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Pierced, Wrapped and Torn

Aspects of String Theory Compactifications

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Abstract

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An outstanding problem in physics is to find a unified framework for quantum mechanics and general relativity. This is required for a better understanding of black holes and the early cosmology of the universe. String theory provides such a unification. In this thesis, we study aspects of compactifications of type IIB string theory. In the first part of the thesis, we study four-dimensional black holes consisting of D3-branes wrapping cycles in the compact dimensions. We discuss the correspondence between these black holes, topological string theory and matrix models. We then study the influence of black holes on the stability of flux compactifications. In the second part of the thesis, we turn to investigations of the type IIB landscape, i.e. the collection of stable and metastable vacua obtained from flux compactifications on conformal Calabi-Yau manifolds. We show that monodromies are important for the topographic structure of the landscape. In particular we find that there are long series of continuously connected vacua in the complex structure moduli space of the internal manifold. We also use geometric transitions to connect the moduli spaces of different manifolds, and create longer series of vacua. Finally, we investigate the stability of string theory vacua by constructing semiclassical instantons. These results have implications for the population of the landscape by eternal inflation.

Keywords: string theory, compactification, supergravity, black holes, Calabi-Yau manifold, cosmology, string theory landscape, eternal inflation

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Till Daniel

List of Papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

I U. H. Danielsson, N. Johansson, M. Larfors, M. E. Olsson and M. Vonk, "4D black holes and holomorphic factorization of the 0A matrix model," *Journal of High Energy Physics* **0510**, 046 (2005) [arXiv:hep-th/0506219].

II U. H. Danielsson, N. Johansson and M. Larfors, "Stability of flux vacua in the presence of charged black holes," *Journal of High Energy Physics* **0609**, 069 (2006) [arXiv:hep-th/0605106].

III U. H. Danielsson, N. Johansson and M. Larfors, "The world next door: Results in landscape topography," *Journal of High Energy Physics* **0703**, 080 (2007) [arXiv:hep-th/0612222].

IV D. Chialva, U. H. Danielsson, N. Johansson, M. Larfors and M. Vonk, "Deforming, revolving and resolving - New paths in the string theory landscape," *Journal of High Energy Physics* **0802**, 016 (2008) [arXiv:0710.0620 [hep-th]].

V M. C. Johnson and M. Larfors, "Field dynamics and tunneling in a flux landscape," *Physical Review D* **78**, 083534 (2008) [arXiv:0805.3705 [hep-th]].

VI M. C. Johnson and M. Larfors, "An obstacle to populating the string theory landscape," *Physical Review D* **78**, 123513 (2008) [arXiv:0809.2604 [hep-th]].

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Contents

1. Introduction

"To explain — since every piece of matter in the Universe is in some way affected by every other piece of matter in the Universe, it is in theory possible to extrapolate the whole of creation — every sun, every planet, their orbits, their composition and their economic and social history from, say, one small piece of fairy cake."

Douglas Adams
The Restaurant at the End of the Universe

Nature, as we perceive it, has three spatial dimensions (up-down, left-right, forward-backward) plus one dimension for time. Does this mean that the physical theories that we use to describe nature must also be four-dimensional? In this thesis, we will argue that this is not necessary. A theory that lives in more than four dimensions can have solutions that describe four-dimensional worlds.

The extra dimensions might challenge our imagination, but do not necessarily make the formulation of the physical theories much more difficult. Furthermore, a higher dimensional theory might provide explanations for physics that we have trouble explaining with four-dimensional theories. For example, it seems that the ten-dimensional string theories can help us understand the mysteries of quantum gravity, i.e. how gravity should be described on very small distances, and do so without a single tunable parameter. This is a marvelous feat, that no other theory has achieved.

The four-dimensional solutions of string theory are known as string theory compactifications, and have been the topic of my research as a PhD student. In particular, I have, together with my collaborators, investigated what happens when the extra dimensions of string theory are pierced by fluxes, wrapped by branes, or torn at singularities. The results of these studies are published in Papers I-VI. The goal of this thesis is to provide a reasonably self-contained introduction to this field of research. The intended audience consists of graduate students in physics with some familiarity with string theory and quantum field theory.

1.1 Setting the Stage

Before we dive into the details that lie behind the research presented in this thesis, it's probably a good idea to give an overview of the field this work has been performed in. We therefore start with two sections that introduce the main subjects of this thesis: string theory and compactifications. The idea is to give the reader a feeling for these concepts without introducing any mathematics. The necessary details will then be fleshed out in the following chapters of the thesis.[1]

1.1.1 String Theory in a Nutshell

So what is string theory? As the name suggests, it is a theory whose basic constituents are strings. These strings move in spacetime and can interact with each other. We will assume that the strings are tiny, so tiny that they can replace point particles as the fundamental building blocks of matter.[2] The beauty of the idea is that a string has many vibrational modes, and each such mode has its definite properties, e.g. mass and spin. If the extent of the string is very small, these modes will be perceived as different particles. In fact, it suffices to have one type of string to obtain a whole spectrum of different particles.

We obtain a quantum mechanical description of the string by studying its worldsheet, i.e. the surface that the string traces out as it moves in spacetime. This is a generalization of the worldline of a point particle, as illustrated in figure 1.1. The motion of a classical, relativistic string will be such that the area of this worldsheet is, in a certain sense, minimized.[3]

A fruitful way to analyze the quantum motion of the string is to view the worldsheet as the fundamental geometrical object in the theory, and the worldsheet coordinates X^M as fields living on the worldsheet. We can then describe the embedding and motion of the string in spacetime in terms of a two-dimensional field theory. By quantizing the action of this two-dimensional theory, we obtain a quantum mechanical description of the dynamics of the vibrating string, and its associated spectrum.

The content of the string spectrum depends on whether the string is closed or open and how it vibrates. The most interesting state in the closed string spectrum is a massless state of spin two, which looks just like a graviton. As we will see in section 2.2, the correct interpretation of this state is indeed that it is a graviton, describing fluctuations of the spacetime metric. As

[1] We will avoid most references to original work in this chapter. Good introductions to string theory, with extensive reference lists, can be found in [GSWa, GSWb, Pola, Polb, BBS].

[2] This means that the length of the string is smaller than 10^{-19} m, or its extent would have been detected in current particle physics experiments.

[3] The world-sheet area is measured in Lorentzian spacetime. This is analogous to the classical, relativistic point particle, whose worldline correspond to a geodesic, i.e. a path of minimal length, in spacetime.

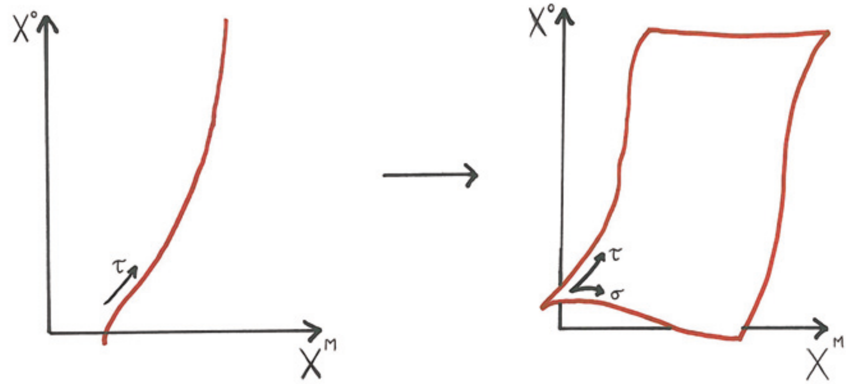

Figure 1.1: On the left is a worldline, describing the evolution of a point particle in spacetime. Here X^0 denotes time, X^M all spatial directions, and τ parametrizes the worldline. The classical motion of the point particle minimizes the length of the worldline. In string theory, point particles are replaced by strings, and worldlines are replaced by worldsheets. This is shown on the right. We now need two parameters τ, σ to describe the worldsheet. The classical motion of the string minimizes the area of the worldsheet.

a consequence, string theory is a quantum theory that contains gravitational interactions, i.e. a candidate for a theory of quantum gravity.

The identification of string theory as a theory of quantum gravity gives a natural suggestion for what the string scale should be. General relativity is expected to be a good description of spacetime down to the Planck length, $l_p \sim 10^{-35}$ m, where the quantum nature of gravity supposedly no longer can be neglected. A natural guess is then that the string length l_s is of this order too. This means that the massive states in the string spectrum will be extremely heavy, and can thus be neglected for practical purposes.[4] Thus, the interesting states in the string spectrum have zero mass.[5]

However, the theory that we have formulated, with the spacetime coordinates being the only fields that live on the world-sheet, has a major problem. The ground states in both the open and closed string spectra turn out to be tachyons, i.e. have imaginary mass. This signals that the theory is unstable. Another problem with the theory is that all states in the spectra are bosonic. There are no states that correspond to the fermionic matter particles of the standard model.

The two problems can be resolved in the superstring theories, where extra fermionic fields are introduced on the world-sheet. With these extra fields,

[4]The string scale could be lower in compactified string theories, as we discuss in chapter 3.

[5]At this point, the reader might ask herself why string theory would then be interesting for particle physics. After all, most particles in the standard model have non-zero masses. We therefore note that the massless states can become massive as a result of symmetry breaking on a lower energy scale.

five different, mathematically consistent string theories can be obtained: type I, type IIA, type IIB, heterotic $E_8 \times E_8$ and heterotic SO(32). All these theories live in ten dimensions and have a symmetry known as supersymmetry that relates fermions and bosons. Their spectra contain the graviton, but also gauge fields and fermions, which means that they could reproduce the standard model fields.[6] We will be interested in the type IIB theory, which we will describe in some detail in chapter 2.

The five string theories are not independent, but are in fact related to each other through various dualties. For example, the type IIA and type IIB theories are related by T-duality; when compactified on circles of radii R and l_s^2/R respectively, they give the same effective theory. Furthermore, yet another theory is included in this web of dualities, namely eleven-dimensional supergravity. These connections have led to the proposal that the five string theories and eleven-dimensional supergravity are really different limits of one unique theory. This theory is called M theory, and is supposedly eleven-dimensional. However, how M theory should be formulated, and what its fundamental constituents are, remains to be discovered.

This concludes our brief introduction to string theory, where we have argued that string theory is a candidate for quantum gravity. It is probably evident from the above discussion that string theory is not yet a complete theory, and many of its aspects are still mysterious to us. One thing that is certain is that string theories live in ten dimensions. This is rather puzzling, since our spacetime is four-dimensional. We discuss the resolution of this puzzle in the next section.

1.1.2 Compactifications: A Landscape of Theories

In the last section we proposed that ten-dimensional string theories are candidates for quantum gravity. Furthermore, they promise to describe the quantum field theories of the standard model as well. The only crux is that the theories live in ten dimensions, whereas all observations to date point to us living in a four-dimensional spacetime.

One method of getting rid of extra dimensions is through compactification. The idea is that some dimension of spacetime is small and compact. Suppose, for concreteness, that space is two-dimensional. Now make one of these dimensions finite, and identify the ends of this finite dimension. The dimension then has the topology of a circle, and the space looks like the surface of a garden hose, i.e. a cylinder. If the length of the hose is much bigger than its diameter, and we look at the hose from some distance, we would think it were one-dimensional. A small ant, on the other hand, could walk in both directions of the surface of the hose, thus experiencing both dimensions.

[6] This is oversimplified. We will see later in this thesis that string theory also contains many other objects than strings, i.e. D- and NS-branes. The particle content of the standard model can be obtained from open strings stretching between intersecting D-branes, see e.g. [CSU01].

Moreover, if the extra dimension is really small, it would go unnoticed also by fundamental fields, at least as long as we are interested in their low-energy behaviour. This will be described in more detail in chapter 3. Therefore, if the six extra dimensions of string theory are small and compact, we would obtain an effectively four-dimensional theory.

In contrast to the simple example where we compactify one spatial direction, there are many different ways to compacitfy six dimensions. Six-dimensional manifolds come in a variety of sizes and shapes. Several parameter fields determine the form of the compactified dimensions. We can construct stable compactifications by introducing objects known as fluxes and branes, that yield a potential for the parameter fields. Classically stable solutions then correspond to the minima of this potential. There is a huge number of such solutions, corresponding to different choices of manifolds, fluxes and branes. Some properties of these solutions will be analyzed in this thesis.

After quantizing, each such solution provides a *vacuum* for a quantum field theory that describes the four-dimensional, low-energy physics of the theory. The collection of these vacua is known as the string theory landscape, and is often thought of as a complicated potential for the many fields that determine the form of the compactified dimensions. The hope is that at least one of these vacua will describe the physics of our universe.

Quantum mechanically, there is a small probability that the potential barriers around a minimum are penetrated by a tunnelling field. This implies that most four-dimensional universes will only be metastable, and decay through the formation of bubbles of a new vacuum phase. If this decay is sufficiently slow, the old phase will not be completely replaced, but will coexist with the new vacuum. In this way, the string theory landscape could give rise to a multiverse, where different vacua are realized in different regions of spacetime. If this is correct, it means that our universe is not unique. Other universes exist as well. Some of these might look like our universe, but most will be radically different — e.g. expand too fast to allow galaxies to form, contract rapidly or have an electromagnetic coupling that does not allow stable atoms.

This stringy multiverse might seem like a very contrived way to describe our world, but it has several benefits. If not all vacua of the landscape are physically realized, we would have to find an argument that explains why our particular vacuum is. From what we know now, this seems difficult; as mentioned above, some properties of our universe are rather atypical in the landscape. It is therefore statistically improbable that a vacuum with just the right properties would be the chosen one. On the other hand, if all vacua are realized, then it is enough that one of these describes our universe. The reason why we find ourselves in this particular vacuum can then be answered in a simple way - we can only make observations that are compatible with our existence.

This kind of anthropic reasoning was first used long before the discovery of the string theory landscape. At this time, physicists were looking for a principle that would explain why the cosmological constant, which, if positive would cause the universe to expand acceleratingly, was zero. However, in [Wei87] it was noted that such a principle was unnecessary. If instead the underlying physical theory allowed for several universes with different cosmological constants to form, then we would necessarily live in a universe where the cosmological constant was very small. This follows from requiring that galaxies could form, which puts strict limits on the value of the cosmological constant. Galaxies are necessary for life (at least of our kind), so this gives an anthropic bound on the cosmological constant. However, there is no need for it to be zero, and we know now that it is indeed not - its value lies just at the boundary of the anthropically allowed region.[7]

We therefore see that the string theory landscape, combined with a mechanism for tunnelling between vacua, gives rise to a multiverse. Some properties of our universe can then be explained by applying anthropic arguments to this multiverse. Naturally, we would not want all properties of the universe to be explained in this way, since this would give us limited ways of testing the predictions of the underlying theory, i.e. string theory. So far, it seems that we need an anthropic explanation of the small value of the cosmological constant, whereas other properties could perhaps be understood by statistical and dynamical arguments based on the properties of the landscape.

1.2 Outline

In addition to the papers I-VI, this thesis contains several chapters that describe the background material of the papers. As any contemporary research in string theory, these papers build on a complex network of ideas, theories and approximations, which can make papers inaccessible for non-experts. The point of the introductory chapters is to explain the necessary concepts used in the papers. Furthermore, the aim is also to put them into context, and to discuss their relevance for the field of string theory.

The outline of the thesis is the following. Chapter 2 contains an introduction to (supersymmetric) string theories. Emphasis is on the low-energy aspects of the type II theories. Chapter 3 introduces the technical details of compactifications (without fluxes) and the resulting four-dimensional theories. In chapter 4 we study black holes in these compactifications, which is the topic of Paper I. We introduce flux compactifications in chapter 5, and also study black

[7]This kind of reasoning is based on the weak anthropic principle, which is uncommon in physics and often criticized. A clarification is therefore in order. The weak anthropic principle does *not* imply that the universe is tuned in a certain way because its purpose is to allow humans to exist. The situation is instead very similar to Darwin's theory of natural selection and the evolution of species. Just as mutations occur randomly in biology, so are universes created through random quantum fluctuations. Natural selection implies that only beneficent mutations will persist in a population, and similarly, only those universes that allow observers will be observed.

holes in such compactifications, which is further discussed in Paper II. Chapter 6 discusses the topography and dynamics of the string theory landscape, and provides a background for Paper III and IV. Finally, chapter 7 deals with some cosmological aspects of the landscape, which is investigated in Papers V and VI.

Part I:
String Theory

2. Type II String Theory

In the previous chapter we introduced string theory and its most important properties. To understand the four-dimensional physics that can be achieved by string theory compactifications, we must study the theory more thoroughly. For string theory to provide a satisfactory description of our world, it should contain both bosons and fermions and be free of instabilities associated with tachyonic modes. There are several superstring theories that achieve this goal. One of them is the type IIB superstring theory, which is the focus of this thesis.

The aim of this chapter is to describe the type IIB superstring theory in some detail. This is a supersymmetric theory with both fermions and bosons, and the critical dimension is ten. We will start by reminding the reader of the classical properties of the theory, in particular its description in terms of the world-sheet action. We then quantize the action, and derive the massless spectrum of the theory. We discuss how the states in the massless sector behave as fields in space-time. In particular, as long as we focus on low-energy processes (as we do in this thesis), the dynamics of the massless sector of the theory is described by a classical field theory called type IIB supergravity. Finally, we describe how non-perturbative objects called D-branes arise in the theory, and what their impact is in the low-energy limit.

This chapter is based on the standard reference books [GSWa, GSWb, Pola, Polb, BBS]. These books contain thorough treatments of the topics that we will touch upon here. They also contain a more complete list of references than we will provide.

2.1 Superstring Theories

2.1.1 The World-Sheet Action

Recall from the preceding chapter that a string moves through space-time in a way that minimizes the area of its world-sheet. Mathematically, this can be described by the minimization of the world-sheet action

$$S_P = -\frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M\partial_\beta X_M, \tag{2.1}$$

This action is known as the Polyakov action. The string tension is given by $T = 1/2\pi\alpha'$, which also defines the string length l_s by $l_s^2 = \alpha'$. The metric on the worldsheet is called $h^{\alpha\beta}$, and we have assumed that space-time is flat. The fields X^M describe the embedding of the string into space-time,[1] as shown in figure 1.1. These are all bosonic fields, and action (2.1) describes a bosonic string theory. We will now describe how it can be modified to accomodate fermionic degrees of freedom.

The starting point of our derivation is the action (2.1). For ease of presentation, we follow [GSWa] and use the symmetries of the action to fix a gauge, before we add the fermions. In particular, the action (2.1) is invariant under general coordinate transformations (diffeomorphisms) and Weyl rescalings of the world-sheet. We can use these symmetries to fix the world-sheet metric $h_{\alpha\beta}$ to a particular gauge. It is convenient to work in conformal gauge $h_{\alpha\beta} = \exp(\phi)\eta_{\alpha\beta}$, where ϕ is a function of the world-sheet coordinates. We then allow a new set of fields ψ^M to propagate on the world-sheet. The world-sheet action becomes[2]

$$S = -\frac{1}{2\pi}\int d^2\sigma \left(\partial_\alpha X^M \partial^\alpha X_M - i\bar{\psi}^M \rho^\alpha \partial_\alpha \psi_M\right), \tag{2.2}$$

Here we put $2\alpha' = 1$ for simplicity, since such factors can later be restored by dimensional analysis. We let σ, τ parametrize the world-sheet, and introduce the two-dimensional Dirac matrices ρ^α that obey

$$\{\rho^\alpha, \rho^\beta\} = -2\eta^{\alpha\beta}. \tag{2.3}$$

From this action we can derive the equation of motion for the fields:

$$\partial_\alpha \partial^\alpha X^M = 0 \quad , \quad \rho^\alpha \partial_\alpha \psi^M = 0. \tag{2.4}$$

Thus ψ^M obey the two-dimensional Dirac equation, and are world-sheet fermions that can be required to be real (i.e. Majorana). The equation of motion for X^M is that of a free scalar field propagating on the world-sheet. In space-time both ψ^M and X^M transform as vectors.

By changing coordinates on the world-sheet to $\sigma^\pm = \tau \pm \sigma$, one can show that the solutions to (2.4) split into left- and right-movers on the world-sheet. Specifically, writing out the world-sheet spinor as (here $\pm$ indicate the chirality of the spinor)

$$\psi^M = \begin{pmatrix} \psi^M_- \\ \psi^M_+ \end{pmatrix} \tag{2.5}$$

one can show that the fields $\psi^M_-, \partial_- X^M$ are right-movers, whereas $\psi^M_+, \partial_+ X^M$ are left-movers. This split into left- and right-movers will be useful for the construction of the spectrum of the theory.

[1] We will use M, N to denote ten-dimensional spacetime indices, reserving μ, ν for four-dimensional spacetime indices.

[2] In two dimensions, the factors of ϕ cancel out in $\sqrt{h}h^{\alpha\beta}$.

The world-sheet action (2.2) has several symmetries, that help in analyzing the theory. It is manifestly invariant under spacetime Poincaré transformations, and is thus a relativistic theory. It is also invariant under translations of the world-sheet. Furthermore it is invariant under the global world-sheet supersymmetry

$$\begin{aligned} \delta X^M &= \bar{\epsilon}\psi^M \\ \delta\psi^M &= -i\rho^\alpha \partial_\alpha X^M \epsilon. \end{aligned} \tag{2.6}$$

As any supersymmetry, this mixes bosons and fermions. We have introduced the symmetry parameter ϵ, which is a constant anticommuting spinor. By using Noether's method one can find the conserved currents associated with these symmetries by allowing the symmetry parameter to be non-constant. In particular, the classical theory has a conserved supercurrent

$$J_\alpha = \frac{1}{2}\rho^\beta \rho_\alpha \psi_M \partial_\beta X^M \tag{2.7}$$

associated with local supersymmetry transformations. Similarly, the conserved current associated with world-sheet coordinate transformations $(\tau, \sigma) \to (\tau'(\tau, \sigma), \sigma'(\tau, \sigma))$ is the world-sheet energy-momentum tensor

$$T_{\alpha\beta} = \partial_\alpha X_M \partial_\beta X^M + \frac{i}{4}\bar{\psi}^M \rho_\alpha \partial_\beta \psi_M + \frac{i}{4}\bar{\psi}^M \rho_\beta \partial_\alpha \psi_M - \text{trace}, \tag{2.8}$$

which decomposes into a boson and fermion part: $T = T_B + T_F$.

In fact, the stress-energy tensor and the supercurrent are not only required to be constant in the classical theory, they must also vanish ($T_{\alpha\beta} = J_\alpha = 0$). As mentioned in the beginning of this section, the bosonic part of the action (2.2) is obtained by gauge-fixing a more symmetric theory, which we will refer to as the superconformal theory. The two theories can only describe the same physics if their fields abide by the same restrictions. In the superconformal theory, the world-sheet metric is a dynamical field, and so is its supersymmetric partner, the gravitino. Thus, there are equations of motion for these fields, and these set the stress-energy tensor and supercurrent to zero. As a result, we should impose

$$T_{++} = T_{--} = J_+ = J_- = 0 \tag{2.9}$$

as constraints on the gauge-fixed theory discussed here.[3]

2.1.2 Quantization

So far, we have focused on the classical aspects of the superstring theories. One of our goals in this chapter is to find the spectrum of the theory. To obtain

[3] $T_{+-} = 0$ follows directly from the tracelessness of the stress-energy tensor and need not be imposed as an extra condition.

this, the theory must be quantized. We will do this following the canonical quantization formalism, which has the advantage that it is relatively straightforward.[4]

The classical propagation of the superstring is found by solving the equations of motion (2.4), subject to the boundary conditions

$$\begin{aligned} \text{closed strings: } & X^M(\sigma,\tau) = X^M(\sigma+\pi,\tau), \\ \text{open strings: } & \partial_\sigma X^M(\sigma^*,\tau) = 0 \quad \text{N}, \\ & X^M(\sigma^*,\tau) = f^M_{\sigma^*}(\tau) \quad \text{D}, \end{aligned} \tag{2.10}$$

where $\sigma^* = 0, \pi$ and

$$\begin{aligned} \text{closed strings: } & \psi^M_\pm(0,\tau) = \psi^M_\pm(\pi,\tau) \quad \text{R}, \\ & \psi^M_\pm(0,\tau) = -\psi^M_\pm(\pi,\tau) \quad \text{NS}, \\ \text{open strings: } & \psi^M_+(0,\tau) = \psi^M_-(0,\tau), \\ & \psi^M_+(\pi,\tau) = \pm\psi^M_-(\pi,\tau) \quad \text{R/NS}. \end{aligned} \tag{2.11}$$

In the last lines we used that the overall relative sign between ψ^M_+ and ψ^M_- is a matter of convention, and so can be fixed at one of the endpoints. Note that the scalar fields are periodic on closed strings, as demanded by their interpretation as the spacetime coordinates of the string. The open string boundary conditions can either be of Dirichlet (D) or Neumann (N) type. The first conditions fix the string endpoint to some hypersurface given by $f^M_{0,\pi}(\tau)$. The second allow the endpoint to move, but makes sure that no momentum flows out of it.[5] It is also possible to have Neumann boundary conditions in some space-time directions and Dirichlet conditions in other. Also the fermionic fields have several consistent boundary conditions, which are known as Ramond (R) and Neveu–Schwarz (NS) boundary conditions. The boundary conditions couple the left-and right-moving sectors on the open string, but not on the closed string.[6]

The solutions to the classical equations of motion can now be expanded in modes satisfying the boundary conditions. E.g. for the bosonic fields on an open string with Neumann conditions:

$$X^M_{open} = x^M + p^M\tau + i\sum_{n\neq 0}\frac{\alpha^M_n}{n}e^{-in\tau}\cos n\sigma. \tag{2.12}$$

[4]The reader is referred to [GSWa] for a description of various ways of quantizing the string.

[5]Momentum will flow out of the string endpoint in the case of Dirichlet conditions. Energy-momentum conservation then implies that this momentum is absorbed by the hypersurface on which the endpoint is fixed. We will discuss what these hypersurfaces are in section 2.4.

[6]On the quantum level there is one level-matching condition that couples the left- and right-movers of the closed string theory, as we will explain below.

As would be expected, the expansion decomposes the motion of an open string into a center of mass translation, described by x^M and p^M, and an infinite set of possible vibrations with frequency set by an integer n. For open strings with Dirichlet boundary conditions, the cosine is replaced by a sine. For the closed string the expansion is

$$\begin{aligned} X^M_{closed} &= X^M_R + X^M_L, \text{ where} \\ X^M_L(\sigma^+) &= \frac{1}{2}x^M + \frac{1}{2}p^M\sigma^+ + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha^M_n}{n}e^{-in\sigma^+}. \end{aligned} \tag{2.13}$$

The expansion for $X^M_R(\sigma^-)$ is identical up to a change of the coefficients α^M_n to $\tilde{\alpha}^M_n$.

Similarly, the expansion of the $\psi^M_\pm$ fields on an open string are

$$\psi^M_\pm = \frac{1}{\sqrt{2}}\sum_r b^M_r e^{-ir\sigma\pm}, \tag{2.14}$$

where r going over integers (half-integers), in the R (NS) sector. For world-sheet fermions on closed strings there are similar expansions for left- and right-movers. This leads to four distinct closed string sectors, corresponding to different combinations of left- and right-moving fermionic modes: NS-NS, NS-R, R-NS and R-R. Thus the closed string is just two copies of the open string, subject to a level matching condition for the zero modes, that we return to in equation (2.21).

We can quantize the theory by promoting the fields to operators and their Poisson brackets to commutator relations. For the theory at hand we obtain the equal-τ (anti)commutators

$$\begin{aligned} [X^M(\sigma), \dot{X}^N(\sigma')] &= i\pi\eta^{MN}\delta(\sigma-\sigma'), \\ \{\psi^M_A(\sigma), {\psi_B}^N(\sigma')\} &= \pi\eta^{MN}\delta_{AB}\delta(\sigma-\sigma'). \end{aligned} \tag{2.15}$$

These translate readily into relations for the expansion coefficients:

$$\begin{aligned} [\alpha^M_m, \alpha^N_n] &= m\eta^{MN}\delta_{m+n}, \\ \{b^M_r, b^N_s\} &= \eta^{MN}\delta_{r+s}. \end{aligned} \tag{2.16}$$

For closed strings, a second copy of these relations is added for the tilded modes.

We now see that the expansion coefficients can be reinterpreted as creation ($n, r < 0$) and annihilation ($n, r > 0$) operators acting on the various sectors of the theory. We define the vacuum in each sector to be the state that is annihilated by all annihilation operators, i.e.

$$\alpha^M_m|0\rangle = b^M_r|0\rangle = 0\,, \forall\, m, r > 0 \tag{2.17}$$

for the two sectors of the open string. These states are also the states of lowest mass in the respective sectors.

In the R sector, the vacuum state is degenerate. This follows from $\{b_m^M, b_0^N\} = 0$ for all positive m, implying that $b_0^M|0\rangle$ are again annihilated by all positive frequency modes. In the NS sector, however, the half-integer $r \neq 0$ and the vacuum state is unique. The spectrum of the theory can then be constructed by acting on the ground state with the creation operators. We are particularly interested in the massless part of the spectrum, to which we now turn.

2.1.3 The Massless Spectrum

In the last section we found the vacua and creation operators of the R and NS sectors. Using this, we can construct the spectrum. However, before we proceed, we first need to determine whether all states in the Fock space we construct are physical. The first indication that this is not the case is given by (2.16) — the 'time-like' creation modes α_m^0 and b_r^0 create negative norm states, a.k.a. ghosts.[7] To get rid of these oddities, recall that the classical theory is a gauge-fixed version of a superconformal theory. As such, there are hidden symmetries in the theory, that should be kept on the quantum level. Particularly, the constraints (2.9) should be imposed in the quantized theory. Any physical quantity, such as the probability for an initial state to evolve into a final state, should be invariant under the superconformal symmetries, which requires

$$\langle\Psi_i|T_{\pm\pm}|\Psi_f\rangle = \langle\Psi_i|J_\pm|\Psi_f\rangle = 0. \tag{2.18}$$

Using the properties of the two-dimensional world-sheet theory, the non-trivial conditions are that $T_{\pm\pm}$ and $J_\pm$ annihilate physical states. As the fields, the currents have expansions in modes that are conventionally denoted L_m for $T_{\pm\pm}$, and G_r for $J_\pm$ (with integer (half-integer) r in the R (NS) sector). The conditions (2.18) are then fulfilled provided that the physical states are annihilated by the positive-frequency modes

$$L_m|\Psi\rangle = 0\,, \; G_r|\Psi\rangle = 0\,, \; r, m > 0. \tag{2.19}$$

The operators L_m and G_r generate the so-called super-Virasoro algebra, so this condition identifies a physical state with the highest weight state of a representation of that algebra.[8]

In addition we have the zero-mode constraint $(L_0 - a)|\Psi\rangle = 0$ and also $G_0|\Psi\rangle = 0$ for the R sector. The first condition turns out to determine the

[7] I.e. the norm of the state $\alpha_{-m}^0|0\rangle$ is negative $\langle 0|\alpha_m^0\alpha_{-m}^0|0\rangle = \langle 0|[\alpha_m^0, \alpha_{-m}^0]|0\rangle = -m$.

