Analysis of Atmospheric Muon Bundles with IceCube

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Abstract

This work is a preliminary study of the background of a search for dark, long-lived particles in the IceCube detector. The high flux of atmospheric muons in IceCube is considered background to the detector’s primary science goal, which is to detect astrophysical neutrinos through the emission of Cherenkov radiation. However, high muon rates may break fresh ground for the detection of hypothetical dark particles. These could be created during the Bremsstrahlung-like interaction of an energetic muon and decay into a muon and an electron. Such an event is expected to produce a characteristic signal in the detector: A track-like signal produced by the Cherenkov-photon emitting muon, followed by a gap due to the electric neutrality of the dark particle. The subsequent decay of the dark particle into a muon produces a further track-like signal.

In order to probe the success rate of this endeavor, two precursory statistical analyses are made using simulated data provided by CORSIKA. In order to obtain a clear track-gap signature that is not diluted by other particles, atmospheric muons should preferably travel in a bundle of few to no other muons. The muon multiplicity in incident bundles is estimated. The study reveals that 50% of all muons are single muons at the point of production, while their relative number increases to 70% by the time they reach the detector boundary.

A more thorough selection taking into account the particle energies as well as IceCube's limited energy resolution is expected to lower the rate.

1 Introduction

This work aims to study the background of a search for long-lived particle (LLP) interactions using atmospheric muons detected in IceCube.

The goal of the project in connection to this study is the indirect detection of the production of a potentially existing non-standard model particle. This hypothetical particle (described e.g. in [6]) is expected to be light, long-lived - with an interaction length on macroscopic scales - and be weakly coupled to the standard model (SM). Especially, this particle is electromagnetically neutral and is thus invisible to a detector sensitive to Cherenkov radiation. LLPs can therefore only be indirectly detected by observing rare, high-energy decays or scattering events of SM particles during which they are produced. IceCube is built to be a neutrino detector, however, a large part of the signal is background produced by atmospheric muons. They give a detection rate of 2.2 kHz [20], corresponding to more than $10^{10}$ detected muons per year. In contrast, neutrinos are detected at a rate of a few mHz [2]. Thus, it is of interest to take advantage of the large background rate and search for interactions involving atmospheric muons that produce LLPs and that can be detected via a unique signature. Hence, the background for IceCube neutrino research is utilized as the beam for the LLP search.

Under consideration here is the Bremsstrahlung-like interaction of a muon to produce a dark, long-lived particle alongside the emission of an electron, as explored in [10]. Subsequently, after a considerable traveled distance the dark particle is to decay back into a muon and an electron, see Fig. [4]. This is a charged lepton flavor violating (CLFV) model and produces a track gap signature in the IceCube detector. The expected signal within the detector is a distinguishable muon track, which then disappears for a brief period of time and reappears at a later point along the extrapolated track. The existence of the dark particle is then indicated by a track gap of detectable length. The search will be dedicated towards events where both the production as well as the decay of the LLP happen inside of the detector volume.

A search for LLPs is currently being attempted at the Large Hadron Collider (LHC) in the ForwArd Search ExpeRiment (FASER) [4]. The aim is to detect hypothetical dark particles produced during high-energy collisions at the interaction point in the LHC that may then pass through many meters
Figure 1: LLP production and decay. A muon decays via a LFV scattering process into an electron and a dark particle. The dark particle then decays into an electron and a muon, producing a track-like signature with a gap corresponding to the existence of the LLP.

of shielding material and enter the FASER detector. Due to the limitations of producing high-energy particles in the collider, energies acquired by subsequent LLPs do not exceed the TeV range. In contrast, IceCube can detect the particles traveling through our atmosphere. These are primarily the products of extensive air showers initiated by the interaction of a cosmic ray with a particle in Earth’s upper atmosphere. During an air shower, the initially highly energetic astroparticle will divide its kinetic energy between all the products of the interaction to produce a large number of lower-energy particles. Many of these are muons, which are typically associated with a bundle of muons and neutrinos stemming from the same primary as they travel through the detector. IceCube’s optical modules detect muons by measuring the Cherenkov radiation emitted as they travel through ice (see Section 2). The muons may have energies up to 100 TeV, thus greatly exceeding the energy limitations of FASER. Furthermore, the setup of FASER limits the detector to capture LLPs of a fixed decay length. LLP detection in IceCube, on the other hand, detects LLPs of various lengths, as long as both the production and decay points are contained within the detector. The large detector volume of a cubic kilometer permits many different decay lengths ranging between approximately 30 to 1000 m. The substantial effective volume increases the chances of detecting rare high-energy muons, despite a limited spatial resolution due to the large separation of the detector units.

This study aims to test muon multiplicity of the incident bundles as well as the rate of LLP background events. In a bundle of several collinear muons with small separation the desired dis- and reappearance of a single muon may not be distinguishable from the through-going track of a non-interacting muon, as the track-gap signal is diluted (see left panel of Fig. 2). Hence, a clear detection of an LLP signature can only be claimed if the multiplicity of muons arriving in a bundle is small. The muon multiplicity and the corresponding energy distribution are studied qualitatively to assess whether the devised goal is realizable. Additionally, collinear neutrinos may pose a serious background to the search, as the disappearance of a muon followed by a subsequent interaction of a neutrino may imitate the desired signal (see right panel of Fig. 2). In order to ascertain whether this coincidental occurrence gives a significant false-positive rate, the rate of such a case is determined by revising simulated muon and neutrino background data. The combination of these checks will help evaluate whether this endeavor to search for LLPs is feasible.
2 The IceCube Detector

IceCube is a particle observatory located on the South Pole, whose primary science goal is to detect high-energy astrophysical neutrinos. The detector is in operation since 2011. The design and technicalities are described in detail in \[3\]. In summary, the detector consists of 86 strings arranged in a nearly-octogonal shape, each holding 60 detector modules. This array is embedded deep within the Antarctic ice, between a depth of 1450 and 2450 m. Specifically, each digital optical module (DOM) is instrumented with a photomultiplier tube, which captures incident photons and transforms them into an electrical signal. Over a short period of about one second, data of the captured light pulses is accumulated and then sent to the IceCube lab. The saved data includes the time of the hit as well as the waveform shape of the incident light. This allows the time reconstruction of photon hits. The DOMs are separated vertically by 17 m along each string and the strings are set 125 m apart, to create an enormous three-dimensional grid of detector units. The array covers a volume of approximately 1 km$^3$. Associated with IceCube, there is also a detector array on the ice surface (IceTop) aiding in the study of air showers, as well as a smaller and denser array of DOMs within IceCube (DeepCore) to lower the detector’s energy threshold. The layout of the detector facilities is shown in Fig. 3.

