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ESTIMATING THE VARIANCE OF A PROPENSITY SCORE MATCHING ESTIMATOR: A NEW LOOK AT RIGHT HEART CATHETERISATION DATA

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Abstract

This study considers the implementation of a variance estimator when estimating the asymptotic variance of a propensity score matching estimator for the average treatment effect. We investigate the role of smoothing parameters in the variance estimator and propose using local linear estimation. Simulations demonstrate that large gains can be made in terms of mean squared error by properly selecting smoothing parameters and that local linear estimation may lead to a more efficient estimator of the asymptotic variance. The choice of smoothing parameters in the variance estimator is shown to be crucial when evaluating the effect of right heart catheterisation, i.e. we show either a negative effect on survival or no significant effect depending on the choice of smoothing parameters.

Keywords: average treatment effect; causal inference; kernel estimator; local linear estimator.

AMS Subject Classification: 62G05; 62P10

1 Introduction

Matching estimators belong to a class of estimators of average treatment effects (ATEs) in observational studies that seek to balance the distributions of covariates in a treatment and control group (Stuart, 2010). In this study we consider one frequently used matching method, the simple nearest-neighbour matching with replacement (Rubin, 1973; Abadie and Imbens, 2006).

Rosenbaum and Rubin (1983) show that instead of matching on covariates directly to remove confounding, it is sufficient to match on the propensity score. In observational studies the propensity score must almost always be estimated. An important contribution is therefore Abadie and Imbens (2012), who derive the large sample distribution of a nearest-neighbour propensity score matching estimator when using the estimated propensity score. Along with the large sample results, Abadie and Imbens (2012) provide an estimator for the asymptotic variance. However, this estimator is given in a general form, including conditional variances and conditional covariances that need to be estimated. Consistent estimators for the conditional variances and covariances are suggested by Abadie and Imbens (2006; 2012), but the choice of smoothing parameters, i.e. the number of matches to estimate the conditional variances and covariances, is not discussed. Abadie and Imbens (2008) study the choice of a smoothing parameter when estimating conditional variances in paired experiments. The authors, however, only regard a few simulation designs and do not look at a setting with an estimated propensity score. Thus, the present study extends previous results on the choice of smoothing parameters for both simulation designs and the propensity score setting. Furthermore, because the asymptotic variance partly consists of conditional expectations, we suggest (and study) non-parametric local linear estimators as alternatives to the estimators in Abadie and Imbens (2012).

Simulations highlight, that by properly selecting the smoothing parameters of the asymptotic variance estimators, large gains can be made in terms of mean squared error (MSE). The simulations also indicate that a residual-based local linear estimator is an attractive option, having a lower MSE in many cases. The drawback is that local linear estimation may be more sensitive to the case when the propensity scores are extreme.

To illustrate the importance of variance estimation we reanalyse the effect of RHC in intensive care units using observational data from the "Study to Understand Prognoses and Preferences for Outcomes and Risks of Treatments" (SUPPORT). Several studies, applying different statistical methods on the same data, show that RHC has a negative effect on survival (e.g., Connors et al., 1996; Hirano and Imbens, 2001; Li et al., 2008; Crump et al., 2009; Vansteelandt et al., 2012), whereas only a few studies find no effect at all (Tan, 2006) or an effect that is sensitive to missing confounders (Lin et al., 1998). These somewhat surprising and conflicting results led to controversy about the use of RHC in critical illness and called for the need of

randomised controlled experiments. A meta-analysis by Shah et al. (2005) of randomised experiments found that the use of the RHC did not increase overall mortality. In fact, the overall conclusion was no effect at all. In this study we show that when reanalysing the RHC data using propensity score matching, the conclusion depends on how the variance is estimated, with two examples of 95% confidence intervals (CIs) for the risk difference of 30-day survival being $(-0.0659, -0.0026)$ and $(-0.0801, 0.0116)$. However, when following the recommendations from the simulation study we conclude that there is no significant effect, i.e. CIs not covering 0.

We proceed as follows. Section 2 provides details of the causal inference framework and the matching estimator. The estimators of the asymptotic variances are described in Section 3. Section 4 includes a simulation study evaluating the estimators. In Section 5 we reanalyse the RHC study applying the different estimators for the asymptotic variances. Section 6 concludes.

2 Framework

A common goal in evaluation research is to evaluate the effect of a binary treatment W on some outcome Y. Let $W = 1$ indicate exposure to a treatment and $W = 0$ indicate exposure to a control. Using the framework introduced by Neyman (1923) and Rubin (1974), each treatment yields a potential outcome, Y_1 and Y_0. In this study we consider the population level average treatment effect (ATE)

$$\tau = E\left(Y_1 - Y_0\right). \tag{1}$$

Imbens and Wooldridge (2009) discuss other estimands that may be of interest, such as the ATE of the treated. However, it should be straightforward to generalise the results for estimators of other causal estimands as long as these estimators include conditional variances and covariances. The issue when estimating treatment effects lies in the fact that we can only observe one of the two potential outcomes for each unit, $Y = WY_1 + (1-W)Y_0$. Hence, a simple comparison of average outcomes in the treatment and control group does not in general estimate the ATE.

Next, we introduce a setting that allows for identification of τ and that is assumed to hold throughout the study. Assume a random sample of N units from the distribution (Y, W, X), where X is vector with k covariates affecting potential outcomes and treatment assignment. Define the propensity score $p(X) = \Pr(W = 1|X)$ and assume that the following assumption holds (Rosenbaum and Rubin, 1983).

Assumption 1 *(i) (Unconfoundedness)* $Y_1, Y_0 \perp\!\!\!\perp W|X$ *and (ii) (Overlap)* $\eta < p(X) < 1 - \eta$*, for some* $\eta > 0$*.*

Under Assumption 1, together with further results in Rosenbaum and Rubin (1983) stating that controlling for the propensity score is sufficient to eliminate

all confounding, τ can be identified with

$$\tau = E\left[\mu(1, p(X)) - \mu(0, p(X))\right], \tag{2}$$

where $\mu(w,p) = E(Y|W = w, p(X) = p)$. Other necessary conditional means and variances that will be used in the sequel are $\bar{\mu}(w,x) = E(Y|W = w, X = x)$, $\bar{\sigma}^2(w,x) = V(Y|W = w, X = x)$ and $\sigma^2(w,p) = V(Y|W = w, p(X) = p)$.

So far, the specification of the propensity score is general. Abadie and Imbens (2012) restrict the propensity score to have a generalised linear specification, although the authors note that the results can be extended to a wider range of parametric models. In this study we choose the propensity score to be specified as a logit model, $p(X) = [1 + \exp(-X'\gamma)]^{-1}$, with γ being a parameter vector and $p(\cdot)$ the standard logistic distribution function. The corresponding logistic density function is $p'(X) = p(X)[1 - p(X)]$. At least one of the covariates in X is assumed to be continuous, implying that $p(X)$ is also continuous. Because $p(X)$ is unlikely to be known in an observational study, it must be estimated from a sample, (e.g., using maximum likelihood). We assume that the estimated propensity score, $\hat{p}(X)$, is correctly specified by the researcher.

2.1 Matching estimator

For the estimation of the ATE, we focus on the matching estimator described in Abadie and Imbens (2006) and Abadie and Imbens (2012). The matching estimator matches each unit to typically a small number of units in the opposite treatment group being closest in terms of some distance. Because each unit can be selected as a match for several times, this estimator is referred to as a nearest-neighbour matching estimator with replacement.

The matching sets that will be used throughout this study are specified as follows, provided that no matching ties are present (Abadie and Imbens, 2012). First, the set of indices for the first L matches for unit i in the opposite treatment group when matching on $\hat{p}(X_i)$ is defined as

$$\mathscr{I}_L(i) = \left\{ j = 1, \ldots, N : W_j = 1 - W_i, \left(\sum_{k:W_k = 1-W_i} \mathbb{1}_{|\hat{p}(X_i) - \hat{p}(X_k)| \leq |\hat{p}(X_i) - \hat{p}(X_j)|} \leq L \right) \right\}. \tag{3}$$

Similarly, we define the set of indices for the first L matches for unit i in the same treatment group as unit i when matching on $\hat{p}(X)$ as

$$\mathscr{H}_L(i) = \left\{ j = 1, \ldots, N : W_j = W_i, \left(\sum_{k:W_k = W_i} \mathbb{1}_{|\hat{p}(X_i) - \hat{p}(X_k)| \leq |\hat{p}(X_i) - \hat{p}(X_j)|} \leq L \right) \right\}. \tag{4}$$

Observe that $\mathscr{H}_L(i)$ includes unit i and that the notation is quite general. Letting $L = M$, a propensity score matching estimator for the ATE with M matches can be compactly written as

$$\hat{\tau} = \frac{1}{N}\sum_{i=1}^{N}(2W_i - 1)\left(Y_i - \sum_{j\in\mathscr{I}_M(i)}\frac{Y_j}{M}\right). \tag{5}$$

A main result in Abadie and Imbens (2012) is that in large samples the distribution of $\sqrt{N}(\hat{\tau}-\tau)$ can be approximated by a normal distribution with mean zero and variance

$$V = \sigma^2 - c'I^{-1}c. \tag{6}$$

Necessary assumptions can be found in Abadie and Imbens (2006) and Abadie and Imbens (2012).

The first part of V is the asymptotic variance of the matching estimator when matching on the true propensity score, $p(X)$, and it may be written

$$\begin{aligned}\sigma^2 =& E\left[(\mu(1,p(X)) - \mu(0,p(X)) - \tau)^2\right] \\ &+ E\left[\sigma^2(1,p(X))\left(\frac{1}{p(X)} + \frac{1}{2M}\left(\frac{1}{p(X)} - p(X)\right)\right)\right] \\ &+ E\left[\sigma^2(0,p(X))\left(\frac{1}{1-p(X)} + \frac{1}{2M}\left(\frac{1}{1-p(X)} - (1-p(X))\right)\right)\right].\end{aligned} \tag{7}$$

The second part in V consists of a $k\times 1$ dimensional vector of covariances between conditional means and covariates,

$$c = E\left[\left(\frac{\mathrm{cov}(X,\bar{\mu}(1,X)|p(X))}{p(X)} + \frac{\mathrm{cov}(X,\bar{\mu}(0,X)|p(X))}{1-p(X)}\right)p'(X)\right], \tag{8}$$

and the $k\times k$ dimensional Fisher information matrix for γ in the propensity score model which, assuming a logistic model, can be simplified to (Lee, 1990):

$$I = E\left[p(X)(1-p(X))XX'\right]. \tag{9}$$

We end this section with a few comments on some properties of the propensity score matching estimator. Propensity score matching is appealing to use for applied researchers, largely because it is an intuitive method of estimation with a smoothing parameter, i.e. number of matches, that is easy to interpret. The practitioner is forced to separate the modelling of the treatment assignment from the outcome and can then rely on straightforward diagnostic checks of the covariate balance and the overlap assumption. As for the formal theoretical properties, Abadie and Imbens (2006) show that although the matching estimator is biased when matching on the vector norm of continuous

covariates, the bias is of sufficiently low order to be ignored when matching on a scalar, such as the propensity score. Further, it follows from the equations above that when estimating the ATE it is more efficient to match on the estimated propensity score, rather than on the true propensity score. However, matching is inefficient in that it does not reach the semi-parametric efficiency bound (Hahn, 1998). As with other estimators of the ATE, the matching estimator also relies on the assumption of unconfoundedness, which may or may not be considered a strong assumption. Still, the propensity score matching estimator has been shown to be robust against misspecification of the propensity score (Waernbaum, 2012). Some other important contributions to the discussion on matching estimators in general are Rubin (1973), Rubin and Thomas (1996), Heckman et al. (1998), Frölich (2004) and Stuart (2010).

3 Estimators of the asymptotic variance of $\hat{\tau}$

The first part of this section describes the estimator for the asymptotic variance of $\hat{\tau}$ (Abadie and Imbens, 2012). The subsequent part introduces the non-parametric local linear estimators for the asymptotic variance.

To estimate V with $\hat{V}$, we estimate the components σ^2, c, and I separately, so that

$$\hat{V} = \hat{\sigma}^2 - \hat{c}'\hat{I}^{-1}\hat{c}. \tag{10}$$

First, a straightforward estimator for the information matrix, I, is

$$\hat{I} = \frac{1}{N}\sum_{i=1}^{N}\left[\hat{p}(X_i)(1-\hat{p}(X_i))X_iX_i'\right]. \tag{11}$$

Second, let $K_M(i)$ denote the number of times unit i is used as a match when estimating the ATE. Abadie and Imbens (2006) and Abadie and Imbens (2012) propose as estimator for σ^2:

$$\begin{aligned}\hat{\sigma}^2 =& \frac{1}{N}\sum_{i=1}^{N}\left[(2W_i-1)\left(Y_i - \sum_{j\in\mathscr{I}_M(i)}\frac{Y_j}{M}\right) - \hat{\tau}\right]^2 \\ &+ \frac{1}{N}\sum_{i=1}^{N}\left[\left(\frac{K_M(i)}{M}\right)^2 + \frac{2M-1}{M}\left(\frac{K_M(i)}{M}\right)\right]\hat{\sigma}^2(W_i,p(X_i)).\end{aligned} \tag{12}$$

Given that the ATE is already estimated, the estimation of σ^2 depends entirely on $\hat{\sigma}^2(W_i,p(X_i))$. To estimate the conditional variance Abadie and Imbens (2012) suggest using the matching estimator

$$\hat{\sigma}^2_L(W_i,p(X_i)) = \frac{1}{L-1}\sum_{j\in\mathscr{H}_L(i)}\left(Y_j - \sum_{k\in\mathscr{H}_L(i)}\frac{Y_k}{L}\right)^2, \tag{13}$$

for $L \geq 2$, yielding $\hat{\sigma}^2_L$ as estimator for σ^2. Although asymptotically unbiased, $\hat{\sigma}^2_L(W_i, p(X_i))$ is not consistent because of L being fixed. However, $\hat{\sigma}^2_L$, being a weighted average of $\hat{\sigma}^2_L(W_i, p(X_i))$, is consistent for σ^2. Third, using that $\text{cov}(X, \bar{\mu}(1,X)|p(X)) = \text{cov}(X, Y|p(X), W = 1)$ and $\text{cov}(X, \bar{\mu}(0,X)|p(X)) = \text{cov}(X, Y|p(X), W = 0)$, the estimator for c suggested by Abadie and Imbens (2012) can be written

$$\hat{c} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\widehat{\text{cov}}(X_i, Y_i|p(X_i), W = 1)}{\hat{p}(X_i)} + \frac{\widehat{\text{cov}}(X_i, Y_i|p(X_i), W = 0)}{1 - \hat{p}(X_i)}\right)\hat{p}'(X_i). \quad (14)$$

Once the propensity score is estimated, the estimation of the conditional covariances completely determines $\hat{c}$. In Abadie and Imbens (2012) the conditional covariances are, for $L' \geq 2$, estimated with

$$\widehat{\text{cov}}_{L'}(X_i, Y_i|p(X_i), W_i = w) = \left(\frac{1}{L' - 1}\right)\sum_{j \in \mathscr{H}_{L'}(i)}\left(X_j - \sum_{k \in \mathscr{H}_{L'}(i)}\frac{X_k}{L'}\right)\left(Y_j - \sum_{k \in \mathscr{H}_{L'}(i)}\frac{Y_k}{L'}\right) \quad (15)$$

if $W_i = w$, and, for $L'' \geq 2$,

$$\widehat{\text{cov}}_{L''}(X_i, Y_i|p(X_i), W_i = w) = \left(\frac{1}{L'' - 1}\right)\sum_{j \in \mathscr{I}_{L''}(i)}\left(X_j - \sum_{k \in \mathscr{I}_{L''}(i)}\frac{X_k}{L''}\right)\left(Y_j - \sum_{k \in \mathscr{I}_{L''}(i)}\frac{Y_k}{L''}\right) \quad (16)$$

if $W_i \neq w$. These two estimators combined yields $\hat{c}_{L'L''}$ and using the same rationale as for $\hat{\sigma}^2_L$, $\hat{c}_{L'L''}$ is consistent for c.

We conclude that when estimating the asymptotic variance, the practitioner must decide on the three parameters, denoted L, L' and L''. Thus, V is estimated with

$$\hat{V}_{LL'L''} = \hat{\sigma}^2_L - \hat{c}'_{L'L''} I^{-1} \hat{c}_{L'L''}. \quad (17)$$

The guidance given by Abadie and Imbens (2012) is that typically $L = 2$, $L' = 2$, and $L'' = 2$ resulting in $\hat{V}_{222}$.

Remark: If assuming constant conditional variance, then the conditional variance is estimated with:

$$\hat{\sigma}^2_0(W, p(X)) = \frac{1}{2N}\sum_{i=1}^{N}\left[(2W_i - 1)\left(Y_i - \sum_{j \in \mathscr{I}_M(i)}\frac{Y_j}{M}\right) - \hat{\tau}\right]^2, \quad (18)$$

which after insertion in σ^2 gives the estimator $\hat{\sigma}^2_0$ (Abadie et al., 2004).

3.1 Local linear estimators

Alternatively, the conditional variance and covariances could be estimated using local linear kernel regression. We study two methods, one that uses variance decompositions and one that performs regressions on residuals. Consider

the univariate non-parametric regression model in treatment group $W_i = w$,

$$Y_i = E(Y|W = w, p(X) = p) + \sigma(w, p(X_i))\varepsilon_i, \tag{19}$$

where $E(\varepsilon_i|p(X_i) = p) = 0$ and $V(\varepsilon_i|p(X_i) = p) < \infty$. Let $\hat{a} = \hat{\mu}_Y(W_i, p(X_i))$ be the local linear kernel estimator for $E(Y|W, p(X))$ that solves the weighted least squares problem in point p (Stone, 1977; Cleveland, 1979):

$$(\hat{a}, \hat{b}) = \arg\min_{a,b} \sum_{i=1}^{N} (Y_i - a - b(\hat{p}(X_i) - p))^2 K_h\left(\frac{\hat{p}(X_i) - p}{h}\right) \mathbb{1}_{W_i = w}, \tag{20}$$

where the regressor is the estimated propensity score and $K_h(\cdot)$ is a kernel function with bandwidth $h > 0$. Because of $\sigma^2(\cdot) = E(Y^2|\cdot) - E(Y|\cdot)^2$ we then estimate the conditional variance with

$$\hat{\sigma}_l^2(W_i, p(X_i)) = \hat{\mu}_{Y^2}(W_i, p(X_i)) - \hat{\mu}_Y^2(W_i, p(X_i)), \tag{21}$$

where $\hat{\mu}_{Y^2}(W_i, p(X_i))$ is the local linear estimator for $E(Y^2|W, p(X))$. The index l refers to using local linear estimation and after plugging $\hat{\sigma}_l^2(W_i, p(X_i))$ into Equation (12) it follows that σ^2 is estimated with $\hat{\sigma}_l^2$.

To avoid extrapolation of the local linear estimators, $\hat{c}$ in (14) is reformulated according to Abadie and Imbens (2009) and c is instead estimated with

$$\hat{c} = \frac{1}{N} \sum_{i=1}^{N} \widehat{\text{cov}}(X_i, Y_i|W_i, p(X_i))\hat{p}'(X_i) \left(\frac{W_i}{\hat{p}(X_i)^2} + \frac{1 - W_i}{(1 - \hat{p}(X_i))^2}\right). \tag{22}$$

A decomposition of the conditional covariance suggests the estimator:

$$\widehat{\text{cov}}_l(X_i, Y_i|W_i, p(X_i)) = \hat{\mu}_{XY}(W_i, p(X_i)) - \hat{\mu}_X(W_i, p(X_i))\hat{\mu}_Y(W_i, p(X_i)), \tag{23}$$

where the local linear estimators $\hat{\mu}_{XY}(W_i, p(X_i))$ and $\hat{\mu}_X(W_i, p(X_i))$ estimate $E(XY|W, p(X))$ and $E(X|W, p(X))$, respectively. Inserting $\widehat{\text{cov}}_l(X_i, Y_i|W_i, p(X_i))$ into Equation (22) yields the estimator $\hat{c}_l$. Thus, the estimator for V when using local linear estimation is

$$\hat{V}_{ll} = \hat{\sigma}_l^2 - \hat{c}_l' \hat{I}^{-1} \hat{c}_l. \tag{24}$$

Modified estimators for the conditional variance and covariance can also be considered, which we denote as residual-based local linear estimators. These estimators are based on Fan and Yao (1998) and Yin et al. (2010). Define

$$r_Y^2 = [Y_i - \hat{\mu}_Y(W_i, p(X_i))]^2 \tag{25}$$

and

$$r_{XY} = [X_i - \hat{\mu}_X(W_i, p(X_i))][Y_i - \hat{\mu}_Y(W_i, p(X_i))]. \tag{26}$$

Furthermore, let $\hat{\mu}_{r_Y^2}(W, p(X))$ and $\hat{\mu}_{r_{XY}}(W, p(X))$ be the local linear estimators for $E(r_Y^2|W, p(X))$ and $E(r_{XY}|W, p(X))$. We then define the residual-based estimators

$$\hat{\sigma}_r^2(W_i, p(X_i)) = \hat{\mu}_{r_Y^2}(W, p(X)) \tag{27}$$

and

$$\widehat{\text{cov}}_r(X_i, Y_i | W_i, p(X_i)) = \hat{\mu}_{r_{XY}}(W, p(X)). \tag{28}$$

We plug $\hat{\sigma}_r^2(W_i, p(X_i))$ and $\widehat{\text{cov}}_r(X_i, Y_i | W_i, p(X_i))$ into Equation (12) and Equation (22) and as a consequence V is estimated with

$$\hat{V}_{rr} = \hat{\sigma}_r^2 - \hat{c}_r' \hat{I}^{-1} \hat{c}_r. \tag{29}$$

The results in Mammen et al. (2011) suggests that under certain assumptions $\hat{\mu}_Y(W_i, p(X_i))$ is consistent for $E(Y|W, p(X))$ (e.g., the model for $\hat{p}(X)$ is parametric and correctly specified, $h \to 0$ and $Nh \to \infty$ as $N \to \infty$). In such a case consistency of $\hat{\sigma}_l^2(W_i, p(X_i))$ follows from that $\hat{\mu}_Y(W_i, p(X_i))$ and $\hat{\mu}_{Y^2}(W_i, p(X_i))$ are consistent. However, in finite samples the estimator using the variance decomposition is not always nonnegative (Härdle and Tsybakov, 1997; Fan and Yao, 1998). Further, as for the estimators of the conditional covariance, Yin et al. (2010) do not study the case of having estimated regressors nor local linear estimation. The authors argue that the local linear estimator for the conditional covariance matrix may destroy the positive-definiteness of the covariance matrix. The consequences are unclear. Regarding the residual-based estimator, Fan and Yao (1998) show that it is a more efficient estimator of the conditional variance than the estimator using the variance decomposition. However, the authors do not consider estimated regressors, making the theoretical properties of $\hat{\sigma}_r^2(W_i, p(X_i))$ and $\widehat{\text{cov}}_r(X_i, Y_i | W_i, p(X_i))$ unknown. Still, through a simulation study we are able to compare the estimators.

4 Simulation study

A simulation study was done to study the aforementioned estimators of σ^2, $c' I^{-1} c$, and consequently, V. The aim was to investigate the role of the smoothing parameters, L, L'', L'', for these estimators and to examine the performance of the alternative local linear-based estimators. The estimators $\hat{\sigma}_L^2$ and $\hat{c}'_{L'L''} \hat{I}^{-1} \hat{c}_{L'L''}$ were studied for the values $L = (2, 3, 5, 9, 15, 25, 50)$ and $L'L'' = (22, 33, 55, 99, 1515, 2525, 4040, 23, 25, 29, 32, 52, 92)$. Furthermore, the simulation study included the estimators $\hat{\sigma}_0^2$, $\hat{\sigma}_l^2$, $\hat{\sigma}_r^2$, $\hat{c}_l' \hat{I}^{-1} \hat{c}_l$ and $\hat{c}_r' \hat{I}^{-1} \hat{c}_r$. The estimators were studied for the sample sizes $N = (250, 1000, 5000)$ to allow for investigation of the finite sample properties of the estimators.

The estimators $\hat{\sigma}^2$ and $\hat{c} \hat{I}^{-1} \hat{c}$ were evaluated for relative bias, $\text{RB}(\hat{\theta}) = E[\hat{\theta}/\theta]$, and mean squared error, $\text{MSE}(\hat{\theta}) = E[(\hat{\theta} - \theta)^2]$, where $\hat{\theta}$ and θ refer to any estimator and corresponding parameter of interest. For $\hat{V}$, the coverage rate of 95% CIs (using the normal approximation) for τ was used to evaluate the estimators. Estimates of RB, MSE, and the coverage rate were obtained by generating 1 008 data sets from each of 240 scenarios, with the scenarios being combinations of five propensity score designs, 16 outcome models, and the three sample sizes. The true parameter values of each scenario were calculated using numerical integration.

