Conceptual reasoning, intuition, and mathematics in physics

Statistical mechanics as a starting point for exploring student reasoning in upper-level university courses

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Abstract

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Learning physics in upper-level university courses introduces various challenges related to increasingly complex concepts and the increasingly integral role of mathematics. However, the research at this educational level remains relatively limited, particularly in topics such as statistical mechanics. This licentiate thesis aims to contribute to our understanding of student reasoning in upper-level physics courses, taking statistical mechanics as a starting point for the exploration. The thesis presents empirical findings from three papers and discusses them in relation to three broad themes: conceptual reasoning, intuition, and mathematics. The first two papers present grounded analyses of video data collected from problem-solving sessions with student groups working through tasks on the topic of statistical mechanics. The first paper presents ten emerged categories of recurring student challenges, suggesting that students had vague ideas about key concepts and struggled to apply their ideas in appropriate ways. The second paper—informed by the first—contributes with a focused exploration of students' use of intuition in upper-level physics and chemistry contexts. Paper II demonstrates how students' responses to their intuitions can lead to either productive or unproductive outcomes regardless of the intuition's appropriateness. The third paper, a phenomenographic study, contributes with insight into the variation of university teachers' views on the role of mathematics within their science, technology, or engineering discipline, in terms of five categories of increasingly integral and decreasingly instrumental roles. In the synthesis of this thesis, I present a model based on the Resources framework to provide a holistic analysis of all my findings. The model employs the process of framing, which involves active and passive activations of conceptual, metacognitive, and epistemological resources. I contrast two distinct frames based on my findings, a limited "stat mech approach" frame and an extended "cross-domain reasoning" frame, and propose the following implications for education. To help upper-level physics students build a more coherent and functional knowledge structure, teachers should: guide students to connect concepts within and between domains of physics; promote students' ability to extend their frame; and consider how their views about intuition and mathematics in physics may be reflected in their teaching.

Keywords: Physics Education Research, Statistical mechanics, Conceptual reasoning, Intuition, Role of mathematics, Resources framework

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Till min mamma,
Catarina

List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I Koerfer, E., & Gregorcic, B. (2024). Exploring student reasoning in statistical mechanics: Identifying challenges in problem-solving groups. *Physical Review Physics Education Research, 20*(1), 010105.

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Author's contribution: I designed the project, collected the data, and carried out several iterations of data analysis, under the supervision of the co-author. I discussed the findings and analysis continuously with the co-author, as well as other members of the research group, as the process evolved. I wrote the original draft of the manuscript and with input from the co-author, as well as suggestions from other colleagues, I finalized the paper.

II Koerfer, E., & Ye, S. (2024). Managing intuition in collaborative problem solving: a case study of beyond-intro chemistry and physics students. *In 2024 Physics Education Research Conference Proceedings* (pp. 218-223).

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Author's contribution: The co-author and I contributed equally to all parts of this work, including the design of the project, several iterations of data analysis, and writing of both the initial and final version of the paper. We conceptualized and conducted this work independently, but received input and feedback from both teams of supervisors.

III Koerfer, E., Polverini, G., Elmgren, M., Eriksson, L., Freyhult, L., Herbert, R. B., Ho, F. M., Solders, A., & Eckerdal, A. (2025). Teachers' conceptions of the role of mathematics in STEM higher education. *Manuscript.*

Author's contribution: AE, ME and FMH initiated and designed the study, and the overall project was led by AE. I contributed to the preparation of interview materials, together my all co-authors, and the data collection was performed by me and all co-authors except AE. The findings are based on an initial analysis by me, GP, LF and AS, which was further developed together with the rest of the co-authors. All authors were involved in writing the first draft of the manuscript and providing comments. All authors read and approved the final manuscript. GP and I contributed equally as first authors.

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Supporting work

Conference presentations

Koerfer, E., Mattson, T., & Samuelsson C. R. (2025). Swimming hall, the Nobel prize and phase transitions: Mapping students' associations through Wikipedia Speedrun. In *Teknisk-naturvetenskapliga fakultetens Universitetspedagogiska Konferens 2025*, Uppsala, Sweden, March 17 [poster]

Koerfer, E., & Gregorcic, B. (2024). Different spins on the two-state paramagnet: Pedagogical advantages and considerations. In *4th World Conference on Physics Education*, Krakow, Poland, August 26-30 [oral presentation]

Polverini, G., Koerfer, E., Eckerdal, A., Elmgren, M., Eriksson, H., Herbert, R., Ho, F., Freyhult, L., & Solders, A. (2024). A phenomenographic study of university teachers' perceptions of mathematics in STEM disciplines. In *SIG 9 Phenomenography and Variation Theory, Biennial Conference 2024*, Uppsala, Sweden, August 21-23 [oral presentation]

Koerfer, E., & Ye, S. (2024). Managing intuition in collaborative problem solving: a case study of beyond-intro chemistry and physics students. In *2024 Physics Education Research Conference*, Boston (MA), United States, July 10-11 [poster]

Koerfer, E., & Gregorcic, B. (2024). Statistical mechanics as a case study for challenges in upper-division physics courses. In *2024 American Association of Physics Teachers (AAPT) Summer Meeting*, Boston (MA), United States, July 6-10 [oral presentation]

Koerfer, E., & Gregorcic, B. (2024). Simplified toy models can make physics harder to grasp. In *2024 American Association of Physics Teachers (AAPT) Summer Meeting*, Boston (MA), United States, July 6-10 [poster]

Koerfer, E., & Ye, S. (2024). "We're like rolling a dice to decide our next step": How students enrolled in beyond intro physics and chemistry courses navigate problem solving. In *Teknisk-naturvetenskapliga fakultetens Universitetspedagogiska Konferens 2024*, Uppsala, Sweden, March 14 [oral presentation]

Koerfer, E., & Gregorcic, B. (2023). From Physics to Physics: An Example of Students' use of Analogies in Statistical Mechanics. In *Teknisk-naturvetenskapliga fakultetens Universitetspedagogiska Konferens 2023*, Uppsala, Sweden, March 15 [oral presentation]

Koerfer, E., & Gregorcic, B. (2022). Student Intuitions in Statistical Mechanics. In *GIREP 2022*, Ljubljana, Slovenia, July 4-8 [oral presentation, best oral presentation award]

Contents

Statements and notes for the reader

The use of language

In many parts of the thesis, I will use the singular pronoun "I" to express that the work within this thesis reflects my own perspectives and ideas. When I mention the work of the three papers, I will use the collective pronoun "we" to refer to myself and my co-authors—at times I refer to "my research" for convenience, but readers should keep in mind that the papers are the result of collaborative efforts. Note that my contributions to each paper is acknowledged in the preceding List of papers.

The use of GenAI

Generative artificial intelligence (GenAI) has not been used in the conceptualization of this thesis, nor in any part of the writing process.

The use of previous work

Section 2.5.1 contains reused text passages from one of the papers comprising this thesis (Paper I), with some minor modifications and restructuring.

Suggested reading for teachers

Teachers who are interested in the topic of statistical mechanics—or similar upper-level courses—and want to focus briefly on the practical recommendations of this thesis, I suggest reading Section 7.2 (Implications for teaching).

Teachers who want some more details and context, I suggest reading the following: Chapter 1 (Introduction); Section 2.5 (Upper-level university physics in PER, in Literature review); Section 3.2 (Resources framework, in Theoretical framework); and Chapter 7 (Synthesis and implications). For more background related to the topic of statistical mechanics, and details of my findings in this context in particular, I refer to Paper I.

Glossary

Conceptual resources – knowledge elements for understanding certain concepts, in line with the Resources framework (e.g., Hammer et al., 2005).

Epistemology -– assumptions regarding how we can know about the nature of reality.

Epistemological resources – knowledge elements for understanding the nature of knowledge and knowledge-building, in line with the Resources framework (e.g., Hammer et al., 2005).

Epistemic agency – the ability or will to shift between *frames* in a situation.

Epistemic game – a coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create knowledge or solve a problem (Tuminaro & Redish, 2007).

Frame – a person's interpretation of "What is going on here?", meaning a set of expectations about a certain situation which affects what they notice and how they act. A situation can be framed with respect to various aspects, such as social, affective, and epistemological (Hammer et al., 2005).

Framing – the activation of a locally coherent set of resources—including conceptual, epistemological, and metacognitive resources—and the use of those resources to make interpretations (Hammer et al., 2005).

Intuition – due to the elusive and complex nature of this term, I refer the reader to Sections 3.2.2 and 7.1 for a description of my perspectives on intuition.

Metacognitive resources – knowledge elements for building and manipulating knowledge, in line with the Resources framework (e.g., Hammer et al., 2005).

Productive – a resource or behavior is productive when it leads to a progression in a student's or a group's thinking, a kind of situated productiveness (Goodhew et al., 2018; Harrer, 2013). This *can* also encompass task success, i.e., arriving at an appropriate answer with respect to the discipline.

(Cognitive) resource – tools and ways of knowing (Redish, 2014), in a cognitive sense, or knowledge elements of different grain sizes—activated by patterns of associations (Redish, 2003).

Abbreviations

CER – Chemistry Education Research
DBER – Discipline-Based Education Research
DPTs – Dual-Process Theories
EU – European Union
ICMJE – International Committee of Medical Journal Editors
KMTG – Kinetic Molecular Theory of Gases
PER – Physics Education Research
STEM – Science, Technology, Engineering, and Mathematics
U-STEP – Upper-level Statistical and Thermodynamics Evaluation for Physics

1. Introduction

Thermal physics—including both thermodynamics and statistical mechanics—aims to find ways to describe the physical properties of large, macroscopic systems, i.e., collections of a very large number of particles. Complete microscopic descriptions of such systems, even classical ones, are practically impossible and not preferable (Schroeder, 2021). Historically, the first approach to this daunting task came with the development of classical thermodynamics in the first half of the 19th century, with its central laws of thermodynamics empirically grounded in experiments on macroscopic systems. The second approach, statistical mechanics, aimed to derive the macroscopic laws and all macroscopic properties of a system from its microscopic properties (Mandl, 1987). Thermodynamics and statistical mechanics cover a wide range of phenomena and have applications across fields such as biology, chemistry, engineering, solid-state physics, and astrophysics (Schroeder, 2021). The two subjects share many terms and concepts, such as temperature, energy, entropy, and equilibrium—which are also crossdisciplinary concepts. Statistical mechanics, however, increases the mathematical complexity, generalizes concepts, and provides an underlying explanation of thermodynamics. Some key concepts of statistical mechanics include macrostates and microstates, distinguishable and indistinguishable particles, the Boltzmann distribution, and the partition function[1].

Learning thermal physics as a university physics student mirrors the historical development mentioned above. That is, physics students encounter thermodynamics at introductory levels and are typically introduced to statistical mechanics content as they advance to upper-level thermal physics courses. The existing research on student learning in thermal physics has mainly considered thermodynamics content at introductory levels (e.g., B. W. Dreyfus et al., 2015). Studies focusing on lower-level courses have historically dominated the field of Physics Education Research (PER). While the upper-level topics have gained more attention over the last two decades, this research remains relatively limited (Loverude & Ambrose, 2015)—particularly on topics other than quantum mechanics, such as thermal physics. In general, extending this area of PER is important as the upper-level physics courses introduce

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[1] Readers should note that the underlying physics theory of statistical mechanics is part of the theoretical framework which has shaped my work, as this is the main context of my studies. For a brief introduction to the physics background most relevant to my studies, I refer the reader to Paper I. For more comprehensive sources, I suggest e.g., Kittel and Kroemer (1980), Mandl (1987), Reif (1965), and Schroeder (2021).

various new challenges for students and teachers. The challenges are often related to increasingly abstract physics concepts and increased demand for mathematical competency. The population of upper-level students is also particularly promising to explore in general—i.e., apart from the topic-specific aspects—since they are at an intermediate stage of becoming physicists (Bing & Redish, 2012). Studies considering students at this stage can also provide valuable insight into various general aspects of learning, for example, from a cognitive or sociocultural perspective. Such research can contribute to our understanding of how students continue to build on their knowledge throughout their physics education, and how they become members of the physics community (Loverude & Ambrose, 2015).

1.1 Overarching aim and themes of the thesis

The overarching aim of my research is to contribute with insight into various challenges faced by students—and ways in which they develop—in upper-level physics courses. To this end, I have considered the context of statistical mechanics as a starting point—since it is remains a limited topic within upper-level PER studies, its content has interesting connections to lower-level thermodynamics, and its relevance covers a variety of applications across different disciplines. From my initial, broad exploration of challenges in problem-solving groups of students, three overarching themes emerged as particularly relevant and encompass all my work so far: **conceptual reasoning**, **intuition**, and **mathematics**. For this reason, the licentiate thesis is structured around these broad themes.

In the thesis, I present and discuss findings from three studies: (Paper I) an exploration of student challenges with reasoning in statistical mechanics; (Paper II) a case study of how chemistry and physics students respond to their intuition; and (Paper III) a study of university teachers' conceptions of the role of mathematics in their STEM discipline.

1.2 Structure of the thesis

The thesis consists of eight chapters and is structured according to the following outline.

Chapter 1 – Introduction

This current chapter introduces the topic of this thesis, including a brief description of its relevance, overarching aim, and structure.

Chapter 2 – Literature review

In this chapter, I provide the following: a brief introduction to the field of PER, including its research trends; a review of the relevant literature with respect to the overarching themes of this thesis (conceptual reasoning, intuition, and mathematics) and studies on upper-level physics content; and a description of how my work is situated within PER, including which research areas I aim to contribute to.

Chapter 3 – Theoretical framework

Here, I outline the theoretical underpinnings of my research projects and the thesis. That is, which perspectives and assumptions I adopt from various theories and models that shape the lens through which I think about and approach my research.

Chapter 4 – Research questions

In this brief chapter, I describe the connections between my overarching research aim, the specific research questions from Paper I-III, and the guiding questions which shaped the synthesis of the thesis.

Chapter 5 – Methodology

This chapter summarizes the methodologies employed in my work to answer the research questions, including both the more theoretical aspects (research approaches and data analyses) and the practical aspects of my research methods (data collection). Additionally, I account for my efforts to establish trustworthiness and address ethical considerations in my studies.

Chapter 6 – Findings and discussion

In this chapter, I summarize the analyses, findings, discussions, and limitations of the three papers comprising this thesis.

Chapter 7 – Synthesis and implications

This chapter considers connections between all three papers to provide a synthesis of the findings as a whole, based on an additional, reflective analysis. I also discuss implications for teaching based on my studies and the synthesis.

Chapter 8 – Future directions

In this final chapter, I briefly describe my ongoing research projects and list some potential directions for future work, including studies that may or will be included in the final doctoral thesis.

2. Literature review

This chapter begins with a brief introduction to the field of Physics Education Research (PER), including some historical aspects and connections to related fields (Section 2.1). I also provide an overview of the research trends in PER in Section 2.1.1, and at the end of that section I make some important remarks about the structure and scope of the following literature review sections. In short, the first part of the review is more general, with respect to physics topics, and is divided into the three broad themes of this thesis: Conceptual reasoning (Section 2.2), Intuition (Section 2.3), and Mathematics (Section 2.4). The second part of the review focuses on studies within upper-level university physics courses (Section 2.5), including separate sections on upper-level thermal physics and statistical mechanics in particular (Section 2.5.1). Finally, I situate my research within the field of PER and account for which areas I aim to contribute to (Section 2.6).

2.1 Physics Education Research

How do people learn the concepts, practices, and ways of thinking within the field of physics? What are the suitable learning objectives in physics, and how can we adjust the instructional approaches to better help students reach those objectives? These questions form the core aims of PER. Just like the other fields within Discipline-Based Education Research (DBER), PER addresses a broad range of questions of learning and teaching specific to its discipline. As such, PER is influenced by—and contributes to—general research on e.g., education, sociology, and cognition, but keeps a firm grounding in discipline of physics through its priorities, knowledge, worldview, and practices (Schweingruber et al., 2012). DBER fields are intrinsically interdisciplinary, and its researchers must face the challenge of bridging various gaps between the natural sciences and the social sciences: being knowledgeable and part of your home discipline, while also adopting research aims and methods of a separate research paradigm.

PER is still a relatively young research field, approaching about 50 years of age, despite being comparatively more developed than its closely related DBER fields (such as biology, chemistry, or engineering education research) (Docktor & Mestre, 2014). Contrasting PER with some of the other DBER fields, it is noteworthy that the content of introductory physics has remained more or less the same the last century, while fields such as the biological sciences, astronomy, and geology has faced considerable changes in the content of textbooks (Docktor & Mestre, 2014). Despite some differences of various kinds, there are similarities in both methodological aspects and subject matter between the DBER fields. The natural sciences, for instance, share some cross-disciplinary concepts and topics. A relevant example for this thesis are the topics of thermodynamics and statistical mechanics, which are featured in several disciplines such as physics, chemistry, and biology. Due to the limited overlap among the DBER literature in these topics in the past, researchers have highlighted the importance of advancing interdisciplinary collaborations (e.g., B. W. Dreyfus et al., 2015). It should also be noted that DBER fields have much in common with science education—whose researchers typically reside in departments of education instead of the department of the studied discipline. Collaborations between researchers from PER and science education can be very valuable, such as in the development of theoretical frameworks (e.g., Hammer et al., 2005) or in studies related to physics content (e.g., Domert et al., 2004).

Before I outline PER and its research trends, it should be noted that most of the published work—and reviews of the field—are unevenly distributed with respect to educational levels and geographical contexts. The most represented contexts in the dominant body of literature, of those I have access to, are studies at undergraduate levels within the United States. This happens to be the educational level that is most relevant for my thesis, which is why I have drawn from existing reviews of PER as a basis for this chapter (e.g., Beichner, 2009; Docktor & Mestre, 2014) as well as the three volumes of the recently published *International Handbook of Physics Education Research* (Taşar & Heron, 2023a, 2023b, 2023c).

While an historical overview of PER is beyond the scope of this thesis[1], I will provide a very brief overview of how the field has developed—as context for the following sections—based on Docktor and Mestre (2014). The origin of PER as a research field can be traced back to the 1970s and 1980s, when the primary research focus was to understand and document students' conceptual difficulties at the introductory level, mainly by adopting a misconceptions perspective[2] and employing quantitative methods. Problem solving was also an early research interest, often involving "expert-novice" comparisons. This initial groundwork led to the development of research-based assessments and instructional interventions in the early 1990s. Students' surprisingly poor performance on such concept inventories focused the research attention to teaching practices, resulting in various research-based instructional strategies with

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[1] Readers interested in a more detailed description of the historical development of PER in the United States: see, for instance, Meltzer and Otero (2015). For a short overview of European PER—centered around thermodynamics topics—I suggest Samuelsson (2020).

[2] I will return to how the approach to studying conceptual reasoning has developed in PER since then in Section 2.2.

an emphasis on active student engagement, as opposed to the traditional, passive modes of teaching. Parallel to technological advancements, PER continued to expand its focus to different aspects of computer-based instruction. Over the last two decades, PER has become increasingly interdisciplinary as its scope has expanded, calling for more incorporation of perspectives and methods from e.g., cognitive science and linguistics. The current state of PER—including research interests, theoretical frameworks, and methods—shows how much we have learned about the complex and multifaceted topic of teaching and learning physics since the 1970s. In the next section, I provide a brief overview of the research trends within PER.

2.1.1 Research trends in PER

Over the past 50 years since the origin of PER as a research field, the research interests have broadened and diversified. This development is reflected in the organization and choices of topical areas in various reviews of the field. Table 2.1 collects three categorizations of research trends in PER from different points in time, as presented in: an introduction to PER (Beichner, 2009), a synthesis of PER at the undergraduate level (Docktor & Mestre, 2014), and the 12 sections of the International Handbook of PER (Taşar & Heron, 2023a, 2023b, 2023c). Note that there are overlaps and connections between the trends from each categorization, and that some of the topical areas are broader than others. As such, Table 2.1 should not be seen as a complete comparison between the three sources. Rather, my point is to illustrate some similarities (e.g., traditional topical areas that persist) and differences (nuances in the more recent topical areas). To see even more contrast in how PER has grown over the years, one can compare with an older snapshot of the field in the resource letter by McDermott and Redish (1999).

My research involves explorations of three very broad themes (conceptual reasoning, intuition, and mathematics), with a starting point in the relatively under-researched topic of upper-level statistical mechanics, in the context of problem-solving groups of students. The projects comprising this thesis also extend to interdisciplinary contexts and teachers' beliefs. I will return to how my work is situated within PER, with respect to the trends outlined here, in Section 2.6. It should also be noted that while my research has mainly revolved around problem-solving situations, the problem solving itself serves as the context of my studies, not the primary focus. As such, the following literature review will not be centered around problem solving[3]. However, problem solving is interrelated to the broad themes of this thesis, and will be more or less explicitly present in the following summaries of the most relevant research within these core themes: *conceptual reasoning* (Section 2.2), *intuition* (Sec-

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[3] For an overview of research on problem solving in introductory physics, see e.g., Maloney (2011).

tion 2.3), and *mathematics in STEM* (Section 2.4). As noted by Sherin (2006): "[...] conceptual understanding and problem solving need not be separate, either in instruction or research." (p. 554). Due to the broad and diverse nature

Table 2.1. *Overview of three different categorizations of research trends in PER. The rows indicate overlaps between the trends, but note that some areas are broader than others and that the table does not serve as a complete comparison.*

Taşar and Heron (2023a, 2023b, 2023c)	Docktor and Mestre (2014)	Beichner (2009)
Subject matter learning	Conceptual understanding	Conceptual understanding
Mathematics in teaching and learning physics	Problem solving	Problem solving
Cognitive, epistemic, and affective theoretical underpinnings of research on physics learning	Cognitive psychology	Epistemology
	Attitudes and beliefs about teaching and learning	Attitudes
Coordination of learning goals and teaching practices	Curriculum and instruction	Evaluation of specific instructional interventions
Physics learning environments		
Physics textbooks		Instructional materials
Assessment of student learning in physics	Assessment	
Educational technologies in physics teaching		Technology
Equity and inclusion in physics education research		Social aspects
Physics teacher education		
The history and philosophy of physics in physics teaching		
Physics education research: history, methodologies, themes		

of my themes, these sections serve as introductions and relevant overviews for the purpose of this thesis, not as exhaustive reviews. I will first provide a brief, general description of each area (when necessary starting outside of PER), to gradually narrow the scope according to the aims and contexts of my research.

2.2 Conceptual reasoning

Before outlining the vast body of literature related to the learning of concepts in physics, I want to highlight a few points. Firstly, I use the term conceptual *reasoning* as it is less specific in terms of the reasoner's intention than e.g., sensemaking or meaning making, to encompass a wide range of events. Secondly, I view reasoning as a dynamic process involving both ontological and epistemological aspects. For the following review, it should be noted that the terms sensemaking and reasoning have often been used interchangeably (Kapon & Berland, 2023) and have been theoretically fragmented in the literature, as addressed by Odden and Russ (2019) through their proposed coherent definition of sensemaking.

