# Quantum Analog of Landau-Lifshitz-Gilbert Dynamics

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The Landau-Lifshitz-Gilbert (LLG) and Landau-Lifshitz (LL) equations play an essential role for describing the dynamics of magnetization in solids. While a quantum analog of the LL dynamics has been proposed in [Phys. Rev. Lett. **110**, 147201 (2013)], the corresponding quantum version of LLG remains unknown. Here, we propose such a quantum LLG equation that inherently conserves purity of the quantum state. We examine the quantum LLG dynamics of a dimer consisting of two interacting spin-$\frac{1}{2}$ particles. Our analysis reveals that, in the case of ferromagnetic coupling, the evolution of initially uncorrelated spins mirrors the classical LLG dynamics. However, in the antiferromagnetic scenario, we observe pronounced deviations from classical behavior, underscoring the unique dynamics of becoming a spinless state, which is nonlocally correlated. Moreover, when considering spins that are initially entangled, our study uncovers an unusual form of revival-type quantum correlation dynamics, which differs significantly from what is typically seen in open quantum systems.

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*Introduction*—The Landau-Lifshitz-Gilbert (LLG) [1] and Landau-Lifshitz (LL) [2] equations describe the dynamics of magnetization in solids on an atomistic, classical level [3]. Several publications have used this approach to describe magnetization dynamics of topological objects [4], the demagnetization of fcc Ni in pump probe experiments [5,6], and the magnetization reversal of ferrimagnetic FeGd alloys [7]. While the LL and LLG equations treat the magnetization as a classical vector, the underlying degrees of freedom are quantum spins. This raises the question whether quantum versions of these equations exist. In the LL case, this problem has been addressed by Wieser [8–10], while the quantum analog of LLG remains unknown.

Here, we extend Wieser's work and propose a quantum analog of the LLG equation. This quantum LLG ($q$-LLG) equation describes the dynamics of the density operator of quantum spins. We examine similarities and differences between the resulting quantum and classical dynamics for a dimer consisting of two spin-$\frac{1}{2}$ particles. We show that the proposed $q$-LLG generally differs from Wieser's quantum LL ($q$-LL) equation for multispin systems in nonpure states, while the two equations may be equivalent up to a rescaling of time in the single spin case, similar to the relation between their classical counterparts.

We first describe the classical LLG and LL dynamics. Consider a system of interacting magnetic moments $\mathbf{m}_k$, being exposed to an external magnetic field $\mathbf{B}$. The LLG and LL equations both describe damped precession of the $\mathbf{m}_k$:s around their local effective magnetic field $\mathbf{B}_k = -(\partial H/\partial \mathbf{m}_k)$. These effective fields contain contributions from $\mathbf{B}$ as well as from all $\mathbf{m}_{l\neq k}$ via the magnetic many-body Hamiltonian $H$, which typically include Heisenberg and Dzyaloshinskii-Moriya terms, i.e., $H = -\sum_k \mathbf{B}\cdot\mathbf{m}_k + \mu_B^{-2}\sum_{l<k}[J_{lk}\mathbf{m}_l\cdot\mathbf{m}_k + \mathbf{D}_{kl}\cdot(\mathbf{m}_k\times\mathbf{m}_l)]$ with $\mu_B$ the Bohr magneton.

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The LLG equations read

$$\dot{\mathbf{m}}_k = \gamma_g \mathbf{m}_k \times \mathbf{B}_k - \frac{\alpha}{|\mathbf{m}_k|}\mathbf{m}_k \times \dot{\mathbf{m}}_k, \qquad (1)$$

with $\gamma_g$ the gyromagnetic ratio and $\alpha$ the dimensionless Gilbert damping. Note that most derivations of Eq. (1) assume that $|\mathbf{m}_k|$ is a constant of the motion. This latter fact can be used in Eq. (1) to find the LL equations (see, e.g., Ref. [3])

$$\dot{\mathbf{m}}_k = \tilde{\gamma}_g \mathbf{m}_k \times \mathbf{B}_k - \frac{\lambda}{|\mathbf{m}_k|}\mathbf{m}_k \times (\mathbf{m}_k \times \mathbf{B}_k) \qquad (2)$$

with the rescaled gyromagnetic ration $\tilde{\gamma}_g = \gamma_g/(1+\alpha^2)$ and LL damping rate $\lambda = \alpha\gamma_g/(1+\alpha^2)$. The way the LL equation can be derived from the LLG equation demonstrates that they are equivalent up to a rescaling of time $t \mapsto (1+\alpha^2)t$ [11].