The detection of particles in IceCube works on a physical basis of detecting Cherenkov radiation \[3\]. Electrically charged particles traveling through a medium at velocities faster than the speed of light in that medium emit Cherenkov photons. The radiation is emitted in a cone around the particle track, the radiation angle being approximately 40° in ice \[16\]. A particle traveling through ice with a velocity greater than the speed of light in ice will produce a continuous emission of photons. In IceCube, these photons may hit the DOMs and produce light pulses in the detector. By searching for patterns in the signals detected by a large number of neighboring DOMs, the track of a particle can be reconstructed. The primary science goal to be achieved with IceCube is the study of astrophysical neutrinos. These are electrically neutral and don’t emit Cherenkov radiation themselves. Instead, a neutrino may interact with nuclei in the ice and produce charged particles - e.g. electrons, muons or taus - with high velocities, which will in turn emit Cherenkov photons. Depending on the neutrino flavor, two different signatures can be expected \[3\]: A track-like signature originating from a muon neutrino producing a muon via a CC interaction. This muon then emits Cherenkov radiation along its track. Alternatively, all neutrino flavors may interact to produce hadronic or electromagnetic showers of different charged particles. The detector then registers Cherenkov photons seeming to stem from a point-source, with a signal of spherical shape due to the scattering on ice.

Figure 2: **Left:** The muon signal is diluted by companion muons in bundle. **Right:** The LLP signature is imitated by a muon decay with a subsequent neutrino interaction.
IceCube’s sensitivity to these secondary particles allows the indirect detection of ultrahigh energy neutrinos, with energies to the order of PeV and above. However, the astrophysical neutrino flux is found to be small in addition to the already inherently small interaction cross section of neutrinos at all energies. This makes the detection of neutrinos very rare, such that a large effective volume is needed to provide a statistically significant detection rate. In this way, IceCube’s large volume optimizes the detector to search for high energy (>$\text{TeV}$) astrophysical neutrinos and detects them at a rate of mHz \cite{2}. These are believed to be emitted by astrophysical sources, such as supernovae ejecta and active galactic nuclei \cite{18}. Hence, studying neutrinos will shed light onto the origin of astroparticles and give insight into the nature of these extremely distant particle accelerators and their highly effective acceleration mechanisms. The second large group of neutrinos stem from air showers initiated by cosmic rays, as previously discussed. These neutrinos dominate at lower energies (>$\text{GeV}$).

However, neutrinos comprise only a small fraction of particles entering the detector and emitting Cherenkov radiation in the ice. In fact, the vast majority of the detected particles are atmospheric muons, created in the same air shower processes as neutrinos. With a detection rate of $2.2 \text{ kHz}$ \cite{20}, atmospheric muons compose a troublesome background to the search for neutrinos, as their tracks are identical to those of neutrino-induced muons. Relative to muons, neutrinos rarely interact in matter and may therefore penetrate Earth without being absorbed. Upgoing muon tracks in IceCube can therefore safely be attributed to neutrinos after interacting, while downgoing tracks are dominated by atmospheric muons. Fig. 4 shows a downgoing atmospheric muon and neutrino traveling through the detector. The Cherenkov light emitted by the muon is shown as the color-coded DOM hits. In addition to being highly abundant, particles of astrophysical origin naturally have larger energies than the ones that can be produced in modern day particle accelerators. Therefore, while being intended for astrophysical science, IceCube also provides the optimal laboratory to study high-energy particle physics. In this case, the atmospheric muon background in IceCube is used to investigate physics beyond the standard model, with focus on searching for hypothetical dark particles produced in muon interactions.
Figure 4: A visualization of a muon (solid line) and a neutrino (dashed line) pathway through the IceCube detector. The detector array is shown as the white structure consisting of strings with the attached DOMs. DOM hits are resembled by colored bubbles. The amount of energy deposited in a DOM is represented by the size of the bubble. The underlying colormap from red over green to blue corresponds to the relative time of the hit. This image is created using the internal IceCube event viewer program Steamshovel [19].

3 Data

This work uses pre-simulated air shower data by the Monte Carlo simulation software Cosmic Ray Simulations for K\textsc{ascade} (CORSIKA) [11]. This program simulates air showers instigated by high-energy cosmic rays interacting in Earth’s atmosphere. The primaries interact to produce lighter nuclei and leptons, which may then undergo further hadronic interactions, annihilate or decay. The Monte Carlo program PROPOSAL (PRopagator with Optimal Precision and Optimized Speed for All Leptons) [14] then propagates muons in the ice and saves the interaction and energy loss data. During propagation, muons are subject to energy loss through pair production, Bremsstrahlung, nuclear interactions as well as continuous ionization. Energy losses by ionization are summed up along the trajectory to give a sequence of larger energy losses. Neutrino interactions are neglected and they traverse the detector without interaction. Furthermore, the photon-tracking code clsim [8] simulates the emission of Cherenkov photons by charged particles and their DOM hits are registered by DOMlauncher [7]. As upgoing muons would be fully absorbed as they travel through Earth, only downgoing ones reach the detector. Therefore only the latter are simulated.