[8] There is much more to say about the super-Virasoro operators L_m, G_r and the algebra that they generate, but since this will not be needed for the further discussion we will not do so here. See e.g. [GSWa, Pola, Polb, BBS] for further discussions.

mass of the state in terms of the oscillator modes as

$$\alpha' M^2 = -\alpha' p^\mu p_\mu = N - a \quad \text{, where}$$
$$N = \sum_{n=1}^{\infty} \alpha_{-n} \cdot \alpha_n + \sum_{r=1}^{\infty} r b_{-r} \cdot b_r. \tag{2.20}$$

The constant a in the zero-mode constraint arises as a normal-ordering ambiguity when expressing L_0 as a product of the creation and annihilation modes. It can be shown that the second condition $G_0|\Psi\rangle = 0$ is the space-time Dirac equation. Consequently, since the ground state in the R sector solves this equation, it must be a space-time fermion.

How can these constraints remove the ghosts of the theory? This question can be answered by fixing the space-time metric to the light-cone gauge. In this gauge, space-time Lorentz invariance is broken by singling out two light-cone coordinates, say $X^{\pm} = (X^0 \pm X^{D-1})/\sqrt{2}$ and reparametrizing the world-sheet so that one of the light-cone coordinates is fixed: $X^+ = x^+ + P^+\tau$. This puts all oscillators α_m^+ to zero for $m \neq 0$. Worldsheet supersymmetry then implies that $\psi^+ = 0$. The super-Virasoro constraints can then be solved for the other light-cone modes X^-, ψ^- in term of X^i, ψ^i. Using these relations, one can show that all remaining modes of the fields have positive norm. After removing the ghosts in this way, we can adjust the space-time dimension D and the normal-ordering constants a so that space-time Lorentz invariance is restored. Requiring that the light-cone theory satisfy the usual Lorentz algebra imposes that $D = 10$ and $a_R = 0$ or $a_{NS} = 1/2$ in the R and NS sectors.[9]

Using these results and equation (2.20), we immediately see that the ground state in the NS sector is a tachyon, with $M^2 = -1/(2\alpha')$. The next mass level is created by the operators $b^N_{-1/2}$ which raise M^2 by half a unit: hence we get a massless state for each M. Remembering that the two light-cone states are unphysical, these states form an eight-dimensional vector representation of SO(8). This is as expected for a massless vector particle in ten dimensions, whose little group is SO(8).

In the R-sector, the vacuum $|0\rangle_R$ is massless. As mentioned above, it cannot be uniquely defined, since any state $|u\rangle = b_0^N|0\rangle_R$ is also a massless state annihilated by all positive frequency modes α_m^N, b_m^N. The b_0^N operators obey the ten-dimensional Clifford algebra, and consequently the states $|u\rangle$ are space-time fermions. The smallest ten-dimensional spinor representation is the real, 16-dimensional Majorana-Weyl spinor, which has definite chirality $\Gamma_{11}|0\rangle_R^{\pm} = \pm|0\rangle_R^{\pm}$. The Dirac equation further reduces the number of states by a factor of two. Hence, the R vacuum consists of two eight-dimensional spinor representations of definite chirality. In fact, all the states in the R sector will be

[9]These conditions can also be derived in other ways. We will not go into the details here, but refer to [GSWa, Pola, Polb, BBS].

sector	SO(8) rep.	M^2
NS-	**1**	$-1/2\alpha'$
NS+	$\mathbf{8}_v$	0
R+	**8**	0
R-	$\mathbf{8}'$	0

Table 2.1: *The low-energy spectrum of the open superstring. The ground state is tachyonic and there are three different massless states. The $\pm$ indicate the G-parity of the states.*

space-time fermions, since the raising operators α_n^N, b_n^N are vectors and raise the spin of the state by an integer number.

The low-energy spectrum of closed strings can now be obtained by combining the states in the four sectors of the open string spectrum. As mentioned above, the right- and left-moving sectors are independent up to the level matching condition

$$(L_0 - \tilde{L_0})|\Psi\rangle = 0, \tag{2.21}$$

where the two operators act on right- and left-moving states respectively. This condition is a consequence of the periodicity of the closed string. It implies that physical states are those where equally many left- and right-movers are excited at each mass level.

The level matching condition cannot be fulfilled by, say, a left-moving NS tachyon and a rightmoving massless state. Consequently, closed string theories have a tachyonic ground state and a large number of massless states corresponding to various combinations of the massless R and NS states. As noted above, the open string states in the R sector are fermions, whereas the NS states are bosons. It follows that the closed string states in the (NS,NS) and (R,R) sectors are bosons, whereas states in the (NS,R) or (R,NS) sectors are fermions.

In summary, we have found a candidate theory for closed and open strings, that have both bosons and fermions. There is one problem left to solve, namely to remove the tachyons from the string spectra. It is possible to fix this problem by truncating the spectrum, a procedure known as the GSO projection [GSO77]. We therefore define the action of the G-parity operator on a state as

$$\begin{aligned} G &= \Gamma_{11}(-1)^{\sum_{n=0}^{\infty} b_{-n}b_n} \text{ , R} \\ G &= -(-1)^{\sum_{r=1/2}^{\infty} b_{-r}b_r} \text{ , NS.} \end{aligned} \tag{2.22}$$

On the massless level, the R states with even G-parity correspond to the positive chirality states. The ground state of the NS sector has odd G-parity, whereas the first excited state has even parity. For the reader's convenience, we list the open string modes, with G-parities, in table 2.1.

The GSO projection consists of truncating the spectrum in the following way. In the NS sector, we keep only the even G-parity states at each mass level. Thus, the tachyon is removed from the spectrum. In the R sector we can choose to keep either the state of even or odd G-parity, but not both. In particular, we can construct a consistent closed string theory by combining the left- and right-moving sectors as (R+,R-), (NS+,R-), (R+,NS+) and (NS+,NS+). This is called type IIA string theory and is a non-chiral theory, since both positive and negative chirality spinors are present in the spectrum.

A chiral theory can be obtained by choosing the same chirality for the left- and right-moving states, i.e. combining (NS+,NS+), (NS+,R+), (R+,NS+) and (R+,R+).[10] This is the massless spectrum of the type IIB string theory. The full spectrum is obtained by acting on the massless states with left- and right-moving creation operators, subject to the G-parity conditions in the respective theory.

The massive states in the spectrum will be of little interest to us, since these states are too heavy to be excited in relevant physical processes.[11] Instead, it is the low-energy behaviour of the theory, and hence the massless spectrum that is of interest to us. We therefore pause to study the states of this spectrum in more detail.

In type IIB string theory, the states in the (NS+,NS+) are products of two vector representations $\mathbf{8}_v\otimes\ \mathbf{8}_v$. These 64 bosonic states decompose into a scalar Φ and two two-tensors: the traceless and symmetric G_{MN} and the antisymmetric B_{MN}. The symmetric mode G_{MN} has spin 2 and therefore have all the properties of a space-time graviton. We will see in the next section that this is indeed the right space-time interpretation of this field. The other two fields are similarly identified with the dilaton and the antisymmetric B-field. The (NS+,R+) states are products of a vector and a spinor representation $\mathbf{8}_v\otimes\ \mathbf{8}'$. These 64 fermionic states decompose into a spin-1/2 spinor λ^- (the dilatino) and a spin-3/2 spinor ψ_M^+ (the gravitino). These have opposite chiralities. The same decomposition happens in the (R+,NS+) sector. The (R+,R+) sector gives us again 64 bosonic states by a product of two spinor representations $\mathbf{8}\ '\otimes\ \mathbf{8}'$ of the same chirality. These decompose into one scalar, $C_{(0)}$, a differential two-form, $C_{(2)}$, and a four-form $C_{(4)}$. All these fields are gauge fields.[12]

[10]The choice of the R chirality is a matter of convention. Changing chiralities on all R states in the two theories give us new copies of the same theories.

[11]Their mass scale is given by the string length, which is expected to be extremely small. Indeed, since string theory is a theory of quantum gravity, the natural guess is that the string scale is of the order of the Planck scale.

[12]Throughout the thesis, differential forms will be used to represent antisymmetric tensor fields. See e.g. [Polb, Nak] for an introduction to this mathematical concept.

Type IIA		TypeIIB	
Sector	Fields	Sector	Fields
(NS+,NS+)	Φ, G_{MN}, B_{MN}	(NS+,NS+)	Φ, G_{MN}, B_{MN}
(NS+,R-)	λ^-, ψ_M^+	(NS+,R+)	λ^-, ψ_M^+
(R+,NS+)	λ^+, ψ_M^-	(R+,NS+)	λ^-, ψ_M^+
(R+,R-)	$C_{(1)}, C_{(3)}$	(R+,R+)	$C_{(0)}, C_{(2)}, C_{(4)}$

Table 2.2: *The massless spectrum of the the type IIA and IIB string theories. The fields in the (NS,NS) and (R,R) sectors are spacetime bosons, the fields in the (NS,R) and (R,NS) sectors are fermions. The $\pm$ indicate the G-parity of the states.*

Similarly, we can write down the massless fields for the type IIA string. The (NS+,NS+) and (R+,NS+) sectors are the same as in type IIB, containing the dilaton, graviton, B-field, negative chirality dilatino and positive chirality gravitino. In the (NS+,R-) sector we find a positive chirality dilatino and a negative chirality gravitino, and the tensor product of the opposite chirality spinors in the (R+,R-) sector combine to a one-form and a three-form gauge field ($C_{(1)}$ and $C_{(3)}$).

We have now achieved one of the goals of this chapter, namely to find the massless states of the type II string theories. This will be the basis for the four-dimensional theories we will derive from compactifications in the following chapters. The massless field content of the two theories is summarized in table 2.2. It is worth noticing that the spectra are very similar. In fact, neglecting the RR gauge fields, the only difference is the choice of parity of the spinors. This hints at an underlying symmetry between these theories. Interestingly, this symmetry, which is known as T-duality, will only be manifest after the theories are compactified. We will return to it in chapter 3.

Note that the number of space-time fermions and bosons match on the zero mass level, exactly as would be required from space-time supersymmetry. This is no coincidence. In fact, theories with interacting spin $\frac{3}{2}$ particles, such as the gravitinos, typically require supersymmetry for consistency. Here we have two gravitinos, and the theory has $\mathcal{N} = 2$ space-time supersymmetry, which explains why the theories are called type II. The supersymmetry persists at all mass levels of the string theory [GSO77] and also non-perturbatively, as can be shown by using the Green–Schwarz formulation for the superstring [GS84]. The space-time supersymmetries of the theories are very important, as we will return to in section 2.3, where we write down the low-energy action that describe the dynamics of the massless fields.

2.2 Spacetime Actions

We now move on to the second goal of this chapter, namely the description of the low-energy dynamics of type II strings in terms of a spacetime action. In this section, we take a first step towards achieving this goal. We show how consistency conditions of the world-sheet action describing an interacting string can be interpreted as equations of motion of the fields. It follows that we can then write down an effective action, whose variation yields the equations of motion.

The string theories described so far have described the motion of strings in flat spacetime; the spacetime metric in the world-sheet action (2.2) is trivial. On the other hand, we have seen that one of the massless states in the superstring spectrum, namely G_{MN}, behaves as a spacetime graviton. Thus, we expect the string theory gravitons should be related to the metric of spacetime, as is consistent with the overall picture of strings building up our world. In this section we summarize how string theory interactions give rise to curved spacetimes, and how the low-energy physics arising from the world-sheet analysis can be described by spacetime actions. We only discuss the spacetime bosons here, returning to the fermionic fields in the next section.

From the spacetime point of view, a natural guess for an action describing a bosonic string propagating in a slightly curved spacetime is

$$S = -\frac{1}{2\pi}\int_{\Sigma} d^2\sigma\big(\eta_{MN} + h_{MN}\big)\partial_\alpha X^M \partial^\alpha X^N. \tag{2.23}$$

The propagation of a string in such a background is described by the path integral

$$\int DXe^{S} = \int DX\left(1 - \frac{1}{2\pi}\int_{\Sigma} h_{MN}\partial_\alpha X^M \partial^\alpha X^N + \mathcal{O}(h^2)\right)e^{S_{flat}}. \tag{2.24}$$

Here S_{flat} is the world-sheet action in flat spacetime.

This is very similar to the path integral of a string that emits gravitons G_{MN} as it propagates through flat spacetime. The emission of a graviton corresponds to the splitting of the string world-sheet as shown in figure 2.2. Using the conformal invariance of string theory, one can show that this split world-sheet is equivalent to an unsplit world-sheet with a puncture (indicated by a cross in 2.2). The process of emitting a graviton can mathematically be described by the insertion of a vertex operator $V(G)$ in the path integral

$$\int DX\int_{\Sigma} V(G)e^{S_{flat}} = \int DX\left(\frac{1}{2\pi}\int_{\Sigma} G_{MN}\partial_\alpha X^M \partial^\alpha X^N\right)e^{S_{flat}}. \tag{2.25}$$

This is just the leading perturbation of the path integral for a string in slightly curved spacetime, as we saw above. Taking into account emissions of any number of gravitons gives the full path integral (2.24). Thus, perturbative

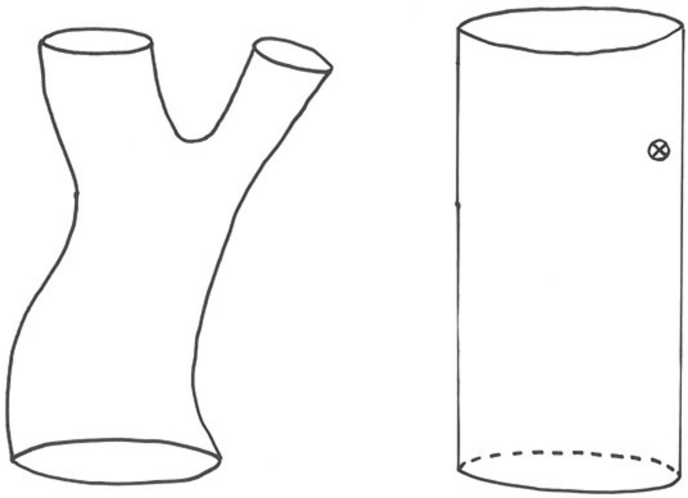

Figure 2.1: String interactions are described by the splitting and joining of world-sheets. A conformal mapping takes the world-sheet in (a) to the world-sheet in (b).

string theory automatically gives the geometry of spacetime, and our interpretation of G_{MN} as a graviton is correct.

We can then describe a string propagating in curved spacetime by an action where the spacetime metric is given by $G_{MN}(X)$

$$S = -\frac{1}{2\pi}\int d^2\sigma G_{MN}(X)\partial_\alpha X^M \partial_\alpha X^N. \tag{2.26}$$

The action (2.26) is an example of a non-linear sigma model. Since the action is no longer quadratic in X, it describes an interacting two-dimensional field theory, in accordance with the discussion above. By expanding the path integral around a classical solution $X^M(\sigma) = x_0^M + Y^M(\sigma)$ the kinetic term can be written

$$G_{MN}(X)\partial_\alpha X^M \partial_\beta X^N \approx [G_{MN}(x_0) + G_{MN,P}(x_0)Y^P + ...]\partial_\alpha Y^M \partial_\beta Y^N. \tag{2.27}$$

The derivatives of the metric at x_0 are related to the radius of curvature in spacetime R_c, so that the expansion coefficients are in fact $\sqrt{\alpha'}R_c^{-1}$. Hence, the nonlinear interaction terms can be neglected if the spacetime curvature is small, so that the curvature radius is much larger than the string length.

Similarly, we can now construct non-linear sigma models by allowing non-trivial background values of the other massless fields. In doing so, we should be careful to check that the non-linear actions respect the symmetries of the original theory. For example, requiring that (2.26) is Weyl invariant leads to the condition [CMPF85]

$$0 = \beta^G_{MN} = \alpha' R_{MN} + \mathcal{O}(\alpha'^2), \tag{2.28}$$

where β^G_{MN} is the beta functional associated with renormalizations of the spacetime metric. For a Weyl-invariant theory, this must be zero. Note that, to leading order, this is just Einstein's vacuum equations. The interacting worldsheet theory is consistent, provided that Einstein's vacuum equations are satisfied. This shows that string theory really determines the spacetime geometry.

For a more general non-linear sigma model, there will be additions to β^G_{MN} from other fields. We also get more beta functionals, one for each field. It is now natural to switch perspective, and interpret these conditions as equations of motion for the spacetime fields. We can then construct a spacetime action whose variation yields these equations of motion. Thus, up, to certain corrections, the low-energy dynamics of string theory can be described as a spacetime field theory.

2.3 Type IIB Supergravity

In the preceding sections it has become clear that, at low energies $E \ll 1/\alpha'$, only the lightest modes of the string theory are excited. Furthermore, the low-energy physics should be described by an effective spacetime action involving only these light degrees of freedom. At higher energies, this description is still useful, provided that corrections are added to the action. These corrections come from higher derivative terms but also the excitation of e.g. heavier fields in loops. The low-energy action thereby describes the leading order behaviour of a string in a double expansion in loop corrections (controlled by the string coupling g_s[13]) and derivative corrections (controlled by α').

Section 2.2 described how low-energy equations of motion can be derived in string theory. Another very useful trick is to use supersymmetry to find these equations. For example, the closure of the symmetry algebra requires that two supersymmetry transformations of a field yield a translation of the field. Likewise, the supersymmetry transformation of an equation of motion should be a combination of equations of motion. In this way the equations of motion for the remaining fields are derived.

Indeed, for type IIB string theory, the large amount of supersymmetry completely determines the equations of motion and hence the low-energy action. It must be the same as the action for the massless sector of type IIB supergravity, whose field content exactly matches the low-energy spectrum of the type IIB string theory. Similarly, the dynamics in the massless sector of the type IIA string theory is described by type IIA supergravity.

[13]The string coupling is not a free parameter in the theory, but is set by the vacuum expectation value of the dilaton field $\langle e^{\Phi} \rangle$.

The bosonic part of the type IIB supergravity action can be written (in the notation of [Polb])

$$
\begin{aligned}
S_{IIB} &= S_{NS} + S_{RR} + S_{CS}, \\
S_{NS} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x (-G)^{1/2} e^{-2\Phi} \left(R + 4\partial_M \Phi \partial^M \Phi - \frac{1}{2}|H_{(3)}|^2 \right), \\
S_{RR} &= -\frac{1}{4\kappa_{10}^2} \int d^{10}x (-G)^{1/2} \left(|F_{(1)}|^2 + |F_{(3)}|^2 + \frac{1}{2}|\tilde{F}_{(5)}|^2 \right), \\
S_{CS} &= -\frac{1}{4\kappa_{10}^2} \int C_{(4)} \wedge H_{(3)} \wedge F_{(3)}.
\end{aligned}
\tag{2.29}
$$

This action is written in terms of p-forms, and $\wedge$ denotes the antisymmetric wedge product of forms. $F_{(p+1)}$ is the RR $p+1$-form field strength, or flux, associated with the p-form gauge field $C_{(p)}$; $F_{(p)} = dC_{(p)}$, where d is the exterior derivative.[14] Similarly, $H_{(3)}$, given by $H_{(3)} = dB_{(2)}$ is the NS three-form flux. The RR five-form flux $\tilde{F}_{(5)}$ is of non-standard form

$$
\tilde{F}_{(5)} = dC_{(4)} - \frac{1}{2}C_{(2)} \wedge H_{(3)} + \frac{1}{2}B_{(2)} \wedge F_{(3)}. \tag{2.30}
$$

It is easy to see that the fluxes fulfill the Bianchi identities

$$
dF_{(3)} = dH_{(3)} = 0 \quad \text{and} \quad d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)}. \tag{2.31}
$$

There is one subtlety with the type IIB action. Taking Hodge dual of a differential p-form living in D dimensions gives a $D-p$ form[15]

$$
*F_{(p)} = F_{(D-p)}. \tag{2.32}
$$

Thus, the Hodge dual of the five-form flux $\tilde{F}_{(5)}$ is again a five-form flux $*\tilde{F}_{(5)}$. However, there is only one four-form gauge field in the theory, so the two field strengths must actually be equal

$$
*\tilde{F}_{(5)} = \tilde{F}_{(5)}. \tag{2.33}
$$

This does not arise as a field equation from the variation of S_{IIB}. Indeed, so far there is no simple covariant action known that yields the self-duality condition as an equation of motion. However, the equations that are given by S_{IIB} are consistent with this constraint, so we will proceed with this action, keeping in mind that the self-duality constraint must be imposed on the level of the equations of motion.

[14]The definitions of the wedge product and the exterior derivative can be found in e.g. [Nak].

[15]The Hodge dual of a p form is defined in e.g. [Nak].

Note that, by Hodge duality, the type IIB theory could equally well be written in terms of the field strengths $\tilde{F}_{(5)}, F_{(7)}$ and $F_{(9)}$, i.e. the Hodge duals of $\tilde{F}_{(5)}, F_{(3)}, F_{(1)}$ respectively, or in a 'democratic' formulation where all field strengths are included and self-duality constraints are imposed to reduce the doubling of the degrees of freedom. Although equivalent, different formulations are useful for studies of different aspects of the theory, as discussed in [BKO$^+$01]. We will mainly use the formulation in terms of $\tilde{F}_{(5)}, F_{(3)}, F_{(1)}$.

It is useful to rewrite S_{IIB}, by defining

$$\begin{aligned} &\tau = ie^{-\Phi} + C_{(0)}\,, G^E_{MN} = e^{-\Phi/2} G_{MN} \\ &G_{(3)} = F_{(3)} - \tau H_{(3)}. \end{aligned} \tag{2.34}$$

This Weyl rescaling of metric absorbs some of the dilaton factors in the action, so that the Ricci scalar term is written as a standard Einstein–Hilbert term. This is known as going to the Einstein frame, whereas the action (2.29) is written in the string frame. The field τ is known as the axio-dilaton. The action can then be written [DG03]

$$\begin{aligned} S_{IIB} &= S_{SUGRA} + S_{CS}, \\ S_{SUGRA} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G^E}\Big(R^E - \frac{1}{2(\mathrm{Im}\tau)^2}\partial_M\tau\partial^M\bar{\tau} \\ &\quad - \frac{1}{12(\mathrm{Im}\tau)}G_{(3)}\cdot\bar{G}_{(3)} - \frac{|\tilde{F}_{(5)}|^2}{4\cdot 5!}\Big), \\ S_{CS} &= -\frac{1}{8i\kappa_{10}^2}\int\frac{1}{(\mathrm{Im}\tau)}C_{(4)}\wedge G_{(3)}\wedge G_{(3)}. \end{aligned} \tag{2.35}$$

We have now achieved the third goal of this chapter, namely to write down the spacetime action for the low-energy dynamics of type IIB string theory. Although this is just the bosonic part of the action, it will suffice for our purposes. The reason for this is that we will only be interested in maximally symmetric spacetimes (de Sitter, Minkowski or anti de Sitter), which are only possible if the expectation values of fermionic fields are zero [CHSW85]. These will be fully $\mathcal{N} = 2$ supersymmetric solutions as long as the supersymmetry variations of the gravitinos and dilatinos are zero (that the supersymmetry variations of the bosonic fields are zero follow from the vanishing of the fermionic fields).

For $\mathcal{N} = 2$ supersymmetry there are two supersymmetry parameters, that we denote $\epsilon^{1,2}$. These are Majorana-Weyl spinors, just as the gravitinos and dilatinos. We label the two gravitinos and dilatinos in the same way, and then collect fermionic fields in column vectors

$$\psi_M = \begin{pmatrix}\psi^1_M \\ \psi^2_M\end{pmatrix}\,, \lambda = \begin{pmatrix}\lambda^1 \\ \lambda^2\end{pmatrix}\,, \epsilon = \begin{pmatrix}\epsilon^1 \\ \epsilon^2\end{pmatrix}. \tag{2.36}$$

The gravitino and dilatino variations are then, in string frame [Gra06]

$$\begin{aligned}
\delta\psi_M &= \nabla_M \epsilon - \frac{1}{8} \not{H}_M \mathcal{P}\epsilon + \frac{1}{16} e^{\Phi} \sum_n \not{F}_{(n)} \Gamma_M \mathcal{P}_n \epsilon \\
\delta\lambda &= \left(\not{\partial}\Phi + \frac{1}{8} \not{H} \mathcal{P} \right) \epsilon + \frac{1}{8} e^{\Phi} \sum_n (-1)^n (5-n) \not{F}_{(n)} \mathcal{P}_n \epsilon
\end{aligned} \tag{2.37}$$

Here we use the democratic formulation of the supergravity of [BKO$^+$01]. We have also introduced the ten-dimensional gamma matrices Γ_M, and a slash denotes contraction with these, e.g. $\not{F}_n = \frac{1}{n!} F_{P_1..P_n} \Gamma^{P_1..P_n}$. The 2×2 matrices $\mathcal{P}$ and $\mathcal{P}_n$ are $\mathcal{P} = -\sigma^3$, $\mathcal{P}_n = \sigma^1$ for even $(n+1)/2$ and $\mathcal{P}_n = i\sigma^2$ for $(n+1)/2$ odd, where σ^i are the Pauli matrices.

The vanishing of the gravitino and dilatino variations put important restrictions on the supersymmetric solutions of the theory. We will return to these conditions at various stages of this thesis. In particular, in chapters 3 and 5 we will see how these supersymmetry restrictions help us to choose suitable compactification manifolds.

2.4 D-branes

So far, we have focused on the perturbative part of the type II string theories. This resulted in a spectrum of massless fields and an effective action describing their dynamics. In the rest of this thesis we will be interested in various solutions to this theory. Some of these solutions will involve non-perturbative aspects of the theories. In this section we will describe one such aspect, namely D-branes, which are extended charged objects.

In the last section it was shown that there are several gauge-fields in the low-energy regime of type II string theories. It is therefore interesting to investigate if there are also objects that are charged under these fields in the theories. In the supergravity approximation such charged objects arise as possible backgrounds, i.e. as solutions to the classical equations of motion derived from the variation of (2.35). Typically, they are extended in spacetime as we now explain.

To find the object charged under a $(p+1)$-form gauge potential, we consider the integral of the potential over a $(p+1)$-dimensional volume:

$$Q_p \int_{\Sigma_{(p+1)}} C_{(p+1)}. \tag{2.38}$$

Here Q_p is the charge of the object we are looking for. This is very similar to the action describing the interaction of a particle of electric charge q with a potential A in electrodynamics

$$q \int_L A, \tag{2.39}$$

where L is the world-line of the particle. By analogy, we expect that $\Sigma_{(p+1)}$ is the world-volume of the object that is electrically charged under a $(p+1)$-form potential. It follows that this is an object with p spatial dimensions, and it is called a p-brane. It has electric charge

$$Q_p = \int *F_{(p+2)} = \int *dC_{(p+1)}, \tag{2.40}$$

where the integral goes over a $(D-p-2)$-dimensional cycle that links $\Sigma_{(p+1)}$. Alternatively, by electric-magnetic duality, the charge can be viewed as the magnetic charge of a $(D-p-3)$-form potential

$$Q_p = \int F_{(D-p-2)} = \int dC_{(D-p-3)}, \tag{2.41}$$

where $F_{(D-p-2)}$ is the Hodge dual of $F_{(p+2)}$, as above. Consequently, we can choose to view the brane either as an electrically or a magnetically charged object, depending on the flux that is present in the formulation of our theory.

It will be important for the rest of this thesis that the brane charges are quantized, just as the charges of ordinary electromagnetism. This follows by a generalised form of Dirac's quantization condition, i.e. by requiring that the wave function of an electric brane is well-defined in the field of a magnetic brane.[16] In ten dimensions we get the condition

$$Q_p Q_{6-p} = 2\pi n, \quad n \in \mathbb{Z}, \tag{2.42}$$

where $n = 1$ for all branes in superstring theory.

Since the supergravity we are interested in is derived from string theory, it is crucial to understand how the branes arise in string theory. Recall that the boundary conditions of open strings can be of either Neumann or Dirichlet type. As for a classical string, conservation of energy and momentum implies that Dirichlet boundary conditions are only possible if the string is attached to another object. These objects can be of any dimension $p = 0 \ldots 9$, where $9-p$ is given by the number of dimensions where the string has Dirichlet boundary conditions.[17] It can be shown that these objects are charged under the RR potential, and that they are in fact the branes described above [Pol95]. From their relation to Dirichlet boundary conditions, the branes are now known as

[16] [BBS, Polb] contain explanations of Dirac's quantization condition and its generalization.

[17] In fact, $p = -1$ is also possible, corresponding to an instanton charged under the C_0 potential of type IIB.

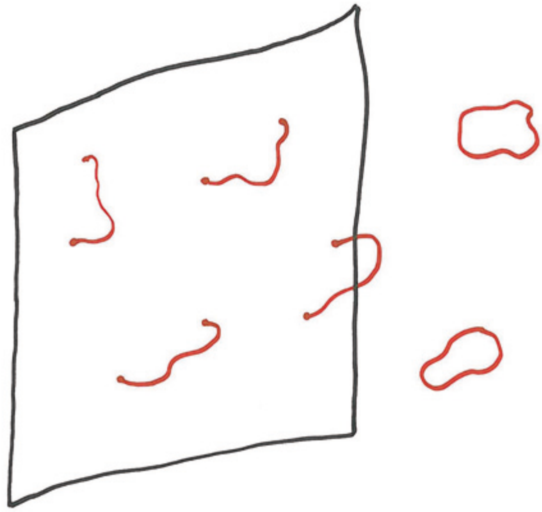

Figure 2.2: D-branes are hypersurfaces on which open strings can end. They are dynamical objects whose spacetime fluctuations are described by open strings. The D-brane interacts with its surroundings via closed strings.

