Magnetic fields and chemical maps of Ap stars from four Stokes parameter observations

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Abstract


Our knowledge of stellar magnetic fields relies almost entirely on circular polarization observations, which has historically limited our understanding of the stellar magnetic field topologies. Recently, it has become possible to obtain phase-resolved high-resolution spectropolarimetric observations in all four Stokes parameters for early-type magnetic stars. Interpretation of such observations with the Magnetic Doppler imaging technique has uncovered a new, previously unknown, level of complexity of surface stellar magnetic fields. This new insight is critical for understanding the origin, evolution and structure of magnetic fields in early-type stars.

In this study we observed the magnetic, chemically peculiar Ap stars HD 24712 (DO Eri, HR 1217) and HD 125248 (CS Vir, HR 5355) in all four Stokes parameters with the HARPSpol spectropolarimeter at the ESO 3.6-m telescope. The resulting spectra have high signal-to-noise ratio and superb resolving power, by far surpassing the quality of any existing stellar Stokes parameter observations.

We studied variation of the spectrum and magnetic observables of HD 24712 as a function of rotational phase (paper I). In the subsequent magnetic Doppler imaging investigation of this star, we interpreted the phase-resolved Stokes line profile observations (paper II). This analysis showed that HD 24712, unlike more massive Ap stars studied in all four Stokes parameters, has a dominant dipolar field component with a negligible contribution of small-scale magnetic structures. Simultaneously with magnetic mapping we derived surface abundance distributions of Fe, Nd, Na, and Ca.

Building upon the technique of Magnetic Doppler imaging, we developed the first three-dimensional abundance inversion code and applied it to reconstruct the abundance distributions of Fe and Ca in three dimensions in the atmosphere of HD 24712 (paper III).

We also performed Magnetic Doppler imaging analysis of the spectropolarimetric observations of HD 125248 (paper IV). The reconstructed detailed maps of the surface abundance distribution and magnetic field topology of HD 125248 revealed a magnetic field with significant deviations from the canonical dipolar field geometry, and strong surface abundance inhomogeneities for Cr and several rare earth elements.

We assessed our inversion results in the context of magnetic Doppler imaging studies of other magnetic, chemically peculiar Ap stars and latest theoretical research on the evolution and stability of magnetic fields in radiative stellar interiors. Our analysis suggests that old or less massive Ap stars have predominantly dipolar magnetic fields while more massive or younger stars exhibit more complicated field topologies. We also compared our three-dimensional chemical abundance maps of HD 24712 to the predictions of theoretical atomic diffusion calculations in magnetized stellar atmospheres, generally finding a lack of agreement between theory and observations.

Keywords: chemically peculiar stars, magnetic fields, spectropolarimetry, Doppler imaging, stellar atmospheres

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Dedicated to my love of Merlot
This thesis is based on the following papers, which are referred to in the text by their Roman numerals.


III  **Rusomarov, N., Kochukhov, O., and Ryabchikova, T.** *Three-dimensional magnetic and abundance mapping of the cool Ap star HD 24712 III. Three-dimensional abundance distribution of iron and calcium*, to be submitted to A&A


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1. Introduction

Magnetic fields are present in a wide variety of stars, from low-mass cool dwarfs and up to hot massive stars. Magnetic fields are known to influence a number of physical processes operating within and around stars. Thus, they play a significant role at practically all evolutionary stages (Mestel, 2001).

Since the discovery of a magnetic field on our own Sun by Hale (1908), our understanding of stellar magnetic fields has been significantly improved thanks to new developments in instrumentation and to the plethora of highly sophisticated numerical techniques for measuring, detecting, and simulating magnetic fields. Thanks to these advancements, it has become possible to study stellar magnetic fields based on detailed analysis and interpretation of the shapes and variability of profiles of spectral lines from high-precision spectroscopic and spectropolarimetric observations (Donati & Landstreet, 2009).

This work is the latest attempt to investigate the atmospheric structure and magnetic fields of chemically peculiar stars. These objects comprise only a small fraction of stars with magnetic fields. However, in contrast to other stars, they feature fairly simple global magnetic fields that show no intrinsic variability on time scales of decades. This provides us with an opportunity to observe and model in great detail both the field topology and the physical processes that govern the interaction between magnetic fields and atmospheric structures.

1.1 Chemically peculiar Ap stars

Chemically peculiar (CP) stars are upper main sequence (hydrogen burning) stars with masses greater than $1.4M_\odot$ that show anomalously strong absorption lines of some elements in their spectra. Relative to the solar abundance of chemical elements, these stars show overabundances of up to several orders of magnitude for some iron peak and rare earth elements, while other elements (e.g. C,N,O) are found to be underabundant (Ryabchikova, 2004). CP stars are separated into two distinct categories: magnetic Ap/Bp, and non-magnetic HgMn and Am stars. The magnetic Ap stars comprise only around 10% of the normal stars with spectral types between B5 and F4. On the Hertzsprung-Russell (HR) diagram all Ap stars are situated on the main sequence and have temperatures ranging from about 7000 K to 17000 K. We schematically illustrate this in Fig. 1.1.

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1The Hertzsprung–Russell diagram is a scatter graph of stars showing the relationship between the stars’ absolute magnitudes or luminosities versus their spectral types or effective temperatures.
Figure 1.1. An illustrative HR diagram showing the approximate position of magnetic Ap/Bp stars. The area (blue) encircled by the ellipse represents roughly the place that magnetic Ap stars occupy on the HR diagram. The most prominent feature of the HR diagram is the diagonal, known as the main sequence, going from upper-left (hot and bright) to the lower-right (cool and dim). Above the main sequence we can find the sub giants, giants and supergiants. The position of the Sun (a main sequence star) is marked with the yellow star.
Figure 1.2. Comparison between spectra of a chemically peculiar Ap star and a normal star of a similar spectral type and solar chemical composition in the wavelength region 6323 Å–6337 Å. The two spectra shown at the bottom are observed intensity spectra of the Ap star HD 24712 obtained at two different rotational phases, when the mean line-of-sight magnetic field reaches its maximum (black) and minimum (red) value. At the top we plot the spectrum of the normal star HD 32115. The spectra have been shifted in the vertical direction relative to each other. The spectral lines belonging to rare earth elements are annotated with red. The identified lines of Fe and Si are marked with black.