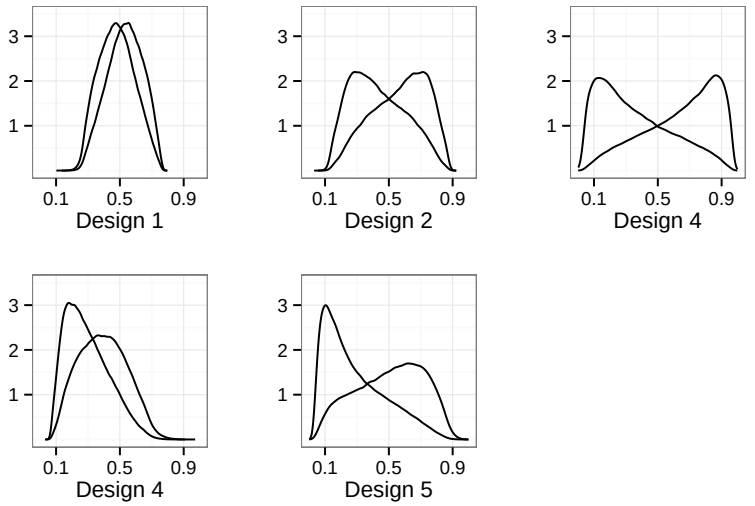


Figure 1: *Propensity score densities (conditional on treated to the right and conditional on controls to the left) in the simulation study.*

All scenarios included four independent covariates (continuous and discrete) chosen to resemble different types of variables present in a realistic setting. The covariates X_1 and X_2 followed a standard uniform distribution, X_3 was exponentially distributed with $\lambda = 1$, and X_4 was a binary variable generated by the decision rule $\mathbb{1}[X_4^* > 0.5]$, where X_4^* followed a standard uniform distribution. The simulation design was then separated into two parts: the distribution of the propensity score in the treatment and control group and the specification of the potential outcome models.

The propensity score designs were chosen to take into account different degrees of symmetry and overlap of the propensity score in the treatment and control group. More specifically, the propensity scores were specified by the models:

- Design 1: $p(X) = [1 + \exp(-1 + 1.25X_1 + 0.5X_2 - 0.1X_3 + 0.5X_4)]^{-1}$
- Design 2: $p(X) = [1 + \exp(-1.9 + 2.5X_1 + X_2 - 0.1X_3 + 0.5X_4)]^{-1}$
- Design 3: $p(X) = [1 + \exp(-3.2 + 4X_1 + 2X_2 - 0.3X_3 + X_4)]^{-1}$
- Design 4: $p(X) = [1 + \exp(0.75 - 2X_1 - X_2 + 0.2X_3 - 0.5X_4)]^{-1}$
- Design 5: $p(X) = [1 + \exp(1.8 - 3.5X_1 - 1.5X_2 + 0.25X_3 - 0.5X_4)]^{-1}$,

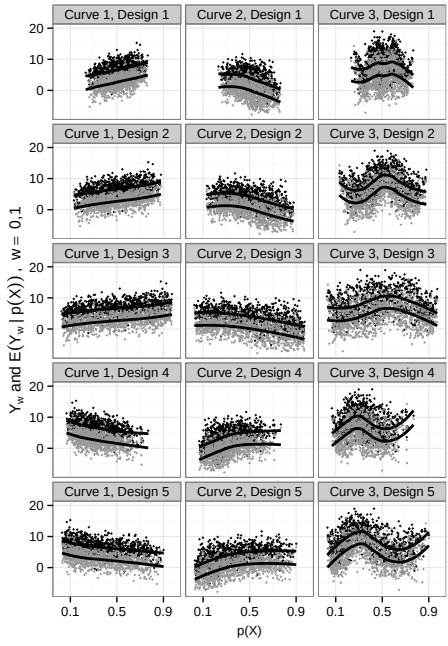
with treatments assigned accordingly by conducting N Bernoulli trials for each design. Figure 1 displays the propensity score distributions conditional on the treatment group for each design.

The first three propensity score designs were expected to yield equally sized treatment and control groups. By changing the degree of overlap, the estimators could be studied under various challenging conditions. In particular, lack

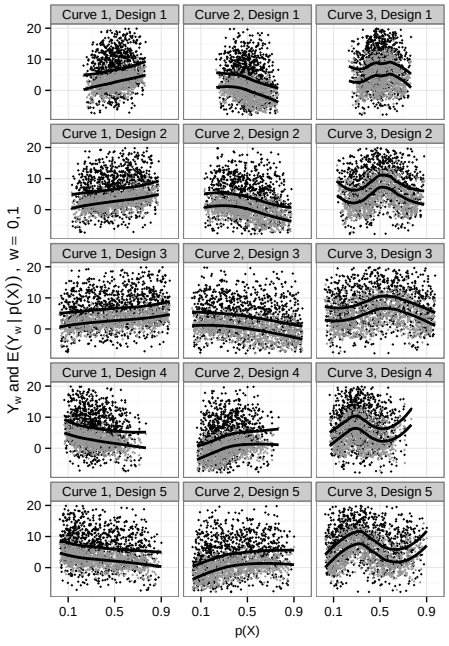
of density overlap was expected to cause a problem for the covariance estimators using matches from the opposite treatment group. Furthermore, Design 3 tested how the estimators handle propensity score values close to each boundary 0 and 1. The means of Design 4 and 5 were 0.33 and 0.37 respectively, leading to a larger control than treatment group. For cases with small treated versus control ratio, the nearest-neighbour matching might discard many observations that are not selected as matches, resulting in less power. In a similar fashion asymmetry could affect the conditional covariance estimator selecting matches from both treatment groups. It should be mentioned that Design 5 was also chosen to mirror the propensity score in the RHC data.

The second part of the design of the simulation study concerned the specification of the potential outcomes. We considered one binary and three continuous outcomes being influenced by four factors: 1) curvature of the conditional expectations, 2) equality of error term distributions of the potential outcomes, 3) constant or non-constant treatment treatment effect, and 4) constant or non-constant variance of the error terms. The outcome models were defined as follows:

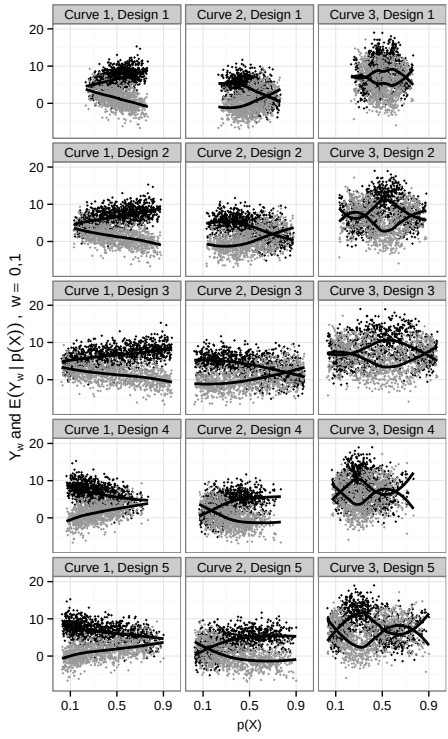
- Outcome models A
  (equal variances, constant treatment effect, constant variance)
  Curve 1: $Y_w = \alpha_w + 3X_1 + X_2 + 0.3\exp(1-X_3) + 0.5X_4 + \varepsilon_w$
  Curve 2: $Y_w = \alpha_w + 3X_1 - (3X_1)^2 + X_2 + 0.5X_3 + 0.5X_4 + \varepsilon_w$
  Curve 3: $Y_w = \alpha_w + 3X_1 - X_2 + (4\cos(4X_1 + 4X_2))^2 + 0.5X_3 + 0.5X_4 + \varepsilon_w$
  Curve 4: $Y_w = \mathbb{1}[\beta_w + 2X_1 + X_2 + 0.2X_3 + X_4 + \varepsilon_w > 4.5]$
  with $\alpha_0 = 0$, $\alpha_1 = 4$, $\beta_0 = 1.5$, $\beta_1 = 2$ and $\varepsilon_w \sim N(0,1)$.
- Outcome models B
  (unequal variances, constant treatment effect, constant variance)
  Curves are the same as in A, but $\varepsilon_0 \sim N(0,1)$ and $\varepsilon_1 \sim N(0,4)$.
- Outcome models C
  (equal variances, non-constant treatment effect, constant variance)
  Curve 1: $Y_1 = 4 + 3X_1 + X_2 + 0.3\exp(1-X_3) + 0.5X_4 + \varepsilon_w$
  Curve 1: $Y_0 = 4 - 3X_1 - X_2 - 0.3\exp(1-X_3) - 0.5X_4 - \varepsilon_w$
  Curve 2: $Y_1 = 4 + 3X_1 - (3X_1)^2 + X_2 + 0.5X_3 + 0.5X_4 + \varepsilon_w$
  Curve 2: $Y_0 = -3X_1 + (3X_1)^2 - X_2 - 0.5X_3 - 0.5X_4 - \varepsilon_w$
  Curve 3: $Y_1 = 4 + 2X_1 - X_2 + (4\cos(4X_1 + 4X_2))^2 + 0.5X_3 + 0.5X_4 + \varepsilon_w$
  Curve 3: $Y_0 = 10 - 2X_1 + 3X_2 - (4\cos(4X_1 + 2X_2))^2 - 0.3X_3 - 0.5X_4 + \varepsilon_w$
  Curve 4: $Y_1 = \mathbb{1}[2 + 2X_1 + X_2 + 0.2X_3 + X_4 + \varepsilon_w > 4.5]$
  Curve 4: $Y_0 = \mathbb{1}[\beta_w + 2X_1 + X_2 + 0.2X_3 + X_4 + \varepsilon_w > 4.5]$, $\varepsilon_w \sim N(0,1)$.
- Outcome models D
  (equal variances, constant treatment effect, non-constant variance)
  Curves are the same as in A, but $\varepsilon_w = \varepsilon_w^*(0.5 + 3p(X))$ and $\varepsilon_w^* \sim N(0,1)$.



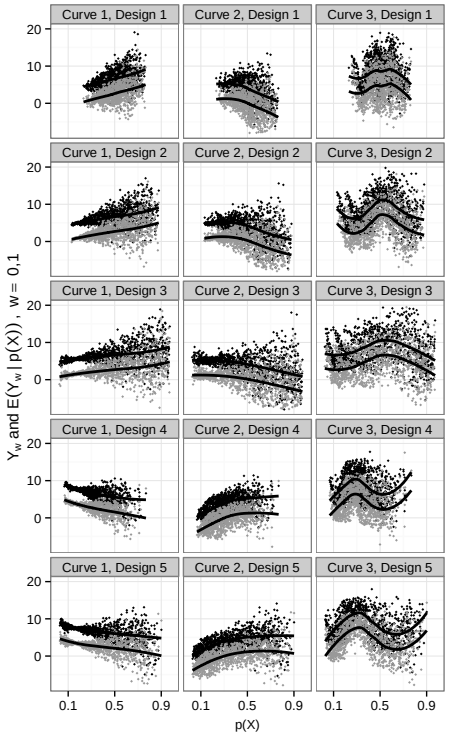
(a) *Outcome models A*



(b) *Outcome models B*



(c) *Outcome models C*



(d) *Outcome models D*

Figure 2: *Realised Y_1 (black) and Y_0 (grey), with $E(Y_1|p(X))$ and $E(Y_0|p(X))$, for all combinations of outcome models and propensity score designs ($N = 1000$).*

Figure 2 illustrates all possible outcome models for each propensity score design. Because previous results in Frölich (2004) suggest that the degree of linearity in $E(Y_w|p(X))$ affects local linear estimators to a larger extent than nearest-neighbour estimators, different curvatures were considered: approximately linear, concave, and a cosine shape. As for the other three factors, $\hat{\sigma}_0^2$ was expected to perform worse in the presence of unequal variances, heteroscedasticity or non-constant treatment. Note that several of these models give small $c'I^{-1}c$, which allows us to evaluate the estimators when the adjustment in fact should be close to zero.

Some remarks on the implementation of the local linear estimators are needed. Local linear estimation was standard, using the Epanechnikov kernel, $K_h(u) = (3/4)(1-u^2)1_{(|u|\leq 1)}$ with bandwidth selected by least squares cross-validation. The cross-validation is described in Li and Racine (2008). Here, we considered a fixed bandwidth window. The choice of kernel is usually not crucial, however bandwidth selection is key. Although a bandwidth selected by least squares cross-validation works fine in general, it does not guarantee a good bandwidth choice for a given data set. Needless to say, looking at every generated data set in a simulation study is not feasible. Still, this implies that, if possible, the asymptotic variance estimators based on local linear estimation could actually be improved upon in practice.

4.1 Simulation results

Figure 3 summarises the performance of the estimators for σ^2 in the 240 designs described earlier, providing the MSE of the estimators relative to the MSE of $\hat{\sigma}_2^2$ and the relative bias of all the estimators. Results for each model and estimator are listed in Tables A.1–A.6.

In Figure 3 we observe that although $\hat{\sigma}_2^2$ has the lowest bias, there is vast improvement in the MSE when selecting $L > 2$. According to the simulation results, we conjecture that L in $\hat{\sigma}_L^2$ should be from 5 to 15, as a larger L would lead to a larger bias that perhaps is of interest to avoid in some applications. Although the bias of the estimators decreases substantially with larger sample sizes, except for the estimator assuming constant variance, $\hat{\sigma}_0^2$, the conclusion regarding L is the same for all sample sizes. We observe that $\hat{\sigma}_0^2$ performs very poorly in several cases with non-constant variance and non-constant treatment effect. Concerning the local linear estimators, $\hat{\sigma}_r^2$ has in general lower MSE and bias than $\hat{\sigma}_l^2$. Also, but not reported, in some cases with smaller sample sizes $\hat{\sigma}_l^2$ resulted in negative variances (which were discarded). In comparison with the other estimators $\hat{\sigma}_r^2$ is comparable to, for instance, $\hat{\sigma}_{15}^2$. It should be noted that $\hat{\sigma}_r^2$ does have some problems with Design 3 (i.e. extreme propensity scores) having larger bias than several other estimators.

Figure 4 displays the MSE and relative bias for the estimators of $c'I^{-1}c$. A main finding is that the estimators have considerable bias in small samples, es-

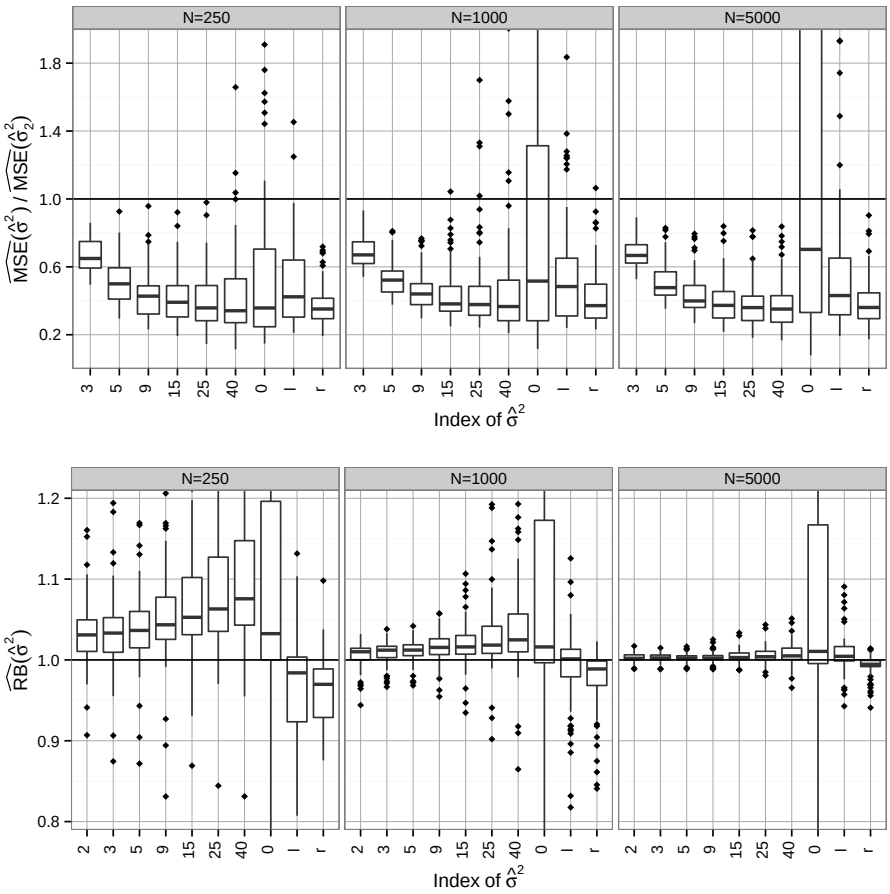
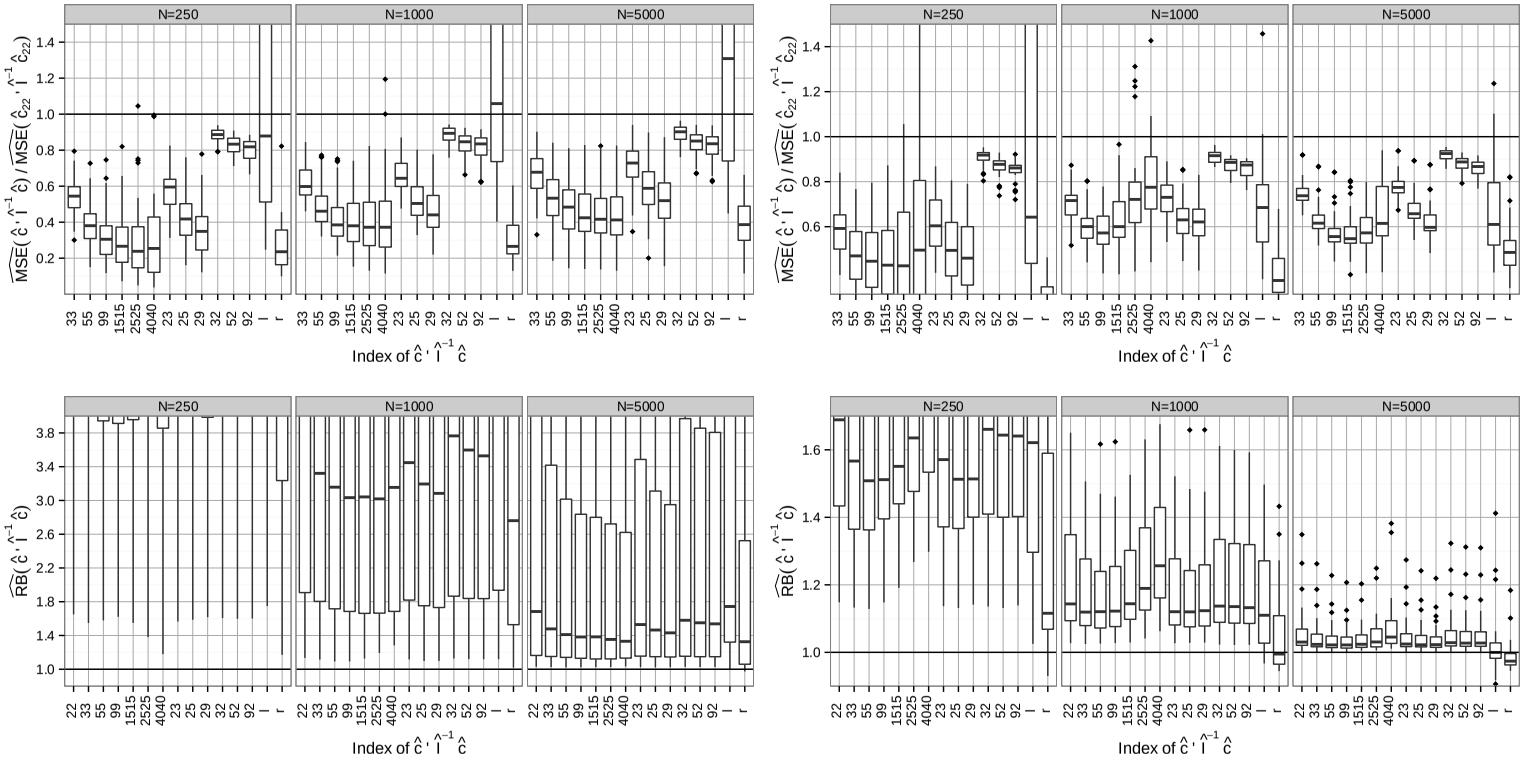


Figure 3: *MSE of $\hat{\sigma}^2$ relative to the MSE of $\hat{\sigma}_2^2$ (above) and relative bias of $\hat{\sigma}^2$ (below). The boxplots summarize results from the 80 designs and three sample sizes in Tables A.1–A.6. Note that the boxplots are cropped for clarity.*

pecially when $c'I^{-1}c$ is small. Numerical results for all models and estimators are given in Tables B.1–B.6. However, as the sample size increases, the bias decreases substantially, especially when $c'I^{-1}c > 0.5$. For both MSE and bias, $\hat{c}_r'\hat{I}^{-1}\hat{c}_r$ has the overall best performance, whereas $\hat{c}_l'\hat{I}^{-1}\hat{c}_l$ has a larger MSE. Among the other estimators, $\hat{c}_{55}'\hat{I}^{-1}\hat{c}_{55}$, $\hat{c}_{99}'\hat{I}^{-1}\hat{c}_{99}$, and $\hat{c}_{1515}'\hat{I}^{-1}\hat{c}_{1515}$ have the lowest MSE and bias. Interestingly, L'' seems to influence the MSE and bias more than L'.

Finally, in Figure 5 we see that the coverage rate for τ when using $\hat{V}$ clearly improves with sample size. For the largest sample size, $N = 5000$, all estimators except for $\hat{V}_{022}$, have a decent coverage rate, but $\hat{V}_{rr}$ performs slightly better than the other estimators.



(a) *45 models with* $c'I^{-1}c < 0.5$ **(b)** *35 models with* $c'I^{-1}c \geq 0.5$

Figure 4: *MSE of* $\hat{c}'\hat{I}^{-1}\hat{c}$ *relative to the MSE of* $\hat{c}'\hat{I}^{-1}\hat{c}$ *(above) and relative bias of* $\hat{c}'\hat{I}^{-1}\hat{c}$ *(below). The boxplots summarise results from the 80 designs and three sample sample sizes in Table B1–B6. Note that the boxplots are cropped for clarity.*

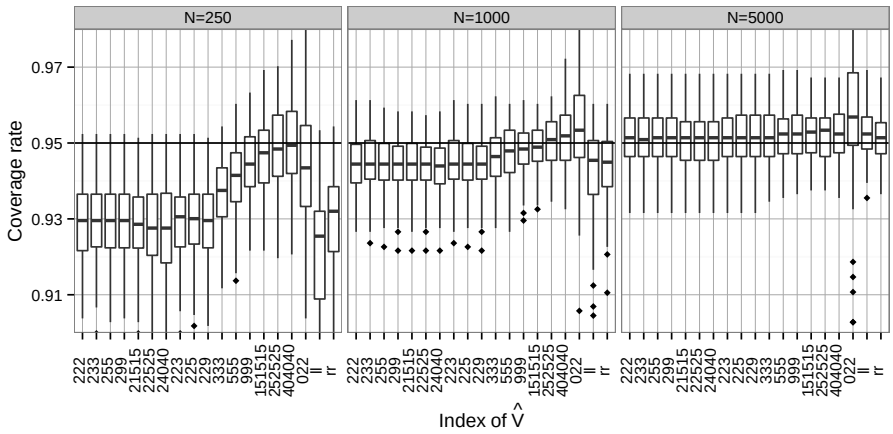


Figure 5: *Coverage rate for τ when using $\hat{V}$. The boxplots summarise results from the 80 designs and three sample sample sizes.*

5 Reanalysing the effect of right heart catheterisation

In this section we evaluate the effect of RHC during the first 24 hours in an intensive care unit on 30-day survival. The analysis is based on the SUPPORT study that gathered data at five US teaching hospitals between 1989 and 1994. In all, 5735 patients receiving intensive care were included in the study, of which 2184 (38%) were managed with RHC and 3551 without. Because the characteristics of a patient influence the decision to use RHC, a large set of relevant covariates was chosen by a panel of seven medical experts, minimising the risk of excluding a confounder. Lists of all covariates are given in the captions of Figures C.1 and C.3 in Appendix C, while a more comprehensive description is found in Hirano and Imbens (2001). Connors et al. (1996) (and references therein) provides a detailed description of the study.

