

# UPPSALA UNIVERSITET

DEGREE PROJECT C IN PHYSICS - 1FA599

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## Modelling the neutron-emission energy spectrum from beam-target T+T fusion reactions

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**Abstract**

Nuclear fusion can be applied to produce energy in tokamaks, where plasma containing deuterium (D) and tritium (T) has to be kept in delicate conditions to allow for a steady rate of exothermic reactions. Neutron spectroscopy is therefore employed to understand plasma properties, by measuring the neutrons emitted by D+D, D+T and T+T fusion. A previous study has computed the kinetic energy spectrum for neutrons exiting the beam-target T+T fusion reaction by utilising detailed nuclear physics models and matching with time-of-flight data gathered by the time-of-flight spectrometer TOFOR, stationed at the now-retired tokamak JET. This spectrum is most appropriate in the COM-frame, yet has been used for models fitted to data gathered by TOFOR (stationed in the LAB-frame) as it is assumed to be sufficiently accurate. This project aims to quantify differences in neutron energies when observed from the different inertial systems, hence testing the assumption. By applying Lorentz transformations and using a pre-existing software called the DRESS code, the kinetic energy spectrum based in the LAB-frame for a neutron emitted towards the time-of-flight spectrometer is produced. The neutron energy spectra is then passed through TOFOR's response function. The spectra depict great variations between the two energy spectra and when transformed into time-of-flight spectra, the same discrepancies diminish drastically. The time-of-flight spectra in the two different frames exhibit almost identical behaviours, except for a small peak at low times of flight. This result highlights its inability to resolve small differences in time-of-flights for neutrons travelling at high-velocities and should be taken into account in future studies involving TOFOR measurements.

## Svensk sammanfattning

Kärnfusion kan tillämpas för att producera energi i tokamaks, där deuterium (D) och tritium (T) bygger upp ett plasma som kräver delikata förhållanden för att upprätthålla en stadig takt av exoterma reaktioner. Neutronspektroskopi används för att förstå plasmats egenskaper genom att mäta neutronerna som emitteras från D+D-, D+T- och T+T-fusion. En tidigare studie har tillämpat detaljerade kärnfysiska modeller samt flygtidsdata som erhållits med en flygtidsspektrometer TOFOR, tidigare stationerad vid tokamaken JET, vilket har producerat ett spektrum för kinetiska energin hos en av neutronerna som emitterats från fusionsreaktionen mellan två tritiumkärnor, en hög-energetisk och en låg-energetisk. Metoderna som tidigare studien använt sig av har producerat ett energispektrum som är mest lämpad för center-of-momentum (COM) inertialsystemet, men har använts för analys av flygtidsdata som tagits med TOFOR, under antagandet att skillnaden inte är för betydande. Detta projekt är ämnad till att uppskatta skillnaden mellan dessa två inertialsystem, om antagandet är applicerbar. Lorentztransformen och befintlig mjukvara kallad DRESS-koden appliceras för att erhålla ett energispektrum i LAB-systemet för en av neutronerna som utsänds mot flygtidsspektrometern. TOFOR's responsfunktion används för att transformera energispektrumen till flygtidsspektrumen, vilket tillåter jämförelsen mellan det som beräknats i tidigare studien och det som TOFOR observerar. Energispektran visar stora skillnader mellan LAB- och COM-baserade observationer, men samma variationer minimeras vid transformation till flygtidsspektra. TOFORs responsfunktion kan producera varierade energispektra från nästintill identiska flygtidsspektra, vilket belyser spektrometers oförmåga att upplösa små skillnader i flygtid för neutroner som färdas med höga hastigheter. Denna egenskap bör tas i hänsyn vid framtida studier som involverar TOFOR-mätningar.

## List of abbreviations

<b>TOFOR</b>	Time-Of-Flight Optimized for Rate
<b>NBI</b>	Neutral Beam-Injection
<b>COM</b>	Center Of Momentum
<b>DRESS</b>	Directional Relativistic Spectrum Simulator

## Acknowledgments

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# 1 Introduction

In this section, a short introduction to the phenomenon of fusion is given.

## 1.1 Nuclear Physics: Fusion

In the center of the stars that light up the night sky, a type of nuclear reaction called fusion takes place. As the light nuclei inside the star are pushed closer to each other, the Coulomb barrier is overcome and two light elements can interact and form a heavier atomic nucleus. The reaction is exothermic for certain elements, which allows the stars to keep shining brightly.

The exothermic property of stellar nuclear fusion stems from the difference in binding energies between the initial and final state particles. The binding energy is defined as the amount of energy required to separate the nucleons in a nucleus from each other [1], generally visualised with the graph in figure 1. This value differs between nuclei, as excitation energy, atomic number and mass number affect it. As light elements with lower binding energy per nucleon fuse into elements with higher binding energy per nucleon, the reaction becomes exothermic. Similarly for fission reactions, heavier elements with low binding energy per nucleon split into lighter elements with higher binding energy per nucleon, thus releasing energy in an exothermic reaction.

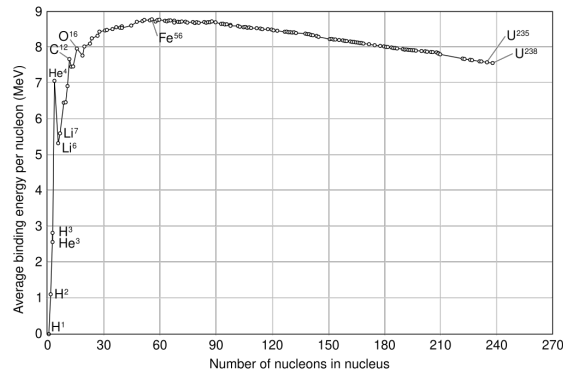


Figure 1: Binding energy per nucleon for a collection of specific nuclei. In this plot, the maximum value is for Fe-56. [2]

## 2 Background - Fusion Power

The potential of fusion power is great, which is the reason that fusion reactors able to generate electricity are being developed. Institutions and countries are working together in many projects with the aim of making the technology commercially available. Currently, the biggest of these projects is ITER, where the largest fusion reactor is being built [3]. The aim of the ITER project is to obtain plasma conditions stable enough for a prolonged generation of fusion power, which might allow for more energy to be harnessed than what is used to power the reactor. Another project, which now holds the world record for most energy produced by fusion is JET, a fusion reactor located in the United Kingdom which has recently been retired.

