Differential cross-section measurements of the production of four charged leptons in association with two jets using the ATLAS detector

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ABSTRACT: Differential cross-sections are measured for the production of four charged leptons in association with two jets. These measurements are sensitive to final states in which the jets are produced via the strong interaction as well as to the purely-electroweak vector boson scattering process. The analysis is performed using proton-proton collision data collected by ATLAS at $\sqrt{s} = 13$ TeV and with an integrated luminosity of $140 \text{fb}^{-1}$. The data are corrected for the effects of detector inefficiency and resolution and are compared to state-of-the-art Monte Carlo event generator predictions. The differential cross-sections are used to search for anomalous weak-boson self-interactions that are induced by dimension-six and dimension-eight operators in Standard Model effective field theory.

KEYWORDS: Hadron-Hadron Scattering

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1 Introduction

The pair production of $Z$ bosons in association with two jets ($ZZjj$ production) in proton-proton collisions is sensitive to a diverse range of physical phenomena. Of particular interest is the purely electroweak (EW) $ZZjj$ process (referred to here as EW $ZZjj$) in which the jets arise through the $t$-channel exchange of electroweak bosons as shown in figure 1 (left). This process is sensitive to the $WWZ$ and $WWZZ$ weak-boson self-interactions, which arise due to the non-Abelian nature of the electroweak interaction. Also of interest is the production of $ZZjj$ with the jets arising from the strong interaction. This is referred to as strong $ZZjj$ production and is shown in figure 1 (right). Theoretical predictions for the strong $ZZjj$ process are very sensitive to the accuracy of the perturbative QCD calculations, both in the overall production rate and in the kinematic properties of the final state. Measurements of $ZZjj$ production can therefore be used to improve our understanding of both the electroweak and strong interactions that underpin the Standard Model (SM) of particle physics.

Measurements of $ZZjj$ production in proton-proton collisions at the LHC have thus far focussed on extracting the electroweak contribution. The ATLAS Collaboration recently published the observation of EW $ZZjj$ production at a significance greater than five standard deviations [1]. Before that, the CMS Collaboration announced evidence for the process...
at a significance greater than three standard deviations [2]. Both of these measurements used multivariate discriminants to separate the electroweak and strong $ZZjj$ components, and reported a fiducial EW $ZZjj$ cross-section within the phase space of the analysis. In the ATLAS analysis, a cross-section for inclusive $ZZjj$ production was also reported. The inclusive $ZZjj$ cross-section is the sum of the strong and electroweak contributions, and the ATLAS measurement provides an important constraint on the overall rate of strong $ZZjj$ production. Measurements sensitive to inclusive $ZZ$ production have also been performed by both experiments, in an inclusive phase space that is not enriched by EW production [3, 4].

In this paper, differential cross-section measurements for the production of four charged leptons in association with two jets are reported ($4\ell jj$ production, with $\ell = e, \mu$). For four-lepton invariant masses greater than twice the $Z$ boson mass, this final state is dominated by the strong $ZZjj$ and EW $ZZjj$ processes, with the $Z$ bosons decaying via $ZZ \rightarrow \ell^+\ell^−\ell^+\ell^−$. Contributions from $\gamma^*$ and $Z^*$ are increasingly important at four-lepton invariant masses that are lower than twice the $Z$ boson mass. Differential cross-sections are particularly useful for probing the electroweak and strong production mechanisms. In the case of EW $4\ell jj$ production, differential cross-sections can be used to search for anomalous $WWZ$ and $WWZZ$ interactions, which have different Lorentz structures to the corresponding interactions in the SM and can therefore distort the kinematic properties of the electroweak process. For strong $4\ell jj$ production, the differential cross-sections allow the perturbative QCD calculations to be confronted in extreme phase space regions such as high dijet invariant mass. The differential cross-sections are measured as a function of three types of observables:

- **Vector boson scattering (VBS) observables** are those that are used to characterise the vector-boson scattering process shown in figure 1 (left). The measured observables are the four-lepton invariant mass, $m_{4\ell}$, the transverse momentum of the four-lepton system, $p_T,4\ell$, the dijet invariant mass, $m_{jj}$, the rapidity interval spanned by the dijet system, $|\Delta y_{jj}|$, and the transverse momentum of the dijet system, $p_T,jj$.

\[ \Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}. \]

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1. ATLAS uses a right-handed coordinate system with its origin at the nominal interaction point (IP) in the centre of the detector and the $z$-axis along the beam pipe. The $x$-axis points from the IP to the centre of the LHC ring, and the $y$-axis points upwards. Cylindrical coordinates $(r, \phi)$ are used in the transverse plane, $\phi$ being the azimuthal angle around the $z$-axis. The pseudorapidity is defined in terms of the polar angle $\theta$ as $\eta = -\ln \tan(\theta/2)$. Angular distance is measured in units of $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$. 

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**Figure 1.** Example Feynman diagrams for EW $ZZjj$ production (left) and strong $ZZjj$ production (right). The scattered quarks (labelled $q$ and $q'$) produce the hadronic jets observed in the final state.
• **Polarisation, charge conjugation, and parity observables**: the polarisation of the Z boson can be probed using the cosine of the angle between the negatively-charged lepton and the Z boson as measured in the centre-of-mass frame of the Z boson. Two angles are measured, $\cos \theta_1^\ell$ and $\cos \theta_3^\ell$, which are sensitive to the polarisation of the leading and subleading Z boson candidates, respectively (the leading Z boson candidate is defined as part of the event selection in section 4). The charge conjugation (C) and parity (P) structure of the WWZ and WWZZ interactions can be probed using the signed azimuthal angle between the two jets, $\Delta \phi_{jj} = \phi_f - \phi_b$, where the jets in the dijet system are ordered such that $y_f > y_b$ [5].

• **QCD-sensitive observables** probe higher-order real emission of quarks and gluons from the 4\ell jj processes. The measured observables are the transverse momentum of the 4\ell jj system, $p_T^{4\ell jj}$, and the scalar sum of transverse momentum of the four leptons and the two jets, $S_T^{4\ell jj}$.

The observables are measured in VBS-enhanced and VBS-suppressed phase space regions. The selection cuts that define the fiducial region of the differential cross-section measurements are given in section 6.

The differential cross-sections are sensitive to anomalous weak-boson self-interactions, which are predicted by effective field theory (EFT) extensions to the SM. Of particular interest are the operators in dimension-eight EFT, which induce anomalous quartic interactions (such as WWZZ) without any corresponding triple gauge coupling (such as WWZ) [6]. Such operators can only be tested using VBS and triboson processes. The differential cross-sections measured in this paper are used in section 9 to constrain the contribution of these purely quartic weak-boson self-interactions.

2 **ATLAS detector**

The ATLAS detector [7] at the LHC covers nearly the entire solid angle around the collision point. It consists of an inner tracking detector surrounded by a thin superconducting solenoid, electromagnetic and hadron calorimeters, and a muon spectrometer incorporating three large superconducting air-core toroidal magnets.

The inner-detector system (ID) is immersed in a 2 T axial magnetic field and provides charged-particle tracking in the range $|\eta| < 2.5$. The high-granularity silicon pixel detector covers the vertex region and typically provides four measurements per track, the first hit normally being in the insertable B-layer (IBL) installed before Run 2 [8, 9]. It is followed by the silicon microstrip tracker (SCT), which usually provides eight measurements per track. These silicon detectors are complemented by the transition radiation tracker (TRT), which enables radially extended track reconstruction up to $|\eta| = 2.0$. The TRT also provides electron identification information based on the fraction of hits (typically 30 in total) above a higher energy-deposit threshold corresponding to transition radiation.

