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Cluster tilting for higher Nakayama algebras

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Abstract

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Auslander-Reiten theory is a fundamental tool to study representation theory from a homological point of view. A higher dimensional analogue, developed by Iyama, is naturally framed in terms of dZ-cluster tilting subcategories in both abelian and triangulated settings. In this thesis, we develop methods to construct and identify such subcategories and show that they witness derived equivalences, with applications to higher Auslander algebras of type A, as well as singular equivalences, with applications to higher Nakayama algebras.

In Paper I, we construct a singular equivalence between d-homological pairs (A, M) and (B, N). An as application, we show that every d-Nakayama algebra is singular equivalent to a self-injective d-Nakayama algebra. This equivalence can be recovered from the combinatorial invariant given by the resolution quiver. Our result generalizes the classical singular equivalence for Nakayama algebras and provides an alternative proof of a similar equivalence of higher Nakayama algebras due to McMahon.

In Paper II, we introduce 2-subhomogeneous d-representation finite algebras and show how they can be constructed using certain tilting complexes over a fractionally Calabi-Yau algebra. As an application, for each higher Auslander algebra of type A satisfying certain coprimality condition, we obtain a new derived equivalence induced by an explicit tilting complex. This yields certain replicated algebras that are 2-subhomogeneous d-representation finite.

In Paper III, we study which d-Nakayama algebras admit an ndZ-cluster tilting subcategory for an integer n>1. The radical square zero case is already covered by results on classical Nakayama algebras due to Herschend-Kvamme-Vaso.For each remaining non-self-injective d-Nakayama algebra, we give a complete classification of its ndZ-cluster tilting subcategories,showing that at most one exists for a suitable integer n. For self-injective d-Nakayama algebras satisfying an additional condition,we show that such a subcategory does exist by constructing an explicit example, applying the methods of Darpö-Iyama to the algebras obtained in Paper II.

Keywords: Representation theory, Homological algebra

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List of papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I Wei Xing, Singularity categories of higher Nakayama algebras, arXiv: 2306.07006.

II Wei Xing, Replicated algebras derived equivalent to higher Auslander algebras of type $\mathbb{A}$, arXiv: 2511.22655.

III Wei Xing, $nd\mathbb{Z}$-cluster tilting subcategories of d-Nakayama algebras, arXiv: 2603.28236.

Contents

1. Introduction

One of the main aims in representation theory is to describe the module category mod$A$ of finite dimensional (right) modules of a finite dimensional algebra A. These A-modules can be decomposed in an essentially unique way into incomposable summands. As homological algebra emerged into representation theory, the seminal work of Auslander and Reiten [1–6] gave a systematical way of studying not only indecomposable modules but also the irreducible morphisms, which are in a sense 'minimal', between them. What was later on called Auslander-Reiten theory became a fundamental tool in representation theory. Moreover, this structure can be visualized by the Auslander-Reiten quiver. This idea connects algebra with combinatorics and geometry.

Alongside this development, Happel introduced Grothendieck's categorical viewpoint into representation theory in the 1980s [22]. The bounded derived category $\mathcal{D}^b(A)$ of A plays a central role and the Auslander-Reiten theory in this setting was developed accordingly. This approach reveals deeper structures of the module categories. Since then, a central topic is to study derived equivalences between two algebras. A breakthrough in this direction was the Rickard's Morita equivalences [41] on derived categories, which states that the derived equivalences between two algebras are induced by tilting complexes. An important result in this direction is when A has finite global dimension. Then $\mathcal{D}^b(A)$ was proved by Happel to be equivalent to the stable module category of its repetitive algebra [22]. This leads to Riedtmann's work on the classification of representation finite self-injective algebras over an algebraically closed field [42–44]. The key tool is Galois covering and the orbit construction, developed by Gabriel and Bongartz [9, 19].

Around the same time, the concept of singularity categories in algebraic geometry merged into representation theory by a series of work of Buchweitz in 1980s [10]. The singularity category was called the stablized derived category, as it detects the stable homological properties of an algebra A. In the case when A has finite global dimension, its singularity category $\mathcal{D}_{sg}(A)$ is trivial. If A is self-injective, then $\mathcal{D}_{sg}(A)$ coincides with the stable module category $\underline{\mathrm{mod}}A$. They both belong to the family of Iwanaga-Gorenstein algebras, whose singularity category coincides with the stable module categories of the Cohen-Macaulay modules. However, if A is not Iwanaga-Gorenstein, the singularity category $\mathcal{D}_{sg}(A)$ can be quite difficult to describe.

In the mid 2000s, classical Auslander-Reiten theory was generalized into higher dimension by Iyama [28, 29, 31]. Instead of the whole category, one restricts to a subcategory called d-cluster tilting for a positive integer d, which turns out to be the suitable framework for higher Auslander-Reiten theory. Such subcategories are defined in various settings, including abelian categories, exact categories, triangulated categories or more generally extriangulated categories. A breakthrough was made to axiomatize such subcategories by d-abelian categories [34] when the ambient category is abelian.

A stronger notion called $d\mathbb{Z}$-cluster tilting was introduced by Iyama-Jasso [32]. In the triangulated setting, $d\mathbb{Z}$-cluster tilting means precisely closed under $(d+2)$-suspensions and $(d+2)$-angulated categories [20] are obtained in this way. When the algebra A has global dimension d, a d-cluster tilting subcategory $\mathcal{C} \subseteq \mathrm{mod}A$ is trivially $d\mathbb{Z}$-cluster tilting and it induces a $d\mathbb{Z}$-cluster tilting subcategory $\mathcal{U}$ in the bounded derived category $\mathcal{D}^b(A)$. In general this doesn't work. However as proved by Kvamme [37], a $d\mathbb{Z}$-cluster tilting subcategory of mod$A$ does induce a $d\mathbb{Z}$-cluster tilting subcategory in the singularity category $\mathcal{D}_{sg}(A)$. Further, as shown by Jasso-Muro [35], a $d\mathbb{Z}$-cluster tilting subcategory of the form $\mathcal{C} = \mathrm{add}C$ of a triangulated category $\mathcal{T}$ determined $\mathcal{T}$ in an essentially unique way. This is known as the derived Auslander-Iyama correspondence.

The existence of $d\mathbb{Z}$-cluster tilting subcategories imposes a strong restriction on the ambient abelian category or triangulated category. For certain families, such as Nakayama algebras, which are representation finite, $d\mathbb{Z}$-cluster tilting is well-understood [26]. Another example is d-Auslander algebras of type $\mathbb{A}$ in which d-clsuter tilting and $d\mathbb{Z}$-cluster tilting coincide because the global dimension is d [31]. As a common generalization, d-Nakayama algebras, constructed by Jasso-Külshammer [36], are equipped with $d\mathbb{Z}$-cluster tilting subcategories but homologically much more complex, making it an ideal common ground to test certain conjectures in higher Auslander-Reiten theory.

The main objects of study in this thesis are $d\mathbb{Z}$-cluster tilting subcategories in both abelian and triangulated settings. We aim to develop methods for constructing and identifying such subcategories in each context. Furthermore, we demonstrate that $d\mathbb{Z}$-cluster tilting subcategories witness derived equivalences, with applications to higher Auslander algebras of type $\mathbb{A}$, as well as singular equivalences, with applications to higher Nakayama algebras.

2. Preliminaries

The aim of this chapter is to provide the background and context necessary to understand and motivate the results presented in this thesis. We introduce the key definitions and results used throughout Paper I-III. We do not include details but give references to interested readers.

2.1 Higher Auslander-Reiten theory

Auslander-Reiten theory studies finite dimensional (right) modules through almost split sequences and the Auslander-Reiten translation. Higher Auslander-Reiten theory which was developed by Iyama [28, 29, 31], extends these ideas to a higher dimensional setting. In both abelian and triangulated settings, d-cluster tilting subcategories provide the natural framework for higher Auslander-Reiten theory.

Definition 2.1.1. *Let d be a positive integer. Let $\mathcal{C}$ be an abelian or a triangulated category, and A a finite-dimensional k-algebra.*

(a) We call a subcategory $\mathcal{M}$ of $\mathcal{C}$ a d-cluster tilting subcategory if it is functorially finite, generating-cogenerating if $\mathcal{C}$ is abelian, and

$$\begin{aligned}\mathcal{M} &= \{C \in \mathcal{C} \mid \operatorname{Ext}^i_{\mathcal{C}}(C, \mathcal{M}) = 0 \textit{ for } 1 \leq i \leq d-1\} \\ &= \{C \in \mathcal{C} \mid \operatorname{Ext}^i_{\mathcal{C}}(\mathcal{M}, C) = 0 \textit{ for } 1 \leq i \leq d-1\}.\end{aligned}$$

If moreover $\operatorname{Ext}^i_{\mathcal{C}}(\mathcal{M}, \mathcal{M}) \neq 0$ implies that $i \in d\mathbb{Z}$, then we call $\mathcal{M}$ a $d\mathbb{Z}$-cluster tilting subcategory.

(b) A finitely generated module $M \in \operatorname{mod} A$ is called a d-cluster tilting module (respectively $d\mathbb{Z}$-cluster tilting module) if $\operatorname{add} M$ is a d-cluster tilting subcategory (respectively $d\mathbb{Z}$-cluster tilting subcategory) of $\operatorname{mod} A$.

Combining the Auslander-Reiten translation τ and the syzygy functor Ω we obtain the higher Auslander-Reiten translation for a finite dimensional algebra A. More specifically the functors

$$\begin{aligned}\tau_d &= \tau\Omega^{d-1} : \underline{\operatorname{mod}}A \to \overline{\operatorname{mod}}A \\ \tau_d^{-1} &= \tau^{-1}\Omega^{-(d-1)} : \overline{\operatorname{mod}}A \to \underline{\operatorname{mod}}A\end{aligned}$$

called the d-Auslander-Reiten translation and the inverse translation respectively. We summarize the properties of d-cluster tilting subcategories in the following Proposition.

Proposition 2.1.2. *[30, Theorem 2.8] [52, Corollary 3.3] Let A be an algebra and let $\mathcal{C}$ be a d-cluster tilting subcategory of* mod$A$. *Then the following statements hold.*

(a) $A \in \mathcal{C}$ *and* $DA \in \mathcal{C}$.

