	1.00	0.50	17.50	13.00	0.26	4351.38	3115.50	0.28
	Jackson	3.00	2.00	0.33	9.00	7.25	0.19	6350.31	4927.76	0.22
	Roszieg	2.25	2.00	0.11	15.25	11.75	0.23	5531.20	4932.16	0.11
Medium	Buxey	2.25	2.00	0.11	34.13	30.12	0.12	5411.60	4946.12	0.09
	Sawyer	3.00	2.75	0.08	24.88	23.13	0.07	7366.03	7171.48	0.03
	Gunther	2.50	2.00	0.20	43.88	36.25	0.17	5781.65	5461.43	0.06
	Kilbridge	2.25	2.25	0.00	33.00	31.63	0.04	5742.24	5871.51	-0.02
Large	Hahn	2.25	2.00	0.11	1102.63	894.00	0.19	8053.75	6039.18	0.25
	Tonge	1.25	1.25	0.00	339.75	323.75	0.05	3450.62	5750.69	-0.67
	Arcus1	1.75	1.00	0.43	7950.25	7480.75	0.06	5487.93	4868.47	0.11
	Arcus2	2.50	1.25	0.50	16085.75	14880.00	0.07	9978.12	NA	NA

instances, and performed worse in 8 % of instances concerning NS^{min} . For Type-2, the U-shaped layout showed superior performance in 88 % of instances, had similar performance in 4 % of instances, and performed worse in 8 % of instances concerning CT^{min} . In Type-rw, the U-shaped layout improved performance in 17 % of instances, had similar performance in no instances, and performed worse in 83 % of instances concerning TC^{min} . Overall, in the large-size test problems, the U-shaped layout was most effective in Type-2, followed by Type-1 and Type-rw. As observed, the worse performance in Type-rw was primarily due to the high GAPs in a few U-shaped results compared to their straight counterparts.

Across all test problems, the U-shaped layout consistently provided higher improvement in Type-2, followed by Type-rw and Type-1. These results demonstrate that the U-shaped layout offers greater flexibility than the straight line, leading to more efficient solutions for JIT production.

5.4. Comparison of results by HRC, objective, and layout

This section investigates the combined effects of HRC, objective, and layout in the case study and test instances. To this aim, the following HRC scenarios are considered:

- NH1-NR1: single human - single robot, per station
- NH1-NR2: single human - two robots per station
- NH2-NR1: two humans - single robot per station
- NH2-NR2: two humans - two robots per station

Considering the above HRC scenarios, the average of the minimum objective values by MILP-based models ($NS_{Avg(HRC)}^{min}$, $CT_{Avg(HRC)}^{min}$, $TC_{Avg(HRC)}^{min}$) at each problem type (Type-1, Type-2, Type-rw) and layout (straight, U-shaped) are compared with each other in Table 10. In this table, the first column shows the size, the second column shows the problem, and the rest represents the average of the minimum objective by MILP models for the HRC scenarios in the straight and U-shaped layouts with their RPD values in the last column. A positive RPD shows improvement by U-shaped compared to the straight layout, while a negative RPD indicates a disimprovement in the average of the HRC scenarios.

According to Table 10, in the case study for Type-1, the $NS_{Avg(HRC)}^{min}$ in U-shaped line (1.75) is less than the straight line (2.00), showing an improvement of 13 %. For Type-2, the $CT_{Avg(HRC)}^{min}$ of U-shaped line (25.60) is less than the straight line (27.45), showing an improvement of 7 %. For Type-rw, the $TC_{Avg(HRC)}^{min}$ of U-shaped line (4185.25) is less than the straight line (4509.90), showing an improvement of 7 %. Thus, in the case instances, the U-shaped layout under HRC scenarios could improve the related measures in all types.

In the small size, for Type-1, the $NS_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems, showing improvement of between 11 % and 50 %. For Type-2, the $CT_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems, showing improvement of between 12 % and 26 %. For Type-rw, the $TC_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems, showing improvement of between 9 % and 28 %. Thus, in small instances, the U-shaped layout under HRC scenarios could improve the related measures in all types.

In the medium size, for Type-1, the $NS_{Avg(HRC)}^{min}$ in the U-shaped line are less than or equal to the straight line for all problems, showing improvement of up to 20 %. For Type-2, the $CT_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems, showing improvement of between 4 % and 19 %. For Type-rw, the $TC_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems (except in Kilbridge), showing improvement of between 3 % and 25 %. In

Kilbridge, a 2 % decline was caused by the large GAPs in the related HRC scenarios. Thus, in the medium size, the U-shaped layout under HRC scenarios could improve the average of objectives in all types in the majority of test problems.