The calorimeter system covers the pseudorapidity range $|\eta| < 4.9$. Within the region $|\eta| < 3.2$, electromagnetic calorimetry is provided by barrel and endcap high-granularity lead/liquid-argon (LAr) calorimeters, with an additional thin LAr presampler covering
$|\eta| < 1.8$ to correct for energy loss in material upstream of the calorimeters. Hadron calorimetry is provided by the steel/scintillator-tile calorimeter, segmented into three barrel structures within $|\eta| < 1.7$, and two copper/LAr hadron endcap calorimeters. The solid angle coverage is completed with forward copper/LAr and tungsten/LAr calorimeter modules optimised for electromagnetic and hadronic energy measurements respectively.

The muon spectrometer (MS) comprises separate trigger and high-precision tracking chambers measuring the deflection of muons in a magnetic field generated by the superconducting air-core toroidal magnets. The field integral of the toroids ranges between 2.0 and 6.0 Tm across most of the detector. Three layers of precision chambers, each consisting of layers of monitored drift tubes, cover the region $|\eta| < 2.7$, complemented by cathode-strip chambers in the forward region, where the background is highest. The muon trigger system covers the range $|\eta| < 2.4$ with resistive-plate chambers in the barrel, and thin-gap chambers in the endcap regions.

Interesting events are selected by the first-level trigger system implemented in custom hardware, followed by selections made by algorithms implemented in software in the high-level trigger [10]. The first-level trigger accepts events from the 40MHz bunch crossings at a rate below 100kHz, which the high-level trigger further reduces in order to record events to disk at about 1 kHz.

An extensive software suite [11] is used in the reconstruction and analysis of real and simulated data, in detector operations, and in the trigger and data acquisition systems of the experiment.

3 Data and simulation

The data used in this measurement correspond to an integrated luminosity of 140 fb$^{-1}$ and were recorded using single-lepton and multi-lepton triggers. The minimum trigger thresholds on transverse momentum depend on the lepton flavour, lepton multiplicity and data-taking periods, and vary between $20 - 26$ GeV and $8 - 24$ GeV for the single-lepton and multi-lepton triggers, respectively. The leptons that fire the trigger are required to be part of the four-lepton system that defines the signal region of the analysis (discussed in section 4). The trigger efficiency for inclusive $4\ell jj$ events is close to unity.

The signal and background processes were modelled using Monte Carlo (MC) event generator simulations. Strong $4\ell jj$ production was modelled using SHERPA 2.2.2 [12]. The fully leptonic final state was simulated using matrix elements at next-to-leading-order (NLO) accuracy in perturbative QCD for up to one additional parton and at leading-order (LO) accuracy for up to three additional parton emissions. Samples for the loop-induced process $gg \to ZZ$ were simulated using matrix elements accurate at leading order (LO) in QCD for up to one additional parton emission, and were normalised using an $m_{4\ell}$-dependent $k$-factor at NLO accuracy in QCD [13, 14], as well as an additional, constant correction of about 1.2 to reproduce next-to-next-to-leading-order (NNLO) calculations for the Higgs-mediated process [15, 16]. Despite the approximation of NNLO effects, the normalisation uncertainty for the $gg \to ZZ$ process is still conservatively taken from the NLO calculation. The matrix element calculations were matched and merged with the SHERPA parton shower
based on Catani-Seymour dipole factorisation [17, 18] using the MEPS@NLO prescription [19–22]. The virtual QCD corrections were provided by the OPENLOOPS library [23–25]. The NNPDF3.0nlo set of parton distribution functions (PDF) were used [26], along with the dedicated set of tuned parton-shower parameters developed by the SHERPA authors. This prediction is referred to as SHERPA strong 4ℓjj.

An alternative prediction for strong 4ℓjj production is obtained using MadGraph5_aMC@NLO [27]. The sample was simulated using matrix elements at NLO accuracy in QCD for up to one parton in the final state, and with the PDF4LHC15 NLO set of PDFs [28]. The MadGraph5_aMC@NLO generator was interfaced to PYTHIA8 to provide parton showering, hadronisation, and underlying-event activity, using the A14 set of tuned parameters [29]. To remove overlap between the matrix element and the parton shower, the different jet multiplicities were merged using the FxFx prescription [30]. EvtGen was used for the properties of the bottom and charm hadron decays. The \( gg \rightarrow ZZ \) contribution is again estimated by using SHERPA. This prediction is referred to as MG5-NLO+Py8 strong 4ℓjj.

Electroweak 4ℓjj production is modelled using MadGraph5 [27]. The Feynman diagrams used in the calculation include the \( t \)-channel exchange of an electroweak boson and the \( s \)-channel processes that contribute at the same order in \( \alpha \text{EW} \), i.e. \( ZZV \) production (with \( V \rightarrow jj \)). The calculation is accurate to LO in perturbative QCD and uses the NNPDF3.0nlo PDF set. The events were passed through PYTHIA8 to provide parton showering, hadronisation, and underlying-event activity using the A14 set of tuned parameters. This prediction is referred to as MG5+Py8 EW 4ℓjj.

An alternative prediction for EW 4ℓjj production is obtained using the Powheg-Box v2 event generator [31–35]. The Feynman diagrams are restricted to the \( t \)-channel exchange of an electroweak boson and the matrix elements calculated at NLO in perturbative QCD, using the NNPDF3.0nlo parton distribution functions. The events are passed through PYTHIA8 to produce the fully hadronic final state, using the A14 set of tuned parameters. The \( s \)-channel contributions from \( ZZV \) production are estimated with the SHERPA 2.2.2 generator, at NLO accuracy for the inclusive process and to LO accuracy for up to two additional parton emissions using the NNPDF3.0nNLO PDF set and the dedicated set of tuned parton-shower parameters developed by the SHERPA authors. The triboson events are added to those from Powheg+PYTHIA8 to produce the alternative EW 4ℓjj prediction. The triboson contribution is about 7% in the fiducial region used in this measurement. This prediction is referred to as Powheg+PYTHIA8 EW 4ℓjj.

Background processes with four prompt leptons arise from \( t\bar{t}Z \) production as well as WWZ and WZZ production. The production of \( t\bar{t}Z \) events was modelled using the SHERPA 2.2.0 generator at LO accuracy, using the MEPS@LO set-up [21, 22] with up to one additional parton. The default SHERPA parton shower was used along with the NNPDF3.0nnlo PDF set [26] PDF set. The sample is scaled such that it reproduces the ATLAS measurement of \( t\bar{t}Z \) production [36]. The production of WWZ and WZZ events were simulated with the SHERPA 2.2.2 generator. The predictions are accurate to NLO in QCD for the inclusive process and to LO for up to two additional parton emissions. The NNPDF3.0nnNLO PDF set was used along with the dedicated set of tuned parton-shower
parameters developed by the Sherpa authors. The sample is scaled such that it reproduces the ATLAS measurement of triboson production [37].

Background processes that do not contain four prompt leptons are estimated by using a data-driven technique as outlined in section 5. The method is cross-checked using simulations of the relevant processes. Production of $WZjj$ with the subsequent leptonic decays of vector bosons was modelled with Sherpa 2.2.2, using the same approach as in strong $4\ell jj$ production. Non-prompt backgrounds arising from $Z$ + jets were simulated using Sherpa 2.2.1 [12] at NLO accuracy in perturbative QCD for up to two partons and LO accuracy for up to four partons. The matrix elements were calculated with the Comix [17] and OpenLoops libraries and matched with the Sherpa parton shower using the MEPS@NLO prescription. The sample was produced with the NNPDF3.0nnlo PDF set, the dedicated set of tuned parton-shower parameters developed by the Sherpa authors, and was normalised to a prediction accurate to NNLO in QCD [38]. The production of $t\bar{t}$ events was modelled using the Powheg-Box v2 [31–34] generator at NLO with the NNPDF3.0nlo PDF set and the $h_{\text{damp}}$ parameter set to 1.5 times the top mass [39]. The events were interfaced to PYTHIA8 to model the parton shower, hadronisation, and underlying event, with parameters set according to the A14 tune.