(b) Denote by $\mathcal{C}_P$ *and* $\mathcal{C}_I$ *the sets of isomorphism classes of indecomposable non-projective respectively non-injective modules in* $\mathcal{C}$. *Then* τ_d *and* τ_d^{-1} *induce mutually inverse bijections*

$$\mathcal{C}_P \underset{\tau_d^{-1}}{\overset{\tau_d}{\rightleftarrows}} \mathcal{C}_I.$$

(c) $\Omega^i M$ *is indecomposable for all* $M \in \mathcal{C}_P$ *and* $0 < i < d$. *Dually,* $\Omega^{-i} N$ *is indecomposable for all* $N \in \mathcal{C}_I$ *and* $0 < i < d$.

(d) The following statements are equivalent.

(i) $\mathcal{C}$ *is* $d\mathbb{Z}$*-cluster tilting.*

(ii) $\Omega^d(\mathcal{C}) \subseteq \mathcal{C}$.

(iii) $\Omega^{-d}(\mathcal{C}) \subseteq \mathcal{C}$.

Let A be a finite dimensional algebra and $\mathcal{M} \subseteq \text{mod}A$ a d-cluster tilting subcategory. We then call $(A, \mathcal{M})$ is a d-homological pair. In particular, if gldim$A \leq d$, then

$$\mathcal{M} = \text{add}M = \text{add}\{\tau_d^{-i}(A) \mid i \in \mathbb{N}\}$$

is the unique d-cluster tilting subcategory of mod$A$. Equivalently, $M \in \text{mod}A$ a d-cluster tilting module. In this case, A is called d-representation finite.

We recall the following characterization of d-cluster tilting modules. In fact, the exact sequences given in (c) provide the d-almost split sequences in add$M$. The key insight is that, via projectivisation, the d-almost split sequences in add$M$ give rise to the projective resolutions of the simple modules over the endomorphism algebra $\text{End}_A(M)$. This phenomenon is addressed by the higher Auslander correspondence.

Lemma 2.1.3. *Let A be a finite dimensional algebra and let* $M \in \text{mod}A$ *be a d-rigid generator-cogenerator. Then the following conditions are equivalent.*

(a) M is a d-cluster tilting module.

(b) $\text{gldimEnd}_A(M) \leq d+1$.

(c) For all indecomposable object $X \in \text{add}M$*, there exists an exact sequence*

$$0 \to M_{d+1} \xrightarrow{f_{d+1}} M_d \xrightarrow{f_d} \cdots \xrightarrow{f_1} M_1 \xrightarrow{f_1} X$$

with $M_i \in \text{add}M$ *for all* $1 \leq i \leq d+1$ *such that the following sequence is exact.*

$$0 \to \text{Hom}_A(M, M_{d+1}) \xrightarrow{f_{d+1}^*} \cdots \xrightarrow{f_2^*} \text{Hom}_A(M, M_1) \xrightarrow{f_1^*} \mathcal{J}_A(M, X) \to 0.$$

The following definition of partial d-cluster tilting subcategories was introduced in Paper III. They are d-rigid but not 'maximal' compared to d-cluster tilting subcategories. Additionally they interact with $\tau_d^{\pm 1}$ and $\Omega^{\pm 1}$ in a nice way.

Definition 2.1.4. *Let A be an algebra and let $\mathcal{C} \subseteq \operatorname{mod} A$ be a subcategory. We call $\mathcal{C}$ a partial d-cluster tilting subcategory if the following conditions hold.*

(a) $A, DA \in \mathcal{C}$.

(b) $\operatorname{Ext}_A^i(\mathcal{C}, \mathcal{C}) = 0$ *for all* $0 < i < d$.

(c) Denote by $\mathcal{C}_p$ and $\mathcal{C}_I$ the sets of isomorphism classes of indecomposable non-projective respectively non-injective modules in $\mathcal{C}$. Then τ_d and τ_d^{-1} induce mutually inverse bijections

$$\mathcal{C}_P \underset{\tau_d^{-1}}{\overset{\tau_d}{\rightleftarrows}} \mathcal{C}_I.$$

(d) $\Omega^i M$ is indecomposable for all $M \in \mathcal{C}_P$ and $0 < i < d$. Dually, $\Omega^{-i} N$ is indecomposable for all $N \in \mathcal{C}_I$ and $0 < i < d$.

We call $\mathcal{C}$ a partial $d\mathbb{Z}$-cluster tilting subcategory if additionally,

(e) $\Omega^d(\mathcal{C}) \subseteq \mathcal{C}$ *and* $\Omega^{-d}(\mathcal{C}) \subseteq \mathcal{C}$.

The definition is motivated by Proposition 2.1.2, which shows that, a d-cluster tilting (respectively $d\mathbb{Z}$-cluster tilting) subcategory is, in particular, partial d-cluster tilting ($d\mathbb{Z}$-cluster tilting).

Remark 2.1.5. *Assume $\mathcal{C}$ is a partial $d\mathbb{Z}$-cluster tilting subcategory. Then Definition 2.1.4 (e) implies the following.*

(e') $\operatorname{Ext}_A^i(\mathcal{C}, \mathcal{C}) \neq 0$ *implies* $i \in d\mathbb{Z}$.

Notice that when $\mathcal{C}$ is $d\mathbb{Z}$-cluster tilting, (e) and (e') are equivalent as shown in Proposition 2.1.2 (d). However, this fails for partial $d\mathbb{Z}$-cluster tilting subcategories as they lack the maximality property.

2.2 Derived categories and singularity categories

Let A be a finite dimensional algebra. We recall the bounded derived category $\mathcal{D}^b(A)$ of A. It is the triangulated category obtained from the bounded homotopy category $\mathcal{K}^b(\operatorname{mod} A)$ of A by localising with respect to the set of quasi-isomophisms. A complex in $\mathcal{D}^b(A)$ is perfect provided that it is quasi-isomorphic to a bounded complex of finitely generated projective A-modules. Perfect complexes form a thick subcategory of $\mathcal{D}^b(A)$, which is denoted by $\operatorname{perf}(A)$.

Two algebras A, B are derived equivalent if $\mathcal{D}^b(A) \cong \mathcal{D}^b(B)$ as triangulated categories. It follows from Rickard's Morita theory for derived categories [41] that two algebras A and B are derived equivalent if and only if there exists a tilting complex $T \in \mathrm{perf}(A)$ such that $B \cong \mathrm{End}_{\mathcal{D}^b(A)}(T)$. In this case, there is a triangle equivalence $F : \mathcal{D}^b(A) \xrightarrow{\sim} \mathcal{D}^b(B)$ such that $F(T) \cong B$. We recall the definition of a tilting complex here.

Definition 2.2.1. *A complex $T \in \mathcal{D}^b(A)$ is a tilting complex if*

(i) $T \in \mathrm{perf}(A)$,

(ii) $\mathrm{Hom}(T, T[i]) = 0$ *for $i \neq 0$, and*

(iii) T *generates* $\mathrm{perf}(A)$*, that is,* $\mathrm{perf}(A)$ *is the smallest thick triangulated subcategory of $\mathcal{D}^b(A)$ containing T.*

We recall a construction of d-cluster tilting subcategories of $\mathcal{D}^b(A)$ via tilting complexes for a τ_d-finite algebra.

Let A be a finite dimensional algebra such that $\mathrm{gldim} A < \infty$. Then $\mathrm{perf}(A) = \mathcal{D}^b(A)$. Let

$$\nu = D \circ \mathbb{R}\mathrm{Hom}_A(-, A) \cong - \otimes_A^{\mathbb{L}} DA : \mathcal{D}^b(A) \to \mathcal{D}^b(A)$$

be the Nakayama functor of $\mathcal{D}^b(A)$. There is the functorial isomorphism [22, Theorem 4.6]

$$\mathrm{Hom}_{\mathcal{D}^b(A)}(X, Y) \cong D\mathrm{Hom}_{\mathcal{D}^b(A)}(Y, \nu X),$$

in other words, the Nakayama functor is the Serre functor of $\mathcal{D}^b(A)$. Denote the d-Nakayama functor by

$$\nu_d = \nu \circ [-d] : \mathcal{D}^b(A) \to \mathcal{D}^b(A).$$

Assume that $\mathrm{gldim} A \leq d$, then

$$\tau_d^{\ell}(M) \cong H^0(\nu_d^{\ell} M)$$

for all $\ell \in \mathbb{Z}$ and $M \in \mathrm{mod} A$, see [31, Lemma 5.5]. Following [31], we say that A is τ_d-finite if moreover $\tau_d^{\ell}(DA) = 0$ for a sufficiently large integer ℓ. We recall the following construction of d-cluster tilting subcategories in $\mathcal{D}^b(A)$. For $T \in \mathcal{D}^b(A)$, set

$$\mathcal{U}_d(T) := \{\nu_d^i(T) \mid i \in \mathbb{Z}\} \subset \mathcal{D}^b(A).$$

Theorem 2.2.2. *[31, Theorem 1.23] Let A be a τ_d-finite algebra. Then $\mathcal{U}_d(A)$ is a d-cluster tilting subcategory of $\mathcal{D}^b(A)$. Moreover, $\mathcal{U}_d(T)$ is a d-cluster tilting subcategory of $\mathcal{D}^b(A)$ for any tilting complex $T \in \mathcal{D}^b(A)$ satisfying* $\mathrm{gldim}\mathrm{End}_{\mathcal{D}^b(A)}(T) \leq d$.

We will later apply the theorem to the bounded derived category of a fractionally Calabi-Yau algebra. We recall the definition here.

Let $\phi : A \to A$ be an algebra automorphism. It induces an autoequivalence

$$\phi^* = - \otimes^{\mathbb{L}}_A {}_{\phi}A : \mathcal{D}^b(A) \to \mathcal{D}^b(A)$$

where ${}_{\phi}A$ is the $A \otimes_{\Bbbk} A^{op}$-module A with the left action changed to $a \cdot b := \phi(a)b$.

Definition 2.2.3. *[23, Definition 0.3] We say that A is twisted fractionally Calabi-Yau (or twisted $\frac{m}{\ell}$-CY) if there exists an isomorphism*

$$\nu^{\ell} \cong [m] \circ \phi^*$$

of functors for some integers $\ell \neq 0$ and m and ϕ an algebra automorphism of A. When $\phi = id$, we say that A is fractionally Calabi-Yau (or $\frac{m}{\ell}$-CY).

We recall the following property.

Theorem 2.2.4. *[23] Let A be a d-representation finite algebra. Then A is twisted fractionally Calabi-Yau.*

We recall the definition of the singularity category of a finite dimensional algebra A.