Dp-branes, or D-branes for short (e.g. [Pola, Polb] contain nice introductions to the topic).

The low-energy behaviour of D-branes can be understood in terms of the open strings that end on it. Indeed, the open string endpoints that lie on the D-brane behave as fields in the brane world-volume. Some of these fields describe the fluctuations of the brane in spacetime. It follows that there should be an open string action that determines the low-energy dynamics of the D-brane. We refer to e.g. [Pola, Polb] for the derivation of this action, which will not be needed in the following. What we do need is that the D-branes have a tension that scales with the inverse of the string coupling. Since the D-branes have a tension, they also have a mass, and as such they interact gravitationally with their surroundings. This interaction can be understood in terms of the emission of closed strings (i.e. gravitons), and generically deforms spacetime around the brane.

Thus, we see that the type II string theories, which were derived as closed string theories in section 2.1, also contain open strings as long as these end on D-branes. The spectrum of the theory is far more complex than the perturbative analysis would make us believe. Luckily, since the D-brane tension is proportional to $1/g_s$, there is a limit where string excitations are light whereas the D-branes are very massive and can consistently be regarded as fixed backgrounds. On the other hand, the D-branes are light in the large string coupling limit, where the string excitations are heavy.[18]

[18]This points to S-duality: a duality of the theory between small and large string coupling. We will not use S-duality in the following, and refer the interested reader to [GSWa, Polb, BBS].

Recall that, in the supergravity approximation, the branes are just backgrounds for the perturbation theory. These backgrounds break some of the symmetry of the original theory. For example, it is easy to see that a D-brane breaks ten-dimensional Lorentz invariance. It can also be shown that a D-brane preserves at most half of the original supersymmetry of the theory. D-branes preserving half of the supersymmetry are so-called BPS objects in the supergravity theory, whose mass is given by their RR charge $M_p = Q_p$. These objects are the lightest possible of the given charge in the supergravity spectrum, and hence are stable. Since type IIB has even form RR potentials, there can be stable odd-dimensional D-branes present in the theory. Similarly, type IIA contain stable even-dimensional D-branes.

For completeness, it should be mentioned that D-branes are not the only charged, extended objects in string theory. The antisymmetric B-field can also be viewed as a gauge potential, but in the NS sector of the theory. The associated electrically charged object is just the string itself, whereas the magnetically charged object is the NS5-brane. Another charged extended object is the orientifold plane, which will be introduced in chapter 5.

This concludes our discussion of D-branes and other basic ingredients of the low-energy dynamics of the type II string theories. In the rest of the thesis, we will use these results to construct effectively four-dimensional solutions of the theories. We will see how background configurations of branes and fluxes yield interesting cosmological solutions with black holes and non-zero cosmological constants. To see how this can be possible, we now turn to compactifications of string theory.

Part II:

Compactifications and Black Holes

3. Calabi–Yau Compactifications

The superstring theories, in particular the type II string theories, live in ten spacetime dimensions. We can use these theories to describe four-dimensional physics by compactifying six spatial dimension. The resulting four-dimensional physics will depend on what the compactification manifold looks like, e.g. how big it is and what topology it has. In this chapter, we discuss type IIB compactifications on Calabi–Yau manifolds. Such compactifications yield four-dimensional theories with $\mathcal{N} = 2$ supersymmetry.

We begin this chapter by reviewing the particle spectrum of a compactified string theory, in particular the massive momentum and wrapping modes. We then discuss the restrictions that supersymmetry puts on the internal manifold of a compactification, and how four-dimensional $\mathcal{N} = 2$ supersymmetry restricts the manifold to be Calabi–Yau. After deriving the low-energy spectrum of the compactified theory, we focus on a subset of these fields, namely the moduli that describe the size and shape of the Calabi–Yau. The metric for the moduli space will be important in the following chapters, so we discuss it in some detail. We end the chapter by a brief discussion on mirror symmetry and how Calabi–Yau manifolds can be constructed.

3.1 Compactifications of String Theories

As mentioned in the introduction, the idea with compactified theories is that some dimensions in spacetime are small and compact, and therefore invisible in practice. A long and thin cylinder, such as the one depicted in Fig. 3.1, has a compact dimension of radius R. For small R this space will look effectively one-dimensional for a field of low energy, as we now explain.

Let's say that there is a field living in a $d+1$ dimensional spacetime, with one compact dimension x^d:

$$x^d \sim x^d + 2\pi R, \tag{3.1}$$

where R is the compactification scale. Suppose for simplicity that the field is scalar. It then satisfies the equation of motion

$$\left(\Delta_{(d+1)} - m^2\right) \Phi = 0, \tag{3.2}$$

Figure 3.1: A field propagating on a cylinder will not notice the curled up dimension x^2 at energies smaller than the compactification scale $1/R$.

where $\Delta_{(d+1)}$ is Laplace–Beltrami operator in $d+1$ dimensions. Now expand this field in variable separated solutions

$$\Phi(x^0, ..., x^d) = \sum_{n \in \mathbb{Z}} \phi_n(x^0, ..., x^{d-1}) \psi_n(x^d). \tag{3.3}$$

It is then straightforward to show that $\psi_n = \exp(inx^d/R)$, which implies that the equations of motion for ϕ_n become

$$\sum_{n \in \mathbb{Z}} \left(\Delta_{(d)} \phi_n - \left(m^2 + \frac{n^2}{R^2} \right) \phi_n \right) e^{inx^d/R} = 0. \tag{3.4}$$

Thus, for each n we get a new d-dimensional field ϕ_n, of higher and higher mass. These fields are known as Kaluza–Klein (KK) modes, or momentum modes. Similar KK expansions can be obtained for spinors and forms, provided that the internal modes ψ_n are changed accordingly.

If the size of the compact dimension is small, then the mass $\frac{n^2}{R^2}$ is very large. At low energies, we can therefore neglect the massive KK modes, and describe the theory by an effective action that only includes the zero modes ϕ_0. We will see several examples of such lower dimensional effective actions in this thesis. Naturally, such actions are only good descriptions at energies smaller than $1/R$ or distances larger than R.

In string theory, compact dimensions can also be wound by strings. Consequently, a string propagating in a compact target space has both winding and momentum modes in the compact dimensions. The mass of a winding mode is given by

$$m_w^2 = w^2 \frac{R^2}{l_s^4}, \tag{3.5}$$

where w is the number of times that the string wraps the compact dimension, and l_s is the length of the string. For $R \ll l_s$, the string can easily wind the compact dimension many times, and the winding modes are light. For large radii, it takes a lot of energy to stretch the string enough, and winding modes are heavy.

In conclusion we see that at large compactification radii, the string theory is adequately described by the momentum mode expansion, whereas at small radii, the winding mode expansion is relevant. In fact, letting $R \to l_s^2/R$ exchanges the winding and momentum modes. Thus a string theory compactified on a large circle describes exactly the same physics as a string theory compactified on a small circle. The two different d-dimensional theories are said to be T-dual. Type IIA and type IIB string theories are examples of T-dual theories.

For practical purposes of string theory compactifications, one should note that the string length l_s is extremely small. A consistent theory of only massless d-dimensional fields can thus be obtained by taking the compactification scale R to be small on macroscopic scales, but still much larger than the string scale. Such a theory will be described by an effective action, which is correct up to $\mathcal{O}(R/l_s)$ corrections.

We now turn to compactifications of type IIB string theory to four dimensions. We restrict the string theory target space to be of the form $X \times M$, where X is the four-dimensional spacetime, and M is a six-dimensional, compact manifold. The spacetime coordinates are denoted x^μ, and six-dimensional compact space has coordinates y^i. Consequently, the (string frame) metric has the block-diagonal form

$$ds_{10}^2 = G_{MN} dz^M dz^N = G_{\mu\nu} dx^\mu dx^\nu + G_{mn} dy^m dy^n. \qquad (3.6)$$

To obtain a four-dimensional effective action for the low-energy physics we insert the block-diagonal metric (3.6) in the type IIB supergravity action (2.35) on page 33. In this chapter we put the expectation value of all fluxes to zero (we will return to the case with flux in the next chapter), and study fluctuations around this background. The effective action is then obtained by expanding the various terms of the action (2.35) in Kaluza–Klein modes, neglecting massive modes and integrating over the internal manifold. We return to this in section 3.3.

Before we can perform this dimensional reduction, more information about the compactification manifold is needed. From now on, we will focus on internal manifolds that lead to supersymmetric four-dimensional theories. There are two motivations for this choice. Supersymmetry facilitates computations; it can protect quantities, meaning that a computation performed at weak coupling can be used to draw conclusions in a strongly coupled regime. Moreover, from a phenomenological point of view, four-dimensional $\mathcal{N} = 1$ supersymmetry is an elegant way to unify the couplings of the fundamental forces, provide dark matter candidates and solve the hierarchy problem among the fundamental forces. Since the ultimate goal of string theory compactifications is to find a description of our world, supersymmetric compactifications seems to be a good corner to look in.

We will start by constructing compactifications that preserve four-dimensional $\mathcal{N} = 2$ supersymmetry. These compactifications are of interest for the black hole compactifications that we will discuss in chapter 4, and are also interesting as a basis for the discussion of $\mathcal{N} = 1$ solutions, as we will see in chapter 5. Our discussion of the supersymmetry of compactified solutions follows [CHSW85].

The purely bosonic solutions of type IIB string theory under study here have ten-dimensional $\mathcal{N} = 2$ supersymmetry if the dilatino and gravitino variations vanish. The relevant conditions were given in equation (2.37), and are reproduced here without background fluxes

$$0 = \delta\psi_M = \nabla_M \epsilon \, , 0 = \delta\lambda = (\partial\!\!\!/ \Phi) \, \epsilon. \tag{3.7}$$

In order to have four-dimensional $\mathcal{N} = 2$ supersymmetry, we require that the variations vanish for a ten-dimensional spinor that can be decomposed into four- and six-dimensional spinors. We therefore restrict our attention to six-dimensional manifolds where a non-vanishing spinor can be defined. If this is possible, we can decompose the ten-dimensional spinors as

$$\epsilon^A = \xi_+^A \otimes \eta_+ + \xi_-^A \otimes \eta_-, \tag{3.8}$$

where $A = 1, 2$ and $\eta_+ = (\eta_-)^*$ is a Weyl spinor on the internal manifold. Similarly, the four-dimensional supersymmetry parameters are denoted by $\xi_-^A = (\xi_+^A)^*$ and are Weyl spinors in spacetime.

With this decomposition, the internal part of the gravitino variation vanishes if the internal spinor is covariantly constant, $\nabla_m \eta_\pm = 0$ [CHSW85]. This is a strong condition on the allowed internal manifold; it should not only harbor a non-vanishing spinor, but this spinor must be covariantly constant. If this condition is met, there are two four-dimensional supersymmetry parameters, ξ^1, ξ^2, and the effective theory has $\mathcal{N} = 2$ supersymmetry.[1] In the next section we describe the manifolds that fulfill this condition, namely Calabi–Yau manifolds.

3.2 Calabi–Yau Manifolds

In this section we will show that Calabi–Yau manifolds have a covariantly constant spinor, and can be used for $\mathcal{N} = 2$ compactifications of type IIB string theory. Calabi–Yau manifolds can be defined in several equivalent ways, and we discuss their most important properties here. Our focus will be on six-

[1] We obtain $\mathcal{N} = 2$ supersymmetric solutions to this effective theory if the dilaton is also covariantly constant, and the four-dimensional spacetime is flat (see [Gra06] for a detailed discussion).

dimensional manifolds, that can be used to construct four-dimensional theories from string theory. For more thorough descriptions and derivations we refer to the vast literature on the subject, e.g. [H$^+$, Gre96, Von05].

In general, it is very hard to explicitly construct the metrics of the compact six-dimensional manifolds needed for string theory compactifications. One useful feature of Calabi–Yau compactifications is that this is not necessary. Instead, if certain requirements are fulfilled, the manifold is Calabi–Yau and the metric is Kähler and Ricci flat.[2] We will now outline how this comes about. In the following, it is assumed that the reader has a basic knowledge of differential geometry, or if not, consults e.g. [Nak] when faced with too many new concepts.

Calabi–Yau manifolds are complex, i.e. they are even-dimensional real manifolds, where the real coordinates can be paired in a consistent way to form complex coordinates. More formally, the manifold has a complex structure, which is a map on the tangent space of the manifold $J : T_pM \to T_pM$ that squares to one and fulfills the Nijenhuis condition

$$J_k^l J_l^k = -1 \text{ and } -J_k^l \partial_m J_n^k + J_k^l \partial_n J_m^k - J_n^k \partial_k J_m^l + J_m^k \partial_k J_n^l = 0. \quad (3.9)$$

For such manifolds, one can define local complex coordinates $w^j, \bar{w}^j$, so that the transition functions between coordinate patches are holomorphic. Hence holomorphic quantities can be globally defined on the manifold, and all forms can be classified as (p, q) forms according to how many holomorphic (p) and antiholomorphic (q) indices they have.

The metric on a complex manifold has pure and mixed components

$$G = G_{ij} dw^i \otimes dw^j + G_{i\bar{j}} dw^i \otimes d\bar{w}^j + G_{\bar{i}j} d\bar{w}^i \otimes dw^j + G_{\bar{i}\bar{j}} d\bar{w}^i \otimes d\bar{w}^j \quad (3.10)$$

with respect to the holomorphic and antiholomorphic basis vectors $\partial_{w^i}, \partial_{\bar{w}^i}$ of the complexified tangent space. We are interested in metrics that are Kähler. Locally, this means that $G_{ij} = G_{\bar{i}\bar{j}} = 0$, $G_{i\bar{j}} = (G_{\bar{i}j})^*$ and moreover

$$G_{i\bar{j}} = \partial_{w^i} \partial_{\bar{w}^j} K(w, \bar{w}). \quad (3.11)$$

Here the Kähler potential K is a function of both holomorphic and antiholomorphic coordinates. One can define a Kähler form, which is a real (1,1)-form

$$J = 2i\partial\bar{\partial}K = i g_{i\bar{j}} dw^i \wedge d\bar{w}^j. \quad (3.12)$$

[2] For Ricci flat manifolds $R_{ij} = 0$. Consequently, the six-dimensional part of Einstein's vacuum equations are automatically satisfied, which makes the manifolds interesting for compactifications.

If this form can be defined globally and is closed $dJ = 0$, the manifold is Kähler.[3]

We would now like to know when the Kähler manifold admits a Ricci flat metric, and hence is Calabi–Yau. This can be reformulated as a topological condition on the manifold. A Ricci flat Kähler manifold always has a vanishing first Chern class $c_1 = 0$. The k^{th} Chern class of a manifold is a topological quantity defined from the expansion

$$c(M) = 1 + \sum c_j(M) = \det(1 + \mathcal{R}) = 1 + \mathrm{tr}\mathcal{R} + ... \tag{3.13}$$

where $\mathcal{R} = R^k_{li\bar{j}} dw^i \wedge d\bar{w}^j$ is the matrix-valued curvature 2-form. The trace of this form is $c_1 = \mathrm{tr}\mathcal{R} = R_{i\bar{j}} dw^i \wedge d\bar{w}^j$. It is easy to see that c_1 is zero if $R_{i\bar{j}} = 0$. Conversely, if the manifold is Kähler and has $c_1 = 0$, there exist a unique Ricci-flat metric for a given Kähler class. That the metric is unique was proven by Calabi [Cal57], and that it always exists was proven by Yau [Yau78].

How can we be sure that Calabi–Yau manifolds have a covariantly constant spinor? To investigate this, we need to study their holonomy groups. Provided that the manifold is Kähler, the only non-zero components for the Christoffel symbols have purely holomorphic or antiholomorphic indices. Hence, under parallel transport around a closed loop, there is no mixing between holomorphic and anti-holomorphic indices. This implies that the holonomy group of the manifold, which would normally be SO(6) for an six-dimensional manifold, is reduced to U(3). If the manifold is also Ricci flat, the U(1) part of the Levi-Civita connection vanishes as well, and the holonomy is further reduced to SU(3).

Reduction of the holonomy of the manifold is important for the existence of covariantly constant spinors. In particular, if the holonomy is reduced to SU(3), the spinor representation of SO(6) is decomposed as **4** $\rightarrow$ **3 + 1**. The SU(3) singlet is a spinor that depends trivially on the tangent bundle of the manifold and is therefore well-defined and non-vanishing. In fact, it can be shown that the spinor is even covariantly constant [CHSW85]. Consequently manifolds of SU(3) holonomy can be used for the supersymmetric compactifications discussed in the end of section 3.1.

In conclusion, we have mentioned three equivalent ways of characterizing Calabi–Yau manifolds. They are Kähler and Ricci flat manifolds, or equivalently manifolds with SU(3) holonomy, or equivalently manifolds with a covariantly constant spinor. These different properties will be used in our construction of four-dimensional effective theories below. In contrast, we will never need the explicit form of the internal metric in our computations.

[3]The Kähler form is obtained by lowering an index of the complex structure, so we use J to denote both quantities.

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{(1,1)} & & 0 & \\
1 & & h^{(2,1)} & & h^{(2,1)} & & 1 \\
 & 0 & & h^{(1,1)} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
$$

Table 3.1: *The Hodge diamond of a Calabi–Yau three-fold.*

Before turning to the moduli space of a Calabi–Yau threefold, we recall some standard results about its cohomology (longer discussions can be found in [Nak, Polb]). The Hodge numbers $h^{(p,q)}$ give the dimension of the Dolbeault cohomology groups $H^{(p,q)}$ of a manifold. On a Calabi–Yau manifold, they are restricted by Kählerity, Hodge duality, complex conjugation and SU(3) holonomy. The result is that $h^{(p,0)} = h^{(0,p)} = h^{(p,3)} = h^{(3,p)} = 0$ for $p = 1, 2$ and $h^{(3,0)} = h^{(0,3)} = h^{(3,3)} = h^{(0,0)} = 1$. It follows that, there are only two unspecified Hodge numbers on a Calabi–Yau manifold. This is usually presented as the Hodge diamond in table 3.1.

Furthermore, a Calabi–Yau manifold always has a nowhere-vanishing global harmonic (3,0)-form Ω, which is defined up to rescalings by a constant. Ω can be written in local coordinates as

$$\Omega = \Omega_{lkj}(w) dw^l \wedge dw^k \wedge dw^j . \tag{3.14}$$

Specifying this form means determining the subspace of $H^3(M)$ that is $H^{(3,0)}(M)$, which is equivalent to choosing the complex structure of the manifold.

3.3 Four-dimensional $\mathcal{N} = 2$ Supergravity

We now have all the information needed to dimensionally reduce type IIB string theory on Calabi–Yau threefolds. To derive the field content of the resulting four-dimensional theory we need the KK expansion of the fluctuations around the expectation values of the type IIB fields, i.e. around the metric G_{MN}, dilaton Φ, NS potential B_{MN} and RR potentials $C_{(0)}, C_{(2)}, C_{(4)}$. The RR expectation values are put to zero. We will denote the fluctuations with lower case letters. The dynamic fields are then g_{MN}, ϕ, the RR fluctuations c, c_{MN}, c_{MNPQ} and the NS fluctuation b_{MN}.[4]

[4]The dynamics of the fermionic fields follow from supersymmetry.

The KK expansion for a form field is similar to the scalar field expansion given in equation (3.3). The Laplacian for forms Δ_d is a generalization of the Laplace–Beltrami operator[5], and we use it in the same way. Thus, a ten-dimensional form field is KK expanded in eigenfunctions to the six-dimensional Δ_d. In particular, massless four-dimensional fields are obtained by expansions in the eigenfunctions with zero eigenvalue, which are called harmonic forms. There is one harmonic form in every cohomology class of the manifold. Thus, there are harmonic forms of (0,0), (1,1), (1,2), (0,3) type, and their duals, on a Calabi–Yau threefold (see section 3.2).

Expanding the ten-dimensional fields in the harmonic (0,0)-form (there is only one, since $h^{(0,0)} = 1$) yields the four-dimensional metric $g_{\mu\nu}$, the axio-dilaton $\tau = i\phi + c$ and two axions $a \cong b_{\mu\nu}$ and $a' \cong c_{\mu\nu}$.[6] Similarly, a complex spacetime vector are obtained from $c_{\mu klm} = c_\mu \Omega_{klm}$ and its complex conjugate.

The harmonic (1,1)-forms give rise to $h^{1,1}$ scalars v^A from $g_{i\bar{j}}$, e.g. for a basis ω_A

$$ig_{i\bar{j}} = v^A(x)\omega_{Ai\bar{j}}. \tag{3.15}$$

Another set of scalars $b^A, c^A, \tilde{c}^A$ is obtained by expanding $b_{i\bar{j}}, c_{i\bar{j}}, c_{\mu\nu i\bar{j}}$ in ω_A. The (2,1)-forms give rise to $h^{(2,1)}$ complex scalars z^I from the pure index fluctuations g_{ij} of the internal metric, expanded in a basis χ_I as

$$g_{ij} = \frac{i}{||\Omega||^2}\bar{z}^I(x)\bar{\chi}_{Ii\bar{k}\bar{l}}\Omega^{\bar{k}\bar{l}}_j, \tag{3.16}$$

where $||\Omega||^2 = \frac{1}{3!}\Omega_{klm}\bar{\Omega}^{klm}$. Furthermore there are $h^{(2,1)}$ complex vectors from $c_{\mu ij\bar{k}} = \tilde{c}^I_\mu \chi_{I,ij\bar{k}}$ and its complex conjugate.

As we argued above the existence of a covariant constant spinor on the Calabi–Yau implies that the four-dimensional theory can have $\mathcal{N} = 2$ spacetime supersymmetry. It is therefore gratifying that the bosonic fields obtained here combine into $\mathcal{N} = 2$ supermultiplets as follows. The four-dimensional metric and the vector c_μ give the bosonic fields (graviton and graviphoton) of the supergravity multiplet. The four scalars ϕ, c, a and a' give the universal hypermultiplet. For each harmonic (1,1)-form, we get four scalars, again combining to a hypermultiplet. Additionally, for each harmonic (2,1)-form there is a complex scalar and a vector; the bosonic content of a $\mathcal{N} = 2$ vector multiplet. Thus, at low energy, the compactified theory is four-dimensional $\mathcal{N} = 2$ supergravity, with $h^{(1,1)}$ extra hypermultiplets and $h^{(2,1)}$ extra vector multiplets. The bosonic field content is summarized in table 3.2.

We have now derived the (bosonic part) of the low-energy spectrum of type IIB string theory compactified on a Calabi–Yau threefold. The next thing to

[5] $\Delta_d = dd^\dagger + d^\dagger d$, see [Nak].

[6] Using Hodge duality, it is straightforward to show that in four dimensions, the field strength $db_{(2)}$ can equally well be obtained from a scalar a.

Multiplet	Multiplicity	Bosonic fields
Supergravity	1	$g_{\mu\nu}, c_\mu$
Universal hypermultiplet	1	ϕ, c, a, a'
Hypermultiplet	$h^{1,1}$	$v^A, b^A, c^A, \tilde{c}^A$
Vector multiplet	$h^{2,1}$	$z^I, \tilde{c}^I_\mu$

Table 3.2: *Bosonic field content of four-dimensional $\mathcal{N} = 2$ supergravity obtained from Calabi–Yau compactification of type IIB string theory. The extra hypermultiplets contain the Kähler moduli of the Calabi–Yau. The vector multiplets contain the complex structure moduli.*

investigate is the dynamics of these fields, and what the effective action looks like. We will be particularly interested in the fields v^A and z^I, which determine the fluctuations of the internal manifold. There is no potential for these fields in the low-energy action, implying that they are moduli. Thus any fluctuation of these fields give a new solution the compactified theory. The moduli will be of important for the compactifications of chapters 4-7, so we pause to for a more detailed study of the metrics for these fields.

3.3.1 Moduli Spaces for Calabi–Yau Threefolds

Recall that the moduli v^A, z^I correspond to deformations $g_{i\bar{j}}, g_{ij}$ of the Calabi–Yau metric G_{mn}. As such they parametrize the size and shape of the three-fold. More concretely, they correspond to the ways one can deform the metric so that it is still Ricci flat:

$$R_{mn}(G_{mn} + g_{mn}) = 0. \tag{3.17}$$

It was shown in [CdlO91] that such variations must be eigenmodes of the Lichnerowicz operator,

$$\tilde{\Delta}_L^2 g_{mn} = \nabla_k \nabla^k g_{mn} + 2R_m{}^p{}_n{}^q g_{pq} = \lambda g_{mn}. \tag{3.18}$$

The zero modes with pure (g_{ij}) and mixed $(g_{i\bar{j}})$ indices solve this equation separately, so there are two types of massless metric fluctuations that span the moduli space of the Calabi–Yau manifold.

It is easy to see that fluctuations with mixed indices correspond to variations of the Kähler structure of the manifold. Indeed, the Kähler form can be expanded as

$$J = v^A \omega_A, \tag{3.19}$$

where v^A are the zero-modes of the mixed index fluctuations. For this reason the $h^{(1,1)}$ fields v^A are known as Kähler moduli. Similarly, the fluctuations with pure indices necessarily change the complex structure, since they cannot be compensated by a holomorphic coordinate transformation $w^i \rightarrow w^i + f^i(w)$. Therefore, the $h^{(2,1)}(M)$ fields z^I are called complex structure moduli.

Since the mixed and pure index deformations solve equation (3.18) separately, the Calabi–Yau moduli space decomposes (at least locally) into the Kähler and complex structure moduli spaces

$$\mathcal{M} = \mathcal{M}_K \times \mathcal{M}_{CS}. \tag{3.20}$$

This decomposition holds also for finite deformations, since in $\mathcal{N} = 2$ supergravity the metric for the scalars in hypermultiplets cannot depend on the vector multiplets and vice versa (see e.g. [Polb]). We can therefore study the two moduli spaces independently.

3.3.1.1 The complex structure moduli space

As discussed in the previous section, the moduli become four-dimensional fields in the compactified theory, and the dimensionally reduced action contains their kinetic terms. These terms are governed by the metrics of the respective moduli space. The geometry of the complex moduli space will play a prominent role in our discussion of black holes and flux vacua in chapters 4-6. In this section we will introduce several important geometrical features of this space, namely the Kähler potential, the periods, the prepotential and the monodromies.

It was shown by [CdlO91] that the metric on complex structure moduli space is given by

$$\mathcal{G}_{IJ} = \frac{1}{2\mathcal{V}} \int G^{k\bar{j}} G^{l\bar{i}} g_{kl} g_{\bar{j}\bar{i}} \sqrt{G_6} d^6 y \tag{3.21}$$

which can be rewritten in terms of three-forms as

$$\mathcal{G}_{I\bar{J}} = -\frac{\int \chi_I \wedge \bar{\chi}_{\bar{J}}}{\int \Omega \wedge \bar{\Omega}}. \tag{3.22}$$

Furthermore, one can show that

$$\frac{\partial \Omega}{\partial z^I} = k_I \Omega + \chi_I, \tag{3.23}$$

so the metric on the complex structure moduli space is in fact $G_{IJ} = \partial_I \partial_J K_{CS}$, where the Kähler potential is

$$K_{CS} = -\ln \left(i \int \Omega \wedge \bar{\Omega} \right). \tag{3.24}$$

Thus the complex structure moduli space of a Calabi–Yau manifold is a Kähler manifold.

Periods and Prepotential

It is useful to rewrite this metric in terms of so-called period integrals of Ω over three-cycles over the manifold. We choose a basis of the homology group $H_3(M)$, which are $N = 2h^{(2,1)} + 2$ three-cycles A^I, B_J that intersect each other pair-wise with intersection number one, as specified by an intersection matrix $Q_{IJ} = A^I \cap B_J$.[7]

The periods of Ω are

$$\Pi_I = \int_{A^I} \Omega = X^I \quad , \quad \Pi_{N+1-I} = \int_{B_I} \Omega = F_I, \tag{3.25}$$

and can be thought of as the "holomorphic volume" of the three-cycles. In terms of the periods, we can rewrite the Kähler potential as

$$K_{CS} = -\ln\left(-i\Pi^\dagger Q^{-1}\Pi\right), \tag{3.26}$$

where we collect the periods in a N-dimensional period vector

$$\Pi(z,\bar{z}) = \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \vdots \\ \Pi_N \end{pmatrix}. \tag{3.27}$$

Furthermore, we can define the dual of the canonical basis on $H^3(M)$ by

$$\int_{A^I} \alpha_J = \int_{B_J} \beta^I = \delta^I_J \,, \int_{B_I} \alpha_J = \int_{A^J} \beta^I = 0, \tag{3.28}$$

and expand Ω in this basis: $\Omega = X^I \alpha_I + F_J \beta^J$.

As we argued at the end of section 3.2, specifying Ω is equivalent to specifying the complex structure. Thus the $N = 2h^{(2,1)} + 2$ expansion modes X^I, F_J should be related to the coordinates z^I on the complex structure moduli space that were defined in that section. However, the dimension of this space is only $h^{(2,1)}$, so not all expansion modes are good coordinates. We can use equation (3.23) to write the F_J as

$$0 = \int \Omega \wedge \frac{\partial \Omega}{\partial X^I} \Rightarrow F_J = \frac{1}{2}\partial_J(X^I F_I) = \partial_J(F). \tag{3.29}$$

The function F is known as the prepotential, and is holomorphic and homogeneous of degree two in X^I. The prepotential determines the Kähler potential, $K_{CS} = -\ln i\left(\bar{X}^I F_I - X^I \bar{F}_I\right)$, and hence the metric of the complex struc-

[7]To avoid a cluttering of indices, we reuse the letters I, J although here they go over the extended interval $0 \ldots h^{(2,1)}$.

ture moduli space. Kähler manifolds that have a holomorphic prepotential are known as special Kähler manifolds.

The X^I are in fact good projective coordinates on the complex structure moduli space, since a rescaling of X^I leads to a rescaling of F_I and hence of Ω, which does not affect the complex structure of the Calabi–Yau. In a particular patch, where, say, $X^0 \neq 0$ we have $h^{(2,1)}$ inhomogeneous coordinates $z^I = (1, X^I/X^0)$, matching the number of complex structure moduli introduced in section 3.2.