*Master equation and properties*—Inspired by the above classical formulation, we propose the trace preserving $q$-LLG analog as

$$\dot{\varrho} = \frac{i}{\hbar}[\varrho, H] + i\kappa[\varrho, \dot{\varrho}], \qquad (3)$$

with $\varrho$ the density operator. The term $i\kappa[\varrho, \dot{\varrho}]$ that modifies the Liouville–von Neumann equation has a *dissipative* (dampinglike) character with $\kappa$ the dimensionless damping rate. In addition to the trace, Eq. (3) can be shown to preserve Hermiticity and non-negativity of $\varrho(t)$. (See Supplemental Material [12], which includes Refs. [13,14], for technical and conceptual details of the $q$-LLG dynamics.)

The first property that can be immediately verified from Eq. (3) is the purity conservation, $(d/dt)\mathrm{Tr}\varrho^2 = 0$. This implies a fundamental difference from the well-known Lindbladian superoperator [15], which also acts as a dissipator, but imposes to the system a much less strict condition, for which the purity is not generally conserved [16]. Thus, in the case of $q$-LLG, the purity of the density operator becomes the quantum analog of the classical magnetization.

The conservation of purity seems to suggest similar results for different entropy measures. Indeed, one can observe that the Rényi entropy $S_\delta(\varrho) = (1-\delta)^{-1}\ln \mathrm{Tr}\varrho^\delta$, $\delta \neq 1$, which tends asymptotically to the von Neumann entropy $S = -\mathrm{Tr}(\varrho \ln \varrho)$ when $\delta \to 1$, is conserved. Thus, although Eq. (3) includes a dampinglike term, the conserved purity ensures that there will be no intrinsic loss of information during the damped dynamics.

*Connection to classical LLG and q-LL*—To illustrate the connection of $q$-LLG with the classical LLG, let us consider the simplest case of a single spin-$s$ particle. Suppose the evolving state of the particle takes the form

$$\varrho = \frac{1}{2s+1}(\mathbb{1} + \boldsymbol{\eta}\cdot\mathbf{S}) \qquad (4)$$

with the spin operator $\mathbf{S}$ and identity $\mathbb{1}$ acting on the $2s+1$ dimensional Hilbert space $\mathcal{H}$ associated with the spin state. We wish to interpret $\boldsymbol{\eta}$ as the quantum analog of the classical magnetization discussed above. This implies that $|\boldsymbol{\eta}|$ should be a constant of the motion and $\boldsymbol{\eta}$ should satisfy a classical LLG-type equation. One can note that $\boldsymbol{\eta}$ is proportional to the expectation value of spin, as Eq. (4) implies

$$\langle\mathbf{S}\rangle = \frac{1}{2s+1}\mathrm{Tr}[(\boldsymbol{\eta}\cdot\mathbf{S})\mathbf{S}] = \frac{s(s+1)}{3}\hbar^2\boldsymbol{\eta}, \qquad (5)$$

where we have used

$$\mathrm{Tr}(S_k S_l) = \frac{s(s+1)(2s+1)}{3}\hbar^2\delta_{kl}. \qquad (6)$$

To see that $|\boldsymbol{\eta}|$ is constant in time, we note that

$$\hbar^2|\boldsymbol{\eta}|^2 = \frac{3(2s+1)}{s(s+1)}\left(\mathrm{Tr}\varrho^2 - \frac{1}{2s+1}\right), \qquad (7)$$

from which immediately follows $(d/dt)|\boldsymbol{\eta}|^2 \propto (d/dt)\mathrm{Tr}\varrho^2 = 0$. To demonstrate that $\boldsymbol{\eta}$ satisfies a classical LLG-type equation, we consider the case where the spin is exposed to a magnetic field $\mathbf{B}$ as described by a Zeeman Hamiltonian $H = -\gamma_g \mathbf{B}\cdot\mathbf{S}$. By inserting Eq. (4) and $H$ into Eq. (3), one finds

$$\dot{\boldsymbol{\eta}} = \gamma_g \boldsymbol{\eta} \times \mathbf{B} - \frac{\kappa\hbar}{2s+1}\boldsymbol{\eta} \times \dot{\boldsymbol{\eta}}, \qquad (8)$$

where the spin commutation relation $[\mathbf{a}\cdot\mathbf{S}, \mathbf{b}\cdot\mathbf{S}] = i\hbar(\mathbf{a}\times\mathbf{b})\cdot\mathbf{S}$ has been used. Clearly, Eq. (8) is identical to Eq. (1) and in this single spin case we find that $\kappa\hbar|\boldsymbol{\eta}|/(2s+1)$ is the dimensionless quantum analog of the Gilbert damping.