Much of the early research within PER and science education was centered around the study of students' conceptual difficulties in terms of scientific "misconceptions", as mentioned in Section 2.1. The misconceptions perspective assumes that students' "intuitive" knowledge—here meaning the knowledge formed through years of experience in the world—is inconsistent with scientific knowledge, and that the rigid nature of these conceptions may explain the observation that they typically outlive teaching (e.g., Viennot, 1979). Early studies in PER were often focused on identifying common student misconceptions, not necessarily considering *how* a student's current ideas interact with new, incompatible ideas. To account for this, (Posner et al., 1982) proposed an advancement *toward a theory of conceptual change*. This was the origin of various theories of *conceptual change*, when researchers turned their attention to what transformations are needed for successful learning of scientific concepts, namely: how does conceptual change occur? Influenced by Piaget, Posner et al. (1982) proposed a model of conceptual change based on the idea that students build new knowledge through *assimilation* or *accommodation*. If students are faced with new ideas that align with their current ideas: they assimilate the knowledge, but when faced with a contradiction: they need to accommodate for the new knowledge, which requires a radical change in terms of replacement of old ideas with new. In PER, this led to studies aiming to identify student misconceptions (or preconceptions) and various instructional strategies to resolve them, such as the Elicit-Confront-Resolve strategy (e.g., McDermott, 2001). Overlapping with the common expert-novice studies on knowledge organization, "ontological" recategorization was suggested as necessary for students to learn concepts. This account for conceptual change claimed that students initially classify a concept within a broad category very different from the category an expert would classify the concept, calling for an "ontological shift" (Amin et al., 2023). In a problem solving context, Chi et al. (1981) reported that experts initially abstract major physics principles to approach and solve a problem, while novices base their interpretations and approaches on the problem's surface features.

Since the introduction of the traditional conceptual change models in the 1980s, various alternative models have been proposed as a response based on the emerging insights and developments of PER[4]. The position that instruction should "replace" or "delete" students' prior conceptions was criticized (e.g., Linder, 1993). Moreover, the idea that students' conceptions are inherently stable across contexts was particularly questioned (e.g., Hammer, 1996). Fragmented perspectives on students' conceptions was introduced in the form of flexible and context-dependent models of conceptual change (e.g., DiSessa, 1993; Hammer, 2000), with the ability to explain why students' reasoning can appear to be inconsistent with scientific views in one situation and consistent in another. In these *knowledge-in-pieces* perspectives, reasoning difficulties are not assumed to stem from stable preconceptions, rather, the context is acknowledged to impact which knowledge elements the student draws from. A certain kind of building blocks of knowledge was proposed by DiSessa (1993), called *phenomenological primitives* or "p-prims" for short. These p-prims are fundamental, intuitive ideas or thought patterns based on experiences from early in life, from which more systematic knowledge may be constructed. Importantly, education should strive toward helping students activate p-prims in appropriate situations, thus supporting the activation of other cognitive elements for the context specified by the p-prims. Learning does not involve restructuring knowledge systems. Instead, learning involves the reorganization of knowledge and the ability to activate knowledge elements in a productive[5] way for a given context. Considering the interplay between mathematics and physics specifically, Sherin (2001) proposed that students learn to connect conceptual content and equation through knowledge elements called *symbolic forms*, a similar construct to p-prims. Each symbolic form includes two parts: (1) the idea that is to be expressed in the equation, and (2) a pattern for how that idea is expressed in symbols. I briefly return to this model in Section 2.4.2.

Another flexible and context-dependent model based on a knowledge-in-pieces perspective is the *Resources framework* (e.g., Hammer, 2000; Redish, 2014). In line with a general trend in the development of conceptual change—that of an increased attention to the roles of multiple types of knowledge elements and their interactions (Amin et al., 2023)—the Resources framework

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[4] An account for the extensive development of conceptual change models over the last five decades is beyond the scope of this thesis. For a recent overview of the area, see (e.g., Amin et al., 2023).

[5] Productive in the sense of arriving at an appropriate answer with respect to the discipline, or as being useful in one's disciplinary progress.

encompasses not only aspects related to conceptual resources and their organization, but how the *framing*[6] of a situation impacts reasoning and learning processes. In short, the concept of framing provides a broad lens of influences related to epistemological, metacognitive, affective, and social aspects. The flexibility and breadth of this framework makes it a useful lens to investigate complex processes—such student reasoning and problem solving in groups—with the ability to approach it both holistically and in fragments of relevant scope and type. Section 3.2 provides more details about the Resources framework, which has been influential in my work and for this thesis in particular.

One example of how students' conceptual change can be supported is via analogical reasoning. Brown and Clement (1989) posited that "students have both useful and detrimental intuitive conceptions" (p. 239) with respect to the scientific theory being taught, and that analogical reasoning can increase the range of application of the useful, intuitive ideas. The aim is to establish an analogical connection between situations that students initially do not view as analogous, through a "bridge", in order for the students to extend their valid intuitions from the anchor situation (called "anchoring intuitions") to the initially troublesome target situation (Brown & Clement, 1989; Clement, 1993; Clement et al., 1989). The "bridging analogy" is deemed necessary to incorporate the appropriate intuitive ideas—which could be considered in terms of p-prims—and the bridge situation should resemble both the anchor and target situations. A similar example is conceptual metaphors, which contributes with an embodied cognition perspective on the mapping between abstract and more concrete conceptual domains (e.g., Lakoff & Johnson, 1980). Conceptual metaphors are implicit mappings in the language of science, that move between abstract scientific concepts and the more concrete image schemata emerging from sensorimotor experiences. For example, the conceptual metaphor *force is an agent*, or a caloric metaphor for heat: *heat flows from location A to location B* (e.g., Brookes & Etkina, 2009, 2015).

Another strategy related to transfer and the use of analogies, specifically in problem-solving contexts, is that of isomorphic problems (i.e., those that require same physics principle to solve them, but vary in surface structure). Based on influences from cognitive theory about transfer, and studies about experts' knowledge structures (e.g., Chi et al., 1981), researchers suggested that the ability to transfer relevant knowledge from one context to another improves with expertise because of the expert's hierarchical knowledge organization. Such organizations can facilitate categorization and recognition based on deep features of the context, rather than surface features. In introductory mechanics, Singh (2008) used isomorphic problems to investigate potential factors that may help or hinder the transfer of problem-solving skills from one problem to the other. Her conclusions illustrate how findings can be

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[6] The term *framing* is borrowed from anthropology (Goffman, 1974) to describe students' interpretation what a situation is about.

interpreted differently when adopting a misconceptions versus a knowledge-in-pieces perspective:

> Misconceptions associated with friction in some problems were so robust that pairing them with isomorphic problems not involving friction did not help students discern their underlying similarities. Alternatively, from the knowledge-in-pieces perspective, the activation of the knowledge resource related to friction was so strongly and automatically triggered by the context, which is outside the conscious control of the student, that students did not look for analogies with paired problems or other aids that may be present. (Singh, 2008, p. 1)

For the purpose of this thesis, it is relevant to consider some research on student reasoning in the topic of thermal physics. Studies within this area mainly concern thermodynamics content at introductory levels, and have been influenced by early work within PER (e.g., Rozier & Viennot, 1991). For an interdisciplinary overview of the topical area, see the extensive review by B. W. Dreyfus et al. (2015) on teaching thermal physics in introductory physics, chemistry, and biology. A more recent outline can be found in Loverude (2023). Many of the studies have focused on central concepts such as temperature, heat, ideal gases, heat engines, and the second law of thermodynamics. Some examples include studies exploring the impact of language in students' reasoning about heat (e.g., Brookes & Etkina, 2015), including the perspective of conceptual metaphors mentioned above. Others have considered reasoning difficulties with the kinetic-molecular theory—for example highlighting challenges with macroscopic versus particulate-level explanations (e.g., Robertson & Shaffer, 2013, 2016). Entropy is another frequently explored topic and has long been considered a challenging concept for students. Several studies have looked at macroscopic perspectives of entropy in introductory courses. One common finding is a tendency among students to "over-apply" the second law of thermodynamics, for example, to argue that the entropy must increase regardless of the context (e.g., Christensen et al., 2009). Another recurring theme is that students struggle to distinguish the system from its surroundings (Christensen et al., 2009; Smith, Christensen, et al., 2015), and are confused regarding which of them the second law should be applied to (Bucy et al., 2006). Language aspects and metaphors for entropy have also been considered from a thermodynamics perspective (e.g., Haglund, 2017; Haglund et al., 2016).

In Section 2.5.1, I elaborate on the related PER literature concerning studies on upper-level thermal physics and statistical mechanics, which is the most relevant to my work.

2.3 Intuition

> Physical intuition is elusive—it is difficult to define, cherished by those who possess it, and difficult to convey to others. Physical intuition is at the same time an essential component of expertise in physics. (Singh, 2002, p. 1103)

In light of the elusive nature of *intuition*, this section will first provide a brief overview of the general research on intuition. Then, I will narrow down to research related to intuition within PER, and related fields, specifically. Intuition has been studied extensively across various disciplines. Particularly in philosophy and psychology, where the empirical grounding mainly has come from studying the speed at which expert chess-players can recognize and select the best moves (Gobet & Chassy, 2009). Other empirical studies on experts' decision-making comes from various domains, including healthcare, business, and firefighting (e.g., Gobet & Chassy, 2008; Patton, 2003). In this general literature, intuition has often been described as the identifying characteristic of expertise: intuition as the ability of an expert to rapidly and easily recognize the key features of a situation (Gobet & Chassy, 2009). From the cognitive perspective, intuition can be seen as a mechanism to reduce cognitive load and aid in rapid interpretation of one's surroundings. This emergent experience is often referred to as a "gut feeling" in everyday language. Within cognitive psychology, Newell and Simon (1972) explained how people can solve problems in this rapid manner—without reasoning analytically—using the concept of "chunking": pieces of knowledge can be chunked into larger components. In this sense, intuition can be equated to pattern recognition, and expertise can be understood as a combination of intuition and a selective search. On the philosophers' side, H. L. Dreyfus and Dreyfus (1986) critiqued this fragmented perspective on intuition due to its lack of contextual considerations with respect to human cognition. They instead argue that intuition is the immediate manifestation of an expert's *holistic* understanding of a given situation. In their proposed "final stage of expertise", both understanding the task and deciding what to do is intuitive—the expert is not making decisions, they are just doing what normally works. To reconcile these two theories of intuition, Gobet and Chassy (2009) used the *template theory* of expert memory to provide a coherent explanation of intuition in expert behavior. In short, chunks that frequently recur in a certain context can turn into a cognitive structure (or pattern of association of knowledge elements) called templates, which allow faster information processing.

Various perspectives have emerged regarding what intuition is and whether it is a trait exclusive to experts, novices, or both. The previous outline serves as a few reference points on a diverse spectrum of ideas, but it should be noted that many other views exist and that intuition is often used in literature without a definition. From the perspective of H. L. Dreyfus and Dreyfus (1986), intuition is seen as characteristic of experts. This has been challenged by Montero and Evans (2011), who argue that the chess experts' so-called "intuitions" are completely rational. There are also perspectives that fall somewhere in the middle, for instance, Gobet and Chassy (2009) and Newell and Simon (1972) who recognize intuition as a part of expertise but not exclusive to experts. The two ends of the spectrum suggest different implications for education: novices develop expert-intuition through experience (H. L. Dreyfus & Dreyfus, 1986); or novices can be explicitly taught heuristics[7] extracted from experts' rational "intuitions" (Montero & Evans, 2011). In my work, I have adopted a middle-ground perspective: all individuals can experience intuition based on their current level of expertise. I elaborate on my perspective on intuition in Section 3.2.2, which is inspired by researchers from mathematics education research (Brady et al., 2022) and the knowledge-in-pieces, or resources, perspective.

2.3.1 Intuition in PER

In PER, the study of intuition has been approached from various angles. In a recent study, Corsiglia et al. (2023) presented four major research strands: intuition as barriers to understanding or problem solving; intuition as an instructional goal; the ontology of intuition, aiming to define and understand intuition; and the epistemology of intuition, where the main focus has been to explore experts' views on intuition. Within the closely related field of chemistry education research (CER), the study of intuition has predominantly centered on students' use of heuristics (e.g., Graulich, 2014; Talanquer, 2014).

Early work on problem solving in PER, which often consisted of expert-novice comparisons, contributed to the empirical support for the general theories described in the previous section. For example, the PER study by Larkin et al. (1980) served as an example in Gobet and Chassy (2009) of how experts can solve routine problems in a matter of seconds. However, as Maloney (2011) highlighted more recently: if we want to compare expert and novice problem solving, the experts should attempt to solve tasks that would actually be considered a problem for them—not the same tasks as given to the students. This was considered in a study by Singh (2002), who found that even when experts' "physical intuition failed them", they still had more non-physics resources available than the students. These resources were described in terms of more general heuristics (e.g., making simplifying assumptions) and specific heuristics (such as thinking in terms of conservation relations). Despite some of these connections between intuition and problem solving, the research on intuitive understanding of physics concepts has mostly been separate from research on problem solving (which has often focused on experts) (Kuo, 2023; Sherin, 2006). Many studies on intuition in physics were concerned with students' prior knowledge, or "everyday intuition", thus over-

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[7] Heuristics can be considered as simple rules ("rules of thumb") that allow problem solver to progress without fully engaging in analytical problem-solving strategies (Kahneman, 2011).

lapping with the broad body of research into student "misconceptions". For example, the Elicit-Confront-Resolve (e.g., Shaffer & McDermott, 1992) and Predict-Observe-Explain (White & Gunstone, 1992) approaches to instructional activities included strategies aiming to help students overcome incorrect ideas which could stem from their intuitions.

Other studies within PER have adapted models from cognitive psychology, dual-process theories (DPTs) (e.g., Kahneman, 2011; Kahneman & Tversky, 1984), to understand certain patterns in students' and experts' reasoning in physics. Key features of DPTs include the modeling of human cognition in terms of two processes: system 1 (a fast, automatic, and subconscious process) and system 2 (a slow, effortful, and deliberate process). The first is often referred to as the "intuitive" process, and the second as the "analytic" process. Intuition can, in these terms, be defined as the outcome of the first process: a quick and unconscious model or impression arising from associations based on past experiences (Kryjevskaia et al., 2021). In some studies adopting DPTs, system 1 has developed a negative connotation compared to the more "sophisticated, formal, and analytical" thinking, because the intuitive process sometimes lead to incorrect responses. For example, Wood et al. (2016) relate intuition to misconceptions: "success [in learning physics] involves rejecting the common, intuitive ideas about the world (often called misconceptions) and instead carefully applying physical concepts." (p. 1). For this reason, the DPTs models have received criticism. However, other work utilizing DPTs within PER (e.g., Gette et al., 2018; Kryjevskaia et al., 2014, 2021), have not aligned with this negative perspective on intuition. Rather, they have underscored the value of both the intuitive and analytical process (and the inevitability of the intuitive process), for students as well as experts. Several researchers, from both PER and CER, have proposed that students need to develop metacognitive strategies in order for the two processes to interact fruitfully in reasoning and problem-solving situations (e.g., Graulich, 2014; Kryjevskaia et al., 2021; McClary & Talanquer, 2010).

As another contrast to the association between intuition and "misconceptions", dynamic perspectives on understanding and problem solving—in terms of knowledge-in-pieces or resources—encompass intuition as well. As phrased by Kuo (2023), such frameworks consider both "formal" and "informal" knowledge resources as relevant to learning and doing physics. For example, Sherin (2006) built on the work by DiSessa (1993)[8] to consider the interplay between intuition and problem solving. He concluded that intuition and common sense are part of solving even traditional textbook problems, and importantly, he suggested that: "Instruction must nurture and refine intuitive physics, not confront and replace it, or simply build up a new set of frameworks." (Sherin, 2006, p. 554). Other researchers have considered ways in which students can

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[8] Who considered "p-prims" as fundamental, intuitive ideas or thought patters that build up knowledge structures (see also Section 2.2).

link between abstract concepts and their concrete experiences and intuition, for example via analogies (e.g., Brown, 1993; Clement, 1993) or via certain conceptual resources in a particular topic (e.g., Sabo et al., 2016).

From an epistemological perspective, studies on experts' views on intuition generally focus on knowledge as a whole, not attempting to define intuition explicitly (Corsiglia et al., 2023). One exception comes from outside of PER, where Marton et al. (1994) studied Nobel prize-winners' views on scientific intuition and found, for example, that scientific intuition is often contrasted to conscious, logical thinking. In PER, investigations considering student epistemologies in quantum physics have received recent attention (e.g., Dini & Hammer, 2017; Gette et al., 2018), but the focus on students' views on intuition in PER remain scarce. Recently, Corsiglia et al. (2023) contributed to this gap in the context of quantum mechanics, reporting six facets of intuition based on the ways students used the word in interviews and responses to questions. The classifications of intuition were based on examples of "something is intuitive when...", for example: you can predict outcomes, or you encounter it in everyday life. As highlighted by the authors, future studies on intuition in PER would be valuable:

> Overall, student statements about intuition were rich, and they often touched on other PER topics, such as math-physics connections or student self-efficacy. We consider intuition a powerful lens for investigations into practitioners' perspectives on learning, knowing, and doing physics. (Corsiglia et al., 2023, p. 14)

Such future work could, for example, examine how students' use of different facets of intuition come into play in problem-solving situations.

2.4 Mathematics

This section concerns a broad and prominent topic within STEM education: the interplay between science and mathematics.

In Section 2.4.1, I will first outline the relevant literature concerning the role of mathematics in STEM, focusing on the disciplines relevant to my research: physics, chemistry, computer science, and geoscience[9]. In Section 2.4.2, I will briefly summarize other aspects related to the interplay between physics and mathematics in PER—from work which has not necessarily focused on defining the role of mathematics itself, but e.g., modeling how students use mathematics in physics.

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[9] Physics is the primary discipline in my work. Paper II also involves chemistry, and the STEM disciplines explicitly included in Paper III (in addition to physics and chemistry) are computer science and geoscience.

2.4.1 The role of mathematics in STEM

A considerable body of research examines the relationship between mathematics and science, often in the context of physics in particular. The topic has been approached in various ways, including historical (e.g., Branchetti et al., 2019), philosophical (e.g., Quale, 2011), and direct or indirect educational perspectives (e.g., Karam et al., 2019; Krey, 2019; Kristensen et al., 2024; Maass et al., 2019; Palmgren & Rasa, 2024; Redish & Kuo, 2015; Tuminaro & Redish, 2007; Uhden et al., 2012). Although mathematics is known to underpin all other STEM disciplines, Maass et al. (2019) highlighted that the role of mathematics is understated in the literature of integrated STEM education, and argue for the need to advance it. It is also well-known within the educational literature that proficiency in mathematics does not necessarily transfer to contexts in other disciplines, in part due to the integration of rich physical meaning in mathematical expressions and representations (e.g., Redish & Kuo, 2015; Uhden et al., 2012). Moreover, how teachers view the role of mathematics in STEM has an impact on their teaching, and consequently influences student learning (de Ataíde & Greca, 2013; Uhden et al., 2012).

Research on the role of mathematics in physics has frequently distinguished between the *structural* and *technical* dimension (e.g., de Ataíde & Greca, 2013; Palmgren & Rasa, 2024; Uhden et al., 2012). The technical dimension is associated with the instrumental character of mathematics, while the structural dimension often encompasses many aspects, such as mathematization, interpretation, reasoning (Pospiech & Karam, 2023). Distinguishing between mathematics as a tool and mathematics as a (conscious) generalization has also been highlighted as an important theme within interdisciplinary mathematics education (e.g., Doig et al., 2019; Goos et al., 2023). More specific roles of mathematics in physics are outlined in Taşar and Heron (2023c). The philosophical debate regarding the interplay between mathematics and physics is arguably important for education: why is mathematics so well suited to describe physical processes? Pospiech and Karam (2023) outline different philosophical positions from the theoretical literature: the world is mathematical; physics and mathematics have a common history and development; physics studies the "mathematizable"; mathematics is analytical priori; and mathematics as embodied cognition. Quale (2011) agrees that foundational beliefs about mathematics—such as Platonist and Formalist positions—and the relationship to physics has implications for the teaching of physics.

While less well-researched compared to physics education, similar issues regarding the technical and structural role of mathematics has gained attention in the chemistry education literature (e.g., Bain et al., 2019; Ye et al., 2024). In computer science, for example, Baldwin et al. (2013) considered the role of mathematics and discussed the gap between its practical and intellectual roles within the discipline. Similarly, within geosciences, students' perceived irrelevance of mathematics within the subject has been discussed. Researchers have argued that geoscience education needs more emphasis on quantitative literacy—which is more than mere calculation, it also includes reasoning with data and problem solving (e.g., Wenner et al., 2009). A few more examples within these other STEM disciplines can be found in Paper III.

Teachers' views about the role of mathematics is important for education, as mentioned previously. There has been some research exploring the connection between teachers' beliefs about teaching and learning and their instructional practices in general (Popova et al., 2020). Considering mathematics specifically, there exists some research on school teachers' views on the role of mathematics in science, showing that the way those views are communicated can influence pedagogical choices and, therefore, also what and how students learn (e.g., Pospiech et al., 2019; Redfors et al., 2016). Pospiech et al. (2019) reported a tendency among the teachers to recognize the importance of separating the technical and structural roles of mathematics. However, less empirical work has considered how STEM university teachers view the role of mathematics in their discipline.

2.4.2 Mathematics-physics interplay

In addition to discussions about the role of mathematics in physics *per se*, the interplay between mathematics and physics has been considered in various ways across the field of PER. Some themes, based on Taşar and Heron (2023b), include: the meanings of physics equations; the meanings of graphs; and visualization and mathematization using digital tools. This research area often overlaps with studies on problem solving. For example, studying how mathematics elements (e.g., symbols, functions, equations, geometry, vectors, and mathematical procedures) are applied and understood during physics problem solving. Specific examples of such studies can be found in reviews such as Docktor and Mestre (2014) and Pospiech and Karam (2023). In terms of reflecting physics modeling in teaching practices, Kuo (2023) highlights that: the non-procedural roles of mathematics—i.e. the structural roles mentioned in the previous section—have been recognized as an important part of modeling approaches to physics teaching (e.g., Brewe, 2008).

Several theoretical frameworks for describing the blending of physics and mathematics have also been developed in PER, such as the following three examples.