Monodromies

Let us now return to the periods Π_I, that give the holomorphic volume of the three-cycles in the Calabi–Yau. At certain loci in the moduli space where one modulus is zero, say $z^1 = 0$, the corresponding cycle has shrunken to zero volume. When this happens the smooth Calabi–Yau threefold turns into a singular manifold, as we will discuss further in section 3.4.

In fact, even just encircling such a locus, say $z^1 = 0$, in the moduli space affects the three-cycles of the Calabi–Yau. Around this path, A^1 is well-defined. The dual cycle B_1 is on the other hand only defined as being the cycle that intersects A^1 once. Therefore, under the transport around $z^1 = 0$, this cycle might very well transform as $B_1 \to B_1 + nA^1$. This is known as a monodromy. Translated to the periods, this induces a monodromy $\Pi_N \to \Pi_N + n\Pi_1$.

More generally the monodromy transforms the period vector as

$$\Pi \to \mathbb{M} \cdot \Pi, \tag{3.30}$$

where the monodromy matrix $\mathbb{M}$ is an integral, symplectic matrix that preserves the intersection matrix Q. Monodromies are not always related to shrinking cycles, but arise because the complex structure moduli space of a Calabi–Yau manifold is usually a quotient of a larger space, known as the Teichmüller space. We will have more to say about this when we discuss the topography of the type IIB landscape in chapter 6, in the line of the discussions of Papers III-V.

The monodromies are very useful in order to derive explicit, local expressions for the metric of the moduli space. The monodromy $\Pi_N \to \Pi_N + n\Pi_1$ around $z^1 = 0$, means that

$$\Pi_N = 2\pi i z^1 \ln|z^1| + \text{regular terms}. \tag{3.31}$$

Since all other periods must be regular near $z^1 = 0$, it is straightforward to show that

$$K(z, \bar{z}) = 2\pi i |z^1|^2 \ln|z^1| + \text{regular terms}. \tag{3.32}$$

It follows the metric diverges as $\ln|z^1|$ at the locus $z^1 = 0$, and that the curvature also diverges. This is one example of the possible singularities in the complex structure moduli space, which we will return to when we discuss black holes and geometric transitions. It is worth mentioning that the curvature divergence is rather mild, and that the distance to the singularity is finite [CGH89].

So far, we have discussed the geometry of the complex structure moduli space from a purely classical perspective. Could it be that some of the features we have derived are altered by quantum mechanical corrections, i.e. the α' and loop corrections of string theory? It turns out that this is excluded by supersymmetry. The expansion parameter of loop corrections is the string coupling $g_s =< \phi >$, the expectation value of the dilaton. This field sits in a hypermultiplet and, lest supersymmetry is broken, it cannot correct the metric for the complex structure moduli, which sit in vector multiplets. Similarly, the expansion parameter for α' corrections is α'/R_c^2, where R_c is the radius of the Calabi–Yau, i.e. a length scale set by the overall volume. Volume rescalings correspond to a Kähler modulus, i.e. a field in a hypermultiplet. Again, supersymmetry forbids such corrections to the metric of the complex structure moduli space. Thus the classical metric for the complex structure moduli space of type IIB Calabi–Yau compactifications is exact also on a quantum level.

3.3.1.2 The Kähler moduli space

The geometry of the Kähler moduli space will not be as important for the rest of the thesis as the complex structure moduli space, but there are some properties we need to derive. In particular, we will need the Kähler potential for this space, which is important for flux compactifications.[8]

Recall that the complex structure metric can be expressed in terms of the holomorphic three-form of the Calabi–Yau. Analogously, we write the Kähler moduli space metric in terms of differential forms, in particular the complexified Kähler form $B+iJ$. A natural metric on the space of deformations of the complexified form is [CdlO91]

$$ds_K^2 = \frac{1}{2\mathcal{V}} \int G^{k\bar{j}} G^{i\bar{l}} \left(g_{k\bar{l}} g_{j\bar{i}} + b_{k\bar{l}} b_{j\bar{i}} \right) \sqrt{G_6} d^6 y = \frac{1}{2\mathcal{V}} \int \tau \wedge *\sigma, \quad (3.33)$$

where the deformations have been rewritten in terms of the real (1,1)-forms τ and σ. The volume of the Calabi–Yau can be expressed as a cubic integral of J

$$\mathcal{V} = \frac{1}{3!} \kappa(J,J,J) \text{ , where } \kappa(\xi,\sigma,\tau) = \int \xi \wedge \sigma \wedge \tau. \quad (3.34)$$

[8] The Kähler moduli are part of a $\mathcal{N}=2$ hypermultiplet, which also includes three other scalar fields. The geometry for the space of scalar fields of such a hypermultiplet is quaternionic Kähler [BW83, HKLR87]. Since the focus of this thesis is really $\mathcal{N}=1$ truncations of this theory (to be described in chapter 5), we will not describe the quaternionic Kähler space, but rather the geometry of the space spanned by the complexified Kähler moduli $b^A + iv^A$ per se.

Furthermore, one can show that [Str85]

$$*\sigma = -J \wedge \sigma + \frac{\kappa(\sigma, J, J)}{\kappa(J, J, J)} J \wedge J, \tag{3.35}$$

which leads to the metric

$$\mathcal{G}(\sigma, \tau) = -3 \left(\frac{\kappa(\sigma, \tau, J)}{\kappa(J, J, J)} - \frac{3}{2} \frac{\kappa(\sigma, J, J)\kappa(\tau, J, J)}{\kappa^2(J, J, J)} \right). \tag{3.36}$$

Expanding $B + iJ = (b^A + iv^A)\omega_A = \tilde{v}^A \omega_A$, where $A = 1...h^{1,1}$, one can show that

$$\mathcal{G}_{A\bar{B}} = -\frac{\partial}{\partial \tilde{v}^A} \frac{\partial}{\partial \tilde{v}^{\bar{B}}} 2 \ln \kappa(J, J, J), \tag{3.37}$$

where $\tilde{v}^{\bar{B}} = (\tilde{v}^B)^*$. Thus there is a Kähler potential $K_K = -2 \ln \kappa(J, J, J)$ for the metric on the complexified Kähler moduli space. By a Kähler transformation, K can equally well be written as [CdlO91]

$$K_K(\tilde{v}, \bar{\tilde{v}}) = -2 \ln i \left(\tilde{v}^{\bar{A}} G_A - \tilde{v}^A \bar{G}_{\bar{A}} \right) \tag{3.38}$$

where

$$G = \frac{1}{3!} \frac{\kappa(e_A, e_B, e_C)\tilde{v}^A \tilde{v}^B \tilde{v}^C}{\tilde{v}^0} \tag{3.39}$$

is a holomorphic prepotential, and as usual $G_A = \partial_A G$. An extra coordinate $\tilde{v}^0$ has been introduced in order for G to be homogeneous of order two.

There are some restrictions on the Kähler moduli space. Naturally, the volumes of all $2p$-cycles in the manifold (including the overall volume) must be positive:

$$\int_{\Sigma_{2p}} J^p = \int_{\Sigma_{2p}} J \wedge ... \wedge J > 0. \tag{3.40}$$

Furthermore the metric on the Calabi–Yau must be positive definite. One can show (see e.g. [Gre96, Von05]) that this restricts the allowed Kähler classes to form a cone. The positivity requirements fail to hold at the boundary of the cone, where cycles may shrink to zero volume, and the Calabi–Yau manifold becomes singular.

In contrast to the complex structure moduli space, supersymmetry does not protect the Kähler moduli space geometry from quantum corrections. Indeed, we saw above that both α' and loop corrections depend on hypermultiplet fields, which can mix with other hypermultiplets. It follows that the Kähler moduli space geometry of a type IIB compactification is corrected by both effects.

	IIA	IIB
Vector multiplet	$h^{(1,1)}$ (Kähler)	$h^{(2,1)}$ (Complex structure)
Hypermultiplet	$h^{(2,1)}$ (Complex structure)	$h^{(1,1)}$ (Kähler)

Table 3.3: *The moduli of a Calabi–Yau compactification group into mirrored $\mathcal{N} = 2$ multiplets in type IIA and IIB compactifications. In addition, both theories contain the supergravity multiplet and universal hypermultiplet.*

3.4 Mirror Symmetry

The discussion so far has hidden a crucial part of type IIB compactifications on Calabi–Yau manifolds, namely its relation to type IIA compactifications on such manifolds. This is indeed a very important manifestation of the connections between the various versions of string theory and deserves comment. To understand it, we first repeat the discussion of section 3.3 for type IIA compactifications.

As we saw in chapter 2.1.2, the type IIA and type IIB theories differ in the RR sector. Thus the bosonic fields that differ are the RR form potentials, which for type IIA are C_M and C_{MNP}. In a type IIA Calabi–Yau compactification, the fluctuations yield $h^{(1,1)}$ vectors (from $c_{\mu i\bar{j}}$) and $h^{(2,1)}$ complex scalars (from $c_{ki\bar{j}}$). These combine with the Kähler and complex structure moduli of the internal manifold, and form $h^{(1,1)}$ vector multiplets and $h^{(2,1)}$ hypermultiplets. There is also a spacetime vector c_μ, which fits into the supergravity multiplet, plus the universal hypermultiplet which is the same as in type IIB. The result is a four-dimensional $\mathcal{N} = 2$ supergravity with $h^{(2,1)}$ extra hypermultiplets and $h^{(1,1)}$ vector multiplets, in correspondence with the type IIB compactifications. This is shown in table 3.3.

Suppose now that there is a Calabi–Yau manifold M with Hodge numbers $h^{(1,1)} = K$ and $h^{(2,1)} = L$. One may wonder if there exists a manifold for which the Hodge diamond 3.1 is mirrored. This would be a new Calabi–Yau manifold W with $h^{(1,1)} = L$ and $h^{(2,1)} = K$. Provided that the two Calabi–Yau manifolds exist, we can compactify type IIA on M and type IIB on W and obtain four-dimensional $\mathcal{N} = 2$ supergravities with identical field content. This does not necessarily imply that the full four-dimensional theories are the same, but this can indeed happen. If this is true, then we say that the manifold W is the mirror of the manifold M, and that the two dual theories are related by mirror symmetry.

To motivate that this is the case, recall that the supergravity theories are low-energy approximations of non-linear sigma models. Calabi–Yau compactifications are thereby non-linear sigma models whose target space is restricted to be of the form $X \times M$, where M is a Calabi–Yau manifold. For such target manifolds, the supersymmetry of the non-linear sigma model is enhanced to $(N_L, N_R) = (2, 2)$ [Gre96]. A lot is known of these highly supersymmetric field theories. In particular, there is a mapping between two sets of operators of these theories and the (1,1) and (2,1) forms on the Calabi–Yau target space. This is suggestive if we want to understand mirror symmetry, which interchange the number of (1,1) and (2,1) forms on the manifold. Thus, we should look for isomorphisms between the field theories that interchanges the two sets of operators. It can be shown that such maps exist [GP90, CdlOGP91].

The discussion so far has been rather abstract, so we pause to discuss an example, following [CdlOGP91] (see also [H$^+$]). The Quintic and the Mirror Quintic are Calabi–Yau manifolds related by mirror symmetry. The Quintic is, as the name suggests, defined as the zero set of a quintic polynomial in $\mathbb{P}_4$. A general quintic polynomial has 126 coefficients, but invariance under linear coordinate transformations can be used to set 25 of these to zero. The remaining 101 coordinates are the complex structure moduli of the Quintic. Furthermore, the Quintic inherits the Kähler class of the surrounding space, and has a one-dimensional Kähler moduli space.[9]

The Mirror Quintic has mirrored Hodge numbers, i.e. $h^{(1,1)} = 101$ and $h^{(2,1)} = 1$. This manifold can be constructed from a one-parameter subfamily of quintic hypersurfaces

$$p = \sum_{k=1}^{5} x_k^5 - 5\psi \prod_{k=1}^{5} x_k = 0, \tag{3.41}$$

where x_k are homogeneous coordinates on $\mathbb{P}_4$ and ψ is the parameter. These hypersurfaces are invariant under a $\mathbb{Z}_5^3$ symmetry group. The Mirror Quintic is identified with the family of manifolds that is obtained by taking the quotient of each quintic hypersurface in the one-parameter submanifold by $\mathbb{Z}_5^3$. In addition, one further step is needed; the quotiented manifolds have singularities coming from the fixed points of the $\mathbb{Z}_5$ actions. These singularities must be resolved, and this introduces the 100 new Kähler moduli [CdlOGP91]. Furthermore, it was shown [CdlOGP91] that $z = \psi^5$ is the complex structure modulus of the Mirror Quintic. In this paper the periods and prepotential of the complex structure moduli space were also computed. The matching between this prepotential and the prepotential of the Kähler moduli space of the Quintic, which is required by mirror symmetry, is also checked in [CdlOGP91] (see also [GP90]). We will use the Mirror Quintic for numerical examples in Papers III-V.

Finally, we should mention how Calabi–Yau manifolds and their mirrors can be constructed more generically. Note that the Quintic is constructed as a hyperplane in the projective space $\mathbb{P}_4$. In this way, the Quintic Kähler class was inherited from the surrounding space, and the complex structure was parametrized by the constants in the defining equation. More generally, a Calabi–Yau can be constructed as a complete intersection of a set of hy-

[9]The Kähler modulus on $\mathbb{C}P_4$ just sets the radius of this space. In general, the Kähler forms for $\mathbb{C}P_4$ and the Quintic are not the same (since the Quintic should be Calabi–Yau and hence Ricci-flat), but they lay in the same Kähler class.

persurfaces in a toric variety, which can be thought of as a generalization of a projective space. Just as above, the hypersurface is a solution to a polynomial equation, whose coefficients become the complex structure moduli of the Calabi–Yau. This can be formalized in the framework of toric geometry, where the toric varieties and polynomial equations are described in terms of polytopes in integral lattices. In particular, these polytopes give a recipe for the construction of mirror manifolds, as described in [Bat94]. Good introductions to this subject can be found in [H$^+$, Gre96].

3.5 Summary

In this chapter, we have seen how type IIB string theory can be compactified on Calabi–Yau three-folds. Putting all background fluxes to zero, we showed that, at low energy, the compactified theory is four-dimensional $\mathcal{N} = 2$ supergravity, with $h^{(1,1)}$ extra hypermultiplets and $h^{(2,1)}$ extra vector multiplets.

The four-dimensional action for the type IIB low-energy theory can be obtained by dimensional reduction of the ten-dimensional theory. Using the fluctuations defined above it is

$$S_{eff}^{\mathcal{N}=2} = \frac{M_p^2}{2} \int d^4x \sqrt{-g_4} \big(R^{(4)} + \mathcal{G}_{I\bar{J}} \partial_\mu z^I \partial^\mu \bar{z}^J + \mathcal{G}_{A\bar{B}} \partial_\mu \tilde{v}^A \partial^\mu \tilde{v}^{\bar{B}} + ...\big) \tag{3.42}$$

where

$$\frac{M_p^2}{2} = \frac{(2\pi)^6 \alpha'^3 \mathcal{V}}{2\kappa_{10}^2}. \tag{3.43}$$

Here $\mathcal{V} = \frac{1}{(2\pi)^6 \alpha'^3} \int \sqrt{G_6} d^6y$ is the volume of the internal manifold, measured in units of the string length $l_s \sim \sqrt{\alpha'}$. The terms that are omitted in $S_{eff}^{\mathcal{N}=2}$ include RR fluctuations, axio-dilaton dependent terms and mixings between the geometric moduli and these fields. The full action can be seen in e.g. [A$^+$96, GL04]; it is rather complicated and will not be written out here.

Local $\mathcal{N} = 2$ supersymmetry protects the complex structure moduli metric $\mathcal{G}_{I\bar{J}}$ from quantum corrections in type IIB compactifications. On the other hand, the metric $\mathcal{G}_{A\bar{B}}$ on Kähler moduli space receives both α' and g_s corrections. This asymmetry is somewhat resolved by mirror symmetry, that equates the effective supergravities obtained from type IIB and IIA compactifications on mirror Calabi–Yau threefolds. Thus, it is enough to study the complex structure moduli spaces of two mirror manifolds to understand the moduli space and low-energy physics of a compactification. Note however, that the complex structure moduli metric of type IIA compactifications is affected by g_s corrections.

Finally, recall that the Calabi–Yau moduli are massless fields, which is problematic from a phenomenological point of view. Such fields would give rise to unobserved fifth forces in the four-dimensional theory. Furthermore, the compactifications discussed here are inherently unstable to changes of the size and shape of the internal manifold. In chapter 5, we show how these problems are resolved by allowing supersymmetry-breaking background flux configurations.

4. Black Holes in Calabi–Yau Compactifications

As our first example of interesting compactifications we will study four-dimensional black hole spacetimes that arise in Calabi–Yau string compactifications with D-branes. An intriguing feature of these models is that the various D-brane configurations give us a way of probing the microstates of black holes. This leads to a microscopic derivation of the black hole entropy.

The focus of this chapter is on the macroscopic properties of a set of supersymmetric black holes. We will see that their electromagnetic charges are determined by the number of times a brane wraps around certain cycles of the three-fold. The black holes are BPS solutions of four-dimensional $\mathcal{N} = 2$ supergravity, and we discuss how the attractor mechanism determines their mass and entropy in terms of the charges. We then turn to the connections between black hole supergravity, topological string theory and matrix models, and how some properties of the black hole can be computed using these relations. This connects to Paper I, where correspondences between the free energies of a black hole and a matrix model are studied.

4.1 Black Hole Thermodynamics

We start our discussion by recapitulating some generic properties that black holes have, irrespective of supersymmetry. In simple words, a classical black hole is a region in spacetime from which nothing can escape. More concretely, it is an asymptotically flat spacetime that contains a region that is not in the past light-cone of future time-like infinity. Black holes have a mass (M) and can in addition carry electromagnetic charges and angular momentum. These quantities completely characterize the black hole. In the following we will study black holes with zero angular momentum, and we will collect the electromagnetic charges in a vector Q. An introduction to black holes in string theory is given in [BBS], and a more detailed discussion can be found in e.g. [Moh01].

For spherically symmetric black holes with $M^2 \geq Q^2$, the horizon area is determined by the charges and the mass:[1]

$$A = 4\pi(M + \sqrt{M^2 - Q^2})^2. \tag{4.1}$$

Solutions with smaller mass lack a horizon, and thereby have naked singularities. Since these are forbidden by the cosmic censorship conjecture, it follows that the lightest black hole of a given charge has mass $M^2 = Q^2$. Such black holes are called extremal.

As matter falls into the black hole, the entropy of the black hole surroundings is reduced. Hence black holes must have an entropy, lest the second law of thermodynamics would be violated. The entropy is given by the Bekenstein–Hawking area law [Bek73, Bek74, Haw75]

$$S_{BH} = \frac{A}{4}. \tag{4.2}$$

This is a very large entropy, several orders of magnitude larger than that of an ordinary star of the same mass [Moh01].

Furthermore, a semiclassical analysis shows that black holes emit thermal radiation [Haw75]. This implies that black holes have a temperature: the Hawking temperature. Since black holes radiate, they will eventually evaporate. The exception to this rule are extremal black holes, which are stable and have zero temperature.

In statistical mechanics, the entropy of a state is determined by the number of microstates that yield the macroscopic properties of that state, i.e.

$$S_{micro} = \log\left[N(M,Q)\right]. \tag{4.3}$$

Yet, classically all the mass of the black hole is squeezed into a singularity and there is only one microstate. This is in obvious conflict with the large entropy in equation (4.2). One of the most important tasks of a theory of quantum gravity is to explain the black hole microstates.

String theory does provide an interpretation of the microstates of e.g. four-dimensional black holes. In string theory compactifications, four-dimensional spacetimes are effective solutions of an underlying higher-dimensional theory. What appears to be a featureless singularity in four dimensions is resolved to a ten-dimensional configuration, or even to several different configurations with the same four-dimensional properties. This could explain the internal structure of the effective black hole. We will now discuss the construction of such solutions, and various ways of calculating their entropy. In the following, we will focus on supersymmetric black holes.

[1] In this chapter, we use units where Newton's constant $G_N = 1$. All areas, lengths and masses are measured in Planck units.

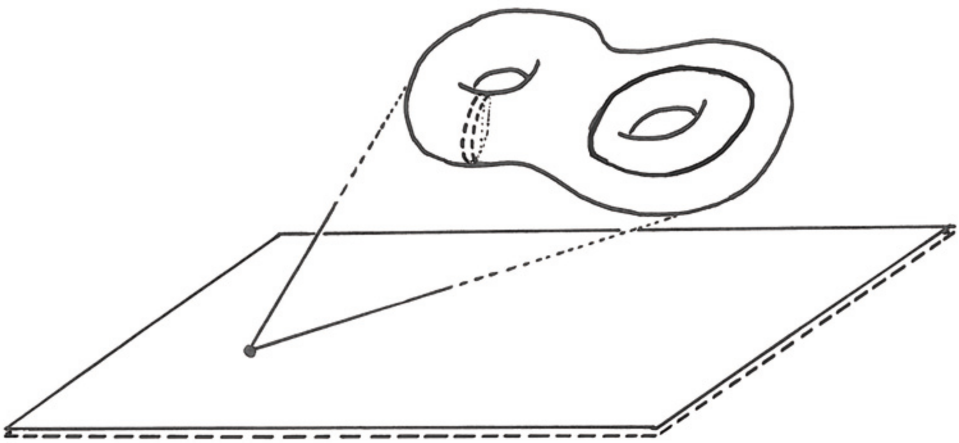

Figure 4.1: Two D-branes in a compactification with six compact dimensions (depicted as a genus two Riemann surface) and a three-dimensional non-compact space (depicted as a flat surface). A D3-brane (solid line) wraps an internal three-cycle and is point-like in space. A D5-brane (dashed line) wraps an internal two-cycle twice and is space-filling. Due to the limited dimensionality of this sheet of paper, both two- and three-cycles are depicted as one-cycles.

4.2 Compactifications with D-branes

Recall that in supergravity, D-branes are solitonic degrees of freedom, that yield new backgrounds for the perturbative theory. It is possible to construct such backgrounds also when some of the dimensions are compact. As usual, we focus on type IIB compactifications, which can have stable background solutions including odd-dimensional D-branes. In chapter 3 we studied compactifications to maximally symmetric four-dimensional spacetimes, but we could equally well look for solutions with less symmetries, e.g. the black hole spacetimes of interest here.

In a compactification, a brane can either be wrapped around some compact dimensions, or stretched out in spacetime. It can also have some directions in compact and some in non-compact dimensions. As a result, different four-dimensional configurations are possible; black holes, cosmic strings, domain walls or completely space-filling branes. An example of a compactified D-brane configuration is depicted in figure 4.2. The D3-brane in this example would appear as a black hole in four-dimensional spacetime.

As in the preceding chapter, the solutions of interest have a block-diagonal metric

$$ds^2_{10} = ds^2_{BH} + ds^2_{CY}, \tag{4.4}$$

but here the four-dimensional metric is a black hole metric. We choose internal manifolds that are Calabi–Yau three-folds, which have non-trivial two-and three-cycles (see section 3.2). As discussed in the last chapter, type IIB Calabi–Yau compactifications result in four-dimensional $\mathcal{N} = 2$ supergravity coupled to $h^{(1,1)} + 1$ hypermultiplets and $h^{(2,1)}$ vector multiplets. The black hole solutions are constructed by D3-branes wrapping three-cycles and do not depend on the hypermultiplets, which are therefore omitted from the following discussion.

In general, compactifications with branes do not preserve any supersymmetry, but by choosing the wrapped cycle carefully, some supersymmetry can be kept. In particular, if the cycle is a so-called special Lagrangian three-cycle, then one can show that the resulting configuration is BPS [BBS95] (for this reason, such cycles are also known as supersymmetric cycles). Thus, half the supersymmetry is preserved, i.e. $\mathcal{N} = 2$ is broken to $\mathcal{N} = 1$. Special Lagrangian cycles are volume minimizing within their homology class, and we will see below that this means that the mass of a BPS black hole is minimized.

It might seem strange to use a $\mathcal{N} = 2$ formulation for the effective four-dimensional theory when the branes preserve at most half of the supersymmetry. Note, however, that the physics far away from the brane, i.e. far away from the black hole, is described by the closed string sector and hence has $\mathcal{N} = 2$ supersymmetry. Furthermore, the full supersymmetry is also restored at the horizon of the black hole by the attractor mechanism [FKS95, Str96, FK96], as we will outline below. Thus, the BPS solution acts as a soliton interpolating between two maximally supersymmetric vacua.

Recall from section 2.4 that D-branes carry RR charge. In particular, D3-branes are sources of the self-dual five form field strength $\tilde{F}_{(5)}$. For a D3-brane wrapped around a cycle $\mathcal{C}$, this translates to being charged under the graviphoton and $h^{(2,1)}$ other gauge fields in the four-dimensional $\mathcal{N} = 2$ supergravity. The four-dimensional electric (q_I) and magnetic (p^I) charges of a wrapped D3-brane are[2]

$$\int_{S^2 \times A^I} \tilde{F}_{(5)} \sim p^I \quad , \quad \int_{S^2 \times B_I} \tilde{F}_{(5)} \sim q_I \quad , \quad p^I, q_I \in \mathbb{Z}. \tag{4.5}$$

Here S^2 denotes a two-sphere in the non-compact space that encloses the black hole and A^I, B_J is the symplectic basis of three-cycles introduced on page 51.[3] The charges can be encoded in the wrapped cycle $\mathcal{C} = p^J B_J - q_I A^I$, from which it can be seen that they have an interpretation as wrapping numbers of symplectic three-cycles. Alternatively, we can use the Poincaré dual three-form $\Gamma = (p^I \alpha_I + q_I \beta^I) \in H^3(M, \mathbb{Z})$ to encode the charges.

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[2]The constant of proportionality in these equations is $\big((2\pi)^2\alpha'\big)^2$. For notational simplicity, we neglect factors of π and α' in this chapter.

[3]Which three-cycle we choose to call electric and magnetic is a matter of taste. The important thing is that there will be one four-dimensional gauge field for each pair of cycles in the symplectic basis. Electromagnetic duality in four dimensions then follows from the invariance under symplectic transformations of the basis.

In $\mathcal{N} = 2$ supergravity, the central charge of Γ is defined as[4]

$$Z(\Gamma) = \int_M \Gamma \wedge \Omega = \int_{\mathcal{C}} \Omega. \tag{4.6}$$

It is convenient to rewrite this using the special geometry of the complex structure moduli space. In terms of the prepotential F and projective coordinates X^I of equation (3.25) on page 51 we have

$$Z(\Gamma) = \sum_I \left(\int_{A^I} \Gamma \int_{B_I} \Omega - \int_{B_I} \Gamma \int_{A^I} \Omega \right) = \left(p^I F_I - q_I X^I \right), \tag{4.7}$$

where $F_I = \partial_{X^I} F$. From this we see that the central charge of a black hole solution depends on the periods, e.g. the holomorphic volumes, of the wrapped cycles, and thus the complex structure moduli of the Calabi–Yau threefold. By studying the supersymmetry algebra, one can now show that the mass of a wrapped brane state is bounded from below by the central charge,

$$M \geq |Z|, \tag{4.8}$$

with equality for BPS states (see e.g. [Soh85]). Note the similarity of this expression with the extremality condition for black holes, $M^2 \geq Q^2$. Identifying the central charge with the charge vector Q, we see that BPS black holes are extremal.

4.2.1 The Attractor Mechanism

The attractor mechanism can be understood from the following observation. Branes are tensionful objects, and so will try to contract any cycle they wrap. The (holomorphic) volume of a three-cycle is given by its period, which are functions of the complex structure moduli in type IIB compactifications. It follows that a wrapped brane creates a potential for the moduli specifying the size of the wrapped cycles, i.e. the complex structure moduli. The moduli are attracted to the minimum of this potential.

This can be made more precise. We write the four-dimensional black hole metric in the form

$$ds^2_{BH} = -e^{2U(r)} dt^2 + e^{-2U(r)} \left(dr^2 + R^2(r) d\Omega_2^2 \right), \tag{4.9}$$

where r is a radial coordinate and the black hole horizon is located at $r = 0$. We assume that spacetime is asymptotically flat, i.e. $U \to 0$ and $R \to r$ as $r \to \infty$. By dimensional reduction, it can be shown that the five-form field strength gives rise to a potential term $R^{-4} V_{bh}(z)$ for the moduli in the

[4] Theories with extended supersymmetry can have central charges, i.e. operators that commute with all other operators in the supersymmetry algebra. See [Soh85] for more details on extended supersymmetry and central charges.

four-dimensional effective action (see e.g. Paper II). This potential term arises from the five-form flux term in the ten-dimensional action (2.35), and can be expressed in terms of the charge three-form Γ as

$$V_{bh} = \int_M \Gamma \wedge *_6 \Gamma, \tag{4.10}$$

where $*_6$ is the Hodge star operator in the internal manifold M. Furthermore, Γ is harmonic, as a consequence of the equation of motion for the five-form flux. Hence, it can be expanded in a basis of harmonic three-forms on M, and the potential can be rewritten as [Den00]

$$V_{bh}(z) = e^K \left(\mathcal{G}^{I\bar{J}} D_I Z D_{\bar{J}} \overline{Z} + |Z|^2 \right). \tag{4.11}$$

Here $D_I Z = \partial_I Z + Z \partial_I K$ and $\mathcal{G}$ is the complex structure moduli space metric derived from the complex structure Kähler potential $K = -\ln i \left(\bar{X}^I F_I - X^I \bar{F}_I \right)$. This potential was first derived in [FGK97] and is of standard $\mathcal{N} = 2$ form.[5]

Using the metric ansatz (4.9) and the potential, one can derive BPS equations for the four-dimensional theory (see e.g. [Den00])

$$\begin{aligned} \frac{dU}{d\sigma} &= e^U |Z| \\ \frac{dz^I}{d\sigma} &= e^U \mathcal{G}^{I\bar{J}} D_{\bar{J}} |Z|, \end{aligned} \tag{4.12}$$

where we have used the parameter $\sigma = 1/r$ and assumed that the complex structure moduli only depend on the distance to the black hole horizon. These equations also arise as the conditions for the vanishing of the supersymmetry variations for the gravitinos and dilatinos (equation (2.37)), showing that supersymmetry is restored when they are satisfied [FKS95, Str96, FK96].[6]

Assuming that the central charge is non-vanishing at its fixed point, we can solve the above equation for U in the near-horizon region. The result is that the near-horizon geometry is $AdS_2 \times S^2$, the black hole mass saturates the BPS condition $M = |Z|_{r=0}$, and the horizon area is given by [FGK97]

$$A = 4\pi |Z|^2_{r=0}. \tag{4.13}$$

[5]Generically, the potential for scalar fields in $\mathcal{N} = 2$ supersymmetric theories is determined by a Kähler potential and a superpotential. Consequently, we see that the central charge can be viewed as a superpotential of the four-dimensional $\mathcal{N} = 2$ supergravity.