The data used for this analysis contains events with primary energies ranging between 600 and $10^8$ GeV. While the true energy spectrum of incident cosmic rays follows a powerlaw profile proportional to $E^{-2.7}$, the generated spectrum follows an $E^{-2}$ law in order to increase the statistics for high-energy cosmic rays. This can be weighted to portray a realistic energy distribution using the weighting package SimWeights [17]. SimWeights is an IceCube specific tool that weighs the spectrum according to the chosen cosmic ray energy spectrum and returns the rate of events in Hz.

Primaries treated in this dataset are protons (p) and the nuclei helium (\(^4\text{He}\)), nitrogen (\(^{14}\text{N}\)), aluminum (\(^{27}\text{Al}\)) and iron (\(^{56}\text{Fe}\)). The defining quantities for the particles and their trajectories that are
used in this analysis are provided in Table 1.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>Unique identifier number</td>
</tr>
<tr>
<td>Type</td>
<td>Particle type (mainly $\mu^+$, $\mu^-$, $\nu_e$, $\nu_\mu$)</td>
</tr>
<tr>
<td>Energy</td>
<td>Energy at the ice surface</td>
</tr>
<tr>
<td>$X, Y, Z$</td>
<td>$x, y, z$ coordinates at the ice surface ($z \approx 1950$)</td>
</tr>
<tr>
<td>$\theta, \phi$</td>
<td>Directional vector consisting of zenith and azimuth angle</td>
</tr>
<tr>
<td>Length</td>
<td>Distance in m traveled by a particle from its starting point to the track's end</td>
</tr>
</tbody>
</table>

Table 1: Excerpt of the data stored by CORSIKA for each particle.

The coordinate system used for IceCube is defined internally [13] to be a Cartesian right-handed one, its $z$-axis is aligned with the cylindrical detector’s long axis. Its origin is set in the center of IceCube, at a depth of 1950 m below the surface of the ice. Consequently, IceTop is situated at $z \approx 1950$ and the upper and lower boundary of IceCube are at $z \approx 500$ and $z \approx -500$, respectively. The definition of zenith $\theta$ and azimuth $\phi$ are equivalent to the definition used in a standard spherical coordinate system, so that $\theta = 0$ corresponds to a particle traveling vertically downward.

An example file containing 9967 events is used to visualize the energy and angle distributions of the data. The energy spectra of the primaries and the propagated muons are shown in Fig. 5. The particles simulated here all have enough energy to reach and trigger IceCube, despite their energy losses on the way. This explains the sudden cutoff below $10^3$ GeV and 100 GeV for primaries and muons, respectively. The particles’ trajectories are defined by a zenith angle $\theta$ and azimuth angle $\phi$. Particles with $\theta = 0$ travel vertically downwards and either hit both IceTop and IceCube or neither. It is implied here that the particles have sufficient energy to reach IceCube. There is of course a fraction that trigger IceTop, but stop before reaching IceCube. While the distribution of azimuth angles is uniform, zenith angles peak at $\pi/2$, as is shown in the left panel of Fig. 6. This is due to the increasing size of the field of view with increasing $\theta$. For an isotropic flux of muons, this will result in a smaller flux for small $\theta$. Strongly inclined trajectories cause muons to traverse longer distances in medium for a fixed vertical distance. Thus, energy losses between two points on a vertical axis for muons with large $\theta$ are greater and the energy threshold required for them to trigger IceCube rises. This gives rise to an increase in energy towards larger zenith angles, shown in the right panel of Fig. 6.

4 Muon Multiplicity Study

A particle air shower often results in numerous muons arriving at the detector simultaneously, i.e. air shower induced muons are expected to frequently come in bundles. The muons of a bundle are then expected to have a small lateral separation and nearly collinear tracks. Due to the large separation of DOMs on the strings, Icecube’s power to resolve individual tracks within a bundle is rather low. Especially when several muons enter the detector at close distances from each other, it may not be possible to distinguish different tracks from the registered light pulses. Consequently, the signature of an interacting muon among a bundle will be invisible to the detector array. These events provide no aid in the search for LLPs, but rather reduce the number of valuable events. This issue is likely to depend on the spacing of incident muons as well as their multiplicity. For instance, several muons of large lateral separation may yet give well defined track signals, corresponding to a single muon each. Similarly, the signature of one stopping muon track among two closely spaced muons with diluted signals may still be visible. In order to determine which events may still provide useful signals, a deeper examination of these two factors with respect to the resulting signal is required.

1/0198000-0198999/detector/IC86.2020_corsika.020904.198302.i3.zst
Figure 5: **Left:** The weighted energy distribution of the primary particles **Right:** The weighted energy distribution of the resulting muons. The muon energies are counted individually, instead of summing the total energy of a bundle.

Figure 6: **Left:** The zenith angle distribution of muon trajectories. **Right:** The zenith distribution with respect to muon energy.

This problem is avoided altogether by exclusively looking at single muon events. While this being the simplest solution, it may come at the cost of reducing the rate of valuable events significantly. Should the majority of events contain a muon multiplicity greater than one this will render a large number of the events useless for the search for LLPs. It is therefore necessary to ascertain the fraction of muon events that involve a single muon. This quantity is thereafter denoted as the muon multiplicity fraction \( X_m \), where \( m \) is an integer referring to a specific muon multiplicity. It is defined as follows:

\[
X_m = \frac{N_m}{N_{\text{tot}}}.
\] 

Here, \( N_m \) refers to the number of events with a muon multiplicity \( m \), and \( N_{\text{tot}} \) resembles the total number of events.