In Connors et al. (1996) concerns were raised that RHC may be harmful to patients, which disagreed with the idea among clinicians that RHC was beneficial. Since then, several other studies (e.g., Hirano and Imbens, 2001; Li et al., 2008; Crump et al., 2009; Vansteelandt et al., 2012), using the same data but applying different statistical methods, have replicated the results in Connors et al. (1996) finding small to moderate, though significant, negative effects of RHC on patient survival. However, further sensitivity analysis by Lin et al. (1998) moderated the results of Connors et al. (1996). Furthermore, Tan (2006), using a distributional approach of the propensity score, and, more importantly, a meta-analysis by Shah et al. (2005) of randomised experiments, concluded that the use of RHC has no effect (positive or negative) on patient survival.

For each patient in the SUPPORT data, we observe treatment status (1 if RHC, 0 if no RHC), an indicator for 30-day survival (1 if survival, 0 if death)

and 21 continuous and 51 binary covariates. The unadjusted estimated ATE of RHC on 30-day survival is -0.0736 (95% CI: $-0.0990; -0.0483$). However, Figures C.1–C.3 clearly show that the covariate distribution in patients receiving RHC differs greatly from the covariate distribution in patients not receiving RHC. To adjust for this we estimate the ATE using 1:1 covariate matching yielding the estimate -0.0809 (95% CI: $-0.1136; -0.0481$). However, the matching estimator is biased when matching on several continuous covariates (Abadie and Imbens, 2006). To overcome the dimensionality problem we match on the propensity score instead.

5.1 Estimating ATE using propensity score matching

We estimate the propensity score using a logit model that includes all 72 covariates (no interactions or quadratic terms). The resulting sample from 1:1 matching on the estimated propensity score is denoted *Matched sample 1*. To evaluate balance we employ some diagnostic measures recommended by Rosenbaum and Rubin (1985), Rubin (2001), Ho et al. (2007) and Austin (2009). We begin by checking the balance of the continuous covariates. As seen in Figure C.1 (comparison of means and variances) and Figure C.2 (Q–Q plots) the propensity score matching improves the balance of many of the covariates in the two treatment groups, although a few covariates become marginally less balanced after matching. Looking at the standardised difference of means, only white blood count (WBC) has a difference larger than 10. With respect to the variance ratios, only albumin and WBC have ratios larger than 1.3. No variance ratio is less than 0.7. Moreover, when regressing the covariates one at time on the linear propensity score, and taking the ratios of the variances of the residuals, all ratios are between 0.94 and 1.05. The graphical comparison using Q–Q plots confirms that the covariate with the most imbalance seems to be WBC. To check the balance of the binary covariates Figure C.3 displays that the proportions in each treatment group are rather similar after matching. In a final check, we look at the propensity score distributions in the treatment groups and conclude that they are very similar in the matched sample (Figure C.4). Furthermore, the propensity score distribution in the original sample suggests that there is common support, although there could be issues with small overlap near the boundaries.

To improve balance in the two covariates with large variance ratios we estimate a second propensity score model by adding the squares of albumin and WBC to the first model (Rosenbaum and Rubin, 1984). Adding these two terms, however, causes the variance ratio of creatinine to drop to 0.65. Therefore, we also add the square of creatinine to the propensity score model. The propensity score model that includes the squares of these three covariates yields the *Matched sample 2*, containing no variance ratio smaller than 0.7 or

larger than 1.3. The other diagnostic measures are roughly similar to the ones based on *Matched sample 1*.

After evaluating the covariate balance, we now estimate the ATE. The estimated ATE in *Matched sample 1* using $\hat{\tau}$ is -0.0343 while the corresponding estimate in *Matched sample 2* is -0.0388. These two estimates are quite close to each other and are considerably smaller than the unadjusted estimate and the covariate matching estimate. To compare, we also estimate ATE with two other commonly used propensity score-based estimators: an inverse probability and a doubly robust estimator ($\hat{\Delta}_{IPW2}$ and $\hat{\Delta}_{DR}$ in Lunceford and Davidian, 2004). The augmented parts of the doubly robust estimator are based on logistic regressions including all covariates (no interaction or quadratic terms) irrespective of propensity score model. The estimates from the inverse probability and the doubly robust estimators vary from -0.0586 to -0.0657 (Table 1). If the RHC has no effect, as suggested by Shah et al. (2005), the propensity score matching estimator performs better than these two estimators.

To establish whether RHC has a significant effect or not we calculate standard errors and CIs. The results in Table 1 reveal that the choice of smoothing parameters when estimating the variance of $\hat{\tau}$ is crucial to the inference we make from the RHC study.

In *Matched sample 1* we observe a large variation in the estimates of σ^2 and that $\hat{\sigma}^2_L$ increases with the size of L, with the largest estimate, $\hat{\sigma}^2_{15} = 3.756$, being almost twice as large as the smallest $\hat{\sigma}^2_2 = 1.947$. Because the simulation study (for $N = 5000$) showed that most estimators for σ^2 performed rather well in terms of relative bias for (the large exception being σ^2_0), then based on the MSE we should at least select $L > 2$ in $\hat{\sigma}^2_L$. Moreover, both local linear-based estimators give similar estimates: $\hat{\sigma}^2_l = 3.495$ and $\hat{\sigma}^2_r = 3.496$. The estimates of $c'I^{-1}c$ do not vary much, except for $\hat{c}'_l\hat{I}^{-1}\hat{c}_l = 1.0053$, which is more than twice as large as any other estimate. Because the simulation study demonstrated that local linear estimation might be unreliable, this estimate is probably too large. On the other hand, the residual-based estimator was found to generally have the lowest MSE. Interestingly, $\hat{c}'_r\hat{I}^{-1}\hat{c}_r = 0.3756$ is the smallest estimate of $c'I^{-1}c$. To conclude, based on the recommendations from the simulation study and the estimates from the RHC data, the first term in V should be estimated with $\hat{\sigma}^2_L$, where $L > 2$, or with $\hat{\sigma}^2_r$. Further, we should not use $\hat{c}'_l\hat{I}^{-1}\hat{c}_l$ to estimate the second term. Following these guidelines, all of the CIs in *Matched sample 1* in Table 1 cover 0, i.e. we conclude that RHC has no effect on survival.

The conclusions from *Matched sample 2* are less distinct, but some results are similar. Again, $\hat{\sigma}^2_L$ varies substantially depending on the choice of L. On the other hand, $\hat{c}'_{L'L''}\hat{I}^{-1}\hat{c}_{L'L''}$ is rather insensitive to the choice of smoothing parameters. Even though $\hat{\sigma}^2_L$ is no longer monotonically increasing with the size of L, it still increases somewhat with L. Furthermore, the local linear-based estimators give similar estimates of σ^2, but considerably different esti-

Table 1: *Estimated ATE, standard errors and interval estimates in RHC study.*

Estimator for ATE	Estimated ATE	Estimator for variance	Estimated σ^2	Estimated $c'I^{-1}c$	Standard error	95% confidence interval
Matched sample 1						
1:1 Matching	-0.0343	$\hat{V}_{222}$	1.947	0.4498	0.0162	$(-0.0659, -0.0026)$
		$\hat{V}_{333}$	2.265	0.4640	0.0177	$(-0.0690, 0.0005)$
		$\hat{V}_{555}$	3.398	0.5007	0.0225	$(-0.0783, 0.0098)$
		$\hat{V}_{999}$	3.687	0.4808	0.0236	$(-0.0806, 0.0121)$
		$\hat{V}_{151515}$	3.756	0.4883	0.0239	$(-0.0810, 0.0125)$
		$\hat{V}_{322}$	2.265	0.4498	0.0178	$(-0.0691, 0.0006)$
		$\hat{V}_{522}$	3.398	0.4498	0.0227	$(-0.0787, 0.0102)$
		$\hat{V}_{922}$	3.687	0.4498	0.0238	$(-0.0808, 0.0123)$
		$\hat{V}_{223}$	1.947	0.4520	0.0161	$(-0.0659, -0.0026)$
		$\hat{V}_{225}$	1.947	0.5018	0.0159	$(-0.0653, -0.0032)$
		$\hat{V}_{229}$	1.947	0.4751	0.0160	$(-0.0657, -0.0029)$
		$\hat{V}_{232}$	1.947	0.4751	0.0161	$(-0.0659, -0.0027)$
		$\hat{V}_{252}$	1.947	0.4436	0.0162	$(-0.0659, -0.0026)$
		$\hat{V}_{292}$	1.947	0.4497	0.0162	$(-0.0659, -0.0026)$
		$\hat{V}_{022}$	3.387	0.4498	0.0226	$(-0.0786, 0.0101)$
		$\hat{V}_{ll}$	3.495	1.0053	0.0208	$(-0.0751, 0.0065)$
		$\hat{V}_{rr}$	3.496	0.3756	0.0233	$(-0.0801, 0.0114)$
IPW	-0.0593				0.0155	$(-0.0896, -0.0290)$
Doubly robust	-0.0657				0.0148	$(-0.0947, -0.0367)$
Matched sample 2						
1:1 Matching	-0.0387	$\hat{V}_{222}$	2.276	0.4615	0.0178	$(-0.0736, -0.0038)$
		$\hat{V}_{333}$	2.196	0.3998	0.0177	$(-0.0734, -0.0040)$
		$\hat{V}_{555}$	2.549	0.4015	0.0194	$(-0.0766, -0.0008)$
		$\hat{V}_{999}$	4.056	0.5031	0.0249	$(-0.0875, 0.0010)$
		$\hat{V}_{151515}$	4.076	0.4577	0.0251	$(-0.0845, 0.0069)$
		$\hat{V}_{322}$	2.196	0.4615	0.0174	$(-0.0728, -0.0046)$
		$\hat{V}_{522}$	2.549	0.4615	0.0191	$(-0.0761, -0.0013)$
		$\hat{V}_{922}$	4.056	0.4615	0.0250	$(-0.0878, 0.0104)$
		$\hat{V}_{223}$	2.276	0.4065	0.0181	$(-0.0741, -0.0033)$
		$\hat{V}_{225}$	2.276	0.4021	0.0181	$(-0.0742, -0.0033)$
		$\hat{V}_{229}$	2.276	0.5013	0.0176	$(-0.0732, -0.0042)$
		$\hat{V}_{232}$	2.276	0.4488	0.0179	$(-0.0737, -0.0037)$
		$\hat{V}_{252}$	2.276	0.4512	0.0178	$(-0.0737, -0.0038)$
		$\hat{V}_{292}$	2.276	0.4509	0.0178	$(-0.0737, -0.0038)$
		$\hat{V}_{022}$	3.664	0.4615	0.0236	$(-0.0850, 0.0076)$
		$\hat{V}_{ll}$	3.760	0.5867	0.0235	$(-0.0849, 0.0073)$
		$\hat{V}_{rr}$	3.774	0.3863	0.0243	$(-0.0864, 0.0088)$
IPW	-0.0586				0.0153	$(-0.0886, -0.0287)$
Doubly robust	-0.0635				0.0150	$(-0.0929, -0.0342)$

mates for $c'I^{-1}c$. The divergence from the previous results in *Matched sample 1* occurs if we in *Matched sample 2* were to use exactly the same recommendations for how to estimate V. As seen in Table 1, when simply using the guideline $L > 2$ several CIs do not cover 0. Only when putting $L \geq 9$, e.g., $\hat{\sigma}_9^2 = 4.056$, do we reach the conclusion of no effect. However, the conclusions from applying the other estimators remain unchanged; for instance, the residual-based estimator in *Matched sample 2* suggests no significant effect.

Moreover, if we follow the same guidelines as before but more strictly, we have a sample size close to $N = 5000$. Using the MSE in the simulation study as the main criterion, we should select $L = 9$ or the residual-based estimator, which would result in the conclusion that RHC has no effect on survival because the confidence interval would not cover 0.

6 Conclusions

In this paper we compare several estimators for the large sample variance of a matching estimator with the intention that the results may act as a guideline to practitioners wishing to implement matching estimators. The results enable us to make recommendations when estimating the large sample variance of the matching estimator, either when using the estimators suggested by Abadie and Imbens (2012) or when using the local linear-based estimators proposed in this paper.

Evidently, the matching sets that are used to estimate the different components of the variance should be specified to include more than two matches, which is in line with the results in Abadie and Imbens (2008). We have also seen that the residual-based local linear estimator is a viable option and that it in several settings is better in terms of MSE, bias and coverage rate compared with the other estimators. Although the residual-based estimator may be sensitive to extreme propensity scores, this forces the practitioner to pay special attention to the distribution of the propensity score. The problem of extreme propensity scores is also indicated for one of the local linear estimators in the application on the RHC data. Some other important findings are that the variance estimator assuming constant variance performs poorly (and thus should be avoided) and that the bias of all the estimators may be substantial in small samples. In particular, the estimators for the adjustment term are biased in small samples. Still, for the coverage rate for the ATE, most estimators work decently in larger samples.

The analysis of the RHC data provides a nice illustration of the importance of specifying the variance estimator. By carefully considering not only the implementation of the matching estimator, but also the estimation of its variance, we arrive at the conclusion that RHC has no effect on survival. This corresponds to findings in several randomised clinical trials (Shah et al., 2005).

Acknowledgements

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Appendix A. Simulation results of $\hat{\sigma}^2$

Table A.1: *MSE and relative MSE of estimators for* σ^2, $N = 250$.

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$								
				Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	A1	23.96	33.60	0.67	0.52	0.46	0.42	0.42	0.44	0.48	0.47	0.35
	A2	36.53	78.18	0.80	0.66	0.62	0.60	0.63	0.71	0.65	0.59	0.51
	A3	77.05	371.78	0.80	0.69	0.62	0.56	0.54	0.53	0.70	0.64	0.72
	A4	1.30	0.05	0.62	0.43	0.37	0.34	0.34	0.35	0.47	0.28	0.33
2	A1	26.81	114.86	0.60	0.39	0.30	0.25	0.23	0.22	0.22	0.37	0.21
	A2	36.98	180.10	0.65	0.53	0.49	0.46	0.47	0.54	0.39	0.42	0.32
	A3	70.15	1037.51	0.61	0.49	0.49	0.54	0.66	0.72	0.40	0.53	0.35
	A4	1.48	0.14	0.58	0.42	0.37	0.34	0.34	0.33	0.33	0.26	0.32
3	A1	40.62	379.05	0.76	0.57	0.44	0.37	0.33	0.30	0.27	0.95	0.40
	A2	56.85	946.57	0.65	0.47	0.44	0.40	0.39	0.40	0.27	0.87	0.40
	A3	112.21	3646.12	0.75	0.69	0.72	0.74	0.68	0.54	0.37	0.64	0.57
	A4	2.21	1.06	0.54	0.38	0.32	0.28	0.26	0.25	0.23	0.27	0.31
4	A1	30.97	196.47	0.84	0.66	0.47	0.41	0.38	0.38	0.31	0.94	0.36
	A2	46.50	772.51	0.63	0.57	0.44	0.42	0.37	0.31	0.21	0.38	0.37
	A3	92.64	1741.57	0.79	0.78	0.74	0.67	0.60	0.67	0.41	0.67	0.49
	A4	1.72	0.49	0.55	0.40	0.32	0.28	0.29	0.28	0.26	0.25	0.30
5	A1	38.18	542.30	0.54	0.41	0.32	0.31	0.27	0.25	0.20	0.58	0.27
	A2	52.94	1215.26	0.66	0.51	0.42	0.44	0.35	0.28	0.19	0.42	0.29
	A3	89.10	5328.46	0.69	0.61	0.58	0.51	0.42	0.55	0.25	0.41	0.26
	A4	2.11	1.17	0.55	0.38	0.30	0.28	0.28	0.27	0.23	0.26	0.29
1	B1	116.25	1264.28	0.66	0.48	0.40	0.36	0.34	0.32	0.31	0.36	0.34
	B2	128.82	1386.62	0.69	0.50	0.42	0.37	0.36	0.36	0.35	0.36	0.35
	B3	169.33	1814.37	0.76	0.59	0.50	0.45	0.44	0.41	0.47	0.49	0.47
	B4	1.37	0.05	0.57	0.39	0.32	0.30	0.28	0.28	0.46	0.27	0.30
2	B1	133.40	4579.10	0.63	0.40	0.31	0.24	0.22	0.19	0.16	0.22	0.22
	B2	143.57	5037.68	0.60	0.40	0.31	0.25	0.22	0.21	0.17	0.24	0.21
	B3	176.74	9545.29	0.50	0.30	0.23	0.21	0.21	0.22	0.15	0.21	0.19
	B4	1.58	0.15	0.54	0.37	0.32	0.30	0.29	0.27	0.33	0.25	0.28
3	B1	202.95	18189.40	0.75	0.47	0.40	0.33	0.29	0.26	0.18	0.41	0.36
	B2	219.19	21617.16	0.70	0.43	0.37	0.32	0.28	0.26	0.18	0.67	0.41
	B3	274.54	27894.17	0.73	0.55	0.48	0.42	0.37	0.31	0.23	0.46	0.36
	B4	2.40	1.00	0.54	0.37	0.31	0.28	0.27	0.26	0.28	0.32	0.34
4	B1	211.21	14674.39	0.82	0.62	0.44	0.38	0.33	0.31	0.33	0.46	0.34
	B2	226.72	18820.01	0.67	0.55	0.41	0.36	0.32	0.29	0.28	0.32	0.33
	B3	272.89	24047.17	0.71	0.61	0.48	0.40	0.35	0.33	0.25	0.42	0.36
	B4	1.81	0.47	0.53	0.38	0.32	0.28	0.29	0.28	0.30	0.26	0.28
5	B1	259.58	40068.31	0.52	0.40	0.31	0.30	0.26	0.23	0.22	0.29	0.29
	B2	274.28	44146.08	0.54	0.41	0.32	0.32	0.27	0.23	0.20	0.29	0.29
	B3	310.53	63074.55	0.55	0.43	0.36	0.32	0.27	0.27	0.16	0.38	0.31
	B4	2.22	1.12	0.54	0.36	0.30	0.28	0.27	0.27	0.26	0.27	0.29

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Table A.1 – *Continued from previous page, $N = 250$.*

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$								
				Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	C1	27.39	32.10	0.67	0.51	0.45	0.42	0.41	0.44	2.05	0.46	0.37
	C2	42.76	64.46	0.75	0.61	0.55	0.53	0.58	0.69	3.82	0.58	0.45
	C3	85.47	243.32	0.73	0.61	0.56	0.58	0.70	0.85	3.12	0.59	0.53
	C4	1.33	0.04	0.61	0.43	0.37	0.36	0.39	0.46	1.51	0.27	0.32
2	C1	30.51	114.94	0.60	0.39	0.31	0.25	0.23	0.22	1.06	0.30	0.23
	C2	45.84	175.26	0.62	0.50	0.45	0.42	0.43	0.50	3.46	0.43	0.31
	C3	88.90	833.21	0.53	0.38	0.37	0.44	0.60	0.74	3.02	0.58	0.26
	C4	1.45	0.12	0.64	0.43	0.39	0.38	0.41	0.48	1.11	0.26	0.32
3	C1	44.28	381.04	0.77	0.58	0.45	0.38	0.34	0.31	1.04	0.65	0.41
	C2	64.37	941.28	0.65	0.47	0.44	0.40	0.39	0.40	1.62	0.88	0.40
	C3	124.98	3459.43	0.74	0.69	0.72	0.75	0.70	0.56	1.57	0.66	0.55
	C4	2.01	0.93	0.62	0.44	0.38	0.35	0.35	0.38	0.61	0.25	0.30
4	C1	34.61	193.33	0.85	0.66	0.47	0.41	0.37	0.38	1.04	0.98	0.36
	C2	54.09	757.19	0.62	0.55	0.42	0.40	0.36	0.31	1.10	0.38	0.35
	C3	103.83	1610.30	0.77	0.78	0.73	0.68	0.64	0.71	1.76	0.66	0.42
	C4	1.83	0.47	0.54	0.39	0.31	0.28	0.29	0.28	0.34	0.25	0.29
5	C1	41.91	539.66	0.54	0.42	0.32	0.31	0.28	0.26	0.72	0.63	0.28
	C2	62.38	1206.41	0.65	0.51	0.43	0.44	0.35	0.29	1.44	0.44	0.29
	C3	111.29	5036.31	0.68	0.59	0.56	0.49	0.40	0.53	1.91	0.40	0.24
	C4	2.27	1.16	0.54	0.37	0.30	0.27	0.27	0.26	0.31	0.26	0.29
1	D1	27.82	63.65	0.69	0.52	0.43	0.37	0.33	0.31	0.42	0.43	0.35
	D2	40.39	120.01	0.77	0.65	0.56	0.53	0.52	0.53	0.55	0.66	0.49
	D3	80.91	421.59	0.78	0.68	0.59	0.53	0.50	0.47	0.66	0.61	0.68
	D4	1.18	0.05	0.62	0.47	0.43	0.42	0.45	0.50	0.47	0.31	0.36
2	D1	36.57	429.07	0.55	0.36	0.26	0.19	0.15	0.11	0.16	0.38	0.25
	D2	46.74	481.70	0.64	0.46	0.39	0.31	0.26	0.22	0.25	0.42	0.32
	D3	79.92	1327.50	0.63	0.50	0.47	0.47	0.50	0.48	0.35	0.42	0.36
	D4	1.22	0.12	0.65	0.47	0.46	0.49	0.60	0.71	0.38	0.31	0.36
3	D1	79.49	2856.56	0.69	0.56	0.35	0.28	0.27	0.27	0.22	0.83	0.63
	D2	95.72	4283.64	0.61	0.50	0.31	0.23	0.19	0.17	0.18	0.62	0.52
	D3	151.07	7067.20	0.69	0.58	0.48	0.38	0.28	0.20	0.26	0.64	0.45
	D4	1.64	0.72	0.57	0.43	0.39	0.42	0.53	0.62	0.23	0.34	0.35
4	D1	10.52	15.24	0.75	0.59	0.50	0.51	0.74	1.66	2.07	2.90	0.61
	D2	26.07	179.88	0.76	0.72	0.69	0.71	0.71	0.75	0.55	0.87	0.53
	D3	72.17	820.31	0.86	0.93	0.96	0.92	0.90	1.15	0.74	1.25	0.69
	D4	1.27	0.36	0.78	0.71	0.75	0.84	0.98	1.00	0.46	0.50	0.50
5	D1	24.04	170.95	0.61	0.39	0.27	0.24	0.25	0.45	0.86	1.45	0.70
	D2	38.82	356.49	0.75	0.63	0.50	0.49	0.47	0.59	0.65	0.84	0.57
	D3	74.93	2519.70	0.84	0.80	0.79	0.72	0.63	1.04	0.55	0.97	0.36
	D4	1.75	1.08	0.63	0.50	0.49	0.49	0.49	0.47	0.30	0.36	0.37

Table A.2: *MSE and relative MSE of estimators for* σ^2, $N = 1000$.

