Modelling and Forecasting Volatility of Gold Price with Other Precious Metals Prices by Univariate GARCH Models

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Yuchen Du, June, 2012

Abstract

This paper aims to model and forecast the volatility of gold price with the help of other precious metals. The data applied for application part in the article involves three financial time series which are gold, silver and platinum daily spot prices. The volatility is modeled by univariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models including GARCH and EGARCH with different distributions such as normal distribution and student-t distribution. At the same time, comparisons of estimation and forecasting the volatility between GARCH family models have been done.

Keywords: gold price, volatility, precious metal, GARCH, EGARCH, volatility forecasting
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1. Introduction

1.1 Background

During the end of the World War II, in 1944, forty-four countries around the world signed the well-known Bretton Woods Agreement to establish a new currency system based on gold. They specified the value of the US Dollar is connected to gold and other currencies are connected to the US Dollar. This system indicated where gold standing, not only just as an investment good. This metal has been the only commodity that served as currency in international financial business for centuries. Gold is so ideal that a number of currencies were based on the value of it. For residents, it is an excellent investment good since it could protect them from assets devaluation. For a country, it is an important reserve asset; the situation is somewhat the same as the other precious metals. The best-known precious metals are gold and silver. While both have industrial uses, they are better known for their usage in art, jewellery as well. Other precious metals include the platinum group metals: palladium, osmium, iridium, and platinum which is the most widely traded one in this group. Historically, precious metals have commanded much higher prices than common industrial metals, like copper and iron. ①

During the past few years, especially after the chaos of the US dollar system caused by the financial crisis, investors who have been able to master the reasonable investment of the precious metals markets have gotten remarkable returns. These metals could play an essential role in a portfolio because of their ability to act as a store of value and their stability against inflation as well. Therefore, the modeling and forecasting of the volatility of gold price is indispensable.

With the common sense that higher profit comes with higher risk, modeling volatility in asset returns is concerned by financial investors. Basically, volatility is widely-used for measuring assets risk; investors want guarantees for investing in risky assets for their investment decision. The demand of assisting investors to handle the volatility of commodities prices need to be supplied by some creative and relative financial instruments.

1.2 Brief Introduction to the methodology

Although it is an urgent need of modelling volatility, there were still some limitations and difficulties. As common financial time series, the usual commodity spot price data always have properties like volatility clustering, fat-tailness, excess kurtosis and skewness. The widely-used classical linear regression model usually considers the residuals have homoscedasticity which means random disturbances have equal variance. That is the exact difficulty using traditional linear tools.

"Models of Autoregressive Conditional Heteroskedasticity (ARCH) form the most popular way of parameterizing this dependence", Teräsvirta (2006), p.2.

The autoregressive conditional heteroskedasticity (ARCH) which is the first model of conditional heteroskedasticity was proposed by Engle (1982). Then the generalized autoregressive conditional heteroskedasticity (GARCH) model was proposed by Bollerslev and Taylor (1986) separately. Since it was brought into use, GARCH model shows better capability of financial time series analysis in some empirical studies. Thereafter, many GARCH family models came up, such as exponential GARCH which was brought up by Nelson (1991), Asymmetric Power ARCH which was carried out by Ding, Z., Engle, R.F. and Granger, C.W.J., (1993). Among the worldwide introduced GARCH family models, Teräsvirta (2006) thought that the overwhelmingly most popular GARCH model in applications has been the GARCH (1,1) model.
2. Methodology


2.1 Standard GARCH Model

Let \( \{r_t\} \) denote a financial time series, presenting returns in this paper.

\[
    r_t = E(r_t | \Omega_{t-1}) + \varepsilon_t
\]

Here, \( \Omega_{t-1} \) is an information set contained all the information through time period \( t-1 \), following Bollerslev (1986). We can see here the conditional mean and variance both change with \( \Omega_{t-1} \).

The function of the conditional mean \( \mu_t(\theta) \),

\[
    E(r_t | \Omega_{t-1}) = \mu_t(\theta) = \phi_0 + \phi_1 r_{t-1} + \cdots + \phi_p r_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}
\]

which can also be considered as an AutoRegressive MovingAverage (ARMA)(p,q) process.

Then an AutoRegressive Conditional Heteroskedasticity - ARCH model proposed by Engle (1982) can be regarded as the variance function,

\[
    \text{Var}(r_t | \Omega_{t-1}) = E(\varepsilon_t^2 | \Omega_{t-1}) = h_t(\theta)
\]

Here, there is an assumption that the conditional distribution of \( \varepsilon_t \) is supposed
to be normal distribution with zero mean and variance equals to $h_t$.