[6]The attractor equations were originally derived from the vanishing of the supersymmetry variations. The reformulation in terms of a potential for the moduli is useful conceptually, and we will use this formulation in Paper II. Furthermore it is necessary for the analysis of non-supersymmetric attractors, see [GIJT05].

It follows that the central charge also determines the macroscopic entropy of the BPS black hole: $S_{BH} = \pi|Z|^2_{r=0}$.

In addition, by extremizing the potential V_{bh} (or rather, by putting $D_I|Z| = 0$ in order to satisfy (4.12)) we fix the complex structure moduli as a function of the charges on the horizon of the black hole [FKS95]

$$p^I = \mathrm{Re}(X^I) \quad q_I = \mathrm{Re}(F_I). \tag{4.14}$$

It is important that the moduli at the horizon, and hence the central charge, are determined exclusively by the charges. This implies that they are independent of the asymptotic value of the moduli far away from the black hole. Since we don't want the mass or the entropy of the black hole to change under changes of its remote surroundings, this is very good.

We have now seen that type IIB compactifications with D3-branes yield four-dimensional extremal black holes, with entropy determined solely by the black hole charges. These charges are given by the wrapping numbers of the D3-branes. One could now go on to investigate the microstates of these black holes, which are different BPS configurations that have the same charges. In particular, this should give a microscopic derivation of the entropy. Such analyses have indeed been performed (see [SV96] for a example), and the macroscopic and microscopic derivations yield the same entropy. Certain non-supersymmetric black holes have also be analysed in this way [DST07a]. This analysis requires a microscopic (e.g. open string) treatment of the D-brane physics that is beyond the scope of this thesis. The interested reader is referred to e.g. [Moh01] for a review.

4.2.2 Topological Strings and Black Holes

The analysis so far has been performed at the supergravity level. Corrections to this action are controlled by parameters proportional to the charges, and can be neglected for large charges. For slightly smaller charges, this is no longer correct, and the effective action should be changed. In particular, one should add terms containing more derivatives to the Lagrangian. As discussed in section 2.2, such corrections arise naturally in string theory. [7]

With the addition of higher derivative terms, the Bekenstein–Hawking area law for the macroscopic entropy must be modified. In $\mathcal{N} = 2$ supergravity with extra vector multiplets, higher derivative terms imply that the holomorphic potential for the vector multiplets is corrected [LCdWM99, LCdWM00, LCdWKM00]

$$F(X^I, \hat{A}) = \sum_{h=0}^{\infty} F_h(X^I)\hat{A}^h, \tag{4.15}$$

[7] In principle, the action is also corrected by hypermultiplet-dependent terms. These are not fixed at the horizon, and are therefore not expected to contribute to the black hole entropy.

where $F_0(X^I, \hat{A})$ is the leading order prepotential used in the last section. $\hat{A}$ is related to the graviphoton field strength and encodes the higher derivative terms. Using the Noether charge formalism of [Wal93], it can be shown that the macroscopic entropy for an $\mathcal{N} = 2$ black hole with such corrections is given by [LCdWM99]

$$S = \pi|Z|^2 + -256\pi \mathrm{Im}(\partial_{\hat{A}} F(X, \hat{A})). \tag{4.16}$$

The attractor equations (4.14) fix $\hat{A}$, the real part of X^I and the corrected F_I at the horizon.

By introducing ϕ^I as the imaginary part of X^I ($X^I = p^I + i\phi^I$), one can rewrite the entropy as the Legendre transform of a function $\mathcal{F}_{BH}(\phi, p)$ [OSV04]:

$$S_{BH}(q,p) = \mathcal{F}_{BH}(\phi, p) - \phi^I \frac{\partial}{\partial \phi^I} \mathcal{F}_{BH}(\phi, p). \tag{4.17}$$

It was shown in [OSV04] that $\mathcal{F}_{BH}$ can be interpreted as the free energy of a microcanonical ensemble of magnetic charges p^I and a canonical ensemble for electric charges q_I with chemical potentials ϕ^I.

The computation of higher genus terms such as the functions F_h, i.e. the modes of the genus expansion of the prepotential, is often difficult. In this case however, it was shown by [AGNT94] that the modes are given by amplitudes in topological string theory, $\mathcal{F}_{top}$. We will not describe what topological string theories are here, since that would double the length of this thesis. The interested reader is instead directed to e.g. [Von05] for a pedagogical introduction to the subject.

To understand why a topological theory might have something to say about type II compactifications on Calabi–Yau three-folds, we should note that these theories have an unusual large amount of global world-sheet supersymmetry, $(N_L, N_R) = (2, 2)$. The generators of the supersymmetries can be used to construct topological string theories.[8] These topological theories deal with a subset of the operators that are present in the physical string theory. The so called topological B-model deals with operators that only depend on the complex structure of the Calabi–Yau. In particular, the genus 0 amplitude of the B-model topological string computes the prepotential of the complex structure moduli space of the Calabi–Yau, and the higher genus amplitudes compute the higher order corrections to the prepotential, $\mathcal{F}_{top,h} \sim F_h$.

Using this connection with the topological string amplitudes, [OSV04] proposed a correspondence between the free energy of a black hole and the real part of a topological string amplitude:

$$\mathcal{F}_{BH}(\phi^I, p^I) = 2\mathrm{Re}\mathcal{F}_{top}(t^I, g_{top}), \tag{4.18}$$

[8] This is a simplification; in order to define topological string theories the generators need to be twisted. See [Von05].

where $t^I = X^I/X^0$ and g_{top} is the topological string coupling constant. The correspondence holds if the topological string and black hole parameters are identified as

$$t^I = \frac{p^I + i\phi^I/\pi}{p^0 + i\phi^0/\pi}, \quad g_{top} = \pm\frac{4\pi i}{p^0 + i\phi^0/\pi}. \tag{4.19}$$

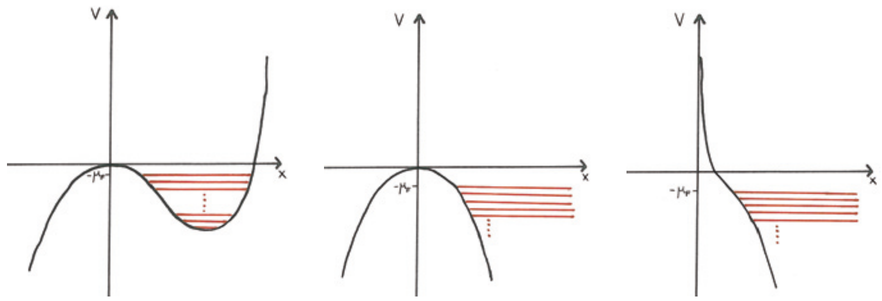
4.2.3 Matrix Models and Black Holes

In Paper I, we use the correspondence between supersymmetric black holes and topological strings to relate the black hole free energy to the free energy of a matrix model. Matrix models are quantum mechanical systems that are related to non-critical two-dimensional string theory, as reviewed in [Kle91]. In particular, a triangulation of the string world-sheet yields a quantum mechanical system of $N \times N$ matrices. The model can also be described by the matrix eigenvalues, that behave like non-interacting, non-relativistic fermions moving in a potential.

The so-called $c = 1$ bosonic string is described by the $c = 1$ matrix model, whose potential is shown in figure 4.2(a). The fermions fill the energy levels of the system up to the Fermi surface, μ_F. The double scaling limit, $\mu_F \to 0, \beta = 1/\hbar \to \infty$ with $\mu = \mu_F\beta$ constant, describes the physics near the local maximum, as shown in figure 4.2(b). In this limit, the Fermi sea is infinitely deep. It turns out that this is the relevant limit to take in order to describe a smooth world-sheet theory.

It is clear from figure 4.2(a) that the $c = 1$ matrix model is unstable to tunnelling through the potential barrier. This non-perturbative instability is an undesired feature. By looking at the double scaling limit, it is clear that there are two ways of obtaining models that are well-defined perturbatively. One could fill the Fermi seas on both sides of the potential barrier, or one could erect an impenetrable wall on the barrier. These models turn out to describe the two-dimensional, non-critical 0B and 0A superstrings, and are therefore known as the 0B and 0A matrix models [TT03, D$^+$03]. Figure 4.2(c) shows the double scaling limit of the 0A matrix model potential, that differs from the $c = 1$ potential by a deformation $(q^2 + 1/4)/x^2$, where q is a real parameter.

Interestingly, it has been shown that matrix models do not only describe two-dimensional physics, but also relate to topological strings used in ten-dimensional string theories. In particular, there are several connections between topological strings on non-compact Calabi–Yau manifolds and matrix models, as reviewed in [Mar04]. One example is [GV95], which shows that the $c = 1$ matrix model, in the double scaling limit, is equivalent to a theory of topological strings propagating on the conifold. To reach this conclusion, one must compactify the matrix model. This is accomplished by going to Euclidean time, which is then compactified with radius R. This is a standard way of obtaining a thermodynamical description of the theory. The free energy of



(a) (b) (c)

Figure 4.2: Matrix models can be described in terms of a Fermi sea. In the double scaling limit, focus is on excitations around the Fermi surface, μ_F. Figure (a) shows the potential of the $c = 1$ matrix model, and (b) shows the double scaling limit of this potential. Figure (c) shows the double scaling limit of the 0A matrix model, which is non-perturbatively stable.

this system can be computed in the double scaling limit, with result[9]

$$\mathcal{F}_{c=1}(\mu, R) = -\frac{R}{2}\mu^2 \ln(\mu) - \frac{1}{24}\left(R + \frac{1}{R}\right)\ln(\mu) + ... \tag{4.20}$$

This expression shows the two first terms in a genus expansion of the string world-sheet. The omitted higher-order terms are all analytic in μ. We are now dealing with a compactified theory, which can be T-dualized. Of particular interest is the self-dual radius $R_{SD}^{c=1} = 1$.

We now turn to the topological strings that should be described by this matrix model. The topological strings propagate on a conifold, which is a non-compact cone in $\mathbb{C}^4$, defined by the polynomial

$$uv - st = 0. \tag{4.21}$$

This is an example of a singular variety of the type described in section 3.3.1.1, that can be reached by shrinking one of the three-cycles of a regular Calabi–Yau three-fold. Conversely, the conifold singularity can be deformed by blowing up a finite size three-cycle, resulting in a regular manifold.[10] The equation for the deformed conifold is

$$uv - st = \mu_c, \tag{4.22}$$

[9] We set $\alpha' = 1$ in this expression.

[10] Somewhat more surprisingly, there is also another way of resolving this singularity, namely by blowing up a two-cycle. The result is another Calabi–Yau manifold. The process of deforming and resolving the conifold is discussed in section 6.4. See also [CdlO90] for more details.

where μ_c is the (complex) size of the blown up three-cycle. For small μ_c the topological string amplitude is given

$$\mathcal{F}_{top}(\mu_c) = \mu_c^2 \ln(\mu_c) - \frac{1}{12} \ln(\mu_c) + ... \tag{4.23}$$

This is just the same expression as the free energy of the $c = 1$ matrix model in equation (4.20). Consequently it seems that the $c = 1$ free energy at self-dual radius is the same as the amplitude of a topological string on the conifold

$$\mathcal{F}_{c=1}(\mu, R_{SD}^{c=1}) = \mathcal{F}_{top}(\mu_c), \tag{4.24}$$

which was proven to all orders in [GV95]. The correspondence holds if μ and μ_c are identified. Note how the non-compactness of the deformed conifold is matched by the infinite Fermi sea in the double scaled matrix model.

If we are now interested in the free energy of a black hole on a deformed conifold, we could use the correspondence (4.18) to compute this in a topological string theory. Furthermore, we could also use the relation between topological string theories and matrix models to compute the free energy. Using equation (4.24) would match the free energy of the black hole to the real part of the $c = 1$ free energy.

In doing this, however, it seems more natural to match the free energy the black hole to the free energy of the matrix model, not just to its real part. This is accomplished if the topological strings on the conifold are matched with the 0A instead of the $c = 1$ matrix model, as argued in [DOV04] and Paper I. We then have the relation

$$\mathcal{F}_{0A}(\mu, R_{SD}^{A}, q) = 2\mathrm{Re}\left[\mathcal{F}_{top}\left(\frac{q + i\mu}{2}\right)\right] = \mathcal{F}_{BH}(\phi^I, p^I), \tag{4.25}$$

which holds to all orders in the perturbative genus expansion, given the proper identification of parameters (see Paper I). Thus, we relate the 0A matrix model at self-dual radius to the deformed conifold, with the equation

$$uv + (\mu - iq) = st. \tag{4.26}$$

Note that the 0A model also has enough real parameters (q, μ) to describe the complex modulus of the conifold. We thereby get a complete matching of the number of (real) parameters on the matrix model and topological string side. Furthermore, recall that the 0A matrix model is non-perturbatively well-defined. Both these properties are in contrast to the $c = 1$ model, and point to that this is the correct identification between the topological theory and the matrix model.

Paper I elaborates on this correspondence between black holes and matrix models. In particular, it clarifies why compactifications on a non-compact manifold such as the deformed conifold can be of any relevance for four-

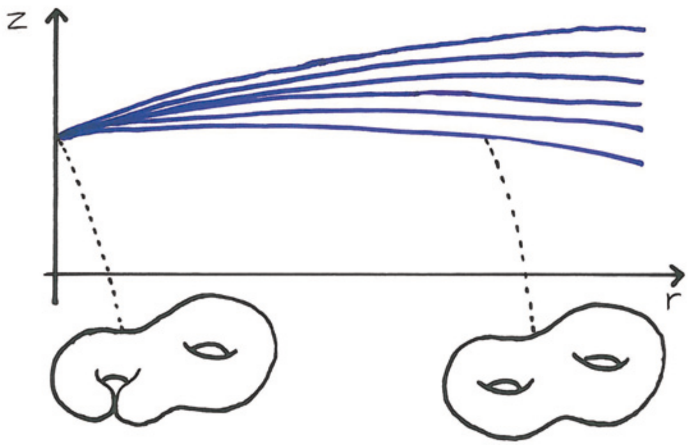


Figure 4.3: This figure shows how the attractor equations can drive the complex structure moduli $z(r)$ of a Calabi–Yau three-fold to a conifold locus on the black hole horizon, irrespective of the asymptotic value of z. It also illustrates how a three-cycle degenerates at the conifold locus, turning a manifold into a singular variety.

dimensional black holes. The key is to view the deformed conifold as a local model for a compact Calabi–Yau. Most compact Calabi–Yau three-folds have conifold singularities, i.e. locii in complex structure moduli space where the Calabi–Yau geometry is locally conifold-like. In such manifolds, we can choose to wrap D3-branes so that the attractor equations drives the moduli to the conifold point at the horizon of the black hole, as shown in figure 4.3. There are then universal terms in the black hole free energy that are given by the deformed conifold, and that are matched by the matrix model computation.

In addition to the universal terms, there will be model-dependent terms that depend on the geometry of the Calabi–Yau away from the shrinking cycle. These terms can also be matched on the matrix model side, by regularizing the matrix model potential. In Paper I (see also Paper II) we compute these terms for the Mirror Quintic manifold, and show that they are of the same form as the large terms of a regulated matrix model. Note how going to the conifold limit in the geometric picture corresponds to taking the double scaling limit on the matrix model side of the correspondence.

In summary, the principal outcome of Paper I is a clarification of the correspondence between matrix models and topological strings. The correspondence to $\mathcal{N} = 2$ black holes served as a guide to how this matching should be made, and linked the 0A matrix model at the self-dual radius to the topological string on the conifold. Furthermore, the 0A matrix model has a geometrical interpretation also at other radii, and Paper I discusses what the appropriate geometries to these models should be. In this matching, the holomorphic factorization and genus mixing of the double scaled 0A matrix model are important. The result is that, in the double scaling limit, the 0A matrix models are all mapped to conifold-like geometries (with the appropriate number of cycles shrinking simultaneously).

In this chapter, we have studied black holes arising from type IIB compactifications with wrapped branes. We have seen how the attractor mechanism fix the complex structure moduli of the internal manifold on the horizon of the black holes. Similar effects arise when fluxes are piercing the internal manifold, as we discuss in the next chapter.

Part III:
Flux Compactifications and the Landscape

5. Flux Compactifications

The free moduli of the $\mathcal{N} = 2$ compactifications studied in chapter 3 pose severe problems for phenomenology and should be fixed in realistic models. In this chapter we show that background values for the RR and NS fluxes create potentials for some of the moduli. Thus by turning on fluxes, a step is taken toward realizing stable or metastable string vacua that could describe four-dimensional universes. The procedure is known as flux compactification, and is one of the cornerstones of modern string theory phenomenology. We will focus on flux compactifications of type IIB string theory on conformal Calabi–Yau manifolds. Several other types of flux compactifications exist, and are reviewed in e.g. [BBS, Gra06, DK07, Den08].

5.1 Background Fluxes

Recall from section 3.3.1, that the complex structure moduli are determined by the holomorphic volume of a basis of homologically inequivalent three-cycles. In the last chapter we saw how the tension of a brane wrapping a cycle introduced a potential for the cycle volume, and hence the associated complex structure modulus. In this chapter, we will show how background fluxes lead to a similar effect.

A manifold with non-trivial p-homology, $H_p(M) \neq 0$, has non-trivial p-cycles C_I. Such a cycle can be pierced by a p-form field strength $F_{(p)}$. By a generalization of Dirac's quantization condition for electromagnetism, the integral of the flux over this cycle must obey

$$\int_{C_I} F_{(p)} \sim F_I \quad \text{where} \quad F_I \in \mathbb{Z}. \tag{5.1}$$

The flux quanta F_I are analogous to the charges of a brane under the associated flux.[1] Such background fluxes $F_{(p)}$ can be present without sources, as long as the cycle it threads has non-trivial topology, which is illustrated in figure 5.1.

Note that the $F_{(p)}$ flux lines are confined to the limited volume of the p-cycle. Confining flux lines costs energy, so this implies that a potential is

[1] In the following we will be interested in three-form fluxes, for which $\int_{C_I} F_{(3)} = (2\pi)^2\alpha' F_I$. For notational simplicity we ignore the factors of π and α'.

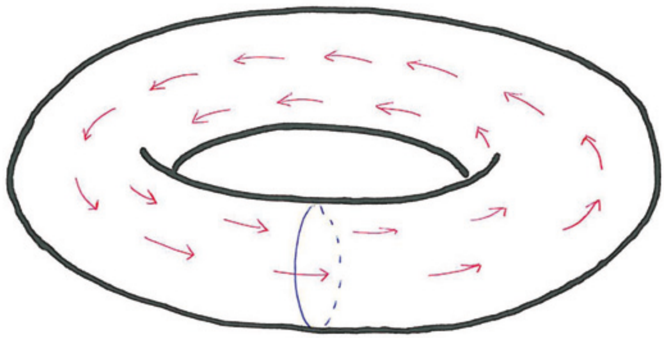


Figure 5.1: A manifold with non-trivial topology can be pierced by a constant flux, even though no sources are present in the manifold. This torus has flux (arrows) through a one-cycle (solid line).

created for the size of the p-cycle, and thereby the modulus determining its volume. Thus, in order to fix the complex structure moduli of a IIB compactification, non-zero background values should be allowed for the three-form fluxes $F_{(3)}$ and $H_{(3)}$. A background configuration is specified by choosing the integers F_I, H_I for a basis of Calabi–Yau three-cycles. The potential induced by the fluxes will be derived by dimensional reduction in section 5.4.

There is one subtlety with this construction. By Gauss' theorem, the overall charge on a compact manifold must vanish. The fluxes introduce a charge in M which must be canceled by other sources. We will show how this is accomplished by studying the ten-dimensional equations of motion in section 5.1.1.

Note that the Kähler moduli of a type IIB compactification cannot be fixed by turning on background fluxes. The Kähler moduli are related to the sizes of two-cycles, or equivalently four-cycles. However, there are no two- or four-form fluxes present in type IIB spectrum that could pierce these cycles and create a potential for their size. Instead, subleading corrections to the supergravity action must be taken into account to show that these moduli can be fixed. This will be discussed in chapter 6.

On the other hand, type IIA contains even-form RR fluxes and a three-form NS flux. This is sufficient to fix all moduli at tree level in certain compactifications. However, for reasons that will be explained in the next chapter, the vacua thus obtained are less interesting for model building than the admittedly more contrived type IIB constructions that we start to describe here.

5.1.1 Background Fluxes and Warping

To make sure that the background flux configurations yield four-dimensional vacua, we must ascertain that this corresponds to a stable ten-dimensional solution. Here we present some of the ten-dimensional equations of motion that restrict the possible configurations. This analysis was first performed by [GKP02], and further discussed in [DG03].

When background fluxes are included, it turns out that the metric ansatz in (3.6) is too restrictive. A less restrictive ansatz that still has four-dimensional Poincaré invariance is possible:

$$ds_{10}^2 = e^{2A(y)}\tilde{G}_{\mu\nu}dx^\mu dx^\nu + e^{-2A(y)}\tilde{G}_{mn}dy^m dy^n. \tag{5.2}$$

This is an example of a warped metric. It is characterized by the warp factor, $e^{2A(y)}$, that multiplies the four-dimensional part of the metric and gives it a dependence on internal coordinates. We will study compactifications where the internal manifold is a conformal Calabi–Yau threefold, i.e. $\tilde{G}_{mn}$ is a Calabi–Yau metric, $e^{-2A(y)}\tilde{G}_{mn}$ is not. The choice of the conformal factor $e^{-2A(y)}$ in front of the Calabi–Yau metric will be explained in section 5.2.

Type IIB string theory has the low-energy action

$$S_{IIB} = S_{SUGRA} + S_{CS} + S_{loc}. \tag{5.3}$$

Here S_{CS} is a Chern-Simons action for the fluxes and S_{loc} accounts for possible sources (e.g. D-branes) that are localized in the compact manifold. S_{SUGRA} contain the Einstein-Hilbert term and kinetic terms. The various terms were defined in equation (2.35) on page 33.

Type IIB supergravity has a five-form flux $\tilde{F}_{(5)}$, two three-form fluxes, which can be combined as $G_{(3)} = F_{(3)} - \tau H_{(3)}$, and a one-form flux $F_{(1)}$. Four-dimensional Poincaré invariance excludes a background expectation value for the one-form flux; there are no internal one-cycles since $h^{(1,0)} = 0$ for conformal Calabi–Yau three-folds. On the other hand, there are plenty of three-cycles for the three-form fluxes to pierce, allowing quantized background values F_I, H_I. The lack of internal five-cycles forbids quantized background five-form flux. Nevertheless, configurations with non-zero five-form flux that is sourced by D3-branes and preserve four-dimensional Poincaré invariance exist. A self-dual ansatz is given by

$$\tilde{F}_{(5)} = (1 + *)d\alpha \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3, \tag{5.4}$$

where $\alpha(y)$ is a function of internal coordinates (as usual, x^μ denote non-compact coordinates).

The supergravity fields (axio-dilaton and fluxes) and localized sources all contribute to the stress energy tensor, $T_{MN} = T_{MN}^{SUGRA} + T_{MN}^{loc}$. The ten-dimensional Einstein equations are, in trace-reversed form

$$R_{MN}^E = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} G_{MN} T \right), \tag{5.5}$$

where E indicates that the Riemann tensor is computed in the Einstein frame (see equation (3.6)). From the non-compact part of this equation, using the warped metric ansatz above, follows the condition [GKP02]

$$\begin{aligned}\tilde{\nabla}^2 e^{4A} = e^{2A}\frac{G_{mnp}\bar{G}^{mnp}}{12\mathrm{Im}\tau} + e^{-6A}[\partial_m\alpha\partial^m\alpha + \partial_m e^{4A}\partial^m e^{4A}] \\ + \frac{\kappa_{10}^2}{2}e^{2A}(T^m_m - T^\mu_\mu)^{loc}.\end{aligned} \tag{5.6}$$

Here a $\tilde{\nabla}^2$ is the Laplacian in the metric $\tilde{G}_{mn}$.

Integrating this constraint over the compact internal manifold, the total derivative on the left hand side vanishes. On the right hand side, the flux and warp terms are positive definite and cannot vanish. Thus the localized sources must cancel these terms. This can be accomplished by objects with negative tension [GKP02].[2] Luckily, such negative tension objects, called orientifold planes, exist in string theory. These are related to an orientifold action that acts both on the world-sheet and the internal manifold. We will return to orientifolds in section 5.2.

Further requirements on the fluxes can be derived from their Bianchi identities and equations of motion. In particular, for the five-form flux

$$\begin{aligned}d\tilde{F}_{(5)} &= F_{(3)}\wedge H_{(3)} + 2\kappa_{10}^2 T_3\rho_3^{loc} \\ \Rightarrow 0 &= \frac{1}{2\kappa_{10}^2 T_3}\int_M F_{(3)}\wedge H_{(3)} + Q_3^{loc}.\end{aligned} \tag{5.7}$$

Here T_3 is the D3-brane tension[3] and ρ_3^{loc}, Q_3^{loc} are the D3 charge density and total charge from orientifold planes and space-filling D3-branes. It is evident that non-zero background fluxes require the introduction of objects that soak up the resulting charge in the compact space.

The tadpole condition (5.7) can be rewritten in terms of $\alpha(y)$. Combined with (5.6), this leads to the condition

$$\begin{aligned}\tilde{\nabla}^2(e^{4A}-\alpha) = \frac{e^{2A}}{6\mathrm{Im}\tau}|iG_{(3)} - *_6 G_{(3)}|^2 + e^{-6A}|\partial(\alpha - e^{4A})|^2 \\ + \frac{\kappa_{10}^2}{2}e^{2A}[(T^m_m - T^\mu_\mu)^{loc} - T_3\rho_3^{loc}].\end{aligned} \tag{5.8}$$

For D3-branes and O3 planes it can be shown that $(T^m_m - T^\mu_\mu)^{loc} = T_3\rho_3^{loc}$, so the last term vanishes.[4] Now, integrate over the internal manifold. Again,

[2] An alternative is given by Dp-branes with $p \geq 7$, since p-branes wrapping a $p-3$ cycle Σ have $(T^m_m - T^\mu_\mu)^{loc} = (7-p)T_p\delta(\Sigma)$ [GKP02]. To maintain four-dimensional Poincaré symmetry, only $p = 7$ is allowed.

[3] In the Einstein frame $2\kappa_{10}^2 T_3 = (2\pi)^{-2}\alpha'^{-2}$.

[4] For brevity, we focus on O3 planes here. Other possible Poincaré symmetry-preserving configurations include O5, O7 and O9 planes. For a discussion of these see [GL04, Gra06].

the left-hand side of the equation vanishes. Thus, consistent solutions have imaginary self-dual three-form flux

$$iG_{(3)} = *_6 G_{(3)} \tag{5.9}$$

and satisfy

$$\alpha = e^{4A}. \tag{5.10}$$

The last equation shows that the warp factor cannot be zero in configurations with non-zero five-form flux, which motivates the metric ansatz above.

5.2 Four-dimensional $\mathcal{N} = 1$ Supergravity

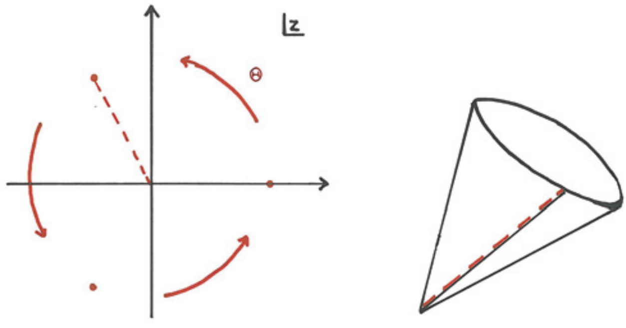
Without background fluxes, four-dimensional $\mathcal{N} = 2$ supersymmetry followed if the internal manifold allowed a covariantly constant spinor, as discussed at the end of section 3.1. This is not a sufficient condition when fluxes are present, since the gravitino and dilatino supersymmetry variations in equation (2.37) on page 34 contain several flux-dependent terms.

However, to preserve some supersymmetry in four dimensions, the ten-dimensional supersymmetry parameter must decompose into a four- and six-dimensional part. Thus, it is still necessary that a nowhere vanishing spinor can exist on the internal manifold. This is one reason for compactifying on a conformal Calabi–Yau; a conformal rescaling does not affect the existence of spinors. What changes is the definition of "conformally constant"; it can be shown that the rescaled manifold has torsion.[5] In fact, by a slight generalization of the decomposition ansatz with respect to equation (3.8) one can categorize the kind of internal manifolds and background fluxes that are allowed in $\mathcal{N} = 1$ compactifications. We will not go through this general analysis, but refer to [Gra06] for a review and references. The result of interest to us is that warped compactifications on conformally Calabi–Yau threefolds have at most $\mathcal{N} = 1$ supersymmetry. It has been shown [GKP02] that $\mathcal{N} = 1$ supersymmetry is preserved if the three-form flux is imaginary self-dual, $*_6 G_{(3)} = iG_{(3)}$, and primitive, i.e. vanishes when contracted with the Kähler form.

A straightforward way to understand the breaking of supersymmetry is to study the effect of orientifold planes. As mentioned above, compactifications with background fluxes require such objects to cancel flux-induced tensions and charges. Orientifold actions O are combinations of an action Ω_p, that reverses the orientation of the string world-sheet, a spacetime involution σ[6] and possibly a sign reversal. The involution leaves four-dimensional spacetime

[5]The torsion tensor is related to the Levi–Civita connection. It measures the failure of closure of a parallelogram made up of small displacement vectors and their parallel transports (see e.g. [Nak]).

[6]This involution should not be confused with the world-sheet coordinate σ.