Next, we argue that the proposed $q$-LLG equation is generally different from the $q$-LL form $\dot{\varrho} = (i/\hbar)[\varrho, H] - (\kappa/\hbar)[\varrho, [\varrho, H]]$ found by Wieser [8]. To see this, we rewrite Eq. (3) as $\dot{\varrho} + \kappa^2[\varrho, [\varrho, \dot{\varrho}]] = (i/\hbar)[\varrho, H] - (\kappa/\hbar)[\varrho, [\varrho, H]]$, which is the desired $q$-LL form if and only if the left-hand side is proportional to $\dot{\varrho}$. This is the case for general pure states ($\varrho^2 = \varrho$) and for a single spin in a nonpure state of the form Eq. (4). Indeed, one finds for these two cases $\dot{\varrho} + \kappa^2[\varrho, [\varrho, \dot{\varrho}]] = (1+\kappa^2)\dot{\varrho}$ and $\dot{\varrho} + \kappa^2[\varrho, [\varrho, \dot{\varrho}]] = (1+\kappa^2\hbar^2|\boldsymbol{\eta}|^2/(2s+1)^2)\dot{\varrho}$, respectively. On the other hand, in other cases, no such simplification is generally possible, and the $q$-LLG and $q$-LL dynamics are therefore generally different. (See Supplemental Material [12] for further details regarding the relation between the $q$-LLG and $q$-LL equations.)

While one can convince oneself that the $q$-LLG proposed in Eq. (3) is the only possible form of master equation that exactly reproduces the classical description for uncoupled magnetic systems, this equivalence is no longer the case when coupling is included. (In Supplemental Material [12], we, however, provide evidence that the $q$-LLG dynamics may, in the large $s$ limit, tend to its classical counterpart also in the multispin case.) To see this, it is sufficient to compare the number of dynamical variables for $N$ particles in LLG and $q$-LLG. For spin-$\frac{1}{2}$, this number is $4^N - 1$ [17], while it is $3N$ in the classical treatment. Thus, the classical-like variables associated with the reduced density operators for the individual quantum spins is only a small portion of the total number of quantum mechanical degrees of freedom. To illustrate this extra richness, we shall in the following examine the $q$-LLG dynamics of a dimer consisting of two spin-$\frac{1}{2}$ particles.

*q-LLG dynamics of a spin dimer*—The spin dimer is described by the tensor state space $\mathcal{H}^{\otimes 2}$ and locally by the dimensionless Bloch vectors $\mathbf{r}_k = (\hbar/2)\boldsymbol{\eta}_k$ associated with the single spin reduced density operators with $k = 1, 2$ labeling the two spins. A general state of the dimer can be written as

$$\varrho = \frac{1}{4} T_{\alpha\beta} \sigma_\alpha \otimes \sigma_\beta, \tag{9}$$

where $\sigma_\alpha \otimes \sigma_\beta$ is a member of the Pauli group $\mathcal{P}_2 = \{\sigma_0 \equiv \mathbb{1}, \sigma_x, \sigma_y, \sigma_z\}^{\otimes 2} \times \{\pm 1, \pm i\}$ and the Einstein summation convention is used. We use Greek and Latin indices to run over $\{0, x, y, z\}$ and $\{x, y, z\}$, respectively. $T_{\alpha\beta}$ are elements of a $4 \times 4$ matrix $T$, for which $T_{00} = 1$, $T_{k0} = r_{1;k}$, and $T_{0l} = r_{2;l}$, where the first constraint ensures normalization and the latter two expressions give the components of the Bloch vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of the two spins. The remaining $T_{kl}$ form the $3 \times 3$ correlation matrix.