The ARCH(q) process function form is like this:

$$e_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i e_{t-i}^2$$

The Generalized ARCH (GARCH) process introduced by Bollerslev (1986) is given below with the same assumption.

$$e_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i e_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$$

where $p \geq 0, q > 0, \alpha_0 > 0, \alpha_i \geq 0, i = 1, ..., q, \beta_i \geq 0, j = 1, ..., p$

We can call $\sum_{j=1}^{p} \beta_j h_{t-j}$ part as GARCH terms. For $p=0$ which means there is no GARCH terms, the process becomes an ARCH(q) process; for $p=q=0$, $e_t$ is simply white noise.

As Teräsvirta said, usually the simplest GARCH(1,1) model works very well:

$$e_t \mid \Omega_{t-1} \sim N(0, h_t)$$

$$h_t = \omega + \alpha e_{t-1}^2 + \beta h_{t-1}$$

with $\omega > 0, \alpha > 0, \beta > 0$.

2.2. Exponential GARCH (EGARCH) Model

The Exponential GARCH(EGARCH) Model was introduced by Nelson (1991) in order to model asymmetric variance effects. It has some special properties which
makes it especially efficient in an asset pricing context. The basic frame for the mean and variance is similar to standard GARCH model, the conditional variance function part is very unique.

Given the EGARCH(p,q) process, with similarity of the conception in standard GARCH,

\[ r_t = E(r_t | \Omega_{t-1}) + \varepsilon_t \]

\[ \varepsilon_t | \Omega_{t-1} \sim N(0, h_t) \]

\[ \ln h_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j g_j(Z_{t-j}) + \sum_{j=1}^{q} \beta_j \ln h_{t-j} \]

where \( g(Z_{t-j}) = \theta(Z_{t-j} + \lambda(|Z_{t-j}| - E(|Z_{t-j}|)), j = 1, ..., q \) and \( Z_t \) could be a standard normal variable. The part relative to \( Z_t \) allows the sign and the magnitude effects affect the volatility separately which is the very property making EGARCH may be more suitable for price return data.

By a more specific way, an EGARCH (p,q) process can also be presented like this,

\[ \varepsilon_t | \Omega_{t-1} \sim N(0, h_t) \]

\[ \ln(h_t) = \omega + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) + \sum_{i=1}^{q} \alpha \left[ \varepsilon_{t-i} | \sqrt{h_{t-i}} - E(|\varepsilon_{t-i}| \sqrt{h_{t-i}}) \right] + \gamma(\varepsilon_{t-i} | \sqrt{h_{t-i}}) \]

2.3 Univariate GARCH Models with extra error terms

In this paper, the aim is to model and forecast volatility of gold price with the help of other precious metals, it is proper to treat the silver and platinum error terms as regressors. The unique process is similar to generalized linear model in this paper. Since we consider these three kind of precious metals, price data are correlated with
each other, we could add the terms we choose of the other two metals to the original GARCH (1,1) Model based on its popularity and applicability. The new univariate GARCH Model with extra error terms behaves like this,

\[ r_t = E(r_t | \Omega_{t-1}^t) + \varepsilon_t \]

\[ \varepsilon_t | \Omega_{t-1}^t \sim N(0, h_t) \]

\[ h_{gold,t} = \omega + \alpha_1 \varepsilon_{gold,t-1}^2 + \beta h_{gold,t-1} + \alpha_2 \varepsilon_{silver,t-1}^2 + \alpha_3 \varepsilon_{platinum,t-1}^2 \]

Here, \( \Omega_{t-1}^t \) stands for a new information set which contains not only gold information during time period t-1, but also both silver and platinum information during that time.

The first equation for the returns and distribution of the residuals are proper to all three series. \( \alpha_1 \) and \( \alpha_2 \) here stand for the special parameters of extra terms.

For a certain pattern of the univariate standard GARCH model case above, the first-order model is the most popular EGARCH model in practice as well. Then it would closely resemble the the situation of standard GARCH(1,1) model, the error terms of silver and platinum return prices are added up to original EGARCH(1,1) model respectively.

2.4 Distributions of Error Terms

2.4.1 Normal Distribution

Normal distribution is one of the most important and common distribution in statistical analysis which is also known as Gaussian distribution.
The probability density function of normal distribution is presented as,

\[ f(\varepsilon) = \frac{1}{\sqrt{2\pi h_i}} e^{-\frac{(\varepsilon - \mu)^2}{2h_i}} \]

where \( \mu \) stands for the mean and \( h_i \) stands for the variance.