In the large size, for Type-1, the $NS_{Avg(HRC)}^{min}$ in the U-shaped line are less than or equal to the straight line for all problems, showing an improvement of up to 50 %. For Type-2, the $CT_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line for all problems, showing improvement of between 5 % and 7%. For Type-rw, the $TC_{Avg(HRC)}^{min}$ in the U-shaped line are less than the straight line in Arcus, showing an improvement of 11 %. In Tonge, a 67 % decline was caused by the large GAPs in the related HRC scenarios, while in Arcus2, no value could be found due to NA status in the related HRC scenarios. Thus, in the large size, the U-shaped layout under HRC scenarios could improve the average of objectives for Type-1 and Type-2, while in Type-rw, no general conclusion can be made due to large GAPs in the related HRC scenario for the corresponding test problems.

Overall, the above results showed that in the considered HRC scenarios, the U-shaped layout could improve more than the straight line regarding the related objective at each problem type. In addition, the detailed results show that the AL-HRC (with both straight and U-shaped layouts) with higher HRC levels results in more efficient solutions for the related problem types compared with the assembly lines with no or limited HRC.

5.5. Research implications

Based on the results, the following managerial insights can be presented.

- Considering the scope of problems in BS-AL-HRC, the Type-1 model usually deals with long-term decisions, particularly when a new AL-HRC is configured. The Type-2 model supports the mid-term decision to re-balance and re-schedule an existing AL-HRC. Finally, the Type-rw model deals with the long-term decision of establishing an AL-HRC from the cost perspective. The Type-rw optimizes the cost of stations, operators (both number and type of HRC), and robot energy expenditure while considering real-world constraints. Moreover, considering the BS-AL-HRC problems, the decision-makers can utilize the proposed straight and U-shaped lines models to investigate which layout is more beneficial for JIT production.
- According to the computational results, compared to the straight one, the U-shaped layout is more promising in obtaining efficient solutions. This can be due to the higher utilization of operators while performing tasks at the entrance and exit of the line. Furthermore, the percentage of improvement in the objective function obtained by different model types shows that Type-2 results in more improvement than Type-rw, followed by Type-1. In addition, the analysis of results showed that the AL with a high HRC level is more promising in obtaining efficient solutions than assembly lines with no or limited HRC.
- Applying the proposed MILP^{LB-UB} models in the real case from the automotive company has proven their applicability in providing high-quality solutions in small to medium-sized problems. This can be due to both efficient lower and upper bounds of objectives in the proposed MILP^{LB-UB} and the advancement of computer technologies enabling decision-makers to solve such complex optimization problems. However, as a future research direction, embedding some initial solutions into the MILP^{LB-UB} as a warm start and integrating MILP^{LB-UB} with customized heuristics can further enhance the solver efficiency, particularly in addressing large-sized problems. Additionally, humans' well-being and environmental aspects while formulating and solving BS-AL-HRC are yet to be explored.

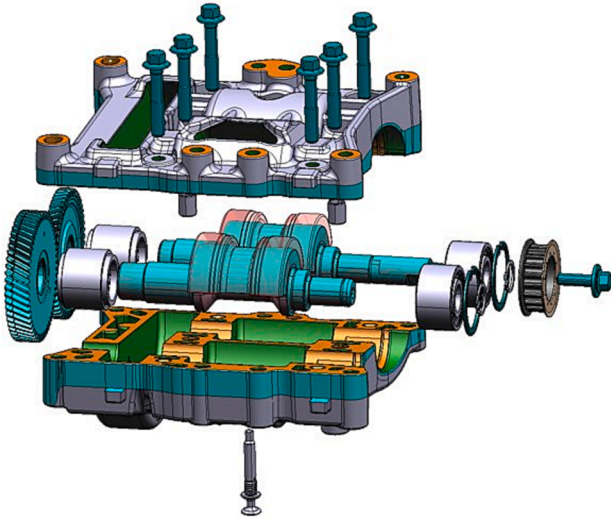


Fig. A1. The explosive map of the MBS unit in the case study.

6. Conclusions

Owing to the growing emergence of cobots in the era of Industry 4.0, recently, many manufacturers have attempted to shift their current manual or robotic assembly lines towards a human-robot collaboration (HRC) environment. In an HRC workplace, human skills and robot capabilities are combined to experience more flexible and agile manufacturing while achieving a human-centric and resilient workplace as initiated by the Industry 5.0 trend. Such advanced assembly line design entails balancing and scheduling tasks and operators among the stations. Additionally, the consideration of line layouts can significantly affect the resulting configuration. This study deals with balancing and scheduling straight and U-shaped AL-HRC with different objectives. The three objectives are categorized into minimizing the number of stations (Type-1), cycle time (Type-2), and the costs of stations, operators, and

robot energy consumption (Type-rw). In the Type-rw, the concept of HRC is enhanced by integrating multiple humans and cobots possessing diverse skills and capabilities, enabling a flexible and efficient utilization of resources while allowing separate, collaborative, and parallel execution of tasks at various stations. A few practical aspects are considered, such as robot energy consumption, robot tool changes, zoning, and technological requirements. Different mixed-integer linear programming (MILP) models with efficient lower and upper bounds for each objective are proposed and tested by solving different case instances and standard test problems from the literature. The computational results show that the proposed MILP^{LB-UB} overcomes the conventional MILP model regarding objectives and gaps. Furthermore, analysis of results shows that the U-shaped layout consistently improved in Type-2, followed by Type-rw and Type-1, while offering greater flexibility than the straight line, leading to more efficient solutions for JIT production. Finally, the higher the HRC level in the straight and U-shaped assembly lines, the better the achievement of the desired objectives compared to the lines with no or limited HRC.