The atmospheres of Ap stars show remarkable enhancements of elements such as Si, Cr, Sr, and Eu relative to the solar chemical composition. Note that this anomalous chemical composition does not apply to their internal chemical composition, but is instead related to the chemical stratification in their atmospheres (see Sect. 1.3). In Fig. 1.2 we show a comparison between spectra of an Ap star and a normal star of a similar spectral type. The spectra of the chemically peculiar Ap star show the presence of numerous spectral lines of rare earth elements, which are absent in the spectrum of the normal star.

Chemically peculiar Ap stars possess unique magnetic characteristics. Their magnetic fields are global and mostly poloidal with strengths varying from several hundred gauss (Aurière et al., 2007) and up to tens of kilogauss for the Babcock’s star HD 215441 (Babcock, 1960). These magnetic fields are stable on time scales of at least several decades. They significantly influence the energy and mass transfer (e.g., convection, diffusion, stellar winds) within the mostly radiative atmospheres of Ap stars, resulting in the formation of atmospheric inhomogeneities in the form of abundance patches and vertical chemical stratification (Alecian & Stift, 2010).

The rapidly oscillating Ap (roAp) stars represent an important subset of Ap stars. They pulsate in high-overtone non-radial p-modes with periods of 6 min to 24 min (Kurtz & Martinez, 2000; Alentiev et al., 2012). Their oscillations have low photometric amplitudes of $\sim 0^m.01$. Stars in this group are
the coolest Ap stars with effective temperatures less than 8000 K. They lie in the δ Scuti instability strip on the main sequence, for which the pulsation excitation mechanism should be the partial helium ionization zone. In contrast, pulsations of roAp stars are thought to be excited by the hydrogen ionization (Balmforth et al., 2001). Oblique pulsator model (Kurtz, 1982) is a geometrical framework explaining the observed characteristics of roAp oscillations. In this model the axis of pulsation does not coincide with the rotation axis, but instead is aligned with the magnetic axis, which is inclined to the rotation axis at a certain angle. This apparent link between the magnetic and pulsation axis suggests that magnetic field plays a major role in the mechanism driving the oscillations.

For most roAp stars pulsations are studied using photometric methods. Recently, it has also become possible to observe their pulsations by measuring radial velocity variations of certain spectral lines (e.g. Kochukhov & Ryabchikova, 2001). Studies of this kind have shown that radial velocities (RV) of some spectral lines are variable with the pulsation period of the star. The measured RV amplitude and phase variations show a strong dependence on the chemical element (Ryabchikova et al., 2007). In most roAp stars lines of rare earth elements, e.g., Nd and Pr, show maximal amplitudes, followed by the cores of H\(\alpha\) and H\(\beta\) lines, while lines of Fe, Si and Cr show little or no pulsational variability. This behavior points at strong vertical chemical inhomogeneities in the atmospheres of roAp stars. The variability of RV amplitudes and phases can be explained by pulsation waves propagating outwards in the chemically stratified stellar atmosphere (Saio et al., 2010). In that case, we find large amplitude oscillations high in the atmosphere, where the density is lower and concentration of rare earth elements is high. This picture explains the observed RV amplitude and phase differences between Si, Ca, and Fe, and rare earth elements.

This complex interplay between the magnetic field, abundance inhomogeneities and pulsations in roAp stars presents us with ideal laboratories for asteroseismology and for detailed analysis of the structure of magnetic field and for studying its role in the appearance of abundance inhomogeneities and driving of non-radial pulsations in these objects.

1.2 Magnetic fields of Ap stars

The fields of Ap stars are measured primarily from circular and linear spectropolarimetric observations. In the presence of even a moderately weak magnetic field, spectral lines split into π and σ components, which are linearly and circularly polarized. This is known as the Zeeman effect. The separation of the π and σ components and their relative intensity are determined by the atomic parameters of individual lines as well as by the orientation of the local magnetic field vector relative to the line-of-sight to the observer. Partially
or fully polarized light can be characterized by the four Stokes parameters, $IQUV$, where $I$ is the total intensity, $QU$ is the intensity of linearly polarized, and $V$ is the intensity of circularly polarized light.

Historically, magnetic fields in stars were detected and measured using circular polarization analyzer (Babcock, 1949). The result of such measurement is an estimate of the longitudinal (line-of-sight) field component averaged over the visible stellar disk $\langle B_z \rangle$. Typical $\langle B_z \rangle$ measurements of Ap stars yield values between 100 G and 10 kG. If the magnetic field is very strong, leading to the appearance of spectral lines with visible splitting in Stokes $I$ in the stellar spectrum, it becomes possible to determine the magnetic field modulus $\langle B \rangle$ averaged over the visible stellar disk.

Analysis of such observations has shown that these magnetic observables exhibit a smooth periodic change on time scales from 12 h and up to many decades. Furthermore, the period of the variability of $\langle B_z \rangle$ and $\langle B \rangle$ was found to coincide with the period of the photometric variability and variability of spectral line intensities; we show an example of this variability for the Ap star HD 24712 in Fig. 1.3. This led to the development of the oblique rotator model (Stibbs, 1950). This model postulates that the photometric, spectroscopic, and magnetic observables change as the result of a stable, dipole magnetic field with its polar axis inclined with respect to the axis of the rigidly rotating stellar atmosphere. Additionally, the star has a stable inhomogeneous horizontal distribution (“spots”) of chemical elements, presumably connected to the magnetic field topology. This phenomenological model explains the periodic variability of the photometric, magnetic field, and spectral lines measurements of Ap stars.