Muon multiplicity fractions for different values of \( m \) are computed according to Eq. 1 for the CORSIKA file \(^\text{2}0198000-0198999/detector/IC86.2020_corsika.020904.198302.i3.zst\) mentioned in the previous Section. Resulting multiplicity fractions at IceTop as well as at the
detector surface are shown in Fig. 7. From the figure it becomes evident that approximately 50% of all events contain a single muon at the surface of the ice. This high single muon fraction is favorable towards the search for LLPs. Even more so is the percentage of single muons at the IceCube’s boundary of 70%. This increased number suggests that a significant number of muons in bundles decay before reaching the IceCube or miss the detector all together. This shifts higher multiplicity events at the ice surface to lower multiplicities at the detector boundary. Thus, of the muons that reach the detector, over 70% of events have "lost" all of their companion muons and now have $m = 1$. Considering the total muon event rate of 2.2 kHz, a back-of-the-envelope calculation implies a rate of 1.5 kHz of single muons or in other words, 50 million events per year. This number is sufficiently large to deem the search for LLPs among this sample reasonable.

In addition, the energy distribution for single muons at the ice surface and at the detector boundary are presented in Fig. 8. With traveled distance, the muon energies shift to lower values, as becomes clear when comparing the two histograms. When comparing the left panel of Fig. 8 to the total muon energy spectrum (Fig. 5 right panel), it can be observed that single muon events occupy the low end of the spectrum. On the other hand, muons in events of larger multiplicities span energies from $10^2$ to $10^5$ GeV and muons with energies greater than 10 TeV almost exclusively occur in bundles.

Figure 7: **Left:** Muon multiplicity fraction $X_m$ for multiplicities from 0 to 30 at the point of interaction. **Right:** Muon multiplicity fraction $X_m$ for multiplicities from 0 to 30 at the detector boundary.

Figure 8: **Left:** Single muon energies at IceTop. **Right:** Single muon energies at the detector boundary.
5 Background Rate Estimation

The aim is to estimate the event rate of an LLP signature imitation due to a stopping atmospheric muon in the detector with a subsequent neutrino interaction (see right panel of Fig. 2). In such cases, the resulting signal in the detector can be mistaken for an LLP event and pose an undesired background to the LLP search. A visualization of an expected LLP track gap signature in the detector is shown in Fig. 9; the same signal is expected of the previously described background. To estimate the significance of this background, a folder consisting of 10,015,261 events is filtered to select those that match this scenario. The selection is done in two steps. First, events are filtered to contain at least one stopping muon that decays visibly. As neutrino interactions in the ice aren’t simulated, the probability of a neutrino interacting after the muon stops is then computed using neutrino cross section data. This section gives a detailed description of the event selection method as well as the computation of interaction probabilities. Follow [9] for the GitHub repository containing the full code. After the final selection, 1,741,911 events, amounting to 2530 s of live-time remain.

Figure 9: Visualization of the desired track gap signal indicative of an LLP event. An atmospheric muon-neutrino pair may potentially produce an identical signal. This image is created using Steamshovel [19].

5.1 Event selection

The selected events should contain at least one muon with a minimum of one collinear neutrino. It is required that at least one of the muons stop inside the detector at an adequate distance from the detector boundaries in order for its signal to be identifiable.

The first selection iteratively chooses events that have a muon associated with at least one neutrino, paying no respect to the collinearity condition yet. This is followed by an iteration through all muons within every selected event, searching for stopping muons within the detector boundaries. Extrapolating the muon track, so that each muon enters and exits the detector volume gives two intersection points with the detector volume - at the point of entry and the point of exit (see left panel of Fig. 10). The two intersection points can now be parameterized by the distance along the particle trajectory (see Appendix A). The entry and exit intersection points are labeled $d_1$ and $d_2$ in Fig. 10 respectively.
A muon decaying inside the detector can now be distinguished by comparing \( d_1 \), \( d_2 \) and \( \text{length} \) (see Table 1). Referring to \( d_{\text{muon\_stop}} \) in the remainder of this section. A number of muons never intersect the detector volume, as they pass by it without entering. Should a muon miss the detector or stop before it, it is ignored and the next muon in the bundle is considered. Should the muon track stop inside of the detector volume, it is saved. Through-going muons, that traverse the detector without stopping create a continuous signal along their tracks, covering any signature of a stopping muon in the same bundle. Therefore, this bundle must be discarded altogether and the next event is considered. Further steps are taken for muons stopping inside the detector. A sketch of this iterative procedure is shown in the following few lines.

\[
\begin{align*}
  d_1 \geq d_{\text{muon\_stop}} & \Rightarrow \text{track stops before entering detector} & \Rightarrow \text{ignore muon} \\
  d_2 \geq d_{\text{muon\_stop}} \geq d_1 & \Rightarrow \text{track stops inside detector volume} & \Rightarrow \text{save muon} \\
  d_2 < d_{\text{muon\_stop}} & \Rightarrow \text{track is through-going} & \Rightarrow \text{discard bundle} \\
  \text{no } d_1, d_2 & \Rightarrow \text{track misses detector volume} & \Rightarrow \text{ignore muon}
\end{align*}
\]

Several muons in a bundle may fulfill the condition \( d_2 \geq d_{\text{muon\_stop}} \geq d_1 \) and of those, only the one that traveled furthest inside the detector volume may be confused with an LLP event. Similar to the case of a through-going muon, the Cherenkov radiation emitted by the furthest traveling muon covers the track of any previously stopping muons and renders their track invisible. Thus, should a bundle contain several muons that stop inside the detector, only the one stopping closest to the exit boundary is saved, i.e. the distance between the stopping point and the second intersection point - \( d_{\text{out}} = d_2 - d_{\text{muon\_stop}} \) - is minimized. This is done by iteratively overwriting the smallest \( d_{\text{out}} \) and saving the corresponding muon object. The variables are then reset after each bundle. The Algorithm 2 in Appendix A shows how this is done. From this point forward, selected events contain at least one stopping muon and no through-going ones. The track and object information of the last stopping muon in each event is saved.

