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AN APPROACH TO CHARACTERIZAION OF THE LORENTZ TRANSFER FUNCTION OF ESS SPOKE CAVITIES AT FREIA

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Abstract

In this memo we discuss the physics of ponderomotive effects in superconducting cavities and identify the preparations needed to be done for performing measurements of Lorentz transfer functions at the FREIA laboratory. Specifically, it turns out that we need a stub tuner for increasing the external quality factor of the spoke cavity in order to be able to measure Lorentz transfer functions. Electronics required for measurements is also discussed.

1 Background

The Lorentz transfer function is a response of the cavity eigen frequency and phase to mechanical excitation of the cavity. An example of such response for a double-spoke measured at the Argonne Laboratory (ANL) is shown in Fig. 1.1. This transfer function tells us that one should

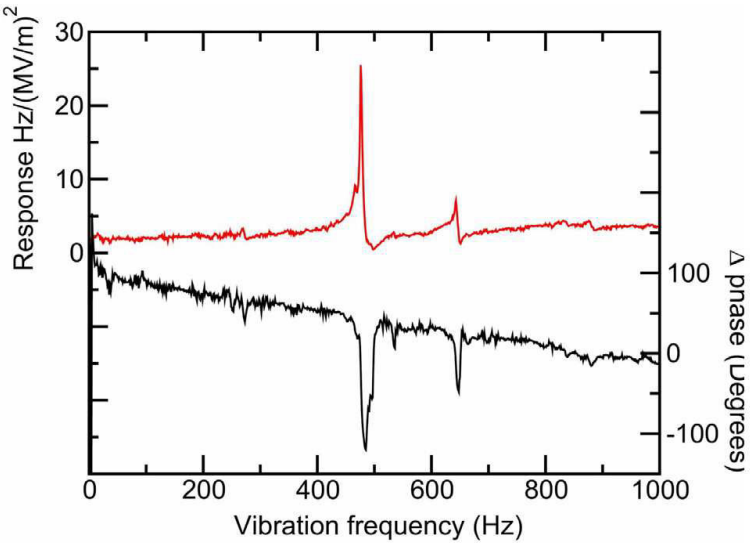

Figure 1.1: Lorentz force transfer function for the double-spoke cavity measured at ANL [1]. Top is the amplitude response; bottom is the phase response.

avoid mechanical vibrations at a frequency of approximately 500 Hz in the cavity environment. Subharmonics of 500 Hz may also be harmful for the cavity operation if the mechanical resonance has a high enough quality factor. Then, a parametric resonance will develop. Mechanical oscillations would detune the cavity and increase RF power overhead if they are no damped. In accelerators with high beam loading like ESS, a strong ponderomotive instability is unlikely because of quite low loaded quality factor of the cavity. However, in view of the stringent requirements on the beam quality may be important. Thus, we discuss below how to measure Lorentz transfer functions.

2 Ponderomotive effects

A detailed analysis of ponderomotive effects in SC cavities can be found in [2] and in what follows we give a brief introduction into this problem following [3].

Any mechanical oscillation of a cavity can be described by excitation of an infinite complete set of eigen mechanical modes of the cavity with some amplitudes q_m. Let Ω_m and τ_m be the eigen frequency and decay time of m^{th} mechanical mode whose equation of motion reads

$$\ddot{q}_m + \frac{2}{\tau_m}\dot{q}_m + \Omega_m^2 q_m = F_m. \tag{2.1}$$

The force F_m can have either systematic nature as, for example, Lorentz force detuning or be stochastic and caused, for instance, by external mechanical noise or turbulence of liquid helium in a cryomodule. Each mechanical oscillation of the cavity changes the cavity shape and results in a variation of the electrodynamical (EM) eigen frequency of the cavity, ω_{cav}. This can be understood if we recall that the ratio of the EM energy stored in the cavity, U, to its EM frequency is an adiabatic invariant so that shape deformation implies a change in energy content and therefore corresponding frequency shift.