The violation of Bell inequalities for certain quantum states gives a precise notion of nonclassical correlations, as such $\varrho$ does not admit a local classical description [18]. To emphasize the nonclassicality of the dynamics in the simulations below, we therefore use Bell nonlocality $B(\varrho)$ [19] as our preferred correlation measure. $B(\varrho)$ is defined in terms of the measurement setting that maximizes the violation of the Clauser-Horne-Shimony-Holt (CHSH) inequality [20] for a given $\varrho$. As shown in Ref. [21], this maximum is given by the two largest singular values $u_1$ and $u_2$ of $T_{kl}$ in the sense that there exists a measurement setting that violates CHSH if and only if $1 < u_1^2 + u_2^2 \leq 2$, where the upper equality corresponds to the Cirel'son bound of maximal violation [22]. This suggests the measure [19] $B(\varrho) = \sqrt{\max\{u_1^2 + u_2^2 - 1, 0\}}$ of Bell nonlocality. A given state $\varrho$ contains nonclassical correlations if and only if $B(\varrho) \neq 0$.

We now consider the dimer Hamiltonian

$$H = -\gamma_g \mathbf{B} \cdot (\mathbf{S}_1 + \mathbf{S}_2) + \frac{4J}{\hbar^2} \mathbf{S}_1 \cdot \mathbf{S}_2 + \frac{4\mathbf{D}}{\hbar^2} \cdot (\mathbf{S}_1 \times \mathbf{S}_2), \tag{10}$$

with $\mathbf{B}$ an external magnetic field, $J$ the Heisenberg exchange coupling strength, and the Dzyaloshinskii-Moriya interaction (DMI) term with $\mathbf{D} = D(0, 0, 1)$ ($D$ is the DMI strength). We take $\kappa = \alpha = 0.5$ to magnify the effects of the nonlinear terms in the $q$-LLG and LLG equations; this choice is also physically consistent with the fact that Gilbert damping increases in low dimensions [23]. Furthermore, we assume spin-$\frac{1}{2}$ and use the standard basis $\sigma_z|\uparrow\rangle = +|\uparrow\rangle$ and $\sigma_z|\downarrow\rangle = -|\downarrow\rangle$.

First, we compare the $q$-LLG and LLG dynamics by examining the Bloch vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of the reduced states $\varrho_1 = \mathrm{Tr}_2\varrho$ and $\varrho_2 = \mathrm{Tr}_1\varrho$, respectively, and the classical magnetizations $\mathbf{m}_1$ and $\mathbf{m}_2$. We consider an antiferromagnetic (AFM) initial state of $q$-LLG dynamics defined as $|\uparrow\downarrow\rangle$ and corresponding initial magnetization $\mathbf{m}_1 = -\mathbf{m}_2 = m(0, 0, 1)$. We examine the resulting dynamics for ferromagnetic (FM, $J < 0$) and AFM ($J > 0$) exchange coupling. We use $\mathbf{B} = B_0(1, 0, 0)$, i.e., an external field in the $x$ direction, and assume that the splitting due to the Zeeman and Heisenberg exchange is of the same order of magnitude by taking $m = J/B_0 = 1\mu_B = 6.58 \times 10^{-2}$ meV/T and $B_0 = 1.00$ T, which corresponds to the timescale $(\gamma_g B_0)^{-1} \sim 10$ ps. Spins in this weak Heisenberg coupling regime can be realized in, e.g., quantum dots (see Ref. [24]).

Figure 1 displays the $q$-LLG and LLG dynamics of the dimer. We show only the evolution of one of the subsystems, since the other behaves in a similar fashion. A striking general feature of the $q$-LLG dynamics can be noticed: while the purity is conserved, the length of the Bloch vector is not generally preserved, even with this simple input state. This can be seen by comparing Figs. 1(a) and 1(b). For an FM exchange, the quantum description is very similar to that of the classical simulation, and both approach a fully saturated moment pointing along the $x$ direction. However, as shown in Fig. 1(b), an AFM-type exchange interaction can lead to that the length of the Bloch vector is completely quenched by the quantum dynamics, effectively becoming a *spinless* state; a behavior that is impossible in the classical description. One may further notice smaller deviations of the two approaches, e.g., that in Fig. 1(a) the $q$-LLG is qualitatively similar to LLG results, with the discernible difference that the dynamics of the $q$-LLG equation is faster than that of the LLG equation. The effect of a nonzero DMI is shown in Figs. 1(c) and 1(d). While the qualitative similarity between $q$-LLG and LLG remains for FM exchange coupling, the asymptotic quantum state is no longer fully quenched, i.e., the Bloch vector has a small but finite magnitude in the AFM case. The latter can be seen as an effect of the singlet-triplet coupling induced by the DMI.