The singularity category of A, denoted by $\mathcal{D}_{sg}(A)$ is the quotient of triangulated categories given as

$$\mathcal{D}_{sg}(A) = \mathcal{D}^b(A)/\mathrm{perf}(A).$$

Recall that $\mathcal{K}^{-,b}(\mathrm{proj}A)$ denotes the upper bounded homotopy category of $\mathrm{proj}A$ and $\mathcal{K}^b(\mathrm{proj}A)$ denotes bounded homotopy category of $\mathrm{proj}A$, which is a thick triangulated subcategory of $\mathcal{K}^{-,b}(\mathrm{proj}A)$. Via the equivalences $\mathcal{K}^{-,b}(\mathrm{proj}A) \cong \mathcal{D}^b(A)$ and $\mathcal{K}^b(\mathrm{proj}A) \cong \mathrm{perf}(A)$, we have that

$$\mathcal{D}_{sg}(A) \cong \mathcal{K}^{-,b}(\mathrm{proj}A)/\mathcal{K}^b(\mathrm{proj}A).$$

Denote by $q' : \mathcal{D}^b(A) \to \mathcal{D}_{sg}(A)$ the quotient functor. Observe that the functor $\mathrm{mod}A \to \mathcal{D}^b(A) \xrightarrow{q'} \mathcal{D}_{sg}(A)$ vanishes on projective modules. Hence it induces a functor $q : \underline{\mathrm{mod}}A \to \mathcal{D}_{sg}(A)$.

By a singular equivalence between two algebras A and B, we mean a triangle equivalence between their singularity categories.

We recall the following theorem by Kvamme [37] which states that a $d\mathbb{Z}$-cluster tilting subcategory in $\mathrm{mod}A$ induces a canonical $d\mathbb{Z}$-cluster tilting subcategory in the singularity category $\mathcal{D}_{sg}(A)$.

Theorem 2.2.5. *Let A be a finite dimensional algebra and $\mathcal{M}$ a $d\mathbb{Z}$-cluster tilting subcategory of* modA. *Then the subcategory*

$$\underline{\mathcal{M}} = \{X \in \mathcal{D}_{sg}(A) \mid X \cong M[di] \text{ for some } M \in \mathcal{M} \text{ and } i \in \mathbb{Z}\}$$

is an $d\mathbb{Z}$-cluster tilting subcategory of $\mathcal{D}_{sg}(A)$. In particular, $\underline{\mathcal{M}}$ is a $(d+2)$-angulated category.

2.3 Galois covering and the orbit constructions

We recall background on covering spaces and orbit categories from [9, 16].

Let $\mathcal{A}$ be a small category. By definition, a (right) $\mathcal{A}$-module is a linear functor $M : \mathcal{A}^{op} \to \mathrm{Mod}\Bbbk$, this is a functor such that for all $x, y \in \mathcal{A}$,

$$M_{yx} : \mathcal{A}(x,y) \to \mathrm{Hom}_{\Bbbk}(M_y, M_x)$$

is a linear map. We denote the abelian category of $\mathcal{A}$-modules and natural transformations between them by $\mathrm{Mod}\mathcal{A}$. We denote by $\mathrm{mod}\mathcal{A}$ the subcategory which consists of finitely presented functors $M \in \mathrm{Mod}\mathcal{A}$.

Recall that an additive categore $\mathcal{A}$ is called Krull-Schmidt if every $x \in \mathcal{A}$ decomposes as $X = \bigoplus_{i=1}^{n} X_i$, where $X_i \in \mathrm{ind}\mathcal{A}$, the isomorphism classes of indecomposable objects in which $\mathrm{End}_{\mathcal{A}}(u)$ is local for all $u \in \mathrm{ind}\mathcal{A}$.

A Krull-Schmidt category $\mathcal{A}$ is called locally bounded if for every $x \in \mathrm{ind}\mathcal{A}$ there are only finitely many $y \in \mathrm{ind}\mathcal{A}$, and $w \in \mathrm{ind}\mathcal{A}$ such that $\mathcal{A}(x,y) \neq 0$ and respectively, $\mathcal{A}(w,x) \neq 0$.

In particular, if $\mathcal{A} = \mathrm{add}P$ with $P \in \mathcal{C}$ for some Krull-Schmidt category $\mathcal{C}$, then $\mathcal{A}$ is locally bounded and

$$\mathrm{Mod}\mathcal{A} \cong \mathrm{Mod}A, \text{ and } \mathrm{mod}\mathcal{A} \cong \mathrm{mod}A$$

where $A = \mathrm{End}_{\mathcal{C}}(P)$. We often identify A and $\mathcal{A}$ in this context.

Let $\mathcal{A}$ be a locally bounded $\Bbbk$-linear Krull-Schmidt category and let G be a group. A $\Bbbk$-linear G-action on the category $\mathcal{A}$ is an assignment $g \mapsto F_g$ of a $\Bbbk$-linear automorphism $F_g : \mathcal{A} \to \mathcal{A}$ such that $F_g \circ F_h = F_{gh}$ for all $g, h \in G$. To simplify notations, we write $g(x) := F_g(x)$ for $g \in G$ and $x \in \mathcal{A}$. A G-action is called admissible if $g(x) \ncong x$ for all $x \in \mathrm{ind}\mathcal{A}$ and $g \in G \backslash \{1\}$. In this case, the orbit category $\mathcal{A}/G$ is also locally bounded $\Bbbk$-linear Krull-Schmidt which is given by the following data.

- The objects of $\mathcal{A}/G$ are the objects of $\mathcal{A}$.
- For $x, y \in \mathcal{A}/G$,

$$\mathrm{Hom}_{\mathcal{A}/G}(x,y) = \bigoplus_{g \in G} \mathrm{Hom}_{\mathcal{A}}(x, g(y)).$$

- For $(a_g)_{g \in G} \in \mathrm{Hom}_{\mathcal{A}/G}(x,y)$ and $(b_g)_{g \in G}\mathrm{Hom}_{\mathcal{A}/G}(y,z)$, set

$$(ba)_g = \sum_{h \in G} h(b_{h^{-1}g}) a_h : x \to g(z),$$

  so $ba \in \mathrm{Hom}_{\mathcal{A}/G}(x,z)$.

The natural functor $F : \mathcal{A} \to \mathcal{A}/G$ induces an exact functor

$$F^* : \mathrm{Mod}\mathcal{A}/G \to \mathrm{Mod}\mathcal{A}$$

called pull-up. This functor has a left adjoint

$$F_* : \mathrm{Mod}\mathcal{A} \to \mathrm{Mod}\mathcal{A}/G$$

called the push-down, which is also exact. Moreover, F_* induces a functor $F_* : \mathrm{mod}\mathcal{A} \to \mathrm{mod}\mathcal{A}/G$.

Notice that a G-action on $\mathcal{A}$ induces a G-action on $\mathrm{mod}\mathcal{A}$ defined by $g(M) = M \circ g$ for $M \in \mathrm{mod}\mathcal{A}$ and $g \in G$. A subcategory $\mathcal{U} \subseteq \mathrm{mod}\mathcal{A}$ is called G-equivariant if $g(\mathcal{U}) \subseteq \mathcal{U}$ for all $g \in G$.

The following result shows that G-equivariant d-cluster tilting subcategories remain d-cluster tilting in the orbit category.

Theorem 2.3.1. *[16] Let $\mathcal{A}$ be a locally bounded $\mathbb{k}$-linear Krull-Schmidt category and G a free abelian group of finite rank acting admissibly on $\mathcal{A}$. If $\mathcal{U} \subseteq \mathrm{mod}\mathcal{A}$ is a locally bounded G-equivariant d-cluster tilting subcategory, then $F_*(\mathcal{U}) \subseteq \mathrm{mod}\mathcal{A}/G$ is a locally bounded d-cluster tilting subcategory.*

The above Theorem is used in [16] to construct self-injective algebras with d-cluster tilting modules. We recall the construction here.

Let A be a finite dimensional algebra with $\mathrm{gldim} A < \infty$. Let $\widehat{A}$ be the repretitive category of A. It can be viewed as an algebra of infinite matrices of the form

$$\widehat{A} = \begin{pmatrix} \ddots & & & & \\ \ddots & A & & & \\ & DA & A & & \\ & & DA & A & \\ & & & \ddots & \ddots \end{pmatrix}.$$

The category $\mathrm{mod}\widehat{A}$ is Frobenius and therefore, $\underline{\mathrm{mod}}\widehat{A}$ is triangulated. Moreover, there exists a triangle equivalence [22]

$$\mathcal{D}^b(A) \cong \underline{\mathrm{mod}}\widehat{A}.$$

Let ϕ be an automorphism of $\widehat{A}$. We denote by $\phi_* : \mathrm{mod}\widehat{A} \to \mathrm{mod}\widehat{A}$ the induced automorphism of the module category, defined by precompostion with ϕ^{-1}. Denote by $F_* : \mathrm{mod}\widehat{A} \to \mathrm{mod}(\widehat{A}/\phi)$ the push-down functor. Following the above discussion, if $\mathcal{U} \subseteq \mathcal{D}^b(A)$ is a d-cluster tilting subcategory and $\mathcal{U}$ is ϕ-equivariant, then $F_*(\mathcal{U}) \subseteq \mathrm{mod}(\widehat{A}/\phi)$ is d-cluster tilting. We illustrate this idea by the following diagram, see [16].

$$
\begin{array}{ccccc}
\mathcal{D}^b(A) & \xrightarrow{\sim} \underline{\mathrm{mod}}\widehat{A} \leftarrow \mathrm{mod}\widehat{A} \xrightarrow{F_*} & \mathrm{mod}(\widehat{A}/\phi) \\
\cup & & \cup \\
\mathcal{U} & \dashrightarrow & \mathcal{V}
\end{array}
$$

If $\widehat{A}/\phi$ has finitely many indecomposable objects then $\mathrm{mod}(\widehat{A}/\phi) \cong \mathrm{mod}\Lambda$ for Λ a self-injective algebra as $\widehat{A}$ is Frobenius. With this construction, many instances of n-fold trivial extension algebras and higher preprojective algebras can be shown to admit d-cluster tilting subcategories.

For later use, we recall the definition of n-fold trivial extension algebras, see [12] for more details.

The n-fold trivial extension of A is given by the following $n \times n$ matrix algebra

$$
T_n(A) = \begin{pmatrix} A & & & & DA \\ DA & A & & & \\ & DA & \ddots & & \\ & & \ddots & \ddots & \\ & & & DA & A \end{pmatrix}
$$

for $n \geq 2$, whilst $T_1(A) = T(A)$ known as the trivial extension algebra.

The fractionally Calabi-Yau property of the algebra A is deeply related to the twisted periodicity of its n-fold trivial extension algebra $T_n(A)$ as shown in [12].

Theorem 2.3.2. *[12, Theorem 1.4] Let A be a finite dimensional algebra over a field $\Bbbk$ such that $A/\mathrm{rad}\,A$ is a separable $\Bbbk$-algebra. The following conditions are equivalent.*

(i) $T(A)$ is twisted periodic.