All samples were passed through a detailed simulation of the ATLAS detector [40] based on GEANT4 [41]. Simulated inelastic $pp$ collisions were overlaid to model additional $pp$ collisions in the same and neighbouring bunch crossings (pile-up). The simulated events were then reweighted to match the pile-up conditions in the data. All simulated events were processed using the same reconstruction algorithms as used in data. Furthermore, the lepton and jet momentum scale and resolution, and the lepton reconstruction, identification, isolation and trigger efficiencies in the simulation were corrected to match those measured in data.

4 Event and object selection

Events are selected for analysis if they were recorded during stable beam conditions and if they satisfy stringent data-quality requirements [42]. Proton-proton interaction vertices are reconstructed using ID tracking information [43] and each reconstructed vertex is required to have at least two associated tracks. Events are required to have at least one reconstructed vertex, with the primary vertex defined as the one with the largest sum of squared track transverse momenta.

Muons are reconstructed from information in the MS and the ID. Baseline muons are required to satisfy the ‘Loose’ identification criteria [44] and are required to be associated with the primary hard-scatter vertex by requiring $|z_0\sin\theta| < 0.5$ mm, where $z_0$ is the longitudinal difference between the primary vertex and the point at which the muon transverse impact parameter is measured. Baseline muons are required to have $p_T > 5$ GeV and $|\eta| < 2.7$. Signal muons are required to satisfy the baseline muon criteria and the ‘Loose’ particle-flow-based isolation working point [44]. They are also required to satisfy $d_0/\sigma_{d_0} < 3$, where $d_0$ is the transverse impact parameter calculated relative to the measured beam-line position and $\sigma_{d_0}$ is its uncertainty.

Electrons are reconstructed from topological clusters of energy deposited in the electromagnetic calorimeter that are matched to an ID track. Baseline electrons are required to satisfy the ‘VeryLoose’ identification criteria [45] and to be associated with the primary
hard-scatter vertex, by requiring $|z_0 \sin \theta| < 0.5$ mm. Baseline electrons are required to have $p_T > 7$ GeV and $|\eta| < 2.47$. Signal electrons are required to satisfy the baseline electron criteria and the ‘LooseAndBLayer’ identification [45] and ‘Loose’ isolation [46] working points. They are also required to satisfy $d_0/\sigma_{d_0} < 5$.

Jets are reconstructed using the anti-$k_t$ algorithm [47, 48] with a radius parameter of $R = 0.4$. The inputs to the algorithm are objects constructed using the particle-flow algorithm [49], based on noise-suppressed positive-energy topological clusters in the calorimeter. Energy deposited in the calorimeter by charged particles is subtracted and replaced by the momenta of ID tracks which are matched to those topological clusters. The jets are initially calibrated using simulations and corrected using in situ measurements of the jet energy scale that is determined from dijet, $\gamma +$ jet and $Z +$ jet events [50]. Jets are required to have $p_T > 30$ GeV and $|\eta| < 4.5$. To reduce the impact of jets that originate from pile-up interactions, jets with $|\eta| < 2.4$ and $p_T < 60$ GeV, or with $2.4 < |\eta| < 4.5$ and $p_T < 50$ GeV, are required to satisfy the ‘Tight’ working points of the jet vertex tagging algorithms [51, 52]. To remove leptons reconstructed as jets, any jets within the range $\Delta R < 0.2$ of an electron are rejected. A similar requirement is applied to jets that overlap with muons, if there are less than three ghost-associated [53] ID tracks within the jet.

Events are required to have at least four (baseline) leptons. The two leptons with the largest transverse momentum are required to satisfy $p_T > 20$ GeV. All possible combinations of same-flavour opposite-charge (SFOC) lepton pairs are formed and each pair is required to satisfy $m_{\ell\ell} > 5$ GeV and $\Delta R(\ell, \ell) > 0.05$, which reduces backgrounds from the leptonic decays of hadrons. The SFOC pairs are then ordered by $|m_{\ell\ell} - m_Z|$. The two $Z$-boson candidates are defined as the two SFOC pairs that have the smallest value of $|m_{\ell\ell} - m_Z|$ and are formed from different leptons. The leading $Z$-boson candidate is defined as the one that has the largest value of $|y_{\ell\ell}|$. The invariant mass of the four leptons is required to satisfy $m_{\ell\ell} > 130$ GeV and each lepton in the quadruplet is required to satisfy the signal lepton definition discussed earlier. Events are also required to contain at least two jets, with the highest transverse momentum jet satisfying $p_T > 40$ GeV. The dijet system is then defined as the two leading (highest transverse momentum) jets in the event that have $\eta_{j1} \times \eta_{j2} < 0$. The dijet system is required to satisfy $|\Delta y_{jj}| > 2.0$ and $m_{jj} > 300$ GeV.

The events that satisfy the selections listed above are then divided into $VBS$-enhanced and $VBS$-suppressed regions using the centrality of the four-lepton system,

$$\zeta = \left| \frac{y_{4\ell} - 0.5(y_{j1} + y_{j2})}{\Delta y_{jj}} \right|,$$  \hspace{1cm} (4.1)

where $y_{4\ell}$ is the rapidity of the four lepton system and $y_{j1}$ $(y_{j2})$ is the rapidity of the leading (subleading) jet in the dijet system. The VBS-enhanced (VBS-suppressed) region is defined as $\zeta < 0.4$ ($\zeta > 0.4$).

The measured and predicted event yields in the VBS-enhanced and VBS-suppressed regions are shown in table 1. The event yields as a function of $m_{jj}$ and $m_{4\ell}$ are shown in figure 2, for both the VBS-enhanced and VBS-suppressed regions. In total, 169 events are reconstructed in the VBS-enhanced region and 53 events in the VBS-suppressed region. In both of the regions, the backgrounds are small and the event yields are dominated by $4\ell jj$...
production. In the VBS-enhanced region, the electroweak process is predicted to be about 17% of the event yield, but this rises to about 40% of the event yield at high-$m_{4\ell}$. There is a modest contribution from background processes that is typically at the 5%–10% level, but reaching 20% at the lowest values of $m_{4\ell}$. The background arising from non-prompt leptons is calculated using a data-driven technique as outlined in section 5. Overall, the data are in good agreement with the signal simulations and predicted backgrounds.

5 Backgrounds

Processes with leptons that do not originate from the decay of a $Z$ boson are considered as backgrounds in this analysis. Background processes with at least four prompt leptons, such as $t\bar{t}Z$ production and the fully leptonic decays of $WWZ$ and $WZZ$ production, are estimated by using the MC simulations presented in section 3.

Backgrounds that contain one or more non-prompt leptons arise from $WZjj$ production (where an additional jet produces a non-prompt lepton) and $t\bar{t}$ production (where the $b$-hadrons produced in the top/anti-top decay are the main sources of the non-prompt leptons). These backgrounds are typically poorly modelled in simulation and are estimated using a data-driven method, whereby their event yield is measured in a control region enriched in non-prompt leptons and extrapolated to the signal region using a scaling factor based on the non-prompt lepton efficiency.