To better understand the behavior depicted in Fig. 1, we next turn to the correlation matrix $T_{kl}$. We again choose an initial AFM-type product state $|\uparrow\downarrow\rangle$ and focus on AFM exchange coupling $J > 0$. Figure 2 shows the $T_{kl}$:s as a function of time. In Fig. 2(a), we see that $T_{kl}$ tends

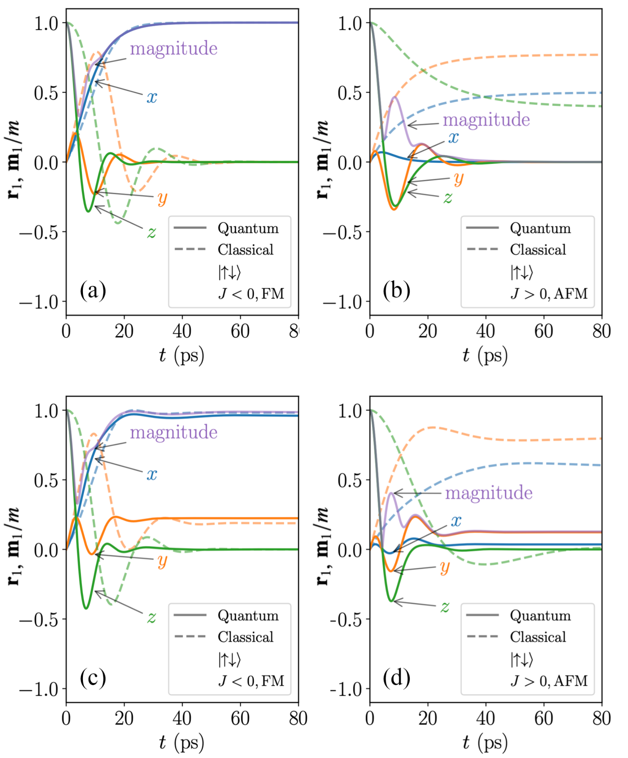


FIG. 1. $q$-LLG and LLG dynamics of a dimer initially prepared in the AFM-type product state $|\uparrow\downarrow\rangle$. The quantum dynamics is illustrated by the Bloch vector (solid lines) and the classical counterpart by the magnetization (dashed lines). (a) and (b) show the dynamics for FM ($J < 0$) and AFM ($J > 0$) exchange coupling, respectively, with in absence of DMI ($D = 0$); (c) and (d) are the corresponding plots for $D/|J| = 0.6$. Because of symmetry, only $\mathbf{r}_1$ (see text for definition) and $\mathbf{m}_1/m$ are shown. Physical parameters $B_0 = 1.00$ T, $|J|/B_0 = 1\mu_B$, and $\kappa = \alpha = 0.5$ are used.

asymptotically to diag$\{-1, -1, -1\}$, which corresponds to the singlet Bell state $|\Psi_-\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$. This explains the steady state seen in Fig. 1(b), as the reduced states of a Bell state have vanishing Bloch vector. In other words, while the Bloch vectors become completely quenched, information has been transferred to the nonlocal correlation of the spins in the steady state limit, so as to conserve the entropy during the process. Furthermore, the numerical results shown in Fig. 2(b) demonstrate the effect of a nonzero DMI on the $q$-LLG dynamics of the correlation matrix elements $T_{kl}$. The effect shows up as a symmetric splitting of the off-diagonal pair $T_{xy}$ and $T_{yx}$. This splitting effect is the explanation why $\mathbf{r}_1$ shown in Fig. 1(d) no longer tends to a state with vanishing Bloch vector.