(1) Symbolic forms (Sherin, 2001)
A framework which considers that students learn to understand equations in an fundamental—or experiential—sense, in terms of a vocabulary of elements called *symbolic forms*. Each symbolic form associates a simple *conceptual schema* (the idea to be expressed in the equation) with a *symbol template* (which specifies a pattern for how that idea is

written in symbols). Sherin compared his symbolic forms to diSessa's p-prims.

(2) **Epistemic games** (Tuminaro & Redish, 2007)
Within the Resources framework, "epistemic games" are proposed as a way to analyze students problem-solving behavior in terms of locally coherent goal-oriented activities, which guide and limit what knowledge students think is appropriate to apply at a given time. (For more details, see Section 3.2 where I describe the Resources framework, framing, and epistemic games.)

(3) **ACER framework** (Caballero et al., 2015; Wilcox et al., 2013)
This analytic framework was developed for characterizing students' use of mathematics in upper-level physics. The framework is structured around the students' Activation of mathematical tools, Construction of mathematical models, Execution of the mathematics, and Reflection on the results (ACER).

As highlighted by Loverude and Ambrose (2015), the interplay between mathematics and physics is one of the recurring themes in the PER literature on upper-level physics courses, where the level of mathematics becomes increasingly challenging. All three frameworks listed above have been employed in upper-level physics content more broadly, while other studies have considered mathematical aspects in specific upper-level content. For the purpose of this thesis, an important point is made by Loverude (2023) regarding the mathematical underpinnings of thermal physics:

> In contrast to other subfields of PER, there is perhaps less focus on the underlying mathematics of thermal physics. This may reflect that, compared to many other upper-division core courses, the mathematics in thermal physics is generally believed to be less difficult mathematically, as it largely stops at "simple" multivariable calculus. However, student data reveal that it is not simple to apply fundamental ideas from calculus in these novel contexts. Statistical physics brings its own set of mathematical complications. (Loverude, 2023, p. 3-6)

A few studies within the topics of upper-level thermal physics and statistical mechanics have considered how student difficulties with mathematics concepts interact with the understanding of physics content (e.g., Loverude, 2009, 2010; Smith, Thompson, & Mountcastle, 2013). I elaborate on studies within these specific content areas, and outline the work on upper-level physics more generally, in the following section.

2.5 Upper-level university physics in PER

Historically, PER has mainly focused on lower-level undergraduate courses. Over the last two decades, however, topics in upper-level courses have gained more attention (Loverude & Ambrose, 2015). Similarly to the initial phases of PER at introductory levels, the existing research at the upper levels have, so far, mainly considered student reasoning and understanding of course-specific content (Loverude & Ambrose, 2015). As such, the studies have had a strong cognitive focus, often aiming to identify student difficulties. Upper-level PER is currently a growing subfield with research trends such as: student learning of content; mathematics-physics interface; instructional reform (e.g., developing anything from textbooks and assessments to courses and programs); and sociocultural or other non-content-focused issues. Many theoretical upper-level courses have been considered, to a varying extent, such as: electricity and magnetism, quantum mechanics, relativity, and thermal physics. An overlapping research trend is mathematics in upper-level physics. As the physics content becomes more advanced, so does the demand on students' mathematical competence. Studies have, therefore, explored the interaction between student difficulties with mathematics concepts and the understanding of physics content, or students use of mathematics more broadly (see Section 2.4). Considering upper-level contexts has also allowed researchers to explore broader issues of instructional reform—such as applying reform frameworks to transform upper-level courses (e.g, Zwickl et al., 2015), or developing assessment instruments suited for the upper levels (e.g, Rainey et al., 2020; Wilcox et al., 2015). Moreover, this context grants the possibility to gain insight into the identity development of physics students through longitudinal studies (e.g., Irving & Sayre, 2015).

Topic wise, quantum mechanics is one of the most mature and represented topics in upper-level PER, encompassing all of these research trends. The literature on quantum mechanics has developed from studies such as Wittmann et al. (2002), to reviews of student difficulties (Singh & Marshman, 2015), to new ways of investigating students' understanding of the relationships between mathematical expressions in the topic (e.g, Riihiluoma et al., 2024). Singh and Marshman (2015) reported that common reasoning difficulties in quantum mechanics are akin to those in introductory physics—and are often the consequence of overgeneralizations of concepts in non-applicable contexts—such as, difficulties with distinguishing between closely related concepts and reasoning about the formalism. Student epistemologies have also been investigated in quantum physics contexts. Interestingly, B. W. Dreyfus et al. (2019) reported a "split" in students' epistemological beliefs about classical versus quantum physics. The authors voiced a concern that quantum physics students are in danger of straying too far away from conceptual reasoning, leaning instead toward a stance of "shutting up and calculating". The authors considered a possible connection to teachers' views and instructional choices:

> Students may internalize messages (intended or not) that quantum mechanics is a place to rely solely on mathematical calculation [...] and that sense-making will not avail them here, which may impede the activation of the productive

> epistemological resources that they bring to classical physics. (B. W. Dreyfus et al., 2019, p. 13).

Compared to the PER literature on introductory physics courses, the research on upper-level courses remains a relatively young and growing area of interest in PER. Upper-level physics introduces various new challenges for students and teachers. Exploring upper-level students—who are at an intermediate stage in the transition from novice to expert physicists (Bing & Redish, 2012)—is interesting from both a cognitive perspective (e.g., how their knowledge develops) and a sociocultural perspective (e.g., how students become members of the physics community). These arguments were highlighted by Loverude and Ambrose (2015). As emphasized by Redish (2003), the learning and teaching of physics is not simply a matter of learning physics content: the "hidden curriculum" includes issues of a second cognitive level (such as metacognition) as well as social and affective aspects (such as motivation, self-image, and emotion). Moreover, Loverude and Ambrose (2015) suggest that instructors of upper-level courses is an interesting and under-researched area of research, stating some relevant issues:

> There is a perception that some instructors at this level are very traditional in their approach and would be uncomfortable with any changing to the existing paradigm for upper-division theory courses. There is little research in this area, however, and the values and expectations of upper-division instructors have not been systematically documented. What, for example, is the purpose of derivations, and are they more important for upper-division courses than for the introductory level? The constraints facing instructors at this level are typically less daunting than is the case in introductory courses; classes are small and students are favorably disposed toward the subject matter, and yet instruction typically remains highly traditional. (Loverude & Ambrose, 2015, p. 6)

As stressed by Linder (1992), epistemological tones that teachers reflect—both implicitly and explicitly—in their teaching can impact students' reasoning and problem-solving approaches, and traditional lecturing may be particularly troublesome in this regard. This has been discussed in the context of quantum physics (e.g., B. W. Dreyfus et al., 2019), as mentioned previously.

For the purpose of my research, the next section provides an overview of the existing upper-level PER literature on the topic of upper-level thermal physics, focusing on statistical mechanics in particular.

2.5.1 Learning upper-level thermal physics

In this section, I start by briefly summarizing studies on upper-level thermal physics (i.e., upper-level thermodynamics and/or statistical mechanics). These studies do not necessarily consider statistical mechanics content explicitly, but may explore the bridge between lower-level thermodynamics and statistical physics. After that, I summarize the PER literature on the topic of statistical mechanics specifically. Note that a few examples of studies on the related topic of introductory-level thermodynamics have been collected in each of the previously reviewed themes, in Sections 2.2, 2.3, and 2.4.

Upper-level thermal physics typically introduces students to some statistical mechanics content, but to what extent may vary considerably across curricula and institutions. Recurring themes from the research on learning thermal physics include: distinctions between closely related quantities (e.g., heat, temperature, internal energy, entropy); mechanistic or statistical reasoning focused on constituent particles (e.g., in the context of the second law of thermodynamics); and mathematical underpinnings (Loverude, 2023). It is also important to note the interdisciplinary connections of thermal physics once more. For example, CER can provide valuable insights into particulate-level mechanistic reasoning, which is often a central focus in chemistry contexts. Moreover, while thermal physics instruction in physics often consider closed systems, some students will encounter the same concepts in more complex biological or chemical contexts (Loverude, 2023). This is reflected in collaborative efforts to develop the instruction of thermal physics in interdisciplinary contexts, such as in physics for the life sciences (e.g., Geller et al., 2014; Redish et al., 2014)

Within the PER literature on thermal physics, a common finding is that connecting macroscopic phenomena to particulate-level models is very challenging for students. A few of these studies have investigated the transition from introductory to upper-level content, which is connected to bridging the macroscopic perspective of thermodynamics to the microscopic perspective of statistical mechanics. Leinonen et al. (2015) studied this in the context of the second law of thermodynamics, looking at how consistently upper-level students use the law at macroscopic and microscopic levels. One of their conclusions was that the macroscopic perspective of entropy is easier to understand than the statistical nature of entropy from the microscopic approach. Another study by Meltzer (2009) observed learning patterns in an upper-level thermal physics course and found that several learning difficulties common among introductory students persisted with the upper-level students. More recent findings, based on studies with a research-based survey instrument, suggest that both introductory and advanced physics students (from upper-level thermodynamics courses) faced similar difficulties with introductory thermodynamics concepts after traditional lecture-based instruction (Brundage et al., 2024, 2025). Students' struggles with concepts in thermodynamics might carry on, even further, to the statistical mechanics context, where the descriptions of the concepts are perhaps even more difficult to grasp. Erceg et al. (2016) investigated students' understanding of concepts related to the microscopic model of gas, as a part of developing the kinetic molecular theory of gases (KMTG) concept inventory. One of their findings indicates that similar student challenges appeared across subgroups from various teaching environments, such as different study programmes, years of study, and curricula with a varying focus on thermodynamics.

Next, I provide an overview of studies that mainly focus on statistical mechanics, which are even rarer.

Learning statistical mechanics

In addition to the difficulties that may follow students from previous thermal physics courses, as mentioned above, statistical physics (or statistical mechanics) introduces new, related concepts and an increased mathematical complexity. As highlighted by Loverude (2023): "In statistical physics, distinctions between microstates, macrostates, and multiplicity are (perhaps unsurprisingly) challenging." (p. 3-5). The majority of the following studies fit into the research trends of subject matter learning (conceptual understanding), instructional materials, and assessment.

Binary systems are a typical way to introduce some key concepts of statistical mechanics. This was considered by Mountcastle et al. (2007), who found that students did not reliably reason that the relative uncertainties of binary outcomes decreases as the number of measurements increases. Loverude (2009) also investigated student understanding of probability concepts in the context of simple systems. He found that students struggled to distinguish microstates from macrostates and apply mathematical relationships for the multiplicity. The same trend was later identified by Loverude in the context of two interacting Einstein solids (Loverude, 2010), which was part of a series of tutorials developed in the study. In the context of a less simple system, Crossette et al. (2021) also found confusion about macrostates and microstates among some of the interviewed graduate students. Considering a wider range of central concepts in statistical mechanics, Lo (2022) interviewed both undergraduate and graduate students on the concepts of basic probability, microstates and macrostates, Einstein solids, the Boltzmann factor, the density of states, and the partition function. Based on the findings, Lo concluded that the students generally could recognize the concepts, but only expressed a superficial understanding of them, and that their reasoning consisted of incomplete and/or loosely connected ideas rather than stable misunderstandings.

Entropy has been explored in several studies. As pointed out by Crossette et al. (2021), entropy is a core concept of thermal physics and despite its precise statistical definition, it is subtle and difficult to understand. Loverude (2015) and Crossette et al. (2021) identified several resources that upper-level and graduate students used to conceptually describe entropy. These include relating entropy to disorder (a common connection for undergraduate students identified in Loverude (2015)), temperature, mixing, or information. Relating entropy to the number of microstates, as stated in the statistical definition, has been reported as strongly preferred by students (Bucy et al., 2006; Crossette et al., 2021).

Other studies have also focused on developing tutorial materials and assessment tools for teaching statistical mechanics. Recently, Rainey et al. (2020, 2022) developed and validated the first upper-level thermal physics assessment that addresses both classical thermodynamics and statistical mechanics content, the Upper-level Statistical and Thermodynamics Evaluation for Physics (U-STEP). In a few different studies, Smith and collaborators developed tutorials and investigated student reasoning about the Boltzmann factor, an essential concept of statistical mechanics. They approached this topic from the following perspectives: students' understanding of Taylor series expansions (Smith, Thompson, & Mountcastle, 2013); comparing relative probabilities of states (Smith, Mountcastle, & Thompson, 2015); and the relation between the Boltzmann factor and the density of states function as expressions of multiplicity (Smith, Mountcastle, & Thompson, 2013). Over several years, these authors have consistently found that students often fail to recognize the situations in which it is justified to apply the Boltzmann factor and struggle to express its physical significance, after lecture instruction alone. However, their results indicate that after participating in the developed tutorial instruction (Smith, Mountcastle, & Thompson, 2015), students were more likely to use the Boltzmann factor appropriately.

2.6 Situating my own research

Against the background of this literature review, I will now position my work within the field of PER and account for which gaps in the existing research I aim to contribute to. Based on the organization of the recently published International Handbook of PER, see Table 2.1, my research fits into several broad trends within the field: Subject matter learning; Mathematics in teaching and learning physics; and Cognitive, epistemic, and affective theoretical underpinnings of research on physics learning.

My research contributes to the relatively limited PER area of subject matter learning at upper-level physics courses. Generally, work within this context provides valuable insight into an intermediate stage of students' transition from novice to expert physicists, including various challenges related to an increasing demand on mathematical competency as well as increasingly abstract physics concepts. Particularly, my work has mainly considered the topic of statistical mechanics (Paper I and II), which has not been investigated to the same extent as upper-level content like quantum physics. By exploring student reasoning in the context of problem-solving groups through an open lens, my work aims to provide insight into several broad themes and identify important aspects related to conceptual reasoning, intuition, and mathematics. This context also provides the possibility to more explicitly consider collaborative, social, and affective aspects of learning physics in future work. More specifically, my research builds on the limited work on students in-the-moment use of intuition in problem-solving contexts, and aims to contribute with new perspectives on the important but elusive concept of intuition (Paper II). In light of the importance of role of mathematics in physics, as well as other STEM disciplines, my interdisciplinary collaboration in Paper III aims to contribute to the empirical gap in the study of STEM university teachers' views on the role of mathematics in their discipline. In addition to increasing the awareness of teachers' views and beliefs, Paper III begins the exploration of how teachers' views about mathematics may affect the teaching and learning in upper-level physics courses.

Overall, my work aligns with the call for expanding the research on upper-level physics courses (Loverude & Ambrose, 2015) and contributing to our understanding of the importance of epistemological aspects (e.g., Bing & Redish, 2012; B. W. Dreyfus et al., 2019) in teaching advanced courses. While the starting point of my work is the topic of statistical mechanics, due to its scope and focus, my research has the potential to contribute to the general body of PER literature on: subject matter learning in upper-level physics; mathematics in teaching and learning physics; and cognitive, epistemic, and affective underpinnings of research on physics learning.

In this thesis, I aim to provide a preliminary contribution of holistic insights into the nature of reasoning and problem solving in physics by exploring: interrelations between conceptual, epistemological, and metacognitive resources; the impact of framing; the role of intuition; and epistemological beliefs with respect to the interplay between physics and mathematics. This aligns with suggested directions for future research on sensemaking and reasoning in science and physics education:

> The first direction has to do with the interrelations between mathematical sensemaking, analogical reasoning, and mechanistic reasoning. While each of these epistemic forms has been extensively studied separately, creative and groundbreaking work in physics often involves iterative smooth transitions between these forms of reasoning, in which one advances the other. (Kapon & Berland, 2023, p. 12-18)

3. Theoretical framework

In this chapter I outline the theoretical underpinnings of my research projects and this thesis. This includes perspectives and assumptions I adopt from different theories and models, that shape the lens through which I view my work, how I think about my research topic, and how I approach my projects. It is crucial to describe my theoretical perspectives, as they influence everything from the questions I ask to the kinds of knowledge claims I make. First, I explain how I interpret the multifaceted term *constructivism* from the perspectives of learning, epistemology, and philosophy (Section 3.1). Next, I describe the theoretical assumptions I draw from constructivism from the different viewpoints: constructivism as a learning perspective (Section 3.1.1); and how constructivist ideas about scientific knowledge and the nature of reality influences the research paradigm I fit into within PER (Section 3.1.2). In Section 3.2, I outline the basis of the Resources framework—a model of knowledge construction that aligns with the previously described theoretical assumptions—from which I have drawn perspectives during my studies and adopted as a framework for the synthesis of this thesis.

3.1 Constructivism

It is important to note that *constructivism* is a multifaceted term that appears across many fields with different meanings. Even within a single field of research, like education, there are several versions of constructivism—as well as disagreements about their interpretation. The word itself implies that something is constructed, but it is important to clarify the focus: *what* is constructed? I found the differentiation by Sjøberg (2010) to be very useful when thinking about this contested topic. He separated it into three different questions, regarding different objects of construction:

(1) **Perspective on learning:** How do people (individually) construct meaning and knowledge about the world?
(2) **Perspective on scientific knowledge:** How is the shared and accepted scientific knowledge about the world constructed?
(3) **Perspective on the world:** How is the world itself constructed?

As highlighted by Sjøberg (2010), the first question is connected to psychology and educational (or learning) theories—as well as epistemology on an individual level—while the second and third are questions of philosophy and epistemology on a broader level (communities and the world). Within DBER, all the perspectives listed above are relevant to consider. Researchers must account for the theoretical assumptions they draw from theories or positions within those perspectives, as those assumptions underpin and shape their work. The first perspective is inherently important, as the research revolves around investigating aspects of learning (and teaching) in a discipline. In the next part, Section 3.1.1, I will describe the ideas and assumptions I have adopted with respect to learning. The second perspective is vital in two different aspects: how scientific knowledge within the discipline being learned (physics) is constructed; and how scientific knowledge within DBER (in my case PER) is constructed. In combination with the third perspective, about the nature of reality, these two aspects can be considered as fundamentally different and calls for separate research paradigms. Moreover, even within PER one can define distinct paradigms. I will elaborate on this issue in Section 3.1.2, and describe the research paradigm I see myself as a part of within PER.

3.1.1 Constructivism as a learning perspective

In this section I will elaborate on constructivism as a learning perspective, and the assumptions I draw from it in my research. This broad perspective is referred to as a family of constructivist-learning theories, encompassing several different theories about how people construct meaning and knowledge. As mentioned previously, both the use of the term constructivism and the interpretation of various branches of constructivism is a debated topic within education. There are, however, some core ideas shared by most of the different versions, as listed by Sjøberg (2010). Here I select and combine the ones I find most relevant:

- Learning involves active construction of knowledge by the learner, it is not passively received from the outside. Learners build on their previous experience and existing ideas to construct new knowledge, they do not come into learning situations as "blank slates".
- Learners have their own individual ideas about the world, based on their experience, but there are similarities between individuals' ideas—some are socially and culturally accepted. These ideas are not always aligned with accepted scientific ideas, and may be persistent and hard to change[1].
- Knowledge is represented in the brain as conceptual structures, which are possible to model and describe to some extent.
- On one hand, knowledge is personal and individual, but learners construct their knowledge through interactions and exchanges with the physical world and other people. (The emphasis on the social, cultural, and

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[1] Modern perspectives on *conceptual change* (see Section 2.2), with strong influences from cognitive science, argue that ideas cannot be "removed" or "deleted". I return to this when describing the Resources framework (Section 3.2).

affective aspects of learning varies between the different versions of constructivism.)

Constructivism encompasses many perspectives and was developed, in parallel, from the ideas of influential scholars such as Piaget and Vygotsky—who were familiar with each others' ideas and commented on both their agreements and criticisms (e.g, Piaget, 2000). The Piagetian perspective on knowledge construction is prominent in constructivism, and other theorists (mainly Vygotsky) contributed with stronger emphasis on the social and collaborative nature of learning. Two of the wide-spread versions of constructivism is *cognitive (or personal) constructivism*, based on Piaget's work, and *social constructivism* based on the ideas of Vygotsky. As highlighted by Sjøberg (2010), while these sometimes portrayed as opposites, they share core constructivist ideas[2].

I draw from both perspectives, starting from a constructivist foundation in the first two points above, and adopting the last two points aligning with influences from both cognitive science, or cognitivism, and the sociocultural perspective (which are not mutually exclusive). Important assumptions that follow for my work include the following: (1) It is possible to gain insights into a person's cognitive structures (in their brain) by investigating their external actions. (2) Language allows people to articulate their thoughts both internally and to others, and this communication can provide insight into their learning (or rather, their reasoning) process on both a cognitive and social level. Moreover, other important influences from cognitive science include insights about how memory functions and cognitive structures in the brain. For example, as stressed by e.g., Redish (2003), the mind works by context-dependent *patterns of association of knowledge elements* (knowledge structures). When information from the context-dependent long-term memory is brought into the working memory, it is an active process and in most cases a response is *constructed* using stored information (prior knowledge/experience) in new ways. These perspectives on learning are also supported by insights from neuroscience. As described by Zull (2002), learning involves *building on* existing neural networks. Understanding these knowledge structures and their function is key to understanding student reasoning. These assumptions underpin the model of knowledge construction that I have adopted several ideas from, the Resources framework, which I summarize in Section 3.2.

3.1.2 A constructivism-inspired research paradigm

How do we construct scientific knowledge about the learning (and teaching) of physics, and what do we assume about the nature of reality? These two ques-

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[2] The variation partly stems from the difference in their research focus: Piaget was interested in epistemology and knowledge, while Vygotsky was interested in the social and cultural conditions for learning.

tions concern epistemological and philosophical assumptions of a research paradigm within PER. A paradigm represents a distillation of what we think about the world, but cannot prove (Lincoln & Guba, 1985). Another way to define a research paradigm, as adopted by Robertson et al. (2018) from Greene and Caracelli (1997), is as a set of assumptions about knowledge, our world, and our ability to know that world (including why) that frame an orientation toward research. This guides a researcher in many regards, including which questions are asked, methods employed, knowledge claims made, and how trustworthiness is established.