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$								
				Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	A1	23.96	6.30	0.67	0.52	0.45	0.42	0.41	0.40	0.51	0.49	0.37
	A2	36.53	18.37	0.78	0.70	0.65	0.63	0.63	0.64	0.76	0.58	0.61
	A3	77.05	116.15	0.89	0.81	0.76	0.71	0.66	0.61	0.82	0.67	0.86
	A4	1.30	0.01	0.62	0.46	0.38	0.34	0.32	0.32	0.47	0.28	0.34
2	A1	26.81	16.08	0.68	0.50	0.44	0.41	0.39	0.36	0.33	0.64	0.35
	A2	36.98	28.94	0.72	0.59	0.56	0.53	0.53	0.54	0.54	0.72	0.48
	A3	70.15	178.10	0.86	0.76	0.76	0.79	0.80	0.83	0.80	0.65	0.63
	A4	1.48	0.03	0.65	0.46	0.39	0.38	0.37	0.36	0.35	0.31	0.36
3	A1	40.62	179.17	0.76	0.59	0.47	0.37	0.33	0.28	0.22	0.83	0.29
	A2	56.85	344.78	0.66	0.53	0.45	0.41	0.37	0.35	0.29	0.67	0.35
	A3	112.21	1477.10	0.60	0.45	0.37	0.38	0.47	0.68	0.37	0.45	0.39
	A4	2.21	0.36	0.65	0.44	0.35	0.33	0.33	0.32	0.27	0.30	0.35
4	A1	30.97	58.20	0.55	0.41	0.35	0.31	0.30	0.28	0.21	1.17	0.26
	A2	46.50	148.44	0.61	0.51	0.41	0.41	0.42	0.44	0.23	0.55	0.40
	A3	92.64	380.74	0.69	0.57	0.54	0.59	0.80	1.11	0.61	0.91	0.58
	A4	1.72	0.09	0.58	0.41	0.34	0.32	0.31	0.32	0.28	0.28	0.34
5	A1	38.18	216.73	0.93	0.56	0.44	0.38	0.31	0.26	0.19	1.24	0.26
	A2	52.94	528.49	0.64	0.48	0.39	0.37	0.35	0.36	0.15	0.40	0.28
	A3	89.10	720.56	0.78	0.63	0.67	0.88	1.33	2.00	0.58	0.61	0.58
	A4	2.11	0.46	0.57	0.39	0.31	0.28	0.27	0.27	0.23	0.26	0.30
1	B1	116.25	249.74	0.65	0.52	0.45	0.42	0.41	0.40	0.36	0.38	0.38
	B2	128.82	273.31	0.66	0.53	0.46	0.43	0.42	0.41	0.40	0.40	0.39
	B3	169.33	448.40	0.72	0.59	0.53	0.49	0.48	0.46	0.52	0.48	0.55
	B4	1.37	0.01	0.54	0.38	0.32	0.29	0.28	0.27	0.48	0.25	0.28
2	B1	133.40	637.97	0.65	0.52	0.48	0.44	0.39	0.37	0.23	0.39	0.36
	B2	143.57	662.74	0.66	0.54	0.50	0.45	0.40	0.38	0.27	0.44	0.38
	B3	176.74	1037.93	0.68	0.54	0.50	0.48	0.45	0.43	0.36	0.42	0.40
	B4	1.58	0.03	0.63	0.42	0.35	0.34	0.32	0.32	0.39	0.30	0.34
3	B1	202.95	8526.93	0.73	0.53	0.38	0.29	0.28	0.23	0.12	0.30	0.26
	B2	219.19	8951.31	0.75	0.54	0.39	0.29	0.28	0.24	0.13	0.31	0.28
	B3	274.54	15188.61	0.57	0.41	0.30	0.25	0.24	0.23	0.13	0.36	0.23
	B4	2.40	0.41	0.57	0.38	0.31	0.29	0.29	0.28	0.27	0.29	0.31
4	B1	211.21	4762.32	0.54	0.40	0.34	0.30	0.28	0.26	0.75	0.31	0.25
	B2	226.72	5547.50	0.55	0.42	0.34	0.30	0.29	0.27	0.67	0.26	0.28
	B3	272.89	6071.07	0.56	0.43	0.38	0.36	0.39	0.41	0.59	0.48	0.34
	B4	1.81	0.10	0.54	0.39	0.31	0.29	0.29	0.28	0.29	0.27	0.29
5	B1	259.58	17497.52	0.92	0.55	0.43	0.37	0.30	0.25	0.34	0.33	0.25
	B2	274.28	21223.97	0.80	0.50	0.39	0.34	0.28	0.24	0.29	0.24	0.24
	B3	310.53	20348.49	0.86	0.55	0.46	0.43	0.41	0.41	0.28	0.43	0.28
	B4	2.22	0.43	0.58	0.38	0.31	0.28	0.27	0.27	0.27	0.27	0.29

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Table A.2 – *Continued from previous page, $N = 1000$.*

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$								
				Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	C1	27.39	6.41	0.68	0.54	0.48	0.44	0.42	0.41	7.56	0.45	0.40
	C2	42.76	13.49	0.72	0.59	0.53	0.50	0.50	0.51	12.82	0.52	0.49
	C3	85.47	52.81	0.74	0.57	0.51	0.48	0.50	0.60	10.46	0.55	0.55
	C4	1.33	0.01	0.64	0.46	0.38	0.35	0.34	0.34	5.40	0.28	0.35
2	C1	30.51	16.93	0.70	0.52	0.47	0.44	0.41	0.38	5.22	0.52	0.38
	C2	45.84	28.18	0.68	0.57	0.52	0.48	0.48	0.48	17.53	0.73	0.45
	C3	88.90	108.93	0.77	0.62	0.61	0.64	0.65	0.69	20.22	0.59	0.47
	C4	1.45	0.02	0.68	0.48	0.41	0.39	0.39	0.42	3.71	0.28	0.34
3	C1	44.28	180.06	0.76	0.59	0.47	0.37	0.33	0.29	1.65	0.54	0.29
	C2	64.37	345.90	0.66	0.54	0.46	0.41	0.37	0.35	3.68	0.67	0.36
	C3	124.98	1449.76	0.59	0.44	0.35	0.36	0.45	0.67	3.24	0.49	0.37
	C4	2.01	0.33	0.71	0.49	0.39	0.34	0.35	0.36	1.36	0.27	0.34
4	C1	34.61	58.81	0.56	0.42	0.36	0.32	0.30	0.29	2.33	1.20	0.26
	C2	54.09	143.41	0.59	0.48	0.39	0.38	0.39	0.42	3.53	0.52	0.37
	C3	103.83	313.92	0.63	0.48	0.44	0.49	0.74	1.16	5.76	0.95	0.47
	C4	1.83	0.09	0.58	0.41	0.34	0.32	0.33	0.33	0.63	0.29	0.35
5	C1	41.91	215.89	0.93	0.56	0.44	0.38	0.32	0.26	1.28	1.28	0.26
	C2	62.38	532.35	0.63	0.48	0.39	0.36	0.35	0.36	2.83	0.40	0.28
	C3	111.29	658.05	0.75	0.59	0.61	0.83	1.31	2.05	13.38	0.62	0.53
	C4	2.27	0.45	0.56	0.38	0.30	0.27	0.26	0.27	0.34	0.25	0.29
1	D1	27.82	14.49	0.65	0.52	0.45	0.40	0.37	0.34	0.39	0.48	0.42
	D2	40.39	28.22	0.74	0.67	0.60	0.57	0.55	0.54	0.62	0.55	0.56
	D3	80.91	125.29	0.89	0.80	0.75	0.70	0.65	0.61	0.80	0.65	0.86
	D4	1.18	0.01	0.64	0.47	0.41	0.38	0.37	0.40	0.48	0.31	0.35
2	D1	36.57	90.00	0.68	0.48	0.39	0.34	0.30	0.25	0.20	0.51	0.38
	D2	46.74	112.59	0.67	0.52	0.45	0.39	0.36	0.32	0.26	0.45	0.43
	D3	79.92	261.18	0.81	0.68	0.66	0.65	0.64	0.63	0.60	0.58	0.58
	D4	1.22	0.02	0.61	0.44	0.38	0.37	0.40	0.49	0.49	0.33	0.34
3	D1	79.49	1690.90	0.81	0.64	0.49	0.38	0.29	0.26	0.20	0.79	0.51
	D2	95.72	2457.35	0.67	0.53	0.42	0.35	0.26	0.21	0.14	0.54	0.41
	D3	151.07	2678.27	0.78	0.60	0.49	0.43	0.42	0.44	0.25	0.58	0.51
	D4	1.64	0.22	0.59	0.42	0.36	0.33	0.33	0.36	0.33	0.41	0.38
4	D1	10.52	3.43	0.67	0.54	0.43	0.38	0.36	0.33	3.86	9.22	1.06
	D2	26.07	23.07	0.86	0.75	0.72	0.76	0.83	0.96	1.00	1.84	0.73
	D3	72.17	201.57	0.80	0.70	0.69	0.74	1.02	1.50	1.10	1.26	0.83
	D4	1.27	0.05	0.70	0.55	0.54	0.66	0.94	1.58	1.30	0.58	0.48
5	D1	24.04	50.86	0.66	0.45	0.36	0.33	0.29	0.28	1.65	3.61	0.93
	D2	38.82	89.97	0.72	0.59	0.51	0.49	0.50	0.51	1.14	1.38	0.51
	D3	74.93	338.12	0.79	0.67	0.77	1.04	1.70	2.81	1.58	1.24	0.72
	D4	1.75	0.30	0.70	0.52	0.47	0.49	0.59	0.76	0.57	0.43	0.51

Table A.3: *MSE and relative MSE of estimators for* σ^2, $N = 5000$.

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$, Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	A1	23.96	1.20	0.73	0.56	0.48	0.41	0.39	0.38	0.56	0.77	0.40
	A2	36.53	4.24	0.85	0.75	0.70	0.62	0.61	0.61	0.82	0.65	0.66
	A3	77.05	23.19	0.89	0.83	0.80	0.84	0.81	0.78	0.81	0.64	0.81
	A4	1.30	0.00	0.57	0.42	0.34	0.30	0.28	0.27	0.56	0.29	0.36
2	A1	26.81	3.10	0.65	0.51	0.43	0.41	0.38	0.36	0.35	0.68	0.34
	A2	36.98	5.78	0.71	0.59	0.51	0.48	0.46	0.45	0.56	0.56	0.48
	A3	70.15	36.69	0.85	0.78	0.76	0.75	0.78	0.84	0.98	0.78	0.69
	A4	1.48	0.01	0.62	0.43	0.36	0.36	0.34	0.34	0.38	0.29	0.35
3	A1	40.62	81.15	0.65	0.40	0.31	0.26	0.23	0.20	0.14	0.48	0.18
	A2	56.85	152.23	0.53	0.36	0.27	0.23	0.21	0.20	0.24	0.44	0.25
	A3	112.21	358.56	0.65	0.52	0.41	0.41	0.42	0.43	0.74	0.43	0.36
	A4	2.21	0.12	0.60	0.43	0.36	0.30	0.29	0.27	0.25	0.27	0.29
4	A1	30.97	11.11	0.62	0.43	0.37	0.29	0.28	0.28	0.22	1.03	0.27
	A2	46.50	24.08	0.63	0.47	0.41	0.41	0.41	0.42	0.30	0.42	0.38
	A3	92.64	74.18	0.74	0.62	0.57	0.50	0.50	0.50	0.83	0.93	0.61
	A4	1.72	0.02	0.65	0.44	0.38	0.33	0.31	0.30	0.33	0.33	0.40
5	A1	38.18	52.33	0.67	0.45	0.36	0.29	0.28	0.26	0.18	1.93	0.22
	A2	52.94	89.65	0.68	0.47	0.40	0.37	0.38	0.38	0.21	0.32	0.35
	A3	89.10	177.38	0.77	0.59	0.52	0.47	0.42	0.43	0.65	0.76	0.47
	A4	2.11	0.08	0.64	0.49	0.41	0.33	0.32	0.32	0.29	0.35	0.39
1	B1	116.25	45.50	0.70	0.55	0.47	0.45	0.44	0.42	0.40	0.40	0.39
	B2	128.82	51.45	0.72	0.57	0.49	0.46	0.44	0.43	0.46	0.42	0.43
	B3	169.33	84.00	0.78	0.64	0.57	0.57	0.55	0.53	0.55	0.47	0.57
	B4	1.37	0.00	0.56	0.40	0.33	0.28	0.26	0.26	0.58	0.28	0.31
2	B1	133.40	131.29	0.65	0.50	0.43	0.39	0.36	0.35	0.23	0.35	0.32
	B2	143.57	136.44	0.66	0.51	0.43	0.39	0.36	0.35	0.26	0.43	0.34
	B3	176.74	199.59	0.71	0.56	0.50	0.48	0.47	0.46	0.39	0.43	0.42
	B4	1.58	0.01	0.56	0.39	0.33	0.31	0.30	0.30	0.44	0.28	0.30
3	B1	202.95	3874.87	0.60	0.39	0.29	0.23	0.20	0.18	0.08	0.19	0.18
	B2	219.19	4340.80	0.58	0.37	0.27	0.22	0.18	0.17	0.09	0.20	0.17
	B3	274.54	4527.09	0.62	0.43	0.33	0.27	0.24	0.24	0.16	0.37	0.27
	B4	2.40	0.14	0.55	0.38	0.33	0.27	0.26	0.25	0.27	0.27	0.29
4	B1	211.21	877.67	0.61	0.43	0.37	0.29	0.28	0.27	3.87	0.34	0.27
	B2	226.72	978.91	0.60	0.43	0.36	0.30	0.29	0.29	3.57	0.33	0.27
	B3	272.89	1128.11	0.63	0.46	0.40	0.32	0.31	0.31	2.68	0.45	0.35
	B4	1.81	0.02	0.59	0.40	0.34	0.29	0.28	0.27	0.34	0.29	0.30
5	B1	259.58	4077.93	0.68	0.44	0.36	0.28	0.28	0.26	1.24	0.32	0.22
	B2	274.28	4302.85	0.69	0.44	0.37	0.30	0.29	0.28	1.20	0.30	0.25
	B3	310.53	4450.88	0.74	0.48	0.41	0.32	0.30	0.28	1.01	0.40	0.28
	B4	2.22	0.10	0.57	0.41	0.34	0.28	0.27	0.26	0.28	0.30	0.31

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Table A.3 – *Continued from previous page, $N = 5000$.*

Design	Curve	σ^2	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{\sigma}^2)/\widehat{\text{MSE}}(\sigma_2^2)$								
				Index of $\hat{\sigma}^2$								
			2	3	5	9	15	25	40	0	l	r
1	C1	27.39	1.16	0.73	0.57	0.49	0.46	0.44	0.43	38.43	0.67	0.42
	C2	42.76	2.81	0.75	0.60	0.53	0.53	0.50	0.50	55.68	0.54	0.50
	C3	85.47	10.52	0.78	0.64	0.58	0.52	0.49	0.47	34.48	0.52	0.56
	C4	1.33	0.00	0.58	0.42	0.34	0.28	0.27	0.26	22.18	0.26	0.34
2	C1	30.51	3.20	0.66	0.52	0.44	0.43	0.40	0.38	23.76	0.52	0.35
	C2	45.84	5.39	0.70	0.58	0.50	0.43	0.42	0.42	84.32	0.57	0.49
	C3	88.90	19.60	0.72	0.57	0.54	0.56	0.61	0.72	104.13	0.61	0.46
	C4	1.45	0.00	0.63	0.44	0.36	0.34	0.32	0.33	16.01	0.29	0.35
3	C1	44.28	81.26	0.65	0.40	0.31	0.27	0.23	0.20	3.03	0.31	0.18
	C2	64.37	152.16	0.53	0.35	0.27	0.23	0.22	0.20	7.72	0.46	0.25
	C3	124.98	348.97	0.65	0.50	0.40	0.41	0.42	0.42	12.01	0.45	0.35
	C4	2.01	0.09	0.69	0.50	0.38	0.37	0.35	0.34	4.12	0.27	0.30
4	C1	34.61	11.26	0.62	0.44	0.37	0.30	0.29	0.29	10.08	1.06	0.27
	C2	54.09	23.27	0.62	0.45	0.39	0.37	0.37	0.38	18.98	0.39	0.36
	C3	103.83	59.17	0.66	0.51	0.46	0.38	0.37	0.37	24.00	0.95	0.49
	C4	1.83	0.02	0.63	0.42	0.37	0.31	0.29	0.29	1.52	0.31	0.38
5	C1	41.91	52.51	0.67	0.45	0.37	0.29	0.28	0.26	4.21	1.93	0.22
	C2	62.38	89.71	0.67	0.47	0.39	0.36	0.37	0.38	14.07	0.32	0.35
	C3	111.29	161.75	0.74	0.54	0.47	0.41	0.37	0.37	50.53	0.79	0.42
	C4	2.27	0.08	0.63	0.47	0.40	0.32	0.31	0.31	0.76	0.34	0.39
1	D1	27.82	2.73	0.73	0.55	0.46	0.40	0.37	0.37	0.45	0.72	0.43
	D2	40.39	6.05	0.82	0.68	0.62	0.58	0.56	0.56	0.66	0.63	0.58
	D3	80.91	25.23	0.88	0.82	0.78	0.80	0.78	0.75	0.78	0.64	0.79
	D4	1.18	0.00	0.61	0.47	0.40	0.33	0.32	0.31	0.56	0.34	0.37
2	D1	36.57	17.03	0.70	0.54	0.45	0.41	0.38	0.35	0.33	0.64	0.45
	D2	46.74	21.62	0.70	0.58	0.47	0.44	0.42	0.39	0.30	0.49	0.43
	D3	79.92	53.99	0.79	0.67	0.64	0.62	0.62	0.64	0.63	0.65	0.58
	D4	1.22	0.00	0.62	0.44	0.37	0.36	0.35	0.36	0.66	0.30	0.32
3	D1	79.49	788.81	0.77	0.46	0.35	0.33	0.27	0.23	0.25	0.38	0.29
	D2	95.72	1056.88	0.63	0.40	0.30	0.27	0.24	0.20	0.14	0.30	0.25
	D3	151.07	1131.58	0.71	0.47	0.36	0.38	0.32	0.26	0.16	0.40	0.34
	D4	1.64	0.08	0.59	0.41	0.33	0.26	0.25	0.23	0.42	0.28	0.30
4	D1	10.52	0.83	0.69	0.52	0.45	0.39	0.36	0.35	11.61	22.75	0.62
	D2	26.07	4.87	0.75	0.65	0.61	0.64	0.64	0.67	1.64	2.60	0.63
	D3	72.17	39.76	0.82	0.75	0.71	0.65	0.65	0.65	2.19	1.74	0.90
	D4	1.27	0.01	0.74	0.63	0.57	0.55	0.54	0.57	6.08	0.49	0.45
5	D1	24.04	15.69	0.65	0.46	0.36	0.37	0.33	0.31	3.78	10.40	0.40
	D2	38.82	27.02	0.67	0.49	0.39	0.40	0.37	0.38	2.11	1.49	0.42
	D3	74.93	91.11	0.79	0.63	0.58	0.56	0.50	0.52	2.64	1.20	0.59
	D4	1.75	0.06	0.83	0.61	0.46	0.41	0.41	0.44	1.68	0.39	0.43

Table A.4: *Relative bias of estimators for* σ^2, $N = 250$.

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	A1	23.96	1.06	1.06	1.05	1.06	1.06	1.06	1.07	1.04	1.02	1.02
	A2	36.53	1.03	1.03	1.03	1.04	1.05	1.07	1.09	1.03	1.01	0.99
	A3	77.05	1.00	1.00	1.01	1.02	1.04	1.08	1.10	0.99	0.99	0.94
	A4	1.30	1.03	1.03	1.03	1.03	1.04	1.04	1.05	1.01	1.00	0.99
2	A1	26.81	1.05	1.05	1.04	1.04	1.05	1.06	1.07	1.03	0.96	0.99
	A2	36.98	1.06	1.06	1.06	1.08	1.09	1.11	1.15	1.06	0.98	1.00
	A3	70.15	1.11	1.10	1.11	1.13	1.18	1.26	1.31	1.09	1.03	0.97
	A4	1.48	1.03	1.04	1.04	1.05	1.05	1.06	1.07	1.01	0.99	0.99
3	A1	40.62	0.98	0.98	0.99	1.00	1.00	1.01	1.02	0.97	0.83	0.91
	A2	56.85	1.02	1.02	1.03	1.06	1.08	1.11	1.15	1.03	0.88	0.95
	A3	112.21	1.03	1.05	1.09	1.17	1.24	1.27	1.25	1.07	0.86	0.88
	A4	2.21	1.01	1.00	1.01	1.03	1.03	1.04	1.04	0.97	0.90	0.92
4	A1	30.97	1.05	1.06	1.06	1.06	1.07	1.08	1.10	1.04	1.01	0.99
	A2	46.50	1.08	1.07	1.10	1.11	1.13	1.15	1.14	1.03	1.06	1.02
	A3	92.64	1.04	1.06	1.10	1.14	1.18	1.19	1.22	1.06	1.02	0.99
	A4	1.72	1.06	1.05	1.05	1.06	1.07	1.07	1.08	1.01	1.00	1.00
5	A1	38.18	1.00	0.99	1.01	1.02	1.03	1.05	1.06	1.00	0.91	0.93
	A2	52.94	1.05	1.07	1.07	1.10	1.14	1.14	1.13	1.00	1.01	0.96
	A3	89.10	1.16	1.18	1.25	1.34	1.37	1.36	1.42	1.13	1.02	0.98
	A4	2.11	1.02	1.03	1.04	1.05	1.05	1.06	1.06	1.00	0.96	0.97
1	B1	116.25	1.03	1.03	1.02	1.03	1.02	1.03	1.03	1.03	0.99	0.99
	B2	128.82	1.02	1.02	1.02	1.02	1.02	1.03	1.04	1.02	0.99	0.99
	B3	169.33	1.01	1.01	1.01	1.02	1.03	1.04	1.05	1.01	0.98	0.96
	B4	1.37	1.02	1.02	1.02	1.02	1.02	1.02	1.02	1.00	1.00	0.99
2	B1	133.40	1.03	1.03	1.03	1.02	1.02	1.03	1.03	1.02	0.96	0.97
	B2	143.57	1.04	1.03	1.04	1.04	1.04	1.04	1.06	1.03	0.96	0.98
	B3	176.74	1.06	1.05	1.05	1.06	1.08	1.11	1.14	1.04	0.97	0.96
	B4	1.58	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.00	0.98	0.98
3	B1	202.95	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.96	0.88	0.91
	B2	219.19	0.98	0.99	0.99	1.01	1.01	1.02	1.02	0.98	0.88	0.92
	B3	274.54	0.98	1.00	1.02	1.06	1.08	1.10	1.09	1.00	0.84	0.88
	B4	2.40	0.99	0.99	0.99	1.00	0.99	0.99	0.98	0.93	0.89	0.90
4	B1	211.21	1.04	1.05	1.04	1.04	1.04	1.04	1.05	0.74	0.97	0.97
	B2	226.72	1.05	1.04	1.05	1.05	1.06	1.06	1.06	0.76	0.98	0.98
	B3	272.89	1.04	1.05	1.06	1.07	1.08	1.09	1.10	0.82	0.98	0.98
	B4	1.81	1.05	1.03	1.03	1.04	1.04	1.04	1.04	1.01	0.98	0.98
5	B1	259.58	0.98	0.97	0.99	0.99	1.01	1.01	1.02	0.72	0.89	0.91
	B2	274.28	0.99	0.99	1.00	1.01	1.03	1.03	1.03	0.73	0.93	0.92
	B3	310.53	1.04	1.03	1.06	1.09	1.11	1.10	1.13	0.80	0.94	0.93
	B4	2.22	0.99	1.00	1.01	1.01	1.02	1.02	1.03	0.99	0.93	0.93

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Table A.4 – *Continued from previous page, $N = 250$.*

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	C1	27.39	1.03	1.03	1.02	1.03	1.03	1.03	1.04	1.22	1.00	0.99
	C2	42.76	1.02	1.02	1.02	1.03	1.03	1.05	1.07	1.29	1.00	0.98
	C3	85.47	1.02	1.02	1.03	1.04	1.06	1.09	1.11	1.24	1.00	0.97
	C4	1.33	1.02	1.02	1.02	1.03	1.03	1.04	1.06	1.14	0.99	0.99
2	C1	30.51	1.03	1.03	1.02	1.03	1.03	1.04	1.05	1.28	0.96	0.98
	C2	45.84	1.03	1.03	1.03	1.04	1.05	1.07	1.10	1.45	0.96	0.98
	C3	88.90	1.06	1.05	1.06	1.08	1.12	1.18	1.22	1.49	1.00	0.95
	C4	1.45	1.03	1.03	1.04	1.05	1.06	1.08	1.10	1.20	0.99	0.99
3	C1	44.28	0.98	0.98	0.99	1.00	1.00	1.00	1.02	1.27	0.83	0.91
	C2	64.37	1.01	1.01	1.01	1.05	1.06	1.09	1.12	1.45	0.88	0.94
	C3	124.98	1.02	1.04	1.08	1.15	1.21	1.24	1.22	1.45	0.86	0.89
	C4	2.01	1.03	1.04	1.06	1.08	1.10	1.13	1.16	1.25	0.94	0.95
4	C1	34.61	1.03	1.04	1.04	1.04	1.05	1.06	1.08	1.29	0.99	0.98
	C2	54.09	1.05	1.05	1.07	1.08	1.10	1.11	1.11	1.39	1.04	1.00
	C3	103.83	1.04	1.05	1.09	1.13	1.16	1.17	1.20	1.39	1.02	0.99
	C4	1.83	1.05	1.05	1.05	1.05	1.06	1.06	1.06	1.09	0.99	0.99
5	C1	41.91	0.99	0.98	1.00	1.01	1.02	1.04	1.05	1.30	0.89	0.93
	C2	62.38	1.03	1.04	1.05	1.07	1.10	1.11	1.10	1.50	0.99	0.95
	C3	111.29	1.10	1.12	1.17	1.24	1.27	1.26	1.31	1.73	0.98	0.96
	C4	2.27	1.02	1.02	1.04	1.04	1.04	1.05	1.04	1.06	0.96	0.96
1	D1	27.82	1.05	1.05	1.04	1.04	1.04	1.03	1.04	1.03	0.99	1.00
	D2	40.39	1.03	1.03	1.03	1.03	1.03	1.05	1.07	1.01	1.00	0.98
	D3	80.91	1.00	1.00	1.01	1.02	1.04	1.07	1.09	0.99	0.99	0.94
	D4	1.18	1.05	1.05	1.05	1.05	1.06	1.07	1.09	1.03	1.02	1.00
2	D1	36.57	1.03	1.03	1.01	1.00	0.99	0.97	0.95	0.98	0.89	0.92
	D2	46.74	1.03	1.04	1.03	1.04	1.03	1.03	1.04	1.02	0.93	0.94
	D3	79.92	1.09	1.08	1.08	1.10	1.13	1.19	1.23	1.06	1.00	0.95
	D4	1.22	1.05	1.06	1.06	1.08	1.10	1.14	1.18	1.06	1.02	1.00
3	D1	79.49	0.91	0.87	0.87	0.83	0.79	0.74	0.70	0.81	0.65	0.70
	D2	95.72	0.94	0.91	0.90	0.89	0.87	0.84	0.83	0.87	0.68	0.74
	D3	151.07	0.98	0.98	0.99	1.03	1.06	1.06	1.02	0.96	0.81	0.78
	D4	1.64	1.01	0.99	1.02	1.06	1.12	1.21	1.27	1.04	0.92	0.91
4	D1	10.52	1.06	1.06	1.06	1.07	1.10	1.17	1.32	1.36	1.07	0.92
	D2	26.07	1.10	1.10	1.13	1.16	1.20	1.24	1.27	1.15	1.10	1.04
	D3	72.17	1.04	1.06	1.11	1.17	1.22	1.24	1.30	1.12	1.06	1.01
	D4	1.27	1.12	1.13	1.17	1.24	1.31	1.37	1.38	1.20	1.13	1.10
5	D1	24.04	0.97	0.96	0.94	0.93	0.93	0.97	1.12	1.27	0.87	0.72
	D2	38.82	1.05	1.06	1.07	1.09	1.11	1.13	1.19	1.17	0.93	0.88
	D3	74.93	1.15	1.19	1.27	1.36	1.41	1.39	1.51	1.24	1.06	0.96
	D4	1.75	1.10	1.10	1.14	1.21	1.25	1.27	1.27	1.15	1.04	1.04

Table A.5: *Relative bias of estimators for* σ^2, $N = 1000$.