There are two main technologies considered when building research reactors, laser-induced implosion and magnetic confinement [4], with only the latter being discussed in this report. The most prevalent type of magnetic confinement fusion reactor is called a tokamak, where the plasma is heated to 100 million Kelvin and kept inside the reactor with strong toroidal and poloidal magnets, which produce a helical magnetic field [5]. A vacuum is created and the fuel is injected into the magnetic field, where it is heated to the required temperature for fusion. The charged nuclei move in helical paths, approximately orbiting the resulting magnetic field (given by the black-colored lines in figure 2).

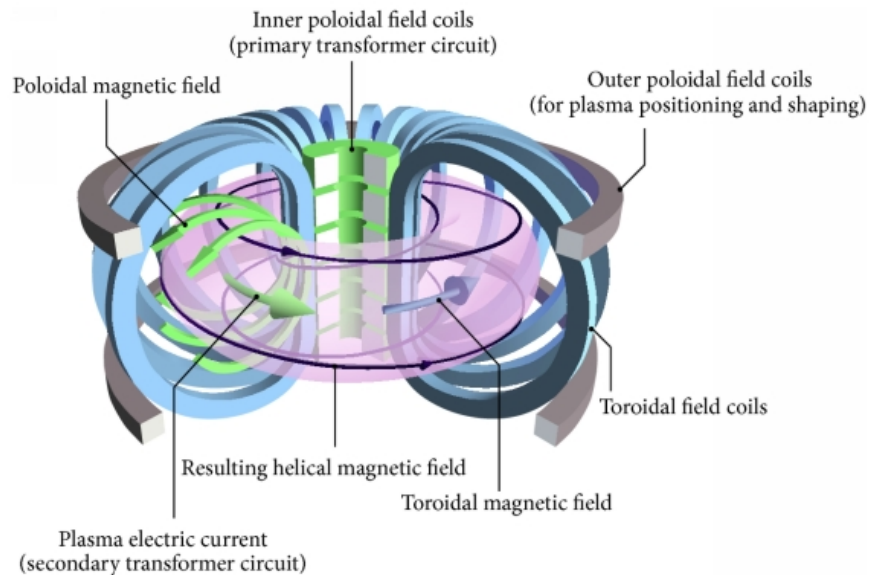


Figure 2: A schematic of a tokamak's magnets and the resulting B-field. [6]

The plasma in the tokamak can be heated with three methods, only one of which is relevant in this report. A beam of neutral particles, such as hydrogen isotopes, is accelerated to high kinetic energies outside of the reactor and then injected into the plasma. This method is called the neutral beam injection (NBI) [5], as the particles are charge-neutral, allowing them to reach further inside the reactor without being affected by the magnetic field. The plasma is heated through Coulomb collisions with these energetic particles.

## 2.1 The Tritium-Tritium reaction

For tokamaks, the preferred fuel is D-T plasma, which contains deuterium (D) and tritium (T) nuclei. The three main reactions with this fuel type are D+D, D+T and T+T, all of them happening between nuclei in the plasma or between a plasma nucleus and a NBI-particle. Of the three possible fusion combinations named above, only the third is considered in this project and presented below:



The total energy released in the TT-fusion reaction is 11 MeV, which is divided between the resulting particles as either kinetic or excitation energy. One of the neutrons can typically receive up to 9.5 MeV as kinetic energy, but not exceed that. It is also important to understand the behaviour of the reaction. The T+T cross section increases rapidly with reactant energy. For this reason, reactions between the beam ions and the thermal plasma ions are typically more frequent than reactions involving only thermal plasma ions. The majority of the neutrons emitted from the JET plasmas considered in this report therefore originate from reactions between an energetic ( $\sim 100$  keV) beam ion and a thermal ( $\sim 10$  keV) plasma ion, commonly referred to as "beam-target reactions". It can be expected that such a reaction includes particles moving differently compared to those in the fusion reaction between two plasma ions if observed from the LAB-frame.

## 2.2 Plasma diagnostics and TOFOR

To maintain the plasma in the delicate state required for fusion, diagnostics are utilised to measure many different variables without directly disturbing the plasma. Neutrons and photons can exit the magnetic field due to their inherent lack of charge and their energy and direction can be recorded with the intent to understand the behaviour of the plasma.

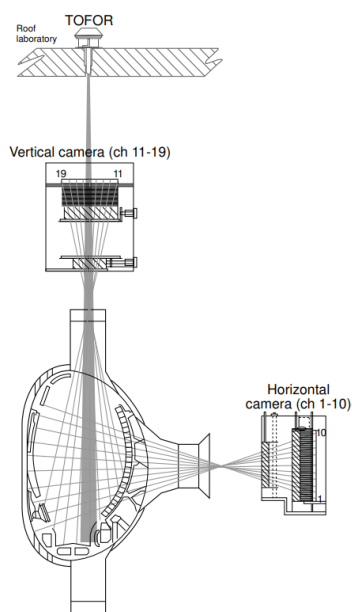


Figure 3: Illustration of TOFOR and its line of sight. Figure reproduced from [5]

TOFOR, an example of such an instrument is a neutron time-of-flight spectrometer which measures the difference in time between two events generated by the same neutron at two different detectors. A time-of-flight spectrum can be generated, which can be related to the incident neutron's energy spectrum by considering the distance between the two detectors and other factors regarding the geometry of TOFOR. This is called the instrument response function [7].

### 2.3 Direction of emission

The neutrons exit the reaction with a nonzero momentum, thus having a direction of travel. To qualitatively describe the neutron's trajectory, one can use the magnetic field lines as a basis for an angle  $\theta$  which will be named exit angle or emission angle. At the point of interaction, a neutron is emitted and its velocity can be denoted by  $\vec{u}$ , while the B-field at that point is given by  $\vec{B}$ . The scalar product of these vectors is given by  $\vec{u} \cdot \vec{B} = uB \cos \theta$ . For example, TOFOR is placed above the reactor and can therefore only measure neutrons emitted perpendicularly to the field lines, i.e. neutrons with an exit angle of  $\theta = 90^\circ$ . If the neutrons are emitted parallel or antiparallel to the B-field, i.e.  $\theta = 0^\circ$  or  $\theta = 180^\circ$ , the direction of emission would be pointing into or outwards of the reactor chamber cross section in figure 3.