Starting from the nature of our world, there are a variety of terms used to distance oneself from a positivist perspective (such as naturalist, constructivist, non-dualist, subjective) with essentially the same adopted assumption: there is no single reality "out there", realities are subjectively and socially constructed. Note that it is the interpretations of the world that are subjective and constructed, not the *physical* world itself (which exists and is interacted with)[3]. The perspective on reality as socially constructed is sometimes portrayed as a constructivist underpinning of qualitative research (e.g., Robson, 2002). However, as argued by Robertson et al. (2018):

> "Qualitative" and "quantitative" are often used to refer to methods, the processes by which researchers obtain and analyze data. A paradigm, in contrast, is a set of assumptions (which may bear out in research aims and methods). The same method may serve different paradigms, and a single paradigm may use multiple methods. (p. 6)

Instead, Robertson et al. (2018) propose two paradigms within PER: case-oriented PER and recurrence-oriented PER. My work fits into the paradigm of case-oriented PER, as it aligns with the assumptions: (1) social actions are guided by the meanings that people are making of their local environments, and (2) reality is subjectively constructed[4]. The first assumption was discussed in the previous section, with respect to my perspective on learning. Importantly, within this research paradigm researchers do not generalize to populations in order to make predictions, rather they generalize to theory by identifying what a given case is a case of, in order to broaden awareness. In this sense, locally identified mechanisms are connected to theory and expected to be useful in similar contexts, but not expected to be representative

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[3] Here I emphasize that I do *not* adopt the following radical stance on the nature of a constructed reality: that reality does not exist until it is created through the construction of an actor. Which is arguable not only radical, but also an unnecessary ontological stance for the purposes of naturalistic research (Lincoln & Guba, 1985, p. 87).

[4] It is subjective in the sense that the locally constructed meanings are themselves what is *real* to participants, therefore important for the research, but the authors also acknowledge the social aspect: "these meanings are complex—tied to social and epistemic expectations and to verbal and nonverbal signals by which participants communicate their expectations to one another." (Robertson et al., 2018, p. 13).

to a population or produce regularities. These assumptions and consequences are reflected in my research aims, approaches, methods, claims, and strategies to establish trustworthiness, as outlined in Chapter 5.

3.2 Resources framework

The *Resources framework* provides a fine-grained, modular, and flexible model of knowledge construction, where the cognitive units are called *resources* (e.g., Hammer, 2000; Redish, 2014). This framework has the flexibility to consider both cognitive and sociocultural knowledge structures, as the "grain-size" of the analysis can be adapted accordingly. As mentioned earlier in this chapter, the framework aligns with insights from neuroscience about how the human brain and memory works: resources (similarly to neurons) are connected in dynamic and emergent networks, not static structures, and the activation of resources (bringing stored knowledge elements from the long-term memory into the working memory) is a context-dependent process. Moreover, learning involves real structural changes among neural connections, which requires repetition, reflection, and integration. This resonates with the constructivist perspective on learning as growth, not information transfer (Redish, 2014; Zull, 2002), and is reflected in the framework: "Resources activate in sets, and a set that activates again and again may eventually become established sufficiently to act as a cognitive unit itself." (Hammer et al., 2005, p. 9).

The idea of *resources* was initially developed as several researchers challenged the misconceptions perspective and proposed a resources perspective as an alternative (Hammer, 1996). One of the criticisms was that the misconceptions perspective offers no account for how students' existing productive ideas may act as resources in the development of new understanding, indicating the need for an alternative that aligns with the constructivist perspective on learning (e.g., Clement et al., 1989; DiSessa, 1993; Hammer, 1996). Moreover, the coherent and robust nature of student misconceptions cannot explain the variation of students' reasoning across different contexts—which calls for a more flexible framework. Within the Resources framework, students' ideas and prior knowledge comprise the resources they bring to a learning situation. These resources are not inherently correct or incorrect, it is the *activation* of a resource (or a set of resources) that can be productive[5] or unproductive depending on the context. Resources that are applied incorrectly in one situation can be valuable in other contexts, and through this view we can understand both how productive activations can support student learning and how inappropriate activations can explain student challenges. The implications, in terms of the focus of a researcher or teacher, aligns with the constructivist views outlined earlier in this chapter:

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[5] See the Glossary.

> A resources-based ontology, by contrast, highlights the students' existing knowledge, and the metaphor of *knowledge as activations* generates questions that focus attention back again on the students, on what resources they have available and how those resources are organized and reorganizable. (Hammer et al., 2005, p. 24)

Hammer (2000) distinguishes between two kinds of resources: *conceptual* and *epistemological*. Conceptual resources are related to knowledge of certain concepts, while epistemological resources concern views about what it means to know something. Although they can be seen as related, Hammer et al. (2005) mention the distinction between *metacognitive resources* (for building and manipulating knowledge) and *epistemological resources* (for understanding the nature of knowledge and knowledge-building). It should also be noted that a single, cognitive resource has a finer grain-size than concepts or abilities as experienced by people: knowledge and experience emerge from the activation of multiple small resources (Hammer et al., 2005). Additionally, learning an idea can be seen as the cognitive state the learner forms at the moment—through the activation of multiple resources—rather than the formation of a cognitive object.

3.2.1 Framing

As mentioned previously, the resources perspective allows explanations for why a person may respond differently in different contexts. Resources are not randomly activated or deactivated, however, which is considered through the concepts of *frames* and *framing*[6]. Following Hammer et al. (2005), a *frame* can be seen as a person's interpretation of "What is going on here?", meaning a set of expectations about a certain situation which affects what they notice and how they act. *Framing* is the activation of a locally coherent set of resources, meaning that the activations are mutually consistent and reinforcing in that moment. A situation can be framed with respect to various aspects, such as social, affective, and epistemological. For example, an individual's or a group's *epistemological framing* of a situation concerns "What do I/we expect to use to answer questions and build new knowledge?". To understand these concepts, let us consider an illustrative and relevant example for my research: a problem-solving group of students. Initially, the students frame the situation individually and may proceed to challenge each others' interpretation of "what's going on here", referred to as *frame negotiation*. To describe what might follow, Hammer et al. (2005) provide a useful context for the relevant concepts from the Resources framework through an example of introductory physics students discussing a problem about elevators:

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[6] The terms "frame" and "framing" are used in various ways in the sociolinguistics literature, but their use within the Resources framework is mainly inspired by a few scholars from linguistics, anthropology (e.g., Goffman, 1974), and discourse analysis, see Hammer et al. (2005).

> The group's definition of the problem type will dramatically affect their next steps; the choice is between one set of resource activations and another, each set including epistemological resources for understanding what sorts of knowledge are relevant, metacognitive resources for forming and manipulating those kinds of knowledge, and of course conceptual resources for understanding motion and forces in elevators. *Framing*, then, is (i) the forming of this set, and then, once it is formed, (ii) the use of those resources to interpret utterances, sensory inputs, and so on. (Hammer et al., 2005, p. 12)

In this episode, the authors identify two examples of epistemological frames as: formal calculation and intuitive sense-making, which answer the question "how should I approach knowledge?". It is also noted that while students sometimes shift between frames easily, at other times frames are resistant to shifts. Student's ability or will to shift frames can be described as an *epistemic agency*, defined by Miller et al. (2018) as "students being positioned with, perceiving, and acting on, opportunities to shape the knowledge building work in their classroom community" (p. 1058).

Another empirical study related to framing comes from Tuminaro and Redish (2007), who observed a tendency among students to rely on "limited sets of locally coherent resources" as they engaged with mathematics while solving physics problems. Consequently, this often restricted the students from activating resources (which they later show to possess) that would have been productive in that context. This "coherent activity that uses particular kinds of knowledge and processes associated with that knowledge to create knowledge or solve a problem" was defined as an *epistemic game* (Tuminaro & Redish, 2007, p. 4). For example, one of the identified epistemic games was named "recursive plug-and-chug", where students plug quantities into physics equation and churn out numerical answers without conceptually reasoning about the physical implications. Also in the context of mathematics use in physics problem solving, Bing and Redish (2009) highlighted how group problem-solving situations can be leveraged to identify students' epistemological framings through *warrants*. In the group discussions, students tend to articulate warrants[7]: reasons or arguments in support of their point of view, which provides insight into their framing of the task.

3.2.2 Resources perspective on intuition

Our exploratory work in Paper II, and ongoing projects, have focused on the elusive concept of intuition. The relevant background and prior work involving intuition are outlined in the literature review (Section 2.3). As mentioned in that overview: there are many different perspectives—across diverse research fields—on what intuition is and who does, or does not, possess it. Here, I outline the resources-inspired perspective on intuition that my co-author and

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[7] Bing and Redish (2009) equate warrants to epistemological resources.

I adopted for Paper II. In the synthesis of this thesis (Section 7.1), I will develop this further by discussing how intuition can be interpreted within the Resources framework more explicitly.

For the purpose of Paper II, we did not redefine intuition, instead we adopted what we referred to as a *resources perspective on intuition*. Our perspective draws inspiration from various existing theoretical ideas. Following Brady et al. (2022), we consider intuition as *the manifestation of one's tacit knowledge, triggered by rapid interpretation of one's surroundings and influenced by one's beliefs and values*. We appreciated this view, as it resonates with the core ideas of the Resources framework—which has framed both mine and my co-author's research projects. Just like resources, intuitions are not classified as "right or wrong": rather, their activation is considered appropriate or inappropriate within a given context[8]. Similarly, we challenge the notion that intuition is static or exclusive to either experts or novices; on the contrary, we view it as a dynamic trait that can evolve over time through practice and experience. This aspect of our perspective is echoed by Sherin (2006) and Kryjevskaia et al. (2021).

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[8] Furthermore, in Paper II we argue for a shift in focus. Since intuition is connected to an automatic and unconscious activation process, we need to consider the *response* to intuition in the moment—not only how to refine the intuitions. We showcase that students can *respond productively* even to an inappropriately activated intuition.

4. Research questions

The overarching aim of my research is to explore how challenges and progressions related to conceptual reasoning, intuition, and mathematics emerge in student problem-solving groups in the context of statistical mechanics—as a case of upper-level university physics. Note that this is the most recent description, as the overarching aim and my studies have developed concurrently. My broad exploration began by asking the following two research questions, specific for Paper I and II, where the theme of intuition emerged as an area of interest from the findings in Paper I and became the focus of an interdisciplinary collaboration in Paper II:

I What challenges do upper-division physics students face when solving statistical mechanics problems in groups? (Paper I)

II How do beyond-intro level chemistry and physics students respond to their intuitions during collaborative problem solving? (Paper II)

The importance of role of mathematics in physics is particularly evident in math-intense upper-level courses like statistical mechanics, but is a relevant topic in broader contexts as well. This led me into an interdisciplinary collaboration in Paper III, aiming to empirically investigate the following research question. In my overarching project, this can be considered as a first step in the exploration of how teachers' views about mathematics may affect the teaching and learning in upper-level physics courses.

III What are university teachers' conceptions of the role of mathematics within the knowledge structure of other STEM disciplines? (Paper III)

Based on all the findings and insights gained throughout these studies, including potential connections and implications, the following two questions emerged. These are related to all the specific research questions above and served as *guiding questions* in my reflection on the findings as a whole, which shaped the synthesis of this licentiate thesis (Chapter 7).

- What insights can be gained from analyzing the students' challenges and progressions through the lens of conceptual, epistemological, and metacognitive resources?

- How may teacher-reflected epistemologies about conceptual reasoning, intuition, and mathematics influence students in these aspects?

5. Methodology

This chapter provides an overview of the methodology adopted in my work, encompassing both theoretical underpinnings and practical aspects in terms of the research approaches and research methods, and how they align with the research questions. Section 5.1 summarizes the specific research approaches employed in my studies: *grounded approach* (mainly Paper I and II) and *phenomenography* (Paper III), and the related data-analysis processes. Additionally, I connect these with my overarching approach to the synthesis of this thesis. Section 5.2 outlines the methods used to collect data: *problem-solving sessions* for Paper I and II (video data) and *semi-structured interviews* for Paper III (audio data). In Section 5.3, I describe how trustworthiness can be established in naturalistic studies, and account for my efforts to establish trustworthiness. Finally, I address ethical considerations in Section 5.4.

5.1 Research approach and data analysis

In this section, I describe the research approaches I have adopted and how they align with the research questions. Since the choice of research approach is closely related to the process of data analysis, I provide an overview of each approach alongside a general description of the corresponding data-analysis procedure. I begin with the grounded research approach, which is a relevant methodological foundation for all three papers, particularly Paper I and II. Then I summarize phenomenography, the methodological approach employed in Paper III. Finally, I comment on my overarching approach to the synthesis of this thesis.

5.1.1 Grounded approach

A grounded approach resembles the starting points of several research approaches within case-oriented research, and it can be characterized as an emergent, subjectivist, inductive, and pragmatic approach. A suitable description of the grounded approach that I have adopted in my work comes from the outset of constructivist grounded theory (e.g., Charmaz, 2000, 2008). Unlike traditional grounded theory (e.g., Glaser & Strauss, 1967), which only considers the research outcome as emergent, the constructivist stance on grounded theory argues that the methodology itself is also emergent. The researcher and their views cannot be separated from the approach. Since a researcher's perspectives may grow and change as they engage with the data, adopting a constructivist grounded approach allows both the methodological decisions and the findings to be emergent and flexible. The researcher must be open to a range of theoretical possibilities during the research process. They must also reflect on, and be transparent about, their own epistemological perspectives and research principles, which inevitably influence the process and outcome.

A grounded approach is particularly well suited for exploring uncharted, complex, or dynamic phenomena. The starting point is therefore an initial topic of exploration, rather than a narrow, predetermined research question which might have turned out unfruitful. The researcher asks "What is happening here?" to examine the empirical world in close detail with an open mind, and "What are these data a case of?" to link to theoretical possibilities. Specific questions may instead arise from the researchers engagement with the data during the study. Moving past the starting point—a grounded approach—to grounded theory means reaching beyond pure induction to iteratively collect and analyze data in phases of constant comparison, to develop increasingly theoretical categories.[1] This involves strategies such as theoretical sampling and theoretical saturation. I will not elaborate on those specific strategies here, since it is beyond the scope of the studies presented in this thesis. Both of the studies (Paper I and II) were exploratory in nature, and considered relatively novel and complex topics, which is why a grounded approach was deemed suitable. We did not move beyond the grounded-approach phases into the systematic theorizing stage of grounded theory, since the emerged findings and process already resulted in extensive studies. Nevertheless, the findings from Paper I and II can inform continued work of that kind in future studies.

Data analysis

Data analyses within a grounded approach are inductive, emergent, and theory-informing processes. In line with my research approach, as described above, the data-analysis process follows the procedures described in the starting point of, for example, constructivist grounded theory (e.g., Charmaz, 2008). The initial analysis is developed by conceptualizing the data rather than imposing a predetermined theoretical framework on them. However, as mentioned above, the researchers must acknowledge the influence of their conscious, and potential unconscious, theoretical perspectives on the data-analysis process and findings. Coding data typically consists of at least two phases: initial coding and focused coding. The initial coding is an open and broad coding of the data, looking for initial themes or ideas. The coding is an inductive, interactive and comparative process, and it is iterative. It starts while gathering data

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[1] In constructivist grounded theory, this is described as moving from *inductive* logic to *abductive* reasoning: the researcher seeks to understand the emergent findings by imagining and empirically testing possible theoretical accounts for the observed data at hand, to yield the most plausible theoretical explanation (Charmaz, 2008).

and continuously emerges as the researcher engages with the data. Depending on the purpose, the data can be considered in various grain sizes, such as analyzing transcribed data with line-by-line coding or coding in larger chunks. Initial codes are constantly compared and scrutinized to evaluate their significance and/or frequency, to develop focused codes. In the focused-coding phase, these codes are tested against the data by using them to analyze large batches of data. The codes that seem to be most important are identified as major focused codes, providing tentative categories that could be raised to theoretical categories through further analytical treatment.

In general, for the purpose of the broad exploration in Paper I and its rich data, our initial phases of data analysis considered chunks of the raw video data. As the process evolved, we transcribed relevant episodes of the data to allow line-by-line coding. The process was similar for Paper II but with a narrower focus on intuition from the start—that had emerged as important from Paper I and my co-author's separate study (Ye et al., 2024)—where we adopted a resources perspective on intuition (see Section 3.2.2). Details can be found in each paper. As mentioned previously, both of these exploratory studies stopped before any further steps to develop theoretical categories, but future studies could continue to refine the abstraction and search for relationships between categories. In such additional phases, a relevant and appropriate external theory can act as a supplement to aid the analysis.

5.1.2 Phenomenography

Phenomenography is a qualitative, empirically based research approach that was originally developed in the late 1970s to tackle certain research questions related to learning and understanding in educational settings. It explores the qualitative variation in the ways a group of people experience[2] a phenomenon (Marton & Booth, 1997). Learning is argued to be related to a change and widening of the ways we see the world, where variation and discernment play vital roles. As a consequence of variation, people experience and discern new critical aspects of a phenomenon. Therefore, the outcome of a phenomenographic study—a set of qualitative, often hierarchical, categories of different ways of experiencing a phenomenon within a certain group—is valuable for teachers in the sense that it raises the awareness of the variation in the understanding of a phenomenon to be learned (Trigwell, 2000).

As mentioned previously, like several other qualitative research approaches, the starting point of phenomenography is similar to a grounded approach—it is non-dualist, qualitative, and emergent. However, as summarized by Trigwell (2000), the essential underpinnings that distinguishes phenomenography from many other qualitative research approaches are the following: (1) Phe-

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[2] To experience is commonly used interchangeably with to conceptualize, perceive, and understand.

nomenography takes a second-order perspective, rather than first-order, as the researchers describe the experience of a phenomenon as described by others. (2) The focus is on the variation of experience, not a full, rich description of the experience itself, such that the sample is selected to maximize the possible variation. (3) It considers the internal—and often hierarchical—relations between different ways of experiencing, rather than listing essentially unrelated categories as one might expect from content analysis or thematic analysis. Moreover, the focus is on identifying variation in ways of experiencing within a defined group of people, not representing the experiences of individuals within that group. Consequently, this specific framework calls for a more fixed study design, despite its grounded and emergent aspects, as I will describe in the subsequent sections. In light of these key aspects of phenomenography, I also wish to highlight the scope of the claims in phenomenographic research. The credibility claim is made in relation to the data at hand, and the emerged categories are the researchers' representations of possible conceptions, as emphasized by Marton and Booth (1997):

> Thus we argue the category of description is a reasonable characterization of a possible way of experiencing something given the data at hand. Whether or not a certain person is really capable of experiencing the phenomenon in question in this particular way, or under what conditions she is capable of doing so, is a question that falls outside phenomenography proper. (p. 136)

In Paper III, our research question was: What are university teachers' conceptions of the role of mathematics within the knowledge structure of other STEM disciplines? Phenomenography was deemed a suitable approach with respect to this question, since our purpose was to gain insight into the *various ways* of conceptualizing the role of mathematics in another STEM discipline *through the lenses of STEM university teachers*.

Data analysis

A phenomenographic data analysis is a grounded, emergent, and iterative process, but it has a predetermined focus and aim. The focus is on the collective meaning of participant responses (rather than individual responses), with respect to the fixed phenomenon under study, particularly the variations and relations among them. The aim of the analysis is to generate a set of empirically grounded and logically related categories representing qualitatively different ways of conceptualizing the phenomena at a collective level (Adawi & Ingerman, 2006). This is referred to as an outcome space. The outcome space can be presented in various formats, such as tables, diagrams or figures, and includes descriptions of each category along with a selection of illustrative examples from the data. The structural relationship between the categories is commonly represented through a hierarchically inclusive outcome space: the categories are ordered to represent increasingly complex or advanced ways of conceptualizing the phenomenon (Han & Ellis, 2019).

In practice, there are several suggested ways of structuring a phenomenographic data analysis process, which bear many similarities in terms of its main stages (see a comparison in Han and Ellis (2019)). For our study in Paper III, we adopted the following procedure, which mainly resembles the stages described in Marton et al. (1992):

1. **Familiarization**
   The researchers engage with the data to develop a good sense of its breadth and depth.
2. **Identification**
   The data is passed through to identify all parts related to the phenomenon.
3. **Sorting**
   The identified data are sorted into preliminary groups ("pools of meaning") according to similarities.
4. **Contrasting and categorizing**
   The sorted data are contrasted to generate an initial set of categories with labels and descriptions. Through several iterations the categories are modified and refined.
5. **Reliability checking**
   Several researchers should apply the categories and analyze the data independently. Disagreement can be discussed and resolved to minimize researcher bias.

This approach required us to transcribe all audio data, such that all parts related to the phenomenon can be identified and to allow fragmentation of individual responses into collective pools of meaning. Details can be found in Paper III.

5.1.3 Approach to the synthesis

I approached the overarching synthesis of this thesis in a grounded way—aligned with the approaches described above—but with the aim of drawing connections on an even broader level than before, to integrate the emerged findings from each of my three studies. The outcome of the synthesis, described in Section 7.1, emerged from an initial approach to search for relationships between all the findings and refine the abstraction. As was pointed out in the earlier section on grounded approach, such phases of the research approach can be supported by an appropriate (informed by the data) external theory acting as a supplement in the analysis. Based on the findings in Paper I, we recognized that a resource perspective could be a valuable theoretical supplement to further investigate both conceptual and epistemological aspects of the reasoning and problem-solving processes. For a subsequent focus on the emerged importance of intuition, we argued for the value of a resources perspective on intuition in our approach for Paper II. The findings in Paper III further suggested the importance of epistemological views, particularly with respect to the relationship between physics and mathematics.

In light of all these indications, and due to the broad and complex nature of the explored contexts, I chose to frame the synthesis based on the Resources framework. This provided a flexible theoretical supplement, that aligns with the theoretical underpinnings of my research projects (assumptions about the world, learning, and constructing scientific knowledge) and has the structure and concepts to encompass all the extensive themes of interest for this thesis: conceptual reasoning, intuition, and mathematics. The relevant theoretical perspectives and frameworks are outlined in Chapter 3.

5.2 Research methods

This section provides a brief overview of the practical aspects of the methodology, that is, the methods I have employed in my studies to collect the data discussed in this thesis. I describe the overall methods of data analysis in Section 5.1, as they are closely related to the respective research approach. Specific details about the methods can be found in Papers I-III, separately. Moreover, I elaborate on issues related to trustworthiness and ethics, with respect to my research methods, in Sections 5.3.2 and 5.4.

5.2.1 Data collection

In this section, I summarize how data were collected in my studies and who the participants were. Note that whenever I refer to the data collection in Paper I, this corresponds to the same set of data that was reanalyzed in Paper II. In terms of the research question, approach, and methods, Paper III differs in several ways from Paper I and will in most cases be described separately.

A common feature in all of my studies is that I collected data in settings that can be considered as neither fully naturalistic (*in vivo*) nor completely experimental/artificial (*in vitro*) to the participants. As highlighted by Dunbar and Blanchette (2001), combining *in vivo* and *in vitro* approaches can be advantageous. In my cases, I chose settings somewhere in the middle of that spectrum to attempt a balance of the benefits and challenges of both simultaneously. An *in vitro* aspect of my settings is that the participants were invited to a separate meeting room for the interview sessions, rather than being recorded in their natural context (which for the students could have been an instructional activity within their course). Despite this, the setting and content of the problem-solving sessions or interviews were still familiar to the participants, and I made efforts to create an environment that felt as safe and natural to them as possible (this is elaborated on in Section 5.4.2). In doing so, I hoped to promote authentic interactions among the participants while still limiting external distractions in the environment and directing their focus to certain questions or tasks.