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	A1	23.96	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.01	1.01	1.01
	A2	36.53	1.00	1.00	1.01	1.01	1.01	1.01	1.02	1.01	1.01	0.99
	A3	77.05	0.97	0.97	0.97	0.98	0.98	0.99	1.01	0.98	0.98	0.95
	A4	1.30	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	0.99
2	A1	26.81	1.01	1.02	1.01	1.02	1.02	1.02	1.02	1.01	0.98	1.00
	A2	36.98	1.01	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.02	0.99
	A3	70.15	1.03	1.03	1.04	1.05	1.06	1.06	1.07	1.04	1.03	0.98
	A4	1.48	1.01	1.01	1.01	1.02	1.02	1.02	1.03	1.00	1.00	1.00
3	A1	40.62	0.98	0.99	1.00	1.00	1.00	1.00	1.01	0.99	0.90	0.96
	A2	56.85	1.00	1.01	1.02	1.03	1.03	1.04	1.06	1.05	0.95	0.99
	A3	112.21	1.02	1.02	1.03	1.04	1.06	1.10	1.18	1.09	0.91	0.89
	A4	2.21	0.99	1.00	1.01	1.01	1.02	1.03	1.04	1.00	0.98	0.99
4	A1	30.97	1.02	1.01	1.02	1.02	1.02	1.02	1.02	1.01	1.04	1.00
	A2	46.50	1.03	1.03	1.03	1.03	1.04	1.05	1.06	0.99	1.06	1.01
	A3	92.64	1.01	1.01	1.01	1.02	1.03	1.07	1.13	1.05	1.02	0.99
	A4	1.72	1.01	1.01	1.02	1.02	1.02	1.03	1.03	1.00	1.01	1.01
5	A1	38.18	1.01	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01	0.99
	A2	52.94	1.03	1.04	1.04	1.05	1.07	1.08	1.11	1.00	1.04	1.01
	A3	89.10	1.02	1.03	1.03	1.06	1.11	1.19	1.30	1.08	0.98	0.97
	A4	2.11	1.03	1.02	1.03	1.03	1.03	1.04	1.05	1.01	1.00	1.02
1	B1	116.25	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.02	1.00	1.00
	B2	128.82	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.02	1.00	0.99
	B3	169.33	0.98	0.99	0.99	0.99	0.99	1.00	1.01	1.00	0.99	0.97
	B4	1.37	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
2	B1	133.40	1.01	1.02	1.01	1.02	1.01	1.01	1.01	1.01	0.99	1.00
	B2	143.57	1.01	1.02	1.01	1.02	1.02	1.01	1.02	1.01	0.99	1.00
	B3	176.74	1.02	1.02	1.02	1.03	1.03	1.03	1.04	1.02	1.00	0.98
	B4	1.58	1.01	1.01	1.01	1.01	1.01	1.01	1.02	0.99	1.00	1.00
3	B1	202.95	0.98	0.99	1.00	0.99	0.99	0.99	0.99	0.99	0.94	0.96
	B2	219.19	0.99	0.99	1.00	1.00	1.00	1.00	1.01	1.00	0.92	0.97
	B3	274.54	0.99	1.00	1.01	1.01	1.02	1.03	1.06	1.03	0.91	0.92
	B4	2.40	0.99	0.99	1.00	1.00	1.01	1.01	1.01	0.97	0.97	0.98
4	B1	211.21	1.02	1.01	1.02	1.02	1.02	1.02	1.02	0.73	1.00	1.00
	B2	226.72	1.03	1.02	1.02	1.02	1.02	1.02	1.03	0.75	1.01	1.00
	B3	272.89	1.02	1.01	1.02	1.02	1.02	1.04	1.05	0.81	1.01	1.00
	B4	1.81	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.99	1.00	1.00
5	B1	259.58	1.01	1.03	1.02	1.02	1.02	1.02	1.02	0.74	0.97	0.99
	B2	274.28	1.02	1.03	1.02	1.03	1.03	1.03	1.03	0.75	1.00	0.99
	B3	310.53	1.01	1.03	1.03	1.03	1.05	1.07	1.10	0.80	0.97	0.98
	B4	2.22	1.01	1.00	1.01	1.01	1.01	1.01	1.02	1.01	0.99	0.99

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Table A.5 – *Continued from previous page, $N = 1000$.*

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	C1	27.39	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.24	1.01	1.00
	C2	42.76	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.29	1.01	0.99
	C3	85.47	1.00	1.00	1.00	1.00	1.01	1.02	1.03	1.24	1.01	0.97
	C4	1.33	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.15	1.00	0.99
2	C1	30.51	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.28	0.99	0.99
	C2	45.84	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.47	1.01	0.99
	C3	88.90	1.01	1.02	1.02	1.03	1.04	1.04	1.05	1.50	1.02	0.97
	C4	1.45	1.01	1.01	1.01	1.02	1.02	1.03	1.03	1.19	1.00	1.00
3	C1	44.28	0.98	0.99	1.00	1.00	1.00	1.00	1.01	1.32	0.92	0.97
	C2	64.37	0.99	1.00	1.01	1.02	1.03	1.03	1.05	1.50	0.95	0.98
	C3	124.98	1.01	1.02	1.03	1.04	1.05	1.09	1.16	1.49	0.93	0.90
	C4	2.01	1.00	1.01	1.02	1.03	1.04	1.06	1.08	1.28	0.99	0.99
4	C1	34.61	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.30	1.03	1.00
	C2	54.09	1.02	1.02	1.02	1.02	1.03	1.04	1.05	1.38	1.04	1.00
	C3	103.83	1.01	1.01	1.02	1.02	1.03	1.07	1.12	1.37	1.02	0.99
	C4	1.83	1.01	1.01	1.02	1.02	1.02	1.02	1.03	1.08	1.01	1.01
5	C1	41.91	1.01	1.02	1.01	1.01	1.01	1.01	1.02	1.33	1.00	0.99
	C2	62.38	1.02	1.03	1.03	1.04	1.05	1.07	1.09	1.56	1.03	1.00
	C3	111.29	1.01	1.02	1.02	1.04	1.08	1.15	1.24	1.79	0.97	0.97
	C4	2.27	1.03	1.02	1.02	1.03	1.03	1.03	1.04	1.07	1.00	1.02
1	D1	27.82	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.01	1.00
	D2	40.39	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.00	1.01	0.99
	D3	80.91	0.97	0.97	0.97	0.98	0.98	0.99	1.01	0.98	0.98	0.95
	D4	1.18	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.01	1.01	0.99
2	D1	36.57	1.01	1.01	1.01	1.00	1.00	0.99	0.98	0.97	0.95	0.97
	D2	46.74	1.01	1.01	1.01	1.01	1.00	1.00	1.00	0.99	1.00	0.96
	D3	79.92	1.03	1.03	1.03	1.04	1.04	1.05	1.05	1.02	1.02	0.97
	D4	1.22	1.01	1.01	1.01	1.02	1.02	1.03	1.04	1.04	1.00	0.99
3	D1	79.49	0.94	0.97	0.97	0.95	0.93	0.90	0.86	0.84	0.82	0.87
	D2	95.72	0.96	0.98	0.98	0.98	0.96	0.94	0.92	0.90	0.83	0.85
	D3	151.07	0.99	1.00	1.01	1.01	1.01	1.02	1.06	0.98	0.89	0.86
	D4	1.64	0.97	0.98	0.99	0.99	1.00	1.01	1.05	1.06	0.91	0.92
4	D1	10.52	1.01	1.00	1.00	1.00	1.00	0.99	1.00	1.30	1.25	0.92
	D2	26.07	1.02	1.02	1.02	1.03	1.04	1.06	1.08	1.09	1.13	1.00
	D3	72.17	1.00	1.00	1.00	1.01	1.03	1.08	1.15	1.10	1.02	1.00
	D4	1.27	1.02	1.01	1.01	1.02	1.04	1.09	1.16	1.16	1.01	0.99
5	D1	24.04	0.98	0.98	0.97	0.96	0.95	0.93	0.91	1.29	1.08	0.84
	D2	38.82	1.01	1.01	1.02	1.03	1.03	1.05	1.06	1.17	1.10	0.95
	D3	74.93	1.01	1.01	1.02	1.04	1.09	1.19	1.31	1.19	0.97	0.93
	D4	1.75	1.03	1.03	1.04	1.06	1.09	1.14	1.19	1.16	1.02	1.02

Table A.6: *Relative bias of estimators for* σ^2, $N = 5000$.

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	A1	23.96	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.02	1.00
	A2	36.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.01	0.99
	A3	77.05	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	0.98
	A4	1.30	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	0.99
2	A1	26.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00
	A2	36.98	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.02	1.01	0.99
	A3	70.15	1.01	1.01	1.01	1.01	1.01	1.02	1.03	1.03	1.03	0.99
	A4	1.48	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00	0.99
3	A1	40.62	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	0.94	0.99
	A2	56.85	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.06	1.00	1.00
	A3	112.21	1.02	1.01	1.02	1.03	1.03	1.04	1.05	1.11	0.98	0.96
	A4	2.21	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.01	1.00	0.99
4	A1	30.97	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.05	1.00
	A2	46.50	1.01	1.01	1.01	1.01	1.00	1.01	1.01	0.98	1.01	1.00
	A3	92.64	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.05	1.02	0.99
	A4	1.72	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.99	1.01	1.00
5	A1	38.18	1.00	1.01	1.00	1.00	1.01	1.01	1.01	1.00	1.09	1.00
	A2	52.94	1.01	1.01	1.01	1.01	1.02	1.02	1.03	0.99	1.00	1.01
	A3	89.10	1.00	1.01	1.01	1.01	1.02	1.02	1.03	1.06	1.03	0.98
	A4	2.11	1.00	1.00	1.01	1.01	1.01	1.02	1.02	1.01	1.01	1.01
1	B1	116.25	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	1.00
	B2	128.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.00	0.99
	B3	169.33	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.01	1.00	0.98
	B4	1.37	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
2	B1	133.40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	B2	143.57	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.99
	B3	176.74	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	0.99
	B4	1.58	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
3	B1	202.95	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.99
	B2	219.19	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.01	0.99	0.99
	B3	274.54	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.04	0.97	0.97
	B4	2.40	1.01	1.01	1.01	1.01	1.01	1.01	1.01	0.98	1.00	1.00
4	B1	211.21	1.01	1.00	1.00	1.00	0.99	1.00	1.00	0.73	1.00	1.00
	B2	226.72	1.01	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.02	1.00
	B3	272.89	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.80	1.00	1.00
	B4	1.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00
5	B1	259.58	1.00	1.01	1.00	1.00	1.01	1.01	1.01	0.73	0.99	1.00
	B2	274.28	1.00	1.01	1.00	1.00	1.01	1.01	1.01	0.75	1.02	1.00
	B3	310.53	1.00	1.01	1.00	1.00	1.01	1.01	1.01	0.80	0.98	1.00
	B4	2.22	1.00	1.01	1.00	1.01	1.01	1.01	1.01	1.01	1.00	1.00

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Table A.6 – *Continued from previous page, $N = 5000$.*

Design	Curve	σ^2	$\widehat{\mathrm{RB}}(\hat{\sigma}^2)$									
			Index of $\hat{\sigma}^2$									
			2	3	5	9	15	25	40	0	1	r
1	C1	27.39	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.24	1.02	1.00
	C2	42.76	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.29	1.00	0.99
	C3	85.47	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.22	1.01	0.99
	C4	1.33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.14	1.00	0.99
2	C1	30.51	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.28	1.00	1.00
	C2	45.84	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.46	1.01	0.99
	C3	88.90	1.00	1.00	1.00	1.01	1.01	1.02	1.02	1.50	1.02	0.99
	C4	1.45	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.18	1.00	0.99
3	C1	44.28	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.34	0.98	1.00
	C2	64.37	1.01	1.00	1.01	1.01	1.01	1.01	1.02	1.52	1.00	1.00
	C3	124.98	1.01	1.01	1.01	1.02	1.03	1.04	1.05	1.50	0.98	0.96
	C4	2.01	0.99	1.00	1.00	1.00	1.01	1.02	1.03	1.29	1.00	0.99
4	C1	34.61	1.01	1.00	1.00	1.00	1.00	1.00	1.00	1.30	1.05	1.00
	C2	54.09	1.01	1.00	1.00	1.01	1.00	1.01	1.01	1.38	1.01	1.00
	C3	103.83	1.00	1.00	1.00	1.00	1.00	1.01	1.01	1.35	1.02	0.99
	C4	1.83	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.07	1.00	1.00
5	C1	41.91	1.00	1.01	1.00	1.00	1.01	1.01	1.01	1.34	1.08	1.00
	C2	62.38	1.01	1.01	1.01	1.01	1.01	1.02	1.03	1.56	1.00	1.01
	C3	111.29	1.00	1.01	1.01	1.01	1.01	1.02	1.02	1.80	1.02	0.98
	C4	2.27	1.00	1.00	1.01	1.01	1.01	1.02	1.02	1.08	1.01	1.01
1	D1	27.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	1.02	1.01
	D2	40.39	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01	0.99
	D3	80.91	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.00	1.00	0.98
	D4	1.18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99
2	D1	36.57	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	1.01	1.01
	D2	46.74	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.02	0.99
	D3	79.92	1.01	1.01	1.01	1.01	1.01	1.02	1.02	1.01	1.02	0.99
	D4	1.22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03	1.00	0.99
3	D1	79.49	1.00	1.00	0.99	0.99	0.99	0.98	0.97	0.84	0.96	0.97
	D2	95.72	1.00	1.00	0.99	0.99	1.00	0.99	0.98	0.90	0.96	0.96
	D3	151.07	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.00	0.96	0.94
	D4	1.64	1.00	1.00	1.00	1.00	0.99	0.99	0.99	1.08	0.98	0.97
4	D1	10.52	1.00	1.00	1.00	1.00	0.99	0.99	0.99	1.28	1.31	1.01
	D2	26.07	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.08	1.06	1.00
	D3	72.17	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.10	1.03	1.00
	D4	1.27	1.00	1.00	1.00	1.00	1.00	1.00	1.01	1.15	1.01	0.99
5	D1	24.04	1.00	1.00	0.99	0.99	0.99	0.98	0.98	1.30	1.40	0.99
	D2	38.82	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.17	1.07	1.01
	D3	74.93	1.01	1.01	1.01	1.02	1.02	1.01	1.02	1.17	1.02	0.97
	D4	1.75	1.01	1.01	1.01	1.01	1.02	1.02	1.04	1.16	0.99	0.98

Appendix B. Simulation results of $\hat{c}'\hat{I}^{-1}\hat{c}$

Table B.1: *MSE and relative MSE of estimators for $c'I^{-1}c$, $N = 250$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$ Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	1	r
1	A1	0.329	2.84	0.74	0.64	0.61	0.65	0.74	0.99	0.78	0.68	0.65	0.89	0.83	0.82	0.71	0.40
	A2	7.845	20.34	0.83	0.77	0.75	0.83	1.06	1.57	0.87	0.81	0.79	0.90	0.85	0.85	0.54	0.46
	A3	6.186	24.43	0.65	0.55	0.52	0.56	0.66	0.73	0.72	0.62	0.58	0.83	0.77	0.75	0.41	0.32
	A4	0.019	0.00	0.70	0.57	0.49	0.47	0.46	0.48	0.72	0.61	0.54	0.88	0.82	0.78	0.50	0.31
2	A1	0.055	3.65	0.58	0.42	0.36	0.33	0.33	0.39	0.64	0.48	0.41	0.87	0.81	0.78	0.56	0.23
	A2	5.821	26.18	0.70	0.66	0.67	0.72	0.91	1.31	0.76	0.70	0.71	0.88	0.85	0.83	0.48	0.34
	A3	5.963	54.59	0.61	0.48	0.45	0.43	0.43	0.50	0.64	0.50	0.46	0.90	0.86	0.84	0.44	0.16
	A4	0.025	0.01	0.62	0.45	0.38	0.33	0.29	0.27	0.67	0.50	0.41	0.85	0.79	0.76	0.52	0.26
3	A1	0.118	12.88	0.50	0.32	0.24	0.19	0.16	0.17	0.52	0.34	0.26	0.90	0.85	0.83	3.80	0.16
	A2	8.085	87.63	0.62	0.47	0.48	0.52	0.63	0.79	0.65	0.49	0.50	0.93	0.89	0.88	0.90	0.22
	A3	7.681	273.41	0.53	0.40	0.36	0.29	0.26	0.30	0.55	0.42	0.38	0.91	0.88	0.86	0.73	0.12
	A4	0.034	0.05	0.48	0.31	0.23	0.17	0.13	0.10	0.50	0.33	0.25	0.91	0.86	0.85	0.80	2.86
4	A1	0.128	7.41	0.60	0.47	0.41	0.39	0.44	0.50	0.62	0.49	0.43	0.90	0.85	0.84	2.69	0.28
	A2	7.683	81.24	0.72	0.66	0.63	0.70	0.71	0.65	0.73	0.66	0.63	0.95	0.93	0.92	0.43	0.28
	A3	7.366	90.93	0.65	0.57	0.58	0.58	0.67	1.06	0.67	0.58	0.59	0.92	0.88	0.87	0.64	0.40
	A4	0.032	0.02	0.59	0.44	0.39	0.37	0.35	0.37	0.63	0.48	0.43	0.88	0.83	0.80	1.11	0.43
5	A1	0.039	11.87	0.43	0.29	0.21	0.19	0.19	0.21	0.44	0.30	0.22	0.92	0.88	0.86	7.40	0.15
	A2	6.102	98.09	0.59	0.46	0.41	0.43	0.36	0.34	0.60	0.47	0.42	0.95	0.91	0.90	11.58	0.23
	A3	6.686	212.92	0.50	0.43	0.38	0.32	0.32	0.57	0.51	0.43	0.37	0.93	0.90	0.87	3.94	0.10
	A4	0.040	0.03	0.60	0.39	0.29	0.25	0.21	0.16	0.61	0.41	0.31	0.92	0.87	0.85	2.51	0.19
1	B1	0.330	22.24	0.60	0.47	0.37	0.38	0.38	0.45	0.66	0.54	0.44	0.79	0.72	0.70	0.39	0.24
	B2	7.844	57.68	0.74	0.64	0.57	0.59	0.72	1.01	0.79	0.71	0.65	0.85	0.78	0.76	0.42	0.38
	B3	6.187	64.94	0.61	0.50	0.45	0.48	0.53	0.57	0.69	0.56	0.50	0.80	0.74	0.72	0.37	0.23
	B4	0.008	0.00	0.59	0.46	0.38	0.37	0.37	0.40	0.64	0.51	0.44	0.82	0.75	0.71	0.43	0.25
2	B1	0.055	69.99	0.45	0.27	0.20	0.18	0.16	0.15	0.50	0.32	0.25	0.88	0.80	0.77	0.25	0.15
	B2	5.819	128.47	0.56	0.47	0.42	0.41	0.46	0.58	0.61	0.52	0.47	0.87	0.82	0.79	0.39	0.20
	B3	5.964	218.15	0.48	0.33	0.28	0.26	0.26	0.27	0.51	0.37	0.31	0.91	0.87	0.84	0.24	0.13
	B4	0.011	0.01	0.54	0.37	0.30	0.26	0.22	0.20	0.60	0.43	0.35	0.83	0.76	0.72	0.46	0.18
3	B1	0.118	625.24	0.30	0.15	0.12	0.07	0.05	0.04	0.31	0.16	0.12	0.94	0.88	0.86	0.37	0.10
	B2	8.085	907.50	0.41	0.25	0.22	0.19	0.18	0.18	0.43	0.26	0.23	0.94	0.89	0.87	1.74	0.13
	B3	7.680	1121.48	0.47	0.32	0.26	0.18	0.15	0.15	0.50	0.34	0.27	0.92	0.88	0.86	0.53	0.12
	B4	0.014	0.04	0.48	0.28	0.20	0.14	0.11	0.08	0.50	0.30	0.22	0.91	0.86	0.84	1.19	0.24
4	B1	0.129	327.51	0.50	0.32	0.24	0.21	0.20	0.18	0.52	0.33	0.26	0.89	0.84	0.83	0.92	0.14
	B2	7.675	641.20	0.58	0.46	0.39	0.43	0.41	0.35	0.59	0.47	0.39	0.93	0.89	0.87	0.46	0.20
	B3	7.372	686.55	0.54	0.41	0.34	0.30	0.32	0.39	0.56	0.42	0.35	0.92	0.88	0.87	0.74	0.15
	B4	0.011	0.01	0.54	0.38	0.32	0.29	0.28	0.25	0.57	0.42	0.35	0.89	0.83	0.80	0.88	3.73
5	B1	0.039	745.64	0.35	0.21	0.14	0.12	0.09	0.07	0.35	0.21	0.15	0.93	0.90	0.88	2.16	0.13
	B2	6.094	1331.76	0.41	0.24	0.19	0.19	0.15	0.12	0.41	0.25	0.20	0.95	0.91	0.89	19.35	0.14
	B3	6.693	1570.48	0.39	0.30	0.23	0.19	0.17	0.20	0.40	0.30	0.23	0.93	0.91	0.89	2.01	0.36
	B4	0.014	0.03	0.49	0.32	0.22	0.18	0.14	0.10	0.49	0.33	0.23	0.93	0.88	0.86	2.69	0.16