The control region is defined using the same event selection as outlined in section 4, but with one (or more) of the four leptons that define the $Z$ boson candidates failing to meet the signal lepton definition. The event yields per bin of a measured distribution are then extrapolated to the signal region by applying a weight of $f/(1-f)$ for each non-signal lepton in each event, where $f$ is the non-prompt lepton efficiency and defined as the fraction of baseline non-prompt leptons that satisfy the signal-lepton definition. The contribution to the event yields in the control region from processes with at least four prompt leptons are

<table>
<thead>
<tr>
<th>Process</th>
<th>Event yield ± stat. ± syst.</th>
<th>VBS-enhanced</th>
<th>VBS-suppressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong $4\ell jj$ (SHERPA)</td>
<td>98.9 ± 0.5 ± 25.2</td>
<td>45.5 ± 0.3 ± 12.9</td>
<td></td>
</tr>
<tr>
<td>EW $4\ell jj$ (MG5+Py8)</td>
<td>24.1 ± 0.1 ± 1.8</td>
<td>2.12 ± 0.02 ± 0.14</td>
<td></td>
</tr>
<tr>
<td>Prompt background</td>
<td>18.8 ± 0.2 ± 2.2</td>
<td>5.5 ± 0.1 ± 0.4</td>
<td></td>
</tr>
<tr>
<td>Non-prompt background</td>
<td>3.0 ± 0.6 ± 3.2</td>
<td>1.1 ± 0.5 ± 1.2</td>
<td></td>
</tr>
<tr>
<td>Total prediction</td>
<td>144 ± 1 ± 26</td>
<td>54 ± 1 ± 13</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>169</td>
<td>53</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Measured and predicted event yields in the VBS-enhanced and VBS-suppressed regions. The background arising from non-prompt leptons is calculated using a data-driven technique as outlined in section 5. The systematic uncertainties in the predictions are estimated by using the procedure outlined in section 7.
Figure 2. Predicted and observed yields as a function of $m_{jj}$ (top) and $m_{4\ell}$ (bottom), measured in the VBS-enhanced (left) and VBS-suppressed (right) regions. The data are represented as black points and the associated error bars represent the statistical uncertainty. The background arising from non-prompt leptons is estimated using a data-driven technique as outlined in section 5. Background processes with four prompt leptons, such as $t\bar{t}Z$ production and the fully leptonic decays of $WWZ$ and $WZZ$ production, are estimated by using simulations and labelled as ‘Other.’ The total uncertainty on the combined signal and background prediction is shown as a grey band (the calculation of these uncertainties is outlined in section 7).
subtracted before the extrapolation. That contribution is typically at the 5%–10% level, though rises to 20% at the largest $m_{4\ell}$.

The non-prompt lepton efficiency is calculated in data using $Z + \text{jet}$ events and $t\bar{t}$ events. Candidate $Z + \text{jet}$ events are required to have a SFOC lepton pair with an invariant mass within 10 GeV of the $Z$ boson mass, whereas the $t\bar{t}$ candidate events are required to have an opposite-flavour opposite-charge lepton pair. The events are then required to have one additional baseline lepton. The non-prompt lepton efficiency is calculated using this additional lepton, after correcting the event yields for contributions from processes that produce at least three prompt leptons, as a function of the lepton $p_T$, the lepton $\eta$ and the number of jets in the event. The non-prompt leptons in $Z + \text{jet}$ events arise mainly from light-flavour decays, whereas they arise mainly from heavy-flavour decays in $t\bar{t}$ events. The two non-prompt lepton efficiency measurements are therefore combined to reflect the expected flavour composition of non-prompt leptons in the signal region of this analysis, as estimated by using the simulations presented in section 3.

6 Correction for detector effects

Particle-level differential cross-sections for $4\ell jj$ production in the VBS-enhanced and VBS-suppressed regions are obtained by correcting the background-subtracted event yields for the effects of detector inefficiency and resolution.

The particle-level fiducial phase space is defined using stable final-state particles with a lifetime of $c\tau > 10$ mm. Dressed leptons are used to define the $Z$ boson candidates. Electrons and muons are required to be prompt (i.e., to not originate from the decay of a hadron) and the dressed lepton is defined as the four-vector sum of the electron (or muon) and all prompt photons within a range of $\Delta R < 0.1$. Dressed electrons are required to have $p_T > 7$ GeV and $|\eta| < 2.47$. Dressed muons are required to have $p_T > 5$ GeV and $|\eta| < 2.7$. Jets are defined using the anti-$k_t$ algorithm with radius parameter set to 0.4. All stable particles except dressed leptons are used as input to the jet-finding algorithm. Jets are required to have $p_T > 30$ GeV and $|\eta| < 4.5$.

To minimise model-dependence in the final result, the kinematic criteria used to define the particle-level fiducial regions are similar to the criteria applied to the reconstructed data in section 4. At least four dressed leptons are required and the two leptons with the largest transverse momenta are required to have $p_T > 20$ GeV. All possible SFOC lepton pairs are formed and later ordered by $|m_{\ell\ell} - m_Z|$. The two $Z$ boson candidates are defined as the two SFOC pairs with the smallest value of $|m_{\ell\ell} - m_Z|$ that are constructed from different leptons. The leading $Z$ boson candidate is defined as the one with the largest value of $|y_{\ell\ell}|$. The invariant mass of the four leptons is required to satisfy $m_{4\ell} > 130$ GeV. Events are required to contain at least two jets. The dijet system is then defined as the two leading (highest transverse momentum) jets in the event that have $\eta_{j_1} \times \eta_{j_2} < 0$. The highest transverse momentum jet is required to have $p_T > 40$ GeV and the dijet system is required to satisfy $|\Delta y_{jj}| > 2.0$ and $m_{jj} > 300$ GeV. Measurements are carried out in two fiducial regions, with the VBF-enhanced and VBF-suppressed regions defined by $\zeta < 0.4$ and $\zeta > 0.4$, respectively, where $\zeta$ is the centrality of the diboson system defined in Equation 4.1.
The event yields are binned to ensure a minimum of 20 (15) events in each bin in the VBF-enhanced (VBS-suppressed) regions at detector level. The bins are also required to be larger than twice the detector resolution, which is determined using the simulated $4\ell jj$ events. For non-angular observables (e.g. $m_{jj}$, $m_{4\ell}$), the last bin is chosen such that less than one $4\ell jj$ event is predicted to lie above the upper bin edge. However, the ‘overflow’ events that lie above that bin edge are included in the last bin. The background-subtracted event yields are then corrected to particle level using an iterative Bayesian unfolding method [54, 55]. The measurements are unfolded using two iterations, except for the $p_T, 4\ell jj$ distribution in the VBF-enhanced region for which three iterations are used. The unfolding method uses simulations to (i) correct for events selected at detector level that do not satisfy the particle-level selection, (ii) correct for migrations between bins of the measured spectrum with a response matrix, and (iii) correct for events selected at particle level but not at detector level.

The default simulations used in the unfolding are the EW $4\ell jj$ sample produced using MG5+Py8 and the strong $4\ell jj$ samples produced using Sherpa. The fraction of detector-level events that are also reconstructed at particle level, referred as the fiducial fraction, is typically between 60% and 80%. About half of the non-fiducial events have at least one pile-up jet reconstructed in the dijet system, and the remaining non-fiducial events arise from low-$p_T$ jets migrating into the fiducial volume due to jet resolution effects. The fraction of particle-level events that are also reconstructed at detector level, referred to as the efficiency, is typically between 40% and 60%. The low overall efficiency is due to inefficiencies in the trigger and the lepton reconstruction plus identification and isolation criteria. The migration among bins is typically small due to the binning methodology, and ranges from 10% to 30%.

Statistical uncertainties in the data are propagated through the unfolding using a bootstrap method [56], with 10 000 pseudo-experiments. For each pseudo-experiment, each event is randomly assigned a weight based on a Poisson distribution with a mean of one. The unfolding is then repeated using the modified distributions created from the event weights. The final statistical uncertainties in the measurement are taken to be the standard deviation of the unfolded values obtained from the ensemble of pseudo-experiments. The statistical uncertainties in the $4\ell jj$ simulations are propagated through the unfolding procedure in a similar fashion, whereby each pseudo-experiment modifies the simulations used in the unfolding procedure.