Recall that for a periodic system ratio U/ω is preserved with an exponential accuracy if a parameter or parameters of the system slowly and smoothly change in time. A slow variation means the variation that occurs on a time scale much larger than the period of oscillations of the system in question. In our case, the mechanical oscillation related to the Loretnz force detuning occur on a time scale of ms whereas EM oscillations are characterized by a ns scale. Due to the adiabatic nature of U/ω, a relative change in frequency is equal to a relative change in energy content

$$\frac{\Delta\omega_{cav}}{\omega_{cav}} = \frac{\Delta U}{U}. \tag{2.2}$$

This result is known as Slater's formula in microwave engineering. The energy change ΔU is determined by a perturbed volume and EM fields in this volume. Assuming EM fields in the perturbed volume be constant, one obtains that frequency shift $\Delta\omega_m$ caused by the m^{th} mechanical mode is proportional to the mechanical mode amplitude q_m, i.e. $\Delta\omega_m \sim q_m$.

Let us stress that the Lorentz force detuning caused by an EM pressure is distributed over the cavity whereas the piezotuner action is localized and will predominantly result in excitation of longitudinal mechanical modes. This imposes a fundamental limit on the compensation of the ponderomotive Lorentz force effects with a piezotuner. In practice, this difference between distributed and localized nature of excitation of mechanical modes appears in rather different behaviour of transfer functions, cf. Fig 2.1 and Fig. 2.2.

Equation 2.1 for the m^{th} mechanical amplitude can be re-written in terms of frequency

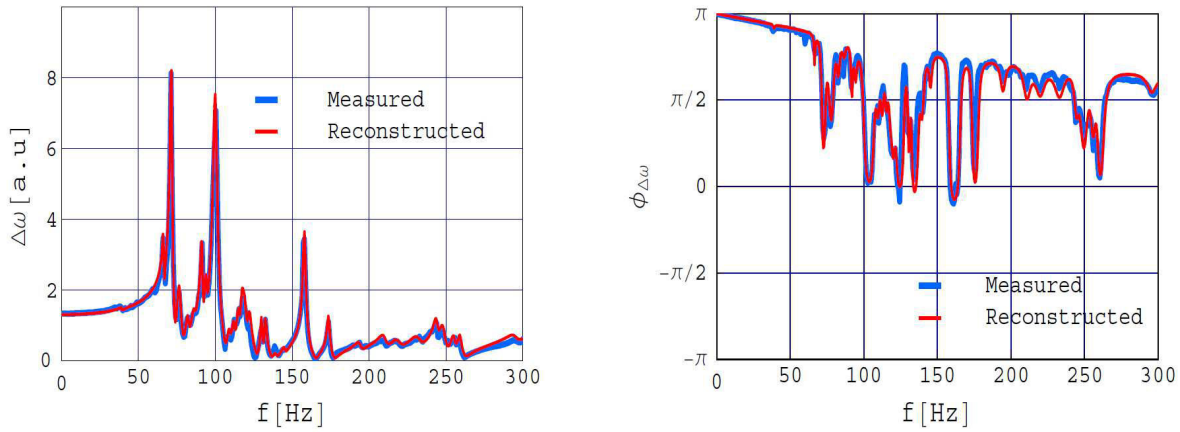

Figure 2.1: Amplitude and phase of the measured and reconstucted transfer function associated with the Lorentz force in the SNS medium beta cavity [2].

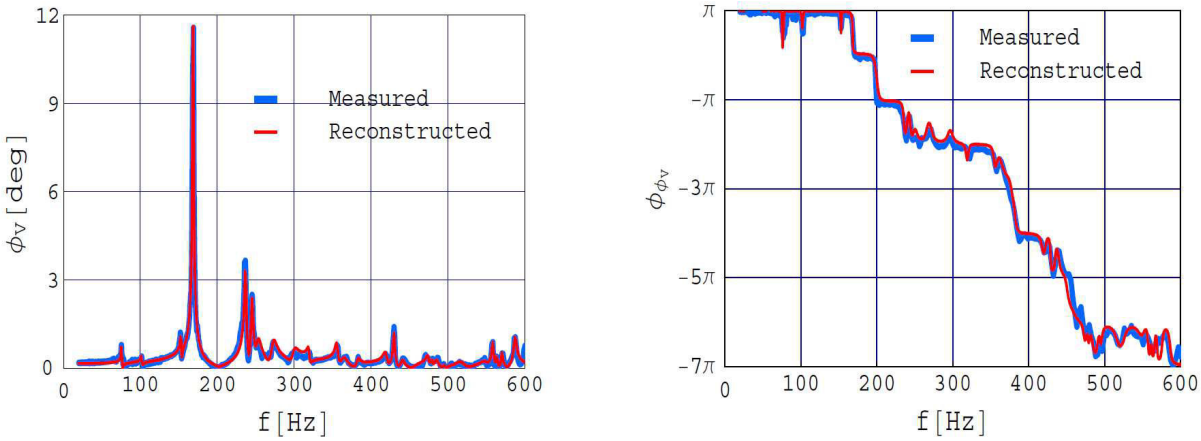

Figure 2.2: Measured and simulated transfer function (amplitude and phase) for the action of the piezoelectric tuner in the SNS medium beta cavity [2].