Normal distribution is a continuous and symmetric distribution with a bell-shape curve. When \( \mu = 1, \sqrt{h_i} = 0 \), the distribution is considered as standard normal distribution, interpreted as N(0,1).

2.4.2 Student-t Distribution

Student-t distribution, also known as t distribution is proposed by Gosset (1908). The little difference to normal distribution is found by him based on a large number of empirical applications. t distribution is also a symmetric, continuous and bell-shaped distribution, all like normal distribution only with a fatter tail which is perfectly corresponded to the financial time series. Estimation GARCH models with student-t distribution were first proposed by Bollerslev (1987).

Let the conditional distribution of \( r_t \), \( t = 1, \ldots, T \), be t distributed with variance \( h_{\nu-1} \) and degrees of freedom \( \nu \).

\[ r_t = E(r_t | \Omega_{t-1}) + \varepsilon_t \]

\[ \varepsilon_t | \Omega_{t-1} \sim f_{\nu}(\varepsilon_t | \Omega_{t-1}) \]

\( f_{\nu}(\varepsilon_t | \Omega_{t-1}) \) denotes the conditional density function for \( \varepsilon_t \).
\[
f_r(e_t | \Omega'_{t-1}) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\sqrt{(\nu-2)\eta_t}} \frac{1}{(1+\left(\frac{e_t^2}{\eta_t(\nu-2)}\right))^{(\nu+1)/2}}
\]

Where \( \nu \) is considered as a number of degree of freedom, \( \Gamma(z) \) is gamma function.

When \( \nu = 1 \), t distribution turns into standard Cauchy distribution, having no mean.

When \( \nu \) is large, t distribution is could approximate standard normal distribution. 

### 2.5 Root Mean Square Error (RMSE)

In this paper, we prefer a criterion called RMSE for goodness of forecasting.

The Root Mean Square Error is also known as Root Mean Square Deviation (RMSD). It is used to measure the differences between the forecast values calculated from a model or an estimator and the values actually observed generally.

Here is the function form of RMSE,

\[
RMSE = \sqrt{\frac{\sum_{t=1}^{N} E_t^2}{N}}
\]

where \( E_t = Y_t - F_t \), \( Y_t \) is the actual value at period \( t \), and \( F_t \) is the forecasting value for period \( t \). \( N \) here is the number of out-of-sample observations.

Finally, the square root of the average is taken. The criterion of RMSE requires that a smaller value which reflects a better forecasting the model completed. But for more accuracy, we cannot definitely judge which model is better with only comparing the quantities of some criterions. Since there hardly is such significance concluded from

\( ^\circ \) More details about t distribution could be seen in *Statistical Inference* text book 2\textsuperscript{nd} Edition (Casella, Berger, 2002)
only criterion comparison.

3. Data

The price of gold is determined through trading in the gold and its derivatives markets, however a procedure known as the Gold Fixing in London which sprang from September 1919, provides a daily benchmark price to the industry. The afternoon fixing was introduced in 1968 to provide a price when US markets are open which is caused by time differences. The other precious metals use the similar pricing method. The data in this paper is downloaded from Kitco via wikiposit.org. All of them are the afternoon fixing prices.

3.1 Returns

Let \{ y_t \} be the financial time series of the daily price of some financial assets. The return on the \( t \)th day is defined as

\[
    r_t = \log(y_t) - \log(y_{t-1})
\]

where \( r_t \) is defined as return of the \( t \)th variable during time period \( t \).

Sometimes the returns are then multiplied by 100 so that they can be treated as percentage changes in the price making some sense in economics. Furthermore, this procedure may decrease numerical errors as the raw returns could be very small numbers and lead to large rounding errors in some calculations, following Cryer and Chan (2008).


\[\text{⑤ The data is downloaded from <http://wikiposit.org/uid?KITCO>}\]
3.2 Plots

Figure 1: Plot of original daily data

Figure 1 shows the plot of the original database used in this paper. It is a plot of daily spot prices including 3 kinds of precious metals, gold, silver and platinum during the time period Oct. 7th. 1996 to Mar. 11th. 2011. We can see that gold price appears to be increasing in general trend, meanwhile silver and platinum prices have more severe up and down than gold price.