Contexts and participants

For Paper I, my participants were volunteering students from a course on statistical mechanics, mainly third (final) year students from a bachelor's program in physics and a few engineering students. I collected data in two phases, corresponding to two different course rounds across two subsequent years. At the time, the course followed the book *Statistical Physics* by Mandl (1987) and consisted of traditional lectures as well as separate tutorial sessions[3]. In total, the participants consisted of nine groups of two to three students.

For Paper III, the participants were volunteering university teachers—in this context, this implies that they are also researchers—from different STEM disciplines. We deliberately sought out more experienced teachers, as their reflective insight increases the likelihood of capturing a broad range of conceptions, aligning with our phenomenographic methodology. This kind of purposeful sampling is inherent to the phenomenographic approach. The researchers in our team interviewed two teachers from their own discipline, to ensure a familiarity with the participants' context, which resulted in a total of 16 participants from the disciplines chemistry, computer science, geoscience, and physics.

Interview tasks and questions

My design of the tasks used in the data collection for Paper I—on the topic of statistical mechanics—was framed by the related physics theory, some prior research within PER, and teaching materials from the course which the participants were recruited from. The aim was to create problems with relevant content, probing the students' reasoning about key concepts in statistical mechanics, while keeping the level appropriate with respect to their progression in the ongoing course. Details regarding the design of the problems, and suggested solutions, are found in Paper I.

For Paper III, we designed questions for our semi-structured interview script based on our phenomenographic methodology and insights gained from our preceding pilot study. We tried to formulate questions that would promote as much reflection about the studied phenomenon as possible, because our aim was capturing the *variation* among the conceptions expressed by the studied group of teachers. With this set of questions, we intended to probe the participants' views about the phenomenon from different angles, without asking leading questions. For example, two of the questions were: "What do you consider to be the role of mathematics in your subject?" and "How would you explain the role of mathematics in your subject to someone outside your subject?".

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[3] The tutorial sessions are typically centered around a teacher working through example exercises on the board, combined with time for the students to work through exercises individually.

Problem-solving sessions (Paper I and II)

The research question in Paper I, and our adopted approach, called for a data-collection setting where the students could discuss and solve our designed problems together, with minimal interference from the researcher. Moreover, it called for collecting audio and video data, to capture anything (including non-verbal communication) which might emerge as interesting throughout the study. We referred to our data-collection method as "group interviews" in Paper I, but to clarify the setting I call them "problem-solving sessions" here. I wish to emphasize that the sessions revolved around the student groups: I did not ask them questions, instead they received the designed interview tasks (one by one) and discussed these until they were satisfied to move on. Before recording started, I clarified that the most important point is to express their reasoning, not to present a certain answer, and I encouraged them to verbalize their thoughts to each other as much as possible. In this sense, the sessions can be described as: collaborative problem solving in a think-aloud manner. My role in sessions was to observe as unobtrusively as possible while also moderating, for instance by asking occasional follow-up questions or encouraging the students to elaborate when necessary. That is, a kind of *reactive observation* approach (Angrosino, 2012).

The choice to collect data in this manner was rooted in its potential advantages with respect to the quality of the data and the well-being of the participants. In a well-functioning group setting, students can articulate their reasoning in a natural way and keep their focus on the task at the same time. It mitigates the risks associated with students explaining their thoughts to an interviewer one-on-one, as it can influence their thinking significantly (Ericsson & Simon, 1998). A disadvantage of collecting data from groups can be that facilitating a group requires considerable expertise (Robson, 2002). This was mitigated by my prolonged experience as a student facilitating other students' group work, and as a teacher readily employing collaborative active learning methods. I return to the advantages and challenges with collecting data in groups in the upcoming sections on trustworthiness and ethical considerations, in Sections 5.3.2 and 5.4.

Semi-structured interviews (Paper III)

For Paper III, we chose to collect audio data by conducting semi-structured interviews, which is a common method of data collection in phenomenographic studies. In general, interviews are an appropriate method for qualitative studies focusing on the meaning or views of a particular phenomenon (Robson, 2002). In particular, a semi-structured interview gave us the advantage of a set of predetermined questions—mitigating the inconsistencies introduced by having eight different interviewers—while allowing for the flexibility to modify the order of questions, or ask follow-up questions, as a response to each interviewee. This was necessary in light of our aim and approach, providing us with the possibility to encourage each participant to reflect on the phenomenon as much as possible, from many perspectives, to yield rich, nuanced data.

In addition to our predetermined interview questions, the research team discussed the importance of creating a natural and welcoming environment from the start of each interview. Moreover, to acquire more depth and reduce bias, each interviewer encouraged elaboration, asked follow-up questions, and invited participants to reflect on colleagues' views as well as their own. This helped to shift the focus from self-presentation to reflection, enabling the participants to explore and articulate a range of views without feeling personally evaluated. Each researcher interviewed two teachers their own discipline to promote a collegial setting—helping the participants feel more at ease—and ensure that we had enough disciplinary knowledge to be able to ask relevant follow-up questions. Again, I will return to the issues regarding trustworthiness and ethical considerations in Sections 5.3.2 and 5.4.

5.3 Trustworthiness

5.3.1 Establishing trustworthiness in naturalistic inquiry

Establishing trustworthiness in research means to persuade yourself and others that your work is meaningful, believable and truthful. Within the positivist and rationalistic paradigm, researchers commonly use four criteria to assess trustworthiness: *internal validity*, *external validity*, *reliability*, and *objectivity*. These criteria have been argued inappropriate to determine trustworthiness within the naturalistic research paradigm, due to the differences in fundamental assumptions about the world and interactions within it. Importantly, the naturalist assumes that "reality" is a multiple set of mental constructions, without an ultimate benchmark, and that it needs to be considered holistically. Lincoln and Guba (1985) proposed the alternative criteria *credibility*, *transferability*, *dependability*, and *confirmability* to consider in naturalistic inquiry. These metrics are explained in Table 5.1, along with suggested techniques for how they can be met in naturalistic inquiry.

It is worth clarifying the measures listed in Table 5.1. The suggested ways to increase credibility include *prolonged engagement*, which means investing sufficient time to learn the "culture" of the context, minimize distortions (introduced by the researcher's presence or by the participants), and building trust (such as taking ethical considerations into account). Immersing oneself in a context for a long time, with an open mind, is useful to provide scope. However, it is important to also keep the observation intentional and focused to provide depth. This is the idea of *persistent observation*, to identify the most relevant elements to focus on in detail, with respect to the purpose of the inquiry. *Triangulation* is another process to improve credibility, where information derived from one source, method, or investigator, is checked against other sources. Within the naturalistic paradigm, this could mean using dif-

Table 5.1. *The four naturalistic criteria for trustworthiness and some techniques for how to actualize them, based on Lincoln and Guba (1985). The techniques are clarified in the main text.*

Naturalistic criteria	Techniques to meet the criteria
Credibility refers to whether the study has been carried out such that the findings and interpretations adequately represent "reality", i.e. the original multiple, mental constructions.	*Prolonged engagement* to provide scope. *Persistent observation* to provide purposeful depth. *Triangulation*, with respect to data. *Peer debriefing* to keep the inquirer "honest". *Member checks*, with respect to constructions. *Reflexive journal.*
Transferability is the alternative to generalizability, since the naturalist only makes time and context-bound hypotheses. Researchers must, therefore, provide enough details of the studied context so that others can judge whether the findings are transferable to another context.	*Thick description* to describe the context and study in detail. *Reflexive journal.*
Dependability accounts for stability and consistency, like *reliability*, but is not achieved by the rationalistic replication method since the studied phenomenon, the human instrument, and the approach are all changeable. Factors of instability and changes in the observed entity or the emergent study design must therefore be considered.	Assure the first criteria, credibility. *Dependability audit*, including the audit trail. *Reflexive journal.*
Confirmability refers to whether the data are confirmable or not, rather than asking whether the researcher is objective or not (as they will always be biased).	*Confirmability audit*, including the audit trail, and *triangulation*. *Reflexive journal.*

ferent sources or methods of data collection, or discussing findings and interpretations among multiple investigators (such as different members of the research team). The researcher should also confer with an uninvolved peer to keep the researcher honest and allow for new perspectives that might otherwise be overlooked, this is the suggested *peer debriefing*. Moreover, *member checks* is a process of testing the researchers' reconstructions against the members who were involved in the data collection. For example, to check if these members recognize the data, interpretations, and conclusions as representative of their own reality.

With respect to transferability, it is the researcher's responsibility to provide a *thick description* of the studied context to give readers the possibility to judge whether a transfer to another context is possible. In practice this means presenting as much details about the study and the context as possible.

Although it is possible to argue that ensuring credibility is sufficient to demonstrate the quality of a study without taking specific measures related to dependability, Lincoln and Guba (1985) suggest additional direct strategies to deal with dependability and confirmability simultaneously. These measures are combined in what is referred to as an inquiry audit, which encompasses the *dependability audit* and the *confirmability audit*. An external, knowledgeable "auditor" inspects and evaluates both the process of the inquiry, to judge its dependability, and the outcomes (e.g., data, findings, interpretations) to judge its confirmability. Whether an auditor will examine it or not, the researcher should keep an *audit trail*: records of the data from the initial raw data to the final published form, along with process notes. Finally, one technique that applies to all the criteria is keeping a *reflexive journal*. This is a kind of diary kept by the researcher to record information about themselves (as the human instrument) and the methods employed (methodological decisions and reasons for making them) on a regular basis. The journal may include the following separate parts: logistics of study, personal reflections (such as speculations about emerging insights), and a methodological log.

5.3.2 Establishing trustworthiness in my studies

Below, I summarize the measures I have taken in my work to ensure trustworthiness according to the naturalistic criteria (bolded) and techniques (italicized) described in Section 5.3.1.

- **Credibility**
  In terms of *prolonged engagement*, I have either been, or am currently, part of the same culture as the participants in my studies. Through my background as a physics student at the same university, just a few years ahead of my participants, I could relate to the students and they were also familiar with me (Paper I and II). Along with the measures I took with respect to my role in the problem-solving sessions, as described in Section 5.2.1, this could decrease the distortions introduced by my presence and help build trust. Specific measures I took to further build trust is described in Section 5.4, about ethical considerations. Similarly, the participants I interviewed for Paper III were other employees at the same physics department. In all my studies, I have also spent a long time on each project, giving myself time to become familiar and engage thoroughly with my data. This has helped me be open to seeing new features as well as identifying the most relevant elements to focus on

in detail (*persistent observation*). Regarding *triangulation*, my general approach has been to discuss my interpretations and findings continuously, throughout the process, with other members of my research team. I have used the strategy *peer debriefing* by also discussing with uninvolved parties, such as other educational researchers (also those from other universities) and physicists from my department who are experts within the studied physics topic (particularly for Paper I). To promote a more natural and internal form of *member checks*, I chose to collect data from groups of students (Paper I and II) in a think-aloud manner. Participants in a group tend to check and ask each other for elaboration, to explain their reasoning in several ways, which gave me more insight into their thought process (Robson, 2002). I also always asked the students to reflect on their own experience in the end of the session, to be able to compare what I interpreted as challenging for them with their self-reported difficulties.

- **Transferability**
  In all of my studies, I have made an effort to provide a *thick description* by presenting as much details of the context, participants, and methods as possible. For example, describing practical details of the data collection and providing the interview materials (interview tasks/questions). In doing so, others can make a more informed judgment about in what ways the findings from my context could be transferred to another context. To conform to the scope of a paper, I have tried to focus on describing the most relevant aspects of the study with respect to my research questions. When describing the participants, I also had to adapt to regulations regarding personal data and other ethical considerations, see Section 5.4. To increase the probability that the findings could be transferable to similar contexts, I designed the problems (for the data collection in Paper I and II) to align with commonly occurring tasks and key concepts within the topic of statistical mechanics.

- **Dependability**
  In addition to the measures described to ensure credibility, I have made other choices to increase the consistency of my studies. For example, all the problem-solving sessions (for Paper I and II) was carried out by the same researcher, me, to increase the probability that the participants were affected in the same way. Of course, my insights and interpretations changed along the way, but I attempted to be consistent with respect to my interactions with the participants (which were quite limited as I mainly observed them). The data collection for Paper III was carried out by several researchers, so we made an effort to design an appropriate protocol for the semi-structured interviews to ensure that the main questions were posed to the participants in the same way. Moreover, with re-

spect to consistency of the interpretations and findings in my studies, the analyses have always been iterative and discussed among the co-authors (sometimes also external people for additional perspectives). To reveal changes in my own interpretations of the data, across my prolonged engagement with it, I returned to the raw data on several occasions to scrutinize and refine my analysis. The process of my work can be tracked through notes I have taken alongside the records of the data itself, a kind of *audit trail*. Such notes include, for example, decisions from meetings with co-authors.

- **Confirmability**
  To ensure that the data are confirmable, I have kept records of the initial raw data (video or audio recordings), versions of processed data (in transcript form) from various iterations, up to the final published form. This is also part of the *audit trail* measure.

Additionally, I have also kept track of my reflections, methodological decisions, and logistics in physical notebooks/diaries, corresponding to a *reflexive journal*. When I have worked closely with collaborators outside my research group, as for Paper II and III, we have also kept track of and shared relevant information digitally.

5.4 Ethical considerations

Studies with human participants always involve ethical dilemmas of some kind, and it is important that the researchers are familiar with, and follow, codes of conduct relevant to their field (Robson, 2002). One such relevant source for my research is the guidelines put forth by the British Educational Research Association (BERA, 2018), where various responsibilities of the researcher are outlined. Note that while such guidelines can be useful, I value the development of my personal moral judgment and ethical decision-making skills—particularly due to the complexity and context-dependency of qualitative research. To me, this development is a continuous process consisting of conversations with colleagues, reading literature, and reflection about my own practices.

Besides my responsibilities to the participants, which I will describe in the subsequent sections, I also have to consider my responsibilities with respect to the research community, my peers, and the wider public. Considering ethical aspects of authorship, I adopt the recommendations from the International Committee of Medical Journal Editors (ICMJE) for determining authorship (ICMJE, 2025). I also make an effort to include a description of author contributions in all of my publications to increase transparency and counteract the ambiguity of author order (Helgesson & Eriksson, 2019). In terms of accessibility of my research, and in accordance with my university's regulations, I always publish my work with open access. Considering some general responsibilities to the community and my peers, I aim to contribute to the field with studies of quality and trustworthiness, and I try to adopt a critical *and* respectful stance toward others' (and my own) work. Finally, I attempt to spread my work and insights within and outside of my community as much as I can, for example through various conferences and teaching seminars at my department.

As a researcher, I have many ethical responsibilities to the participants in my studies. In the following sections, I summarize those related to recruitment, informed consent, ethical approval, risks and benefits for participants, and data management.

5.4.1 Recruitment, informed consent, and ethical approval

In the recruitment stage, it is important to consider the sampling of participants. Not only with respect to the research question, but also the representation of people in research. These two aspects can be difficult to balance. For the two phases of data collection for Paper I (reanalyzed in Paper II), my research question and practicalities led me to recruit students who were enrolled in a course on statistical mechanics that I had access to. I was not looking for any particular students within this group, so I recruited participants by presenting myself and the study during a lecture, asking for volunteers. As an incentive, participants were offered a cinema ticket as compensation. At this stage, I clarified that the study was not a part of the course, particularly that it is not an examination process, and that they have the right to withdraw from the study at any time. Students were given the chance to sign up as interested, with their name and email address, under the premise that this was only an indication of interest, not consent to participate. For Paper III, each co-author was responsible for recruiting and interviewing two experienced teachers from their own discipline. Since we were sampling more purposefully, for the methodological reasons described earlier in this chapter, this meant emailing suitable colleagues or acquaintances from our respective departments to inform about the study and ask if they were interested. We attempted to find volunteers from different divisions within each discipline, and we tried to represent gender as equally as possible.

In general, I have tried to adapt my data collections to the participants' schedules to make the process as convenient as possible for them. Prior to all the interviews I conducted, the participants were given an information sheet about the study and its purpose (Appendix A). This document states, for example, how their identity will be protected and that they can withdraw their consent at any time. I also encouraged the participants to ask questions about these matters before they signed the informed consent form (Appendix B).

None of my studies have involved collection of sensitive personal data, therefore, I have not been required to seek ethical approval from the Swedish Ethical Review Authority. The considerations I have taken with respect to collecting personal data within the European Union (EU), and at my institution, is described in Section 5.4.3.

5.4.2 Risks and benefits for the participants

My studies have not involved any risk of physical harm of the participants. Moreover, the sessions/interviews did not include any sensitive topics that might otherwise introduce more risks for the participants. Unintentionally collecting sensitive personal data was also unlikely in these contexts, but if it would happen those parts of the data would have been redacted. Participating did not introduce any external risks, such as affecting the students' grade in the related course, and their identities remain protected by several measures (described in the following section). Overall, my efforts mainly concern mitigating negative emotional consequences or risks for the participants during the data collection process. I also gave the participants an opportunity to discuss and review the tasks or questions with me at the end of the session.

There are many reasons for considering the social environment and dynamics between individuals in the data collection process. Both the trustworthiness of the study and the personal experience of the participants are influenced by this. It is, therefore, crucial to promote a setting where participants feel comfortable and safe to express their thoughts, to collect credible data and consider the participants well-being simultaneously. For these reasons, among others, I chose to collect data from groups of students—who already knew each other from other contexts—for Paper I. A successful group session with peers can create a safer environment for the students to express themselves, despite being recorded in an *in vitro* setting, and reduces the risk of the students feeling interrogated. My role was to observe as unobtrusively as possible, while also moderating the sessions. I intentionally positioned myself in the corner of the room to further encourage the students to focus on each other rather than me. In general, before recording, I tried to set a comfortable tone and minimize any hierarchical gap between us, for example by engaging casually with the participants and clarifying that my purpose is not to judge their performance. Many of the participants were also already familiar with me from my background as a student a few years ahead of them. Moreover, I have never had a teacher-student relationship with them, which otherwise could have increased the power imbalance between us. During the problem-solving sessions, I was also wary of the dynamic between the participants in the groups—to make sure that everyone was treated respectfully and had the opportunity to participate comfortably.

For Paper III, I conducted one-on-one semi-structured interviews with two senior physicists from my department. My role was naturally more interactive, but I made an effort to make the interview feel like a conversation among colleagues rather than an interrogation—for example by getting to know each other a bit before starting. I was not as concerned about potential power imbalances affecting the participants negatively in this case, since they were more experienced researchers and teachers than I. Nevertheless, to mitigate the participants' potential stress regarding their "performance" during the interview, I clarified that we were looking for variation among perspectives—not identifying their personal conceptions. I encouraged the participants to voice any idea they, or their colleagues, may have or had.

My studies have also led to potential benefits for the participants. In the case of the student groups, the problem-solving sessions also served as a collaborative learning opportunity, where they discussed concepts and problem solving relevant to the course they were enrolled in. This was even articulated by many of the students after the sessions. The students were also compensated with a cinema ticket, which was judged to be an appropriate compensation with respect to the participation—not insignificantly small, but also not so big it could be considered coaxing. For the data collection in Paper III, the participating teachers were given the opportunity to reflect thoroughly on a topic relevant to their discipline and their teaching, which many of them highlighted as both intellectually enjoyable and rewarding.

5.4.3 Data management and GDPR

In all my studies I have collected, processed, and stored personal data within the EU. I have, therefore, complied with the EU General Data Protection Regulation (GDPR). This includes ensuring that any collection or processing of personal data has a clear and specified purpose, that the participants have given their informed consent, and that the data is protected. My institution, Uppsala University, is ultimately responsible in this regard, which is why I have also followed the university's requirement to report projects in which personal data will be managed.

In the process of transcribing my collected raw data (video and audio recordings), I replaced the participants' names with pseudonyms. Whenever I have shared findings outside my research teams, such as during presentations or in publications, the participants have been referred to by these pseudonyms to protect their identity. When describing the context and participants, I have also made an effort to ensure that those descriptions are not so detailed that the participants could be identified anyway.

The raw data collected for Paper I and II are stored on external hard drives in a secure safe, and the consent forms kept in a locked space, which only authorized people in the research group have access to. The raw data collected for Paper III are stored on the platform *Allvis*—a secure platform for storing and sharing research data in collaborative projects, administered by the university. In accordance with the Swedish Archives Act about storing public records (which includes research data), the raw data will be retained for at least 10 years. This is also to ensure that the quality of the research can be assessed. Following the guidelines on the deletion of data, documents that are temporary or of little importance are deleted, and the raw data are thinned out on a regular basis after the required retention.

6. Findings and discussion

In this chapter, I provide an overview of the analyses, findings, and discussions of the three papers. Note that Paper I and II consider different perspectives on the same data sets, in addition to a similar data set from a chemistry context in Paper II. Paper III involves a separate approach and data set. General aspects of my methodology, including the research approach, context, data collection, and data analysis, are described in Chapter 5. Specific details can be found in Papers I-III. Each following section will, therefore, only briefly introduce the paper and the relevant methodological aspects, and provide a summary of the findings and certain points of the discussions. Finally, I discuss the limitations of my studies in Section 6.4. Note that in this thesis, the main reflection and discussion of my findings is the focus of the synthesis in Chapter 7.

6.1 Paper I

Exploring student reasoning in statistical mechanics: Identifying challenges in problem-solving groups

Published in *Physical Review Physics Education Research.*

Motivated by the need to expand research on upper-level courses in physics, this paper aimed to explore potential challenges encountered by groups of problem-solving students on the topic of statistical mechanics. In light of the complexity of the content and context, and relative novelty within PER, we adopted a grounded approach to qualitatively analyze video recordings of nine small student groups solving problems involving key concepts on the topic. The groups consisted of upper-level physics students at a Swedish university. We asked the following broad research question to guide the emergent study design and analysis.

RQ: What challenges do upper-division physics students face when solving statistical mechanics problems in groups?

6.1.1 Data analysis and findings

Details regarding the general methods of data analysis in my work can be found in Section 5.2, and the relevant physics content is summarized in Paper

I. Here I will briefly summarize the context, analysis, and findings of Paper I, in order to highlight some discussion points in the following section. Details regarding the analysis, findings, and discussion can be found in Paper I.