To ensure that the unfolding procedure is unaffected by the presence of beyond-the-SM physics in the data, a signal injection test has been performed using the effective field theory (EFT) predictions presented in section 9. Pseudo-data from the EFT simulations are unfolded with the response matrices constructed from the nominal SM simulation, and the result is compared with the particle-level EFT simulation. The unfolding procedure is shown to introduce very little bias, with differences between the unfolded and particle-level distributions being much smaller than the experimental uncertainties in the measurement.

The number of iterations is optimised so the quadrature sum of the statistical uncertainty and the unfolding bias uncertainty (discussed in section 7) is minimised, whilst also requiring that the per-bin statistical uncertainty after unfolding is not smaller than that observed at detector level.
7 Systematic uncertainties

Experimental systematic uncertainties arise from differences between data and simulation in the trigger efficiency, the lepton reconstruction, the jet reconstruction, the luminosity determination, and the average number of proton-proton interactions per beam bunch crossing. These uncertainties affect the normalisation of the background processes estimated by using simulations and the simulations of the signal processes used in the unfolding procedure. For each source of uncertainty, the simulations are varied by ±1 standard deviations and the analysis repeated. The systematic uncertainty is defined as the change in the differential cross-section relative to the nominal analysis with the default simulations.

The luminosity of the data sample is known to an accuracy of 0.83% using a combination of van der Meer beam separation scans in dedicated running periods and luminosity-sensitive detectors in standard data-taking periods [57].

The average number of proton-proton interactions per bunch crossing varies during the different data-taking periods, and the simulations are corrected to reproduce the distribution observed in the data. This procedure is impacted by an uncertainty in the ratio of the predicted and measured inelastic cross-sections within the ATLAS fiducial volume [58]. A systematic uncertainty is introduced to account for this, by scaling the average number of proton-proton interactions per bunch crossing in the simulation.

The lepton reconstruction, identification, isolation and trigger efficiencies in simulation are corrected, using scale factors, such that they match those observed in the data, as outlined in section 3. Systematic uncertainties in this procedure are estimated by modifying the scale factors by their associated uncertainties [44, 46]. Uncertainties in the lepton momentum scale and resolution are estimated by scaling and smearing the lepton transverse momentum by the known differences between data and simulation [46, 59].

Jets are calibrated using a combination of MC-based and data-driven corrections, as discussed in section 4. The jet energy scale and jet energy resolution uncertainties are estimated by scaling and smearing the jet four-momentum in the simulation by the associated uncertainties in the calibration procedure [50]. Furthermore, the JVT algorithm introduces an inefficiency in the jet reconstruction. The uncertainty that arises from imperfect modelling of the JVT in simulation is estimated by varying the JVT requirement [51, 52].

Backgrounds containing at least four prompt leptons are normalised to ATLAS measurements, as outlined in section 3. The uncertainty that arises from subtracting these backgrounds is estimated by varying the normalisation of the simulated samples by an amount commensurate with the experimental precision of the ATLAS measurements. Backgrounds containing non-prompt leptons are estimated by using the data-driven method outlined in section 5. The non-prompt background estimate contains statistical uncertainties from the finite event yields in the control regions and systematic uncertainties from the subtraction of prompt-lepton processes from those event yields. The uncertainty associated with subtracting the non-prompt backgrounds is estimated by varying the background estimates by an amount commensurate with the statistical and systematic uncertainty. Uncertainties associated with the expected flavour composition of non-prompt leptons in the signal region have been checked and found negligible.
**Theoretical systematic uncertainties** in the simulation of $4\ell jj$ production impact the measurement via the unfolding procedure. Three sources of uncertainty are investigated, arising from (i) the renormalisation and factorisation scale dependence in the $4\ell jj$ calculations, (ii) the parton distribution functions, and (iii) the choice of event generator.

The default simulations used in the unfolding procedure are MG5+Py8 (EW $4\ell jj$) and SHERPA (strong $4\ell jj$). These predictions are produced with a default choice for factorisation scale, $\mu_F$, and renormalisation scale, $\mu_R$, but also have additional weights that allow the impact of renormalisation and scale variations to be assessed. There are six variations in total, corresponding to $\{\mu_R, \mu_F\}$ being scaled by $\{0.5, 0.5\}$, $\{0.5, 1.0\}$, $\{1.0, 0.5\}$, $\{1.0, 2.0\}$, $\{2.0, 1.0\}$, or $\{2.0, 2.0\}$. The systematic uncertainty due to scale choice for each generator prediction is then taken to be the envelope of the six scale variations. However, for the loop-induced $gg \rightarrow 4\ell$ contribution to the strong $4\ell jj$ process, the normalisation uncertainty is taken from the higher-order calculations to which the MC sample is normalised, and the scale variations are only used to assess the additional uncertainty in the shape of each distribution.

Uncertainties associated with PDFs are evaluated using the PDF4LHC recommendation [28]. First, the nominal prediction for each generator is reweighted by the 100 variations of the NNPDF PDF set (including the associated $\alpha_s$ variations). The standard deviation of the 100 predictions that are obtained is taken as the uncertainty in the NNPDF prediction. The nominal prediction is then reweighted to reproduce the nominal MMHT and CT14 NNLO PDF sets. The total PDF uncertainty for a given process is then taken as the envelope of the NNPDF uncertainty and the MMHT / CT14 predictions.

The dependence on the choice of event generator is evaluated by unfolding the Asimov data sample constructed from the MG5+Py8 strong $4\ell jj$ simulation with the nominal response matrix. The MG5+Py8 sample is first reweighted on an event-by-event basis to reproduce the SHERPA prediction at particle level, which avoids double counting uncertainties associated with the sample normalisation that is obtained from the factorisation and renormalisation scale variations. The difference between the unfolded event yield and the reweighted MG5_NLO+Py8 particle-level prediction is taken as a symmetric systematic uncertainty in the differential cross-section measurement.

The effect of interference between the EW $4\ell jj$ and strong $4\ell jj$ processes is estimated at leading-order in perturbative QCD using MadGraph5_aMC@NLO 2.6.1. The contribution is found to be far smaller than the theoretical uncertainties in the EW $4\ell jj$ and strong $4\ell jj$ calculations and no additional uncertainty is added.

**Systematic uncertainties in the unfolding method** are evaluated in two ways, using bias tests implemented in simulation. In the first approach, 10 000 pseudo-experiments are created by independently fluctuating each bin of the expected particle-level event yield for each observable. Each pseudo-experiment is then folded using the nominal response matrix and the nominal yield of ‘non-fiducial’ events is added (non fiducial events are those that satisfy the detector-level selection but not the particle-level selection). Each pseudo-experiment is then unfolded using the nominal response matrix and the bias for that pseudo-experiment is taken to be the difference between the unfolded yield and the particle-level yield for that pseudo-experiment. The bias in the unfolding method is taken to be the bias estimated when the particle-level yield has varied by ±1 standard deviation. In the second approach,
Table 2. Systematic uncertainties in the differential cross-section measurements. The range given for each uncertainty reflects the fact that the uncertainties depend on the underlying kinematics of the process and are therefore larger in some bins of certain distributions.