(a) (b)

Figure 5.2: An orbifold is obtained when points related by an internal symmetry are identified. Figure (a) shows the z-plane which has a $\mathbb{Z}_3$-symmetry that can be orbifolded. Identifying points related by the action $\Theta z = e^{2\pi i/3} z$ yields a cone, shown in figure (b). Note how the dotted line $re^{2\pi i/3}$ is identified with the real axis. The origin is a fixed point under Θ, and becomes a singularity in the orbifold.

unchanged and orbifolds the compactification three-fold by an internal symmetry, i.e. uses the symmetry action to identify parts of the internal space. An example of an orbifold is shown in figure 5.2. Orientifold planes are the non-dynamical fixed planes under an orientifold action O. Since four-dimensional spacetime is invariant under the orientifold actions considered here, the O planes are space-filling.

For consistency, σ must be isometric and holomorphic on the internal space [GL04]. Isometry implies that the Kähler form J is preserved by the action: $OJ = J$. Holomorphicity guarantees that O is compatible with the complex structure, and the holomorphic three-form must obey $O\Omega = \pm\Omega$. The lowest dimensional O plane compatible with these conditions is an O3 plane, which is pointlike in the internal space. It has been shown (see [GL04] and references therein) that the orientifold action leading to O3 planes is[7]

$$O = (-1)^{F_L} \Omega_p \sigma^* \text{ , where } \quad \sigma^* \Omega = -\Omega. \tag{5.11}$$

Here F_L is the spacetime fermion number in the left-moving sector, which is non-zero for the fields in the RR sector in type IIB. As usual, σ^* denotes the pullback of σ to the space of forms.

5.3 Spectrum

We now turn to the low-energy spectrum of flux compactifications. Just as in chapter 3, we find this by KK expanding the various fields in the harmonic

[7]This action could equally well lead to O7 planes, see [GL04].

$$
\begin{array}{ccccccc}
 & & & h_+^{(3,3)} = 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h_+^{(1,1)} + h_-^{(1,1)} & & 0 & \\
h_-^{(3,0)} = 1 & & h_+^{(2,1)} + h_-^{(2,1)} & & h_+^{(2,1)} + h_-^{(2,1)} & & h_-^{(0,3)} = 1 \\
 & 0 & & h_+^{(1,1)} + h_-^{(1,1)} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & h_+^{(0,0)} = 1 & & &
\end{array}
$$

Table 5.1: *The Hodge numbers of an orientifolded Calabi–Yau three-fold decompose into the dimensions of the eigenspaces of the orientifold action* (5.11).

forms of the internal manifold. As we will see below a lot of the work is already done when it comes to the massless sector. We need only consider which of the massless $\mathcal{N} = 2$ fields remain in the spectrum after orientifolding.

First, we should however show that the harmonic forms on the conformal Calabi–Yau are the same as on the original manifold. Given the metric ansatz (5.2) it is straightforward to show that, for a field Φ

$$\Delta_{(10)}\Phi = e^{-2A(y)}\tilde{\Delta}_{(4)}\Phi + e^{2A(y)}\tilde{\Delta}_{(6)}\Phi. \tag{5.12}$$

where the three Δ are the differential operator for the field on the various spaces, and a tilde denotes the use of the tilded metrics in the metric ansatz (5.2). To obtain this result, it is necessary that the metric ansatz is of the form (5.2), with the conformal factor multiplying the (tilded) Calabi–Yau metric. It follows that the four-dimensional zero-mass spectrum is once again found by expansion in the harmonic forms of the internal Calabi–Yau. Thus, the low-energy effective theory can be found by studying the effect of the orientifold action (5.11) on the harmonic forms of the Calabi–Yau.

The ten-dimensional type IIB fields $\Phi, G_{MN}, C_{(2)}$ are even under the world-sheet orientation reversal Ω_p, whereas $B_{(2)}, C_{(0)}, C_{(4)}$ are odd. Furthermore, the NS states are invariant under $(-1)^{F_L}$, while the RR states change sign. Thus, the massless fields that remain in the truncated theory result from expansions in harmonic forms with the appropriate σ^* eigenvalue.

Since σ is compatible with complex structure, harmonic (p, q)-forms must be either even or odd eigenstates of the action. Consequently, invariant four-dimensional fields are obtained by Kaluza–Klein expansions of the ten-dimensional fluctuations in harmonic forms, just as in the $\mathcal{N} = 2$ Calabi–Yau compactifications in section 3.3. The only difference is that the expansion will be in harmonic forms of definite σ eigenvalue, chosen so that the ten-dimensional field is invariant under the orientifold action. This reduces the number of four-dimensional fields. Table 5.1 show how the Hodge numbers of the conformal Calabi–Yau manifold are divided

Multiplet	Multiplicity	Bosonic fields
Supergravity	1	$g_{\mu\nu}$
Universal chiral multiplet	1	ϕ, c
Chiral multiplet	$h_-^{(1,1)}$	b^a, c^a
Chiral/linear multiplet	$h_+^{(1,1)}$	$v^\alpha, \tilde{c}^\alpha$
Vector multiplet	$h_+^{(2,1)}$	$\tilde{c}_\mu^\kappa$
Chiral multiplet	$h_-^{(2,1)}$	z^k

Table 5.2: *Bosonic field content in type IIB compactifications with O3/O7-planes on a conformal Calabi–Yau threefold. The complex structure moduli z^k form a chiral multiplet and the Kähler moduli v^α form a chiral/linear multiplet. Note how the orientifold action in equation* (5.11) *projects out many of the states in table 3.2.*

into eigenspaces of σ^*, where it has been taken into account that the holomorphicity of σ^* relates the dimensionality of some of the eigenspaces [GL04]. For compactifications which only have O3-planes, it can furthermore be shown that three- and two-forms must be even and odd respectively, i.e. $h_+^{(2,1)} = h_-^{(1,1)} = 0$.

Proceeding as in section 3.3, one can show that the four-dimensional fields no longer yield the bosonic field content of $\mathcal{N} = 2$ multiplets. For example, the graviphoton in the $\mathcal{N} = 2$ gravity multiplet ($c_{\mu klm} = c_\mu \Omega_{klm}$) is odd under the orientifold action, and is projected out. The graviton is even and remains in the spectrum, giving the bosonic field content of the $\mathcal{N} = 1$ gravity multiplet. In a similar way the hypermultiplets decompose to chiral/linear multiplets, and the vector multiplets give rise to $\mathcal{N} = 1$ vector and chiral multiplets. The total effect is that the $\mathcal{N} = 2$ multiplets decompose into multiplets of four-dimensional $\mathcal{N} = 1$ supergravity [GL04]. The resulting spectrum is given in table 5.2.

There is one caveat in the above construction, that we must consider before proceeding. The analysis of the low-energy spectrum performed above is perfectly all right as long as the warp factor e^{2A} is constant. Suppose that the warp factor instead varies. This does not affect the massless solutions to (5.12), but it changes the mass of massive Kaluza–Klein modes

$$\tilde{\Delta}_{(4)}\phi_n(x) = -e^{4A} M_n^2 \phi_n(x). \tag{5.13}$$

Since the KK expansion is not in mass eigenstates, the effective low-energy theory becomes complicated. Different fields should be included in the theory depending on the location on the internal manifold.

In the following, we will study large-volume compactifications where warping is constant [GKP02].[8] Thus, all massive KK modes will be neglected. Another interesting approach is to use eigenfunctions of the Laplacian on the warped internal manifold for KK compactifications. These are mass eigenstates in the warped compactification. However, the four-dimensional field theory obtained from such compactifications is more involved (i.e. the potential is harder to find). We refer to [DT08, STUD08, DST07b, FTUD08] for a thorough discussion of these issues.

5.4 Action

The action for the four-dimensional theory is obtained by dimensional reduction of the ten-dimensional type IIB supergravity action given in equation (2.35), just as in the $\mathcal{N} = 2$ case. In this section we show that the action is determined by a Kähler potential K and a holomorphic superpotential W, as expected for four-dimensional $\mathcal{N} = 1$ supergravity.

The low-energy scalar fields consist of the complex structure moduli z^k, the complexified Kähler moduli, $\rho^\alpha = iv^\alpha + \tilde{c}^\alpha$, and the axio-dilaton, $c + i\phi$. The kinetic terms for these fields are given by

$$\mathcal{G}_{U\bar{V}} \partial_\mu \Upsilon^U \partial^\mu \bar{\Upsilon}^V, \tag{5.14}$$

where Υ collectively denotes the different fields. The field space is Kähler, $\mathcal{G}_{U\bar{V}} = \partial_U \partial_{\bar{V}} K$, and factorizes at leading order.[9] Thus the Kähler potential K is given by the sum

$$K = K_{CS}(z, \bar{z}) + K_{dil}(\tau, \bar{\tau}) + K_K(\rho, \bar{\rho}). \tag{5.15}$$

Here $K_{CS} = -\ln\left(i \int \Omega \wedge \bar{\Omega}\right)$, as derived in section 3.3.1, and

$$K_{dil} = -\ln\left(i(\tau - \bar{\tau})\right) \tag{5.16}$$

is straightforwardly derived by dimensional reduction.

The Kähler potential for the Kähler moduli is still given by $K_K = -2\ln\kappa(J, J, J) = -2\ln\mathcal{V}$, as in section 3.3.1. With more than one Kähler modulus, one has to change coordinates on the moduli space before the metric is explicitly Kähler. This has been studied in some detail in [BBHL02, GL04]. If there is instead only one Kähler modulus,

---

[8] To understand why this is the case, consider the scaling of the various terms of equation (5.8) on page 78 under volume rescalings of the internal metric $\tilde{G}_{mn} \to \lambda^2 \tilde{G}_{mn}$. The terms involving A scale as λ^{-2} whereas the flux term scale as λ^{-6}, yielding $A = 1 + \mathcal{O}(\lambda^{-4})$. It follows that A is constant when λ is large.

[9] As in the $\mathcal{N} = 2$ case, the Kähler and complex structure moduli do not mix when the warping is constant. There are however metric mixings between Kähler moduli and the axio-dilaton, as discussed in chapter 6.

parameterizing the overall volume of the Calabi–Yau, the Kähler potential can explicitly be written as

$$K_K = -3 \ln \left(i(\rho - \bar{\rho}) \right). \tag{5.17}$$

There are several corrections to K_K, that are subleading at large volume and small string coupling. These will be discussed in chapter 6.

In addition to the kinetic terms for the fields in the low-energy spectrum, the action contains the four-dimensional Einstein-Hilbert term and a potential term. This potential arises from the background fluxes $F_{(3)}$ and $H_{(3)}$,[10] which satisfy the ten-dimensional equations of motion and Bianchi identities:

$$dG_{(3)} = 0 \quad d * G_{(3)} = 0. \tag{5.18}$$

We saw above that four-dimensional Poincaré invariance only allows background three-form flux in the internal manifold. Equation (5.18) then requires the three-form flux to be harmonic. This implies that background fluxes can be expanded in a basis of harmonic three-forms. For the large-volume compactifications of interest here, these are just the harmonic three-forms of the Calabi–Yau.

Since the ten dimensional integrals depend on the compactification volume through the metric determinant, the following ansatz for the ten-dimensional metric is useful

$$ds_{10}^2 = e^{-6u} \hat{G}_{\mu\nu} dx^\mu dx^\nu + e^{2u} \hat{G}_{mn} dy^m dy^n. \tag{5.19}$$

For notational simplicity, we omit possible warp factors, that would anyway be constant in the large volume limit. The parameter u sets the internal volume scale, $\mathcal{V} = \exp(6u)$. Keeping in mind that the Calabi–Yau volume is set by a Kähler modulus, we promote u to a four-dimensional field. We must then rescale the four-dimensional metric in order to decouple u from the four-dimensional Einstein–Hilbert term.

Inserting this metric and the harmonic expansion of the flux in the ten-dimensional action (2.35) leads to a potential term in the four-dimensional action

$$\begin{aligned} V = \frac{1}{\mathcal{V}^2 (\mathrm{Im}\tau) \int \Omega \wedge \bar{\Omega}} \Big[&\int G_{(3)} \wedge \bar{\Omega} \int \bar{G}_{(3)} \wedge \Omega \\ &+ G^{k\bar{l}} \int G_{(3)} \wedge \chi_k \int \bar{G}_{(3)} \wedge \bar{\chi}_{\bar{l}} . \Big] \end{aligned} \tag{5.20}$$

where it has been used that $\mathcal{V}$ and τ are independent of internal coordinates.

[10] If warping is non-constant, there will be contributions to the potential term also from the five-form flux and the Ricci scalar, which are related to the three-form flux by the ten-dimensional equations of motion. See [GKP02, DG03] for a discussion.

Defining the Gukov–Vafa–Witten superpotential [GVW00]

$$W = \int \Omega \wedge G_{(3)} \tag{5.21}$$

and using the Kähler potential in equation (5.15), allows us to rewrite the potential in the standard $\mathcal{N} = 1$ form[11]

$$V = e^K \left(\mathcal{G}^{U\bar{V}} D_U W D_{\bar{V}} \overline{W} - 3|W|^2 \right), \tag{5.22}$$

where $D_U W = \partial_U W + W \partial_U K$. This potential has supersymmetric minima when all $D_U W = 0$. Note that the superpotential depends on complex structure moduli and the axio-dilaton, but is independent of all Kähler moduli. Furthermore, as long as $h^{(1,1)} = 1$ it is straightforward to show that $\mathcal{G}^{\rho\bar{\rho}} K_\rho K_{\bar{\rho}} = 3$, which implies that

$$\mathcal{G}^{\rho\bar{\rho}} D_\rho W D_{\bar{\rho}} \overline{W} - 3|W|^2 = 0. \tag{5.23}$$

This identifies the low-energy theory as so-called no-scale models [CFKN83, ELNT84].[12] The no-scale condition (5.23) holds also for compactifications with more than one Kähler modulus, as shown in [GL04].

The no-scale property implies that the potential V is positive semi-definite, with minima when $V = 0$. These minima can, but need not be supersymmetric. Since $D_\alpha W \sim W$, this might be non-zero for some α, even if $V = 0$. The supersymmetry breaking scale is then set by the value of the superpotential in the minimum. In addition the potential can have minima at non-zero values of V, which are necessarily supersymmetry breaking. Note that neither of these minima fix the Kähler moduli, which only enter in an overall scale of the potential. These moduli can only be stabilized by non-classical corrections, as we discuss in the next chapter.

On the other hand, the potential V will generically fix all the complex structure moduli and the axio-dilaton in a given compactification. To see this note that

$$\partial_\tau V = \partial_k V = 0 \tag{5.24}$$

are $h^{(2,1)} + 1$ equations for $h^{(2,1)} + 1$ variables for each choice of flux. Of course, since we derived these expressions from the supergravity approximation, which is valid at large volume and small coupling, not all such vacua need be consistent. In particular, we have to check that the axio-dilaton is fixed so that the string coupling is small. Even with this extra constraint, finding vacua is not very difficult.

[11] See e.g. [GGRS83, Soh85] for a discussion of four-dimensional $\mathcal{N} = 1$ supersymmetry and supergravity.

[12] The name 'no-scale' was introduced in [ELNT84] to emphasize that the supersymmetry breaking scale is not put in by hand in these models, but arise from the dimensional reduction.

Rewriting the superpotential in terms of the flux quanta and the periods

$$W = \int \Omega \wedge G_{(3)} \equiv (F - \tau H) \cdot \Pi \tag{5.25}$$

it follows that $V = 0$ if

$$\begin{aligned} D_k W = 0 \leftrightarrow (F - \tau H) \cdot D_k \Pi = 0 \text{ and} \\ D_\tau W = 0 \leftrightarrow (F - \bar{\tau} H) \cdot \Pi = 0, \end{aligned} \tag{5.26}$$

where the last equality is a consequence of the simple Kähler potential (5.16) for the axio-dilaton τ. These equations can be interpreted geometrically; given a choice of integral flux F, H, a potential is created that forces complex structure moduli to a point where the (3,0) and (1,2) parts of the flux vanish, leaving an imaginary self-dual $G_{(3)}$. Supersymmetric minima are furthermore possible if $W = 0$, i.e. $G_{(3)}$ is a (2,1)-form.

Summarizing, the four-dimensional effective field theory obtained by compactifying type IIB string theory on a conformal Calabi–Yau manifold, in the presence of background three-form fluxes and O3-planes, yields an $\mathcal{N} = 1$ supergravity. The action is (after restoring factors of α' and π)

$$\begin{aligned} S_{eff}^{flux} = \frac{M_p^2}{2} \int d^4x \sqrt{-g_4} \big(R^{(4)} + \mathcal{G}_{k\bar{l}} \partial_\mu z^k \partial^\mu \bar{z}^l + \mathcal{G}_{\alpha\bar{\beta}} \partial_\mu \rho^\alpha \partial^\mu \bar{\rho}^\beta \\ + \mathcal{G}_{\tau\bar{\tau}} \partial_\mu \tau \partial^\mu \tau - \frac{1}{(2\pi)^2 \alpha'} V \big), \end{aligned} \tag{5.27}$$

where $\mathcal{G}$ is given by the Kähler potentials in equation (5.15), and V is given by the Kähler and superpotential as specified in equation (5.22). The four-dimensional Planck mass is given by

$$\frac{M_p^2}{2} = \frac{(2\pi)^6 \alpha'^3 < \mathcal{V} >}{2\kappa_{10}^2}, \tag{5.28}$$

where $< \mathcal{V} >$ is the expectation value of the compactification volume. In addition, there can be local sources, i.e. space-filling D3-branes, present. Such sources would add a second term to the action, describing the low-energy action of the open strings propagating on the brane.

Once the complex structure moduli and axio-dilaton are fixed at a minimum of the potential V, they decouple from the low-energy four-dimensional theory. The potential term turns intro a cosmological constant, whose value is be zero when the minimum preserves $\mathcal{N} = 1$ supersymmetry.

5.5 Black Holes in Flux Compactifications

We have now seen that both background fluxes and wrapped D3-branes introduce potentials for the complex structure moduli in IIB Calabi–Yau compactifications. A general compactification could include both fluxes and branes, and it is interesting to investigate how such combinations behave. This is the topic of Paper II, where the influence of a black hole on the stability of a flux compactification is estimated.

Recall that there is a crucial difference between the black hole and flux potentials; the black hole fix the moduli at the horizon of the black hole, leaving free moduli at infinity. The flux potential, on the other hand, exists at all points in spacetime, and hence the moduli are massive also at infinity. It is therefore expected that the influence from the black hole could only be important near the horizon of the black hole.

Since the effective four dimensional physics is determined by the value of the moduli, it would be very interesting if the black hole did have a substantial impact on the fixing of the moduli. The near-horizon region would then correspond to another string theory vacuum than the faraway region, as discussed in Paper II and [GSS06]. Moreover, the formation of a black hole that couples to fixed background fields in a vacuum, could, under certain assumptions, catalyze the nucleation of an expanding vacuum bubble in the vicinity of the black hole [GSS06]. This suggests that matter contracting to form black holes could even foster the creation of new universes.

We will study a flux compactification with additional D3-branes wrapped around internal three-cycles. Such a configuration will typically result in a non-supersymmetric four-dimensional black hole solution. Solutions with $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetry could also be constructed, but our focus will be on the general case. The black hole is charged under the self-dual five-form field strength, which gives rise to a vector field for each pair of non-trivial three-cycles. Table 5.2 shows the gauge fields that remain after orientifolding the four-dimensional theory, as is required to cancel flux-induced tadpoles.

A compactification with both branes and fluxes is necessarily complicated, as the different sources might couple to each other and also back-react on the geometry. There are also topological restrictions that limit the combined brane and flux configuration. In particular, the number of cycles admitting stable wrapped D3-branes is reduced by the Freed–Witten anomaly [FW99]. This renders the problem difficult to solve in detail. The interest here is however to investigate under which conditions the black hole attractor mechanism could possibly overcome a potential induced by fluxes. A qualitative answer to this question is obtained by studying a black hole immersed in a background potential induced by fluxes.

For a generic compactification we then obtain an effective four-dimensional potential (see Paper II)

$$S_{pot} = -\frac{1}{2\kappa_{10}^2}\int d\mathrm{Vol}_4 \left(\frac{1}{R(r)^4}V_{bh}\left(z\right) + V_f\left(z\right)\right), \tag{5.29}$$

where $R(r)$ measures the radial distance to the black hole center (see (4.9) on page 63).[13] The black hole potential V_{bh} arises from the five-form flux term in the ten-dimensional type IIB supergravity action, and is quadratic in the black hole charges q. The flux potential V_f is given by the three-form flux and is quadratic in the flux quanta, Q.

Note the black hole potential is suppressed by a factor of R^{-4}, and obtains its largest value at the horizon, $R = R_{BH}$. It is clear that this term will be subdominant, at least for macroscopic black holes. The radius of the black hole horizon is bounded from below by the charges, with $R_{BH} \sim q$ for extremal black holes. Since the potentials are proportional to the square of the corresponding flux quanta Q and charge q, the effect of the black hole is suppressed by a numerical factor $\sim 1/(Q^2q^2)$. Thus for large black hole charges, and reasonable flux quanta, the effect of the black hole potential is clearly subdominant all the way in to the horizon.[14]

However, these generic arguments do not exclude that the different functional forms of the potentials, or appropriately chosen charges and flux quanta, could make the black hole potential significant in the near-horizon region. This was investigated in Paper II, under the assumption that the black hole and flux potentials can be expressed in terms of superpotentials and Kähler potentials, as derived in sections 4.2.1 and 5.4. This assumption immediately implies that the functional forms of the two potentials are very similar, which closes one of the above loopholes. Moreover, Paper II contains an explicit example where the requirements for the displacement of a minimum is analyzed. The result of this analysis is indeed, that very small black hole charges or certain fine-tuned flux can change the location of the minimum in a noticeable way.[15] So, although the general result is that flux compactifications are stable against the introduction of wrapped D3-branes, choosing the fluxes and charges carefully might result in a destabilization. Note however, that it is unclear if such fine-tunings are possible in a more realistic setting, where the topological restrictions on the solution are taken into account.

[13]From the supergravity analysis we have that R is measured in terms of M_p. Moreover, the flux quantization conditions sets the scale of $V_{bh} \sim (\alpha')^4$ and $V_f \sim (\alpha')^2$.

[14]This is in contrast with the analysis of [GSS06], where the background potential is not restricted to be a flux potential, which e.g. implies that the scale of the potential is a free parameter. Here, the scales are given by flux quantization as mentioned in the previous footnote.

[15]This conclusion depends also on the size of the extra dimensions. For very large extra dimensions, the black hole potential could compete with the potential induced by fluxes even for larger black holes.

An interesting question left open in Paper II is how the stability and macroscopic properties of the black hole are affected by the surrounding flux potential. For example, one would expect the black hole entropy to depend both on the charges and on the flux quanta of the surroundings, since the moduli are now fixed by the fluxes. This has been confirmed by [LBO07] for a supersymmetric flux/brane configuration. Furthermore, [LBO07] finds that these black holes are unstable and can decay to spacetime localized flux configurations. It is possible that similar instabilities can appear for non-supersymmetric black holes in flux configurations.

In summary, the conclusion of Paper II is that only certain fine-tuned black holes could destabilize flux vacua. We now leave the discussion of black holes in supergravity compactifications. In the following chapters we will instead study flux compactifications, and the flux-induced potential, in more detail. Our focus will also in the following be on the stability of flux vacua.

6. The Type IIB Landscape

In this chapter we introduce the string theory landscape, i.e. the collection of metastable string theory vacua. The different vacua represent low-energy effective field theories of different dimensionality, geometry and field content. Of particular interest is the effective potential that fixes the moduli, whose minimal value determines the cosmological constant of the effective spacetime.

Our focus will be on four-dimensional vacua arising from type IIB compactifications. One reason for our interest in type IIB is that de Sitter vacua can be constructed in this way, and de Sitter vacua are interesting from a cosmological perspective, as we discuss in chapter 7. Furthermore, the special geometry of the complex structure moduli space gives us a powerful tool to study not only the vacua, but also the topography of this part of the landscape. This is the topic of Papers III and IV.

6.1 Stabilizing Kähler Moduli

In the previous chapter we saw how background three-form fluxes can fix the complex structure moduli and axio-dilaton, but not the Kähler moduli, of a type IIB compactification. The reason for this is that the flux-induced potential is of $\mathcal{N} = 1$ no-scale type. Nevertheless, we should recall that the superpotential and Kähler potential have been derived from the type IIB supergravity action (2.35) on page 33. This is a low-energy approximation of string theory valid at weak coupling and large volume. We saw in chapter 2 that the action receives corrections from subleading terms both in the α' and g_s expansions. As a result, both the superpotential W and the Kähler potential have quantum corrections.

The superpotential is not renormalised at any order in perturbation theory and is thus protected from perturbative corrections [BEQ06]. Non-perturbative corrections, on the other hand, are possible. These arise either from instantonic D3-branes [Wit96] or gaugino condensations on stacks of D7-branes [KKLT03]. In both cases, the corrections are of the form

$$W_{np} = \sum_{\alpha} A_{\alpha}(z, \tau) e^{i a_{\alpha} \rho_{\alpha}}, \tag{6.1}$$

where the constants a_α and the functions A_α depend on the type of correction and the geometry of the compactification manifold. As in the previous chapters, z denotes complex structure moduli, ρ denote Kähler moduli and τ is the axio-dilaton.

The Kähler potential receives perturbative corrections, which introduce a dependence of the axio-dilaton in the metric for Kähler moduli space. To first order, the corrected Kähler potential is

$$\begin{aligned} K &= K_{CS}(z,\bar{z}) + K_{dil}(\tau,\bar{\tau}) + K_K^{\alpha'}(\rho,\bar{\rho},\tau,\bar{\tau}) \\ K_K^{\alpha'} &= -2\ln\left[e^{-3\phi_0/2}\mathcal{V} + \frac{\xi}{2}\left(\frac{-i(\tau-\bar{\tau})}{2}\right)^{-3/2}\right], \end{aligned} \tag{6.2}$$

where ξ is proportional to the Euler number of the compactification manifold, and ϕ_0 is the vacuum expectation value for the dilaton [BBHL02]. The Kähler potentials K_{CS} and K_{dil} were discussed in section 5.4. Equation (6.2) only includes the leading order α' correction. In principle, g_s corrections should also be taken into account, but they are smaller than the α' corrections at large volume [BHK06, BHP07].

Assuming that the three-form fluxes first stabilize the complex structure moduli, one can then use the above effects to stabilize also the Kähler moduli.[1] As long as the tree level superpotential is small in the minimum, the corrections to the Kähler potential yield negligible contributions to the potential. This is used in the KKLT model [KKLT03], where Minkowski or anti de Sitter vacua are constructed at a reasonably large compactification volume, using only the non-perturbative corrections W_{np}. These vacua can be uplifted by space-filling anti-D3-branes [KKLT03] and/or non-zero F-terms (i.e. $D_iW \neq 0$ for some z^i) [SS04] to create (non-supersymmetric) de Sitter vacua.

For more generic values of the tree level superpotential, both (6.1) and (6.2) are important. It was shown in [BBCQ05] that by utilizing these corrections, Kähler moduli can be fixed at exponentially large compactification volume.[2] These vacua are non-supersymmetric and result in four-dimensional anti de Sitter spacetimes. As in the KKLT model, anti-D3-branes can be added to uplift the minima to yield a positive cosmological constant. [3]

[1]This two-step fixing of moduli simplifies the analysis considerably. Given a large separation in energy scales between the tree-level potential and corrections, the fixing of Kähler moduli will not affect the flux-fixed fields considerably. Scenarios without this separation are also possible. See [Dim08] for a categorization of different possibilities and [BMP08] for a discussion of the two-step assumption in from a phenomenological point of view.

[2]The volume is very large when measured in string units. Compared to more mundane length scales, the compactified dimensions are still very small. Thus there is no clash with particle physics experiments.

[3]One cannot use F-terms to uplift these vacua, since their absence is used to prove the existence of a minimum.

6.2 Predictions and Statistics of the Landscape

By combining the flux-induced potential with the quantum corrections discussed in the previous section, we see that a large number of four-dimensional vacua can be constructed. Indeed, a generic Calabi–Yau has Euler number around 100 and thus something of the order of 100 three-cycles that can pierced by flux. Each such cycle can have up to 10 units of flux through it, without (necessarily) being in conflict with the tadpole condition (5.7). We thus have in the order of 10^{100} possible flux configuration on a generic Calabi–Yau, and the simplicity of the stabilization conditions (5.24) implies that most such configurations yield some vacuum. Taking all possible Calabi–Yau manifolds into account increases this number. Furthermore, the inclusion of branes in the compactifications adds a large open string sector to the landscape, thereby increasing the number of vacua. In addition, we can construct vacua using other string theories as a starting point.[4]

The large number of possible vacua is of course a drawback for those who would like string theory to be a theory of everything that gives unique four-dimensional predictions. One could wonder if any four-dimensional predictions could be derived from such a landscape. It is therefore important to note that the landscape does not allow everything. On quite general grounds one can look for conditions that need to be fulfilled by the effective theories that arise in the landscape. Such conditions set the boundaries to the surrounding swampland of field theories that do not have a quantum gravity completion [Vaf05, OV07].

From another point of view, the proliferation of vacua has the benefit that it can naturally accommodate fine-tuned parameters that we observe. The prime example of this is the cosmological constant, which has been measured to be approximately $\Lambda \sim 10^{-120} M_p^4$, which is about 120 orders of magnitude smaller than would be expected by naive dimensional reasoning. As mentioned above, in the landscape, the cosmological constant is determined by the potential value in a vacuum. Given more than 10^{120} vacua, the probability that at least one of these vacua has the right cosmological constant is no longer so small. Furthermore, in a landscape of vacua, one can use anthropic reasoning to argue that it is likely that we observe such a fine-tuned value, as discussed in the introduction to this thesis.