In order for the track gap signal to be unmistakable, both muon tracks (the disappearing and the reappearing) must be clearly visible in the detector. Hence, the saved muons are required to stop a minimal distance from the first intersection point, so as to assure a clear signal upon entering. Further, they are required to stop a minimal distance from the second intersection point, so that a minimum track gap and the reappearing of the signal are still inside the detector. Similarly, the gap in the signal must be large enough for it to be visible. In practice, this requires both of the following two statements to be true. First, the muon travels at least a minimum distance after entering the detector. Secondly, the muon stops before a minimum distance from the exit boundary. This is accomplished by comparing distance from the muon stopping point to both detector boundaries - \( d_{\text{in}} \) and \( d_{\text{out}} \) - with set limit values. The limits are given by a minimal distance between the muon stopping position and the first intersection point, as well as the minimal distance between the muon stopping point and the second intersection point, respectively. The former is equivalent to \( d_{\text{from\_boundary}} \) mentioned in Table 2. The latter comprises both the minimal track gap length, \( d_{\text{min\_trackgap}} \) and the minimal distance required for a neutrino interaction to be away from the detector boundary, \( d_{\text{to\_boundary}} \). The values used for these limits are given in Table 2. An illustration of this setup is shown in the right panel of Fig. 10. The corresponding code is presented in Algorithm 3 in Appendix A.

### 5.2 Cross sections and probability calculation

As neutrino interactions are not simulated, it is necessary to compute their interaction probabilities using neutrino cross section data.

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\[\text{Distances between muons in a bundle are assumed to be small.}\]
Figure 10: **Left:** Lengths from initial coordinates to the intersection points of the muon track vector with the entry boundary (d1) and exit boundary (d2). Similarly, the length from initial coordinates to muon stopping point (\(d_{\text{muon\_stop}}\)). **Right:** Globally defined variables for the selection process are shown in blue. These are \(d_{\text{from\_boundary}}\), \(d_{\text{min\_trackgap}}\), \(d_{\text{to\_boundary}}\) (see Table 2 for details). The lengths from muon stopping point to entry boundary (\(d_{\text{in}}\)) and exit boundary (\(d_{\text{out}}\)) are shown in red.

<table>
<thead>
<tr>
<th>variable</th>
<th>description</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
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<td>(d_{\text{from_boundary}})</td>
<td>minimum length for muon in detector before stopping</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>(d_{\text{to_boundary}})</td>
<td>minimum distance from neutrino interaction to exit boundary</td>
<td>50</td>
<td>m</td>
</tr>
<tr>
<td>(d_{\text{min_trackgap}})</td>
<td>minimum track gap length</td>
<td>30</td>
<td>m</td>
</tr>
<tr>
<td>max_cone_angle</td>
<td>maximum angle of cone with muon track as axis</td>
<td>20</td>
<td>deg</td>
</tr>
</tbody>
</table>

Table 2: Defined variables for detector geometry

**Neutrino selection**

Before the probabilities are calculated, neutrinos are subjected to a selection based on their offset from the muon track. The motivation for this is that a neutrino interacting at a large offset from the muon track will produce a signal that is too far away from the original muon track to be mistaken for an LLP event.

Even if the angle difference between the muon and the neutrino is small - as for two almost collinear tracks - their lateral separation at a certain time depends on the separation of their starting points as well as their traveled distance.\(^5\) Per definition, two collinear particles have an angle distance of zero, they will therefore always have the same distance from each other. In this case, their distance is determined by the distance of their starting points. Two particles of a non-zero, yet small angle difference will either diverge or converge until they pass a plane of minimal distance. In this sense, this method checks the lateral distance of the particles at a certain point rather than their collinearity. As the particles generated in a CORSIKA event stem from the same primary, their tracks mostly diverge with a small angle, thus simplifying the problem to finding the distance between the intersection points.

\(^5\)More precisely, the lateral distance of two particles on a track depends on the full set of the particles’ initial conditions: their initial positions at time \(t\) and their velocities. The initial positions are given in CORSIKA as their starting positions and the time at which this point is reached. Here, this problem is simplified by assuming that the time difference of two particles passing a certain point is irrelevant, as the generated particles stem from the same air shower and time differences are small. Additionally, velocities are approximately equal. This reduces the distance determination to finding the spacial separation of two points on a plane. The time between muon stop and detected neutrino signal would matter if it was a neutrino from a different air shower that coincidentally passes the muon track at the right time. These events cannot be tested using the given files and are therefore neglected.
of two diverging tracks with a specified plane. The aim is to reject neutrinos that are too far away from the muon track within the available interaction length. The tolerated lateral distance for neutrinos to interact in should increase linearly with the distance to the muon stopping point. The geometry of this problem resembles a cone with a fixed angle \( \text{max\_cone\_angle} \) (value given in Table 2) and an axis aligned with the muon track (see right panel of Fig. 11). The cone’s vertex is fixed on the muon stopping point and its opening faces downward. The neutrino is allowed to interact between \( \text{d\_muon\_stop} + \text{d\_min\_trackgap} \) and \( \text{d\_muon\_stop} + \text{dp} \) along the track of the muon, where \( \text{dp} = \text{d\_out} - \text{d\_to\_boundary} \). A sketch of the available interaction length is shown in the left panel of Fig. 11. The constraint set by \( \text{d\_to\_boundary} \) is to ensure that track-like signals induced by neutrino interaction products are distinguishable close to the detector boundaries. At the length \( \text{d\_muon\_stop} + \text{dp} \), a cross section of the cone is constructed. This results in a circular plane with a radius of \( \text{rp} = \tan(\alpha) \cdot (\text{dp}) \), where \( \alpha \) is the angle between the muon track and the neutrino track. Every interacting neutrino with a track intersecting this plane within the radius \( \text{rp} \) will produce a signal that is close enough to the muon track to be mistaken for an LLP event. Only these neutrinos are accepted and their interaction probability within \( \text{d\_muon\_stop} + \text{d\_min\_trackgap} \) and \( \text{d\_muon\_stop} + \text{dp} \) is computed in the next step.