The moment technique was proposed as a way to extract more information from the circular polarization spectra of Ap stars (Mathys, 1988; Mathys & Hubrig, 1997). In practice, this method allowed to obtain two additional magnetic observables (the mean quadratic field and the field asymmetry) sensitive to surface distribution of the field strength and the projection of the field vector on the line of sight. Other studies attempted to detect and interpret the broadband net linear polarization produced by the Zeeman effect as a way to characterize the transverse component of the stellar magnetic field (Landolfi et al., 1993; Leroy, 1995). Studies of magnetic Ap stars with these methods showed that the majority of stars appear to possess simple, often axisymmetric magnetic fields that can be represented with a superposition of dipolar, quadrupolar and octupolar geometries (Landstreet & Mathys, 2000; Bagnulo et al., 2002). At the same time, it was demonstrated that magnetic field configurations recovered in this way are incapable of reproducing the observed Stokes parameter profiles (Bagnulo et al., 2001). Additionally, these models are questionable due to a non-uniqueness of the description of non-dipolar components and a systematic bias introduced by neglecting chemical spots.

The deficiencies of this modeling approach were overcome with the appearance of spectropolarimeters capable of acquiring phase-resolved high-
Figure 1.3. Rotational variability of the mean line-of-sight magnetic field, equivalent line widths, and photometric measurements of HD 24712. On the left panel we show $\langle B_z \rangle$ measurements, obtained by Rusomarov et al. (2013), from Fe-peak and rare earth elements with red “+” and blue “x” symbols, respectively. Variations of the equivalent line width (a measure of line strength) normalized to its mean value for a selection of Fe-peak and rare earth element lines, used by Rusomarov et al. (2015), are shown in the right top panel. The colors and symbols carry the same information as in the previous plot. Relative broadband light variations (Kurtz, 1982) are shown in the right bottom plot.

resolution observations in circular and linear polarization (Wade et al., 2000; Silvester et al., 2012). Examples of such observations, obtained in this thesis for the Ap star HD 125248, are shown in Fig. 1.4, for a narrow spectral window centered around the spectral lines Cr II 4812.337 Å and Eu II 6437.64 Å. Analysis of such full Stokes vector observations using indirect imaging techniques provided distribution of magnetic field and chemical spots on the surface of Ap stars without using a priori assumptions about geometry of surface structures, (e.g., Kochukhov et al., 2004a; Kochukhov & Wade, 2010; Silvester et al., 2014a). These studies gave convincing evidence that the real structure of magnetic fields on the surfaces of Ap stars is far more complex than a simple axisymmetric low-order multipole. Instead, the fields appear to be close to a dipole only on the largest spatial scales but are distorted by significant small-scale magnetic field features.

Surveys of magnetic Ap stars in open clusters with known distances and ages have shown that Ap stars arrive on the zero age main sequence with a strong magnetic field, which then slowly decreases in strength during the main sequence evolution (Landstreet et al., 2007). A second important discovery was recently made for a subset of pre main-sequence intermediate and high-mass stars. These objects, known as Herbig Ae/Be stars, are found in regions of recent star formation and show evidence of circumstellar material and accretion disks. The HR diagram position of these objects makes them direct progenitors of A and B stars. Studies of the magnetic fields in these stars (Alecian et al., 2013) established that a few percent of Herbig Ae/Be stars have
rotational modulation of the Stokes $IQUV$ profiles for the Cr II 4812.337 Å (left panel) and Eu II 6437.64 Å (right panel) spectral lines for the Ap star HD 125248. The spectra for different rotational phases are offset vertically. Rotational phases are indicated in the leftmost column of each panel. The Stokes $QU$ and $V$ profiles are expanded by factors 9 and 3, respectively, relative to Stokes $I$.

magnetic fields with the incidence, strength and geometrical properties similar to those of magnetic Ap/Bp stars. Therefore, A and B stars acquire their fields at the earliest phase of their evolution, even before the Herbig Ae/Be phase, but the exact origin of these fields remains currently unknown.

Thus, it is accepted that magnetic fields in Ap stars are remnants of a magnetic field that was created by a mechanism active before the pre main sequence. This is known as the fossil field theory. Historically, the main weakness of the fossil field hypothesis was the lack of stable field configurations that can survive inside a star over its lifetime. Braithwaite & Spruit (2004) have solved this problem demonstrating with numerical simulations that a stable equilibrium can actually exist in a main sequence star with radiative outer envelope. Braithwaite & Spruit (2004) and Braithwaite & Nordlund (2006) evolved a non-rotating, self-gravitating body of electrically conducting plasma with the pressure and temperature profiles similar to those of a real star, seeded with an initially random small-scale magnetic field. They found that this field quickly reaches an equilibrium state after several Alfvén timescales, which are on the order of tens of years for the observed field strengths of Ap stars. Once it has settled into an equilibrium state the field then starts decaying on very long timescales on the order of $10^{10}$ years due to Ohmic diffusion, which is caused by the finite electrical conductivity of the plasma.
Braithwaite & Nordlund (2006) and Braithwaite (2008) discovered that depending on the initial conditions the starting magnetic field evolves into two distinct equilibrium configurations: an axisymmetric and a non-axisymmetric one. Both can be represented in terms of twisted flux tubes comprised of a poloidal (meridional) field component wrapped around a toroidal (azimuthal) one. The axisymmetric configuration corresponds to a single twisted tube wrapped around the star, and the non-axisymmetric configuration to one to several flux tubes arranged in a more complex pattern. The axisymmetric configuration presents itself in observations roughly as a dipole, while the non-axisymmetric configuration apparently is responsible for the highly complex magnetic field geometries found for τ Sco (Donati et al., 2006) and HD 37776 (Kochukhov et al., 2011).

In summary, current theoretical results properly reproduce key observations: the static nature of Ap star fields on timescales of decades and mostly simple global field topologies. Still, there is a need for more quantitative understanding of the effect of initial conditions on the resulting equilibrium and the diffusive evolution of these equilibria. Finally, we note that fossil field theory addresses the geometry of stable interior magnetic fields in Ap stars, while the problem of their origin is still an open question.