The aim of this exploratory study was to characterize challenging aspects for the students, with the overarching goal to find areas of student thinking and reasoning patterns that would be interesting to explore further. It was our intent to approach this study in a grounded way, allowing the data speak for itself as much as possible. By asking a very broad research question, we avoided asking unfruitful questions *a priori*. We adopted a flexible study design, and the analysis was developed by conceptualizing the data rather than imposing a predetermined theoretical framework on them[1]. The emerged findings could then inform more specific research questions that would be interesting to explore in future research, with a suitable theoretical perspective in mind. Against this methodological approach, we then designed problems on the topic of statistical mechanics and collected data by video and audio recording groups of students discussing and attempting to solve these problems together in a think-aloud manner. The groups consisted of two to three student peers, mainly in their third and final year of a bachelor's program in physics, and the problem-solving sessions lasted approximately 1.5 h. The student volunteers were recruited from a statistical mechanics course at a Swedish university in two phases, in two consecutive years, for a total of nine recorded group sessions.

Common student challenges emerged through our prolonged engagement and iterative analysis of the data, which we finally reduced to ten categories of challenges. We divided the categories into two broad groupings: seven related to *challenges with concepts* (A1-A7) and three related to *challenges with problem-solving strategies* (B1-B3). The final versions of the categories are presented in Table 6.1. We provide a full summary of the categories, with transcribed excerpts as examples to illustrate what led to each category, in Paper I. Zooming out to the broadest patterns, we noticed through our engagement with all the data that the students' ideas and reasoning patterns on this topic seemed to be context dependent. Individual students also shifted between ideas frequently. Overall, our findings indicate that the topic of statistical mechanics seemed to present various challenges for the students. These challenges involved a broad range of relevant terms and concepts, as well as their approaches to solving problems. The findings related to the former grouping illustrate challenges involving the following key concepts of statistical mechanics: (A1) *macrostates and microstates*, (A2) *entropy and temperature*, (A3) *distinguishable and indistinguishable*, (A4) *equilibrium*, (A5)

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[1] Despite our efforts to observe the data through an open and broad lens, our theoretical perspectives on both learning and the physics content (conscious and unconscious) inevitably influence the research process and outcome. I provide the relevant physics theory in Paper I and describe my theoretical perspectives in Chapter 3. Awareness of the prior studies in this area also influenced the design and possibly the analysis of our study, these are reviewed in Section 2.5.1.

energy, (A6) *temperature and heat bath*, and (A7) *the Boltzmann distribution and partition function*. The latter grouping is naturally broad, since there are many aspects related to approaching and solving problems. Paper I focused on three of the most recurring student challenges across our datasets, involving the application of typical mathematical relations, the management of conflicts between qualitative reasoning and mathematical results, and the definition of one's own concepts.

Table 6.1. *Overview of the ten emerged categories of student challenges from Paper I, divided into the groupings (A) challenges with concepts and (B) challenges with problem-solving strategies.*

Challenges with concepts	Challenges with problem-solving strategies
A1 Struggling to define and disentangle macrostates and microstates and attributing statistical weight inappropriately	**B1** Inappropriately applying typical mathematical relations from statistical mechanics and thermodynamics
A2 Treating macroscopic quantities like entropy and temperature as microscopic properties of a system's components	**B2** Having difficulties managing conflicts between quantitative results and qualitative reasoning
A3 Confusing distinguishable and indistinguishable and not recognizing the cause nor the consequences of that distinction	**B3** Defining new concepts that led to more confusion due to their inconsistency with established relations
A4 Associating an equilibrium state with increased order	
A5 Struggling to recognize the consequences and causes of a system having discrete energy states when the states are bounded or the total energy is kept fixed	
A6 Struggling to conceptualize temperature through the statistical definition and misinterpreting the meaning and effects of a heat bath	
A7 Reasoning inconsistently about the Boltzmann distribution and the partition function, and their applicability	

6.1.2 Discussion

In the following, I will focus the discussion on some general findings and discussion points regarding the overarching groupings of categories.

Firstly, I point out some underlying broad patterns in the first grouping of categories, *challenges with concepts*. Our data suggests that the students seldom had a definite perception of key concepts, and if they did, it was often incomplete or inappropriate in the considered context. Moreover, the students often articulated experiences of *intuition* during the reasoning process, seemingly connected to recognizable terms from prior physics courses and everyday experiences. The students frequently faced conflicts between their intuition and their loosely connected ideas about the key concepts in the course. Despite demonstrating metacognitive awareness about their intuitions, reconciling the conflicting ideas was very challenging. In one way or another, intuition seemed to guide the students significantly in their reasoning and problem-solving process. This observation led to the focused exploration in Paper II, considering students' use of, and response to, intuition. Another point to highlight is the potential connections between the challenges in categories A2, A4, A5, and A6. That is, through the subtle relationship between the central concepts of entropy, temperature, and energy through the statistical definition of temperature. It is also noteworthy that the students articulated their confusion with the many different and abstract concepts involved in the topic. Sometimes unsure of the terms themselves, but more often the conceptual meaning of the terms and how they are related. The students also emphasized that this was extra challenging due to the lack of focus on conceptual understanding in the course, which they felt was mainly focused on long derivations of mathematical expressions. These self-reported difficulties support our findings of observed challenges and show that the students were aware of them in many cases.

Secondly, keeping a zoomed-out perspective, I highlight some general trends in the second grouping of categories, *challenges with problem-solving strategies*. In general, our findings suggest that the students struggled to interpret the problem statements and recognize the relevant assumptions of the contexts. As self-reported by the students, they were unsure of in which situations they could apply certain methods and relations. One of the most evident patterns that emerged from the data was a tendency to apply a certain mathematical relation, or set of relations, in an inappropriate context. Sometimes by default or by making unjustified assumptions based on surface features of the problems. In many cases, the students referred to this recipe-like strategy as "the stat mech approach." All these difficulties were often magnified by the intrinsic mathematical demands of the exercises, as well as their abstractness. However, students were sometimes able to succeed in the mathematical calculations but nevertheless struggled to solve the problem due to inappropriate aspects of their qualitative reasoning. As mentioned in for the previous set of categories, the challenges in grouping B were often explicitly articulated by the students—which is interesting from a metacognitive perspective, and provides some support for our interpretation of the data. In Paper I we limited our analysis to challenges, but we also made important initial observations regarding productive problem-solving strategies that emerged from the data. For example, certain ways in which the student groups sometimes overcame a barrier or resolved a conflict, which I discuss in the synthesis of this thesis (Section 7.1) and intend to explore further in future work.

Overall, the challenges in the first and second grouping are likely coupled in various ways. General challenges in this course, and potentially in similar upper-level physics courses, seem to be connecting key concepts (both to other relevant concepts and to prior experiences), bridging between micro and macro perspectives, and discerning the underlying physics of highly idealized and abstract contexts. In light of the suggested context-dependency of student ideas, and how they shifted frequently, the Resources framework could be suitable as a theoretical supplement to further analyses. The students' reasoning processes could be considered more holistically using the concept of *framing*, including conceptual, metacognitive, and epistemological resources, to account for challenges from both groupings and potential progressions. A preliminary analysis of this kind is discussed in the synthesis of this thesis, in Section 7.1.

6.2 Paper II

Managing intuition in collaborative problem solving: a case study of beyond-intro chemistry and physics students

Published in *2024 Physics Education Research Conference Proceedings*.

This case study is part of a larger ongoing project exploring the elusive concept of *intuition* in problem-solving contexts, together with collaborators in CER. In our prior, separate work on collaborative problem solving in upper-level chemistry and physics, intuition emerged as a recurring pivotal aspect in guiding the students' problem-solving approaches. Informed by these prior findings, we employed a grounded approach aiming to explore how students use intuition when solving problems in groups. We adopted a dynamic, resources-perspective on intuition to inductively reanalyze our previously collected video data of problem-solving sessions. The data sets included student groups from beyond-intro level chemistry and physics courses, respectively, at a Swedish university. For the purpose of this short paper, and in line with our perspective on intuition, we narrowed our focus to the following guiding question.

RQ: How do beyond-intro level chemistry and physics students respond to their intuitions during collaborative problem solving?

6.2.1 Data analysis and findings

Details regarding the general methods of data analysis in my work can be found in Section 5.2, and our theoretical perspective on intuition is described in Section 3.2.2. The full analysis and presentation of the findings can be found in Paper II.

For this case study specifically, we reanalyzed data sets that we were already familiar with from our prior work. We approached the rich chemistry and physics datasets by first pinpointing instances where students explicitly used their intuition (such as articulating "my intuition is..." or "my gut-feeling tells me...") or seemingly implicitly relied on intuition (by phrases such as "I feel like..."). From the subsequent inductive analysis, the emerging trends led us to categorize the students' responses to their intuitions into *productive* or *unproductive*. In this work we considered a response to their intuition as productive mainly if it created a learning opportunity, but also if it led to task success. To illustrate our findings within the limitations of this short paper, we selected illustrative cases from both the chemistry and the physics context.

The findings of Paper II are summarized in Table 6.2, where the various cases productive and unproductive responses are described on a general level. A brief account of the problem-solving contexts, in which these cases occurred, can be found in Paper II with references to the precise descriptions in Paper I and Ye et al. (2024). Notably, our findings demonstrate that students' responses to their intuition can be productive or unproductive regardless of whether their intuition was appropriate or not in that context. In the following section, I discuss the findings of Paper II and highlight some of the arguments we put forth based on these findings. Additionally, I discuss some preliminary reflections on the consequences of ontological and epistemological perspectives on intuition held by students, educators, and educational researchers.

6.2.2 Discussion

The findings from Paper II demonstrate how students' responses to their intuitions could lead to either productive or unproductive outcomes regardless of the intuition's appropriateness. This indicates the value of helping students to develop strategies to manage their intuitions, rather than purely focusing on refining their intuitions about the disciplinary content[2]. A necessary first step to consciously respond to one's intuition is to be aware of it in the first place. This requires certain kinds of strategies, but also a belief that it is relevant to do so. However, as our findings demonstrated, awareness of one's intuition is not always enough to respond productively. The two physics cases provided examples where both student groups were aware of their inappropriate intuition, but only the group who questioned the intuition, and engaged in a

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[2] Exploring how this happens, looking into the processes involved in the development of students' intuitions, is another elusive and interesting topic for future research.

Table 6.2. *Summary of the empirical findings from Paper II: cases of productive and unproductive student responses to their intuition in problem-solving situations, from both the physics and chemistry groups.*

Productive response to intuition	Unproductive response to intuition
Physics case: Engaging in metacognitive reflection about an inappropriate intuition (regarding a physical phenomenon), turning the situation into a collaborative learning opportunity via analogical reasoning.	**Physics case:** Consciously relying on their inappropriate intuition without scrutinizing it (despite explicit discussions about it), modifying their initially correct calculations to align with the intuitive prediction.
Chemistry case: Engaging in a conceptual discussion in response to an appropriate intuition (regarding the complexity of their solution), leading them to successfully revise their approach and solve the task.	**Chemistry case:** Blindly following their inappropriate intuition regarding the approach, without explicitly recognizing the intuition or scrutinizing it, leading them down the wrong path (justified through confirmation bias).

conceptual reflection about it, was able to achieve a productive outcome. In this case it was productive in terms of apparent collaborative learning, task success, and the students' own satisfaction with the solution. These cases illustrate that awareness alone is not enough to respond productively to one's intuition, without strategies for how to critically reflect on it.

In light of the elusive but important topic of intuition in our disciplines, and in educational contexts more generally, we utilized these cases from our empirical findings to discuss and reflect on various implications related to intuition. One such important point of discussion, which we started to reflect on throughout the study, is the significance of the stance which someone adopts with respect to intuition. Namely, we believe that it is important to consider the various ontological and epistemological perspectives on intuition held by different actors within education. As mentioned above, someone's beliefs and values regarding intuition will influence their decisions about how they approach and respond to it. Existing literature has mainly focused on experts' perspectives in this regard, as described in Section 2.3, and research on students' views on intuition remains limited. This aspect was beyond the scope of this case study, but we highlighted the importance of future work in this direction since students' beliefs and attitudes regarding intuition likely play pivotal roles in shaping their progression toward more expert-like intuitions within a discipline. We also argued that students would benefit from a flexible stance toward intuition. Flexible in the sense that one neither always disregards nor always trusts it, but rather seeks to understand and leverage it in productive ways. Teachers may be tempted to leave implicit matters—such as intuition—implicit, and hope that students develop the necessary perspectives and strategies along the way, somehow. Based on our findings—observing students' frequent use of intuition with notably different outcomes—I believe that it might be even more important for teachers to be explicit about the implicit matters, even when it is as elusive as intuition. Teachers' own perspectives and choices in the classroom are influential to students' progression. I discuss some preliminary recommendations for teachers in Section 7.2.

Continuing the discussion of the importance of understanding different perspectives on intuition held by different actors in education, we also emphasized the consequences for educational researchers. As researchers exploring intuition and its role in a discipline, our views on intuition shape the questions we might pose, the methodologies we employ, and our analyses. For example, if researchers adopt a static stance on intuition—considering it as a trait exclusive to experts—it may lead to an underlying assumption that students' intuitions are flawed. An associated recommendation could then be that students should try to avoid their intuitions, or always discard them. We believe that such research can reinforce negatively biased conceptions about students' abilities, and lead to unproductive implications for teaching. If intuition is instead viewed as a universal, dynamic phenomenon, it allows us to ask pedagogically valuable questions—such as how intuition evolves from a novice to an expert stage. Research based on this perspective has the opportunity to promote more inclusive teaching, as mentioned above. Importantly, we can attribute the insights gained in this study to the grounded approach and our flexible, resources-perspective on intuition. If we would have approached the same data by merely categorizing the students' intuitions as disciplinary "correct" or "incorrect", we would probably never have recognized the significance of the students' strategies for managing their intuitions. Moreover, our dynamic stance toward intuition helped turn our attention to the question we posed in this work in the first place. Observing students' reliance on intuition in our previous work was something we deemed important, and not a phenomenon that should be avoided purely based on its elusive nature—perhaps it is even more important to study for this very reason. It is also worth highlighting that our context, students in more advanced level courses, granted us the opportunity to explore an intermediate stage of students' transition from novices to experts. Notably, our data suggests that the students possess intuitions about disciplinary content that may not stem purely from everyday experiences. We further observed that students, at times, leverage such intuitions through analogies between different areas within the discipline. This observation supports the stance that intuition is dynamic and can evolve over time, and it opens up for valuable future directions.

Paper II serves as a preliminary study and has laid the groundwork for future research exploring various dimensions of intuition and its role in problem solving. Based on the findings and the discussion above, I suggest some preliminary implications for teaching in Section 7.2. I also reflect on how intuition may be introduced in a synthesis model of my findings, based on the

Resources framework, in Chapter 7, and discuss potential future directions in Chapter 8.

6.3 Paper III (manuscript)

Teachers' conceptions of the role of mathematics in STEM higher education

This paper is a manuscript that has not yet been published.

The interplay between mathematics and other STEM disciplines is an important topic within educational research. While the role of mathematics in other STEM disciplines has been considered from a theoretical perspective, or students' conceptions, less is known about how teachers conceptualize its role. In this paper, we aimed to fill this gap by employing a phenomenographic approach to empirically investigate variations among STEM university teachers' conceptions of the role of mathematics in their discipline. We performed a phenomenographic analysis of 16 semi-structured interviews with STEM teachers at a Swedish university, guided by the following research question, aiming to construct an outcome space of qualitatively different categories of conceptions.

RQ: What are university teachers' conceptions of the role of mathematics within the knowledge structure of other STEM disciplines?

6.3.1 Data analysis and findings

Paper III is framed by a phenomenological methodology, including both the theoretical underpinnings and commonly associated methods for data collection and specific process of data analysis. This is described in Sections 5.1.2 and 5.2, respectively. The full analysis and presentation of the findings can be found in Paper III.

In line with the phenomenographic tradition, our study focused on a central phenomenon: the role of mathematics in STEM higher education from the teachers' point of view. Moreover, we aimed to identify conceptions among teachers from these disciplines as a group, not differences between disciplines or individuals. To this end, we first conducted a pilot study aiming to construct the interview protocol for the semi-structured interviews and gauge whether the STEM teachers could be considered as group in this context (i.e., whether the views across disciplines seemed consistent enough). Informed by our pilot study, we then collected audio data for the full phenomenographic study through semi-structured interviews with 16 teachers from chemistry, computer science, geoscience, and physics at a Swedish university. We approached the iterative data-analysis process according to the phenomenographic methodology, this is described in Section 5.1.2. Several rounds of analysis and group discussions resulted in a final set of categories of qualitatively different conceptions of the phenomenon, including their descriptions and the relationships between them. We also reached consensus regarding suitable representative quotes for each category to illustrate the findings in an outcome space.

Our phenomenographic analysis resulted in an outcome space of five categories of qualitatively different ways of conceptualizing the role of mathematics in STEM disciplines, from university teachers' perspectives. The outcome space is hierarchically inclusive, meaning that the categories are increasingly broad since each category includes the attributes of all previous ones, in addition to its own. It is worth to point out, once more, that the outcome space describes conceptions of the phenomena on a collective level, not individuals' conceptions. To provide an overview of our findings, I list the five emerged categories below, with the names and descriptions we formulated to represent them.

1. **A computational tool**
   The role of mathematics in the discipline is conceptualized as a tool for calculations.
2. **A language for communication**
   In addition, the role of mathematics in the discipline is conceptualized as a language to describe, formulate and explain.
3. **A key to understanding**
   In addition, the role of mathematics in the discipline is conceptualized as something fundamental to understanding the discipline. In this sense, mathematics is a key to reasoning, finding relationships, drawing conclusions, providing structure and modeling.
4. **An agent for development**
   In addition, the role of mathematics in the discipline is conceptualized as a vehicle to develop the discipline, formulate new questions and drive research forward.
5. **A philosophical foundation**
   In addition, the role of mathematics in the discipline is conceptualized as an inherent part of the knowledge structure. In this sense, mathematics is experienced as a philosophical foundation since it is not simply utilized for a purpose within the discipline, but it is how the discipline functions.

More detailed descriptions of each category, along with representative quotes from our data, can be found in Paper III. The outcome space and its hierarchically inclusive nature is illustrated in Figure 6.1. In addition to breadth, we also argued that the categories describe increasingly nuanced and complex conceptions (what we referred to as increasingly sophisticated) of the role which mathematics plays in the discipline. The first category starts from a general instrumental role, then the role of mathematics becomes decreasingly instrumental and increasingly integral to the other discipline, to finally being considered as intertwined with the discipline at its core in the fifth category. In this way, the latter categories provide more potential to capture nuances in the role of mathematics and how these various roles are connected. Some points of discussion regarding these findings are highlighted in the following section.

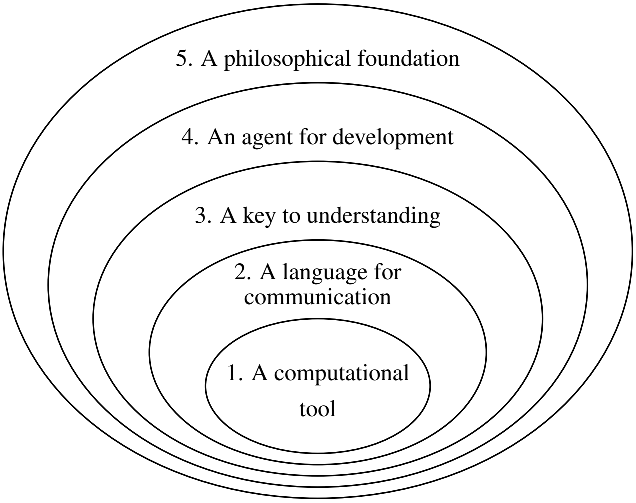

Figure 6.1. A visual representation of the emerged outcome space from the phenomenographical analysis in Paper III, including the five categories of university teachers' conceptions of the role of mathematics in their STEM discipline. The figure illustrates the hierarchical relationships between the categories of conceptions.

6.3.2 Discussion

The role of mathematics in other STEM disciplines is a widely recognized and explored topic, including studies that have taken historical, philosophical, and educational perspectives. Overviews of this relevant body of research can be found in Section 2.4 and in Paper III—which also relates our emerged categories to existing research. However, scholars may provide these various descriptions of the roles which mathematics assumes in other disciplines, but how do university teachers in STEM actually view its role in their discipline? This is the gap which the empirical findings in Paper III contribute to.

Importantly, prior empirical work considering students' views of the role of mathematics in other STEM disciplines suggest a mismatch between students' conceptions and the teachers' conceptions presented here. For example, de Ataíde and Greca (2013) studied the views of undergraduate students in their final year of a high school physics teacher programme, in the topic of thermodynamics, and found views consistent with our first three categories—but often limited to the first category. Importantly, they observed relations between the students' problem-solving proficiency and the epistemic view of the role of mathematics they held, highlighting that a more developed epistemic view is important for successful learning in physics. Therefore, efforts to better help students develop beyond the first categories of conceptions should be pursued. Expanding students' view of mathematics beyond simple computation to these broader roles could enhance STEM education and increase interest in STEM research. As such, teachers need to be aware of most of the categories, and that it is meaningful to communicate the role of mathematics to the students, as echoed by some of our participants. By employing a phenomenographic approach in this study, we could capture nuance and variations among the teachers' conceptions, including structural relationships between the conceptions, to increase the awareness of the diversity of perspectives on this important topic. A mismatch between students' and teachers' views could suggest that teachers' views are not effectively implemented or reflected in their practice. The role of teacher-reflected epistemologies are further discussed with respect to the mathematics-intense course of statistical mechanics, and similar upper-level physics courses, in the synthesis and implications of this thesis (Chapter 7). Future work could provide concrete recommendations for *how* teachers could apply the findings presented here in their practice, in order to help students advance their conceptions of the role of mathematics in their STEM discipline. For instance, suggesting suitable contrasting contexts that could be utilized to allow students to experience variation, in order to discern new relevant aspects of the role of mathematics—i.e., to learn, in the variation theory of learning sense (e.g., Marton & Booth, 1997).

The fifth category, a philosophical foundation, contains some subtle and important points worth discussing. This conception adds a perspective on the deep relationship between mathematics and the discipline, touching upon the domain of philosophy of science by speculating about the nature of mathematics and the discipline. As highlighted by one interviewee, if this philosophical position is taken too far—i.e., viewing mathematics as synonymous with the discipline—it can be problematic since it can lead to deviation from the principles of natural sciences. In short, certain philosophical positions regarding the role of mathematics in another STEM discipline could be problematic if it makes one loose touch with the physical world and the inevitable limitations of mathematical modeling. The following quote comes from our findings, where one of the interviewed physics teachers highlights this issue:

Ph3: I think there is a risk there that you get misled a little bit. That you imagine that the universe must align itself according to certain principles. That you get the idea that since mathematics is often associated with some form of beauty and elegance and such things, then you have to imagine that the universe must respect the same principle of simplicity in some way [...] but I think that essentially, it's about a limited point of view, which means that you can miss certain

> important aspects of the world. That you choose to see the world in a certain way, which rather reflects one's own prejudices than something real out there.