(ii) There exists $d, r \geq 1$ such that $T_r(A)$ is d-representation finite.

(iii) A has finite global dimension and is twisted fractionally Calabi-Yau.

2.4 Higher Nakayama algebras

We recall the definition of higher Nakayama algebras introduced in [36]. A d-Nakayama algebra is uniquely determined by its Kupisch series, which records the Loewy lengths of certain indecomposable projective modules over the algebra. We begin by recalling the definition of a Kupisch series.

A tuple of non-negative integers $\ell_\infty = (\ldots, \ell_{-1}, \ell_0, \ell_1, \ldots)$ is called a Kupisch series if $\ell_i \leq \ell_{i-1} + 1$ for all $i \in \mathbb{Z}$.

- ℓ_∞ is ℓ-bounded if $\ell = \max\{\ell_i \mid i \in \mathbb{Z}\}$.
- ℓ_∞ is connected if either
  - (a) $\ell_r = 1$ for a unique $r \in \mathbb{Z}$ and $\ell_i = 0$ for $i < r$ or

(b) $\ell_i \geq 2$ for all $i \in \mathbb{Z}$.

- ℓ_∞ is of width m if $|\{\ell_i \neq 0 \mid i \in \mathbb{Z}\}| = m$.
- ℓ_∞ is m-periodic if $\ell_i = \ell_{i+m}$ for all $i \in \mathbb{Z}$.

In this thesis, we always assume that

(i) ℓ_∞ is ℓ-bounded for some integer ℓ.

(ii) ℓ_∞ is connected of width m or ℓ_∞ is connected and m-periodic.

We recall the definition of ordered sequences $(os^d_{\ell_\infty}, \preccurlyeq)$ from [36]. Define

$$os^d_{\ell_\infty} := \{x = (x_1, x_2, \dots, x_d) \in \mathbb{Z}^d \mid x_1 < \cdots < x_d \text{ and } x_d - x_1 + 1 \leq \ell_{x_d - d+1} + d - 1\}$$

to be the ordered sequence determined by ℓ_∞. Additionally $os^d_{\ell_\infty}$ is endowed with the relation $\preccurlyeq$ defined as

$$x \preccurlyeq y \Longleftrightarrow x_1 \leq y_1 < x_2 \leq y_2 < \cdots < x_d \leq y_d$$

for $x = (x_1, \dots, x_d), y = (y_1, \dots, y_d) \in os^d_{\ell_\infty}$.

We define the $\mathbb{k}$-linear category $\mathcal{A}^d_{\ell_\infty} = \mathbb{k}Q^d_{\ell_\infty}/I$ determined by ℓ_∞ and the integer d via quivers with relations. Let $\{e_i \mid 1 \leq i \leq d\}$ be the standard basis of $\mathbb{Z}^d$.

- The vertices set of the quiver $Q^d_{\ell_\infty}$ is given by $os^d_{\ell_\infty}$.
- There is an arrow $a_i(x) : x \to x + e_i$ whenever $x + e_i \in os^d_{\ell_\infty}$.
- The ideal I of the path category $\mathbb{k}Q^d_{\ell_\infty}$ is generated by

$$a_i(x+e_j)a_j(x) - a_j(x+e_i)a_i(x), \text{ with } 1 \leq i, j \leq d.$$

By convention, $a_i(x) = 0$ whenever x or $x + e_i$ is not in $os^d_{\ell_\infty}$, hence some of the relations are indeed zero relations.

By construction in [36], $\mathcal{A}^d_{\ell_\infty}$ has a distinguished $d\mathbb{Z}$-cluster tilting subcategory

$$\mathcal{M}^d_{\ell_\infty} = \text{add}\{M(x) \mid x \in os^{d+1}_{\ell_\infty}\}.$$

Here as a representation $M(x)$ assigns k to vertex $z \in os^d_{\ell_\infty}$ if $(x_1, \dots, x_d) \preccurlyeq z \preccurlyeq (x_2 - 1, \dots, x_{d+1} - 1)$ and 0 otherwise. All arrows $k \to k$ act as identity, while other arrows act as zero. By convention, $M(x) = 0$ if $x \notin os^{d+1}_{\ell_\infty}$. Then

$$\text{Hom}_{\mathcal{A}^d_{\ell_\infty}}(M(x), M(y)) \cong \begin{cases} kf_{yx} & x \preccurlyeq y \\ 0 & \text{otherwise.} \end{cases}$$

Here f_{yx} is given by $k \xrightarrow{1} k$ at vertices z where $M(x)_z = M(y)_z = k$ and 0 otherwise. The composition of morphisms in $\mathcal{M}^d_{\ell_\infty}$ is completely determined by

$$f_{zy} \circ f_{yx} = \begin{cases} f_{zx} & x \preccurlyeq z \\ 0 & \text{otherwise.} \end{cases}$$

We call $f_{yx} \neq 0$ an arrow morphism if there is no $z \in os^{d+1}_{\ell_\infty}$ such that $f_{yx} = f_{yz} \circ f_{zx}$.

If ℓ_∞ is of width m, we use the notation $\underline{\ell} = (\ell_1, \ldots, \ell_m)$ instead. In this case, the d-Nakayama algebra $A^d_{\underline{\ell}}$ of type $\mathbb{A}_m$ is defined as

$$A^d_{\underline{\ell}} = \mathcal{A}^d_{\ell_\infty}$$

with the distinguished $d\mathbb{Z}$-cluster tilting subcategory given by

$$\mathcal{M}^d_{\underline{\ell}} = \mathcal{M}^d_{\ell_\infty}.$$

In particular, the $(d-1)$-Auslander algebra A^d_m of type $\mathbb{A}$ is defined by

$$A^d_m = A^d_{\underline{\ell}}$$

where $\underline{\ell} = (1, 2, \ldots, m)$.

Now assume ℓ_∞ is m-periodic, i.e. $\ell_\infty = (\ldots, \ell_m, \ell_1, \ldots, \ell_m, \ldots)$. Consider the group

$$G = \langle \sigma = (m, m, \ldots, m) \rangle \subseteq \mathbb{Z}^d$$

and the G-action on $\mathcal{A}^d_{\ell_\infty}$ via $F_\sigma : x \mapsto x + \sigma$. We obtain the orbit category $\mathcal{A}^d_{\ell_\infty}/G$. Notice that $\mathcal{M}^d_{\ell_\infty}$ is G-equivariant. Thus by Theorem 2.3.1,

$$\mathcal{M}_{\ell_\infty}/G \subseteq \operatorname{mod}(\mathcal{A}^d_{\ell_\infty}/G)$$

is a $d\mathbb{Z}$-cluster tilting subcategory.

The d-Nakayama algebra of type $\widetilde{\mathbb{A}}_{m-1}$ is defined as

$$A^d_{\underline{\ell}} = \mathcal{A}^d_{\ell_\infty}/G \text{ where } \underline{\ell} = (\ell_1, \ldots, \ell_m).$$

The distinguished $d\mathbb{Z}$-cluster tilting subcategory of $\operatorname{mod} A^d_{\underline{\ell}}$ is given by

$$\mathcal{M}^d_{\underline{\ell}} = \mathcal{M}^d_{\ell_\infty}/G.$$

We summarize basic homological properties of $M(x) \in \mathcal{M}^d_{\ell_\infty}$ in the following proposition.

Proposition 2.4.1. *([36, Proposition 2.22, Proposition 2.25, Theorem 3.16] [16, Lemma 3.5(b)]) Let $x \in os^{d+1}_{\ell_\infty}$. The following statements hold true.*

(i) $M(x)$ is simple if and only if $x = (i, i+1, \ldots, i+d)$ for some integer i.

(ii) $M(x)$ is projective if and only if $x_1 = \min\{y \mid (y, x_2, \ldots, x_{d+1}) \in os^{d+1}_{\ell_\infty}\}$.

(iii) $M(x)$ is injective if and only if $x_{d+1} = \max\{y \mid (x_1, \ldots, x_d, y) \in os^{d+1}_{\ell_\infty}\}$.

(iv) If $x_1 > x_{d+1} - \ell_{x_{d+1}-d} - d + 1$, then there exists an exact sequence

$$0 \to \Omega^d(M(x)) \to P^d \to \cdots \to P^1 \to M(x) \to 0$$

with $P^i = M(x_{d+1} - \ell_{x_{d+1}-d} - d + 1, x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_{d+1})$ for $1 \leq i \leq d$ and $\Omega^d(M(x)) = M(x_{d+1} - \ell_{x_{d+1}-d} - d + 1, x_1, \ldots, x_d)$.

(v) We use the notation $\tau_d(x) = (x_1 - 1, x_2 - 1, \ldots, x_{d+1} - 1)$*. Then* $\tau_d(M(x)) = M(\tau_d(x))$ *for* $M(x)$ *not projective.*

(vi) If ℓ_∞ *is m-periodic, then we have*

$$\dim_{\Bbbk} \operatorname{Ext}_A^d(M(y), M(x)) = |\{x \mid \sigma^i x \preccurlyeq \tau_d(y) \text{ for some } i\}|.$$

If ℓ_∞ *is of width m, then*

$$\operatorname{Ext}_A^d(M(y), M(x)) \cong \begin{cases} k & x \preccurlyeq \tau_d(y), \\ 0 & \text{otherwise.} \end{cases}$$

3. Summary of papers and results

In this chapter, we summarize the main results of the articles that comprise this thesis. The shared motivation behind them is the characterization and construction of $d\mathbb{Z}$-cluster tilting subcategories for certain algebras. Furthermore, we explore the role of $d\mathbb{Z}$-cluster tilting subcategories in derived equivalences and singular equivalences.

In Paper I, we construct a singular equivalence between d-homological pairs $(A, \mathcal{M})$ and $(B, \mathcal{N})$. An as application, we show that every d-Nakayama algebra is singular equivalent to a self-injective d-Nakayama algebra. This equivalence can be recovered via a combinatorial invariant of d-Nakayama algebras, known as the resolution quiver. Our result generalizes the classical singular equivalence for Nakayama algebras and provides an alternative proof of a similar equivalence previously obtained for higher Nakayama algebras [40].

In Paper II, we introduce the notion of 2-subhomogeneous d-representation finite algebras and show how they can be constructed using certain tilting complexes over a fractionally Calabi-Yau algebra. As an application, for each higher Auslander algebra of type $\mathbb{A}$ satisfying certain coprimality condition, we obtain a new derived equivalence induced by an explicit tilting complex. As a consequence, we obtain certain replicated algebras that are 2-subhomogeneous d-representation finite.