Figure 2: Plot of return data

Figure 2 shows the plot of the results of log-difference. The analysis, consist of estimation and forecast, we will do needs return data instead of the raw price data through the way that using the first-order log-difference result multiplied by one hundred of each daily afternoon fixing prices.
3.3 Data Analysis

Table 1 presents some summary statistics of the return data. All these three time series have the same amount of observations. The mean of each is very close to zero, as well as the median and a not very large standard deviation. Among them, only gold return data does have a no-negative skewness and each of them has excess kurtosis. The ARCH-LM test here is a methodology to test for the lag length of ARCH errors using the Lagrange multiplier test which was proposed by Engle (1982)\(^\circ\). This test should be done before we apply GARCH models to the data. The p-values for this test are all very small that the null hypothesis, say the dataset has no ARCH effects should be rejected. Based on this test, it is secured and proper that we can fit this data to a GARCH model.

Table 1: Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Gold Returns</th>
<th>Silver Returns</th>
<th>Platinum Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>-7.97200</td>
<td>-16.08000</td>
<td>-17.28000</td>
</tr>
<tr>
<td>Max.</td>
<td>7.00600</td>
<td>18.28000</td>
<td>11.73000</td>
</tr>
<tr>
<td>Median</td>
<td>0.01829</td>
<td>0.05153</td>
<td>0.00000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.03281804</td>
<td>0.03916584</td>
<td>0.04063242</td>
</tr>
<tr>
<td>Std.Dev</td>
<td>1.105005</td>
<td>1.957008</td>
<td>1.593869</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.01957904</td>
<td>-0.3763566</td>
<td>-0.5944549</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>8.451281</td>
<td>10.75905</td>
<td>13.11895</td>
</tr>
<tr>
<td>ARCH-LM Test</td>
<td>p-value &lt; 2.2e-16</td>
<td>p-value &lt; 2.2e-16</td>
<td>p-value &lt; 2.2e-16</td>
</tr>
</tbody>
</table>

Note: The data is collected since 1996-10-07 to 2011-03-11 and each series has 3624 observations in total.

\(^\circ\) More details about ARCH-LM Test could be seen in Appendix.
The Figure 3 demonstrates the autocorrelation function results. As a financial time series, having the autocorrelation means it can be predictable because future values depend on current and past value and this dependence could last for long term. Teräsvirta (2006) said that observations in return series of financial assets observed at weekly and higher frequencies are not independent. While observations in these series are uncorrelated or nearly uncorrelated, the series contain higher-order dependence. This can explain the autocorrelation function results below precisely.

Figure 3: Autocorrelation Function of Returns, Returns Square and absolute value of Return data

We can see that gold return data is not correlated; silver and platinum either. However, return square data shows that both gold and silver data
are highly correlated and also have a long-term memory. The return square data shows autocorrelation since it is formed as square which is exactly corresponded to the form of conditional variance. The absolute values of return data turns out higher correlation than both two before.

4. Estimation

We apply univariate GARCH models and EGARCH models to the data for estimation distinguishingly with different error term distributions.

4.1 GARCH Models

Table 2 below presents the results of estimation of univariate GARCH model, only from log-likelihood and AIC values of the models we can see that when we both add silver and platinum error terms, the model has a smaller AIC value and larger log-likelihood. From the point of view, different distributions make a difference on estimation indeed. With a student-t distribution, the models all provide smaller AIC and larger log-likelihood. Besides, the parameter of silver term often turns out to be very small, sometimes not significant which suggests that we could model the volatility without silver term.
Table 2: Estimation of Standard GARCH Models with different distributions

<table>
<thead>
<tr>
<th></th>
<th>Normal distribution</th>
<th>t distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ω</td>
<td></td>
</tr>
<tr>
<td></td>
<td>α₀</td>
<td></td>
</tr>
<tr>
<td></td>
<td>β</td>
<td></td>
</tr>
<tr>
<td></td>
<td>α₁</td>
<td></td>
</tr>
<tr>
<td></td>
<td>α₂</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Log-likelihood</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AIC</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>standard +Plat +Silver</th>
<th>+Plat standard +Silver</th>
<th>+Plat +Silver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>standard +Silver</td>
<td>+Silver</td>
<td></td>
</tr>
</tbody>
</table>

|                | 0.026                  | 0.022                  | 0.026         | 0.021         | 0.012         | 0.012         | 0.013         |
|                | (0.002)                | (0.002)                | (0.002)       | (0.003)       | (0.003)       | (0.003)       | (0.003)       |