Continued on next page

Table B.1 – *Continued from previous page, $N = 250$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$													
				Index of $\hat{c}'\hat{I}^{-1}\hat{c}$													
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	C1	0.000	0.54	0.54	0.40	0.33	0.32	0.31	0.33	0.61	0.47	0.39	0.79	0.72	0.68	0.51	0.23
	C2	0.040	1.64	0.60	0.43	0.33	0.34	0.35	0.38	0.64	0.48	0.40	0.81	0.71	0.67	0.48	0.20
	C3	0.136	7.31	0.59	0.43	0.37	0.40	0.47	0.48	0.65	0.51	0.45	0.81	0.73	0.68	0.46	0.33
	C4	0.000	0.00	0.61	0.43	0.35	0.33	0.32	0.34	0.65	0.50	0.43	0.81	0.72	0.67	0.38	0.24
2	C1	0.000	1.76	0.45	0.31	0.24	0.20	0.18	0.15	0.48	0.35	0.28	0.84	0.78	0.75	0.54	0.15
	C2	0.089	4.47	0.52	0.31	0.23	0.21	0.21	0.18	0.54	0.36	0.27	0.87	0.78	0.73	0.81	0.40
	C3	0.144	20.81	0.55	0.42	0.43	0.54	0.73	0.52	0.61	0.47	0.48	0.84	0.77	0.74	0.53	0.16
	C4	0.000	0.00	0.57	0.40	0.31	0.27	0.23	0.20	0.61	0.45	0.35	0.83	0.77	0.73	0.42	0.26
3	C1	0.002	13.66	0.39	0.24	0.16	0.12	0.08	0.05	0.42	0.25	0.17	0.90	0.86	0.83	2.56	0.11
	C2	0.258	30.95	0.41	0.25	0.19	0.14	0.09	0.05	0.43	0.25	0.20	0.92	0.87	0.85	3.16	0.20
	C3	0.873	206.40	0.48	0.37	0.43	0.50	0.35	0.08	0.49	0.38	0.43	0.92	0.87	0.86	1.07	0.10
	C4	0.001	0.04	0.47	0.27	0.16	0.12	0.09	0.06	0.48	0.28	0.17	0.92	0.88	0.86	0.79	3.32
4	C1	0.016	5.30	0.50	0.31	0.25	0.23	0.24	0.26	0.52	0.33	0.27	0.88	0.82	0.82	2.92	0.18
	C2	1.402	29.59	0.59	0.51	0.47	0.51	0.46	0.34	0.60	0.51	0.46	0.93	0.91	0.89	0.42	0.18
	C3	0.879	50.76	0.57	0.45	0.41	0.38	0.35	0.41	0.59	0.48	0.43	0.89	0.85	0.85	0.74	0.44
	C4	0.002	0.01	0.56	0.38	0.33	0.30	0.27	0.27	0.57	0.38	0.33	0.90	0.84	0.83	1.27	0.46
5	C1	0.004	10.95	0.43	0.27	0.20	0.15	0.13	0.11	0.45	0.28	0.21	0.92	0.89	0.87	7.78	0.15
	C2	1.041	48.46	0.50	0.31	0.26	0.26	0.18	0.12	0.51	0.32	0.27	0.93	0.89	0.87	22.90	0.18
	C3	0.693	127.06	0.47	0.35	0.32	0.30	0.21	0.14	0.47	0.35	0.31	0.93	0.90	0.87	5.99	0.13
	C4	0.002	0.03	0.53	0.34	0.25	0.19	0.15	0.11	0.55	0.36	0.27	0.91	0.86	0.85	1.95	0.17
1	D1	0.329	2.91	0.74	0.64	0.62	0.66	0.75	0.99	0.78	0.69	0.66	0.88	0.83	0.82	0.74	0.41
	D2	7.845	21.55	0.84	0.76	0.74	0.82	1.04	1.53	0.87	0.79	0.78	0.90	0.85	0.85	0.54	0.45
	D3	6.187	25.50	0.66	0.55	0.52	0.56	0.65	0.72	0.72	0.61	0.57	0.84	0.78	0.76	0.41	0.32
	D4	0.019	0.01	0.70	0.59	0.53	0.53	0.52	0.56	0.74	0.64	0.58	0.89	0.83	0.80	0.51	0.34
2	D1	0.055	5.59	0.56	0.39	0.32	0.27	0.26	0.29	0.60	0.43	0.35	0.87	0.80	0.77	0.55	0.20
	D2	5.822	30.22	0.68	0.62	0.63	0.67	0.82	1.16	0.73	0.65	0.67	0.87	0.85	0.83	0.43	0.44
	D3	5.964	61.46	0.62	0.50	0.45	0.43	0.41	0.46	0.64	0.52	0.47	0.90	0.85	0.84	0.46	0.16
	D4	0.021	0.01	0.71	0.54	0.47	0.46	0.43	0.43	0.74	0.58	0.50	0.89	0.83	0.81	0.89	0.26
3	D1	0.118	43.12	0.41	0.27	0.15	0.10	0.07	0.07	0.43	0.29	0.16	0.90	0.85	0.82	15.09	0.15
	D2	8.087	164.44	0.49	0.36	0.32	0.33	0.37	0.45	0.51	0.38	0.33	0.92	0.87	0.86	1.55	0.17
	D3	7.681	363.86	0.50	0.36	0.31	0.24	0.21	0.23	0.52	0.38	0.32	0.92	0.88	0.85	0.97	0.11
	D4	0.034	0.06	0.49	0.27	0.21	0.17	0.14	0.12	0.50	0.28	0.22	0.94	0.91	0.89	1.24	0.36
4	D1	0.127	1.29	0.79	0.73	0.75	0.82	1.05	1.54	0.83	0.76	0.78	0.91	0.87	0.84	4.75	0.46
	D2	7.687	44.01	0.78	0.74	0.80	0.87	0.93	0.90	0.79	0.74	0.79	0.95	0.92	0.92	0.57	0.34
	D3	7.362	61.06	0.63	0.59	0.64	0.69	0.81	1.39	0.66	0.62	0.65	0.89	0.84	0.83	1.01	0.26
	D4	0.109	0.02	0.65	0.63	0.64	0.58	0.53	0.49	0.70	0.66	0.66	0.89	0.87	0.85	1.25	0.82
5	D1	0.038	3.57	0.52	0.32	0.23	0.23	0.29	0.47	0.55	0.35	0.26	0.86	0.79	0.75	50.41	0.41
	D2	6.108	45.20	0.71	0.68	0.60	0.63	0.58	0.58	0.73	0.69	0.61	0.92	0.90	0.87	3.00	0.30
	D3	6.678	139.83	0.59	0.50	0.46	0.41	0.43	0.82	0.60	0.49	0.45	0.92	0.88	0.85	1.49	0.14
	D4	0.157	0.06	0.60	0.45	0.39	0.31	0.21	0.12	0.61	0.46	0.41	0.93	0.89	0.86	2.14	0.17

Table B.2: *MSE and relative MSE of estimators for $c'I^{-1}c$, $N = 1000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$													
				Index of $\hat{c}'\hat{I}^{-1}\hat{c}$													
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	1	r
1	A1	0.329	0.25	0.84	0.77	0.75	0.74	0.75	0.81	0.85	0.80	0.78	0.93	0.88	0.88	1.07	0.54
	A2	7.845	5.23	0.85	0.80	0.78	0.77	0.81	0.87	0.89	0.85	0.83	0.93	0.89	0.88	0.69	0.68
	A3	6.186	3.70	0.73	0.61	0.58	0.57	0.63	0.75	0.78	0.67	0.65	0.87	0.80	0.77	0.53	0.46
	A4	0.019	0.00	0.75	0.66	0.66	0.63	0.61	0.62	0.80	0.70	0.68	0.87	0.84	0.85	0.99	0.53
2	A1	0.055	0.16	0.65	0.52	0.48	0.47	0.47	0.48	0.68	0.57	0.53	0.87	0.81	0.79	1.13	0.37
	A2	5.821	3.26	0.85	0.77	0.76	0.78	0.84	0.94	0.87	0.79	0.78	0.93	0.90	0.89	0.74	0.58
	A3	5.963	5.68	0.70	0.60	0.56	0.63	0.74	0.83	0.76	0.67	0.64	0.86	0.81	0.79	0.46	0.35
	A4	0.025	0.00	0.73	0.61	0.57	0.54	0.52	0.52	0.74	0.64	0.60	0.92	0.87	0.86	0.78	0.42
3	A1	0.118	0.61	0.57	0.41	0.33	0.28	0.27	0.26	0.58	0.42	0.34	0.92	0.88	0.86	3.01	0.23
	A2	8.085	8.22	0.72	0.64	0.57	0.59	0.67	0.84	0.73	0.65	0.59	0.94	0.92	0.89	0.70	0.36
	A3	7.681	21.88	0.72	0.60	0.62	0.71	0.80	0.77	0.75	0.63	0.64	0.92	0.90	0.88	0.59	0.25
	A4	0.034	0.00	0.68	0.51	0.45	0.41	0.37	0.33	0.70	0.53	0.46	0.94	0.91	0.90	1.01	0.25
4	A1	0.128	0.26	0.75	0.66	0.59	0.60	0.59	0.63	0.77	0.68	0.61	0.91	0.87	0.85	2.60	0.48
	A2	7.683	9.47	0.82	0.68	0.66	0.70	0.76	0.87	0.82	0.71	0.69	0.96	0.91	0.90	0.60	0.48
	A3	7.366	10.38	0.68	0.54	0.50	0.57	0.73	0.96	0.72	0.58	0.55	0.89	0.85	0.83	0.69	0.35
	A4	0.032	0.00	0.74	0.61	0.52	0.51	0.51	0.51	0.75	0.63	0.55	0.92	0.87	0.86	1.06	0.38
5	A1	0.039	0.47	0.51	0.39	0.35	0.32	0.31	0.28	0.54	0.41	0.37	0.90	0.86	0.85	5.25	0.22
	A2	6.102	9.10	0.66	0.60	0.56	0.58	0.66	0.74	0.68	0.60	0.57	0.94	0.91	0.90	0.51	0.39
	A3	6.686	12.92	0.69	0.64	0.72	0.92	1.22	1.43	0.71	0.65	0.71	0.92	0.89	0.89	0.76	0.33
	A4	0.040	0.00	0.62	0.50	0.42	0.41	0.39	0.38	0.63	0.52	0.44	0.93	0.89	0.88	1.52	0.29
1	B1	0.330	1.50	0.67	0.53	0.48	0.45	0.44	0.46	0.71	0.60	0.55	0.85	0.76	0.75	0.54	0.38
	B2	7.844	10.53	0.80	0.71	0.65	0.64	0.66	0.70	0.84	0.77	0.72	0.90	0.85	0.82	0.52	0.50
	B3	6.187	8.67	0.73	0.60	0.55	0.54	0.60	0.68	0.77	0.65	0.62	0.87	0.79	0.76	0.52	0.40
	B4	0.008	0.00	0.68	0.56	0.53	0.50	0.49	0.50	0.76	0.62	0.58	0.83	0.78	0.78	0.74	0.42
2	B1	0.055	2.46	0.52	0.35	0.30	0.28	0.28	0.28	0.56	0.41	0.35	0.83	0.74	0.73	0.41	0.22
	B2	5.819	10.78	0.74	0.63	0.59	0.59	0.62	0.68	0.77	0.66	0.63	0.90	0.85	0.85	0.57	0.39
	B3	5.964	15.07	0.69	0.57	0.53	0.56	0.61	0.66	0.73	0.63	0.59	0.88	0.81	0.80	0.46	0.32
	B4	0.011	0.00	0.67	0.54	0.48	0.45	0.44	0.44	0.70	0.60	0.53	0.89	0.82	0.80	0.77	0.35
3	B1	0.118	20.42	0.46	0.32	0.21	0.15	0.13	0.11	0.48	0.33	0.22	0.92	0.89	0.87	1.07	0.13
	B2	8.085	46.47	0.65	0.55	0.41	0.39	0.40	0.45	0.65	0.55	0.43	0.94	0.91	0.89	1.46	0.25
	B3	7.680	72.56	0.63	0.48	0.48	0.48	0.51	0.48	0.66	0.50	0.49	0.91	0.89	0.87	0.57	0.21
	B4	0.014	0.00	0.59	0.45	0.39	0.34	0.30	0.27	0.62	0.47	0.40	0.92	0.89	0.87	1.72	0.21
4	B1	0.129	7.86	0.60	0.46	0.38	0.38	0.37	0.37	0.62	0.49	0.41	0.88	0.82	0.80	1.56	0.29
	B2	7.675	43.60	0.73	0.56	0.52	0.55	0.58	0.62	0.73	0.59	0.54	0.95	0.89	0.88	0.54	0.38
	B3	7.372	37.60	0.72	0.60	0.55	0.60	0.66	0.78	0.73	0.61	0.57	0.91	0.87	0.86	0.88	0.38
	B4	0.011	0.00	0.69	0.51	0.43	0.41	0.41	0.39	0.71	0.54	0.46	0.92	0.85	0.84	1.05	0.31
5	B1	0.039	21.69	0.46	0.33	0.29	0.25	0.24	0.22	0.49	0.35	0.30	0.90	0.88	0.87	3.11	0.17
	B2	6.094	67.05	0.52	0.44	0.40	0.40	0.42	0.44	0.53	0.45	0.41	0.93	0.91	0.90	0.73	0.29
	B3	6.693	67.94	0.60	0.55	0.57	0.62	0.72	0.74	0.62	0.56	0.57	0.92	0.89	0.89	0.65	0.27
	B4	0.014	0.00	0.58	0.45	0.37	0.36	0.32	0.30	0.60	0.47	0.40	0.92	0.87	0.85	1.76	0.23

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Table B.2 – *Continued from previous page, $N = 1000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$													
				Index of $\hat{c}'\hat{I}^{-1}\hat{c}$													
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	1	r
1	C1	0.000	0.03	0.56	0.37	0.29	0.26	0.25	0.25	0.64	0.47	0.40	0.76	0.66	0.62	0.52	0.34
	C2	0.040	0.10	0.60	0.41	0.35	0.31	0.31	0.33	0.68	0.52	0.45	0.78	0.67	0.64	0.48	0.23
	C3	0.136	0.35	0.55	0.42	0.36	0.34	0.40	0.54	0.63	0.50	0.45	0.76	0.67	0.63	0.44	0.27
	C4	0.000	0.00	0.55	0.40	0.32	0.29	0.26	0.26	0.65	0.50	0.42	0.76	0.67	0.63	0.40	0.22
2	C1	0.000	0.07	0.55	0.36	0.29	0.26	0.26	0.25	0.61	0.43	0.36	0.80	0.71	0.68	0.64	0.20
	C2	0.089	0.17	0.56	0.38	0.31	0.29	0.29	0.29	0.65	0.47	0.40	0.79	0.71	0.67	0.41	0.19
	C3	0.144	0.74	0.65	0.48	0.40	0.42	0.63	1.19	0.71	0.55	0.48	0.83	0.75	0.71	0.67	0.35
	C4	0.000	0.00	0.56	0.39	0.32	0.29	0.28	0.26	0.63	0.46	0.39	0.80	0.72	0.69	0.45	0.22
3	C1	0.002	0.52	0.46	0.35	0.25	0.19	0.15	0.14	0.49	0.37	0.27	0.89	0.84	0.82	2.41	0.14
	C2	0.258	1.21	0.55	0.46	0.34	0.30	0.26	0.24	0.58	0.49	0.37	0.90	0.86	0.84	1.20	0.24
	C3	0.873	7.11	0.62	0.46	0.56	0.72	1.25	2.78	0.64	0.48	0.57	0.91	0.87	0.87	0.92	0.25
	C4	0.001	0.00	0.51	0.34	0.27	0.21	0.17	0.16	0.54	0.37	0.29	0.89	0.85	0.82	0.74	0.15
4	C1	0.016	0.13	0.60	0.46	0.38	0.38	0.37	0.38	0.63	0.50	0.42	0.87	0.80	0.77	3.05	0.29
	C2	1.402	1.89	0.70	0.56	0.51	0.55	0.61	0.73	0.73	0.61	0.56	0.91	0.85	0.84	0.43	0.33
	C3	0.879	2.76	0.61	0.48	0.48	0.58	0.79	1.09	0.65	0.53	0.51	0.88	0.83	0.82	1.01	0.30
	C4	0.002	0.00	0.59	0.44	0.35	0.34	0.37	0.36	0.63	0.49	0.40	0.86	0.80	0.77	1.11	0.26
5	C1	0.004	0.34	0.51	0.36	0.30	0.26	0.25	0.22	0.55	0.39	0.32	0.88	0.84	0.83	6.52	0.17
	C2	1.041	2.52	0.52	0.44	0.39	0.41	0.46	0.54	0.55	0.46	0.42	0.91	0.89	0.87	0.37	0.28
	C3	0.693	3.94	0.57	0.53	0.60	0.81	1.18	1.68	0.61	0.55	0.62	0.88	0.85	0.85	0.93	0.31
	C4	0.002	0.00	0.55	0.40	0.32	0.29	0.28	0.26	0.58	0.44	0.35	0.89	0.85	0.83	2.49	0.23
1	D1	0.329	0.26	0.82	0.76	0.74	0.72	0.73	0.78	0.84	0.79	0.77	0.92	0.88	0.87	1.06	0.51
	D2	7.845	5.27	0.85	0.80	0.78	0.78	0.81	0.87	0.89	0.85	0.83	0.92	0.88	0.87	0.68	0.68
	D3	6.187	3.85	0.73	0.61	0.58	0.58	0.63	0.74	0.79	0.67	0.66	0.87	0.80	0.78	0.54	0.46
	D4	0.019	0.00	0.81	0.75	0.74	0.73	0.71	0.72	0.85	0.78	0.76	0.91	0.88	0.88	0.98	0.60
2	D1	0.055	0.21	0.69	0.53	0.47	0.46	0.46	0.44	0.73	0.59	0.52	0.88	0.81	0.78	1.27	0.34
	D2	5.822	3.67	0.83	0.75	0.74	0.75	0.80	0.89	0.86	0.78	0.77	0.92	0.90	0.88	0.69	0.55
	D3	5.964	6.09	0.72	0.62	0.58	0.64	0.76	0.85	0.78	0.69	0.66	0.87	0.82	0.79	0.50	0.36
	D4	0.021	0.00	0.81	0.66	0.62	0.59	0.60	0.63	0.83	0.70	0.66	0.93	0.88	0.87	0.77	0.45
3	D1	0.118	1.37	0.57	0.42	0.35	0.31	0.25	0.22	0.59	0.44	0.36	0.90	0.85	0.83	4.82	0.18
	D2	8.087	13.25	0.63	0.57	0.55	0.54	0.54	0.63	0.65	0.58	0.56	0.94	0.92	0.90	2.06	0.37
	D3	7.681	28.64	0.69	0.60	0.61	0.67	0.72	0.67	0.72	0.63	0.63	0.92	0.90	0.88	0.81	0.25
	D4	0.034	0.00	0.61	0.53	0.46	0.43	0.38	0.38	0.63	0.55	0.48	0.94	0.92	0.92	0.87	0.26
4	D1	0.127	0.09	0.84	0.76	0.70	0.71	0.73	0.81	0.87	0.80	0.75	0.93	0.89	0.87	4.08	0.59
	D2	7.687	6.20	0.87	0.76	0.75	0.78	0.86	1.01	0.88	0.79	0.77	0.96	0.91	0.90	0.69	0.55
	D3	7.362	8.33	0.66	0.50	0.45	0.53	0.73	0.99	0.71	0.56	0.52	0.88	0.83	0.81	0.86	0.33
	D4	0.109	0.00	0.78	0.65	0.63	0.69	0.82	1.00	0.80	0.68	0.66	0.92	0.88	0.87	0.80	0.48
5	D1	0.038	0.19	0.58	0.41	0.35	0.30	0.26	0.25	0.62	0.44	0.37	0.87	0.81	0.77	6.84	0.27
	D2	6.108	5.44	0.77	0.67	0.64	0.66	0.77	0.88	0.79	0.70	0.67	0.93	0.89	0.87	0.71	0.46
	D3	6.678	9.97	0.73	0.63	0.71	0.97	1.31	1.61	0.75	0.65	0.72	0.92	0.87	0.87	0.87	0.35
	D4	0.157	0.01	0.68	0.51	0.47	0.51	0.63	0.74	0.70	0.53	0.49	0.93	0.88	0.88	0.77	0.36

Table B.3: *MSE and relative MSE of estimators for $c'I^{-1}c$, $N = 5000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$														
				Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	A1	0.329	0.03	0.90	0.84	0.81	0.82	0.82	0.82	0.94	0.90	0.87	0.93	0.90	0.88	1.37	0.65
	A2	7.845	1.27	0.92	0.87	0.84	0.80	0.79	0.79	0.94	0.89	0.88	0.95	0.93	0.92	0.83	0.82
	A3	6.186	0.79	0.72	0.61	0.55	0.52	0.50	0.49	0.80	0.71	0.65	0.87	0.80	0.78	0.47	0.50
	A4	0.019	0.00	0.80	0.67	0.62	0.59	0.59	0.58	0.84	0.74	0.69	0.90	0.85	0.83	0.95	0.57
2	A1	0.055	0.01	0.72	0.61	0.54	0.56	0.54	0.54	0.77	0.67	0.60	0.88	0.83	0.81	1.33	0.49
	A2	5.821	0.66	0.80	0.70	0.66	0.68	0.67	0.68	0.85	0.76	0.72	0.91	0.87	0.86	0.78	0.66
	A3	5.963	0.88	0.75	0.59	0.53	0.55	0.55	0.60	0.79	0.66	0.60	0.90	0.84	0.82	0.52	0.43
	A4	0.025	0.00	0.75	0.60	0.55	0.56	0.55	0.56	0.79	0.67	0.61	0.91	0.85	0.84	0.82	0.49
3	A1	0.118	0.04	0.54	0.38	0.34	0.29	0.29	0.28	0.57	0.41	0.36	0.91	0.88	0.88	1.73	0.30
	A2	8.085	1.49	0.75	0.64	0.56	0.49	0.51	0.52	0.77	0.67	0.59	0.93	0.89	0.88	0.65	0.52
	A3	7.681	2.78	0.74	0.61	0.56	0.55	0.64	0.87	0.75	0.63	0.58	0.94	0.90	0.89	0.58	0.40
	A4	0.034	0.00	0.75	0.63	0.54	0.48	0.47	0.43	0.77	0.65	0.56	0.95	0.92	0.91	1.31	0.35
4	A1	0.128	0.02	0.77	0.65	0.60	0.59	0.58	0.58	0.80	0.69	0.65	0.92	0.88	0.86	1.36	0.49
	A2	7.683	1.54	0.83	0.73	0.67	0.75	0.75	0.76	0.85	0.76	0.71	0.94	0.90	0.88	0.69	0.60
	A3	7.366	1.66	0.72	0.61	0.55	0.50	0.49	0.51	0.76	0.65	0.60	0.90	0.87	0.85	0.86	0.48
	A4	0.032	0.00	0.74	0.66	0.58	0.51	0.50	0.49	0.77	0.68	0.62	0.92	0.89	0.86	1.08	0.52
5	A1	0.039	0.02	0.66	0.50	0.45	0.48	0.47	0.46	0.68	0.55	0.48	0.92	0.85	0.85	6.79	0.39
	A2	6.102	1.32	0.74	0.63	0.58	0.58	0.59	0.63	0.77	0.67	0.62	0.93	0.90	0.89	0.57	0.49
	A3	6.686	1.68	0.74	0.64	0.58	0.55	0.60	0.85	0.75	0.66	0.59	0.94	0.91	0.90	0.69	0.54
	A4	0.040	0.00	0.71	0.59	0.55	0.49	0.49	0.49	0.74	0.64	0.58	0.93	0.88	0.88	1.77	0.46
1	B1	0.330	0.11	0.77	0.64	0.58	0.57	0.57	0.56	0.85	0.74	0.69	0.85	0.78	0.75	0.98	0.36
	B2	7.844	2.15	0.83	0.73	0.71	0.69	0.69	0.68	0.87	0.79	0.77	0.91	0.86	0.85	0.68	0.68
	B3	6.187	1.59	0.71	0.59	0.54	0.54	0.52	0.52	0.79	0.70	0.65	0.86	0.79	0.77	0.47	0.47
	B4	0.008	0.00	0.73	0.61	0.54	0.49	0.48	0.46	0.80	0.69	0.63	0.86	0.81	0.78	0.92	0.49
2	B1	0.055	0.10	0.59	0.45	0.36	0.39	0.36	0.35	0.64	0.51	0.42	0.85	0.78	0.75	0.67	0.31
	B2	5.819	1.78	0.74	0.62	0.56	0.57	0.56	0.57	0.80	0.69	0.63	0.88	0.84	0.82	0.56	0.50
	B3	5.964	2.04	0.74	0.59	0.53	0.59	0.58	0.61	0.79	0.66	0.60	0.88	0.83	0.81	0.51	0.42
	B4	0.011	0.00	0.69	0.53	0.48	0.49	0.47	0.48	0.75	0.61	0.55	0.88	0.82	0.80	0.83	0.43
3	B1	0.118	1.29	0.33	0.19	0.14	0.14	0.14	0.13	0.35	0.20	0.16	0.93	0.89	0.89	3.00	0.12
	B2	8.085	6.56	0.65	0.54	0.45	0.39	0.39	0.40	0.67	0.56	0.48	0.94	0.90	0.89	0.42	0.34
	B3	7.680	8.42	0.66	0.52	0.47	0.44	0.50	0.61	0.68	0.54	0.49	0.94	0.90	0.89	0.40	0.33
	B4	0.014	0.00	0.70	0.54	0.49	0.43	0.42	0.40	0.72	0.57	0.51	0.94	0.91	0.89	1.78	0.31
4	B1	0.129	0.37	0.61	0.44	0.36	0.37	0.36	0.36	0.65	0.48	0.41	0.87	0.82	0.79	2.40	0.32
	B2	7.675	5.61	0.78	0.63	0.56	0.61	0.60	0.61	0.80	0.67	0.60	0.93	0.88	0.86	0.49	0.44
	B3	7.372	5.56	0.73	0.61	0.55	0.57	0.55	0.56	0.75	0.63	0.58	0.92	0.89	0.87	0.68	0.45
	B4	0.011	0.00	0.70	0.61	0.53	0.42	0.42	0.42	0.74	0.64	0.57	0.90	0.87	0.84	1.46	0.46
5	B1	0.039	0.56	0.56	0.40	0.33	0.35	0.34	0.33	0.59	0.43	0.36	0.90	0.84	0.84	6.33	0.28
	B2	6.094	6.24	0.70	0.59	0.53	0.54	0.55	0.57	0.73	0.62	0.56	0.92	0.89	0.88	0.61	0.53
	B3	6.693	6.43	0.73	0.62	0.56	0.55	0.57	0.68	0.74	0.64	0.58	0.95	0.92	0.91	0.61	0.46
	B4	0.014	0.00	0.68	0.55	0.49	0.42	0.42	0.41	0.70	0.58	0.52	0.93	0.88	0.87	1.66	0.38