In Paper III, we investigate which d-Nakayama algebras admit an $nd\mathbb{Z}$-cluster tilting subcategory for an integer $n > 1$. The radical suqare zero case is already covered by results on classical Nakayama algebras due to Herschend-Kvamme-Vaso [26]. For each remaining non-self-injective d-Nakayama algebra, we provide a complete classification of its $nd\mathbb{Z}$-cluster tilting subcategories. In fact, there exists at most one for a suitable integer n. For self-injective d-Nakayama algebras satisfying an additional condition, we show that such a subcategory does exist by constructing an explicit example. This example is obtained by applying the methods of [16] to the algebras obtained in Paper II.

3.1 Paper I

3.1.1 Summary

The singularity category of an algebra A was introduced by Buchweitz [10], which captures the stable homological properties of A. The Buchweitz-Happel theorem states that if A is Iwanaga-Gorenstein, then the singularity category $\mathcal{D}_{sg}(A)$ is triangulated equivalent to the stable category of maximal Cohen-Macaulay modules over A. However, for non Iwanaga-Gorenstein algebras, the singularity category is more difficult to describe.

In this paper, we give a construction of singular equivalences between two d-homological pairs, based on the construction of d-wide subcategories introduced in [25].

Theorem 3.1.1. *Let $(A,\mathcal{M})$ be a d-homological pair. Let $\mathcal{W}\subset\mathcal{M}$ be an additive subcategory. Let $P\in\mathcal{W}$ be a module and set $B=\operatorname{End}_A(P)$, so that P becomes a B-A-bimodule. Assume the following:*

(i) As an A-module P has finite projective dimension.

(ii) $\operatorname{Ext}^i_A(P,P)=0$ for all $i\geq 1$.

(iii) Each $W\in\mathcal{W}$ admits an exact $\operatorname{add}P$-resolution

$$\cdots\to P_m\to\cdots\to P_1\to P_0\to W\to 0,\ P_i\in\operatorname{add}P.$$

(iv) $(B,\mathcal{N})$ is a d-homological pair where $i_\rho=\operatorname{Hom}_A(P,-):\operatorname{mod}A\to\operatorname{mod}B$ and $\mathcal{N}=i_\rho(\mathcal{W})$.

Then the following statements hold

(a) $\mathcal{W}$ is a wide subcategory of $\mathcal{M}$ and there is an equivalence of categories

$$i_\lambda=-\otimes_B P:\mathcal{N}\to\mathcal{W}.$$

(b) $i_\lambda:\operatorname{mod}B\to\operatorname{mod}A$ induces a fully faithful triangle functor between the singularity categories of A and B,

$$\mathcal{D}_{sg}(i_\lambda):\mathcal{D}_{sg}(B)\to\mathcal{D}_{sg}(A).$$

Moreover, $\mathcal{D}_{sg}(i_\lambda)$ is a triangle equivalence if for any indecomposable $M\in\mathcal{M}$, there exists an integer $n\in\mathbb{N}$ such that $\Omega^n(M)\in\mathcal{W}$.

(c) If in addition $\mathcal{M}$ is a $d\mathbb{Z}$-cluster tilting subcategory of $\operatorname{mod}A$, then $\underline{\mathcal{M}}\subset\mathcal{D}_{sg}(A)$, $\underline{\mathcal{N}}\subset\mathcal{D}_{sg}(B)$ are $d\mathbb{Z}$-cluster tilting and hence $(d+2)$-angulated. Moreover, $\mathcal{D}_{sg}(i_\lambda)$ restricts to an equivalence between $(d+2)$-angulated categories

$$\mathcal{D}_{sg}(i_\lambda):\underline{\mathcal{N}}\to\underline{\mathcal{M}}.$$

We apply our construction to higher Nakayama algebras, which generalizes the singular equivalences between classical Nakayama algebras studied by Chen-Ye [15] and Shen [49]. A similar singular equivalence was previously established by McMahon [40] via iterative contractions along fabric idempotent ideals. Our approach provides a more explicit realization of this equivalence. Moreover, this singular equivalence restricts to an equivalence between the distinguished $(d+2)$-angulated categories.

Theorem 3.1.2. *Let A be a d-Nakayama algebra. Then the singularity category $\mathcal{D}_{sg}(A)$ is triangulated equivalent to the stable module category $\underline{\mathrm{mod}}B$ with B a self-injective d-Nakayama algebra. More precisely,*

$$\mathcal{D}_{sg}(i_\lambda) : \underline{\mathrm{mod}}B \to \mathcal{D}_{sg}(A)$$

is a triangle equivalence. In addition, $\mathcal{D}_{sg}(i_\lambda)$ restricts to an equivalence between $(d+2)$-angulated categories

$$\mathcal{D}_{sg}(i_\lambda) : \underline{\mathcal{N}} \to \underline{\mathcal{M}}$$

where $\underline{\mathcal{N}} \subseteq \underline{\mathrm{mod}}B$ and $\underline{\mathcal{M}} \subseteq \mathcal{D}_{sg}(A)$ are the distinguished $d\mathbb{Z}$-cluster tilting subcategories induced by the distinguished $d\mathbb{Z}$-cluster tilting subcategories $\mathcal{N} \subseteq \mathrm{mod}B$ and $\mathcal{M} \subseteq \mathrm{mod}A$.

To construct B, we generalize the notion of resolution quivers to higher Nakayama algebras, from which the singular equivalences described above can be recovered. Resolution quivers for classical Nakayama algebras were introduced by Ringel [45] and further studied in [48–50].

More precisely, let A be a d-Nakayama algebra with Kupisch series $\underline{\ell} = (\ell_1, \ldots, \ell_n)$. The vertex set of the resolution quiver $R(A)$ is $\{1, 2, \ldots, n\}$, and there is an arrow from i to j if $j \equiv i - \ell_i - d + 1 (\mod n)$. When $d = 1$, this definition coincides with the classical resolution quivers for Nakayama algebras defined in [45]. Let $J \subseteq \{1, 2, \ldots, n\}$ be the set of vertices that lie in a cycle of $R(A)$ and let $I = J + n\mathbb{Z}$. It turns out that B is the endomorphism algebra of the direct sum of indecomposable projective A-modules whose coordinates lie in I. Here, by coordinates, we follow a slightly modified version of the notation in [36] where indecomposable projective A-modules are indexed by ordered sequences of length $d+1$ with certain restrictions given by $\underline{\ell}$.

We give an example here.

Example 3.1.3. *Let $d = 2$, $n = 5$ and $\underline{\ell} = (3, 4, 4, 4, 4)$ be a periodic Kupisch series. Then the Gabriel quiver $Q^2_{\underline{\ell}}$ of the 2-Nakayama algebra $A = A^2_{\underline{\ell}}$ is given as follows. Here the leftmost and the rightmost lines should be identified.*

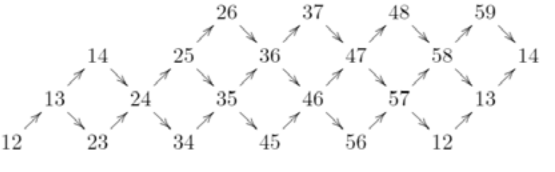

Figure 3.1. Gabrial quiver of A

The Auslander-Reiten quiver of the distinguished $2\mathbb{Z}$-cluster tilting subcategory $\mathcal{M} \subseteq \mathrm{mod}A$ is given below.

Figure 3.2. The AR-quiver of $\mathcal{M}$

The resolution quiver of $R(A)$ is given as follows.

Figure 3.3. The resolution quiver of A

By picking up the indecomposable projective A-modules whose coordinates lie in $\{2,3,4,5\}$ and taking the endomorphism algebra B of their direct sum. We conclude that A and B are singular equivalent. Indeed, B is the self-injective 2-Nakayama algebra $A^2_{4,3}$ whose Gabriel quiver is given as follows.

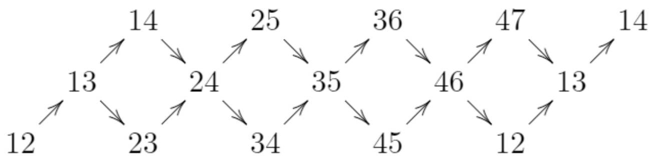

Figure 3.4. The Gabriel quiver of B

We give the Auslander-Reiten quiver of the distiguished $2\mathbb{Z}$-cluster tilting subcategory $\mathcal{N} \subseteq \mathrm{mod}B$ below. The interested reader may compare the Auslander-Reiten quiver of $\mathcal{M}$ consisting objects whose coordinates lie in $\{2,3,4,5\}$, with the Auslander-Reiten quiver of $\mathcal{N}$. In fact, the two quivers coincide.

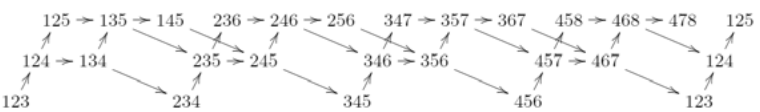

Figure 3.5. The AR-quiver of $\mathcal{N}$

3.1.2 Discussion

These results demonstrate that d-cluster tilting subcategories, together with their wide subcategories, are useful in constructing singular equivalences. Furthermore, if additionally they are $d\mathbb{Z}$-cluster tilting, the resulting equivalence restricts to an equivalence of $(d+2)$-angulated categories.

Further, our result provides concrete examples of the derived Auslander-Iyama correspondence introduced by Jasso-Muro [35]. This correspondence provides a method of recognizing such a triangulated category $\mathcal{T}$ with a basic $d\mathbb{Z}$-cluster tilting object c via the endomorphism algebra $\mathrm{End}_{\mathcal{T}}(c)$. In particular, [35, Theorem 6.5.2] gave a recognition theorem of the stable module categories of self-injective higher Nakayama algebras. Our result extends this recognition theorem to the singularity categories of all higher Nakayama algebras.

Finally, we introduced the resolution quiver of a d-Nakayama algebra A, a purely combinatorial invariant that encodes important homological properties of A. This invariant can be studied further, potentially uncovering homological features of the algebra.

3.1.3 Future work

The singularity category of an algebra A captures the asymptotic behavior of the syzygy functor Ω_A. More precisely, it is triangulated equivalent to the stabilization of the stable module category obtained by formally inverting Ω_A. Our construction is expected to apply to algebras with $d\mathbb{Z}$-cluster tilting subcategories, since in this setting the behavior of the syzygy functor is well understood. A natural direction for future work would be to apply our construction to specific algebras admitting $d\mathbb{Z}$-cluster tilting subcategories, such as those constructed by Vaso [52–55].