Figure 11: **Left:** The available length for neutrinos to interact is shown in green. On one side, it is constrained by the minimum track gap length (at distance \( \text{d\_muon\_stop} + \text{d\_min\_trackgap} \)). On the other side, it is limited by the required minimum distance from the exit boundary (at distance \( \text{d\_muon\_stop} + \text{dp} \), where \( \text{dp} = \text{d\_out} - \text{d\_to\_boundary} \)). The available track gap length is then given by the available interaction length plus \( \text{d\_min\_trackgap} \). **Right:** The neutrino selection criterion is described by a downward facing cone fixed on the muon stopping point and an axis aligned with the muon track vector. The neutrino track vector is required to intersect the circular area defined by the cross section of the cone at the length \( \text{dp} \) from the cone vertex. The radius of the circle is \( \text{rp} \). The angle between the muon track and neutrino track is given by \( \alpha \).
Interaction probability

Neutrino and antineutrino cross sections for both charged current (CC) and neutral current (NC) interactions and their corresponding errors are taken from [5], Tables I and II. A linear interpolation of the cross sections between the discrete energy values is performed using `scipy.interpolate.interp1d`.

This results in two functions, one for neutrino and one for antineutrino cross sections with energies between 50 and $5 \cdot 10^{11}$ GeV. A plot of the function obtained for neutrino cross sections is shown in Fig. 12.

![neutrino cross sections](image)

**Figure 12:** Neutrino CC and NC cross sections. The data points taken from [5] are marked with x. The interpolated functions are shown as the solid lines. They cover the energy range prevalent in the simulated neutrino sample. For these energies, antineutrino cross sections are slightly lower than those of their antiparticle.

The probability that a neutrino of energy $E$ (corresponding to a cross section $\sigma$) interacts with a nucleon in matter of nucleon density $n$ decreases exponentially with the traveled distance $x$,

$$P_\nu = n \sigma \int \exp(-n \sigma x) dx.$$  \hspace{1cm} (2)

This assumes that $n$ and $\sigma$ remain constant with traveled distance. Nucleons in ice of density $\rho_{\text{H}_2\text{O}} = 0.92 \text{ g cm}^{-3}$ [4] have

$$n = \rho_{\text{H}_2\text{O}} N_A = 5.5 \cdot 10^{23} \text{ cm}^{-3}.$$  \hspace{1cm} (3)

Here, the molar mass of nucleons is implicitly assumed to be 1 g/mole. $N_A$ is Avogadro’s number. Thus, a neutrino traveling between the points $l_1$ and $l_2$ will interact with a probability of

$$n \sigma \int_{l_1}^{l_2} \exp(-n \sigma x) dx = \exp(-n \sigma l_2) - \exp(-n \sigma l_1),$$  \hspace{1cm} (4)

where $l_1 = d_{\text{muon\_stop}} + d_{\text{min\_trackgap}}$ and $l_2 = d_{\text{muon\_stop}} + dp$.

Stopping muons are often associated with several collinear neutrinos. Any one of these will interact with a certain probability to produce a background signal. Thus, the probability for a false positive LLP signal through the interaction of a neutrino is given by the sum over all the interaction probabilities. It is thus natural to compute the total neutrino interaction probability as

$$P_{\text{tot}} = \sum_i P_{\nu,i}.$$  \hspace{1cm} (5)
It is noteworthy, however, that the cross section uncertainties provided by [5] are asymmetric. Consequently, they do not conform to the standard error propagation formalism, which is valid only for Gaussian distributions. In order to take this into account, the Python package add_asym is used for the computation of the total interaction probabilities. This method for the summation of values with non-Gaussian probability density functions has been suggested by [15] and their code is used for the computation. Effectively, the probability sum is then computed iteratively. A detailed description of how the errors are processed can be found in Appendix A.

The LLP production model considered here allows for CC track-like interactions only. However, other models might have different decay modes, such as CC cascade-like or NC interactions. For the sake of completeness, all the following scenarios are considered separately: NC interaction probabilities for all flavor neutrinos, CC cascade-like interaction probabilities (produced by electron neutrinos), CC track-like interaction probabilities (produced by muon and tau neutrinos) and total interaction probabilities (of all neutrinos). Thus, we arrive at four total interaction probabilities corresponding to the aforementioned classes.

The resulting interaction probabilities are converted into probability rates by multiplying with the respective weights using SimWeights.

6 Results and Discussion

Filtering 10 015 261 events for the criteria described in Section 5 leaves 1 741 911 events consisting of a muon accompanied by a neutrino from the same cosmic ray interaction. These amount to a livetime of 2500 s. In view of these numbers, it becomes apparent that 17% of all events may pose a troublesome background to the LLP search according to this selection process. It is notable here that this number is not yet the event rate, but merely the fraction of events that are composed of at least one muon and one neutrino that fulfill the selection criteria. They can be labeled background events, as their signal may imitate the track gap signature induced by the creation and decay of a dark particle. When including the neutrino interaction probability, the resulting total event rate is $5.5 \cdot 10^{-5}$ Hz. This number is mainly part due to track-like CC interactions by muon neutrinos, which constitute a probability rate of $4 \cdot 10^{-5}$ Hz. Only these events produce a track-like signal akin to the LLP decay. The neutrino rate of mHz sets an upper limits to the background rate, and the obtained order of magnitude falls below this rate. Given the 17% selection fraction and the small neutrino interaction cross sections, the results are considered reasonable. It is important to mention that the selection criteria imposed here are far from complete, but rather provide a conservative and preliminary sorting. In order to determine the true background rate, follow-up measurements at IceCube should be made and compared to the results by this theoretical estimate.