Similarly, the tritium ions being used in the NBI enter the reactor at an angle with respect to the magnetic field lines. This angle is called the pitch angle and can influence the direction of emission of neutrons. At JET, the pitch angle of the NBI ions is generally around 60 degrees. See figure 4 for illustration of the angles, although the exit angle is not presented correctly since the neutron path should travel upwards in figure 3. Upwards in figure 4 would be outside of the figure.

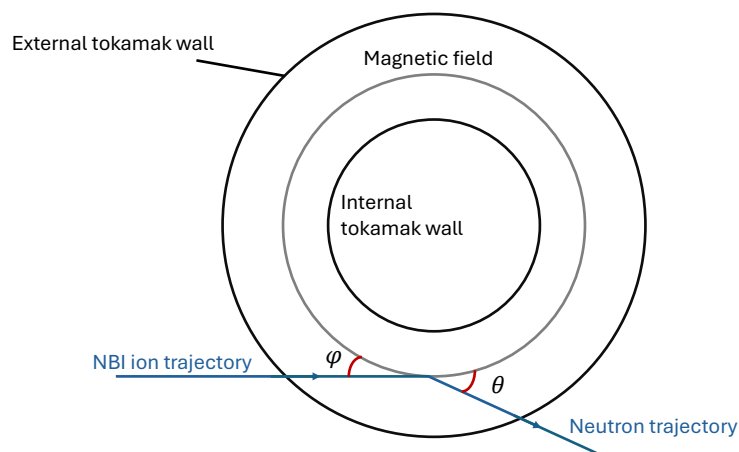


Figure 4: Illustration of the definition of the pitch angle  $\varphi$  and exit angle  $\theta$ . It should be considered that the exit angle is applied in the direction which can be regarded as normal to the paper itself.

## 2.4 Aim of Project

A previous study [8] has produced an energy spectrum for the emitted neutrons by fitting a parameterized R-matrix model to the time-of-flight data measured by TOFOR [8], as shown in figure 5a.

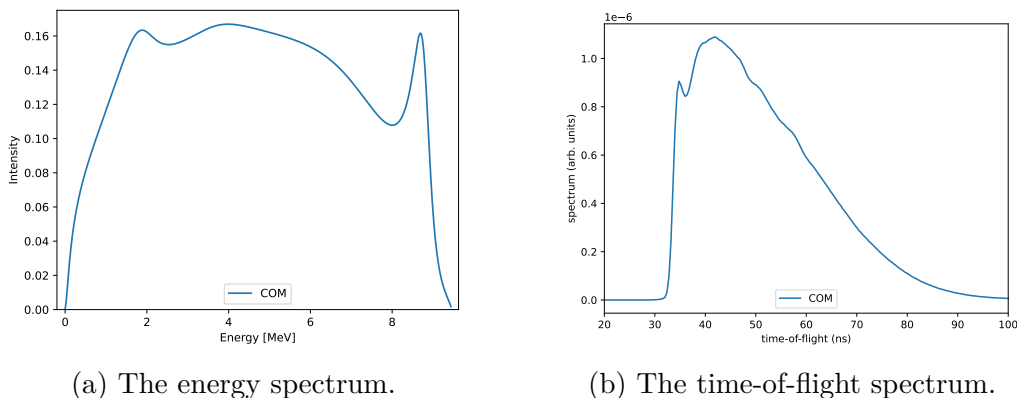


Figure 5: The relevant spectra in the COM-frame of neutrons emitted from NBI-plasma T+T fusion, as determined in [8].

The R-matrix framework outputs an isotropic spectrum for neutron observations

in the center-of-momentum (COM) frame, which has been utilised under the assumption that it is representative of the corresponding spectrum observed from the LAB-frame, where TOFOR is stationed. As mentioned in section 2.1, observing the movements of the particles from the LAB-frame would exhibit differing behaviours than COM-based observations. The aim of this study is therefore to quantify the difference between the spectrum observed in these two frames, in order assess whether the TOFOR analysis in [8] needs to be revised or if the COM/LAB differences are too small to have a significant impact.

### 3 Theory

Some concepts from Special Relativity and useful formulas are presented in this section of the report.

#### 3.1 Relativistic Kinematics

For a completely accurate description of particle kinematics, valid at all possible particle velocities, Newtonian mechanics is not enough. Thus, the theory of special relativity has to be implemented.

A particle's four-momentum is defined as

$$p^\mu = \begin{pmatrix} p^0 \\ p^1 \\ p^2 \\ p^3 \end{pmatrix} = \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix} = m_0 \gamma \begin{pmatrix} c \\ v_x \\ v_y \\ v_z \end{pmatrix} \quad (2)$$

with  $p^0$  being the time-component and the other three being the space-components, all with units of eV/c [9]. The  $\gamma$  in equation 2 is given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Each particle has a unique four-momentum for each inertial system, which yields increasingly convoluted computations. Thus, invariant quantities such as  $(p^\mu)^2 = \frac{E^2}{c^2} - p^2$  are used, as their numerical value does not change between reference frames. A useful result of invariants is the relation between energy and momentum. Since the four-momentum can be written with two quantities, the square can be written as follows:

$$(p^\mu)^2 = (p^0)^2 - (p^1)^2 - (p^2)^2 - (p^3)^2 \quad (3)$$

$$= m_0^2 c^2 = \frac{E^2}{c^2} - p^2. \quad (4)$$

Solving the expression for  $E$  yields the energy-momentum relation:

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}. \quad (5)$$

Another fundamental concept in the theory of relativity is the Lorentz transform [10], which takes in a particle's four-momentum and computes its equivalent in another inertial frame. There are a few transformations that can be done, depending on the movement of the inertial frame B relative to a rest frame A. One of these transformation is called a boost, which changes the velocity of frame A to that of frame B. If the boost is done in the x-direction only, i.e. the velocity of frame B relative to frame A is only pointing in the x-direction, the transform is of the form:

$$\Lambda_{\mu}^{\mu'} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (6)$$

where  $\beta$  is defined as follows:

$$\beta = \frac{v}{c} = \frac{\mathbf{p} \cdot \mathbf{c}}{E}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}. \quad (7)$$

The inverse of the Lorentz transform is of the same shape, with the difference being the sign of the  $\beta\gamma$  elements not being negative. A boost from inertial frame A to inertial frame B is done with the transform  $\Lambda_{\mu}^{\mu'}$ , and a boost from B to A uses the inverse transform  $\Lambda_{\mu'}^{\mu}$ .