We next consider the effect of the DMI term for initially correlated quantum spins. An AFM coupled dimer with an initial Bell state $|\Psi_+\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$ is considered. In Fig. 3, nonlocality is shown for different DMI strength $D$. After an intermediate oscillatory phase, the system evolves asymptotically to a maximally nonlocal

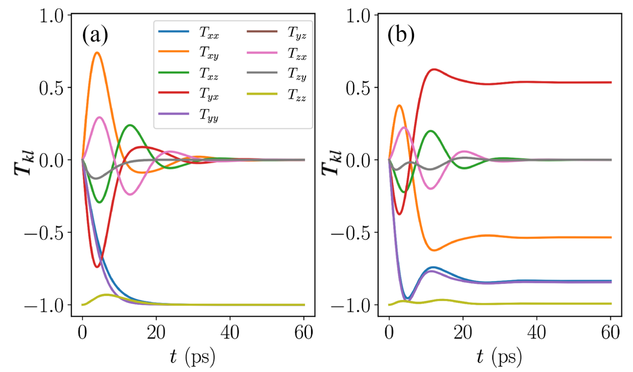


FIG. 2. The correlation matrix elements $T_{kl}$ with (a) $D/|J| = 0$ and (b) $D/|J| = 0.6$, driven by $q$-LLG dynamics for an AFM ($J = 1$) coupled dimer with initial product AFM-type state $|\uparrow\downarrow\rangle$. Physical parameters $B_0 = 1.00$ T, $|J|/B_0 = 1\mu_B$, and $\kappa = 0.5$ are used.

steady state for $D \neq 0$, while it decays monotonically to zero when $D = 0$. These results can be understood as follows. In absence of DMI, the singlet Bell state $|\Psi_-\rangle$ is decoupled from the triplet states, which means that the system, initially in the triplet Bell state $|\Psi_+\rangle$, will remain in the triplet subspace. The dissipative dynamics thereby forces the spins to approach the FM-type product state $\frac{1}{2}(|\uparrow\rangle + |\downarrow\rangle)^{\otimes 2}$, which is the lowest energy eigenstate in the triplet subspace. In contrast, a nonzero $D$ couples the singlet and triplet states, thereby opening up a route toward the energetically favorable $|\Psi_-\rangle$, which is maximally nonlocal.

We now consider nonpure input states. As we shall see, this opens up for the possibility of having revival-type quantum nonlocality effects. Suitable states to examine are of Werner type [18]: $\varrho_W^{\psi} = [(1-p)/4]\mathbb{1} + p|\psi\rangle\langle\psi|$ with $|\psi\rangle$ any maximally entangled spin state. We take $\varrho(0) = \varrho_W$ with $p = 0.9$, which is sufficiently large for the input state to be nonlocal [25], and consider each of the four maximally entangled standard Bell states $|\Psi_\pm\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$ and $|\Phi_\pm\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$.

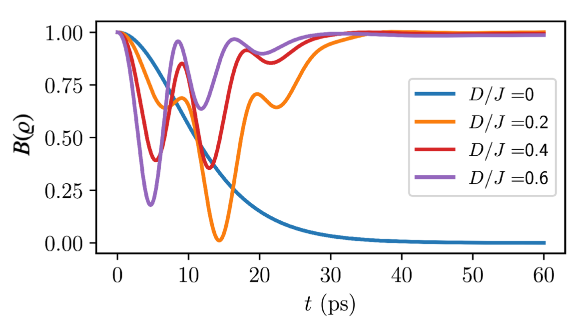


FIG. 3. Quantum nonlocality driven by the $q$-LLG dynamics with zero and nonzero DMI strength, in an AFM ($J > 0$) exchange coupling dimer. We use $\mathbf{D} \perp \mathbf{B}$ and the initial state taken to be the Bell state $|\Psi_+\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$. Physical parameters $B_0 = 1.00$ T, $|J|/B_0 = 1\mu_B$, and $\kappa = 0.5$ are used.