The focus of the selection described in Section 5 is mainly on the geometry of particle tracks with respect to the detector and with respect to each other. The program filters for events with muons with recognizable tracks, which implies that they stop at adequate distances from the detector boundaries. Further, the probability calculation accounts for neutrinos that pass within an appropriate distance from the reconstructed muon track after its stopping point. Lacking still is the filtering for energies. So far, muons and associated neutrinos of all energies are permitted. Whereas, in order for the disappearing and reappearing muons of a background event to be confused for an LLP event, their energies should be constrained. During the production of an LLP, the parent muon’s energy is split between the different products of the interaction. While the mass of the dark particle remains an unknown, its neutrality suggests that the energy loss during propagation is negligible. During its decay into an electron and a muon, its energy is once again divided among the products. As a consequence of the unknown LLP mass, the muon energy after the LLP decay cannot be determined precisely. The only requirement that can be made is that the energy of the neutrino at the point of interaction must be smaller than the energy of the muon before stopping.
Thus, energy constraints on the neutrinos are still to be implemented. This should be followed by subsequent constraints on the neutrino energies. In theory, it is important to distinguish between muons stopping due to gradual energy losses and those that undergo spontaneous decay. In the former case, the muon stops due to its low energy and the DOMs may capture the signal of declining energy, rendering the track recognizable as such. Only the latter case produces a muon signal as expected in an LLP event. However, given IceCube’s low energy resolution, it is unclear whether these two classes of events are separable in real IceCube measurements. A scenario that has not been included in the background rate is the imitation of an LLP signature due to the interaction of a neutrino stemming from a different primary than the muon. So far, all background candidates are assumed to originate from particles of the same bundle. These events must be selected separately and then added to the existing filters in order to estimate the background more precisely.

For the calculation of neutrino interaction probabilities, the neutrino track is required to pass through a cone with vertex on the muon stopping point and the opening of a fixed angle \( \alpha \). This is done to ensure that neutrinos that pass too far away from the muon track to be mistaken aren’t included in the probability calculation. In this version of the event selection program, \( \alpha \) is set to a preliminary value only. The exact cone angle depends on the angle between the reconstructed neutrino track and muon track and the precision to which this angle can be determined. In practice, one should also take into account the uncertainties of the muon and neutrino track angles when doing this calculation. When using real data, the uncertainties for quantities as such are determined by the energy- and angular resolution of the signal. Even though the signal resolution is still unknown for the purpose of this work, it can be expected that the cone angle of \( \alpha = 20^\circ \) is an overestimation of the real angular constraint between neutrino and muon track. A reduction of \( \alpha \) will further reduce the number of selected background candidates. Generally, the cone approximation is likely an overestimation of the background, as neutrinos may interact at a large lateral offset from the muon track, while close to the muon stopping point. In order to remove such events, one could impose the requirement for the neutrino to pass through two cross sections of the cone, instead of only one. The first one is to be set closer to the muon stopping point than the second one and will therefore have a smaller radius. This constrains the neutrino track to be almost collinear to that of the muon. Lastly, this event selection was done before the "working-group specific" processing, i.e. level 3 of the suggested IceCube level filtering system. Level 3 filtering is usually done prior to any project-specific event selection. In this stage of data processing, a set of filters is applied to the events and the reconstruction is processed anew. This typically reduces the event rate to below 1Hz. Finally, the background event selection as described in Section 5 can be applied on the remaining events to achieve the most efficient background selection.

The primary energy spectrum and the zenith distribution of the resulting sample of background events are shown in Fig. [13]. The interaction length available for each neutrino (shown and explained in the left panel of Fig. [11]) is a measure for the probability that a neutrino will interact. It implies an available track gap length, i.e. the maximum possible track gap length, produced by a neutrino interacting at the latest point possible. A histogram of available track gap lengths is shown in the left panel of Fig. [14] revealing that this quantity remains mostly below 200 m. A small number of neutrinos have available track gap lengths of over 100 m. The distribution of zenith angles (shown in the right panel of Fig. [14]) peaks at intermediate zenith angles and declines towards larger and smaller zenith angles, similarly to the initial distribution.
Figure 13: **Left:** The primary energy distribution of selected background events. **Right:** The primary zenith distribution resulting from the event selection.

Figure 14: **Left:** Histogram of available track gap lengths. **Right:** Zenith angle distribution with respect to the available track gap lengths.

7 Summary and Conclusion

Using simulated data by CORSIKA, the atmospheric muon flux incident to the particle detector IceCube is studied. This work is in connection with the planned search for LLPs produced in rare muon interactions, as predicted by [10]. The LLPs are invisible to the Cherenkov detector due to their electrical neutrality, and can thus only be observed indirectly. In this case, the detection can be achieved by observing a muon track with a considerable gap somewhere along the track. The track gap then follows the production of the LLP from a muon interaction. The subsequent reappearance of a muon further along the track corresponds to the decay of the LLP. The analysis is split into two parts, consisting of the determination of the muon multiplicity of bundles and a rough estimation of the background to the LLP search.

Atmospheric muons often travel in bundles alongside other particles produced in the same air shower. Inside the detector array, these bundles appear to travel almost collinearly and have a close separation with respect to that of the DOMs. Thus, the signal of a bundle consisting of multiple muons will likely hide the disappearance of an individual muon. However, a study of the simulated data reveals that 50% of the muons at the ice surface are single muons. Extrapolating the muon tracks down to the IceCube boundaries increases this number to 70%, as higher multiplicity events escape the detector. It can be
assumed that the fraction of single muons increases from the ice surface to the detector boundary at the cost of higher multiplicity events. In a bundle of multiple muons, several of them will decay before hitting the detector or diverge from its companions and miss the detector, thus reducing the multiplicity. The energies associated with single muons at the ice surface peak at 1 TeV and reduce to 200 GeV by the time they reach IceCube. Single muon events typically occur at lower energies than higher multiplicity events, as this corresponds to the energy of the primary at the time of air shower. Low energy cosmic rays will divide their energy among fewer product particles than high energy cosmic rays.