Using the transform, one gets the four-momentum in one frame as the product of the transform and the four-momentum in another frame. This expression is shown below:

$$\begin{aligned} p^{\mu'} &= \Lambda_{\mu}^{\mu'} p^{\mu} \\ &= \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix} \\ &= \begin{pmatrix} \gamma \frac{E}{c} - \beta\gamma p_x \\ -\beta\gamma \frac{E}{c} + \gamma p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \frac{E'}{c} \\ p'_x \\ p'_y \\ p'_z \end{pmatrix}. \end{aligned} \quad (8)$$

In particle interactions, some quantities are conserved. Generally, energy and momentum are conserved in collisions in an inertial frame but not between frames.

### 3.2 The DRESS equation

The Division of Applied Nuclear Physics at Uppsala University has developed a software for computing the energy spectra of particles produced in nuclear reactions, called the Directional Relativistic Spectrum Simulator (DRESS) code [11]. This code can be used to solve a specific equation which originates from the conservation of four-momentum in a two-particle collision. This code uses natural units, where  $c = 1$  is used to simplify the equations through omission of specific constants. As such, this formalism will be applied from this point in the report and onward.

In an inertial frame  $S$ , two tritons  $A$  and  $B$  collide and three particles exit the interaction. The emitted particles each have four-momentum  $p_k^\mu$ ,  $k = 1, 2, 3$ . The total initial and final four-momenta of the the frame  $S$  are defined as follows:

$$p_i^\mu = \begin{pmatrix} E_A + E_B \\ \vec{p}_A + \vec{p}_B \end{pmatrix}$$

$$p_f^\mu = \sum_{k=1}^j \begin{pmatrix} E_k \\ \vec{p}_k \end{pmatrix} = \sum_{k=1}^j p_k^\mu.$$

The conservation of total four-momentum gives

$$p_i^\mu = p_f^\mu = P_S^\mu = \begin{pmatrix} E_S \\ \vec{P}_S \end{pmatrix}, \quad (9)$$

where  $P_S^\mu$  is the four-momentum of  $S$ .

To find the energy of one of the emitted particles, for example a neutron,  $p_1^\mu = p_n^\mu$  is subtracted from equation 9 and then squared. This yields:

$$(P_S^\mu - p_n^\mu)^2 = \left[ \left( \sum_{k=1}^j p_k^\mu \right) - p_n^\mu \right]^2 = \left( \sum_{k=2}^j p_k^\mu \right)^2 \quad (10)$$

Using the definition of invariant four-momentum in equation 4, then equation 10 is expanded:

$$(P_S^\mu - p_n^\mu)^2 = (P_S^\mu)^2 + (p_n^\mu)^2 - 2(P_S^\mu \cdot p_n^\mu)$$

$$\left( \sum_{k=2}^j p_k^\mu \right)^2 = M_R^2,$$

where  $M_R^2$  is the invariant mass of the emitted particles except for the neutron on the left hand side of the expression above, and  $(P_S^\mu)^2$  is the invariant mass of the initial tritons  $A$  and  $B$ . This leaves the upper part of the expression to be further expanded:

$$(P_S^\mu)^2 + (p_n^\mu)^2 - 2(P_S^\mu \cdot p_n^\mu) = M^2 + m_n^2 - 2E_S E_n + 2\vec{P}_S \cdot \vec{p}_n$$

Using the conservation of invariant four-momenta in equation 10, the above expression is equal to  $M_R^2$ , which allows for rearranging of terms. If one defines  $\vec{p}_n$  in the final term above as

$$\vec{p}_n = \frac{\vec{p}_n}{|\vec{p}_n|} |\vec{p}_n| = \hat{u}_1 \sqrt{E_n^2 - m_n^2}$$

with  $\hat{u}_1$  as the unit vector of the direction of emission and applies the energy-momentum relation given in equation 5, the final expression becomes:

$$E_n - \sqrt{E_n^2 - m_n^2} \frac{\vec{P}_S \cdot \hat{u}_1}{E_S} = \frac{M^2 + m_n^2 - M_R^2}{2E_S}. \quad (11)$$

This will be called the DRESS equation, which can be solved using the DRESS code. It can give the kinetic energy of the neutron in question depending on the emission direction and the invariant masses of all other emitted particles and the four-momenta of the initial particles in any inertial reference frame  $S$ . Furthermore, if the velocity distributions of the reactants  $A$  and  $B$  are provided, DRESS can calculate the energy spectrum of the chosen product particle (in this project, the product of interest will be the neutron from the T+T reaction)

### 3.2.1 The simplified DRESS equation

The DRESS equation is applicable in all inertial frames, but might be simplified for some frames. If one chooses to observe the reaction in the COM-frame of the two initial particles, the equation can be simplified significantly, as  $\vec{P}_S = \vec{0}$ . This also means that, due to equation 5,  $E_S = M$ . Applying these changes to equation 11 yields the simplified DRESS equation:

$$E_1 = \frac{M^2 + m_1^2 - M_R^2}{2M}, \quad (12)$$

which is strictly valid in the reaction's COM-frame.

## 4 Part 1 - Isotropic spectra

The project is divided into two different parts, with different results and different methods used to attain those results. The report will reflect this by presenting each part individually, beginning with the computations on the isotropic energy spectra.

### 4.1 Procedure

In general, this project is about comparing between two inertial frames. This first part is all about building the groundwork on which the second part can be built on. A Lorentz transformation is applied to a large number of modelled neutrons.

#### 4.1.1 Sampling the spectrum

The centre-of-momentum inertial frame's energy spectrum presented in figure 5a is a probability distribution function. To compute the same function but as if observed from the LAB-frame, the Lorentz transform is used, which cannot be applied to the function itself. Instead, a large amount of neutrons are sampled from the distribution, and the neutron energies are transformed. This is done through a Monte Carlo simulation, which generates random energies inside the gray box shown in figure 6 and only saves those under the curve.

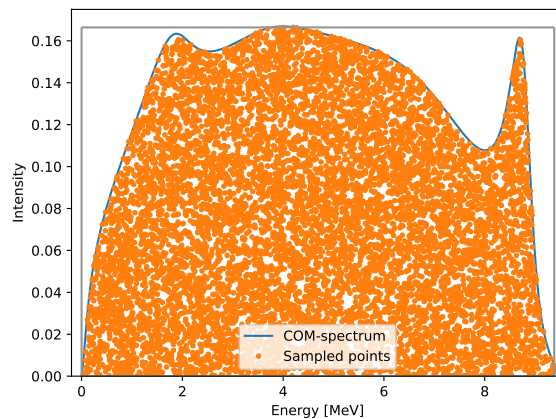


Figure 6: The COM-spectrum and the sampled points under the curve. The gray line represents the rectangle in which the points were uniformly sampled. This plot visualises ten thousand accepted points, but one million will be used for the computations as it gives a better fit.