1.3 Chemical structures in Ap star atmospheres

The presence of chemical peculiarities in Ap stars is believed to be limited to only their atmospheres and outer, mostly radiative envelopes and does not reflect their bulk chemical composition. This can be deduced from several arguments. The diversity of measured chemical compositions of Ap stars by far exceeds the scatter of chemical composition of interstellar clouds from which these stars form. Abundance patterns of Ap stars also do not match the composition of any stars of lower or higher mass. It is difficult to imagine the conditions under which gas clouds with exceptionally strong anomalies in chemical composition could have formed. Thus, it is believed that the observed chemical anomalies for Ap stars are products of physical processes active in their atmospheres and subphotospheric layers.

Another important fact to consider is the observed spectroscopic variability of Ap stars, i.e., rotational variability of strengths of spectral lines associated with different chemical peculiarities. This has been interpreted as a non-uniform distribution of chemical elements on the surfaces of these objects, in contrast to the uniform surface distribution of chemical elements for normal A and B stars. It has been inferred from observations that different elements are concentrated in different surface spots, as the line strengths of different species have different behavior with rotational phase. These abundance spots have been found to correlate with the presence of magnetic fields.
Atomic diffusion is a process known to be able to create this complex picture (Michaud, 1970). In a hydrostatically stable atmosphere the gravitational attractions towards the center of the star felt by a given atom or ion is counteracted by the combined gas and radiation pressure. The latter is an outwards push experienced by atoms and ions when they absorb photons coming from the stellar interior. The net force might be non-zero for certain atoms or ions. Depending on which force is dominant, an element can appear to be depleted (if it sinks out of the stellar atmosphere) or overabundant (if it accumulates in the atmosphere).

This process competes with numerous mixing processes active in the stellar outer layers. In Sun-like stars the outer envelope is fully convective, so all diffusion processes are suppressed and therefore all Sun-like stars exhibit a similar chemical composition. In intermediate-mass normal A and B stars fast rotation creates large-scale mixing currents that inhibit strong atomic diffusion effects to appear. Most of the peculiar intermediate-mass stars tend to be slow rotators with projected rotational velocity $\lesssim 90$ km s$^{-1}$, which explains why many of them have observable chemical anomalies.

In the atmospheres of Ap stars their magnetic fields also play a significant role in atomic diffusion. They help to stabilize the atmosphere against any large-scale motions, thus facilitating diffusion. Charged particles traveling in the presence of magnetic fields experience Lorentz force, which compels them to spiral around the magnetic field lines. When the magnetic field lines are horizontal relative to the stellar surface, this effect leads to reduced vertical motion of ions. If the field lines are vertical to the surface, then the effect of the magnetic field on atomic diffusion is greatly reduced. It is believed that atomic diffusion in the presence of magnetic field is responsible for the appearance of patchy abundance distribution of chemical elements in the atmospheres of Ap stars. Theoretical models (Alecian & Stift, 2010) for the case of purely dipolar magnetic field suggest the existence of narrow overabundance belts of most metals around the magnetic equator. If the field is not dipolar, abundance patches would be created at places where the magnetic field lines are parallel to the local surface.

However, atomic diffusion in the presence of a magnetic field is not a straightforward process to model. It depends on atomic physics, thermodynamic structure of the stellar atmosphere, magnetic field topology, and on other factors. Although the basic physical mechanism responsible for chemical anomalies and formation of surface spots in Ap stars appears to be securely identified, theoretical diffusion calculations still fail to quantitatively reproduce observations of individual Ap stars. Recent surface abundance imaging studies (e.g., Kochukhov et al., 2004a,b; Lüftinger et al., 2010; Nesvacil et al., 2012; Silvester et al., 2014b) have revealed a great diversity of chemical distributions with no apparent correlation between the field geometry and spots. This contrasts theoretical studies, which expect a tight relation between the magnetic and chemical structures.
Atomic diffusion in the presence of magnetic field not only creates surface abundance inhomogeneities but it also produces vertical abundance gradients. Theoretical developments have enabled calculation of model atmospheres with diffusion included (LeBlanc et al., 2009). For stars with effective temperatures typical of cool Ap stars these calculations have suggested the existence of a rapid abundance change with depth that has a shape of a step-function. A number of observational studies have confirmed the presence of such abundance gradients in Ap star atmospheres (see e.g., Kochukhov et al., 2006).

The methods for studying surface abundance inhomogeneities and vertical chemical gradients have been improved considerably in the last decade. Historically, these methods were limited to comparing variations of line-strength of certain chemical elements to photometric observations and magnetic field measurements such as $\langle B_z \rangle$ and $\langle B \rangle$ (e.g., Leone & Catanzaro, 2001; Ryabchikova et al., 2007). Nowadays, the preferred method for studying horizontal abundance inhomogeneities in Ap stars is magnetic Doppler imaging (see Chapter 2), which is capable of inferring abundance and magnetic field distribution on the stellar surface by analyzing observed intensity and polarization spectra (see e.g., Silvester et al., 2014a; Kochukhov & Wade, 2010; Kochukhov et al., 2004a,b; Lüftinger et al., 2010).

Methods that investigate vertical abundance gradients typically involve iterative fitting the relative strengths and profiles of spectral lines that are sensitive to chemical stratification. Current techniques are based on finding parameters of a step-function that best describes observed data (Ryabchikova et al., 2005) or on solving a regularized vertical inversion problem, which recovers the vertical stratification profile of a given chemical element without making a priori assumptions about the shape of vertical distribution (Kochukhov et al., 2006). In some cases, when these studies were coupled to calculation of model atmospheres, it was shown that vertical stratification of chemical elements can lead to changes in the temperature distribution with depth in the atmospheres of Ap stars (see e.g., Shulyak et al., 2009).