Muons often have one or multiple neutrinos associated with them. While these do not threaten to dilute the track-gap signature, a neutrino interacting along the track of a stopped muon may imitate the signal. In order to estimate the significance of this effect, the CORSIKA data is filtered to select these cases. Unlike for muons, neutrino interactions aren’t simulated in the data set. Instead, the probability of a neutrino interacting is added manually. Hence, the filtering process searches for muons that both stop inside the detector and have at least one associated neutrino. For these selected candidates, the neutrino interaction probability is estimated and converted into a probability rate. Given the small interaction cross section of neutrinos and their small detection rate in IceCube, the background rate is found to be $5 \cdot 10^{-5}$ Hz. Electron neutrinos involved in CC interactions constitute the majority of these events. The selection process implemented here is but the first stage of the final background estimation. A number of steps will still be taken to achieve more precise results, like setting energy constraints on the particles involved. Further background producing scenarios must be taken into account, such as a signal imitation due to muons and neutrinos of different primaries. The geometric constraints should be revisited in considering their realisability using real IceCube data, which requires knowledge of the detector limitations. Finally, the data must be filtered according to the IceCube-specific leveled filtering system. The background rate derived here is considered to be a conservative estimation of the true background rate and will be backed up by measurements at a later stage.
References


A Appendix to Background Estimation

Detector geometry

The geometry of the IceCube detector is defined to be an extruded polygon underneath the surface of the ice. The position of the DOMs in the detector are set to be the grid points of an array within the detector volume. The option to set a padding around the detector boundaries is set to 60 m. This increases the detector volume in order to account for light reaching the detectors from muons passing by just outside the detector boundaries.

Position parameterization

The information given in the CORSIKA files contain the coordinates of the particles as they trigger IceTop, as well as a directional vector (given by zenith and azimuth) and the track length (defined as the length of the track from a particle’s starting point to end point). These are summarized in Table 1. The position of a particle in the detector at a certain time is parameterized by the distance it has traveled along its trajectory. The particle’s track is defined as a vector passing through its initial coordinates \((X, Y, Z)\) in Table 1 with an associated direction (given by the corresponding \(\theta\) and \(\phi\) in Table 1). Thus, a particle having traveled 0 m is found at its initial coordinates and a particle having traveled its entire \textbf{length} (see Table 1) is found at its decay/stopping point.

Codes

Algorithm 1 shows the general structure of the muon selection process. Muons stopping inside the detector are subject to further geometrical checks. These functions are shown in the following two code listings.

```python
for muon in bundle:
    if d1 >= d_muon_stop:
        continue  # track stops before entering detector
    elif d2 >= d_muon_stop:
        savelastmuon ( muon )
        distancecheck ( muon )
    elif d2 <= d_muon_stop:
        break  # track is through-going
    else:
        continue  # track misses detector volume
```

Listing 1: Selecting muons that stop inside the detector volume.

Algorithm 2 shows a function selecting the muon in a bundle that travels furthest in the detector.

```python
shortest_d_to_boundary = 99999  # large number to ensure that first muon in bundle is saved
furthest_traveled_muon = None

def savelastmuon(muon):
    d_out = d2 - d_muon_stop
    if d_out < shortest_d_to_boundary:
        shortest_d_to_boundary = d_out
        furthest_traveled_muon = muon
```

Listing 2: Selecting the muon with the shortest distance to the exit boundary.

Algorithm 3 ensures that muons have at least a set distance from the entry detector boundary before stopping, and that the stopping point is at least a certain distance from the muon’s exit boundary.
d_in = d_muon_stop - d1  
d_out = d2 - d_muon_stop  
d_in_min = d_from_boundary  
d_out_min = d_min_trackgap + d_to_boundary  
def distancecheck(muon):  
    if d_in > d_in_min and if d_out > d_out_min:  
        return True  

Listing 3: Requiring the muons to have at least a certain distance from the detector boundaries at their stopping points.

Error handling during probability calculations

Each cross section has associated upper and lower $1\sigma$ errors, which are asymmetric and therefore do not conform to the standard Gaussian probability distribution function. In order to avoid confusion with the notation for the cross section, the errors corresponding to a variable $x$ are denoted $\Delta x_-$ and $\Delta x_+$. Thus, each cross section value is given as $\sigma \Delta \sigma$. 

First, the upper and lower errors are interpolated between the given energy range in the same manner as the neutrino cross sections. Hence, every cross section function is associated with two functions for the upper and lower $1\sigma$ bound. In order to acquire propagated uncertainties for the probability calculation via [4], the errors are propagated via the standard error propagation formalism,

\[ \Delta P_- = \left| \frac{\partial P}{\partial \sigma} \right|_{\sigma_c} \Delta \sigma_- \]  
\[ \Delta P_+ = \left| \frac{\partial P}{\partial \sigma} \right|_{\sigma_c} \Delta \sigma_+ \]  

where the derivative is evaluated at the true cross section value $\sigma_c$. This gives asymmetric upper and lower bounds to each probability value, which relate directly to the upper and lower cross section uncertainties.

When performing a sum, the central values are added in the fashion of Eq. 5 and Gaussian errors are conventionally added in quadrature, e.g. $\Delta P_{\text{tot}} = \sqrt{\Delta P_{\nu,1}^2 + \Delta P_{\nu,2}^2}$. However, in this case, the underlying probability distribution function is not known, and the asymmetric errors are added using the python function add_asym provided by [15]. In the justification of their method [15] argue that the "central" values (in this case $P_{\nu,i}$) are no longer a good estimate of the distribution center. Therefore, their linear addition as well as the quadratic addition of errors has no statistical foundation. Specifically, add_asym sums variables of non-Gaussian probability distributions by transforming each distribution function into a Gaussian function. The package offers a linear and a quadratic transformation, both of which are deemed equivalently accurate. In this case, the quadratic relationship is used. In the transformed state, linear addition is valid and a total probability distribution can be obtained, described by the total mean, variance and skewness. This distribution is then transformed back to obtain the final probability distribution, returning the parameters $P_{\text{tot}}, \Delta P_+$ and $\Delta P_-$. As the backward transformation has no analytical solution, this step is done numerically, resulting in iterative solutions for the final distribution parameters.