A million neutron energies are generated through a Monte Carlo simulation, which give rise to the histogram in figure 7.

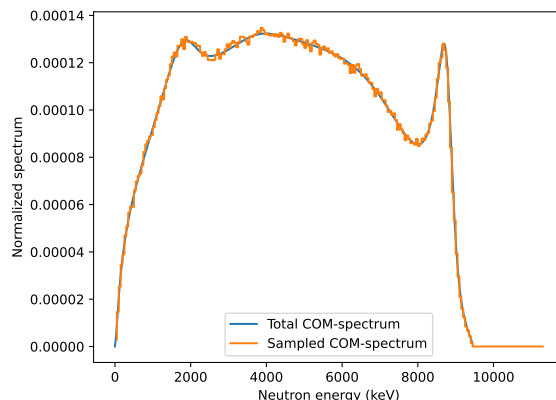


Figure 7: The sampled points from figure 6 arranged in a histogram. The original spectrum is plotted for reference. The histogram includes 1 million successfully sampled points, and fits well to the spectrum from [8].

### Beam-target reaction

Since this report focuses on the fusion reaction between tritium nuclei in the plasma and tritium nuclei in the neutral beam injection, the approximation of a beam-target reaction is made. Since the nuclei in the beam generally move with much higher kinetic energy compared to the plasma fuel, the nuclei in the plasma can be regarded as being still (this approximation will be relaxed later in the report). This emulates a beam interacting with a stationary target, which allows for the definition of the following four-momenta in the LAB-frame for the two tritium nuclei.

$$p_{T1}^\mu = \begin{pmatrix} E_{T1}/c \\ p_{T1,x} \\ p_{T1,y} \\ p_{T1,z} \end{pmatrix} \quad p_{T2}^\mu = \begin{pmatrix} m_{T2} \cdot c \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (13)$$

The target ion has zero momentum since it is at rest, and its time-component is only given by the tritium's rest mass in units of  $eV/c$ . In the expression above, the beam energy is given by  $E_{T1} = m_T c^2 + E_K$ , where  $E_K$  is the kinetic energy of an ion in the beam.

If  $E_K$  is known, then the velocity of the COM-frame can be computed. The  $\beta$  given in equation ?? can be found for the entire COM-frame, by choosing  $p = \sum_i p_i = p_{T_1} + p_{T_2}$  and  $E = \sum_i E_i = E_{T_1} + E_{T_2}$ . It is important to take values based in the LAB-frame, as the sum will give the momentum and energy of the center-of-momentum of the two tritium nuclei included in the beam-target reaction. The following expression emerges:

$$\frac{v_{COM}}{c} = \beta = c \frac{p_{T_1}}{E_1 + E_2} = c \frac{\sqrt{E_{T_1}^2/c^2 - m_T^2 c^4}}{2m_T c^2 + E_K/c}.$$

With this result,  $\gamma$  can be computed and the Lorentz transform can be constructed. If the beam ions are assumed to have identical kinetic energies, then one  $\Lambda_\mu^{\mu'}$  is needed. Otherwise, each different value of kinetic energy has to have a corresponding Lorentz transform with different numerical values. In this case, only one value for the beam's kinetic energy is considered. The chosen value is  $E_K = 100$  keV, as it represents the injection energy of the ions heating the plasma through NBI at JET.

### The Lorentz Transformation

Each sampled point represents a sampled neutron from the COM-based neutron emission. In figure 6, each random energy is for an emitted neutron, for which the rest mass is known. Each neutron is also characterised by a unique four-momentum, with a known momentum and unknown direction of emission. The latter factor is important to include, as transforming  $v\hat{x}$  and  $-v\hat{x}$  yields inverse effects, hence directions of neutron emission in the COM-frame has to be simulated. Here, this will be assumed to be isotropic, hence a number of polar and azimuthal angles equal to the amount of saved random energies are generated, with the azimuthal angles being uniformly sampled between 0 and  $2\pi$  and the cosine of the polar angle sampled uniformly between  $-1$  and  $1$ . These are then rewritten to cartesian coordinates, which yields the unit vector for each neutron's three-momentum.

After the sampling of directions, each sampled neutron energy can give rise to a four-momentum:

$$E_n = E_{Kn} + m_n c^2, \quad p_n = \sqrt{E_n^2/c^2 - m_n^2 c^2}$$

$$\Rightarrow p_n^\mu = \begin{pmatrix} E_n \\ \vec{p}_n \end{pmatrix}$$

Having these quantities defined, the spectrum in figure 7 can be boosted. The speed of the COM-frame is unchanged by the fusion reaction and  $\Lambda_{\mu}^{\mu'}$  is the same for the emitted neutron as for one of the tritium nuclei. Therefore, each sampled four-momentum in the COM-frame can now be transformed to the LAB-frame using the expression in equation 8, where the individual values for each neutron's kinetic energy in the LAB-frame can be extracted.

## 4.2 Results

For the first part, the total LAB-spectrum was generated by a Lorentz transformation of the total COM-spectrum according to section 4.1.

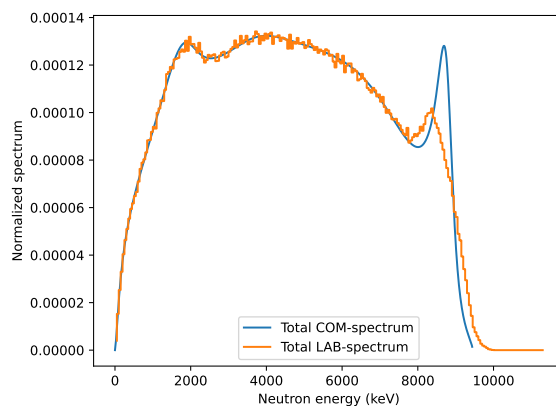


Figure 8: The total LAB-spectrum and the total COM-spectrum juxtaposed, with the LAB-spectrum being the histogram due to the random sampling used in the procedure.