The many string vacua also allow a statistical reasoning about whether certain properties are more common than others in the landscape. For example, one could scan the vacua to see what the typical scales are for the cosmological constant and the supersymmetry breaking scales. Such statistical surveys have been undertaken in the type IIB landscape, see e.g. [DD04, Dou03, DD05]. It

[4]Some of these will be related to the type IIB constructions discussed here by mirror symmetry or other dualities. Nevertheless, there are e.g. several type IIA compactifications that are mapped to large coupling, small volume type IIB constructions by mirror symmetry. Such vacua are not accounted for here.

is also interesting to investigate conditional probabilities, i.e. statistical distributions in a subsection of the landscape that fulfills some predefined requirements [AD06]. The conclusions from this type of surveys are interesting since they can lead to a definition of "stringy naturalness" (see e.g. [DK07] for a discussion). Thus, we learn about the structure of the underlying theory. On the other hand, if we are interested in the statistical distribution of vacua in a cosmological setting, we also need to take the stability of various vacua into account. We will return to this question in chapter 7.

6.3 Dynamics and Topography

Given that there is a vacuum in the landscape with the properties of our universe, one can wonder if the rest of the landscape is of any relevance. Maybe it is just superfluous information about physics we could never measure? There are several indications that this is not the case. For example, a popular way of explaining the homogeneity and isotropy of our universe is assuming that it has gone through a period of inflation (i.e. rapid expansion) [Gut81]. A slowly rolling scalar field could be the source of such an expansion. This could possibly happen in the string theory landscape, e.g. if there is an almost flat direction in the potential for the moduli near our vacuum in the landscape. Slow roll inflation could then be obtained by a modulus rolling in this direction. There could also be other features of the landscape that result in inflation, and we will return to this in the next chapter. For now we note that, to investigate if inflation occurs, we need to study the potential also away from the vacua.

Another interesting aspect of the existence of other vacua is that they could allow us to study the history of our universe before its nucleation. In the landscape this is just a question of stability. The vacua are just local minima in a very complicated potential, and as such can be unstable to quantum mechanical tunneling to other minima. To estimate the stability of minima, we need to investigate the topography of the landscape, i.e. the distances and barriers between minima. Topographic properties are also of relevance for statistical discussions, especially if there could be dynamical principles in the landscape that make certain vacuum properties more likely. This is discussed further in Paper III, to which we also refer for further references.

A problem with topographic studies of the landscape is that these properties tend to be model dependent. Therefore, we should aim for a simplified model that captures the general topographic features. Such a model can be found by restricting our attention to the flux-induced potential in type IIB compactifications. This sector is sufficiently complex (it is multidimensional and involves fluxes) to be interesting, but simple enough to obtain results. In doing this, we neglect the crucial fixing of Kähler moduli, which is of course equally important as the fixing of complex structure moduli. We should therefore view this simplification as a first step toward a better understanding of the topography of the landscape. If we find any interesting structure at this level, we should investigate if the structure persists after adding quantum corrections. We return to this discussion in the end of this section.

A topographic question that is relatively easy to answer regards how vacua are distributed in the complex structure moduli space. We only need to solve the equations (5.24). Thus it seems straightforward to determine the distance between vacua, and, assuming that potential barriers are of similar scales, one would suppose that vacua in regions with many vacua are more prone to decay than those situated in more sparsely populated regions. This might sound plausible, but there is one very important caveat in the above reasoning. The potential is a function on a parameter space that is a combination of quantized flux directions and continuous moduli directions. Two minima that are close in moduli space might have different flux configurations, and therefore lie far apart in flux space. Furthermore, there is usually not a continuous potential barrier between the two, which makes transitions between vacua difficult to handle.

The main result of Paper III is the identification of series of continuously connected minima in the flux-induced potential on the complex structure moduli space. Recall that the potential and the metric on this space are determined by a superpotential W and a Kähler potential K_{CS}. Moreover, we have seen in section 3.3.1 that these quantities can be expressed in terms of the periods

$$W = (F - \tau H) \cdot \Pi \quad , \quad K_{CS} = -\ln(i\Pi^{\dagger}Q^{-1}\Pi). \tag{6.3}$$

Now, we should recall from section 3.3.1 that the complex structure moduli space is a fundamental domain of a larger Teichmüller space. As such its topology can be rather complex. In particular, the periods are not single-valued on the complex structure moduli space, but suffer from monodromies around singular points. The monodromy transformations correspond to the transitions between different fundamental domains in Teichmüller space. A nice description of these concepts can be found, for the example of the Mirror Quintic, in the seminal paper [CdlOGP91]. A simplified discussion that illustrates the general idea by studying the torus is given in Paper V.

The astute reader might now worry that the monodromies in section 3.3.1 were derived in the complex structure moduli space of $\mathcal{N} = 2$ compactifications, and might not prevail in $\mathcal{N} = 1$ compactifications. This is a subtle point, which we can handle as follows. Recall from chapter 5 that supersymmetry is broken by orientifold planes, which are necessary to cancel flux-induced tadpoles. Orientifolding projects out some of the periods, thereby reducing the size of the complex structure moduli space. Nonetheless, in the generic case, we expect that the singular points and monodromies are not all projected out from the resulting moduli space. In the following, we will therefore assume that there are monodromies in the orientifolded moduli space. When we study explicit examples, we will tacitly assume that these are the moduli spaces obtained after orientifolding.

As discussed in section 3.3.1 the monodromies transform the N-dimensional period vector as

$$\Pi \to \mathbb{M} \cdot \Pi, \tag{6.4}$$

where the monodromy matrix $\mathbb{M}$ is an integral, $N \times N$ symplectic matrix. The monodromy matrices form a subgroup of the symplectic group Sp($N, \mathbb{Z}$). This guarantees that $\mathbb{M}$ preserves the intersection matrix Q, and leaves the Kähler K_{CS} potential invariant. Hence the geometry of the different fundamental domains are the same.

The superpotential, on the other hand, transforms as

$$W = (F - \tau H) \cdot \mathbb{M} \cdot \Pi. \tag{6.5}$$

Consequently, the superpotential in the new fundamental domain is different from the superpotential in the old fundamental domain. The superpotential is continuous across the borders between fundamental domains. Hence, if the potential has minima in the two domains, there will be a continuous potential barrier between the two. Now we see the difference to the naive inter-minima distance discussed above. The distance between the minima has to be measured on the full Teichmüller space.

Note now that we would obtain the same transformed superpotential if we remained in the fundamental domain, but let the vectors of flux quanta transform instead. Clearly, substituting

$$(F - \tau H) \to (F - \tau H) \cdot \mathbb{M} \tag{6.6}$$

into W, would yield the same transformed superpotential (6.5). Thus we can continuously connect minima with different fluxes on the same fundamental domain. The minima lie on different sheets of a multiple-valued potential, of the type shown in figure 6.1. The distance between the minima is the length of the curved path that takes us from one minimum on the first sheet, around a singular point and across a branch cut to the second minimum on the next sheet.

In Paper III, the analytical properties of these sequences of minima are studied in more detail, and several examples are found by numerical studies of the Mirror Quintic. A generic question that is addressed regards the length of these series, in particular if they are finite or not. This is worth investigating, since it could have implications on the dynamics on the landscape. The question can be reformulated as a requirement on the monodromy group, as discussed in Papers III and IV. In general, infinite series of minima are possible if the mon-

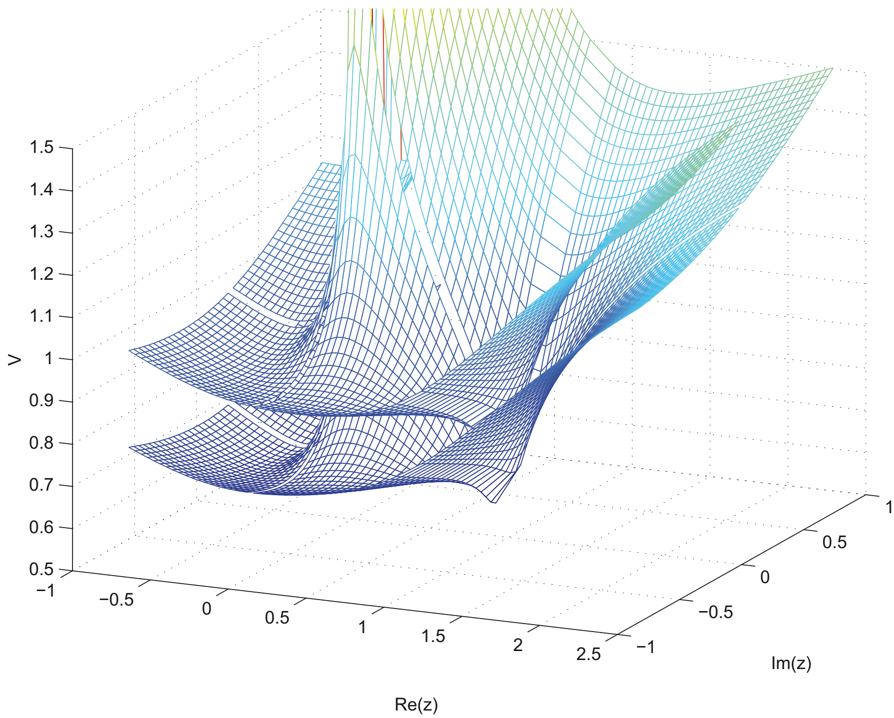


Figure 6.1: A flux-induced potential on the complex structure moduli space of the Mirror Quintic. The figure shows two sheets of the potential that are continuously connected across the real z axis. There is a minimum on each sheet.

odromy group is a subgroup of finite index in the symplectic group.[5] Whether this is the case is an unresolved mathematical problem, also for the simple example of the Mirror Quintic.

In discussions of finiteness, it is important to remember that the minima we consider here need not be vacua in the full string theory. As argued above, we need to check that Kähler moduli can be fixed. This can be problematic. The series will typically contain some minima with non-zero F-terms, which are problematic in the BBCQ fixing of Kähler moduli. Furthermore, since the superpotential changes along the series, it will generically leave the region where KKLT stabilization works. Thus, even if we would find infinite series of minima, they might yield long, but not infinite, series of vacua. Nevertheless, the fact that vacua are connected is already interesting. In addition, these

[5]The index is a measure on how ’big’ the subgroup is in comparison with the full group. More technically, it is the number of left cosets associated with the subgroup in the full group. See Paper III for further discussions.

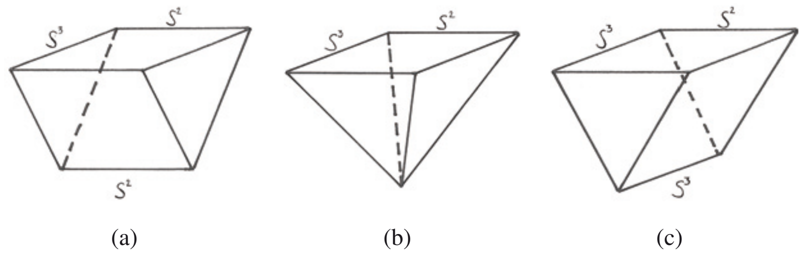


Figure 6.2: The conifold (b) is a cone with base $S^2 \times S^3$. It is a singular variety that can be “desingularized” in two ways: (a) by blowing up a two-cycle (resolving) or (c) by blowing up a three-cycle (deforming).

sequences serve as nice toy models for cosmological discussions, as we return to in the next chapter.

In conclusion, by utilizing monodromy transformations, we can find continuous potential barriers between minima in the flux-induced potential in type IIB compactifications. It is likely that similar results hold for type IIA compactifications, although this has not yet been investigated. Note however, that monodromies cannot connect all minima in the complex structure moduli space. Monodromy matrices are symplectic, and can only connect a restricted set of flux configurations. Naturally, one wonders if there are ways of extending the moduli space to connect more minima. This leads us to consider geometric transitions, and is the topic of the next section and Paper IV.

6.4 Geometric Transitions

The simplest example of geometric transitions connects the deformed and resolved conifolds. We therefore return to the conifold, that was introduced in chapter 4 as the solution to a quartic equation (4.21) in $\mathbb{C}^4$. The conifold is a non-compact singular variety, which has the form of a cone with base $S^2 \times S^3$. We also noted that the conifold is the singular limit of the deformed conifold, which is a non-compact Calabi–Yau manifold. By shrinking a three-cycle in the deformed conifold, i.e. by moving in its complex structure moduli space, we reach the conifold. This is illustrated in figure 6.2.

In addition, there is another way to smoothen the conifold singularity, which leads to a different Calabi–Yau manifold, called the resolved conifold. By shrinking a two-cycle in the resolved conifold, i.e. by moving in its Kähler moduli space, we reach the conifold. Again, figure 6.2 illustrates the two manifolds.

From this we conclude that the moduli spaces of the deformed and the resolved conifolds are connected. Furthermore, it has been shown that the conifold locus, i.e. the point in moduli space where the cycle vanishes, is at finite distance from points describing smooth manifolds [CGH89]. This is non-trivial since the metric on moduli space has a curvature singularity at the conifold locus (see section 3.3.1). The finite distance implies that transitions between the deformed and resolved conifolds can occur (see [CdlO90] for details).

However, before we continue, we should motivate why such a transition could actually make sense. We just noted that there is a field space curvature singularity at the conifold point. Consequently, the low-energy effective field theory is singular (recall that the kinetic term for the moduli is given by this diverging metric). We can no longer describe the transition by classical geometry alone.

The solution of the problem is that string theory perceives the transition differently. Recall from chapter 4 that three-branes wrapped around three-cycles correspond to four-dimensional black holes. The masses of these black holes are given by the periods of the wrapped cycles, and go to zero at the conifold point. This implies that there are new light fields at this point. These must be included in the low-energy effective action. Doing so results in a smooth effective field theory [Str95]. Concretely, by incorrectly integrating out these light states from the smooth action, one obtains a singular theory that exactly matches that of the conifold [Str95].

Conifold transitions, or more general geometric transitions where several cycles shrink, arise also in the moduli space of compact Calabi–Yau manifolds. It is important to note that the topology of the internal manifold changes in a geometric transition. Thus, geometric transitions result in a web of connected, distinct Calabi–Yau manifolds [GMS95].[6]

As an example, it has been shown [CGH90, GMS95] that by shrinking three-cycles in the Quintic $\mathcal{M}_{(1,101)}$, which has Hodge numbers $h^{(1,1)} = 1$ and $h^{(2,1)} = 101$, a singular variety is obtained. This variety can be resolved by blowing up two-cycles, resulting in a manifold $\mathcal{M}_{(2,86)}$, with Hodge numbers $h^{(1,1)} = 2$ and $h^{(2,1)} = 86$. The change in Hodge numbers implies that we have to shrink 16 three-cycles in 15 different homology classes and blow up one two-cycle in the process.

Both the Quintic $\mathcal{M}_{(1,101)}$ and the manifold $\mathcal{M}_{(2,86)}$ have mirror manifolds. These are the Mirror Quintic $\mathcal{M}_{(101,1)}$ and the manifold $\mathcal{M}_{(86,2)}$ respectively.[7] Thus, it is to be expected that there is a geometric transition between the mirrored manifolds as well. Concretely, by shrinking three-cycles in $\mathcal{M}_{(86,2)}$, and then blowing up two-cycles, we expect to reach the moduli space of the Mirror

[6] That all Calabi–Yau threefolds are connected by geometric transitions is knows as Reid's fantasy [Rei87], and has yet to be proven.

[7] Recall from chapter 3 that mirroring a manifold implies that the Hodge numbers are interchanged.

Quintic. Paper IV shows that this is indeed the case. By using toric geometry[8] the periods and monodromies of the $\mathcal{M}_{(86,2)}$ are constructed, and it is explicitly shown how the complex structure moduli space of the Mirror Quintic is embedded in that of $\mathcal{M}_{(86,2)}$.

Let us now return to the landscape, and the effects geometric transitions has on its topography. The web of Calabi–Yau manifolds is very attractive from this perspective, since it unites distinct moduli spaces into one connected landscape. Consequently, it is expected that vacua on different moduli spaces can be connected, and the evolution in the landscape would no longer be limited to a particular Calabi–Yau compactification.

This idea is studied in more detail in Paper IV. In particular, it is investigated if the larger monodromy group on the moduli space of $\mathcal{M}_{(86,2)}$ can be used to construct continuous paths between the minima in type IIB compactifications on the Mirror Quintic. These paths would go through the geometric transition locus, encircle a singular point of the moduli space of $\mathcal{M}_{(86,2)}$, and then return to the Mirror Quintic slice of the moduli space. Although these paths are generically much longer than paths remaining in the Mirror Quintic moduli space, it is still interesting that they exist, as we now explain.

The larger monodromy group allows to connect more minima on the Mirror Quintic. In particular, Paper IV shows that infinite series of minima can be constructed. As noted above, such series may very well be cut off at some finite length, when the Kähler moduli fixing is taken into account. Nonetheless, this length might be long, and hence provide an interesting topographic setting for dynamical studies.

Paper IV also discusses the effect that background fluxes have on the geometric transitions. Concretely, if there is flux through the cycles involved in the transition, we have to worry about charge and flux conservation. For example, if there is three-form flux through the shrinking cycle before the transition, there should be flux present also after the transition. But since the flux-lines are now cut open, there must be sources for the flux. This can be obtained by wrapping space-filling five-branes on the blown-up two-cycles. How this type of transitions should be treated is still an open question. Similar reasoning can be applied to the cases with flux through the torn cycle that intersects the shrinking cycle, or if there is flux through both. We refer to Paper IV for a detailed discussion on the various possibilities, and references to the literature.

Different flux configurations also lead to different scalar potentials. For general flux configurations, the potential diverges at the conifold locus. Consequently, there is a large barrier between minima. In the absence of flux through the shrinking cycle, the potential instead tends to zero on the conifold locus

[8]Toric geometry is a mathematical framework for the construction of Calabi–Yau manifolds. We will not describe it here, but refer to chapter 2 in Paper IV for the necessary concepts. Introductions to the subject can also be found in e.g. [H$^+$, Gre96].

(see Paper III for a derivation). These different scenarios will certainly affect the transition between minima, as is argued in Paper IV.

In summary, we have argued that monodromy transformations yield series of continuously connected minima in the flux-induced potential. In the next chapter, we will use these sequences of minima in studies of landscape dynamics. In particular, the model provides generic inter-minima distances and potential barriers, that can be used for studies of the stability of minima. Furthermore, we investigate the cosmological properties of the model, in particular if any kind of inflation occurs.

7. Landscape Cosmology

We now turn to cosmological aspects of the string theory landscape. In any model with several vacua, we should expect that quantum mechanical tunnelling mediates transitions between vacua. Applied to the string theory landscape, this means that many vacua (in particular all de Sitter vacua) are only metastable when quantum mechanical effects are taken into account. Consequently, there is a slow migration between the vacua of the landscape. If our universe is described by one of these vacua, its creation could be understood as a mere transition from another vacuum. Similarly, we should expect that the universe will eventually tunnel to another phase. Needless to say, it is very interesting to estimate the probability for these processes, i.e. how stable various vacua are.

Just as in the preceding chapter, we view the string theory landscape as an effective field theory (coupled to gravity), with moduli fields trapped in minima of a complicated potential. We can then use well-known methods from quantum field theory to calculate the stability of minima. This analysis is used in Papers V and VI for the de Sitter and Minkowski vacua of the type IIB model landscape introduced in the last chapter.

7.1 Tunnelling Between Vacua

We start this chapter by recalling various mechanisms of tunnelling. As noted above, the moduli-fixing potential of string theory has many meta-stable false vacua. If such a vacuum is accessed, it gives rise to, say, a four-dimensional spacetime of a given geometry. The decay of such a spacetime proceeds by the nucleation of a bubble of a lower energy phase. After the transition, spacetime consists of three regions: a true vacuum region inside the bubble, a bubble wall of some thickness and a surrounding false vacuum region.

It will often be energetically favourable for the new vacuum bubble to grow, thus converting more and more of spacetime into the less energetic phase.[1] It follows that, even though the bubble nucleation is a localized process, the

[1] Since the potential energy is lower in the true vacuum, there is a gain in energy when the false vacuum is replaced. On the other hand, the bubble wall has tension, and its potential energy grows with its size. The energy cost associated with the growing wall depends on the tension and the geometry of spacetime [CDL80].

subsequent growth of the bubble can create very large patches of the new phase, perhaps even substituting the false vacuum completely.

7.1.1 Instantons in the Flux Potential

There are two properties of the string theory vacuum that must be understood in order to compute tunnelling probabilities. The first is the quantum behaviour of the moduli fields. This quantifies how a field can tunnel through a potential barrier. The second important feature is how the gravitational properties of de Sitter spacetime affect the fields. It can be shown that this also results in a migration between vacua. We will discuss the two processes in turn.

For ease of presentation, our description will focus on a theory with only one field. The action is

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) , \tag{7.1}$$

where $V(\phi)$ is a potential with two minima; one false, high-energy minimum and one true minimum of lower energy. An example potential $V(\phi)$ is shown in figure 7.1. We will mainly focus on positive definite potentials, that are similar to the no-scale potential induced by fluxes in type IIB string theory.

The Coleman–DeLuccia Instanton

To calculate the transition probability out of a vacuum in quantum field theory, we should compute the path integral over all paths that take us to the other side of the potential barriers surrounding the vacuum. Clearly, this is a formidable task — even for the simple potential of figure 7.1 there are infinitely many paths in field space that take us from one vacuum to the next. However, as found by Coleman and collaborators [CC77, CDL80, CGM78, Col77], there is a considerable simplification in the semiclassical limit, where it can be shown that one path dominates the transition. This path goes between the basins of attraction of the two vacua and extremizes the Euclidean action.[2] We find the path by solving the Euclidean equations of motion, subject to the appropriate boundary conditions, i.e. that the field starts and ends its Euclidean evolution with zero velocity in the basins of attraction of the two minima.

The solution to the Euclidean equations of motion is a field and metric configuration known as an instanton. The Euclidean action S_I of the instanton determines the probability for the tunnelling process by [Col77]

$$\Gamma = A e^{-(S_I - S_{BG})/\hbar} \left[1 + \mathcal{O}(\hbar) \right] . \tag{7.2}$$

[2]The Euclidean action is the analytic continuation of the standard action to imaginary time. It is not bounded from below, and we should therefore look for its saddle points, not its minima, to find instantons [CGM78].

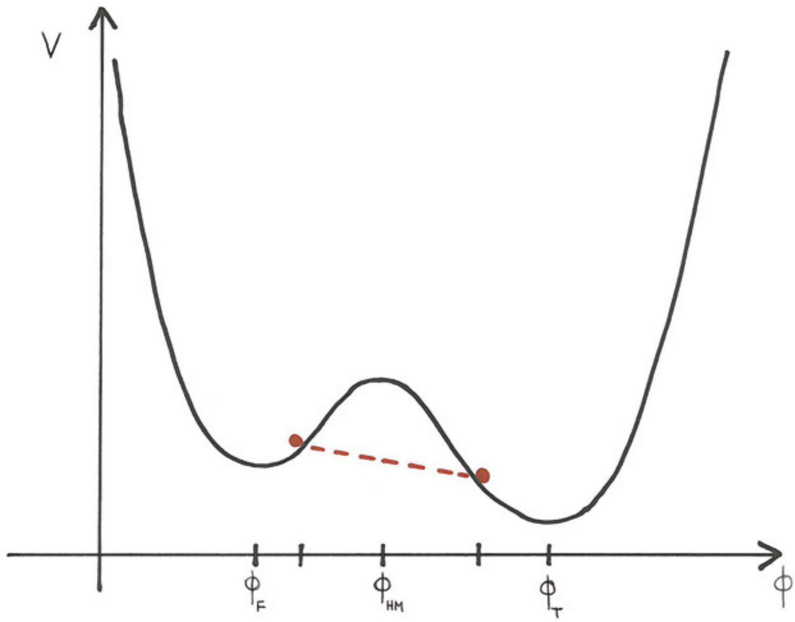


Figure 7.1: This figure shows a potential $V(\phi)$ with one false minimum, at ϕ_F, and one true vacuum, at ϕ_T. There is also a local maximum at ϕ_{HM}, which is relevant for the Hawking–Moss instanton. The two dots represent the endpoints of an Coleman–DeLuccia instanton, that mediates transitions between the minima. For theories with gravity, tunnelling is possible in both directions.

Here S_{BG} is a background term, given by the Euclidean action of the vacuum that we tunnel from. Interestingly, in a theory with gravity, tunnelling is allowed in both directions between the true and false vacua [LW87], although the downwards transition to the true vacuum is more probable.[3] In the following, we will not discuss the prefactor A, which encodes the first quantum corrections to the rate. We will also set $\hbar$ to one and focus on transitions from the false to the true vacuum. We will refer to this solution as the Coleman–DeLuccia instanton

The Euclidean equations of motion are the equations of a scalar field coupled to a dynamical metric. For general metrics, they are prohibitively difficult to solve. On the other hand, it is tempting to assume that bubble spacetimes which preserve a lot of symmetry give the highest transition probabilities, and this has also been proved in flat spacetimes [CGM78]. There is no proof that this is the case when spacetime is curved, but we will assume so in the following. If this assumption is false, the instantons computed here will only put a lower bound on the transition rate.

[3]The relative probability for the processes is determined by the difference in background terms in the true and false vacua $S_{BG}[\phi_{T/F}] \sim 1/V(\phi_{T/F})$ [LW87].

With this assumption, there are limits where the problem can be solved analytically. One example of this is the thin wall limit. Here, the bubble wall, defined as the region in spacetime where the field ϕ changes its value from $\phi_i \approx \phi_F$ to $\phi_f \approx \phi_T$ (cf. figure 7.1), is very thin in comparison with the radius at which the bubble is nucleated.

In the thin wall limit, one can calculate the transition probability exactly. The tension of the bubble wall, σ, is defined by

$$\sigma = \int_{\phi_F}^{\phi_T} d\phi \sqrt{2\left(V_0(\phi) - V_0(\phi_F)\right)}. \tag{7.3}$$

Here V_0 is a function that is identical to V except in the vicinity of ϕ_F, where the potential is deformed such that $V_0(\phi_T) = V_0(\phi_F)$ and $dV_0/d\phi(\phi_{T,F}) = 0$ [CDL80]. Using this, one can show that, in the regime where gravitational effects are small

$$S_I - S_{BG} = \frac{27\pi^2\sigma^4}{2\left(V(\phi_F) - V(\phi_T)\right)^3}. \tag{7.4}$$

The gravitational corrections to the thin wall actions have also been computed [Par83]. The simplicity of the thin wall formula (7.4) renders it very useful for estimating the transition rates between vacua. We will return to discuss its relevance for the landscape, and in particular our type IIB landscape model introduced in chapter 6. Outside the thin wall limit, one can use numerical methods to compute the instantons, as discussed in Paper V.

The Hawking–Moss Instanton

The endpoints of a Coleman–DeLuccia instanton are represented by red dots in figure 7.1. For field theories coupled to gravity, the location of these endpoints can be anywhere along the barrier sides. In fact, the endpoints could even merge at the top of the potential, since this is also a solution to the Euclidean equations of motion. For concreteness we reproduce these equations here, for the O(4) invariant case with Euclidean metric $ds_E^2 = d^2\chi + r^2(\chi)d\Omega_3^2$:

$$\begin{aligned} \dot{r}^2 &= 1 + \epsilon^2 r^2 \left(\frac{1}{2}\dot{\phi}^2 - V\right), \\ \ddot{\phi} + \frac{3\dot{r}}{r}\dot{\phi} &= \frac{\partial V}{\partial \phi}. \end{aligned} \tag{7.5}$$

Here a dot denotes differentiation with respect to the Euclidean time χ, and ϵ is a numerical constant. These equations can clearly be solved when the field sits at one of the extrema of the potential. The solution when the field sits at the maximum is known as the Hawking–Moss instanton [HM82].

When the moduli reach the maximum of the potential (at $\phi = \phi_{HM}$) they can subsequently roll to either of the two vacua, thus mediating a transition between the vacua. The action for the process is given by

$$S_I - S_{BG} = \frac{8\pi^2}{3\epsilon^2}\left(-\frac{1}{V(\phi_{HM})} + \frac{1}{V(\phi_F)}\right). \tag{7.6}$$

One possible interpretation of the Hawking–Moss instanton relies on the stochastic behaviour of a field in de Sitter spacetime [Sta86]. Since de Sitter vacua have a temperature [GH77], there can be a thermal excitation of the moduli, allowing them to random walk up the potential V.

The spacetime realization of this process is that the field in a horizon-sized patch fluctuates to the local maximum of the potential, and subsequently relaxes to the true vacuum. The process divides spacetime into a horizon-sized bubble surrounded by a false vacuum region. There is no sharp bubble wall, describing the transition of the field from the false to the true vacuum value. On the contrary, most of the bubble is 'wall-like' since the field has not settled into one the true vacuum. In this way, one can view the Hawking–Moss instanton as an extremely thick-walled limit of the Coleman–deLuccia instanton.

Depending on the shape of the potential and the strength of the gravitational effects, either the Hawking–Moss or the Coleman–DeLuccia instanton dominates the decay. Loosely speaking, if the potential barrier is low and wide, stochastic effects could easily lift the field to the local maximum, and the Hawking–Moss instanton dominates. On the other hand, if the potential barrier is narrow and high, tunnelling through the barrier will be easier. The transition probability is then given by the Coleman–DeLuccia instanton.[4]

Instantons in a Model Landscape

We now turn to the construction of instantons in the sequences of minima of the type IIB no-scale potential discussed in chapter 6. This analysis is performed in Paper V, to which we refer for details. In this paper, the generic topographic features of the series are quantified. Simple scaling arguments imply that the potential barriers are broad, and therefore that most instantons are thick-walled. This is also confirmed in the example of the Mirror Quintic, where some instantons are constructed numerically.

An important part of the construction of instantons in this toy landscape, and the landscape as a whole, is that the field space is multidimensional. Thus, we should look for saddle points to the Euclidean action describing several scalar fields coupled to gravity. Furthermore, the geometry of the field space is, as we discussed in chapters 3 and 5, rather complicated (e.g. it is not flat). The multidimensionality and non-trivial geometry imply that it is even more difficult to find the instanton path that connects two vacua, e.g. because we

[4] A quantitative formulation of these conditions can be found in [HW05], see also Paper V.

now have to check all possible directions in field space. Several paths that yield finite probabilities for tunnelling are possible, although it is plausible that one instanton will give the dominant contribution.

How can we find the dominant instanton? Checking all possible paths for instantons is clearly impossible, so we need a method for finding the path that is most relevant. In this analysis, there is one very useful analogue for the instanton problem. As first noted in [Col77], the field equation in (7.5) is equivalent to the equation of motion for a particle moving in the *upside-down* potential $-V$, subject to a friction force proportional to $\dot{r}/r$. Similarly, for a multidimensional landscape, we obtain the equations of motion for several coupled particles, moving in the upside-down potential under friction.