|                | 0.085                  | 0.082                  | 0.085         | 0.082         | 0.072         | 0.071         | 0.071         |
|                | (0.005)                | (0.005)                | (0.005)       | (0.009)       | (0.009)       | (0.009)       | (0.009)       |

|                | 0.894                  | 0.895                  | 0.895         | 0.895         | 0.922         | 0.922         | 0.924         |
|                | (0.006)                | (0.007)                | (0.007)       | (0.009)       | (0.010)       | (0.010)       | (0.009)       |

|                | 0                      | 0.003                  | 0              | 0.003         | 0              | -0.001        | 0              |
|                | (0.001)                | (0.001)                | (0.001)       | (0.001)       | (0.001)       | (0.0006)      | (0.0006)      |

|                | 0                      | -0.00028              | -0.0001       | 0              | 0              | 0.001         | 0.002         |
|                | (0.001)                | (0.001)                | (0.001)       | (0.001)       | (0.001)       | (0.001)       | (0.001)       |

|                | -4895                  | -4889                  | -4894         | -4889         | -4740         | -4737         | -4881         | -4738         |
|                | -4895                  | -4889                  | -4894         | -4889         | -4740         | -4737         | -4881         | -4738         |

|                | 2.781                  | 2.780                  | 2.782         | 2.779         | 2.694         | 2.694         | 2.713         | 2.694         |

Notes: $\alpha_1$ in the model equals to zero suggests that there is no platinum error term in it, $\alpha_2$ stands for the parameter of silver error term. Both of them equal to zero means standard GARCH process.
4.2 EGARCH Models

Table 3: Estimation of Univariate EGARCH models

<table>
<thead>
<tr>
<th></th>
<th>Normal Distribution</th>
<th>t distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>standard</td>
<td>+plat</td>
<td>+Silver</td>
</tr>
<tr>
<td></td>
<td></td>
<td>+silver</td>
<td></td>
</tr>
<tr>
<td><strong>ω</strong></td>
<td>-0.124</td>
<td>-0.130</td>
<td>-0.131</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.008)</td>
<td>(0.008)</td>
</tr>
<tr>
<td><strong>α₀</strong></td>
<td>0.169</td>
<td>0.158</td>
<td>0.167</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.010)</td>
</tr>
<tr>
<td><strong>γ</strong></td>
<td>0.052</td>
<td>0.061</td>
<td>0.057</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td><strong>β</strong></td>
<td>0.970</td>
<td>0.961</td>
<td>0.962</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.003)</td>
</tr>
<tr>
<td><strong>α₁</strong></td>
<td>0</td>
<td>0.001</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td><strong>α₂</strong></td>
<td>0</td>
<td>0.003</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-4897</td>
<td>-4884</td>
<td>-4891</td>
</tr>
<tr>
<td>AIC</td>
<td>2.783</td>
<td>2.778</td>
<td>2.781</td>
</tr>
</tbody>
</table>

Notes: γ here is the parameter in the $g(Z_{t-j})$ part, the same as the θ in the

$$g(Z_{t-j}) = a Z_{t-j} + \lambda (|Z_{t-j}| - E(|Z_{t-j}|)), j = 1, ... , q.$$  

Following the same pattern of section 4.1, we can get the estimation results shown in
Table 3. Based on AIC and log-likelihood, we can get a similar result as univariate standard GARCH models. When adding both platinum and silver return's error terms since it has a smaller AIC value and larger log-likelihood. Only under the assumption with t distribution, adding silver error term makes the model change a little bit; it has a smaller AIC value.

4.3 Comparison

Compared to the univariate GARCH models, the average of all the six univariate EGARCH models gives a better performance in both log-likelihood and AIC value.

With different distribution assumption, although the sample size is large enough that we could approximate student-t distribution to standard normal distribution, the estimation still is influenced by distribution difference.

Among all these models, the model based on EGARCH(1,1) model with silver terms under the student-t distribution has both the smallest AIC and largest log-likelihood value.

5. Forecast

Besides the estimation comparison of models, we can tell the goodness of fitting by using that model forecasting and then compared to some existing data, such as an 100-out-of-sample forecasting. Through that way, as mentioned before, RMSE is a quite useful tool and has more exactness.