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Table B.3 – *Continued from previous page, $N = 5000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\text{MSE}}$	$\widehat{\text{MSE}}(\hat{c}'\hat{I}^{-1}\hat{c})/\widehat{\text{MSE}}(\hat{c}'_{22}\hat{I}^{-1}\hat{c}_{22})$, $N = 5000$														
				Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	1	r	
1	C1	0.000	0.00	0.60	0.40	0.32	0.27	0.25	0.24	0.67	0.50	0.42	0.79	0.67	0.63	0.74	0.25	
	C2	0.040	0.01	0.59	0.44	0.37	0.34	0.33	0.33	0.70	0.57	0.50	0.78	0.71	0.68	0.50	0.26	
	C3	0.136	0.03	0.57	0.43	0.36	0.33	0.31	0.30	0.68	0.54	0.47	0.76	0.69	0.66	0.47	0.34	
	C4	0.000	0.00	0.57	0.41	0.34	0.30	0.28	0.27	0.67	0.51	0.44	0.76	0.67	0.63	0.65	0.24	
2	C1	0.000	0.00	0.56	0.40	0.32	0.26	0.24	0.24	0.65	0.48	0.40	0.80	0.73	0.71	0.80	0.26	
	C2	0.089	0.01	0.65	0.53	0.45	0.39	0.39	0.40	0.74	0.61	0.54	0.83	0.78	0.75	0.61	0.30	
	C3	0.144	0.05	0.66	0.51	0.44	0.39	0.37	0.36	0.73	0.59	0.51	0.82	0.77	0.74	0.73	0.45	
	C4	0.000	0.00	0.63	0.48	0.39	0.32	0.30	0.30	0.72	0.56	0.47	0.79	0.73	0.70	0.45	0.30	
3	C1	0.002	0.02	0.42	0.29	0.22	0.18	0.17	0.16	0.44	0.31	0.24	0.93	0.88	0.86	2.19	0.15	
	C2	0.258	0.09	0.56	0.43	0.37	0.33	0.33	0.34	0.58	0.46	0.40	0.94	0.90	0.88	0.52	0.22	
	C3	0.873	0.51	0.67	0.55	0.50	0.54	0.64	0.80	0.70	0.59	0.54	0.91	0.86	0.86	1.24	0.44	
	C4	0.001	0.00	0.59	0.45	0.36	0.27	0.25	0.23	0.62	0.48	0.40	0.90	0.85	0.83	0.91	0.21	
4	C1	0.016	0.01	0.59	0.43	0.35	0.36	0.35	0.36	0.63	0.47	0.40	0.85	0.79	0.76	2.44	0.33	
	C2	1.402	0.23	0.75	0.62	0.55	0.57	0.57	0.59	0.78	0.65	0.59	0.92	0.87	0.86	0.54	0.42	
	C3	0.879	0.25	0.69	0.58	0.51	0.53	0.52	0.55	0.73	0.63	0.57	0.90	0.86	0.83	2.11	0.40	
	C4	0.002	0.00	0.63	0.50	0.42	0.38	0.39	0.41	0.67	0.54	0.46	0.86	0.82	0.79	1.37	0.39	
5	C1	0.004	0.01	0.55	0.39	0.33	0.35	0.34	0.32	0.58	0.43	0.37	0.88	0.82	0.81	10.15	0.28	
	C2	1.041	0.21	0.69	0.58	0.52	0.54	0.58	0.63	0.71	0.61	0.54	0.93	0.90	0.88	0.45	0.43	
	C3	0.693	0.29	0.65	0.52	0.45	0.48	0.56	0.84	0.69	0.57	0.49	0.91	0.86	0.86	0.61	0.35	
	C4	0.002	0.00	0.58	0.44	0.38	0.41	0.40	0.40	0.61	0.47	0.40	0.89	0.84	0.83	4.33	0.36	
1	D1	0.329	0.03	0.89	0.83	0.80	0.83	0.82	0.83	0.94	0.89	0.86	0.92	0.89	0.87	1.67	0.63	
	D2	7.845	1.28	0.92	0.87	0.84	0.81	0.80	0.80	0.94	0.89	0.87	0.95	0.93	0.92	0.87	0.82	
	D3	6.187	0.81	0.73	0.61	0.56	0.52	0.50	0.49	0.80	0.71	0.65	0.87	0.81	0.78	0.48	0.50	
	D4	0.019	0.00	0.84	0.75	0.71	0.65	0.64	0.63	0.87	0.79	0.76	0.93	0.91	0.89	1.11	0.66	
2	D1	0.055	0.01	0.71	0.58	0.52	0.53	0.53	0.52	0.77	0.65	0.59	0.87	0.81	0.79	2.85	0.47	
	D2	5.822	0.72	0.79	0.69	0.64	0.66	0.65	0.66	0.84	0.75	0.70	0.91	0.86	0.85	0.85	0.64	
	D3	5.964	0.99	0.73	0.59	0.52	0.53	0.53	0.59	0.77	0.65	0.59	0.90	0.85	0.82	0.51	0.42	
	D4	0.021	0.00	0.80	0.70	0.64	0.64	0.64	0.65	0.84	0.74	0.69	0.92	0.88	0.87	0.57	0.54	
3	D1	0.118	0.06	0.65	0.53	0.47	0.37	0.36	0.35	0.69	0.56	0.50	0.91	0.88	0.86	12.11	0.39	
	D2	8.087	2.00	0.73	0.62	0.56	0.53	0.53	0.55	0.76	0.65	0.60	0.93	0.90	0.89	0.81	0.53	
	D3	7.681	3.11	0.78	0.66	0.61	0.62	0.72	0.93	0.78	0.67	0.63	0.94	0.90	0.89	0.60	0.43	
	D4	0.034	0.00	0.81	0.72	0.61	0.57	0.52	0.47	0.82	0.74	0.63	0.96	0.94	0.94	0.51	0.44	
4	D1	0.127	0.01	0.88	0.81	0.78	0.76	0.76	0.75	0.90	0.85	0.83	0.95	0.92	0.90	2.50	0.65	
	D2	7.687	1.12	0.82	0.76	0.73	0.78	0.78	0.81	0.85	0.79	0.76	0.94	0.91	0.90	0.82	0.72	
	D3	7.362	1.28	0.71	0.61	0.53	0.47	0.46	0.48	0.76	0.65	0.59	0.89	0.86	0.83	1.10	0.49	
	D4	0.109	0.00	0.79	0.69	0.62	0.61	0.61	0.62	0.82	0.73	0.67	0.92	0.89	0.86	1.11	0.57	
5	D1	0.038	0.01	0.68	0.55	0.50	0.44	0.44	0.42	0.73	0.60	0.55	0.87	0.80	0.79	15.35	0.42	
	D2	6.108	0.92	0.76	0.66	0.60	0.59	0.60	0.63	0.80	0.70	0.65	0.92	0.90	0.88	0.68	0.56	
	D3	6.678	1.31	0.75	0.64	0.58	0.56	0.64	0.94	0.76	0.66	0.59	0.94	0.91	0.90	0.75	0.54	
	D4	0.157	0.00	0.78	0.66	0.59	0.53	0.53	0.58	0.79	0.68	0.61	0.94	0.91	0.89	0.64	0.52	

Table B.4: *Relative bias of estimators for $c'I^{-1}c$, $N = 250$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	A1	0.329	4.55	4.09	3.89	3.85	3.97	4.23	4.82	4.16	3.98	3.92	4.36	4.26	4.24	4.25	3.19
	A2	7.845	1.15	1.13	1.13	1.15	1.19	1.29	1.44	1.14	1.13	1.14	1.14	1.13	1.14	1.03	0.93
	A3	6.186	1.27	1.20	1.19	1.21	1.23	1.27	1.30	1.21	1.20	1.20	1.24	1.23	1.23	1.12	0.96
	A4	0.019	3.33	2.98	2.80	2.69	2.71	2.75	2.84	3.01	2.86	2.78	3.18	3.09	3.04	2.76	2.26
2	A1	0.055	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	A2	5.821	1.38	1.33	1.33	1.35	1.41	1.50	1.66	1.34	1.33	1.36	1.36	1.36	1.35	1.18	1.07
	A3	5.963	1.61	1.49	1.45	1.46	1.47	1.46	1.49	1.50	1.46	1.46	1.57	1.55	1.55	1.38	1.04
	A4	0.025	3.39	2.89	2.61	2.49	2.42	2.36	2.37	2.97	2.72	2.58	3.19	3.10	3.05	2.65	2.04
3	A1	0.118	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	A2	8.085	1.49	1.40	1.36	1.41	1.48	1.61	1.77	1.41	1.36	1.40	1.47	1.45	1.46	1.36	1.07
	A3	7.681	1.98	1.80	1.72	1.69	1.63	1.64	1.74	1.81	1.73	1.70	1.92	1.91	1.89	1.82	1.10
	A4	0.034	4.76	3.80	3.25	3.00	2.72	2.55	2.35	3.86	3.34	3.08	4.56	4.44	4.39	4.14	2.58
4	A1	0.128	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	A2	7.683	1.43	1.37	1.37	1.41	1.47	1.53	1.55	1.38	1.37	1.40	1.42	1.41	1.41	1.29	1.08
	A3	7.366	1.55	1.45	1.45	1.48	1.53	1.60	1.83	1.46	1.45	1.47	1.52	1.50	1.50	1.44	1.09
	A4	0.032	3.46	2.97	2.70	2.64	2.62	2.62	2.73	3.03	2.77	2.70	3.30	3.22	3.18	3.40	2.19
5	A1	0.039	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	A2	6.102	1.67	1.57	1.51	1.51	1.55	1.55	1.57	1.57	1.51	1.51	1.65	1.63	1.63	1.68	1.18
	A3	6.686	1.99	1.79	1.81	1.83	1.78	1.77	2.11	1.80	1.79	1.81	1.95	1.94	1.93	2.24	1.12
	A4	0.040	3.69	3.13	2.76	2.55	2.41	2.31	2.24	3.16	2.80	2.59	3.55	3.47	3.42	4.21	2.01
1	B1	0.330	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	7.844	1.40	1.33	1.30	1.30	1.33	1.43	1.58	1.34	1.31	1.30	1.36	1.34	1.34	1.19	1.06
	B3	6.187	1.59	1.44	1.39	1.38	1.41	1.45	1.48	1.46	1.42	1.40	1.52	1.49	1.49	1.30	1.12
	B4	0.008	>5	4.61	4.18	3.94	3.96	4.03	4.16	4.74	4.37	4.16	>5	4.95	4.84	4.17	3.24
2	B1	0.055	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	5.819	2.03	1.81	1.73	1.71	1.73	1.79	1.95	1.84	1.77	1.74	1.95	1.92	1.90	1.60	1.36
	B3	5.964	2.22	1.93	1.78	1.74	1.73	1.71	1.72	1.96	1.83	1.78	2.12	2.07	2.05	1.62	1.28
	B4	0.011	>5	>5	4.35	4.06	3.85	3.66	3.59	>5	4.62	4.28	>5	>5	>5	4.53	3.24
3	B1	0.118	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	8.085	2.64	2.25	2.01	1.97	1.94	2.00	2.08	2.27	2.05	2.00	2.56	2.51	2.49	2.28	1.54
	B3	7.680	3.13	2.66	2.41	2.29	2.12	2.05	2.07	2.70	2.45	2.32	3.01	2.96	2.93	2.52	1.57
	B4	0.014	>5	>5	>5	>5	>5	4.58	4.05	>5	>5	>5	>5	>5	>5	>5	4.49
4	B1	0.129	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	7.675	2.56	2.25	2.11	2.06	2.13	2.15	2.14	2.27	2.13	2.07	2.48	2.44	2.42	2.21	1.62
	B3	7.372	2.70	2.39	2.22	2.17	2.17	2.23	2.44	2.40	2.25	2.19	2.60	2.54	2.52	2.52	1.61
	B4	0.011	>5	>5	>5	>5	>5	>5	4.94	>5	>5	>5	>5	>5	>5	>5	4.77
5	B1	0.039	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	6.094	3.68	3.05	2.67	2.52	2.51	2.37	2.28	3.08	2.71	2.55	3.57	3.50	3.46	4.28	2.06
	B3	6.693	3.81	3.07	2.90	2.75	2.59	2.49	2.79	3.09	2.90	2.74	3.69	3.65	3.61	4.06	1.99
	B4	0.014	>5	>5	>5	>5	4.61	4.19	3.86	>5	>5	>5	>5	>5	>5	>5	3.93

Continued on next page

Table B.4 – *Continued from previous page, $N = 250$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.040	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C3	0.136	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C4	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
2	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.089	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C3	0.144	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C4	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
3	C1	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.258	>5	>5	>5	>5	>5	>5	4.39	>5	>5	>5	>5	>5	>5	>5	>5
	C3	0.873	>5	>5	>5	>5	>5	>5	4.85	>5	>5	>5	>5	>5	>5	>5	3.94
	C4	0.001	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
4	C1	0.016	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	1.402	2.72	2.39	2.29	2.32	2.45	2.49	2.37	2.41	2.31	2.33	2.62	2.58	2.57	2.33	1.65
	C3	0.879	>5	4.50	4.24	4.27	4.38	4.27	4.39	4.61	4.36	4.32	>5	>5	4.96	>5	3.10
	C4	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
5	C1	0.004	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	1.041	4.13	3.50	3.04	2.95	2.95	2.74	2.42	3.53	3.08	2.98	3.98	3.90	3.85	4.73	2.26
	C3	0.693	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	4.27
	C4	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
1	D1	0.329	4.61	4.15	3.94	3.91	4.04	4.29	4.89	4.22	4.03	3.99	4.40	4.30	4.29	4.31	3.24
	D2	7.845	1.15	1.14	1.13	1.15	1.20	1.30	1.45	1.14	1.14	1.15	1.14	1.14	1.14	1.03	0.93
	D3	6.187	1.28	1.21	1.20	1.21	1.24	1.27	1.30	1.22	1.20	1.21	1.25	1.23	1.24	1.14	0.96
	D4	0.019	3.59	3.23	3.08	3.01	3.06	3.10	3.24	3.28	3.15	3.09	3.44	3.35	3.33	2.96	2.50
2	D1	0.055	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	D2	5.822	1.42	1.36	1.34	1.37	1.42	1.51	1.67	1.37	1.35	1.37	1.39	1.38	1.38	1.23	1.09
	D3	5.964	1.65	1.53	1.49	1.49	1.50	1.48	1.50	1.54	1.50	1.49	1.61	1.59	1.59	1.43	1.06
	D4	0.021	3.61	3.21	2.96	2.90	2.92	2.93	3.04	3.27	3.05	2.97	3.43	3.34	3.31	2.94	2.25
3	D1	0.118	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	D2	8.087	1.69	1.52	1.45	1.47	1.53	1.64	1.79	1.53	1.46	1.47	1.66	1.64	1.64	1.67	1.14
	D3	7.681	2.19	1.91	1.80	1.75	1.66	1.65	1.76	1.94	1.82	1.76	2.13	2.10	2.08	2.13	1.17
	D4	0.034	4.27	3.41	3.00	2.94	2.89	2.87	2.83	3.48	3.07	3.00	4.09	4.01	3.97	4.45	2.25
4	D1	0.127	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	D2	7.687	1.30	1.28	1.29	1.34	1.40	1.46	1.49	1.28	1.29	1.33	1.30	1.29	1.30	1.24	1.04
	D3	7.362	1.44	1.35	1.36	1.41	1.47	1.54	1.78	1.36	1.37	1.40	1.40	1.39	1.39	1.43	1.02
	D4	0.109	1.65	1.55	1.58	1.66	1.69	1.70	1.68	1.57	1.59	1.66	1.61	1.60	1.60	1.75	1.31
5	D1	0.038	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	D2	6.108	1.49	1.42	1.42	1.43	1.46	1.48	1.52	1.43	1.43	1.43	1.47	1.46	1.46	1.62	1.12
	D3	6.678	1.82	1.70	1.71	1.77	1.74	1.73	2.09	1.70	1.70	1.74	1.79	1.78	1.77	2.01	1.09
	D4	0.157	1.73	1.58	1.58	1.62	1.55	1.38	1.18	1.59	1.59	1.62	1.70	1.68	1.68	1.94	1.17

Table B.5: *Relative bias of estimators for $c'I^{-1}c$, $N = 1000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	A1	0.329	1.88	1.78	1.73	1.73	1.72	1.74	1.78	1.79	1.75	1.74	1.84	1.81	1.82	2.10	1.53
	A2	7.845	1.03	1.03	1.03	1.03	1.03	1.04	1.06	1.03	1.03	1.03	1.02	1.02	1.02	0.98	0.94
	A3	6.186	1.06	1.05	1.04	1.05	1.06	1.09	1.13	1.05	1.05	1.05	1.05	1.05	1.05	1.02	0.95
	A4	0.019	1.49	1.41	1.37	1.37	1.36	1.36	1.37	1.43	1.40	1.39	1.44	1.42	1.42	1.60	1.27
2	A1	0.055	>5	>5	4.66	4.45	4.41	4.42	4.44	>5	4.85	4.66	>5	>5	>5	>5	4.00
	A2	5.821	1.06	1.06	1.06	1.06	1.07	1.08	1.11	1.06	1.06	1.06	1.06	1.06	1.06	0.99	0.95
	A3	5.963	1.12	1.09	1.09	1.10	1.14	1.18	1.22	1.10	1.09	1.11	1.10	1.10	1.10	1.06	0.96
	A4	0.025	1.47	1.38	1.34	1.33	1.32	1.33	1.35	1.38	1.35	1.34	1.44	1.42	1.42	1.45	1.22
3	A1	0.118	>5	4.42	3.93	3.66	3.48	3.42	3.37	4.49	4.02	3.74	>5	>5	>5	>5	3.14
	A2	8.085	1.10	1.09	1.08	1.09	1.11	1.14	1.19	1.08	1.08	1.09	1.09	1.09	1.09	1.02	0.97
	A3	7.681	1.22	1.19	1.20	1.25	1.30	1.35	1.34	1.19	1.20	1.25	1.21	1.21	1.20	1.16	0.99
	A4	0.034	1.73	1.58	1.48	1.47	1.46	1.44	1.42	1.59	1.50	1.48	1.69	1.67	1.66	1.88	1.27
4	A1	0.128	3.73	3.32	3.16	3.03	3.04	3.02	3.09	3.36	3.20	3.08	3.59	3.53	3.49	>5	2.76
	A2	7.683	1.10	1.09	1.08	1.08	1.09	1.11	1.15	1.09	1.08	1.08	1.10	1.10	1.09	1.02	0.97
	A3	7.366	1.11	1.09	1.08	1.09	1.13	1.19	1.27	1.09	1.08	1.09	1.10	1.10	1.10	1.06	0.98
	A4	0.032	1.51	1.42	1.36	1.32	1.32	1.33	1.35	1.42	1.37	1.33	1.48	1.46	1.45	1.66	1.21
5	A1	0.039	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	A2	6.102	1.14	1.12	1.12	1.12	1.14	1.17	1.21	1.12	1.12	1.12	1.14	1.14	1.13	1.07	1.03
	A3	6.686	1.19	1.15	1.16	1.22	1.30	1.40	1.47	1.16	1.16	1.21	1.18	1.18	1.18	1.16	0.98
	A4	0.040	1.61	1.45	1.39	1.36	1.37	1.37	1.38	1.47	1.41	1.37	1.58	1.56	1.55	1.91	1.23
1	B1	0.330	3.46	2.98	2.72	2.64	2.59	2.60	2.64	3.05	2.84	2.76	3.25	3.13	3.10	2.92	2.32
	B2	7.844	1.09	1.07	1.07	1.07	1.06	1.08	1.10	1.08	1.07	1.07	1.08	1.07	1.07	0.99	0.96
	B3	6.187	1.14	1.11	1.09	1.09	1.10	1.14	1.17	1.12	1.10	1.10	1.13	1.12	1.11	1.08	1.00
	B4	0.008	2.02	1.81	1.72	1.69	1.66	1.66	1.69	1.86	1.77	1.73	1.91	1.86	1.86	1.94	1.55
2	B1	0.055	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	5.819	1.20	1.17	1.16	1.15	1.15	1.16	1.19	1.17	1.15	1.15	1.19	1.18	1.18	1.11	1.02
	B3	5.964	1.23	1.18	1.15	1.17	1.20	1.24	1.28	1.19	1.17	1.18	1.21	1.20	1.20	1.14	1.02
	B4	0.011	2.05	1.81	1.70	1.67	1.65	1.65	1.67	1.84	1.75	1.71	1.97	1.91	1.90	1.93	1.51
3	B1	0.118	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	8.085	1.35	1.28	1.25	1.23	1.23	1.25	1.30	1.28	1.25	1.23	1.34	1.33	1.33	1.33	1.09
	B3	7.680	1.49	1.38	1.37	1.39	1.43	1.47	1.45	1.39	1.38	1.40	1.46	1.45	1.44	1.32	1.13
	B4	0.014	2.99	2.52	2.30	2.21	2.16	2.10	2.03	2.56	2.35	2.24	2.88	2.82	2.80	3.46	1.85
4	B1	0.129	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	7.675	1.34	1.27	1.22	1.22	1.22	1.24	1.28	1.27	1.23	1.22	1.33	1.31	1.31	1.21	1.12
	B3	7.372	1.33	1.26	1.23	1.23	1.26	1.33	1.40	1.27	1.24	1.23	1.31	1.30	1.29	1.32	1.12
	B4	0.011	2.42	2.13	1.95	1.85	1.83	1.82	1.83	2.14	1.98	1.89	2.34	2.27	2.25	2.54	1.66
5	B1	0.039	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	6.094	1.58	1.44	1.40	1.38	1.38	1.40	1.45	1.45	1.41	1.39	1.55	1.54	1.53	1.50	1.27
	B3	6.693	1.58	1.43	1.41	1.45	1.53	1.63	1.68	1.44	1.42	1.45	1.54	1.53	1.53	1.49	1.20
	B4	0.014	2.70	2.25	2.06	1.97	1.96	1.94	1.92	2.28	2.10	2.01	2.60	2.55	2.53	3.34	1.71

Continued on next page

Table B.5 – *Continued from previous page,* $N = 1000$.