In Paper V, this analogue results in a method for constructing (approximate) instantons. It is observed that most directions of the upside-down potential do not allow instanton paths, e.g. because of steep slopes that kick the particle away to infinity instead of allowing it to come to rest near the new vacuum.[5] Taking these restrictions into account, suggests that instanton paths will follow e.g. ridges in the upside-down potential. It also points to the importance of saddle points in the potential, since these can harbor Hawking–Moss instantons. The main result is that there will be a family of effectively one-dimensional instantons (i.e. one field tunnels while the others remain close to their minima) associated with such saddle points. Some numerical examples of such instantons are also computed in Paper V. The instantons are thick-walled, as expected, and the actions for the Coleman–DeLuccia and Hawking–Moss instantons are of the same order of magnitude. However, since the transition probability depends exponentially on the action, it is clear that the Coleman–DeLuccia instanton will be the dominant decay mode. In the large volume limit, the transition rates are exponentially suppressed.

7.1.2 Domain Walls and Branes

In addition to the expanding bubbles described in the last section, we could also imagine spacetimes where two vacuum phases are separated by a static domain wall. For example, supersymmetric vacua are stable against vacuum decay [DT77, Wit81, Hul83], but can coexist if separated by a BPS domain wall (see [CS97] for a review). These BPS walls correspond to Coleman–DeLuccia bubbles of infinite radius [CGR93], and the stability of the vacua can be understood from the fact that it takes infinite energy to nucleate a bubble of infinite radius.

In string theory compactifications with background fluxes, it is natural to interpret the BPS domain walls as BPS configurations of D- or NS-branes [GVW00]. As reviewed in section 2.4 D5(NS5)-branes are charged under RR(NS) three-form fluxes. Type IIB supersymmetric vacua with different flux

[5] This reasoning is inspired by Coleman's undershoot/overshoot analysis of one-dimensional instantons, see [Col77].

configurations can then be separated by five-brane domain walls, constructed from branes that wrap special Lagrangian three-cycles of the internal manifold (see section 2.4). The two non-compact directions of the five-brane yield a static BPS domain wall, whose tension is determined by the change in superpotential between the vacua [GVW00]. This is very similar to the mass of the BPS black holes, discussed in chapter 4, which is determined by the central charge/superpotential of $\mathcal{N} = 2$ supergravity.

The BPS domain walls are interesting for several reasons. Four-dimensional domain wall spacetimes can be constructed by BPS domain walls between Minkowski and anti de Sitter vacua [CDG$^+$06, CGR93, CS97]. Furthermore, near-BPS domain walls can be used to estimate the stability of vacua with a small breaking of supersymmetry [CDG$^+$06]. For example, in the type IIB flux landscape, there are vacua where supersymmetry is only broken by the subleading effects stabilizing Kähler moduli. The stability of these vacua can be estimated by this kind of analysis, as further discussed in Paper V.

In addition, the bubble walls constructed in Paper V separate vacua with different fluxes as well, so it is natural to interpret them as configurations of five-branes with badly broken supersymmetry. An indication that this might be the case is that the scale of the tension for the BPS domain and bubble walls are similar. A problem with this interpretation is, however, that the bubble walls in the model can be very thick, and it is not immediately clear how this should be interpreted in terms of the brane configurations. This is further discussed in Paper V.

Alternatively, one can try to construct the bubble walls directly from five-branes, as proposed in [FMRSW01, BP00, FLW03]. The transition from one vacuum to another is then interpreted as the nucleation of a charged brane bubble. The process is similar to the neutralization of an electromagnetic field by the creation of electron-positron-pairs. Specifically, a non-zero background four-form flux in four dimensions can be shielded by the nucleation of a charged membrane [BT88]. In string theory the charged membrane can be identified with the non-compact part of a wrapped five-brane, as suggested in [FMRSW01, BP00, FLW03]. The transition probability can then be estimated using equation (7.2). This process is natural from a string theory perspective, but it is not completely clear how to embed it in a supergravity picture, as discussed in e.g. [dA06, dA07].

7.2 Inflation

As mentioned in the last chapter, inflation provides a neat explanation for the homogeneity and isotropy of our universe.[6] There are different ways of obtaining inflation, the simplest being to provide the universe with a positive

[6]Inflation also solves the flatness and monopole problems and provides seeds for structure formation. See [Gut04, LR99] for reviews on inflation and inflationary models.

cosmological constant. Any spacetime with a positive cosmological constant inflates, i.e. expands at an exponential rate $\exp(Ht)$. Here the Hubble parameter H is given by the cosmological constant Λ as $H \propto \sqrt{\Lambda}$. Thus, the false vacua discussed above, that have positive $\Lambda = V(\phi_F)$, are inflating.

We mentioned above that the true vacuum bubble that is created as the field tunnels out of the false vacuum, will grow and might eventually replace the false vacuum phase. This conclusion must be revised in light of the expansion of the resulting spacetimes. Recall that the vacuum energy is decreased as the field tunnels down to the true vacuum. Consequently, the Hubble parameter of the true vacuum will be smaller than the false vacuum, and the false vacuum spacetime grows faster than the true vacuum spacetime. It seems that a complete transition can never occur.

What might change this picture is taking several tunnelling events into account. As the false vacuum phase expands, new true vacuum bubbles can form. Since all these bubbles grow, they might eventually cover the whole false-vacuum phase. After a little thought, one can convince oneself that this will happen if the decay rate is larger than the expansion rate, as measured by the false vacuum Hubble parameter.[7] If this condition is satisfied, the bubbles will expand, overlap and percolate, thus replacing the false vacuum completely.

If instead the decay rate is smaller than the expansion rate, a part of the false vacuum phase prevails. It continues to inflate and produce new true vacuum bubbles. In a more complicated potential with many different vacua where gravitational effects also allow 'upward' transitions, tunnelling gradually populates more and more vacua. This process is known as eternal inflation [Lin86b, Lin86a, Vil83], and results in a fractal multiverse composed of bubbles within bubbles, as shown in figure 7.2.[8] Since the process goes on forever, all vacua of the potential are eventually realized and the dependence on initial conditions is washed away.

Eternal inflation is very interesting from the point of view of the landscape. In section 6.2 of the preceding chapter we discussed that the vastness of the string theory landscape allows statistical and anthropic reasoning about the physical properties of our world. There is one important underlying assumption for this reasoning, namely that the various vacua are actually realized. To achieve this, a mechanism that populates the vacua of the landscape is needed. As we saw in the preceding paragraph, eternal inflation provides such a mechanism.

[7]To make this conclusion more precise one should compare the physical volumes of the false and true vacuum phases [TWW92]. This comparison shows that a false vacuum region remains if the decay probability per physical time and volume is small compared to H_F^4.

[8]Note that inflation is only eternal on a global scale. Locally, tunnelling and the subsequent relaxation of the field to the true vacuum will end inflation, and possibly produce the initial conditions for a Big Bang scenario. See [Agu07, Gut07, Win08] for introductions to eternal inflation.

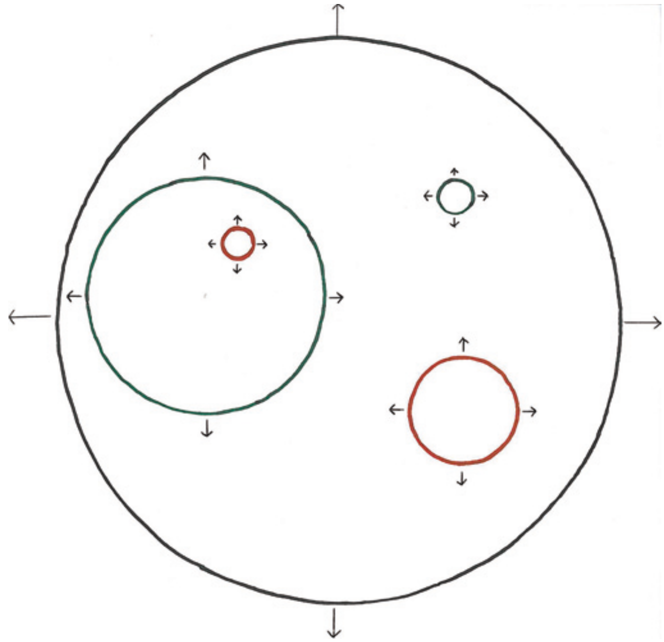


Figure 7.2: The vacua in the string theory landscape could result in a multiverse composed of metastable bubble universes. The bubbles correspond to different vacua, and have different four-dimensional physics. The arrows indicate that the bubbles are expanding.

As showed in Paper V, and mentioned in the previous section, the transition probabilities between the minima in the monodromy sequences are typically exponentially suppressed. In particular, the tunnelling rate is much smaller than the expansion rate of the false vacuum. Therefore, the landscape model leads to eternal inflation, and a multiverse where the various minima are realized in different spatiotemporal regions. The implications and limitations of this conclusion is further discussed in Paper V.

Although generic transitions in the monodromy sequences are slow, there might be regions in this toy landscape where faster transitions are allowed. For example, in Paper V it is argued that series of minima near a conifold locus, can be closer to each other than minima in generic series. It could thus be the case that a transition between minima near the conifold results in percolation and a total transition to the new phase.

This implies, as noted in Paper III and thoroughly investigated in [CD08b], that certain monodromy sequences could provide settings for chain inflation [FLS06, FS05, CD08b]. Chain inflation is a model where the inflationary expansion of the universe is sourced by a scalar field tunnelling through a series consecutive vacua. Since the field gets temporarily stuck in every vacuum, the transition down the potential is slow, giving the universe enough time to inflate. On the other hand, if the downwards tunnelling is not too slow, percolation will occur at every step of the chain. Long chains of vacua make a balancing of these conditions possible, and the result is a homogeneneous universe (given by the state at the bottom of the chain of vacua).

As emphasized in [FLS06] the fast downward transition implies that chain inflation is a dynamical model that selects a particular final state.[9] This is very different from the eternal inflation scenario, where several states coexist at once. However, if a landscape, such as the monodromy landscape discussed in Papers III-V, allows both slow and fast transitions, it is plausible that chain inflation does not completely remove the multiverse. The conclusions depend on where in the landscape the cosmological evolution starts. If it starts at the top of a suitable chain, chain inflation might produce a homogeneous universe. For any other initial condition, eternal inflating phases will be present. Chain inflation can occur locally, whereas the global evolution is determined by eternal inflation, and hence results in a multiverse.

Before we leave this discussion of inflation in landscape models,[10] which might have seemed rather academic to the reader, we note that there could actually be observational ways of separating between eternal and chain inflation. If our universe is described by a vacuum in the string landscape, both eternal and chain inflation could give rise to observable cosmological signatures. As an example, it is plausible that the bubbles in an eternally inflating multiverse collide. If our universe has undergone such a collision, there might be imprints of it left in the cosmic background radiation [AJS07, AJ07].[11] Similarly, if the universe is the result of a chain inflation cascade, there will be extra sources for cosmological perturbations (in addition to the quantum fluctuations that are always present) [CD08b]. This might yield other detectable signals in the cosmic background radiation [CD08a].

7.2.1 An Obstacle for Tunnelling

So far we have shown that the monodromy model for the landscape harbors instantons mediating transitions between vacua, allows statistical reasoning based on eternal inflation, and possibly also chain inflation. It is interesting to see if these features prevail in the sector fixed by fluxes in the type IIB landscape. To investigate if this is the case, we need to consider both open string moduli, associated with D-branes, and Kähler moduli. In the real string theory landscape, all these moduli are present, and we need to account for

[9]This has been debated, since fast downward tunnelling does not necessarily exclude upwards transitions or other less probable transitions out of the chain. Such transitions could lead to eternally inflating scenarios.
[10]We have only mentioned some of the many aspects of inflation in the landscape. See e.g. [MS08, Dan05] for more complete treatments.
[11]This is a topic under current research, see e.g. [AJT08, FKNS09].

their dynamics under a tunnelling that changes the flux configuration of the vacuum.

In this section we will discuss the Kähler moduli. As noted above, the classical potential induced by background fluxes (see equation (5.20) on page 84) does not fix the Kähler moduli. Note however, that the potential scales with the compactification volume $\mathcal{V}$. Since the volume is determined by the Kähler moduli, it is clear that these moduli will be affected by tunnelling events when the flux, and thereby the potential, changes. Therefore, to find instantons in the type IIB landscape, we must consider the transition of a coupled system of complex and Kähler moduli in the quantum corrected potential[12] defined on the total moduli space.[13]

This problem is investigated in Paper VI, which focuses on the transition between different flux configurations. These vacua are connected by a barrier in complex moduli space, possibly generated by a monodromy.[14] Now, as the complex structure moduli tunnel out of the false vacuum, the potential for the Kähler volume modulus changes, and it starts rolling. Importantly, since this is an instanton process, the volume modulus moves according to its Euclidean equations of motion, and rolls in the upside-down potential. As shown in Paper VI, the upside-down potential is unbounded from below, leading to an ever-increasing kinetic energy of the volume modulus. Thus, not only will the instanton destabilize the volume modulus, thereby not connecting the two vacua, it will also result in a curvature singularity.[15]

In fact, this is very similar to the dilatonic domain wall spacetimes studied in supergravity theories by Cvetic and collaborators [CS95, CY95]. These are theories where one field is dilatonic, i.e. it is light and couples to the other fields of the theory, thereby modulating the strength by which these other fields are fixed. As shown by these authors, domain wall spacetimes with dilatonic fields will be singular. This holds for the domain walls associated with tunnelling bubbles, but also for spacetimes with static domain walls. Thus the spacetime configuration that a tunnelling bubble would relax to, i.e. two regular vacuum regions separated by a domain wall, does not exist. What might be possible is a configuration where there is a singularity in the region on one side of the wall.

Paper VI also discusses possible tunings of the potential that could result in regular instantons and domain walls, and concludes that there could be regions

[12] The corrections to the classical potential were discussed in section 6.1.

[13] Recall that the monodromy model already is multidimensional, and the inclusion of Kähler moduli just adds more directions to the field space of this model. Thus, nothing conceptual changes when the Kähler moduli are added. Instantons are still found by solving the Euclidean equations of motion, as described above and in Paper V.

[14] We could also consider domain walls associated to the nucleation of a D-brane. What is important for the conclusions is the tension of the spacetime wall between the two phases, which is roughly the same in both pictures.

[15] Whether these singular instantons have infinite actions, and thereby remove one decay channel out of the false vacuum is still an open question. See Paper VI for a further discussion.

in the type IIB landscape where tunnelling proceeds according to the above mechanisms. Although such regions would be exceptional, they would be very interesting to study. Another interesting possibility is to relax the symmetries of the four-dimensional spacetimes. The construction of instantons and spacetimes that preserve less symmetry is very difficult, but might be necessary to understand the mechanism that populates the string landscape.

To conclude we note that the disparate scales fixing the complex structure moduli and the Kähler moduli pose a severe problem for the population of the type IIB landscape. Recall that the vastness of the landscape is a consequence of the many different flux configurations. Moreover, important features of the vacua, such as their cosmological constant, are primarily determined by the fluxes. Thus, to move between most vacua, flux-changing transitions are necessary.[16] Although the required processes can be found and understood in toy models, they are prohibitively difficult to embed in the full landscape. Whether this is an artifact of the approximate methods we use remains to be shown. Indeed, the problems found here might give clues to how a more stringy description of the landscape could be constructed.

[16] Note that, even though the low-energy transitions associated with Kähler or open string moduli are under better control, they could never change the flux-fixed properties of a vacuum.

8. Epilogue

In this thesis, we have studied some aspects of the rich field of research that is string theory. Our attention has been focused on different types of four-dimensional effective field theories, that arise as compactified, low-energy approximations to the theory. These effective field theories, or supergravities, describe universes that are similar, but often simpler, than our own.

We have seen that by adding branes to the supergravity solution, four-dimensional black holes can be described. Some properties of these black holes can be computed using topological string theories and matrix models, and we have studied this correspondence in some detail. Furthermore, we have studied how stable and metastable theories are obtained by compactified supergravities with background fluxes. This results in the string theory landscape, which is to a large extent uncharted territory. By investigating the effect of period monodromies we started to draw a map of a corner of this landscape. This map showed that there are long sequences of minima, connected by continuous potential barriers. These series can also include minima in the moduli spaces of different compactification manifolds, indicating that the cosmological evolution of a universe could include topological transitions of the extra dimensions. The monodromies also helped us investigate the stability of the vacua in this part of the landscape through the construction of semi-classical instantons. These instantons compute the probability for transitions between various vacua in the landscape. We further connected this discussion of transitions to eternal inflation and the multiverse.

During the course of these investigations, we have learned more about string theory, but new questions have also emerged. String theory is still far from being complete, and many new discoveries and surprises lie ahead, before we know if it provides a useful description of our world. In the forthcoming years, input from cosmological observations and particle physics experiments will hopefully provide tests of the theory, and certainly serve as inspiration for further theoretical investigations.

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Summary in Swedish
Aspekter på kompaktifieringar av strängteori

Vad är egentligen poängen med strängteori? Jo, som med all grundforskning i fysik så är motivet för forskning inom strängteori att vi vill hitta en bättre beskrivning av världen. Denna beskrivning, eller modell, ska kunna förklara våra observationer, och helst vara baserad på så enkla principer som möjligt, så att vi kan förstå den.

Vi har idag väl testade teorier som beskriver nästan allt i världen. Med hjälp av stora partikelacceleratorer och detektorer tränger vi djupare in i mikrokosmos, och undersöker de partiklar som bygger upp materien, och de krafter som påverkar dessa. Våra observationer beskrivs mycket väl av kvantmekaniska teorier[1] vilka sammanfattas i Standardmodellen[2] för partikelfysik.

Vidare låter avancerade teleskop oss se längre och längre ut i världsrymden, och därmed kartlägga de stjärnor, galaxer och svarta hål[3] som finns därute. Dessa observationer kan vi förklara med hjälp av Einsteins allmänna relativitetsteori, som beskriver gravitionen genom att göra tiden och rummet till dynamiska objekt, som kröks av sitt materieinnehåll. Med den allmänna relativitetsteorin kan vi till och med förstå hur vårt universum har utvecklats över årmiljarderna sen Big Bang.

Vi har alltså jättebra teorier som beskriver såväl de små, lätta och snabba partiklarna i mikrokosmos, som de stora, tunga objekten i makrokosmos. Behöver vi något mer? Ja, det visar sig faktiskt att vårt universum också innehåller objekt som är både små och tunga, även om vi inte kan återskapa dessa objekt i experiment som vi kan göra här på jorden. I de svarta hål som finns idag och vid tiden för Big Bang trängs en enorm mängd materia i en mycket liten rymd. För att beskriva detta krävs en teori som tar hänsyn både till rumtidens dynamik och kvantmekaniska effekter.

Det är här strängarna kommer in i bilden.

---

[1] Dessa teorier kallas kvantfältteorier och beskriver materia och krafter med hjälp av fält. Fälten följer kvantmekanikens lagar och excitationer i fälten ser ut som punktformiga partiklar. En viktig egenskap hos teorierna är att krafter beskrivs som ett utbyte av partiklar.

[2] Standardmodellen beskriver hur kvarkar, elektroner och andra partiklar växelverkar via tre fundamentala krafter: den elektromagnetiska, den svaga och den starka kärnkraften.

[3] Svarta hål är regioner med enormt stark gravitation som bildas då riktigt tunga stjärnor dör.

Figur: Grundidén i strängteori är att de till synes punktformiga partiklar som bygger upp vår värld egentligen är endimensionella strängar. Strängarna är dock så små att vi inte kan se deras utsträckning.

Strängteori och dess kompaktifieringar

Idén bakom strängteorin är väldigt enkel. I de kvantmekaniska teorierna ser vi partiklar som punktformiga, d.v.s. utan någon som helst utsträckning. Vad händer om de här partiklarna egentligen är små strängar, som visas i figuren ovan? Jo, till skillnad från den punktformiga partikeln kan en endimensionell sträng svänga och vibrera. Om nu strängen är pytteliten, så kommer vi inte kunna se att strängen har en längd och att den vibrerar. I stället kommer vi uppfatta strängen som en punktformig partikel med vissa egenskaper. Dessa egenskaper bestäms av hur strängen svänger, precis som olika svängningar hos en gitarrsträng ger upphov till olika toner. En typ av sträng som svänger på en massa olika sätt kan på så sätt ge upphov till alla partiklar vi hittills har hittat (och många, många fler som vi inte har hittat ännu).

Förbluffande nog, så fungerar det här enkla idén väldigt bra. Bland alla partiklar som strängen ger upphov till finns en som har precis de egenskaper som krävs för att beskriva gravitationen på ett kvantmekaniskt sätt. Det visar sig också att om vi studerar hur strängteorin beter sig vid låga energier, så reproducerar den Einsteins relativitetsteori. Från en kvantmekanisk beskrivning av en sträng får vi de lagar som beskriver dynamiken hos hela universum!

För att också reproducera partikelfysikens Standardmodell från strängteori, så måste man studera strängarna lite närmare. Detta leder till en hel del överaskningar. För det första så visar det sig att strängteori inte bara innehåller strängar, utan även objekt som kallas bran.[4] Vidare så måste teorin också vara supersymmetrisk för att kunna beskriva materiepartiklar. Supersymmetri är inte något som vi har observerat i världen, men det är möjligt att en sån symmetri finns vid riktigt höga energier. Det finns fem supersymmetriska strängteorier som är matematiskt välfungerande. De fem teorierna är dock inte oberoende av varandra, utan verkar i viss mån vara olika beskrivningar av samma sak.[5]

[4]Bran är objekt av olika dimensioner som är laddade under fält som finns i strängteori (precis som en elektron är laddad under det elektromagnetiska fältet). Namnet bran kommer från ordet membran, som betecknar ett tvådimensionellt objekt.

[5]Lite mer precist, så är de fem strängteorierna länkade till varandra via olika dualiteter. Detta medför att man tror att strängteorierna bara är approximationer till en mer fundamental teori, som man kallar M-teori. Vad denna fundamentala teori är för något är lika okänt som orsaken till att den kallas just M-teori.

Den mest intressanta aspekten är dock att strängteorin faktiskt ger en förutsägelse för hur många dimensioner världen har. Detta är en stor skillnad mot exempelvis relativitetsteorin, som fungerar lika bra i vilken dimension som helst. Tyvärr är det här inte bara intressant, utan också problematiskt. För att strängteorin ska vara matematiskt välfungerande, måste rumtiden nämligen vara tiodimensionell.[6]

Detta är ju en anledning så god som någon att kasta bort teorin, eftersom alla våra observationer tyder på att vårt universum har tre rumsdimensioner (upp/ned, bak/fram, höger/vänster) och en tidsdimension. Fler dimensioner kan vi knappt tänka oss. Men under fysikens utveckling har vi fått lära oss att acceptera ett antal otänkbara fenomen, och vi vill därför inte dra några förhastade slutsatser. En tiodimensionell teori skulle ju kunna beskriva vår värld, om vi bara kunde förklara hur de extra dimensionerna blir osynliga för oss. Vi behöver alltså ett sätt att gömma dimensioner, och ett sånt sätt är att helt enkelt rulla ihop dem.[7]

Denna process kallas kompaktifiering och kan förstås på följande sätt. Pappret som den här texten är skriven på har två dimensioner, upp/ned och höger/vänster. En myra som kryper på pappret har alltså två dimensioner att röra sig i. Vi kan nu rulla ihop pappret, så att det bildar en cylinder. Den hoprullade dimensionen blir då som en cirkel och om myran springer i denna riktning så kommer den så småningom tillbaka till den punkt där den började. Detta är vad som menas med en kompakt dimension. Rullar vi ihop pappret riktigt hårt blir den kompakta dimensionen mycket liten. Till slut blir den så liten att myran efter ett enda myrsteg i den kompakta riktningen direkt kommer tillbaka till startpunkten. Det är som om pappret vore en endimensionell linje som myran balanserar på.

På samma sätt kan man tänka sig att en tiodimensionell teori, såsom strängteori, kan användas för att beskriva vår värld. Vi rullar ihop sex av de tio dimensionerna riktigt hårt. Om vi antar att de extra dimensionerna är mycket mindre än ett myrsteg så kommer de inte märkas av så värst mycket i mikrokosmos heller, åtminstone inte så länge vi tittar på lågenergetiska processer.[8]

Vi ser alltså att en kompaktifierad tiodimensionell strängteori kan beskriva ett fyrdimensionellt universum som vårt eget. Men den kan beskriva så mycket mer än det! Sex dimensioner kan kompaktifieras på en mängd olika sätt, som

[6]M-teorin, som binder samman de olika strängteorierna, kräver dock en dimension till, så att det blir elva dimensioner totalt. Detta är också den högsta dimension där supersymmetri och relativitetsteori kan kombineras.

[7]Ett annat populärt alternativ är att tänka sig att vår värld beskrivs av ett tredimensionellt bran i en högredimensionell rymd. Om vi sitter fast på branet kan de extra dimensionerna vara riktigt stora utan att vi märker dem.

[8]Det som blir skillnaden är att en teori med extra dimensioner ger upphov till nya, tunga partiklar, som skulle kunna detekteras vid höga energier. Varje fundamental partikel i teorin kan nämligen snurra runt den kompakta dimensionen. Rörelseenrgin hos den snurrande partikeln tolkas som en massa i den lägredimensionella teorin.

svarar mot rum av olika former och storlekar. I de här rummen kan det också finnas bran och så kallade flöden.[9] Varje sådan uppsättning av bran och flöden har en viss potentiell energi, och de extra dimensionerna kommer vrida och vända på sig så att denna energi blir så liten som möjligt, precis som en kula på ett lutande plan kommer rulla nedåt för att minimera sin potentiella energi.

Vi kan tänka oss den här potentiella energin som ett landskap, med många berg och dalar. Varje dal i landskapet svarar mot en ny fyrdimensionell modell, som beskriver ett nytt universum med nya fysikaliska lagar. Även egenskaperna hos själva den fyrdimensionella rumtiden kan skilja mellan de kompaktifierade teorierna i de olika dalarna. Till skillnad från de landskap vi är vana vid, har strängteorins landskap många fler dimensioner[10] men fysiken fungerar på samma sätt: liksom kulan på det lutande planet, kommer en kompaktifierad teori rulla ner mot dalarna i landskapet. Vilken av alla dessa dalar som svarar mot vårt universum är än så länge en öppen fråga. Den stora mängden dalar gör oss dock hoppfulla om att minst en av dem kommer att beskriva vår värld.

Det visar sig också att även de teorier i landskapet som inte beskriver vårt universum är av intresse. Från kvantmekaniken har vi nämligen lärt oss att potentialbarriärer inte är omöjliga att överbrygga eller tunnla igenom. Det finns alltså en liten chans att ett universum som finns i en dal strängteorilandskapet, tunnlar igenom ett bergsmassiv och rullar ut i en annan dal, där den fyrdimensionella fysiken ser annorlunda ut. För en invånare i detta universum skulle detta vara ganska obehagligt — helt plötsligt skulle till exempel alla atomer kunna bli instabila, därför att den elektromagnetiska kraftens styrka är annorlunda i den nya dalen. Det är därför intressant att undersöka hur en sån här tunnlingsprocess skulle se ut, och hur ofta den sker.

För det första kan man konstatera att ett helt universum inte skulle tunnla samtidigt. Istället skulle en liten bubbla bildas någonstans. Den här bubblan skulle utvidga sig, men den behöver inte helt ersätta det gamla universumet.[11] Inuti bubblan finns dock ett helt nytt universum, som kan expandera och så småningom kanske fyllas av stjärnor och galaxer. Själva tunnlingen skulle i det nya universumet faktiskt kunna vara ganska likt en Big Bang. Genom att noggrannt studera det landskap som strängteorins kompaktifieringar ger upphov till, kan vi alltså hoppas lära oss mer om vårt universums födelse, och kanske till och med vad som föregick vår värld.

[9] Vi använder ordet flöde för att beteckna högredimensionella generaliseringar av elektromagnetiska fält, det vill säga sådana fält som skapas kring en magnet eller en antenn.

[10] Antalet dimensioner hos landskapet ges av antalet parametrar som behövs för att beskriva formen på de extra dimensionerna. I en två-dimensionell analogi kan vi beskriva formen av ett klot med en parameter — klotets diameter. Formen av en torus, till exempel en cykelslang, beskrivs av två parametrar — slangens diameter och tjocklek.

[11] Huruvida detta gör att invånarna i det gamla universumet kan pusta ut, avgörs av ifall de bor tillräckligt långt från den region där bubblan bildas.

Här är vi nu. Bygget av strängteorin är långt ifrån färdigt, och en del skulle nog hävda att teorin alltmer börjar likna ett kråkslott, om än stående på en stabil och enkel grund. För att undersöka vad som faktiskt är relevant i teorin, och vad vi behöver för att beskriva vår värld, räcker det inte med ord. Vi måste använda den kraftfulla matematiska formalism som är teorins stomme. Med hjälp av denna kan vi undersöka olika aspekter av teorin, och om de verkar rimliga.

Denna avhandling handlar just om olika aspekter av strängteorins kompaktifieringar, och de fyrdimensionella teorier vi får därifrån. Mer specifikt studeras kompaktifieringar av den så kallade typ IIB-strängteorin, eller snarare av denna teoris approximation vid låga energier, som kallas typ IIB-supergravitation. Vi undersöker hur kompaktifierade bran ger upphov till fyrdimensionella svarta hål i artikel I, och hur vissa egenskaper hos dessa svarta hål kan beräknas. I artikel II kontrollerar vi om sådana svarta hål gör kompaktifieringar med flöden instabila, och på så sätt skulle kunna skynda på skapandet av nya universa. Artiklarna III och IV kartlägger en specifik del av strängteorilandskapet och upptäcker långa serier av sammanlänkade dalar. Detta används sen i artikel V och VI för att beräkna sannolikheten för tunnling i denna region, och huruvida ett multiversum kan skapas.

Gemensamt för alla dessa artiklar är att de ger upphov till minst lika många frågor som svar. Strängteorin, och dess landskap, är fortfarande fullt av outforskade områden. För varje ny expedition in i den outforskade terrängen får vi lite mer information. Allt detta ingår sedan i den beskrivning strängteori ger av världen.

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