Both spectra are for the neutron emission integrated over all angles. Clear deviations can be seen between the two spectra in figure 8, with the most prominent being the peak at roughly 8.5 MeV. In the interval between 0 and 8 MeV, the two curves differ only by the noise inherent to Monte Carlo methods, an expected result. Yet, above 8 MeV, greater discrepancies can be observed, as the prominent peak in the total COM-spectrum is seemingly diminished and slightly spread out to lower and higher energies. However the total (i.e. angle integrated) LAB-spectrum is not what TOFOR observes, thus motivating the development of results for the second part.

## 5 Part 2 - Angular-dependent spectra

In this section, the methods used to improve the results from the first part are presented, alongside the improved results.

### 5.1 Procedure

The modelled system used for the computations of the isotropic spectra is improved in this part. This is done with the DRESS equation, which allows for extraction of angular-dependent information. The isotropic emission assumed earlier is not entirely correct due to the direction of travel of the tritium nuclei before the reaction, which means that some emission directions are more likely than , and not isotropic.

A further improvement to the model is the application of relevant energy distributions for the tritium nuclei.

#### Sample neutrons

One of the parameters that can be changed in the DRESS code is the neutron's emission direction  $\hat{u}_n$ , which can be chosen by the user. Five values for the exit angle are selected,  $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$ , to give a simple but representative illustration of the differences between the spectra for different emission directions. Rewritten to cartesian coordinates, the five emission angles are:

$$\hat{u}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{u}_2 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \hat{u}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \hat{u}_4 = \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \quad \hat{u}_5 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}.$$

These values are used as input to the DRESS calculations. For the purpose of fulfilling the aim of this project, only the  $\hat{u}_3$  emission direction is needed. Though it is also of interest to test other emission directions for comparison.

#### Model the tritium fusion reaction

So far, the tritium fusion reaction has been treated as a pure beam-target reaction, where the NBI triton moves with a kinetic energy of 100 keV and the plasma-based tritium nucleus has 0 keV of kinetic energy. The chosen values represent an approximation, which can be refined by switching to energy distributions that represent the conditions for experiments at JET. The DRESS code has built-in functions which generate numerical values matching relevant distributions.

The plasma-based ion should not have 0 keV of kinetic energy in either the COM-frame and LAB-frame. Change this value by using a Maxwellian energy distribution with its centre at plasma temperature 7 keV.

The NBI tritons do not move uniformly in the beam, with parallel direction of movement and only 100 keV. To model a more realistic behaviour of the beam ions, a slowing down distribution between 20 keV and 100 keV is used to sample the kinetic energies for the tritons and a variation in direction of travel is introduced by sampling the cosine of the pitch angle between 0.5 and 0.7 (which approximately translates to 45° and 60°).

### Compute the DRESS equation

In order to compute the neutron energies  $E_n$  in equation 11, it is necessary to know the corresponding values of  $M_R^2$ , the invariant mass of the  $\alpha$ -particle and the residual neutron combined. Since  $M_R^2$  is invariant, its distribution is the same in all reference frames and can therefore be computed using the simplified DRESS equation 12. Since the simplified DRESS equation is valid specifically for the COM-frame, the points sampled from the total COM-spectrum (see figure 7) can be used to extract a spectrum for  $M_R^2$ , which can be inserted into equation 11 when computing the LAB-spectra. Rewriting the simplified DRESS equation gives:

$$M_R^2 = M^2 + m_n^2 - E_1 \cdot 2M$$

where  $E_1$  is the sampled neutron's COM-based energy from the first part of the procedure (see figure 6), thus allows for a corresponding spectrum to be generated. Since  $M_R^2$  is an invariant, it can be originated with COM-based energies and still be useful in computations in any inertial frame such as the LAB-frame.

## 5.2 Results

At  $\theta = 90^\circ$ , the DRESS calculations described in section 5.1 have produced the following plots:

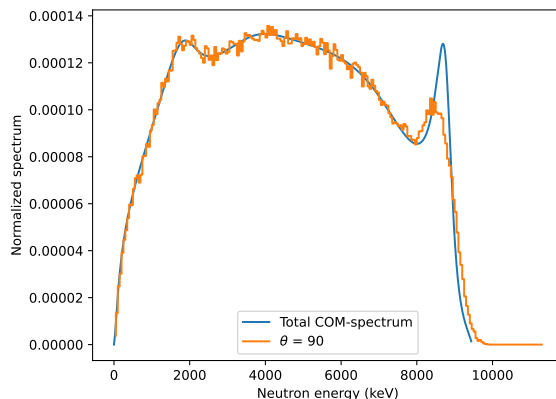


Figure 9: Comparison of the total energy spectrum in the COM-frame and the LAB-spectrum for the perpendicular emission of neutrons.

Similarly to the result for part 1, the LAB-spectrum deviates most from the total COM-spectrum near the peak at 8.5 MeV. Those differences in figure 9 are slightly less pronounced than in figure 8.

The orange histogram in figure 9 is for the emission of neutrons perpendicular to the B-field lines in the reactor, while the total COM-spectrum is assumed to look the same for any emission angle. The spectra have been normalised so that the general shapes can be more easily juxtaposed, which leads to the loss of the comparison between the flux of emitted neutrons. The isotropic spectrum has much higher counts than the perpendicular LAB-spectrum.

### 5.3 Time-of-flight spectra

TOFOR does not observe neutron energies, it measures time-of-flight. Thus, the spectra computed so far will be fed through the instrument's response function, which can transform the energies in figures 8 and 9 into time-of-flights of the neutrons.

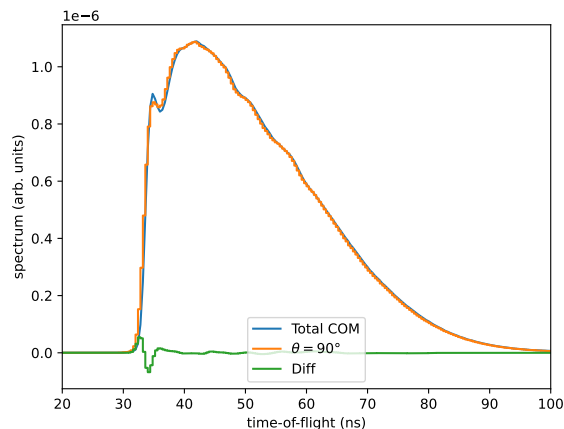


Figure 10: Time-of-flight spectra, one for the perpendicular emission of neutrons in LAB and one for the isotropic emission in COM. The green curve gives the difference between the two spectra.