Moreover, the interviewee commented that such views are not rare among their colleagues or famous spokespersons for science, and cautioned against it: "I think it can even be dangerous to believe in the independence of mathematics, it's not as innocently philosophical as you might think". In several ways, this category provides an important point of view to consider, since one's philosophical position in this matter could deeply affect the way in which research and teaching activities are carried out. Overall, unpacking the subtleties of the intricate relationship between mathematics and the other STEM discipline, corresponding to the higher categories of conceptions here, might be even more important for students in upper-level courses relying heavily on mathematics. In this thesis, my synthesis (Section 7.1) touches upon these issues in the context of statistical mechanics, where mathematics and physics are noticeably intertwined. Future research might explore more systematically how philosophical positions about mathematics are held, communicated, or acted upon in STEM higher education.

6.4 Limitations

As a consequence of engaging in case-oriented research—as theoretically and methodologically outlined in Chapters 3 and 5—the findings from Paper I-III cannot be considered as complete categorizations of the studied phenomena and are not meant to be generalizable to their respective populations. Instead, the rich, detailed data serve as cases that contribute to theory. The findings must be considered in relation to the specific contexts, which is why I have made an effort to provide as much details about the contexts as possible—to enable others to judge if and how the findings can be transferred to another context. It should also be noted that the findings are based on the researchers' interpretations of the participants' expressed thoughts and behaviors, which inevitably is influenced by the researchers' own ideas and beliefs. The interpretations do not necessarily reflect the whole picture or—in the case of groups—their individual perspectives. All these issues were discussed with respect to my efforts to establish trustworthiness in Section 5.3. While details regarding the limitations of each study can be found in the respective paper, I elaborate on some of these points here.

For the data collection in Paper I, it should be noted that there is a selection bias due to volunteering students—which could impact the scope and nature of the identified challenges. Moreover, while all student groups had the chance to attend the lectures covering the necessary content for our designed problems, the problem-solving sessions were spread out during the course, more so in the first phase. This is another potential bias, due to the unequal amount of time the participants had to process the course content before the sessions. It is also important to acknowledge that performing a large-grained analysis, such as in Paper I and II, implies that nuances and details in the data can be overlooked. However, my focus has been on common themes and trends so far, and the findings presented in this thesis provide insights and suggestions for further studies where such trends can be investigated deeper.

In Paper II, our identification of episodes where students used intuition in their problem solving has potential limitations. In the cases of students' explicit articulation (e.g., "my intuition tells me"), we have not considered the student's own views of what intuition means to them. This is however part of our ongoing work, which is briefly described in Chapter 8. In the cases of implicit use—such as when students relied on a feeling which they could, or would, not justify—the findings may be more sensitive to the researchers' interpretations. Nevertheless, we believe that observing students' naturally emergent use of intuition in an authentic problem-solving situation was valuable for our purposes. Similarly, in Paper III we did not consider the participating teachers' views about what "counts" as mathematics or not—a non-trivial question, which could impact their stance on the role of mathematics in their discipline. As such, in addition to the other suggestions for future work, teachers' conceptions of mathematics itself could be interesting to explore.

7. Synthesis and implications

In this chapter, I first provide a preliminary and reflective analysis of the findings from Paper I-III as a whole. This serves as the synthesis of my findings (Section 7.1) shaped by the guiding questions of thesis, with the purpose of drawing connections between the papers. In Section 7.2, I discuss implications for teaching based on the findings from my studies and the discussion in the preceding synthesis.

7.1 Synthesis of findings

At this stage, it might be helpful to recall the overarching aim of my research. That is, to explore how challenges and progressions related to conceptual reasoning, intuition, and mathematics emerge in student problem-solving groups in the context of statistical mechanics—as a case of upper-level university physics. In this section, I will reflect on all the findings and insights I gained throughout the studies in Paper I-III. The following *guiding questions* helped shape this synthesis:

- What insights can be gained from analyzing the students' challenges and progressions through the lens of conceptual, epistemological, and metacognitive resources?

- How may teacher-reflected epistemologies about conceptual reasoning, intuition, and mathematics influence students in these aspects?

In this preliminary and reflective analysis, I adopted the Resources framework (see Section 3.2) as a theoretical supplement. The following model serves as a guide to help think about the big picture of my combined findings, and possible connections between them. As such, this section includes some speculations—which can suggest ideas for future directions (Chapter 8).

Overall, my findings have illustrated student challenges related to increasingly complex and intricate physics *concepts*, increasingly integrated *mathematics*, and potentially increasingly difficult to develop *intuitions*, as well as a lack of strategies with which students would manage their intuitions. As highlighted by Singh and Marshman (2015) in their review of student difficulties in upper-level quantum mechanics, learning upper-level physics is challenging because one must continue to build on all the prior knowledge learned at the introductory and intermediate levels, in addition to the increasing mathematical demands. However, they also point out that little is known about how students develop during their transition from introductory to intermediate to advanced physics courses.

In this synthesis chapter, I present a graphical depiction of a model (see Figure 7.1) based on the Resources framework. The model describes student reasoning in an upper-level physics course, such as statistical mechanics, through the process of *framing*, involving activations of *conceptual*, *metacognitive*, and *epistemological resources*.

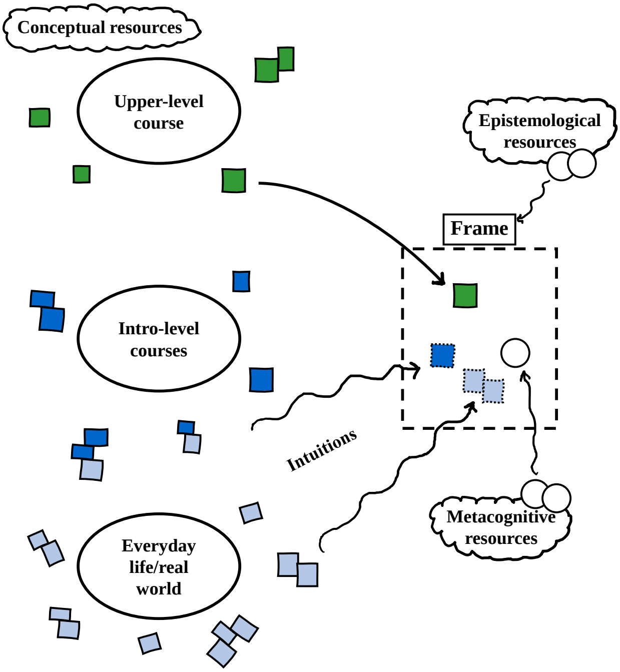

Figure 7.1. A graphical depiction of a model based on the Resources framework, and the findings from my studies, to describe student reasoning in an upper-level physics course through the process of *framing*. The different parts and functions of the model are described in the main text.

Below, I describe the various parts of this model and why they are relevant to describe such contexts, based on the findings from my studies. First, I describe how the conceptual resources are represented in the model. This representation provides an explicit consideration of student ideas associated with intro-level, classical physics courses as an intermediate step between ideas associated with everyday experiences (and real-world applications) and those associated with the upper-level physics course. Then, I provide a description of how the model represents the process of framing, including how intuition is interpreted within this framework. The model depicts student intuitions as also being based on prior experience in earlier physics courses, not only from everyday experiences as is often the focus in constructivism-inspired research.

Finally, I also comment on the challenges of appropriate resource activations in increasingly abstract and mathematics-heavy contexts, which upper-level physics courses tend to be.

Representing the conceptual resources. Many concepts in upper-level courses, like statistical mechanics, have evolved and been generalized since their introduction in earlier physics courses. As such, those concepts have to be reconciled not only with students' everyday experience, but also with ideas developed in various prior physics courses. For this reason, the model contains three areas, representing three different domains of conceptual resources *within* the same discipline (see the rectangular shapes with different colors in Figure 7.1). My findings suggest it is challenging for students to connect their everyday ideas and experiences with both their ideas about concepts from prior physics instruction and with the more abstract, generalized versions of the same concepts in the more advanced context of an upper-level physics course. A concept such as temperature, for example, is prevalent in everyday life, in intro-level courses like classical thermodynamics, and in statistical mechanics, but with seemingly different meaning or purpose attributed to it. That is, the coherent development of the concept between these steps is not obvious to most students. Moreover, existing research on intro-level thermodynamics content suggests that connecting between the first two steps is already challenging for students in itself. Not surprisingly, those challenges carry on with the students to the upper-level courses (e.g., Brundage et al., 2024; Leinonen et al., 2015), where students are also faced with additional challenges of bridging between macroscopic and microscopic perspectives. For all these reasons, my model depicts the conceptual resources from the advanced domain as separated from the other two domains. Based on my own and others' research findings, the students' weak connections between key concepts *within* statistical mechanics is represented in the model by disconnected resources *within* that domain.

My model also accounts for the experience of intuition, which emerged as a prevalent and important aspect of the students' reasoning processes in my findings—as often articulated by the students themselves. Here, intuition is described as an experience emerging from more tacit, but inevitable patterns of association when interpreting a situation—based on any part or parts of one's current knowledge. As such, even if the students are not consciously drawing on their prior ideas of physics concepts in the context of an upper-level course, their thinking is influenced by them via intuitions. The students' simultaneous activation of several ideas can lead to conflicts between advanced concepts and one's intuitions. In the following, I describe how intuition, along with other subprocesses involved in framing, are represented in my model.

Representing the process of framing. The model as a whole illustrates the parts and functions involved in the process of framing a situation (here reasoning in a problem-solving context). Following the Resources framework, framing refers to the forming and use of a set of resources, including activated epistemological resources for understanding what sorts of knowledge are relevant, metacognitive resources for forming and manipulating those kinds of knowledge, and conceptual resources for understanding the concepts deemed relevant. In this sense, the epistemological and metacognitive resources can be seen as more procedural in nature than the conceptual resources. The activated epistemological resources determine the nature of the *frame*, meaning a set of expectations about the situation which affects what is noticed and acted upon. The frame is represented by the dashed rectangle in the model, which implicitly represents the epistemological resources that shaped it. If a particular epistemological resource is activated during the reasoning process, and is worth highlighting explicitly, this is represented as circles within the frame (with a symbol to characterize it). The intentional, conscious activations of conceptual resources are represented by straight-lined arrows, which draws a resource (or an already constructed unit of several resources) from one of the domains into the frame. The more unconscious activation of conceptual resources is represented by wavy arrows that "leak" resources into the frame, indicating the inevitable tacit interpretation which cues resource activations. I use this to represent the emergent experience of *intuition*, as a kind of subprocess of the framing. These "passively" activated resources are depicted as more transparent than the consciously activated ones, since it is not necessarily possible to trace one's emergent experience of intuition back to specific resources. Activated metacognitive resources are represented as circles within the frame—similarly to explicitly highlighted epistemological resources—with a symbol to represent its function in the moment, a formation or manipulation of other resources within the frame.

Comments about the challenges of appropriate resource activations in increasingly abstract and mathematical contexts. Mathematics plays an explicitly integral role in topics like statistical mechanics, but the intricate nature of the interplay between physics and mathematics seems to be challenging for students to recognize. When the line between physics and mathematics is more blurred, there are potential risks of losing touch with the physics concepts and real-world connections. Students are faced with abstract, idealized contexts where the physics concepts are more implicit (e.g., crucial in the underlying assumptions), in which a productive *framing* of the situation becomes very challenging. Moreover, my findings suggested that many students relied on a very limited frame when solving the problems—based on a belief about what reasoning and problem-solving in this particular course looks like—which led to inappropriate activations of resources and challenges to manage internal conflicts of course-specific concepts with their intuitions. To illustrate the impact of students' framing, I provide two contrasting cases based on my findings in the following section.

7.1.1 Two illustrative cases of distinct frames

Below, I give two examples of how the framing of an activity impacts its outcome. In both cases, the starting point of the students' conceptual resources and their organization can be considered roughly the same. This interpretation is supported by the common student challenges identified in Paper I, which indicated that students' ideas about key concepts in statistical mechanics were vague or haphazardly constructed. My findings suggested that these conceptual knowledge organizations tend to be incoherent in two aspects: weak connections between key concepts within statistical mechanics; and weak connections between the same concept as it has evolved from everyday experiences, through intro-level courses, to statistical mechanics. However, in several cases from my data I observed how differences in the students' framing of a situation seemed to determine whether they progressed or not. Therefore, I contrast a case of a limited frame to that of an extended frame, based on the model described above and insights from my findings. The two different frames are illustrated in Figure 7.2, and described one after the other in the following text.

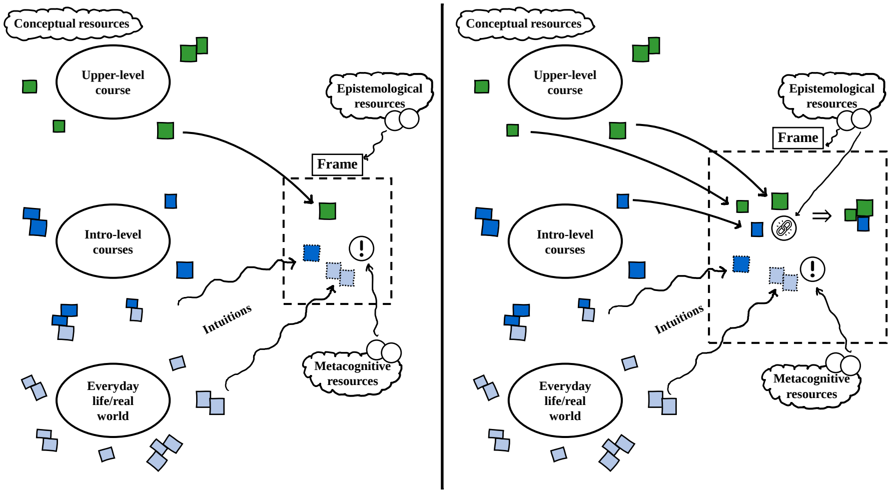

Figure 7.2. Based on the model in Figure 7.1, this image provides contrasting cases of the process of framing for two distinct frames: the limited "stat mech approach" frame (on the left); and the extended "cross-domain reasoning" frame (on the right). These cases are described in the main text.

Limited frame: the "stat mech approach"

My findings indicated a recurring limited frame, formed by a frequently activated epistemological resource: only a limited part of their knowledge associated to statistical mechanics is relevant. Several students seemed to have developed a particular belief about reasoning and solving problems in the topic of statistical mechanics. Namely, a belief that there is a standard procedure—

which the students' referred to as the "stat mech approach"—that can be used in all situations within the topic, and encompasses all the knowledge they need[1]. This can also be interpreted as a kind of "course-general" *epistemic game* (see Section 3.2). The limited frame was warranted by the students based on their perception that this is what they always do within the course, not recognizing that the approach actually requires certain conditions and assumptions for it to be appropriate. In this kind of framing, the consciously activated conceptual resources would often be based on surface features of the task at hand. (Example utterances from students in Paper I include: "*Should we take it in a 'stat mech' approach? Because they are talking about lattices, epsilons?*" and "*we assume it's Boltzmann distributed, we always do that in stat mech.*") Due to the incoherent organization of the conceptual resources related to the topic, the activated conceptual resources are represented as entering the frame in smaller fragments—without a clear connection beforehand. However, the more unconscious activation of conceptual resources—also highly based on strong associations to surface features—lets resources from everyday life or intro-level courses *leak* into the frame. This emergent subprocess of the framing could be described as experiencing an *intuition*. It is noteworthy that many students were aware of this, in a sense activating a metacognitive resource of recognizing and articulating the experienced intuition. However, within this limited frame and without further epistemological and metacognitive resources, the students struggled to connect and manipulate the set of activated resources. Despite being aware of conflicts between the activated conceptual resources and their intuition, these could not be reconciled. Epistemologically, this frame does not consider the conscious activation of resources from other domains, or strategies to connect between them in the reasoning process. Instead, students are left with limited options to attempt a resolution of such conflicts, such as trusting their intuition blindly and relying on confirmation bias: manipulating their calculations to align with their intuition (see the physics case of an unproductive response to an inappropriate intuition demonstrated in Paper II[2]). Resolving these conflicts, I argue, requires productive strategies of responding to one's intuitions—as the next case will illustrate. Moreover, within this limited frame, mathematics is used much like a "black-box" tool for calculations in a superficial way. Considering the role of mathematics in physics, this corresponds to the lowest category of conceptions found in Paper III.

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[1] This bears resemblance to the epistemological framing *disciplinary siloing*—e.g., "Since this is a physics course, I don't have to bring in any knowledge from chemistry." (Redish, 2014)—but here limited to a certain part of a domain *within* the discipline. As such, this limited framing could be thought of as a *domain siloing*, or even *subdomain siloing*, if the domain in this case corresponds to the topic of statistical mechanics.

[2] This refers to the physics case of an unproductive response to intuition, with the student group of Fran, Fiona, and Filip. This example is most conveniently read in Paper II (p. 221), as this particular episode has been isolated and described with sufficient context there.

Extended frame: the cross-domain reasoning

In contrast to the previous case, this example illustrates that when other epistemological resources are activated in the framing process, the frame can be extended such that cross-domain reasoning occurs. One such resource could be the belief that their knowledge from prior physics courses, and everyday experiences, is relevant and useful for reasoning in statistical mechanics. Or, more generally, in the sense that there should be a coherence between concepts and phenomena in physics. Moreover, in this kind of extended frame, there is not a belief that a small set of resources constitute all of statistical mechanics—even though the connections between concepts are still weak, and often recognized as such by the students themselves. We can consider a case which is initially the same as the previous one, i.e., the activated conceptual resources might be based on surface features of the task and intuitions allow other resources to leak into the frame. As before, students' activated metacognitive resource allows them to recognize the intuition and that there is a conflict with the consciously activated statistical mechanics resources. The crucial difference is that, epistemologically, the students believe that it is relevant and valuable to attempt to build a connection between conceptual resources from different domains to reconcile their conflicting ideas. In this case, their frame is extended to explicitly consider the problem of the conflict, and engage in conceptual reasoning with the aim to understand the inconsistency. That is, they had the *epistemic agency* to extend their frame spontaneously. For example, they intentionally activate conceptual resources from prior physics courses, in combination with an epistemological resource of analogical reasoning[3], in order to build a bridge between the resources from the advanced context and the introductory/classical context. The analogical reasoning becomes a valuable resource for building connections between conceptual resources, and can be the key to overcoming the conflict by making sense of the inconsistency. One example of this was the case of a productive response to an inappropriate intuition in Paper II[4]. This particular example demonstrates that some of the students had the epistemic agency to extend their frame in this way—to leverage it as an collaborative opportunity to extend and reorganize their knowledge structure—even though they did not encounter a mistake in their current problem-solving context[5]. In terms of connecting to everyday experiences, analogical reasoning was also used to draw connections between the advanced

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[3] Analogical reasoning can be seen as a process combined by an epistemological resource—in the sense that it is queued by a belief that analogies are relevant for knowledge building in the situation—and a metacognitive resource for forming and manipulating conceptual resources in a particular, coherent way.

[4] This refers to the physics case of a productive response to intuition, with the student group of Alex, Adam, and Alice. This example is most conveniently read in Paper II (p. 220), as this particular episode has been isolated and described with sufficient context there.

[5] This kind of proficient response more rare than the limited framing. As a finding highlighted in a review of studies on upper-level quantum mechanics suggests: "many advanced students

physics concepts and the mathematics concepts via tangible examples—such as drawing connections between microstates and macrostates via an example of rolling dice.

7.1.2 Role of teacher-reflected epistemologies

Fiona: I feel like [the problem-solving session] shows a bit of the problem I have with statistical mechanics [...] I always have a problem of knowing why to use which formula, because we derive them and I understand the derivation of it [...] But it's never explained, or I never get why we apply which model for which situation, and what is the difference between the situations. [...] it's more focused on the derivation of the math, not on the concept behind. So it's not talked in the way of "this is the concept, and therefore we do this" but "we do this", and initially we might say we apply this concept.

Filip: Yeah, and the math is also quite hard sometimes [laughs].

As the above excerpt from my data[6] exemplifies, many of the students' self-reported difficulties within the topic of statistical mechanics were attributed to issues related to functionality of their knowledge and the interplay between conceptual physics reasoning and mathematics. Importantly, this particular student group frequently employed the limited "stat mech approach" frame, described in the previous section. It is important to consider how the students' framing might be influenced by teacher-reflected epistemologies, whether those are implicit or explicit in the teaching (e.g., Linder, 1992). For example, if the teaching activities is perceived by the students as mostly focusing on mathematical operations rather than the physics concepts behind it, this may lead students to believe that conceptual reasoning will not help them in problem-solving situations—and/or will not help guide students in developing the resources necessary to reason productively. This can be reflected by the presentation of content by the teacher—such as what is made explicit and what connections are drawn—but also in the choice of learning activities. Many teachers in upper-level physics courses seem to continue using traditional lecturing (e.g., Loverude & Ambrose, 2015), which could augment the message that the students do not need to practice reflecting and reasoning. Similar concerns were put forth by B. W. Dreyfus et al. (2019) in the context of classical versus upper-level quantum physics: teachers' views and instructional choices may influence students to believe that quantum mechanics is a place to rely solely on mathematical calculation, which may impede productive activations of resources they bear with them from classical physics.

Moreover, in the highly idealized and mathematical contexts often utilized in the teaching of upper-level topics, the physics may be "hidden" to students

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may not automatically exploit their mistakes as an opportunity for repairing, extending, and organizing their knowledge structure." (Singh & Marshman, 2015, p.19).

[6] A part of this excerpt was featured in Paper I (p. 16).

in implicit underlying assumptions. If these are not focused on and made explicit in the teaching, including the connection to real-world applications, it may contribute to student epistemologies such as "*what we are doing here is not real physics*"—which was also articulated by students in my data. Relating this to the role of mathematics in physics, as was discussed for the fifth category of teacher conceptions in Paper III: there is a risk of losing connection to the physical world if the integration of mathematics in physics is taken too far. Even if teachers do not conceptualize the role of mathematics in physics in this way, it is important to consider how such epistemologies may still be reflected through their choices in teaching. Again, the teaching choices in these topics may further insinuate that the students should and can succeed in the course by solely relying on mathematical proficiency. If the assessment aligns with this idea, the students may pass the course despite not having developed a functional understanding of the key physics concepts. This has also been discussed in the context of quantum physics:

> In fact, most physics faculty, who teach both introductory and advanced courses, agree that the gap between conceptual and quantitative learning gets wider in a traditional physics course from the introductory to advanced level. Therefore, students in a traditionally taught and assessed quantum mechanics course can hide their lack of conceptual knowledge behind their mathematical skills even better than students in introductory physics. (Singh & Marshman, 2015, p. 21)

Additionally, as illustrated by the opening quote to this section, the mathematics in upper-level courses can also be challenging for the students in and of itself. My findings suggest that the students' mathematical conceptual resources are not necessarily well-connected to the physics resources either (e.g., students were often not activating their mathematical knowledge about probability distributions in general when reasoning about the Boltzmann distribution). A potential difference between learning within various upper-level physics courses, such as quantum physics and statistical mechanics, could be that statistical mechanics has many concepts that are recognizable—but has since evolved—from both everyday experience and/or intro-level courses (e.g., temperature, energy, and entropy). My findings suggest that the students may not be able to "hide" as easily behind mathematical proficiency due to experienced strong intuitions. It may, therefore, be even more important to consider teacher-reflected epistemologies regarding intuition explicitly, as was argued for in more detail in Paper II.