<table>
<thead>
<tr>
<th>Source</th>
<th>VBS-enhanced region</th>
<th>VBS-suppressed region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>0.8–2.1</td>
<td>0.8–2.0</td>
</tr>
<tr>
<td>Leptons</td>
<td>0.8–1.6</td>
<td>1.0–1.5</td>
</tr>
<tr>
<td>Jets</td>
<td>2.7–18</td>
<td>3.4–13</td>
</tr>
<tr>
<td>Pile-up</td>
<td>0.0–2.5</td>
<td>0.0–0.7</td>
</tr>
<tr>
<td>Backgrounds</td>
<td>0.9–9.0</td>
<td>1.2–7.0</td>
</tr>
<tr>
<td>Theory modelling</td>
<td>0.6–7.5</td>
<td>1.2–8.8</td>
</tr>
<tr>
<td>Unfolding method</td>
<td>0.9–12</td>
<td>1.2–12</td>
</tr>
<tr>
<td>Total systematic</td>
<td>6–22</td>
<td>5–17</td>
</tr>
</tbody>
</table>

Figure 3. Systematic uncertainties in the differential cross section for $4\ell jj$ production in the VBS-enhanced region as a function of $m_{4\ell}$ (left) and $m_{jj}$ (right).

The particle-level events are reweighted such that the simulation better matches the data at detector level. The reweighted detector-level distribution is then unfolded using the nominal response matrix. The bias in the unfolding is taken to be the difference between the unfolded spectrum and the reweighted particle-level prediction. The two approaches for assessing bias in the unfolding method give similar results, and the uncertainty in the unfolding method is defined using the first approach.

A summary of the experimental and theoretical uncertainties in the differential cross-section measurements is shown in Table 2. The kinematic dependence of these uncertainties as a function of $m_{4\ell}$ and $m_{jj}$ are shown in Figure 3. The dominant systematic uncertainties are those associated with the jet reconstruction, the theoretical modelling, and the unfolding method.
8 Results

The differential cross-sections for inclusive $4\ell jj$ production in the VBS-enhanced region as a function of $m_4\ell$, $m_{jj}$, $\Delta\phi_{jj}$, $\cos\theta^*_1$, $p_T$, and $S_T$ are shown in figures 4, 5, and 6. The differential cross-sections as a function of $m_4\ell$ and $m_{jj}$ are examples of observables that are typically used to study vector-boson scattering processes. The differential cross-sections as a function of $\cos\theta^*_1$ are sensitive to the polarisation of the leading $Z$ boson candidate, whereas the differential cross-section as a function of $\Delta\phi_{jj}$ is sensitive to the charge-conjugation and parity structure of the $WWZ$ and $WWZZ$ interactions. The differential cross-sections as a function of $\cos\theta^*_1$ are sensitive to the higher-order real emission of quarks and gluons from the $4\ell jj$ processes. Differential cross-sections as a function of other kinematic variables (or in the VBS-suppressed region) are presented in the appendix.

The data are compared with two theoretical predictions, constructed from MG5+Py8 for the EW $4\ell jj$ process and either SHERPA or MG5_NLO+Py8 for the strong $4\ell jj$ process. Scale uncertainties in each $4\ell jj$ prediction are estimated by varying the renormalisation and factorisation scales used in the matrix-element calculation independently by a factor of 0.5 or 2.0. Uncertainties in the $4\ell jj$ predictions due to PDFs are estimated for SHERPA by reweighting the nominal sample to reproduce the 100 variations of the NNPDF PDF sets (including the associated $\alpha_s$ variations) and taking the RMS of these variations, and for MG5+Py8 following the PDF4LHC recommendations. The impact of PDF-related uncertainties are found to be much smaller than the impact of scale uncertainties.

The prediction obtained using SHERPA for strong $4\ell jj$ production is found to be in satisfactory agreement with the data for all measured distributions in the VBS-enhanced region. However, the prediction obtained using MG5+Py8 for strong $4\ell jj$ production is found to underestimate the inclusive $4\ell jj$ cross-section in all distributions, but the disagreement is especially noticeable at low $m_{jj}$, low $m_4\ell$ and low $|\Delta\phi_{jj}|$. The central values of the SHERPA prediction are also below the data, but the agreement with data is improved for SHERPA due to the larger theoretical uncertainty in the prediction as estimated from scale variations. The formal accuracy of the calculations is similar, but not identical. Both the predictions are accurate at LO in QCD for the $4\ell jj$ final state. However, SHERPA simulates additional jet activity at LO in QCD whereas MG5+Py8 relies on the PYTHIA8 parton shower, which is only accurate to leading logarithm. This may have an impact because the event selection identifies the two ‘tagging’ jets as the highest transverse momentum jets that satisfy $\eta_{j_1} \times \eta_{j_2} < 0$, which may select the third-highest transverse momentum jet in some instances.

The EW $4\ell jj$ contribution is also shown separately in figures 4–6 to show the sensitivity of the measurements to the electroweak process. The electroweak contribution is about 20% of the measured $4\ell jj$ cross-section in the VBS-enhanced region. However, it is much larger at high $m_{jj}$, where it accounts for about 50% of the measured cross-section, and much lower at low $m_{jj}$, where it accounts for just 5%. Furthermore, the POWHEG+PYTHIA8 EW $4\ell jj$ prediction is in very good agreement with the MG5+Py8 EW $4\ell jj$ prediction for all measured distributions, demonstrating that the choice of EW $4\ell jj$ model has very little impact on the inclusive $4\ell jj$ prediction.

The differential cross-sections for inclusive $4\ell jj$ production in the VBS-suppressed region as a function of $m_4\ell$ and $m_{jj}$ are shown in figure 7. The data are again compared with the two
Figure 4. Differential cross-sections for inclusive $4\ell jj$ production in the VBS-enhanced region as a function of $m_{4\ell}$ (left) and $m_{jj}$ (right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The data are compared with two theoretical predictions, estimated by using Sherpa (triangles) and MadGraph5 (circles) for the strong $4\ell jj$ contribution and MG5+Py8 for the EW $4\ell jj$ contribution. The band on the theoretical predictions represents the theoretical uncertainty from renormalisation/factorisation scale choices and PDF choice. The dashed lines show the contribution of EW $4\ell jj$ production to the differential cross-section as predicted by MG5+Py8 and Powheg+Pythia8. The s-channel contributions from ZZV production are missing from the Powheg+Pythia8 prediction and are estimated with Sherpa. ‘Overflow’ events that lie above the upper bin edge of the last bin are included in that bin.

Figure 5. Differential cross-sections for inclusive $4\ell jj$ production in the VBS-enhanced region as a function of $\Delta\phi_{jj}$ (left) and $\cos\theta_{12}^*$ (right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The theoretical predictions are constructed in the same way as in figure 4.
Figure 6. Differential cross-sections for inclusive 4ℓjj production in the VBS-enhanced region as a function of \( p_{T,4\elljj} \) (left) and \( S_{T,4\elljj} \) (right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The theoretical predictions are constructed in the same way as in figure 4. ‘Overflow’ events that lie above the upper bin edge of the last bin are included in that bin.

Theoretical predictions that are estimated by using MG5+Py8 for the EW 4ℓjj process and either Sherpa or MadGraph5 for the strong 4ℓjj process. The electroweak contribution is less than 5% of the measured 4ℓjj cross-section in this region, and remains below 15% even at the highest values of \( m_{jj} \). The prediction obtained using Sherpa for strong 4ℓjj production is found to be in satisfactory agreement with the data for the measured distributions, except at the largest values of \( m_{jj} \) where the predicted cross section is too large. This feature has been observed in previous measurements sensitive to vector-boson fusion and vector-boson scattering [60]. The prediction obtained with MG5+Py8 is in good agreement with the data at low-\( m_{4\ell} \), but underestimates the inclusive 4ℓjj cross-section at high-\( m_{4\ell} \) and low-\( m_{jj} \).