For d-Nakayama algebras, the resolution quiver appears to be the appropriate combinatorial tool for capturing their singularity category. As in the classical case, many homological properties of Nakayama algebras can be deduced from the combinatorial structure of their resolution quivers. It is therefore natural to hope for analogous results for the resolution quivers of d-Nakayama algebras.

3.2 Paper II

3.2.1 Summary

In this paper, we introduce the concept of 2-subhomogeneous d-representation finite algebras. Such an algebra has global dimension d. Moreover, there is a unique d-cluster tilting subcategory which consists of direct sums of projective and injective modules. Since we assume global dimension d, such d-cluster tilting subcategories are trivially $d\mathbb{Z}$-cluster tilting.

Definition 3.2.1. *Let A be a finite dimensional algebra. We say A is 2-subhomogeneous d-representation finite if* $\mathrm{gldim} A = d$ *and $A \oplus DA$ is a d-cluster tilting module.*

Such algebras appear in the classification of d-representation finite acyclic Nakayama algebras [52]. In fact, they are all 2-subhomogeneous d-representation finite. A similar result for acyclic higher Nakayama algebras is established in Paper III. More precisely, among all homogeneous acyclic d-Nakayama algebras, those that are nd-representation finite for some integer $n > 1$ are all 2-subhomogeneous. In this paper, we provide a construction of such algebras via tilting complexes over a fractionally Calabi-Yau algebra.

Theorem 3.2.2. *Let A be a finite dimensional algebra such that* $\operatorname{gldim} A < \infty$ *and A is $\frac{d}{a+1}$-CY. Suppose $X \in D^b(A)$ is such that $T = \bigoplus_{i=0}^{a-1} \nu^i X$ is a tilting complex. Denote by $B = \operatorname{End}_{D^b(A)}(T)$ the endomorphism algebra of T. Then the following statements hold.*

(i) There is a triangle equivalence $F : \mathcal{D}^b(A) \xrightarrow{\sim} \mathcal{D}^b(B)$ which restricts to an equivalence between $d\mathbb{Z}$-cluster tilting subcategories $F : \mathcal{U}_d(T) \xrightarrow{\sim} \mathcal{U}_d(B)$.

(ii) B is 2-subhomogeneous d-representation finite.

Our construction applies to all $(d-1)$-Auslander algebras A^d_{n+1} for which n and d are coprime. In this setting, we construct a tilting complex of the form described in Theorem 3.2.2, where X is projective. The resulting endomorphism algebra B is 2-subhomogeneous and nd-representation finite. As a consequence, we obtain an $nd\mathbb{Z}$-cluster tilting subcategory of $\mathcal{D}^b(A^d_{n+1})$ which was previously unknown.

Theorem 3.2.3. *Let $A = A^d_{n+1}$ be a $(d-1)$-Auslander algebra of type $\mathbb{A}$ with* $\gcd(n,d) = 1$. *There is a tilting complex $T = \bigoplus_{i=1}^{n+d} \nu^i P$ where P is the basic projective module with $|P| = \frac{1}{n+d}\binom{n+d}{d}$ defined in Paper III Definition 4.18. Denote by $B = \operatorname{End}_{\mathcal{D}^b(A)}(T)$ the endomorphism algebra of T. Then the following statements hold.*

(i) There is a triangle equivalence $F : \mathcal{D}^b(A) \xrightarrow{\sim} \mathcal{D}^b(B)$ which restricts to an equivalence between $nd\mathbb{Z}$-cluster tilting subcategories $F : \mathcal{U}_{nd}(T) \xrightarrow{\sim} \mathcal{U}_{nd}(B)$.

(ii) B is 2-subhomogeneous $nd\mathbb{Z}$-representation finite, i.e. $\operatorname{gldim} B = nd$ *and $B \oplus DB$ is an $nd\mathbb{Z}$-cluster tilting module.*

(iii) B is $\frac{nd}{n+d+1}$-Calabi-Yau.

We describe the indecomposable projective A-modules in terms of lattice paths from the bottom left corner to the top right corner in a $(d \times n)$-rectangle. The condition $\gcd(n,d) = 1$ ensures that no lattice points lie on the diagonal of the rectangle. We then consider lattice paths that lie strictly below the diagonal, known as rational Dyck paths. Finally, we define P to be the direct sum of the indecomposable projective modules indexed by these rational Dyck paths.

It turns out the endomorphism algebra $B_0 = \mathrm{End}_A(P)$ is of independent interest, as it can be realized as a quotient algebra of the incidence algebra of the poset of rational Dyck paths. Moreover, it reveals the structure of B together with its nd-Auslander algebra [28] and its $(nd+1)$-preprojetcive algebra [33]. More precisely, B (respectively, the nd-Auslander algebra of B) can be described as the $(n+d)$-replicated algebra $B_0^{(n+d)}$ (respectively the $(n+d+1)$-replicated algebra $B_0^{(n+d+1)}$) of B_0. Furthermore, the $(nd+1)$-preprojective algebra of B is given by the $(n+d)$-fold trivial extension $T_{n+d}(B_0)$ of B_0. This yields the fractionally Calabi-Yau property of B_0 by applying [12, Theorem 1.4].

The results discussed above are collected in the following theorem.

Theorem 3.2.4. *Assume* $\gcd(n,d) = 1$. *Let* $s = \lceil \frac{d}{n} \rceil$. *The following statements hold true.*

(i) $\mathrm{gldim} B_0 = d - s$ *and* B_0 *is* $\frac{(n-1)(d-1)}{n+d+1}$*-Calabi-Yau.*

(ii) $B_0^{(n+d)}$ *is nd-representation finite and* $\frac{nd}{n+d+1}$*-Calabi-Yau.*

(iii) $\mathrm{gldim} B_0^{(n+d+1)} \leq nd + 1 \leq \mathrm{domdim} B_0^{(n+d+1)}$.

(iv) $T_{n+d}(B_0)$ *is self-injective. Moreover,* $\underline{\mathrm{mod}} T_{n+d}(B_0)$ *is* $(nd+1)$*-Calabi-Yau and admits an* $(nd+1)$*-cluster tilting module.*

3.2.2 Discussion

The methods and results in this paper are interesting for several reasons.

First, we provide a construction of 2-subhomogeneous d-representation finite algebras as endomorphism algebras of tilting complexes of a particular form over fractionally Calabi-Yau algebras. This construction has potential applications in other fractionally Calabi-Yau categories, which naturally arise in areas such as combinatorics, algebraic geometry and symplectic geometry.

Furthermore, the new derived equivalences of higher Auslander algebras of type $\mathbb{A}$ are of particular interest due to their connections with the wrapped Fukaya categories of symmetric products of disks with finitely many stops [18], as well as with Fukaya-Seidel categories associated to certain singularities [17]. The derived equivalence constructed in this paper may offer new perspectives on the understanding of these categories.

Moreover, we introduce a new combinatorial model for higher Auslander algebras A_n^d of type $\mathbb{A}$ in terms of lattice paths in a rectangle. The Gabriel quiver of A_n^d can be embedded into a d-dimensional simplex, and thus becomes increasingly complex and difficult to visualize as d grows. In contrast, the lattice path model remains inherently two-dimensional, offering a more accessible perspective. This approach may prove useful in understanding further homological properties of these algebras.

Finally, the construction yields a derived equivalence between higher Auslander algebras and replicated algebras, generalizing the result in [38]. Moreover, certain m-fold trivial extensions can be realized as the higher preprojective algebras of specific d-representation finite algebras. These connections merit further investigation.

3.2.3 Future work

This work naturally leads to several directions for further research.

First, it is an interesting problem to study derived equivalenes between the incidence algebra of a poset and its quotient by monomial relations. This question was investigated in [14] and [21]. In context of this paper, the algebra B_0 arises as such a quotient of the incidence algebra of the lattice D of rational Dyck paths. When $d = 3$, applying the construction in [14] shows that B_0 and D are derived equivalent. However, for $d > 3$, additional machinery needs to be developed. Computational experiments on certain large examples indicate that the Coxeter polynomials of B_0 and D coincide, suggesting that a derived equivalence between B_0 and D may exist in general.

Secondly, the fractionally Calabi-Yau property of lattices naturally arises as a question. As conjectured by Chapoton [13], the derived category of a fractionally Calabi-Yau poset may be equivalent to the Fukaya-Seidel category of certain singularities. Positive results in this direction have been obtained for Tamari lattices [46] and for the lattice of order ideals of the product of two chains [21]. In the context of this paper, the algebra B_0 is fractionally Calabi-Yau. Therefore, if B_0 and D are derived equivalent, one can conclude that the lattice of rational Dyck paths is fractionally Calabi-Yau as well.

Moreover, the combinatorial method developed in this paper for verifying tilting complexes demonstrates that combinatorial techniques provide a powerful tool for addressing problems in homological algebra. The construction of d-representation finite algebras could potentially be applied in other fractionally Calabi-Yau categories that admit a combinatorial description. A natural starting point would be to investigate the singularity categories of the boundary algebras arising from the dimer model on a disk [11].

3.3 Paper III

3.3.1 Summary

In this paper, we investigate which d-Nakayama algebras admit an $nd\mathbb{Z}$-cluster tilting subcategory for $n > 1$. The radical square zero case is already covered by results on classical Nakayama algebras due to Herschend-Kvamme-Vaso [26]. For each remaining non-self-injective d-Nakayama algebra, we provide a complete classification of its $nd\mathbb{Z}$-cluster tilting subcategories. In fact, there exists at most one for a suitable integer n. A self-injective d-Nakayama algebra is determined by two positive integers m and ℓ. We prove that an $nd\mathbb{Z}$-cluster tilting subcategory is only possible if n divides both m and $(\ell-2)$. Moreover, in the case $n=\ell-2$, we show that such a subcategory does exist by constructing an explicit example.

Depending on the shape of the quivers and relations, or equivalently the shape of the Kupisch series, we divide the results into four cases: acyclic homogeneous case, acyclic non-homogeneous case, cyclic non-homogeneous case and self-injective case.

The classification result for the acyclic homogeneous case is given as follows. Notice that in the case when $\ell \geq 3$, such d-Nakayama algebras which admit an $nd\mathbb{Z}$-cluster tilting subcategory are 2-subhomogeneous.

Theorem 3.3.1. *Let $A = A^d_{\ell,m}$ be a homogeneous acyclic d-Nakayama algebra. There exists an $nd\mathbb{Z}$-cluster tilting subcategory $\mathcal{C} \subseteq \operatorname{mod} A$ if and only if one of the following conditions is satisfied.*

(a) $\ell = 2$ and $n \mid (m-1)$.