Figure 10 is the main result for this project. It visualises the difference between the time-of-flight spectra of the different reference frames treated in this project and shows that the two spectra closely resemble each other.

In contrast to the two energy spectra shown in figure 9, there is no difference between the two time-of-flight curves except for minimal deviation at  $t_{tof} = 35$  ns. The green curve

### 5.3.1 Other exit angles

More than one emission angle was included in the computations and, although not directly relevant for the TOFOR measurements, in this section, energy spectra and time-of-flight spectra are presented for those angles.

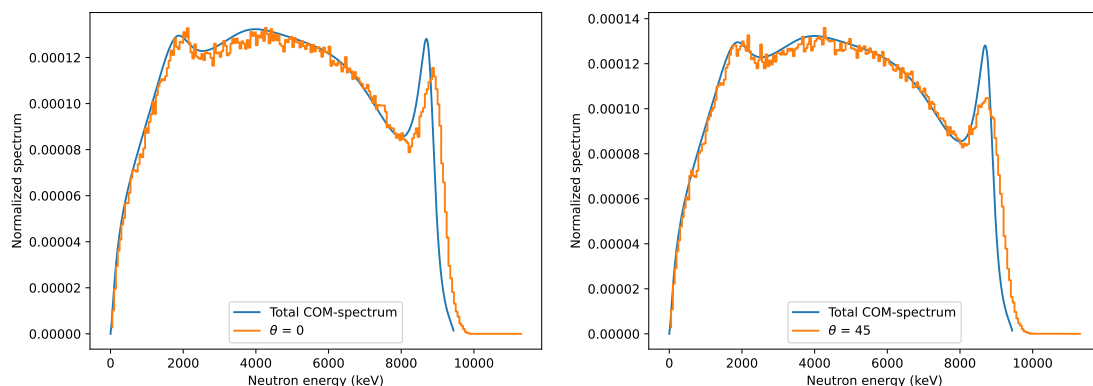
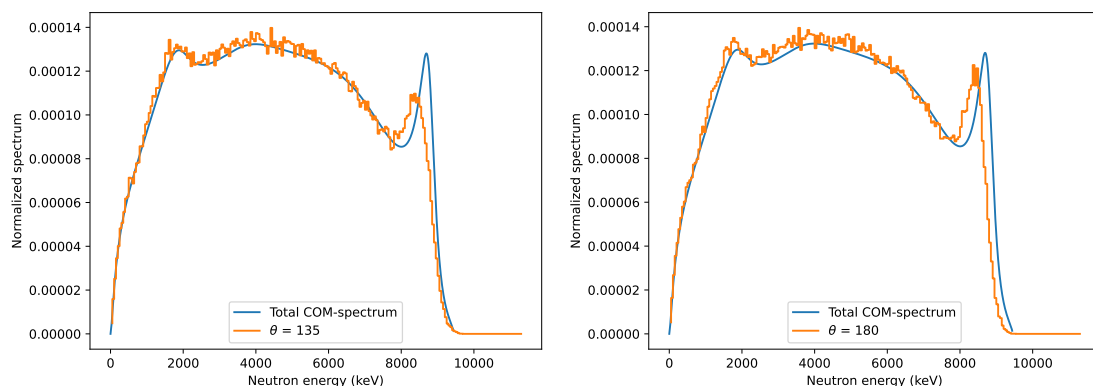
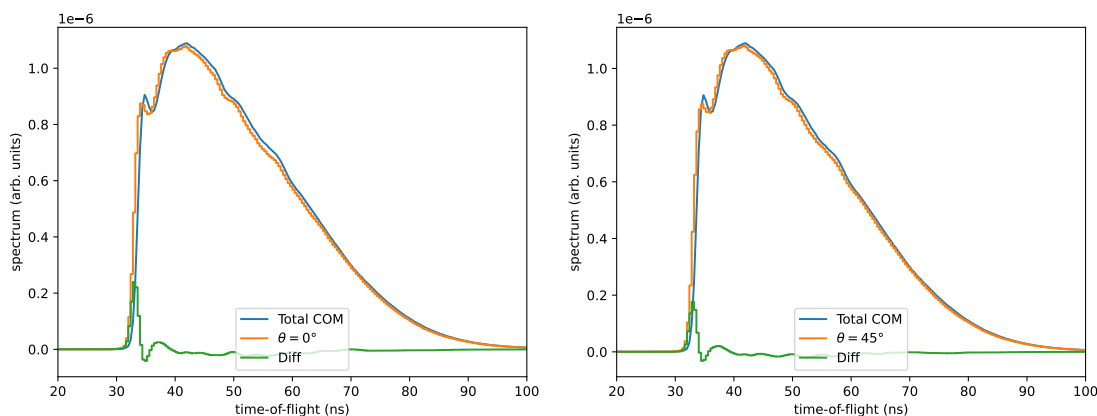
(a) Energy spectrum for neutron emission at  $\theta = 0^\circ$  (parallel emission)(b) Energy spectrum for neutron emission at  $\theta = 45^\circ$  (semi-perpendicular emission)(c) Energy spectrum for neutron emission at  $\theta = 135^\circ$  (semi-perpendicular emission)(d) Energy spectrum for neutron emission at  $\theta = 180^\circ$  (anti-parallel emission)

Figure 11: Plots for other neutron exit angles.

In figures 11a - 11d, the energy spectra for the neutrons emitted in the remaining emission directions considered in this report are shown. Some obvious properties are the tendencies of the LAB-based spectra with respect to the COM-spectrum. The neutrons emitted with  $\theta = 0^\circ$  and  $\theta = 45^\circ$  tend to have higher energies, while those in the interval  $\theta = 135^\circ$  and  $\theta = 180^\circ$  seem to have lower energies. This stems from the constructive or destructive addition of velocities depending on whether a neutron is emitted parallel or anti-parallel to the direction of movement of the COM-frame. If a neutron exits the reaction with a non-zero projection of its velocity upon the path of the COM-frame (i.e. if it is not emitted perpendicularly), an observation from the LAB-frame will include the energy of the moving inertial frame. This can