1.4 Motivation for this thesis

From the above discussion we can make the following conclusions: The magnetic field topology of Ap stars cannot be in general considered as a low-order axisymmetric multipole. Four Stokes parameter studies hint that most Ap stars have much more complex field configurations. Secondly, the theory of atomic diffusion in the presence of magnetic field gives only a qualitative explanation of observations but has little quantitative predicting power. Thirdly, there is only a handful of detailed imaging studies of Ap stars from four Stokes parameter observations. More such studies are required to better constrain the parameter space of current theoretical models of magnetic fields and atomic
diffusion in Ap stars. Finally, abundance structures in the atmospheres of Ap stars are currently studied using two methods that analyze only one aspect (either vertical or horizontal) of chemical inhomogeneities. We believe that by combining vertical stratification and surface imaging methods into one modeling framework we can provide comprehensive and definitive observational constraints for theoretical studies.

In order to achieve these goals we have carried out an observational program aimed at observing a small sample of Ap stars in all four Stokes parameters with the HARPSpol spectropolarimeter (Piskunov et al., 2011) at the ESO 3.6-m telescope. HARPSpol is one of the three currently available night time spectropolarimetric instruments that have resolving power greater than $5 \times 10^4$–$10^5$, and polarimetric sensitivity of $10^{-5}$.

We aim to simultaneously map the magnetic field and abundance distribution using magnetic Doppler imaging in our target stars. We also evaluate the possibility of applying a self-consistent technique for studying vertical stratification and horizontal abundance inhomogeneities. Finally, we attempt to confront our observational results for a couple of carefully studied Ap stars with theoretical models of magnetic fields in radiative stellar envelopes and try to get a new insight into the relation between magnetic fields and abundance structures in the atmospheres of Ap stars.
2. Indirect imaging techniques

2.1 Doppler imaging

The technique of Doppler imaging is a widely used method of extracting information about surface inhomogeneities using time-series of high-resolution stellar spectra. The basis of this technique is the rotational Doppler broadening mechanism. As a star rotates, an observer sees one side of the stellar surface moving towards and the other away from him. This results in a redshift of the receding side and a blueshift for the approaching side, which causes the spectral lines to become broadened around the line center. If there is a localized spot on the stellar surface, this spot would manifest itself as a distortion in the observed spectral line that changes its position with rotational phase.

The main idea behind Doppler imaging is illustrated in Fig. 2.1, which shows how a spot with increased line strength produces a characteristic dip in the observed stellar spectrum. Furthermore, we also see that the behavior of the dip with time gives information about its position on the stellar surface. Namely, a spot close to the stellar equator produces a feature in the spectrum that is visible during half the rotational period of the star; a spot positioned at higher latitudes produces features in the spectral line that might be present in the spectrum for the whole time and migrate back and forth relative to the line center.

An interesting aspect of Doppler imaging is the maximum theoretical resolution that can be achieved. This can be approximately expressed as the number of resolution elements across the stellar disk

$$N_{\text{max}} \simeq \frac{2 v_e \sin i}{W},$$

(2.1)

where $v_e \sin i$ is the projected equatorial rotational velocity and $W$ is the width of a spectral line measured in the absence of rotation. In general, $W$ depends on the spectral type of the star and the resolving power of the spectrograph. We note that Eq. (2.1) estimates the resolution that can be achieved from a single snapshot of the stellar spectrum. In the case when we have multiple snapshots obtained at different rotational phases the maximum resolution of the method is significantly greater than what is given by Eq. (2.1).

It is obvious that Doppler imaging, unlike other imaging methods, e.g., interferometry, does not directly depend on the distance to the star. Using modern spectrographs readily available on medium-size telescopes we can achieve spatial resolution $\sim 1000$ km on the stellar surface for a typical rapidly rotating
Figure 2.1. Schematic illustration of the Doppler imaging principle. On the left panel we show stellar surface with two spots with an enhanced line strength for four rotational phases indicated above each spherical plot. The right panel shows the resulting rotational modulation of the Stokes $I$ profile of a typical spectral line. The profiles are offset vertically for different rotation phases indicated on the far right. The dotted line shows how the distortions in Stokes $I$ caused by the smaller spot, which is visible the whole time, move across the profile. The distortions caused by the bigger spot are indicated by the long-dashed thick line. Because the big spot is visible for part of the rotational period, the distortions are present for phases 0.25 to 0.85.

early-type star, which corresponds to an angular resolution of $\leq 10^{-6}$ arcseconds. Doppler imaging is the only remote sensing technique in astronomy that allows for such high spatial resolution.

In general, Doppler imaging can be formulated as the following inverse problem:

**Given a time series of spectral line profiles find a surface distribution map of the given quantity that best reproduces the observed data.**

Mathematically, we can treat this as a non-linear least squares problem with respect to the unknown surface distribution $x$, which reduces to the minimization problem:

$$
\Psi(x) = \sum_{\varphi} \sum_{\lambda} \left( \frac{I^{\text{obs}}_{\varphi \lambda} - I^{\text{calc}}_{\varphi \lambda}(x)}{\sigma_{\varphi \lambda}} \right)^2 + \mathcal{R}(x) \rightarrow \min,
$$

(2.2)

where $I^{\text{obs}}_{\varphi \lambda}$ and $I^{\text{calc}}_{\varphi \lambda}$ are the observed and synthetic spectra, and $\sigma_{\varphi \lambda}$ are the error bars of the observed spectra. The summation is carried over all available rotational phases $\varphi$ and wavelength points $\lambda$. Because Doppler imaging is an inherently ill-posed problem, i.e., an arbitrary observational data set does not correspond to a unique surface distribution $x$, a regularization in the form of a functional $\mathcal{R}(x)$ that incorporates a priori information about the expected solution must be used. In practice, $\mathcal{R}(x)$ guarantees the simplest possible solution that best describes a given observed data set.
In this form, Doppler imaging has been successfully applied to the study of surface abundance inhomogeneities in chemically peculiar stars (e.g., Adelman et al., 2002; Makaganiuk et al., 2011) and temperature spots in late-type active stars (e.g., Piskunov et al., 1990; Lindborg et al., 2014). In these cases one solves the problem defined by Eq. (2.2) for different quantities. For chemically peculiar stars we image surface abundance inhomogeneities of chemical elements, while for late-type active stars we are interested in the distribution of surface temperature or brightness spots.