9 Effective field theory interpretation

The differential cross-sections can be used to search for signatures of physics beyond the SM. For measurements sensitive to vector-boson scattering, dimension-eight effective field theory (EFT) modeling can be a tool, whereby the SM Lagrangian is extended with new interactions encoded in dimension-eight operators, i.e.,

\[
\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{f_{T,i}}{\Lambda^4} \mathcal{O}_{T,i}
\]

where \( \mathcal{L}_{SM} \) is the SM Lagrangian, \( \mathcal{O}_{T,i} \) are a set of the dimension-eight operators, and the \( f_{T,i}/\Lambda^4 \) are Wilson coefficients that specify the strength of the anomalous interactions. The \( \mathcal{O}_{T,i} \) operators are particularly interesting as they only induce anomalous quartic weak-boson self-interactions [6] and can only be tested using vector-boson scattering processes or triboson.
production. In focussing on the dimension-eight operators, it is implicitly assumed that the contribution from dimension-six operators is zero, i.e., that they are already constrained from measurements of diboson production \cite{4, 61–64} and vector-boson fusion \cite{60}. However, constraints on the Wilson coefficients of operators in a dimension-six effective field theory are presented in the appendix for completeness.

The squared scattering amplitude for the effective field theory prediction can be written as

\[ |M|^2 = |M_{SM}|^2 + 2 \text{Re}(M_{SM}^* M_{d8}) + |M_{d8}|^2, \]

where \( M_{SM} \) is the SM scattering amplitude and \( M_{d8} \) is a non-SM scattering amplitude that contains anomalous interactions. The cross-section therefore contains three contributions: the SM contribution, the interference between the SM amplitude and the dimension-eight amplitude, and a pure dimension-8 contribution. The interference term and the pure dimension-eight term contribute at order \( f_T^2 i / \Lambda^4 \) and \( f_T^2 i / \Lambda^8 \), respectively. At large values of the Wilson coefficients, the contribution of the pure dimension-eight term can therefore be larger than the interference contribution, and the theoretical prediction is then sensitive to missing higher-orders in the EFT expansion that contribute at order \( 1 / \Lambda^8 \).

Theoretical predictions for the interference and pure dimension-eight contributions to EW \( ZZjj \) production are produced using MadGraph5. Simulated events are produced for each operator separately with the associated Wilson coefficient set to unity. The events are passed through Pythia8 to produce the \( ZZ \rightarrow \ell^+ \ell^- \ell^+ \ell^- \) decay channel and to simulate the fully hadronic final state using the A14 set of tuned parameters. For each operator, the effective field theory prediction is given by the sum of the interference contribution, the pure-dimension-8 contribution, the MG5+Py8 EW \( 4\ell jj \) prediction, and the Sherpa

\[ \text{Figure 7.} \] Differential cross-sections for inclusive \( 4\ell jj \) production in the VBS-suppressed region as a function of \( m_{4\ell} \) (left) and \( m_{jj} \) (right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The theoretical predictions are constructed in the same way as for figure 4. ‘Overflow’ events that lie above the upper bin edge of the last bin are included in that bin.
| Wilson coefficient | $|\mathcal{M}_{ds}|^2$ Included | Expected | Observed |
|--------------------|--------------------------------|----------|----------|
| $f_{T,0}/\Lambda^4$ | yes                            | [-1.00, 0.97] | [-0.98, 0.93] |
|                    | no                             | [-19, 19]   | [-23, 17]  |
| $f_{T,1}/\Lambda^4$ | yes                            | [-1.3, 1.3]  | [-1.2, 1.2]  |
|                    | no                             | [-140, 140] | [-160, 120] |
| $f_{T,2}/\Lambda^4$ | yes                            | [-2.6, 2.5]  | [-2.5, 2.4]  |
|                    | no                             | [-63, 62]   | [-74, 56]   |
| $f_{T,5}/\Lambda^4$ | yes                            | [-2.6, 2.5]  | [-2.5, 2.4]  |
|                    | no                             | [-68, 67]   | [-79, 60]   |
| $f_{T,6}/\Lambda^4$ | yes                            | [-4.1, 4.1]  | [-3.9, 3.9]  |
|                    | no                             | [-550, 540] | [-640, 480] |
| $f_{T,7}/\Lambda^4$ | yes                            | [-8.8, 8.4]  | [-8.5, 8.1]  |
|                    | no                             | [-220, 220] | [-260, 200] |
| $f_{T,8}/\Lambda^4$ | yes                            | [-2.2, 2.2]  | [-2.1, 2.1]  |
|                    | no                             | [-3.9, 3.8]×10^4 | [-4.6, 3.1]×10^4 |
| $f_{T,9}/\Lambda^4$ | yes                            | [-4.7, 4.7]  | [-4.5, 4.5]  |
|                    | no                             | [-6.4, 6.3]×10^4 | [-7.5, 5.5]×10^4 |

Table 3. Expected and observed 95% confidence interval for the dimension-eight Wilson coefficients, using a two-dimensional fit to the $4\ell jj$ differential cross-sections measured as a function of $m_{4\ell}$ and $m_{jj}$. Results are presented when including or excluding the pure dimension-eight contributions to the EFT prediction.

Strong $4\ell jj$ prediction. The interference contribution has a linear dependence on the Wilson coefficient, whereas the pure-dimension-8 contribution has a quadratic dependence. This enables modelling of theoretical predictions for any value of a given Wilson coefficient.

The measured differential cross-section as a function of $m_{4\ell}$ and $m_{jj}$ in the VBS-enhanced region and the associated EFT-dependent theoretical predictions are used to define a likelihood function, assuming Gaussian-distributed uncertainties. Statistical correlations between the bins of the differential cross-section measurements are estimated by using a bootstrap procedure (as outlined in section 6) and included in the covariance matrix in the likelihood function. Each source of systematic uncertainty in the measurement is also included in the covariance matrix and is assumed to be fully correlated between the bins of $m_{4\ell}$ and $m_{jj}$. Scale and PDF uncertainties in the SHERPA strong $4\ell jj$ and MG5+PY8 EW $4\ell jj$ predictions are implemented as Gaussian-constrained nuisance parameters. The confidence level at each value of Wilson coefficient is calculated using the profile-likelihood test statistic [65], which is assumed to be distributed according to a $\chi^2$ distribution with one degree of freedom [66]. From this, 95% confidence intervals are constructed for each Wilson coefficient. The first bin of the $m_{4\ell}$ distribution is not used in the statistical test. This bin is insensitive to the dimension-eight operators, but removing the bin prevents a possible overconstraint on the Wilson coefficients, which arises due to one very small eigenvalue of the combined $m_{4\ell}-m_{jj}$ covariance matrix.
The 95% confidence intervals on the Wilson coefficients in the dimension-eight effective field theory are shown in table 3. For each Wilson coefficient, confidence intervals are shown when including or excluding the pure dimension-eight contribution in the theoretical prediction. In all cases, the Wilson coefficients are consistent with zero. The Wilson coefficients associated with the $O_{T,0}$ and $O_{T,1}$ operators are the most tightly constrained. Furthermore, the constraints obtained with the pure dimension-eight contribution included in the theoretical prediction are more stringent than those obtained with only the interference contribution included in the theoretical calculation. This means that the limits are only valid if higher-order terms in the EFT expansion do not contribute significantly.

Constraints are also placed on each Wilson coefficient after restricting the interference- and pure dimension-eight- contributions to have $m_{4l} < E_c$, where $E_c$ is a cut-off that prevents unitarity being violated at large energy scales. The dependence of the 95% confidence intervals on the value of $E_c$ is shown in figure 8 for the $O_{T,0}$ and $O_{T,1}$ operators. The 95% confidence intervals degrade by a factor of 4–5 when the energy scale cut off is reduced from $E_c = \infty$ to $E_c = 1$ TeV. A similar trend is seen for the other operators and shown in the appendix. Unitarity bounds are also shown in figure 8, to indicate the consistency between the 95% confidence intervals and the partial-wave unitarity considerations for vector boson scattering processes [67].