deviations $\Delta\omega_m$ of the EM cavity frequency as

$$\ddot{\Delta}\omega_m + \frac{2}{\tau_m}\dot{\Delta}\omega_m + \Omega_m^2\Delta\omega_m = -k_m\Omega_m^2 V(t)^2 + n(t), \tag{2.3}$$

where $V(t)$ and $n(t)$ are the cavity voltage and external mechanical excitation, k_m is the coupling between the RF field and the m^{th} mechanical mode, which we will refer to as the Lorentz force detuning coefficient. The total EM frequency shift is $\Delta\omega = \sum \Delta\omega_m$ and $k = \sum k_m$ is the static Lorentz force detuning coefficient of the cavity.

One should distinguish between the static and dynamic Lorentz detuning observed during CW and pulsed tests. In general, increasing the boundary stiffness will decrease the static Lorentz force detuning. Contrary to the static Lorentz force detuning behaviour, increasing the boundary stiffness does not necessarily decrease the dynamic Lorentz force detuning [4].

3 Measurement of Lorentz transfer function

A convenient method to determine the parameters of the mechanical modes and the Lorentz force detuning coefficients is to modulate the radiation pressure at angular frequency Ω_m in order to excite the m^{m} mode only. To this end, one can be drive the cavity in CW mode at some relatively high gradient V_0, introduce a small periodic modulation of the accelerating gradient ΔV and sweep the modulation frequency. This will allow to measure the amplitude and phase of the cavity frequency modulation as a function of sweep frequency ω, known as the Lorentz transfer function. The latter for an individual mechanical modes takes the form

$$\frac{\delta\omega_m}{\delta\nu} = -\frac{2k_m\Omega_m^2 V_0^2}{(\Omega_m^2 - \omega^2) + (2i/\tau_m)\omega} \equiv \Gamma_m(\omega). \tag{3.1}$$

Here $\delta\nu = \Delta V/V_0$ is the relative modulation of accelerating gradient and $\delta\omega_m$ is the EM frequency deviation caused only by the modulation. The total Lorentz transfer function for a cavity, $\Gamma(\omega)$, is the sum of transfers functions for the individual mechanical modes, $\Gamma_m(\omega)$. Using a measured Lorentz transfer function, in principle, one should be able to reconstruct the mechanical parameters of cavity. Unfortunately, the procedure used in [2] is not explained but one can probably try to fit a multi-peak Lorentzian curve [5] to measured $\Gamma(\omega)$.

Let us now estimate the RF power required for measuring the Lorentz transfer function. Actually, we need to estimate how large RF modulation should be so that we able to measure the cavity frequency response. A standard method of measuring the cavity frequency as a function of time is the Cavity Resonance Monitor [6]. For many years it was based on analog technology but progress on digital signal processing made possible development of a Digital Cavity Resonance Monitor with a resolution down to 0.2 Hz. Then, we will target a cavity frequency modulation of 2 Hz, which results in an accelerating gradient modulation of around 0.6 MV/m (0.4 MV cavity voltage). The gradient corresponding to a steady-state CW operation must be a factor of 10 larger, which means 6 MV/m (3.84 MV cavity voltage). The RF power required to create cavity accelerating voltage V_0 reads

$$P_i = \frac{V_0^2}{8Q_{ext}(R/Q)}, \tag{3.2}$$

where Q_{ext} and R/Q are the external quality factor and normalized shunt impedance, respectively. One finds that 3.84 MV cavity voltage calls for almost 50 kW RF power!

Let us stress that the cavity has to be operated in CW mode so that we will not be able to measure the Lorentz transfer function with the current test stand setup. The only way is to increase the external quality factor by 100 times using a matching network, for example, in the form of a waveguide with adjustable stubs. So, the work to be done is as follows

- study a possibility of increasing Q_{ext} by a factor of 100
- develop a digital Cavity Resonance Monitor
- develop an algorithm of reconstruction of the mechanical parameters of the cavity from a measured Lorentz transfer function

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