In the application of Doppler imaging to the problem of restoring chemical abundance maps for chemically peculiar stars we use Tikhonov regularization (Tikhonov & Arsenin, 1977) in the form

$$\mathcal{R}(x) = \Lambda \sum_i ||\nabla x_i||^2,$$

where the index $i$ runs over all surface elements, and $\Lambda$ is a coefficient that determines the contribution of $\mathcal{R}(x)$ to the total discrepancy function $\Psi(x)$. We choose the value of $\Lambda$ on the basis of a balance between the goodness of fit and smoothness of the solution.

### 2.2 Magnetic Doppler imaging

The idea of Doppler imaging has been extended to mapping magnetic field topology of chemically peculiar magnetic Ap stars (Piskunov & Kochukhov, 2002). The magnetic fields of these stars are strong and globally organized, which leads to the appearance of polarization signatures in all four Stokes parameters. Fig. 2.2 illustrates the effect of a strong magnetic field on the Stokes profiles of a spectral line. Piskunov & Kochukhov (2002) showed that, if we have a time series of full Stokes vector spectropolarimetric observations we can not only reconstruct surface abundance maps of several chemical elements but also recover the distribution of the magnetic field.

The technique of magnetic Doppler imaging is a fairly recent development because it imposes challenging observational and computational requirements. The polarization signatures, produced by a magnetic field, depend not only on the field strength, but also on its orientation relative to the line-of-sight to the observer which changes as the star rotates. These signatures are typically on the order of $10^{-3}$ to $10^{-2}$ for Stokes $V$ and up to one magnitude smaller for Stokes $QU$ for the typical magnetic Ap stars. Recently, it has become possible to record these signatures with sufficient precision using the new generation of spectrographs that have resolving power greater than $5 \times 10^4$ and polarimeters capable of achieving a polarization sensitivity on the order of $10^{-5}$ (e.g., HARPSpol, Piskunov et al., 2011).

In order to model the polarization signatures in a realistic way one needs to solve the equations of polarized radiative transfer for a large number of
Figure 2.2. Variations of the Stokes $IQUV$ profiles of the Fe I 6336.824 Å line for an oblique dipolar magnetic field. The first three rows illustrate the surface structure of the magnetic field with a polar field strength of 8 kG, inclined at an angle of 90° to the stellar rotational axis. The spherical plots show the maps of the field modulus (a), radial field component (b), and field orientation (c) for five rotational phases, indicated above each spherical plot. The bars on the right indicate the field strength in kG. The contours are plotted with a 2 kG step. Different colors distinguish between the outward (red) and inward (blue) directed magnetic field vectors. The arrow length in the third row of spherical plots is proportional to the field strength. The synthetic Stokes $IQUV$ profiles are shown in the bottom four panels. The horizontal axis corresponds to the distance from the line center $\lambda_0$ measured in Å. The profiles are shifted vertically according to rotational phases indicated to the right of the last panel. Note the different scale used for the Stokes $IV$ and $QU$ parameters.
points on the stellar surface on a dense wavelength grid. This presents a formidable computational difficulty. With the advent of powerful computer clusters and programming tools specifically designed to take advantage of such hardware the computational requirements of magnetic Doppler imaging have been largely met.

The mathematical formulation of magnetic Doppler imaging is a generalization of the Doppler imaging presented in Sect. 2.1 with the exception that the set of free parameters includes a surface abundance distribution of one or more chemical elements and a vector map of the magnetic field. The observational data now consist of time series of Stokes $IQUV$ parameter spectra. This inverse problem is solved by treating it as a non-linear least squares minimization problem:

$$
\Psi(\varepsilon, B) = \sum_{\phi\lambda} w_I(I^\text{obs}_{\phi\lambda} - I^\text{calc}_{\phi\lambda}(\varepsilon, B))^2 / \sigma_{I\phi\lambda}^2 \\
+ \sum_{\phi\lambda} w_Q(Q^\text{obs}_{\phi\lambda} - Q^\text{calc}_{\phi\lambda}(\varepsilon, B))^2 / \sigma_{Q\phi\lambda}^2 \\
+ \sum_{\phi\lambda} w_U(U^\text{obs}_{\phi\lambda} - U^\text{calc}_{\phi\lambda}(\varepsilon, B))^2 / \sigma_{U\phi\lambda}^2 \\
+ \sum_{\phi\lambda} w_V(V^\text{obs}_{\phi\lambda} - V^\text{calc}_{\phi\lambda}(\varepsilon, B))^2 / \sigma_{V\phi\lambda}^2 \\
+ \Lambda_a R_a(\varepsilon) + \Lambda_f R_f(B) \rightarrow \min,
$$

(2.4)

where $IQUV$ denote the Stokes parameters. The relative quality of the observed data for each wavelength point $\lambda$, rotational phase $\phi$, and Stokes $IQUV$ parameter is characterized by $\sigma_{\phi\lambda}$. $R_a$ and $R_f$ are the regularization functionals for the abundance and magnetic field maps respectively, and $\Lambda_a$ and $\Lambda_f$ are the corresponding regularization parameters. The weights $w$ are introduced to ensure that the relative contributions to the total discrepancy function $\Psi$ of different Stokes parameters are approximately the same.

Magnetic Doppler imaging, as in the case of regular Doppler imaging, is an ill-posed problem, which is solved with the help of regularization. In contrast to abundance Doppler imaging, which is an inherently ill-posed problem, magnetic Doppler imaging based on four Stokes parameter data is not an ill-posed problem per se. In other words, there is one-to-one correspondence between a noise-free data set and a magnetic field distribution (Piskunov, 2005). In this case, we use regularization because of the incompleteness of observational data and not because this inverse problem has inherently multiple solutions for the magnetic field distribution.