(b) $\ell \geq 3$, $(d+1)|(n-2)$ and $m = \frac{n-2}{d+1}(\ell+d-1)+\ell+1$.

Moreover, we have in case (a) that

$$\begin{aligned}\mathcal{C} &= \operatorname{add}(\{A\} \cup \{\tau_{nd}^{-k}(M(1,2,\dots,d+1))\}) \\ &= \operatorname{add}(\{A\} \cup \{M(1+kn,\dots,d+1+kn) \mid 1 \leq k \leq \frac{m-1}{n}\})\end{aligned}$$

and in case (b) that $\mathcal{C} = \operatorname{add}(A \oplus DA)$.

To get the classification result for acyclic non-homogeneous case, we introduced the sink-source gluing of two algebras defined by quivers with relations. Such a gluing can be viewed as a special case considered in [8, 27, 39, 53, 54].

Definition 3.3.2. *Let $A = \Bbbk Q_A/\mathcal{R}_A$ be an algebra in which there is a sink vertex a in Q_A and $B = \Bbbk Q_B/\mathcal{R}_B$ be an algebra in which there is a source vertex b in Q_B. The sink-source gluing of A and B, denoted by $\Lambda := A \Delta B$, is defined to be the algebra $\Lambda = \Bbbk Q_\Lambda/\mathcal{R}_\Lambda$ given by a quiver with relations where*

$$\begin{aligned}(Q_\Lambda)_0 &= ((Q_A)_0 \cup (Q_B)_0)/(a \sim b) \\ (Q_\Lambda)_1 &= (Q_A)_1 \cup (Q_B)_1.\end{aligned}$$

Note that Q_A and Q_B are full subquivers of Q_Λ. We set $e_A := \sum_{i\in(Q_A)_0} e_i$ for the sum of all primitive idempotents of Λ corresponding to the vertices of Q_A, and similarly we set $e_B := \sum_{i\in(Q_B)_0} e_i$. The relations $\mathcal{R}_\Lambda$ are given as follows.

$$\mathcal{R}_\Lambda = \langle \mathcal{R}_A + \mathcal{R}_B + (1-e_A)\Bbbk Q_\Lambda(1-e_B) \rangle.$$

Let $\Lambda = A\Delta B$. We have a pull-back diagram as follows.

$$\begin{array}{ccc} \Lambda & \xrightarrow{{}^{\Lambda}_{\mathrm{B}}\pi} & B \\ {\scriptstyle {}^{\Lambda}_{\mathrm{A}}\pi}\big\downarrow & & \big\downarrow \\ A & \longrightarrow & \Bbbk. \end{array}$$

Here

$${}^{\Lambda}_{\mathrm{A}}\pi : \Lambda \to \Lambda/\langle 1 - e_A\rangle \cong A \text{ and } {}^{\Lambda}_{\mathrm{B}}\pi : \Lambda \to \Lambda/\langle 1 - e_B\rangle \cong B$$

are ring epimorphisms. In particular,

$${}^{\Lambda}_{\mathrm{A}}\pi_* : \mathrm{mod}A \to \mathrm{mod}\Lambda \text{ and } {}^{\Lambda}_{\mathrm{B}}\pi_* : \mathrm{mod}B \to \mathrm{mod}\Lambda,$$

are fully faithful embeddings which are also exact.

Let $\mathcal{C} \subseteq \mathrm{mod}A$ and $\mathcal{D} \subseteq \mathrm{mod}B$. We may view $\mathcal{C}$ and $\mathcal{D}$ as subcategories of $\mathrm{mod}\Lambda$ and define the gluing

$$\mathcal{C}\Delta\mathcal{D} = \mathrm{add}\{\mathcal{C}, \mathcal{D}\} \subseteq \mathrm{mod}\Lambda.$$

To be more precise, we write $\Lambda = A\Delta_S B$ where S is the simple Λ-module at vertex a or equivalently b in the gluing process. Denote by

$$d\text{-}CT(A) = \{\mathcal{M}_A = \mathrm{add}M_A \subseteq \mathrm{mod}A : d\text{-cluster tilting}\}$$

the set of d-cluster tilting subcategories of $\mathrm{mod}A$ which admit an additive generator. Respectively we use $d\text{-}CT(B)$ for B. Let

$$d\text{-}CT_S(\Lambda) = \{\mathcal{M} = \mathrm{add}M \subseteq \mathrm{mod}\Lambda : d\text{-cluster tilting and } S \in \mathcal{M}\}.$$

We obtain the following Proposition.

Proposition 3.3.3. *Let $\Lambda = A\Delta_S B$. Then there is a bijection*

$$\begin{aligned} d\text{-}CT(A) \times d\text{-}CT(B) &\leftrightarrow d\text{-}CT_S(\Lambda) \\ (\mathcal{M}_A, \mathcal{M}_B) &\mapsto \mathcal{M}_A\Delta\mathcal{M}_B \\ (\mathcal{M} \cap \mathrm{mod}A, \mathcal{M} \cap \mathrm{mod}B) &\leftarrow\!\shortmid \mathcal{M} \end{aligned}$$

Moreover, replacing d-CT by $d\mathbb{Z}$-CT, the above bijiection restricts.

The sink-source gluing of two acyclic d-Nakayama algebras is again an acyclic d-Nakayama algebra. By applying the above bijection we obtain the classification result for acyclic non-homogeneous case.

Theorem 3.3.4. *Let $A = A^d_\ell$ be an acyclic d-Nakayama algebra. Then* $\mathrm{mod}A$ *has an $nd\mathbb{Z}$-cluster tilting subcategory $\mathcal{C}$ if and only if the following conditions hold.*

(i) $A = A_1 \Delta A_2 \Delta \cdots \Delta A_t$ *with* $A_i = A^d_{\ell_i, m_i+1}$ *for* $1 \leq i \leq t$.
(ii) For $1 \leq i \leq t$*, one of the following conditions holds.*
(a) $\ell_i = 2$ *and* $n = m_i$.
(b) $(d+1)|(n-2)$ *and* $m_i = \frac{n-2}{d+1}(\ell_i + d - 1) + \ell_i$.
Moreover, in that case $\mathcal{C} = \mathrm{add}(\{A \oplus DA\} \cup \{S_i \mid 1 \leq i \leq t-1\})$ *where*

$$S_i = M(1 + \sum_{k=1}^{i} m_i, 2 + \sum_{k=1}^{i} m_i, \ldots, d+1 + \sum_{k=1}^{i} m_i).$$

In particular, it was shown in Paper III that for acyclic d-Nakayama algebras, the set of $nd\mathbb{Z}$-cluster tilting subcategories and the set of partial $nd\mathbb{Z}$-cluster tilting subcategories coincide.

Now we continue with the cyclic non-homogeneous case.

For an acyclic d-Nakayama algebra A, we obtain a cyclic d-Nakayama algebra A^c by identifying its sink with its source in a similar way to how we defined gluing. We call this a self-gluing. Reversing this process leads to the notion of self-degluing, which requires the choice of a deglue point (assuming such exists).

As shown in Paper III, the existence of an $nd\mathbb{Z}$-cluster tilting subcategory of a d-Nakayama algebra imposes certain restrictions on the Kupisch series. Consequently each such cyclic non-homogeneous d-Nakayama algebra A admits a self-deglue point. We consider the self-degluing of A, denoted by A^a which is an acyclic non-homogeneous d-Nakayama algebra. Moreover an $nd\mathbb{Z}$-cluster tilting subcategory of mod$A$ induces an $nd\mathbb{Z}$-cluster tilting subcategory $\mathcal{C}^a \subseteq \mathrm{mod}A^a$. By applying Theorem 3.3.4, we obtain a necessary condition for A. We further show the necessary condition is in fact sufficient by considering the infinite sink-source self-gluing Λ of A^a following [53]. It was shown in [53] that $\mathcal{C}^a$ naturally induces an $nd\mathbb{Z}$-cluster tilting subcategory $\mathcal{C} \subseteq \Lambda$ which is obtained as the infinite self-gluing of $\mathcal{C}^a$. Moreover, $\mathcal{C}$ is $\mathbb{Z}$-equivariant and thus by applying the orbit construction, we have $\mathcal{C}/\mathbb{Z} \subseteq \mathrm{mod}\Lambda/\mathbb{Z}$ is $nd\mathbb{Z}$-cluster tilting. As $\Lambda/\mathbb{Z} \cong A$, the above construction gives an $nd\mathbb{Z}$-cluster tilting subcategory for A. We obtain the following classification result for cyclic non-homogeneous case.

Theorem 3.3.5. *Let* $A = A^d_{\underline{\ell}}$ *be a cyclic non-self-injective* d*-Nakayama algebra. Then* mod$A$ *admits an* $nd\mathbb{Z}$*-cluster tilting subcategory* $\mathcal{C}$ *if and only if the following conditions hold true.*
(i) $A = (A_1 \Delta A_2 \Delta \cdots \Delta A_t)^c$ *with* $A_i = A^d_{\ell_i, m_i}$ *for* $1 \leq i \leq t$.
(ii) For $1 \leq i \leq t$*, one of the following conditions holds true.*
(a) $\ell_i = 2$ *and* $n = m_i$.
(b) $(d+1)|(n-2)$ *and* $m_i = \frac{n-2}{d+1}(\ell_i + d - 1) + \ell_i$.

Moreover, in that case $\mathcal{C} = \operatorname{add}(\{A \oplus DA\} \cup \{S_i \mid 1 \leq i \leq t\})$ *where*

$$S_i = M(1+\sum_{k=1}^{i} m_i, 2+\sum_{k=1}^{i} m_i, \ldots, d+1+\sum_{k=1}^{i} m_i).$$

For a positive integer s, recall that an algebra Γ is called s-representation finite if $\operatorname{gldim}\Gamma = s$ and there exists an s-cluster tilting subcategory $\mathcal{C} = \operatorname{add} C \subseteq \operatorname{mod}\Gamma$ with $C \in \operatorname{mod}\Gamma$. Notice that in this case, $\mathcal{C}$ is trivially $s\mathbb{Z}$-cluster tilting. In [47, Example 7.4], the following question was posed: whether every s-representation finite d-Nakayama algebra with $d > 1$ is given by the acyclic homogeneous ones satisfying the numerical condition in Theorem 3.3.1 (b). We give an affirmative answer to this question as a consequence of Theorem 3.3.4 and Theorem 3.3.5.