In this paper, we complete forecasting following this method:

First of all, there are 3624 observations in total in the dataset. Now we reserve the last 100 observations in order to compare to the forecasting value. Then we should regard the front 3524 values as a “window” which has a certain length and 3524 observations. Secondly, we estimate the model with this “window”, after that we can do a one-step
forecast of volatility. Take the univariate standard GARCH as an example of one-step forecast,

\[ h_{gold,t} = \omega + \alpha_0 \epsilon_{gold,t-1}^2 + \beta h_{gold,t-1} + \alpha_1 \epsilon_{silver,t-1}^2 + \alpha_2 \epsilon_{platinum,t-1}^2 \]

\[ h_{gold,t+1} = \omega + \alpha_0 \epsilon_{gold,t}^2 + \beta h_{gold,t} + \alpha_1 \epsilon_{silver,t}^2 + \alpha_2 \epsilon_{platinum}^2 \]

Next, we repeat the first step except for creating a new window from 3524 observations to 3525 observations. The “window” gets wider and wider. We can keep moving forward by repeating this two steps one-hundred times and then we can get 100 forecast values. Eventually, we can calculate RMSEs of all the values from all the models and compare them.

First, we compare RMSEs of GARCH models.

**Table 4: The results of RMSE of GARCH models**

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<tr>
<th></th>
<th>Standard + plat + silver</th>
<th>Standard + plat</th>
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<th>Standard + plat + silver</th>
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**Note:** RMSE is short for Root Mean Square Error.

From Table 4, adding platinum and silver error terms at the same time seems has a much better result than every model else, meanwhile the group with t distribution makes better performance than the other. The most important thing is that extra error terms actually can help with forecasting gold price volatility since most of them turn out a better result than using standard GARCH models without all the regressors no matter what distribution we assume.
Next, we compare the results of EGARCH models. The results in Table 5 do not have a spectacular better number among all. But in both normal distributions and t distribution, adding only silver term shows its efficiency in helping forecasting gold volatility. Similarly the group with t distributions performs better. Also same as GARCH models, most models with extra terms have smaller RMSE.

Compared the results from the view of the kinds of model, by average, EGARCH models provide a better final result in forecasting than GARCH models no matter with or without regressors, especially excellent with t distribution.

Table 5: The results of RMSE of EGARCH models

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<tr>
<th></th>
<th>Standard</th>
<th>+plat</th>
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<th>+plat</th>
<th>Standard</th>
<th>+plat</th>
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<td>Student-t Distribution</td>
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Note: RMSE is short for Root Mean Square Error.

6. Conclusion

In summary, this paper aims to forecast gold prices under the help of other precious metals such as silver and platinum. Among all the results we get, for models, EGARCH models seem to be more efficient in forecasting volatility; for distributions, unique properties of t distribution makes it more appropriate than normal distribution in forecasting.

The result actually confirms a hypothesis we did not mention much, whether the precious metals could help with forecasting volatility of gold price or not. Now we can know it for sure that adding silver and platinum error terms makes GARCH
models better. The extra regressors increase the accuracy of the models in forecasting.

But there are still existing limitations.

First we assume all the estimation and forecasting are processed under the assumption that the error term denotes a symmetric distribution such as normal distribution which is not that enough for real situation.

Second, we consider all the data do not contain any breaks, however, after the world-wide financial crisis, there may be some structural breaks during that time. This needs many more steps of analysis before we applied the data to the models.

There have been a lot empirical applications of analyzing financial time series, they seem to be the same kind of financial data and all applied to GARCH family models, however the combination of different data and different point of views leads to totally different results. In this paper, it is too complicated to explain that how and why the silver and platinum could help with modeling and forecasting this way. We can dig it deeper and deeper and find more details to improve the whole estimation and forecast system of GARCH family models.
Reference


Appendix

ARCH - Lagrange multiplier (LM) test

The ARCH - Lagrange multiplier (LM) test is a methodology to test for the lag length of ARCH errors, in the other word, The ARCH-LM Test is for testing whether the series has ARCH effects at all (Engle, 1982).

In the ARCH-LM test, there is a regression, 

\[ \varepsilon_i^2 = \alpha_0 + \left( \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 \right) + \varepsilon_i \]

In this regression, \( \alpha_1 = \alpha_2 = \ldots = \alpha_q = 0 \) is the null hypothesis, meanwhile, the alternative hypothesis \( H_1: \alpha_1 \neq 0 \) or \( \alpha_2 \neq 0 \) or \( \ldots \alpha_2 \neq 0 \) means that not all \( \alpha_i \) being not significant at the same time. Besides, the test statistics follows \( \chi^2 \) distribution with \( p \) degrees of freedom.