Piskunov & Kochukhov (2002) in their formulation of magnetic Doppler imaging searched for the magnetic field distribution as a discrete vector map defined on the stellar surface. For a given surface point with longitude $\phi$ and latitude $\theta$ the magnetic field vector has radial, meridional, and azimuthal components defined in the local coordinate system. Accordingly, the mag-
netic field vector map, defined on a discrete \((\theta, \phi)\)–grid, is comprised of two-
dimensional maps of the radial \(B_r\), meridional \(B_m\), and azimuthal \(B_a\) com-
ponents. On the other hand, Donati et al. (2006) and Kochukhov et al. (2014)
represented the magnetic field as a superposition of a poloidal and a toroidal
component, each expressed as a spherical harmonic expansion. Using this
approach, we can compute the radial, meridional, and azimuthal maps of the
magnetic field using the spherical harmonics expansion

\[
B_r(\theta, \phi) = -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} \alpha_{\ell,m} Y_{\ell,m}(\theta, \phi)
\]

(2.5)

\[
B_m(\theta, \phi) = -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} [\beta_{\ell,m} Z_{\ell,m}(\theta, \phi) + \gamma_{\ell,m} X_{\ell,m}(\theta, \phi)],
\]

(2.6)

\[
B_a(\theta, \phi) = -\sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} [\beta_{\ell,m} X_{\ell,m}(\theta, \phi) - \gamma_{\ell,m} Z_{\ell,m}(\theta, \phi)],
\]

(2.7)

where the real spherical harmonic functions describing the mode with degree
\(\ell\) and order \(m\)

\[
Y_{\ell,m}(\theta, \phi) = -C_{\ell,m} P_{\ell,|m|}(\theta) K_m(\phi),
\]

(2.8)

\[
Z_{\ell,m}(\theta, \phi) = \frac{C_{\ell,m}}{l+1} \frac{\partial P_{\ell,|m|}(\theta)}{\partial \theta} K_m(\phi),
\]

(2.9)

\[
X_{\ell,m}(\theta, \phi) = -C_{\ell,m} \frac{P_{\ell,|m|}(\theta)}{\sin \theta} m K_{-m}(\phi),
\]

(2.10)

are expressed using associated Legendre polynomials \(P_{\ell,m}(\theta)\) and

\[
C_{\ell,m} = \sqrt{\frac{2\ell + 1}{4\pi} \frac{(\ell - |m|)!}{(\ell + |m|)!}}
\]

(2.11)

\[
K_m(\phi) = \begin{cases} 
\cos(|m|\phi), & m \geq 0 \\
\sin(|m|\phi), & m < 0
\end{cases}
\]

(2.12)

In this formalism, instead of searching for a vector map \(\{B_r, B_m, B_a\}\), we re-
construct the magnetic field vector in terms of the spherical harmonic coefficient
\(\alpha_{\ell,m}, \beta_{\ell,m}\) and \(\gamma_{\ell,m}\) that represent the radial poloidal, horizontal poloidal
and toroidal components, respectively. We proceed with the spherical har-
monic expansion of the magnetic field components up to terms with \(\ell \leq \ell_{\text{max}}\).

This formalism has several benefits over the direct mapping method: it is
easy to test a specific magnetic field configuration, e.g., dipole, dipole plus
quadrupole; the divergence-free condition for the magnetic field is automati-
cally satisfied; and we can easily calculate the relative energies of individual
poloidal and toroidal components of the magnetic field according to the sur-
face integrals \(\int B_{\ell,m}^2\).
We regularize the magnetic field with the help of the penalty function

\[
\mathcal{R}_f = \sum_{\ell=1}^{\ell_{\text{max}}} \sum_{m=-\ell}^{\ell} (\alpha_{\ell,m}^2 + \beta_{\ell,m}^2 + \gamma_{\ell,m}^2) \ell^2,
\]

which prevents the code from introducing high-order modes not justified by the observational data. The abundance maps are still constrained using the Tikhonov regularization

\[
\mathcal{R}_a = \sum_i \sum_j ||\nabla \epsilon^{(i)}_j||^2,
\]

where the index \( j \) runs over all surface elements and \( i \) goes through all abundance maps \( \epsilon^{(i)} \).

Magnetic Doppler imaging was initially applied to Ap stars in a simplified form, using only circular polarization data (Stokes \( IV \) parameters) for magnetic field mapping. This required multipolar regularization or parametrization of the magnetic field topology (e.g., Kochukhov et al., 2002; Lüftinger et al., 2010). After high-resolution spectropolarimetry in all four Stokes parameters became feasible, this limitation was lifted. The application of magnetic Doppler imaging to four Stokes parameter spectropolarimetric observations of 53 Cam (Kochukhov et al., 2004a) and \( \alpha^2 \) CVn (Kochukhov & Wade, 2010; Silvester et al., 2014a) showed its obvious advantages over previous methods (e.g., Landolfi et al., 1998; Bagnulo et al., 1995) used to diagnose magnetic fields of chemically peculiar magnetic stars, and over magnetic Doppler imaging in the Stokes \( IV \) parameters. For both objects the authors found magnetic field configuration and abundance distribution that successfully reproduced the observed Stokes parameters. It was demonstrated that these Ap stars do not have strictly dipolar fields. Instead, they exhibit field topologies that are quasi-dipolar on large scales but have some small-scale features necessary to explain the linear polarization observations.