Corollary 3.3.6. *A d-Nakayama algebra A is s-representation finite if and only if $A = A^d_{\ell,m}$ and $s = nd$ for some integer $n > 1$ such that*

$$(d+1)|(n-2) \text{ and } m = \frac{n-2}{d+1}(\ell+d-1)+\ell+1.$$

For the cyclic homogeneous case, the algebra $A = \widetilde{A}^d_{m-1,\ell}$ is self-injective. When $\ell = 2$, we may apply the classification for classical Nakayama algebra to get the following result.

Proposition 3.3.7. *Let $A = \widetilde{A}^d_{2,m-1}$ be a self-injective d-Nakayama algebra. Then* $\operatorname{mod} A$ *admits an $nd\mathbb{Z}$-cluster tilting subcategory $\mathcal{C}$ if and only if $n|(m-1)$. In this case,*

$$\begin{aligned}\mathcal{C} &= \operatorname{add}(\{A\} \cup \{\tau_{nd}^{k} M(i,\ldots,i+d) \mid k \in \mathbb{Z}\}) \\ &= \operatorname{add}(\{A\} \cup \{M(i+knd,\ldots,i+(kn+1)d) \mid k \in \mathbb{Z}\}).\end{aligned}$$

When $\ell \geq 3$, we have the following necessary condition such that an $nd\mathbb{Z}$-cluster tilting subcategory exists for A.

Proposition 3.3.8. *If* $\operatorname{mod} A$ *admits an $nd\mathbb{Z}$-cluster tilting subcategory, then $n \mid m$ and $n \mid (\ell-2)$.*

It seems to be infeasible to get a classification in this case. However as A can be viewed as the orbit category of the repetitive category of a $(d-1)$-Auslander algebra A' of type $\mathbb{A}$, by considering the tilting complex constructed in Paper II, we give an explicit example of an $nd\mathbb{Z}$-cluster tilting subcategory for each A with $\gcd(\ell-2,d) = 1$ and $n = \ell-2$.

Proposition 3.3.9. *Assume* $\gcd(\ell-2,d)=1$. *There is a tilting complex* $T \in \mathcal{D}^b(A')$ *such that*

$$\mathfrak{U} = \operatorname{add}\{\nu^i_{(\ell-2)d}(T) \mid i \in \mathbb{Z}\} \subseteq \mathcal{D}^b(A')$$

is an $(\ell-2)d\mathbb{Z}$*-cluster tilting subcategory. Moreover,* $\mathfrak{U}$ *is* $\mathbb{Z}$*-equivariant and thus* $\underline{\mathcal{U}} = \mathfrak{U}/\mathbb{Z}$ *is an* $(\ell-2)d\mathbb{Z}$*-cluster tilting subcategory of* $\underline{\operatorname{mod}}A$*. Consequently, its preimage under* $\operatorname{mod}A \to \underline{\operatorname{mod}}A$

$$\mathcal{U} \subseteq \operatorname{mod}A$$

is an $(\ell-2)d\mathbb{Z}$*-cluster tilting subcategory.*

3.3.2 Discussion

This paper makes the following contributions.

Firstly, we gave a full classification of $nd\mathbb{Z}$-cluster tilting subcategories for a non-self-injective d-Nakayama algebra and any $n>1$. Although there are examples of algebras with $n\mathbb{Z}$-cluster tilting subcategories, there are few results that give a complete classification for a given family. Notable exceptions for which a classification result has been given are the classical Nakayama algebras [26]. Our classification recovers the results for classical Nakayama algebras and extends it to a larger family, where most algebras are of wild representation type.

Secondly, it is an interesting question to ask for an algebra A, if there exists an n-cluster tilting subcategory and an m-cluster tilting subcategory for $m \neq n$. Vaso [52] gave an answer for homogeneous Nakayama algebras with certain pairs (m,n). Our result gives examples for d-Nakayama algebras for (d,nd) for a suitable $n>1$.

Furthermore, the sink-source gluing of two algebras with d-cluster tilting (respectively $d\mathbb{Z}$-cluster tilting) subcategories can be applied to construct more algebras with d-cluster tilting (respectively $d\mathbb{Z}$-cluster tilting) subcategories. A more complicated gluing procedure was considered in [53] but restricted to representation-directed algebras. The main difficulty is to verify certain glued subcategories are d-cluster tilting. The sink-source gluing is much simpler but we can deal with much more complicated algebras by making use of the properties of d-cluster tilting subcategories.

Lastly, we introduce the concept of partial d-cluster tilting subcategories to tackle certain technical difficulties in verifying d-cluster tilting. They are weaker notions of d-cluster tilting and closed under taking intersections. In other words, partial d-cluster tilting subcategories of an algebra form a lattice, with d-cluster tilting subcategories being maximal elements. The benefit is that if a partial $d\mathbb{Z}$-cluster tilting subcategory exists, we can consider the smallest one which leads to combinatorial constraints. As it happens, for non-self-injective Nakayama algebras these minimal partial $d\mathbb{Z}$-cluster tilting subcategories are in fact $d\mathbb{Z}$-cluster tilting, which yields the classification.

3.3.3 Future work

As mentioned in the discussion, one possible future research topic would be to apply the sink-source gluing to other families of algebras with d-cluster tilting subcategories but not representation-directed, for example, the infinite families of 2-representation finite algebras obtained from self-injective quivers with potentials [24], radical square zero algebras [55], and d-Nakayama algebras with different d.

Secondly, although d-cluster tilting subcategories encode essential homological properties of the algebra, it is in general very difficult to verify if a given subcategory is d-cluster tilting or not. However, partial d-cluster tilting subcategories are much easier to verify. An interesting topic would be to study the poset structure of partial d-cluster tilting subcategories which might give clues to the existence or even classification of d-cluster tilting subcategories as they appear as maxima in the poset.

4. Sammansfattning på svenska

En algebra är en mängd vars element kan adderas och multipliceras på ett sätt som är kompatibelt med skalning via skalärer i någon kropp. En representation av en algebra representerar dess element som linjära avbildningar på vektorrum över samma kropp. På så sätt kan abstrakta algebraiska strukturer studeras genom hur de agerar på mer konkreta objekt. Representationsmorfismer bevarar dessa aktioner och visar hur den algebraiska strukturen avspeglas i förhållandet mellan olika representationer.

Kategoriteori utgör ett naturligt ramverk för att organisera matematiska objekt och förhållanden mellan dem. Representationerna av en algebra tillsammans med tillhörande morfismer utgör det som kallas modulkategorin. En fundamental egenskap hos denna kategori är att varje modul kan delas upp på ett essentiellt unikt sätt i en direkt summa av ouppdelbara moduler. Att förstå dessa grundläggande byggstenar och hur de interagerar med varandra är ett centralt problem i representationsteori.

Ett fundamentalt verktyg i representationsteori är Auslander-Reiten teori, som utforskar representationer från ett homologiskt perspektiv. Teorin ger metoder för att analysera modulkategorin, och ger ett sätt att beskriva ouppdelbara moduler via Auslander-Reiten translation och nästa kluvna följder.

Perspektiven från Auslander-Reiten teori utvidgades till högre dimension av Iyama. Istället för att studera hela modulkategorin är fokus på att studera en viss delkategori som kallas d-kluster tilting. Denna delkategori tar vara på essentiell homologisk information, men är på vissa sätt lättare att få grepp om, jämfört med hela modulkategorin. I denna version av teorin spelar högre varianter av de klassiska begreppen, såsom d-Auslander-Reiten translationen och d-nästan kluvna följder, en central roll. En mer strikt version av begreppet d-kluster tilting är $d\mathbb{Z}$-kluster tilting som introducerades av Iyama och Jasso. Detta begrepp fångar djupare strukturella egenskaper och är mer användbart i studiet av relaterade kategorier som den deriverade kategorin och singuläritetskategorin.

Förekomsten av en $d\mathbb{Z}$-kluster tilting delkategori begränsar kraftigt vilka algebror som kan förekomma och klassifikationsresultat är typiskt sett rättframma eller högst icketriviala. I denna avhandling utvecklar vi metoder för att konstruera och identifiera sådana delkategorier. En konstruktion är via såkallade 2-subhomgena d-representations ändliga algebror. Ett klassifikationsresultat är för d-Nakayama algebror. Dessutom ger förekomsten av $d\mathbb{Z}$-kluster tilting delkategorier upphov till ekvivalenser av både deriverade kategorier och singuläritetskategorier för vissa högre Nakayama algebror.

I Artikel I konstruerar vi ekvivalenser av singuläritetskategorier mellan d-homologiska par $(A, \mathcal{M})$ och $(B, \mathcal{N})$. Singuläritetskategorin beskriver hur syzygy-funktorn beter sig asymptotiskt, vilket också kan beskrivas inom $d\mathbb{Z}$-kluster tilting delkategorierna $\mathcal{M}$ och $\mathcal{N}$. Genom att utnyttja detta visar våra resultat hur sådana delkategorier kan användas för att studera singuläritetskategorier. Som tillämpning visar vi att varje d-Nakayama algebra är singulärt ekvivalent med en självinjektiv d-Nakayama algebra. Denna ekvivalens kan återskapas från upplösningskogret som är en kombinatorisk invariant vi tillordnar varje d-Nakayama algebra. Våra resultat generaliserar klassiska resultat för Nakayama algebror och ger ett alternativ bevis för en liknande ekvivalens beskriven av McMahon.

I Artikel II introducerar vi begreppet 2-subhomgena d-representationsändliga algebror. Dessa är algebror vars modulkategorier innehåller en $d\mathbb{Z}$-kluster tilting delkategori bestående av direkta summor av projektiva och injektiva moduler. Vi visar också att sådana algebror kan konstrueras genom vissa tiltingkomplex över fraktionella Calabi-Yau algebror. Som tillämpning visar vi att varje högre Auslander algebra av typ $\mathbb{A}$ som uppfyller ett visst villkor är deriverat ekvivalent med en viss replikerad 2-subhomgen d-representationsändlig algebra.

I artikel III studerar vi vilka d-Nakayama algebror som har $nd\mathbb{Z}$-kluster tilting delkategorier för $n > 1$. Bland dessa är algebrorna med radikalkvadrat noll redan utredda av Herschend, Kvamme och Vaso. För varje återstående icke-självinjektiv d-Nakayama algebra ger vi en fullständig klassifikation av dess $nd\mathbb{Z}$-kluster tilting delkategorier. Mer specifikt visar vi att det finns som mest en för ett lämpligt heltal n. För självinjektiva d-Nakayama algebror som uppfyller ett visst villkor visar vi genom exempel att en $nd\mathbb{Z}$-kluster tilting delkategori existerar. Vi uppnår detta genom att tillämpa metoderna som utvecklats av Darpö och Iyama på algebrorna som vi konstruerade i artikel II.

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