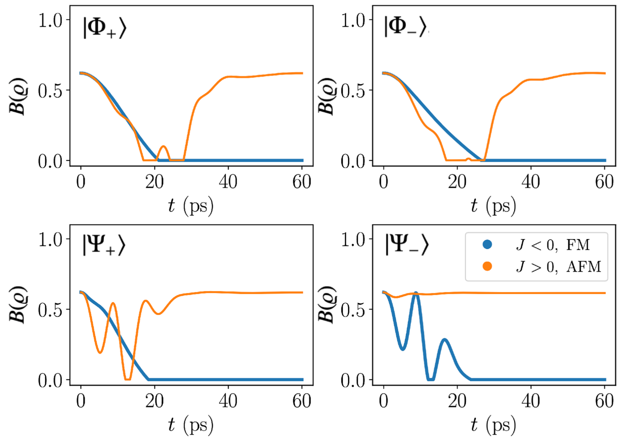


FIG. 4. $q$-LLG dynamics of nonlocality for initial Werner states $\varrho_W = [(1-p)/4]\mathbb{1} + p|\psi\rangle\langle\psi|$ with $|\psi\rangle$ being the Bell states $|\Phi_\pm\rangle = (1/\sqrt{2})(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)$ and $|\Psi_\pm\rangle = (1/\sqrt{2})(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$. We set the mixing parameter to $p = 0.9$ so as to ensure a nonlocal input. $\mathbf{D}$ is taken along the $z$ axis and $B = B_0(1/\sqrt{3})(1,1,1)$. We consider FM and AFM Heisenberg exchange for each of the four input Werner states. Physical parameters $B_0 = 1.00$ T, $|J|/B_0 = 1\mu_B$, $D/|J| = 0.4$, and $\kappa = 0.5$ are used.

The pairs $|\Psi_\pm\rangle$ and $|\Phi_\pm\rangle$ are of AFM type and FM type, respectively. We compare $q$-LLG dynamics of FM ($J < 0$) and of AFM ($J > 0$) coupled dimers, with nonzero DMI $\mathbf{D} = D(0,0,1)$ and external magnetic field $\mathbf{B} = (B_0/\sqrt{3})(1,1,1)$.

Figure 4 shows the resulting nonlocality $B(\varrho)$ for the Werner input states. The simulations confirm a revival-type behavior of nonlocal correlation in the $q$-LLG dynamics. This is very different from what one expects in Lindbladian type dynamics, where the nonlocality typically is irreversibly lost after a finite duration [26]. The origin of the effect seen in Fig. 4 is the perfect balance between entropy production and loss associated with the $q$-LLG dynamics. We further see that the nonlocal correlation is generally more profound for AFM coupled ($J > 0$) dimer. This is due to the fact that the lowest energy state in the AFM case is expected to have higher entanglement than for FM coupled spins.

*Conclusions and outlook*—We propose a master equation that is purity conserving and can be regarded as the quantum analog of the classical LLG. Although the connection to both LLG and $q$-LL [8] may be demonstrated for a single spin-$s$ particle, no equivalent comparison is in general possible for the case of several quantum spins. The proposed master equation is applied to a spin-$\frac{1}{2}$ dimer. In contrast to the classical formulation, the Bloch vector magnitude is typically not conserved, and depends strongly on the type of spin-spin coupling. The most striking example is an AFM-type product state with AFM exchange coupling, that evolves in time due to an external magnetic field, in which the Bloch vector becomes completely quenched, meanwhile the information is stored in the correlation matrix so as to create a highly robust resource for entanglement-based quantum information applications. Our results also demonstrate that a quantum description produces a dynamics that may be faster than that of the classical one, which might be of relevance to understand pump-probe experiments and experiments on ultrafast magnetization dynamics; in this context, thermal effects could be accounted for by introducing a stochastic noise operator, which models random thermal fluctuations of the environment. The evolution of mixed state input states reveals an unusual finite time behavior of quantum correlations that profoundly differs from what is typically seen in open system dynamics. To deal with the exponential scaling of quantum degrees of freedom, an interesting extension would be to develop a hybrid multiscale method that couples a quantum based description of magnetism to a classical one. This will open up for several applications, such as for the time evolution of molecular magnets or single atoms supported on magnetic substrates that can be studied by using pump-probe techniques. In addition to its natural applications in the realm of spin dynamics, we foresee the $q$-LLG framework as an alternative optimization method for probing the phase space in a problem of quantum spins.

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[1] T. L. Gilbert, A phenomenological theory of damping in ferromagnetic materials, IEEE Trans. Magn. **40**, 3443 (2004).
[2] L. D. Landau and E. M. Lifshitz, On the theory of the dispersion of magnetic permeability in ferromagnetic bodies, Phys. Z. Sowjetunion **8**, 153 (1935).