In the next section, I continue this discussion with a more practice-oriented reflection, suggesting implications for teaching based on my findings and this synthesis.

7.2 Implications for teaching

This section summarizes some recommendations for teachers based on the findings from my work. Overall, in line with the research paradigm I adhere to (see Section 3.1.2), the separate findings from Paper I-III all contribute to raising awareness about each of the studied phenomena through context-rich cases[7]. Readers should, however, keep in mind that my studies have not investigated any educational interventions specifically. Therefore, all these implications for teaching serve as recommendations based on the findings—along with my reflections in the synthesis of this thesis—and should be considered in relation to the knowledge and context of each teacher reading this. These recommendations can also suggest directions for future research, which is necessary to propose more concrete advice for teachers.

The specific suggestions from each study can be found in its corresponding paper. Instead, the following focuses on the implications for teaching based on the synthesis of all my findings in the previous section—which naturally overlaps with the paper-specific recommendations. The overarching goal of the first two points below are echoed by the general goals of physics education as proposed by Redish (2003): *concepts* should be rooted in the physical world; knowledge should be linked into *coherent* physical models; and knowledge should be *functional*—i.e., students should learn how and when to use it. My findings further support that we should not assume that upper-level students can efficiently tackle these challenges without purposeful guidance, particularly considering the various additional challenges associated with the more advanced physics courses.

Teachers should guide students to build a more coherent and functional knowledge structure: increase the focus on conceptual reasoning and bridge *within* and *between* domains. Building a coherent knowledge structure becomes increasingly challenging as students advance to upper-level courses: e.g., more connections between knowledge elements are required and various ideas need to be reconciled. To guide students in this, it is important for teachers to spend time on and emphasize the importance of conceptual reasoning. In line with the ideas of cognitive apprenticeship (Collins et al., 1989), teachers could model their thinking in class by, for instance, demonstrating how connections between concepts can be made explicitly. Particularly, teachers could build bridges *between* different domains of physics. That is, they could showcase how concepts evolve from everyday contexts to prior physics courses to the upper-level course. Analogical reasoning is one way in which teachers could build such connections, where the intermediate domains could serve as the bridge for *bridging analogies* (e.g., Clement, 1993). Teachers should also guide students to draw connections between concepts

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[7] Paper I is mainly relevant to teachers in statistical mechanics courses, or similar upper-level physics courses. Paper II and III makes points that are less content specific and is, therefore, relevant to a broader range of teachers (mainly within STEM higher education).

within the domain, such as unpacking the subtle relationship between entropy, temperature, and energy *within* statistical mechanics. Moreover, other kinds of analogies can be leveraged by teachers to model the connection between physics concepts and mathematics concepts, such as bridging via an everyday experience (e.g., connecting the concepts of microstates and macrostates to mathematical resources related to probability via tangible examples of rolling dice).

Since students need to construct these connections themselves, teachers should also create opportunities for students to engage in conceptual reasoning. For example, through various collaborative learning activities—or peer learning based models such as Supplemental Instruction—with the purpose of building these bridges. Collaborative discussions and negotiations between members of a well-functioning group[8] is, as we know, a valuable approach to facilitate learning. I will return to this point in the following recommendations.

Teachers should guide students to build a more coherent and functional knowledge structure: promote students' epistemic agency. As highlighted in the suggestions above, it is important that students also learn how and when to use what they learn, i.e., develop a functional knowledge structure. In this regard, students need the necessary strategies to draw connections, frame situations appropriately, and navigate challenges—also in the absence of a teacher. This is not a simple task either. Again, in line with cognitive apprenticeship (Collins et al., 1989), teachers can model their strategies by emphasizing important aspects of their *framing*—such as choices related to metacognitive and epistemological resources—in their reasoning and problem-solving process. For instance, teachers could model the *epistemic agency* to extend one's frame when faced with a conflict or inconsistency. That is, shift the attention to the problem of the inconsistency, rather than the task at hand, and turn the situation into a learning opportunity. This kind of shift can be achieved, for example, by extending the frame and engaging in cross-domain reasoning, as illustrated in the previous section. When highly idealized and abstract contexts are utilized in the teaching, it may be particularly important for teachers to draw attention to the purpose of using such examples, make the implicit underlying concepts and assumptions explicit, and consider relevant real-world applications for credible context and connection to the physical world. That is, highlight the *physics* and the modeling aspects of the context[9]. The aim is to encourage students to strive for coherence between concepts and realize the value of engaging in conceptual reasoning in these mathematics-intense topics, instead of "shutting up and calculating". Similarly to the previous point,

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[8] It is an important but challenging task to facilitate fruitful dynamics in group work, as was discussed in Section 5.4. There is a large body of literature on active learning with recommendations in this regard, for a practical summary I suggest Chasteen (2017).

[9] Inspiration could be drawn from Modeling Instruction (e.g., Brewe, 2008) and adapted to upper-level physics contexts.

teachers should create opportunities for students to actively engage in activities that allow them to practice and develop such beliefs and strategies.

Teachers should consider how their own views are reflected in teaching, intentional or not, about intuition and the interplay between mathematics and physics. In line with my discussion in Section 7.1.2, I encourage teachers to reflect on the issues presented there. These are related to teachers' views of the role of mathematics and conceptual reasoning in physics as well as their perspective on intuition. Moreover, teachers should consider what views they may reflect through their choices of teaching and learning formats, and what they make explicit and emphasize in their teaching. Teacher may, for example, ask themselves: Do my reflected views align with the intended learning outcomes? Does the chosen educational format provide students with opportunities to practice the skills we expect them to develop? Do I model authentic reasoning processes in my teaching? A general recommendation in this regard could be to initiate and engage in collegial discussions about these issues, both within and between disciplines.

More specifically, based on my findings I encourage teachers to not shy away from bringing attention to implicit and elusive aspects of learning physics. These aspects may be even more important to discuss and attempt to make more explicit. For example, teachers may guide students to develop strategies specifically related to intuition, and promote a flexible stance toward intuition: one neither always disregards nor always trusts it, but rather seeks to understand and leverage it in productive ways. Again, collaborative learning environments could allow for students to practice managing their intuitions through recognizing, articulating, and critically discussing them. With respect to their perceived role of mathematics, teachers may consider how this role, or roles, is implicitly reflected in their teaching and in the tasks provided to students. Teachers can explicitly integrate the different perspectives in their teaching to help students develop more nuanced conceptions, or discern the subtleties of the integral role that mathematics undoubtedly plays in abstract, upper-level courses. That is, teachers could bear in mind the potential challenges—discussed in the previous section—that students may face when the line between mathematics and the discipline is blurred. Here, teachers can guide students toward extending their frame to include cross-domain reasoning, as opposed to resorting to a limited frame where mathematics mostly assumes a superficial, instrumental role.

8. Future directions

This chapter begins with a short description of the directions which I am currently pursuing further (Section 8.1). Due to the broad, exploratory nature of my work so far, the findings presented in this licentiate thesis suggest various potential aspects to investigate in more depth. I conclude by outlining a few such ideas for future research in Section 8.2, some of which I may consider in upcoming studies to be included in the doctoral thesis.

8.1 Ongoing research

In this section, I briefly describe two main projects that comprise my ongoing research. Firstly, a project which mainly extends on the work in Paper I and II, to provide a more in-depth exploration of some interesting aspects identified in those projects. Secondly, a project which continues the work in Paper II to explicitly consider students' and teachers' views on intuition.

Focused exploration of student challenges related to connecting the concepts of entropy, temperature, and energy to prior experience in the context of statistical mechanics: the affordances and limitations of commonly used idealized models. This work builds upon the findings from Paper I and II, such as that students' ideas about key concepts in statistical mechanics are challenging to reconcile with their intuitions about the same terms from everyday experience and intro-level physics courses. With a more practice-oriented focus, we aim to explore the utility of the two-state paramagnet as an example—which has been recommended as a pedagogical tool to demonstrate some subtle aspects of the relationship between entropy, energy, and temperature (Moore & Schroeder, 1997). To this end, I intend examine some cases in which the two-state paramagnet was leveraged as an analogy by the students and helped them to extend their frame, activate other sets of resources, and overcome the conflict—suggesting a spontaneous bridging between their ideas from one physics course to another. Additionally, I plan to analyze in what ways these productive cases differ from cases in which the students struggled to recognize the similarities in the underlying structure and could not resolve their conflict.

Exploring how teachers and upper-level students in physics and chemistry view intuition, how they perceive its role in their discipline, and how they respond to their intuition in problem-solving contexts. Building on our work in Paper II, my co-author and I are aiming to contribute to the limited research on students' and teachers' epistemological views on intuition. This would provide insight into an overlooked aspect of Paper II, that is, what beliefs and meanings students may attribute to statements such as "*my intuition tells me...*" in reasoning situations. Additionally, our attention to teachers' views resonates with the work presented in Paper III. In line with the arguments put forth in Paper II, we also aim to explore students' and teachers' perceptions about how they respond to their intuitions in problem-solving contexts. For example, what strategies they might employ to recognize and manage their intuitions. From the teachers' point of view, we are also interested in what ways, if any, they address aspects related to intuition in their teaching.

8.2 Future research

Below, I list a sample of potential directions for future research, based on existing work and the findings presented in this licentiate thesis.

- Future work could continue the exploration of the complex interplay between various kinds of resources in students' *framing* of reasoning and problem-solving situations in upper-level physics courses. Such studies could advance the approach and preliminary analysis presented in the synthesis of this thesis, including the model based on the Resources framework (Section 7.1). A focused analysis of students' productive patterns of activations could provide valuable suggestions for teachers on how to, for example, vary the context of exercises to help students develop strategies to frame such physics tasks productively.

- In line with the broad point of exploration above, future studies could consider these aspects with respect to collaboration, communication, and representations. For example by introducing further suitable theoretical supplements to guide the analysis in this regard, such as social semiotics (e.g., Airey & Linder, 2017).

- Future research could explore the processes involved in the development of students' intuitions as they transition from introductory to advanced levels of university physics, or other science subjects. Additionally, future studies could explicitly investigate ways to integrate intuition into teaching practices, supporting teachers in fostering students' intuition and helping them develop metacognitive strategies to manage intuition.

- Another interesting direction for future research is to more closely investigate the role of teacher-reflected epistemologies in upper-level physics courses. This could, for example, be studied with respect to: the interplay between physics and mathematics; and the role of conceptual reasoning and intuition. Questions to ask could be: How may teachers' views about these aspects affect their choice of educational format? How can teacher-reflected views influence students' epistemological views within the course, or the discipline as a whole?

- On a similar but broader note, future work could investigate the relationship between teachers' and students' views on the role of mathematics in other STEM disciplines, and how education could help students to develop their views. These studies could consider questions such as: What are students' and teachers' conceptions of *what mathematics is*, and how does it vary in different levels of education?; and How do those ideas relate to the conception of the role of mathematics in other contexts, and vice versa?

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Appendix A.
Information sheets

Paper I (and II)

Informed Consent – Information sheet UUPER

Students' conceptual understanding in statistical mechanics

INFORMATION SHEET FOR PARTICIPANTS

You are invited to participate in a research project in Physics Education Research (PER).
Ebba Koerfer, PhD student in physics with specialization in PER at Uppsala University, is the project leader.

Before you decide to participate, it is important to understand why the research is carried out and what it involves. Please take your time to read through the following information.

What is the purpose of the project?

The purpose of the study is to investigate students' understanding and reasoning about conceptually advanced concepts in the field of statistical mechanics.

Since PER is relatively limited in more advanced levels of physics, like statistical mechanics, it is relevant to study how students comprehend and learn the subject in order to improve the education and teaching.

Why have I been invited to participate?

You are invited to participate because you are taking the course "Statistical mechanics".

What are the advantages of participating?

By participating, you help Ebba Koerfer and Bor Gregorcic at Uppsala University to deepen their understanding of students' learning in statistical mechanics. This might in turn help improve the education of this subject in the future. The study can also give yourself insight into your current conceptual understanding.

Are there any risks with participation?

There are no particular risks with participating in this project.

Do I have to participate?

No – it is entirely up to you. If you choose to participate you should keep this information sheet and fill out the consent form, to show that you have understood what rights you have towards the research, and that you are happy to participate.

Until the data has been completely anonymized, you may withdraw from the project and the raw data in which you are included will then be destroyed.

Collected data will be preserved until the quality of the research has been ensured (in accordance with Uppsala University's data protection policy: see https://www.uu.se/om-uu/dataskyddspolicy). Raw data will then be thinned out on a regular basis.

The data may be used to produce various research publications (such as articles, conference presentations, dissertations and reports). If you want to withdraw from the study, we advise you to contact the research leader as soon as possible after completed participation.

To withdraw, we ask you to contact the research leader on ebba.koerfer@physics.uu.se. In case the research leader is absent, you can contact Bor Gregorcic on bor.gregorcic@physics.uu.se so that your request can be answered as quickly as possible. You do not have to state any reason why you withdraw, and it will not affect you in any way.

What happens if I choose to participate?

You will have the chance to discuss a couple of questions with one or two other students outside of lecture time, the exact time based on when it suits everyone involved. It will take about 1-1.5 hours. The activity is recorded with a video camera and audio recording to get a complete picture of the communication (we need your permission for this). Collected data will then be transcribed (video and audio data are transferred to text form and anonymized in the transcript).

In the analysis of the data, we will focus on your understanding and communication of concepts in statistical mechanics, but this will not form the basis for any type of grading in your education.

Data protection and confidentiality

Your data will be processed in accordance with the General Data Protection Regulation (GDPR) (EU) 2016/679. All information collected about you will be kept confidential. If you agree to be video or audio recorded, the recordings will be saved until the quality of the research has been ensured and then thinned out on a regular basis. Your raw data/recordings will only be shown to the researchers previously mentioned, the research group for PER at the Department of Physics and Astronomy, Uppsala University. The data published cannot be used to identify you. All electronic data will be stored on a password-protected hard drive. The consent forms will be stored in such a way that only authorized people in the research group have access to them.

What happens to the results of the project?

The results may be summarized in published articles, reports and presentations. Quotations and important findings will always be completely anonymized regarding the participants in these publications.

Complaints

If you are dissatisfied with something in the research project, please contact the project leader Ebba Koerfer (ebba.koerfer@physics.uu.se). If you are still dissatisfied after this and want to file a formal complaint, you can contact Bor Gregorcic, associate senior lecturer in Physics education research, (bor.gregorcic@physics.uu.se).
Specify the research project, the name of the researcher and what the complaint is about.

Data protection policy

Information classification: KRT 211 (Confidentiality, Accuracy, Accessibility)

Databank register

UU processes personal data in accordance with regulation (EU) 2016/679 of the European Parliament and Council, the General Data Protection Regulation (GDPR).

You have the right to request an answer as to whether Uppsala University processes personal data about you, and to receive a copy, free of charge, of the personal data that is processed once a year. It is often called a register extract. Along with such a request, Uppsala University also provides additional information about the processing, such as its purpose, categories of personal data being processed, anticipated storage time, etc. We collect the information from our list of personal data processing, that we carry out on an ongoing basis.

You have the right to request that your personal information at Uppsala University be corrected if it is false. You can, for example, do this with a supplementary statement to your contact person, course leader, manager or research leader at Uppsala University. We are obliged to correct false personal data without undue delay. However, we do not need to correct your information if it is only processed to document completed research.

You have the right to have your personal data deleted from Uppsala University's system if it is not a public document. Data that we can delete can for example be if it is temporary or of little importance, as if you are dropping out of your education. Then the data can be deleted after a while (thinning deadline).

The right of deletion is severely limited by the regulations on public documents or by requirements for documentation of research or studies.

If Uppsala University is unable to delete your information for legal reasons, we will limit the processing of your information to include only what is required to fulfill our obligations under those provisions.

You have the right to request that the processing of your personal data be restricted with us. This means that we only process your personal for certain specific purposes. We will limit the treatment in the following cases:

- If you claim that the personal data is incorrect, and we need time to check the accuracy of the data.
- If you object to the treatment performed by us. In that case, the processing is limited until a balance has been struck between your reasons for the objection and our obligations for processing, which we may have for legitimate reasons.
- If you believe that we should delete the personal data, but we cannot do so for some reason.

You have the right to object to the processing of your personal data with us in certain cases, for example in research or educational activities. We will then interrupt the processing if we do not have compelling reasons to continue with it, or if the processing is required to utilize legal claims that we may have (e.g. against a contracting party/supplier).

If you have questions about data protection, you can always contact your contact person at Uppsala University, the person responsible for a project or course, or Uppsala University's data protection representative (dataskyddsombud@uu.se). See other contact information below:

Uppsala University
Box 256

Dag Hammarskjölds väg 7, 752 37 Uppsala
751 05 Uppsala, Sweden
Switchboard: 018-417 00 00
Organisation identification number: 202100-2932

If you believe that you have not been processed in accordance with the Data Protection Act and related national legislation, you have the right to send a complaint to the Swedish Authority for Privacy Protection (imy@imy.se).

Paper III

Information about participation in the research project “The role of mathematics in higher STEM education”

1. Information about the project and how research subjects are selected

This research project is initiated by MINT, the faculty of science and technology’s centre for discipline-based education research, see mint.uu.se, with researchers and doctoral students from several departments at the faculty of science at technology, Uppsala University. I want to ask you if you would be willing to participate in a study within a research project called “The role of mathematics in higher STEM education”, where STEM means Science, Technology, Engineering, and Mathematics. Through this project we wish to investigate how faculty at our departments view the role of mathematics within their disciplines.

The background of the project is that previous research shows that how students understand the role of mathematics in their discipline influences their learning. However, much less is known about how teachers in higher STEM education view the role of mathematics within their disciplines, which can influence how they design, approach and implement their teaching activities. The project aims at shedding some light on this and thus improve teaching and learning at the faculty.

The reason we ask you to participate is because you are a researcher/doctoral student at X. The research principal (forskningshuvudman) of the project is Uppsala University. The research principal is the organization responsible for the project.

2. What participation in the study would involve for you

If you agree to participate, this means that you are willing to be interviewed once before the middle of April 2023, time and place decided by you and the interviewer. The interview will take maximum one hour. You will be asked questions on how you view the role of mathematics within your discipline, and if you don’t like to answer a question, you don’t have to. The interview will be recorded and transcribed. The recording and transcription will be stored in Allvis, a secure storage of research data at Uppsala University. Your name will neither appear in the recording nor in the transcript. Your voice is personal data, and will not be shared outside the research group and stored securely. Data will be analysed at a group level to ensure that no individual participant can be identified. The results of the analyses will be published in peer reviewed conferences and/or journals.

3. How to learn about study results

You will be able to partake of the results of the study by contacting the principal investigator, see below. Preliminary analysis is planned to be finished by the end of May, 2023.

4. Participation is voluntary

Participation in the project is completely voluntary. At any time, you can choose to no longer participate and you do not have to say why. If you no longer wish to participate, you must notify the person in charge of the project, see contact details below.

5. Contact information to those responsible for the project

The principal investigator of this project is Anna Eckerdal, Dept. of information technology, Uppsala University.

Email: Anna.Eckerdal@it.uu.se

Phone number: 073-469 70 09

Appendix B.
Consent forms

Paper I (and II)

UPPSALA
UNIVERSITET

UPPSALA UNIVERSITY
PER

Consent for study within the project ”Students’ conceptual understanding in statistical mechanics”

The purpose of the study and what the participation entails:

Physics education research (PER) is relatively limited in certain areas, especially at more advanced levels, one of which is statistical mechanics, often perceived as conceptually difficult.

The purpose of the study is to examine students’ reasoning about some central concepts in statistical mechanics, which were found to be confusing for students in recent studies, to provide insight into students’ conceptual understanding and ways of thinking. The project is intended to shed light on particularly problematic areas, in order to develop the teaching of this subject in the future. This is a continuation of an exploratory case-study, and the goal is to investigate research questions that arose in the previous study. The results would also be valuable for instance when developing/expanding on a questionnaire that tests students’ conceptual understanding of statistical mechanics. Participation involves discussing a couple of questions in groups of 2-3 people outside lecture time, for no more than 1.5 hours.

Collected data will only be analyzed by us in the research group, e.g. by writing out (transcribing) and interpreting what you say and do. Photo and video data will not be displayed to anyone other than us researchers. Processed data (transcripts) may come to be presented at conferences and in scientific articles, but not in other contexts, such as social media. Your name will never be mentioned in any context outside the researchers’ work with the data.

Participation in the study is completely voluntary. You can at any time during or after the study choose to drop out, in accordance with the Data Protection Act, by notifying us via the email address at the bottom of the document. If we would like to use the data in another way, we will contact you for an extended permit.

Thank you for your participation!
Ebba Koerfer, PhD student in physics with specialization in PER at Uppsala University
Supervisor: Bor Gregorcic, associate senior lecturer in PER, at the Department of Physics and Astronomy, Uppsala University

- ☐ I have read the accompanying information sheet for the project “Students’ conceptual understanding in statistical mechanics” and have been orally informed about the participation.
- ☐ I give my consent for survey, video and audio data to be collected during my participation.
- ☐ I only allow those mentioned in the information sheet to take part in the collected video and audio data.
- ☐ I understand that published data cannot be used to identify me.
- ☐ I have understood that I can withdraw from the study at any time and that the raw data in which I’m included will then be destroyed.

I give my consent to participate in the scientific study.

Signature:____

Name clarification:_______________________________________

Email: ____

If you want to know more, feel free to contact ebba.koerfer@physics.uu.se

Paper III

Consent to participating in the research project “The role of mathematics in higher STEM education”

I have read and understood the information about the study in the document “Information about participation in the research project ‘The role of mathematics in higher STEM education’”. I have been given the opportunity to ask questions and I have had them answered. I may keep the written information.

- ☐ I consent to participating in the study described in the document “Information about participation in the research project ‘The role of mathematics in higher STEM education’”

- ☐ I consent to the processing of my personal data, the recorded interview, as described in the document “Information about participation in the research project ‘The role of mathematics in higher STEM education’”

Place and date	**Signature and clarification of signature**
....	....