10 Conclusion

Differential cross-sections are measured for the production of four charged leptons in association with two jets, using proton-proton collision data collected by the ATLAS experiment at a centre-of-mass energy of $\sqrt{s} = 13$ TeV and with an integrated luminosity of $140 \text{ fb}^{-1}$. This final state is sensitive to the strong $4\ell jj$ process, in which the jets arise from the strong interactions, and the electroweak $4\ell jjj$ process, which is characterised by the $t$-channel exchange of an electroweak boson. The measurement of $4\ell jj$ production can therefore be used to improve the understanding of both the electroweak and strong interactions that underpin
the Standard Model of particle physics. The differential cross-sections for $4\ell jj$ production are measured as a function of observables that collectively (i) characterise vector-boson scattering processes, (ii) probe the polarisation, parity and charge conjugation properties of the $4\ell jj$ process, and (iii) probe the real emission of quarks and gluons from the $4\ell jj$ process. The differential cross-sections are compared with various state-of-the-art Monte Carlo event generator predictions and the measurements are found to be sensitive to the event-generator modelling of strong $4\ell jj$ production and the EW $4\ell jj$ process at high dijet invariant mass and high values of $|\Delta \phi_{jj}|$. The differential cross-section measurements are consistent with Standard Model expectations and are used to set constraints on anomalous weak-boson self-interactions induced by dimension-six and dimension-eight operators in Standard Model effective field theory.

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A Additional results and interpretations

The differential cross-sections for inclusive $4\ell jj$ production in the VBS-enhanced region as a function of $p_T, 4\ell, p_T, jj, |\Delta y_{jj}|$, and $\cos \theta_{34}^*$ are shown in figure 9. The differential cross-sections for inclusive $4\ell jj$ production in the VBS-suppressed region as a function of $p_T, 4\ell, p_T, jj, S_T, 4\ell jj, |\Delta y_{jj}|, \Delta \phi_{jj}, \cos \theta_{12}^*$ and $\cos \theta_{34}^*$ are shown in figures 10 and 11. The data are compared with two theoretical predictions, constructed from MG5+Py8 for the EW $4\ell jj$ process and either SHERPA or MG5_NLO+Py8 for the strong $4\ell jj$ process. The EW $4\ell jj$ contribution is also shown separately to show the sensitivity of the measurements to the electroweak process.

The dependence of the 95% confidence intervals on the value of $E_c$ is shown in figure 12 for the $\mathcal{O}_{T,2}, \mathcal{O}_{T,5}, \mathcal{O}_{T,6}, \mathcal{O}_{T,7}, \mathcal{O}_{T,8}$ and $\mathcal{O}_{T,9}$ operators. The 95% confidence intervals degrade by a factor of 4–5 when the energy scale cut off is reduced from $E_c = \infty$ to $E_c = 1$ TeV.

Table 4 shows 95% confidence intervals on the Wilson coefficients of operators in a dimension-six effective field theory [69]. The method to constrain the Wilson coefficients is identical to that used for the dimension-eight operators in section 9, with the interactions from the dimension-six operators provided by the SMEFTSim package [69]. In the case of CP-odd operators, the interference contribution is zero for parity-even observables such as $m_{jj}$ and $m_{4\ell}$. However, the interference contribution produces large asymmetric effects in the parity-odd $\Delta \phi_{jj}$ observable. Constraints are therefore placed on the CP-odd Wilson coefficients using the measured differential cross-section as a function of $\Delta \phi_{jj}$ when the pure dimension-six contribution to the EFT is excluded. The constraints obtained on the dimension-six Wilson coefficients in this analysis are much weaker than those obtained measurements of diboson production [4, 61–64] and vector-boson fusion [60].
Figure 9. Differential cross-sections for inclusive $4\ell jj$ production in the VBS-enhanced region as a function of $p_{T,\ell jj}$ (top left), $p_{T,jj}$ (top right), $|\Delta y_{jj}|$ (bottom left) and $\cos(\theta^*_4)$ for the second Z boson candidate (bottom right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The data are compared to two theoretical predictions, estimated by using SHERPA (triangles) and MADGRAPH5 (circles) for the strong $4\ell jj$ contribution and $\text{MG5+PY8}$ for the $4\ell jj$ contribution. The band on the theoretical predictions represents the theoretical uncertainty from renormalisation/factorisation scale choices and PDF choice. The dashed lines show the contribution of $\text{EW 4}\ell jj$ production to the differential cross-section as predicted by $\text{MG5+PY8}$ and Powheg+Pythia8. The $s$-channel contributions from $ZZV$ production are missing from the Powheg+Pythia8 prediction and are estimated with SHERPA. For the $p_{T,\ell jj}$, $p_{T,jj}$ and $|\Delta y_{jj}|$ measurements, any ‘overflow’ events that lie above the upper bin edge of the last bin are included in that bin.
Figure 10. Differential cross-sections for inclusive 4ℓjj production in the VBS-suppressed region as a function of \( p_{T,4ℓ} \) (top left), \( p_{T,4ℓjj} \) (top right), \( p_{T,4ℓjj} \) (bottom left) and \( S_{T,4ℓjj} \) (bottom right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The data are compared to two theoretical predictions, estimated using SHERPA (triangles) and MadGraph5 (circles) for the strong 4ℓjj contribution and MG5+Py8 for the EW 4ℓjj contribution. The band on the theoretical predictions represents the theoretical uncertainty from renormalisation/factorisation scale choices and PDF choice. The dashed lines show the contribution of EW 4ℓjj production to the differential cross-section as predicted by MG5+Py8 and POWHEG+PYTHIA8. The s-channel contributions from ZZV production are missing from the POWHEG+PYTHIA8 prediction and are estimated with SHERPA. ‘Overflow’ events that lie above the upper bin edge of the last bin are included in that bin.
Figure 11. Differential cross-sections for inclusive 4ℓjj production in the VBS-suppressed region as a function of |Δyjj| (top left), Δφjj (top right), cos θjj (bottom left) and cos θ12 (bottom right). The data are represented as black points and the associated error bars represent the statistical uncertainty. The total uncertainty in the measurement is represented as a grey hatched band. The data are compared to two theoretical predictions, estimated using SHERPA (triangles) and MADGRAPH5 (circles) for the strong 4ℓjj contribution and MG5+PY8 for the EW 4ℓjj contribution. The band on the theoretical predictions represent the theoretical uncertainty from renormalisation/factorisation scale choices and PDF choice. The dashed lines show the contribution of EW 4ℓjj production to the differential cross-section as predicted by MG5+PY8 and POWHEG+PYTHIA8. The s-channel contributions from ZZV production are missing from the POWHEG+PYTHIA8 prediction and are estimated with SHERPA. For the |Δyjj| measurement, any ‘overflow’ events that lie above the upper bin edge of the last bin are included in that bin.
Figure 12. Expected and observed 95% confidence interval for the $f_{T,2}$, $f_{T,3}$, $f_{T,6}$, $f_{T,7}$, $f_{T,8}$ and $f_{T,9}$ Wilson coefficients as a function of a cut-off scale, $E_c$, which restricts the interference- and pure dimension-eight- contributions to have $m_{4\ell} < E_c$. The constraints are obtained using a two-dimensional fit to the $4\ell jj$ differential cross-sections measured as a function of $m_{4\ell}$ and $m_{jj}$.
Table 4. Expected and observed 95% confidence interval for the dimension-six Wilson coefficients. Results are presented when including or excluding the pure dimension-six contributions to the EFT prediction. The constraints are obtained using a two-dimensional fit to the $4\ell jj$ differential cross-sections measured as a function of $m_{4\ell}$ and $m_{jj}$, except for the constraints on CP-odd operators when the pure dimension-six contribution to the EFT is excluded. Those constraints, denoted by a (*), are obtained in a fit to the differential cross-section as a function of $\Delta \phi_{jj}$.

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