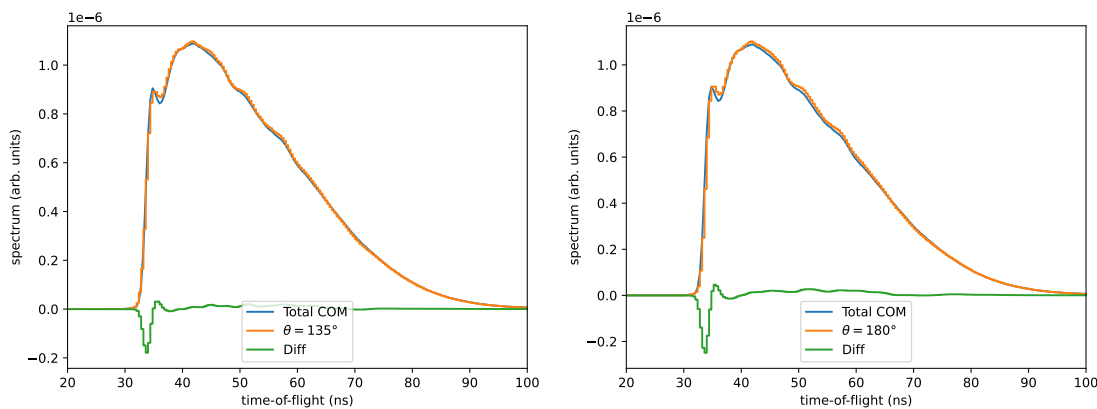
be thought of as a Doppler shift of the particle, which affects the observed energy from the LAB-frame. As such, neutrons emitted with  $\theta = 0^\circ$  are "boosted" by the total momentum of the tritium ions before interaction, while neutrons emitted with  $\theta = 180^\circ$  are "dampened".

The energy spectra are then subject to TOFOR's response function, which transforms them into time-of-flight spectra.



(a) Time-of-flight spectrum for neutron emission at  $\theta = 0^\circ$  (parallel)

(b) Time-of-flight spectrum for neutron emission at  $\theta = 45^\circ$  (semi-perpendicular)



(c) Time-of-flight spectrum for neutron emission at  $\theta = 135^\circ$  (semi-perpendicular)

(d) Time-of-flight spectrum for neutron emission at  $\theta = 180^\circ$  (anti-parallel)

Figure 12: Results for other neutron emission angles.

Similarly to the result in figure 10, discrepancies between the time-of-flight spectra are minimal outside of the 35 ns peak. The angular dependent energy shift which

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was observable in figures 11 is similarly on display in figures 12, although much less pronounced. Since higher energies lead to shorter time-of-flights, the LAB-based spectra in figures 12a and 12b tend to the left of the blue curve while the LAB-based spectra in figures 12c and 12d tend to the right of the COM-based spectrum. This is clearer when subtracting the two spectra, which is presented with the green curve in the figures above. The discrepancy is mostly positive for the parallel and semi-parallel emission spectra, which stems from the shift of the LAB-spectrum towards shorter time-of-flights, while the difference is mostly negative for the anti-parallel and semi-anti-parallel emission spectra. The reason is the same as discussed above.

## 6 Discussion

A short discussion about the results will be presented in this section.

### 6.1 The spectra

The result from the previous study (given in [8]), i.e. the isotropic COM-spectrum in figure 5a, has been derived under the assumption that it is representative of what TOFOR can observe in the LAB-frame. This assumption has been tested in this report, and the conclusion is that it is appropriate for a beam-target-dominated T+T fusion reaction, yet is not completely correct.

First of all, there are discrepancies between results in the two inertial frames considered in this project. Both the energy spectra and the time-of-flight spectra in the LAB-frame deviate from the isotropic COM-spectrum, with the energy spectra in the LAB-frame exhibiting greater deviations from the isotropic energy spectrum in the COM-frame than the time-of-flight spectra in the LAB-frame and the isotropic time-of-flight COM-spectrum. Moreover, the results show that the neutron's exit angle also does affect the spectra. The curve visualising the differences in figures 10 and 12 has smaller values for the perpendicular emission of neutrons than any other considered. As such, the assumption that the COM-spectrum is representative of LAB-based TOFOR observation is more appropriately applicable for neutrons emitted with  $\theta = 90^\circ$ .

There is another interesting behaviour to discuss. The spectra in figures 9 and 11 deviate significantly from each other, yet when TOFOR's response function is applied, those large differences are diminished to minimal ones as seen in figures 10 and 12. This is due to TOFOR's inability to measure with the required precision to resolve discrepancies of such magnitude. Thus, great care has to be taken when reviewing results from previous study, as any small change to a time-of-flight spectrum can yield different energy spectra.

### 6.2 Future Work

Using the simplified DRESS equation allowed for the  $M_R^2$ -spectrum to be found. This computation used the total COM-based energy spectrum, which is extracted in previous study [8] while only considering one value for the energy of the COM-frame,  $E_{COM} = 50$  keV. A problem arises, as the LAB-spectra which depend on the neutron's exit angle are computed for a multitude of values for  $E_{COM}$ , due to the usage of energy distributions. Yet, this computation used the constant  $M_R^2$ -values, which should have been dependent on the energy of the COM-frame. As

such, the interpretation of the result is limited by this drawback due to the DRESS equation being constructed to handle only one energy for the COM-frame at a time. The solution would be to retrace the steps of the previous study and compute a three-dimensional spectrum consisting of a multitude of isotropic COM-based energy spectra. This would allow for the correct value of  $M_R^2$  to match the energy of the tritium atoms (or more correctly  $E_{COM}$ ) when computing the DRESS equation.

## 7 Ethical and Societal Aspects of the project

This project is made to aid in the research of neutron spectroscopy at the Division for Applied Nuclear Physics at Uppsala University. This report has also striven to adhere to correct scientific conduct guidelines.

The societal changes originating from the results achieved by this project are not envisioned to be significant, hence not being discussed. The reason is that the difference between two inertial reference frames in experiments at JET does not affect the rights, the economical situation or the privacy of individuals.

Similarly, ethics is not applicable to this specific project but could be discussed regarding nuclear fusion in general.

## 8 Conclusion

This report employs relativistic kinematics and pre-existing software called the DRESS code to determine if the COM-based energy spectrum is appropriate for analysis of LAB-based TOFOR observations. The results presented in this report indicate that no large differences exist between time-of-flight spectra observed from either the COM inertial frame and the LAB inertial frame, yet those minimal discrepancies turn into greater differences for the equivalent energy spectra. Though the methods in this report could be improved and the results can only be interpreted as guidelines, the behaviour visualised in this project should be considered in future neutron spectroscopy data and models.

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