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.040	>5	>5	4.34	3.99	3.84	3.83	3.92	>5	4.78	4.47	>5	>5	>5	4.73	3.34
	C3	0.136	3.97	3.11	2.77	2.61	2.58	2.74	3.15	3.31	3.01	2.89	3.53	3.36	3.27	2.86	2.30
	C4	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
2	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.089	4.13	3.28	2.83	2.64	2.58	2.59	2.56	3.45	3.06	2.87	3.77	3.60	3.53	3.03	2.16
	C3	0.144	>5	4.24	3.74	3.48	3.56	4.23	>5	4.39	3.94	3.70	4.71	4.51	4.42	4.32	3.29
	C4	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
3	C1	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.258	3.61	2.90	2.67	2.51	2.42	2.32	2.26	2.95	2.73	2.57	3.44	3.37	3.33	3.41	2.20
	C3	0.873	2.68	2.30	2.16	2.30	2.67	3.62	>5	2.34	2.21	2.33	2.58	2.52	2.51	2.67	1.82
	C4	0.001	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
4	C1	0.016	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	1.402	1.41	1.32	1.26	1.26	1.29	1.33	1.41	1.33	1.28	1.28	1.38	1.36	1.36	1.12	1.10
	C3	0.879	1.97	1.72	1.62	1.62	1.76	2.04	2.38	1.75	1.66	1.66	1.88	1.83	1.82	1.98	1.43
	C4	0.002	>5	>5	>5	4.67	4.57	4.67	4.74	>5	>5	4.92	>5	>5	>5	>5	4.10
5	C1	0.004	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	1.041	1.65	1.51	1.47	1.46	1.49	1.56	1.65	1.52	1.48	1.48	1.61	1.60	1.59	1.31	1.35
	C3	0.693	2.57	2.21	2.12	2.28	2.62	3.22	3.92	2.24	2.15	2.30	2.46	2.43	2.41	2.79	1.77
	C4	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
1	D1	0.329	1.91	1.80	1.75	1.74	1.74	1.75	1.80	1.82	1.78	1.76	1.87	1.84	1.84	2.11	1.53
	D2	7.845	1.03	1.02	1.03	1.03	1.03	1.04	1.06	1.03	1.03	1.03	1.02	1.02	1.02	0.97	0.94
	D3	6.187	1.06	1.05	1.04	1.05	1.06	1.09	1.13	1.05	1.05	1.05	1.06	1.05	1.05	1.02	0.96
	D4	0.019	1.56	1.49	1.47	1.46	1.46	1.47	1.48	1.51	1.49	1.48	1.52	1.50	1.50	1.63	1.35
2	D1	0.055	>5	>5	>5	4.98	4.96	4.96	4.92	>5	>5	>5	>5	>5	>5	>5	4.42
	D2	5.822	1.07	1.06	1.06	1.07	1.07	1.09	1.11	1.06	1.06	1.06	1.07	1.07	1.07	1.02	0.96
	D3	5.964	1.12	1.10	1.09	1.11	1.14	1.19	1.23	1.11	1.10	1.12	1.11	1.11	1.11	1.08	0.97
	D4	0.021	1.55	1.47	1.42	1.42	1.42	1.44	1.50	1.48	1.44	1.43	1.52	1.50	1.50	1.42	1.29
3	D1	0.118	>5	>5	>5	>5	4.88	4.59	4.33	>5	>5	>5	>5	>5	>5	>5	3.95
	D2	8.087	1.13	1.11	1.10	1.12	1.13	1.15	1.20	1.10	1.10	1.12	1.13	1.13	1.13	1.11	1.00
	D3	7.681	1.26	1.22	1.23	1.27	1.32	1.36	1.35	1.23	1.23	1.27	1.25	1.24	1.24	1.23	1.02
	D4	0.034	1.58	1.45	1.41	1.39	1.38	1.40	1.48	1.46	1.42	1.40	1.55	1.54	1.53	1.69	1.26
4	D1	0.127	2.45	2.31	2.23	2.18	2.18	2.19	2.27	2.33	2.26	2.22	2.39	2.36	2.34	4.41	2.05
	D2	7.687	1.07	1.07	1.06	1.07	1.08	1.10	1.14	1.06	1.06	1.07	1.07	1.07	1.07	1.03	0.96
	D3	7.362	1.09	1.07	1.06	1.07	1.11	1.18	1.26	1.07	1.06	1.07	1.08	1.08	1.08	1.06	0.97
	D4	0.109	1.13	1.11	1.09	1.09	1.13	1.19	1.28	1.12	1.10	1.10	1.13	1.12	1.12	1.13	1.02
5	D1	0.038	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	4.97
	D2	6.108	1.11	1.09	1.10	1.10	1.11	1.15	1.18	1.09	1.10	1.10	1.10	1.10	1.10	1.09	1.03
	D3	6.678	1.17	1.14	1.14	1.20	1.28	1.38	1.45	1.14	1.15	1.20	1.16	1.16	1.16	1.14	0.98
	D4	0.157	1.14	1.12	1.11	1.12	1.17	1.26	1.33	1.12	1.11	1.12	1.14	1.13	1.13	1.12	1.03

Table B.6: *Relative bias of estimators for $c'I^{-1}c$, $N = 5000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	A1	0.329	1.16	1.15	1.14	1.14	1.16	1.16	1.16	1.16	1.15	1.15	1.15	1.15	1.15	1.33	1.03
	A2	7.845	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
	A3	6.186	1.01	1.01	1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	0.98
	A4	0.019	1.11	1.09	1.08	1.07	1.05	1.05	1.05	1.10	1.09	1.08	1.10	1.10	1.10	1.17	1.05
2	A1	0.055	1.96	1.80	1.70	1.65	1.68	1.67	1.67	1.83	1.75	1.70	1.89	1.86	1.84	2.50	1.61
	A2	5.821	1.01	1.01	1.01	1.01	1.01	1.01	1.02	1.01	1.01	1.01	1.01	1.01	1.01	0.97	0.97
	A3	5.963	1.03	1.02	1.02	1.02	1.02	1.03	1.05	1.02	1.02	1.02	1.03	1.03	1.03	1.03	0.98
	A4	0.025	1.10	1.07	1.06	1.05	1.07	1.07	1.08	1.08	1.07	1.06	1.09	1.09	1.09	1.15	1.02
3	A1	0.118	1.91	1.68	1.57	1.52	1.49	1.48	1.47	1.70	1.59	1.54	1.86	1.84	1.84	2.26	1.51
	A2	8.085	1.02	1.02	1.02	1.02	1.02	1.02	1.03	1.02	1.02	1.02	1.02	1.02	1.02	0.97	0.96
	A3	7.681	1.05	1.04	1.04	1.04	1.05	1.08	1.13	1.04	1.04	1.04	1.05	1.05	1.05	1.02	0.97
	A4	0.034	1.16	1.12	1.11	1.10	1.10	1.10	1.10	1.13	1.11	1.10	1.15	1.14	1.14	1.32	1.06
4	A1	0.128	1.56	1.46	1.40	1.38	1.40	1.39	1.39	1.47	1.42	1.40	1.53	1.51	1.51	1.86	1.31
	A2	7.683	1.02	1.02	1.02	1.02	1.02	1.02	1.03	1.02	1.02	1.02	1.02	1.02	1.02	0.97	0.96
	A3	7.366	1.02	1.02	1.02	1.01	1.01	1.02	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.02	0.97
	A4	0.032	1.10	1.08	1.08	1.07	1.06	1.07	1.07	1.08	1.08	1.07	1.09	1.09	1.09	1.18	1.05
5	A1	0.039	3.12	2.65	2.40	2.29	2.36	2.34	2.34	2.69	2.47	2.35	3.00	2.91	2.89	>5	2.23
	A2	6.102	1.04	1.03	1.03	1.03	1.03	1.03	1.04	1.03	1.03	1.03	1.04	1.04	1.03	0.98	0.99
	A3	6.686	1.03	1.02	1.02	1.02	1.02	1.05	1.09	1.02	1.02	1.02	1.03	1.03	1.03	1.03	0.95
	A4	0.040	1.11	1.08	1.07	1.06	1.08	1.08	1.08	1.09	1.07	1.07	1.10	1.10	1.10	1.24	1.05
1	B1	0.330	1.49	1.40	1.35	1.34	1.33	1.33	1.32	1.43	1.39	1.37	1.44	1.41	1.41	1.65	1.17
	B2	7.844	1.01	1.01	1.00	1.00	1.01	1.01	1.01	1.01	1.00	1.00	1.01	1.01	1.01	1.00	0.96
	B3	6.187	1.03	1.02	1.01	1.01	1.02	1.02	1.02	1.02	1.02	1.01	1.02	1.02	1.02	1.01	0.98
	B4	0.008	1.22	1.16	1.14	1.13	1.09	1.09	1.09	1.18	1.16	1.15	1.19	1.18	1.18	1.27	1.10
2	B1	0.055	4.82	3.95	3.48	3.22	3.34	3.25	3.24	4.10	3.69	3.44	4.46	4.30	4.22	4.55	3.05
	B2	5.819	1.04	1.03	1.03	1.03	1.02	1.03	1.03	1.03	1.03	1.03	1.04	1.04	1.04	0.97	0.95
	B3	5.964	1.06	1.05	1.04	1.04	1.05	1.05	1.06	1.05	1.04	1.04	1.05	1.05	1.05	1.05	1.00
	B4	0.011	1.22	1.16	1.13	1.12	1.12	1.12	1.13	1.17	1.14	1.13	1.20	1.19	1.18	1.28	1.09
3	B1	0.118	>5	4.56	3.81	3.50	3.46	3.41	3.37	4.65	3.92	3.60	>5	>5	>5	>5	3.12
	B2	8.085	1.08	1.06	1.05	1.05	1.04	1.04	1.05	1.07	1.06	1.05	1.08	1.08	1.08	1.00	0.96
	B3	7.680	1.12	1.09	1.08	1.07	1.09	1.11	1.16	1.09	1.08	1.08	1.11	1.11	1.11	1.05	1.01
	B4	0.014	1.39	1.30	1.26	1.23	1.23	1.24	1.24	1.31	1.26	1.24	1.38	1.36	1.36	1.73	1.17
4	B1	0.129	4.20	3.42	3.02	2.84	2.84	2.81	2.82	3.49	3.11	2.95	3.97	3.86	3.81	>5	2.67
	B2	7.675	1.06	1.05	1.05	1.04	1.04	1.04	1.05	1.05	1.05	1.04	1.06	1.06	1.06	0.98	1.02
	B3	7.372	1.07	1.05	1.05	1.04	1.04	1.04	1.05	1.05	1.05	1.04	1.06	1.06	1.06	1.05	1.01
	B4	0.011	1.28	1.22	1.19	1.17	1.15	1.15	1.15	1.23	1.20	1.19	1.26	1.25	1.25	1.47	1.14
5	B1	0.039	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	B2	6.094	1.12	1.09	1.09	1.08	1.08	1.08	1.09	1.09	1.09	1.08	1.12	1.12	1.11	1.06	1.10
	B3	6.693	1.09	1.07	1.05	1.05	1.06	1.08	1.13	1.07	1.05	1.05	1.09	1.08	1.08	1.06	1.00
	B4	0.014	1.32	1.25	1.20	1.18	1.18	1.18	1.18	1.25	1.21	1.19	1.30	1.29	1.29	1.52	1.13

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Table B.6 – *Continued from previous page, $N = 5000$.*

Design	Curve	$c'I^{-1}c$	$\widehat{\mathrm{RB}}(\hat{c}'\hat{I}^{-1}\hat{c})$														
			Index of $\hat{c}'\hat{I}^{-1}\hat{c}$														
			22	33	55	99	1515	2525	4040	23	25	29	32	52	92	l	r
1	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.040	2.10	1.77	1.64	1.57	1.56	1.55	1.54	1.87	1.75	1.70	1.93	1.88	1.85	1.77	1.38
	C3	0.136	1.68	1.48	1.41	1.38	1.31	1.30	1.31	1.53	1.46	1.43	1.58	1.55	1.54	1.39	1.33
	C4	0.000	>5	>5	>5	4.91	4.67	4.53	4.44	>5	>5	>5	>5	>5	>5	>5	4.21
2	C1	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.089	1.58	1.44	1.38	1.34	1.32	1.32	1.33	1.47	1.42	1.38	1.52	1.49	1.48	1.45	1.13
	C3	0.144	1.85	1.63	1.54	1.49	1.38	1.35	1.32	1.67	1.58	1.54	1.76	1.73	1.71	1.74	1.46
	C4	0.000	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
3	C1	0.002	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	0.258	1.51	1.37	1.31	1.28	1.29	1.29	1.30	1.38	1.32	1.30	1.47	1.46	1.45	1.35	1.09
	C3	0.873	1.35	1.26	1.23	1.21	1.20	1.25	1.36	1.27	1.24	1.22	1.32	1.31	1.31	1.41	1.18
	C4	0.001	4.64	3.75	3.30	3.01	2.80	2.72	2.62	3.82	3.39	3.12	4.43	4.30	4.26	4.09	2.52
4	C1	0.016	4.34	3.53	3.09	2.90	2.87	2.85	2.86	3.61	3.21	3.04	4.08	3.95	3.89	>5	2.80
	C2	1.402	1.07	1.06	1.05	1.05	1.05	1.06	1.07	1.06	1.05	1.05	1.06	1.06	1.06	0.92	0.95
	C3	0.879	1.19	1.14	1.12	1.10	1.09	1.09	1.12	1.14	1.13	1.11	1.17	1.16	1.16	1.22	1.01
	C4	0.002	2.18	1.91	1.77	1.68	1.64	1.65	1.68	1.94	1.80	1.73	2.08	2.04	2.01	2.59	1.66
5	C1	0.004	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5	>5
	C2	1.041	1.13	1.10	1.10	1.09	1.10	1.11	1.14	1.10	1.10	1.09	1.13	1.12	1.12	0.91	1.04
	C3	0.693	1.26	1.19	1.14	1.12	1.15	1.22	1.38	1.19	1.15	1.13	1.24	1.23	1.23	1.24	1.00
	C4	0.002	3.25	2.65	2.39	2.26	2.35	2.36	2.39	2.70	2.46	2.33	3.10	3.02	3.00	>5	2.26
1	D1	0.329	1.16	1.15	1.14	1.14	1.16	1.16	1.16	1.15	1.15	1.15	1.15	1.15	1.15	1.42	1.03
	D2	7.845	1.00	1.00	1.00	1.00	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.97
	D3	6.187	1.01	1.01	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	0.98
	D4	0.019	1.14	1.12	1.10	1.10	1.07	1.07	1.07	1.12	1.11	1.11	1.13	1.12	1.12	1.24	1.06
2	D1	0.055	2.10	1.90	1.79	1.74	1.75	1.75	1.75	1.94	1.84	1.80	2.01	1.97	1.96	3.56	1.68
	D2	5.822	1.02	1.01	1.01	1.01	1.01	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	0.95	0.96
	D3	5.964	1.03	1.02	1.02	1.02	1.02	1.03	1.05	1.03	1.02	1.02	1.03	1.03	1.03	1.03	0.98
	D4	0.021	1.10	1.08	1.07	1.06	1.08	1.09	1.10	1.08	1.08	1.07	1.10	1.09	1.09	1.04	0.99
3	D1	0.118	2.24	1.97	1.86	1.80	1.70	1.68	1.67	2.00	1.89	1.83	2.17	2.15	2.14	4.64	1.76
	D2	8.087	1.02	1.02	1.02	1.02	1.02	1.03	1.04	1.02	1.02	1.02	1.02	1.02	1.02	0.99	0.96
	D3	7.681	1.05	1.05	1.04	1.04	1.06	1.08	1.13	1.05	1.04	1.04	1.05	1.05	1.05	1.03	0.97
	D4	0.034	1.11	1.09	1.08	1.07	1.08	1.07	1.07	1.09	1.08	1.07	1.10	1.10	1.10	1.00	1.05
4	D1	0.127	1.31	1.28	1.26	1.25	1.26	1.26	1.25	1.29	1.27	1.26	1.30	1.29	1.28	1.74	1.20
	D2	7.687	1.01	1.01	1.01	1.01	1.02	1.02	1.03	1.01	1.01	1.01	1.01	1.01	1.01	0.97	0.96
	D3	7.362	1.02	1.02	1.02	1.01	1.01	1.01	1.03	1.02	1.02	1.01	1.02	1.02	1.02	1.02	0.97
	D4	0.109	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.10	0.98
5	D1	0.038	2.48	2.21	2.08	2.01	1.93	1.91	1.88	2.25	2.13	2.06	2.38	2.32	2.30	>5	1.91
	D2	6.108	1.03	1.02	1.02	1.02	1.02	1.03	1.04	1.02	1.02	1.02	1.03	1.03	1.03	0.99	0.99
	D3	6.678	1.03	1.03	1.02	1.02	1.02	1.05	1.09	1.02	1.02	1.02	1.03	1.03	1.03	1.02	0.95
	D4	0.157	1.03	1.03	1.02	1.02	1.03	1.03	1.05	1.03	1.02	1.02	1.03	1.03	1.03	1.01	0.97

Appendix C. Check of covariate balance in RHC study

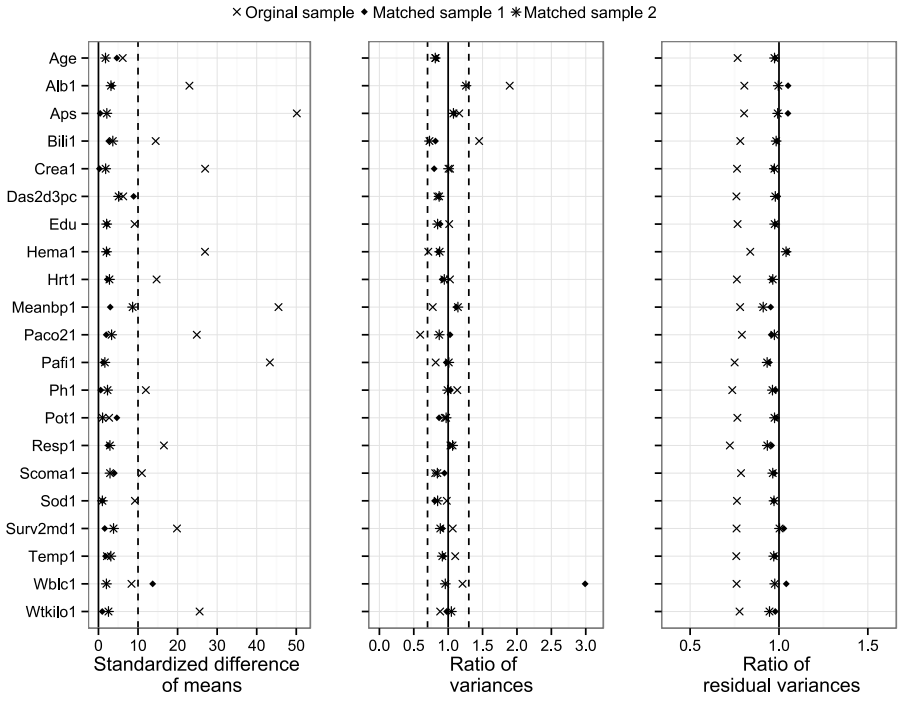

Figure C.1: *Comparison of means, variances and residual variances of continuous variables in the treated and control group. Note that Albumin is off the chart (variance ratio* $= 4.38$*). List of variables: 1. Age (years), 2. Albumin, 3. Apache score, 4. Bilirium, 5. Creatinine, 6. DASI - Duke Activity Status Index, 7. Education (years), 8. Hematocrit, 9. Heart Rate, 10. Mean Blood Pressure, 11. PaCO2, 12. PaCO2/FI02 ratio, 13. PH, 14. Potassium, 15. Respiratory Rate, 16. Glasgow coma score, 17. Sodium, 18. Estimate of probability of surviving 2 months, 19. Temperature, 20. WBC, 21. Weight.*

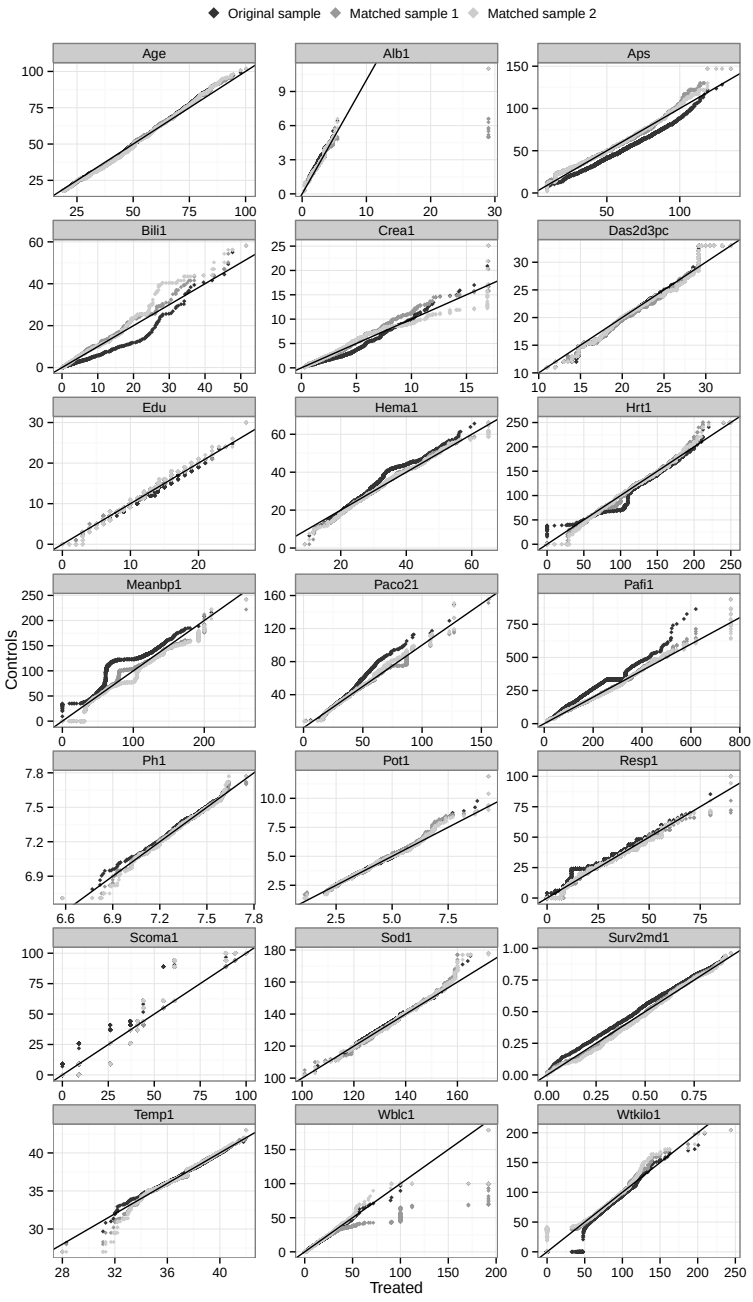

Figure C.2: *Q–Q plots comparing the distributions of continuous covariates in the treated and control group in the original sample and the two matched samples.*

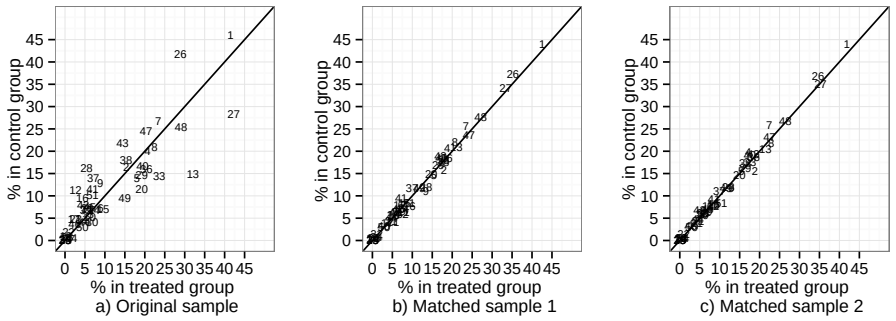

Figure C.3: *Comparison of binary covariates in the treated group and the control group in the original sample and the two matched samples. List of variables: 1. Female, 2. Race: Black, 3. Race: Other, 4. Income: $11-$25k, 5. Income: $25-$50k, 6. Income: > $50k, 7. Insurance: Medicare 8. Insurance: Private & Medicare, 9. Insurance: Medicaid, 10. Insurance: No Insurance, 11. Insurance: Medicare & Medicaid, 12. Category 1: COPD, 13. Category 1: MOSF w/Sepsis, 14. Cateogory 1: MOSF w/Malignancy, 15. Category 1: CHF, 16. Category 1: Coma, 17. Category 1: Cirrhosis, 18. Category 1: Lung Cancer, 19. Category 1: Colon Cancer, 20. Category 2: MOSF w/Sepsis, 21. Category 2: Coma, 22. Category 2: MOSF w/Malignancy, 23. Category 2: Lung Cancer, 24. Category 2: Cirrhosis, 25. Category 2: Colon Cancer, 26. Respiratory diagnosis, 27. Cardiovascular diagnosis, 28. Neurological diagnosis, 29. Gastrointestinal diagnosis, 30. Renal diagnosis, 31. Metabolic diagnosis, 32. Hematological diagnosis, 33. Sepsis diagnosis, 34. Trauma diagnosis, 35. Orthopedic diagnosis, 36. Cardiovasular symptoms, 37. Do not resuscitate on day 1, 38. Cancer: Localised, 39. Cancer: Metastatic, 40. Congestive heart failure, 41. Dementia, stroke or cerebral infarct, Parkinson's disease, 42. Psychiatric history, active psychosis or severe depression, 43. Chronic pulmonary disease, severe pulmonary disease, 44. Chronic renal disease, chronic hemodialysis or peritoneal dialysis, 45. Cirrhosis, hepatic failure, 46. Upper GI bleeding, 47. Solid tumour, metastatic disease, chronic leukemia/myeloma, acute leukemia, lymphoma, 48. Immunosuppression, organ transplant, HIV, Diabetes Mellitus, Connective Tissue disease, 49. Transfer (> 24 h) from another hospital, 50. Definite myocardial infarction, 51. Weight = 0.*

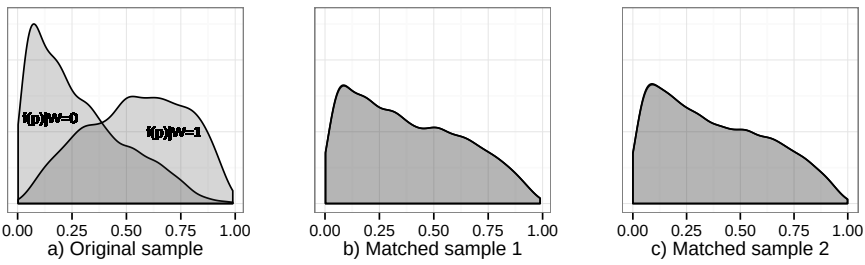

Figure C.4: *Propensity score distributions for patients receiving (treated) and not receiving RHC (controls).*

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