[3] O. Eriksson, A. Bergman, L. Bergqvist, and J. Hellsvik, *Atomistic Spin Dynamics: Foundations and Applications* (Oxford University Press, Oxford, UK, 2017).
[4] M. Pereiro, D. Yudin, J. Chico, C. Etz, O. Eriksson, and A. Bergman, Topological excitations in a kagome magnet, Nat. Commun. **5**, 4815 (2014).
[5] R. F. L. Evans, U. Atxitia, and R. W. Chantrell, Quantitative simulation of temperature-dependent magnetization dynamics and equilibrium properties of elemental ferromagnets, Phys. Rev. B **91**, 144425 (2015).
[6] M. Pankratova, I. P. Miranda, D. Thonig, M. Pereiro, E. Sjöqvist, A. Delin, O. Eriksson, and A. Bergman, Heat-conserving three-temperature model for ultrafast demagnetization in nickel, Phys. Rev. B **106**, 174407 (2022).
[7] R. Chimata, L. Isaeva, K. Kádas, A. Bergman, B. Sanyal, J. H. Mentink, M. I. Katsnelson, T. Rasing, A. Kirilyuk, A. Kimel, O. Eriksson, and M. Pereiro, All-thermal switching of amorphous Gd-Fe alloys: Analysis of structural properties and magnetization dynamics, Phys. Rev. B **92**, 094411 (2015).
[8] R. Wieser, Comparison of quantum and classical relaxation in spin dynamics, Phys. Rev. Lett. **110**, 147201 (2013).
[9] R. Wieser, Description of a dissipative quantum spin dynamics with a Landau-Lifshitz/Gilbert like damping and complete derivation of the classical Landau-Lifshitz equation, Eur. Phys. J. B **88**, 77 (2015).
[10] R. Wieser, Derivation of a time dependent Schrödinger equation as the quantum mechanical Landau-Lifshitz-Bloch equation, J. Phys. Condens. Matter **28**, 396003 (2016).
[11] M. Lakshmanan and K. Nakamura, Landau-Lifshitz equation of ferromagnetism: Exact treatment of the Gilbert damping, Phys. Rev. Lett. **53**, 2497 (1984).
[12] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.133.266704, which includes Refs. [13,14], for technical and conceptual details of the $q$-LLG dynamics; further details regarding the relation between the $q$-LLG and $q$-LL equations; and evidence that the $q$-LLG dynamics may, in the large $s$ limit, tend to its classical counterpart also in the multispin case.
[13] https://github.com/UppASD/UppASD
[14] https://github.com/yuefeiliuphys/quanLLG_two_qubits.git
[15] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, UK, 2007).
[16] D. Manzano, A short introduction to the Lindblad master equation, AIP Adv. **10**, 025106 (2020).
[17] The operator space of a spin-$\frac{1}{2}$ particle is spanned by four operators, identity $\sigma_0$ and the three Pauli operators $\sigma_x$, $\sigma_y$, $\sigma_z$. This implies $4^N - 1$ operators for $N$ such spins with the normalization condition removed.
[18] R. F. Werner, Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model, Phys. Rev. A **40**, 4277 (1989).
[19] B. Horst, K. Bartkiewicz, and A. Miranowicz, Two-qubit mixed states more entangled than pure states: Comparison of the relative entropy of entanglement for a given nonlocality, Phys. Rev. A **87**, 042108 (2013).
[20] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, Phys. Rev. Lett. **23**, 880 (1969).
[21] R. Horodecki, P. Horodecki, and M. Horodecki, Violating Bell inequality by mixed spin-$\frac{1}{2}$ states: Necessary and sufficient condition, Phys. Lett. A **200**, 340 (1995).
[22] B. S. Cirel'son, Quantum generalizations of Bell's inequality, Lett. Math. Phys. **4**, 93 (1980).
[23] D. Steiauf and M. Fähnle, Damping of spin dynamics in nanostructures: An ab initio study, Phys. Rev. B **72**, 064450 (2005).
[24] B. Trauzettel, D. V. Bulaev, D. Loss, and G. Burkard, Spin qubits in graphene quantum dots, Nat. Phys. **3**, 192 (2007).
[25] A Werner state is known to be nonlocal if $p > (1/\sqrt{2})$.
[26] C. Bengtson, M. Stenrup, and E. Sjöqvist, Quantum nonlocality in the excitation energy transfer in the Fenna-Matthews-Olson complex, Int. J. Quantum Chem. **116**, 1763 (2016).