This study can be further extended by including the ergonomic risks and environmental aspects while addressing the considered problems. Moreover, another topic to explore is integrating the proposed MILP^{LB-UB} models with initial solutions or developing customized heuristics for them.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

I have shared the data in the Attach file step

Table A1

The input data for the considered case study.

Task	Name	HT (s)		RT (s)		RE (cost)		PR	ZC	TER	COT	RTR
		1	2	1	2	1	2					
1	Place axle 1 in MBS 1 bottom	8	4	16	8	2.52	2.20			2		2
2	Place axle 2 in MBS 1 bottom	12	6	24	12	3.78	3.30	1		2		2
3	Place axle 1 in MBS 2 bottom	8	4	16	8	2.52	2.20			2		2
4	Place axle 2 in MBS 2 bottom	12	6	24	12	3.78	3.30	3		2		2
5	Place MBS 1 top on MBS 1 bottom	12	12	12	12	1.89	3.30	2			6	1
6	Place MBS 1 top on MBS 1 bottom	12	12	12	12	1.89	3.30	2			5	1
7	Place MBS 2 top on MBS 2 bottom	12	12	12	12	1.89	3.30	4			8	1
8	Place MBS 2 top on MBS 2 bottom	12	12	12	12	1.89	3.30	4			7	1
9	Pick and place screws 1&2 on MBS 1	2.2	1.1	15.2	7.6	2.39	2.09					3
10	Pick and place screws 3&4 on MBS 1	2.2	1.1	15.2	7.6	2.39	2.09					3
11	Pick and place screws 5&6 on MBS 1	2.2	1.1	15.2	7.6	2.39	2.09					3
12	Pick and place screws 1&2 on MBS 2	2.2	1.1	15.2	7.6	2.39	2.09					3
13	Pick and place screws 3&4 on MBS 2	2.2	1.1	15.2	7.6	2.39	2.09					3
14	Pick and place screw 5&6 on MBS 2	2.2	1.1	15.2	7.6	2.39	2.09					3
15	Tighten screws 1&2 on MBS 1	7	3.5	15	7.5	2.36	2.07	5,6,9				4
16	Tighten screws 3&4 on MBS 1	7	3.5	15	7.5	2.36	2.07	5,6,10				4
17	Tighten screws 5&6 on MBS 1	7	3.5	15	7.5	2.36	2.07	5,6,11				4
18	Tighten screws 1&2 on MBS 2	7	3.5	15	7.5	2.36	2.07	7,8,12				4
19	Tighten screws 3&4 on MBS 2	7	3.5	15	7.5	2.36	2.07	7,8,13	20			4
20	Tighten screws 5&6 on MBS 2	7	3.5	15	7.5	2.36	2.07	7,8,14	19			4
21	Pick and place pulley on MBS 1	4	2	8	4	1.26	1.10	2				1
22	Pick and place the pulley screw and washer on MBS 1	4	2	8	4	1.26	1.10	21	23			3
23	Brace the axle and tighten the pulley screw on MBS 1	7	3.5	14	7	2.20	1.93	15,16,17,22	22			4
24	Pick and place pulley on MBS 2	4	2	8	4	1.26	1.10	4				1
25	Pick and place the pulley screw and washer on MBS 2	4	2	8	4	1.26	1.10	24				3
26	Brace the axle and tighten the pulley screw on MBS 2	7	3.5	14	7	2.20	1.93	18,19,20,25				4
27	Perform rotational control on MBS 1	4.2	2.1	8.4	4.2	1.32	1.16	23				1
28	Perform rotational control on MBS 2	4.2	2.1	8.4	4.2	1.32	1.16	26				1

Appendix

The explosive map of the MBS unit assembled in the application case is shown in Fig. A.1. This assembly unit mainly consists of the top and bottom halves, two axles, six screws, and a modular set of pulleys with screws and washers.

The input data for the assembly of two required MBS units are summarized in Table A.1. The table includes the task number, name, human time (HT), robot time (RT), robot energy expense (RE), precedence relationships (PR), zoning constraints (ZC), technological requirements (TER), collaborative tasks (COT) and robot tool requirement (RTR).

Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.cie.2023.109775>.

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