This work uses the INVERS10 code (Piskunov & Kochukhov, 2002) which is implemented following the principles of magnetic Doppler imaging methodology presented here. INVERS10 solves the minimization problem defined by Eq. (2.4) using the Levenberg-Marquardt minimization algorithm simultaneously for the abundance maps and for the vector magnetic field distribution. A major advantage of INVERS10 over other magnetic mapping codes is the use of detailed polarized radiative transfer calculations based on realistic model atmospheres. In contrast, other codes use simplified analytical methods, such as Gaussian profiles or Milne-Eddington atmosphere (e.g., Donati & Brown, 1997). INVERS10 is designed for parallel execution using the Message Passing Interface and uses its Open MPI implementation (Gabriel et al., 2004). This allows us to perform magnetic Doppler inversions using many computing nodes.
2.3 Three-dimensional Doppler imaging

Existing methods that are applied for studies of abundance inhomogeneities in the atmospheres of Ap stars can be roughly split into two types: two-dimensional (horizontal) magnetic Doppler imaging and one-dimensional vertical stratification analysis. Magnetic Doppler imaging has been used extensively to study the magnetic fields and abundance distribution of chemical elements on the stellar surface (e.g., Kochukhov et al., 2004a; Kochukhov & Wade, 2010; Silvester et al., 2014a, 2015). One-dimensional abundance stratification analysis has been applied in the context of vertical stratification in the atmospheres of Ap stars (e.g., Shulyak et al., 2010; Shulyak et al., 2009; Kochukhov et al., 2009). Unfortunately, each of these techniques addresses only one aspect of a stellar atmosphere: magnetic Doppler imaging does not take into account vertical chemical stratification, while vertical stratification analysis does not consider horizontal abundance inhomogeneities.

Three-dimensional magnetic Doppler imaging lifts the limitations of these methods by incorporating vertical stratification of chemical elements directly into magnetic Doppler imaging. This makes it possible to probe chemical inhomogeneities in three spatial dimensions and provide detailed observational constraints for theoretical studies of atomic diffusion in the atmospheres of Ap stars.

We implemented three-dimensional Doppler imaging based on the established technique of magnetic Doppler imaging (e.g., Piskunov & Kochukhov, 2002); we fit observations with synthetic spectra by iteratively adjusting a set of free parameters (maps) defined on a discrete latitude-longitude grid on the stellar surface.

Following the parametrization successfully employed by one-dimensional stratification studies, we approximate vertical stratification at a point on the stellar surface with a function of four parameters: abundance high in the atmosphere $\varepsilon^{up}$, abundance deep in the atmosphere $\varepsilon^{lo}$, and the position $d$ and width $\delta$ of the transition region between the over- and underabundant layers. The vertical scale of the stellar atmosphere is defined as the logarithm of the column mass $m$. In Fig. 2.3, we schematically illustrate this approximation of vertical stratification. Accordingly, we can characterize the three-dimensional
distribution of a given chemical element in the stellar atmosphere with four scalar maps, \( \varepsilon^{up}, \varepsilon^{lo}, d, \) and \( \delta \), defined on a discrete surface grid.

The maps of the unknown parameters, \( \varepsilon^{up}, \varepsilon^{lo}, d, \) and \( \delta \), are determined by solving the following non-linear minimization problem:

\[
\Psi = \sum_{\varphi \lambda} \left( \frac{I_{\varphi \lambda}^o - I_{\varphi \lambda}^c (\varepsilon^{up}, \varepsilon^{lo}, d, \delta)}{\sigma_{\varphi \lambda}} \right)^2 + \mathcal{R}(\varepsilon^{up}, \varepsilon^{lo}, d, \delta) \rightarrow \min, \tag{2.15}
\]

where \( I_{\varphi \lambda}^o \) and \( I_{\varphi \lambda}^c \) are the observed and calculated spectra, and \( \sigma_{\varphi \lambda} \) are the error bars of the observed spectra. The summation is carried over all available rotational phases \( \varphi \) and wavelength points \( \lambda \).

Three-dimensional Doppler imaging is an ill-posed problem; for an arbitrary observational data set, there is an infinite number of non-unique solutions, i.e., a set of maps of the unknown parameters \( \varepsilon^{up}, \varepsilon^{lo}, d, \) and \( \delta \), that fit the observed spectra within the required \( \chi^2 \) level. We regularize this problem with first-order Tikhonov regularization (Tikhonov & Arsenin, 1977) for the individual maps. We also introduce an additional convex penalty function that minimizes the difference between \( \varepsilon^{up} \) and \( \varepsilon^{lo} \) for each point on the stellar surface. The regularization functional \( \mathcal{R} \) from Eq. (2.15) can be written as:

\[
\mathcal{R} = \Lambda_{up} \sum_i ||\nabla \varepsilon_{i}^{up}||^2 + \Lambda_{lo} \sum_i ||\nabla \varepsilon_{i}^{lo}||^2 + \Lambda_d \sum_i ||\nabla d_i||^2 + \Lambda_\delta \sum_i ||\nabla \delta_i||^2 + \Lambda_{up, lo} \sum_i (\varepsilon_{i}^{up} - \varepsilon_{i}^{lo})^2, \tag{2.16}
\]

where the index \( i \) runs over all surface elements of the discrete surface grid. The regularization coefficients \( \Lambda \) determine the contribution of the respective regularization terms from Eq. (2.16) to the total discrepancy function \( \Psi \). They are usually determined by the amount of the complexity of the solution that is necessary to fit the observed spectra. In practice, the contribution of the regularization functional \( \mathcal{R} \) is not allowed to drop less than \( \sim 5\% - 10\% \) of the total discrepancy function.

In this study, we used a modified version of the magnetic Doppler imaging code INVERS10 (Kochukhov & Piskunov, 2002; Piskunov & Kochukhov, 2002) to perform the three-dimensional Doppler imaging analysis of the roAp star HD 24712. A general description of INVERS10 has been provided in Section 2.2.
3. Summary of papers

3.1 Paper I

In the first paper we present an analysis of the full Stokes vector spectropolarimetric data set obtained for the cool roAp star HD 24712 (HR1217, DO Eri). The spectropolarimetric observations were performed with the recently commissioned polarimetric unit (Piskunov et al., 2011) attached to the HARPS spectrograph (Mayor et al., 2003) at the 3.6-m telescope at La Silla, Chile. This data set represents the highest quality Stokes parameter observations ever obtained for a star other than the Sun. These spectra have a resolving power exceeding $10^5$ and a signal-to-noise ratio of 300–600. These observations enable an unprecedented study of the magnetic field topology of HD 24712 and the three-dimensional distribution of chemical elements in its atmosphere.

Based on these spectra, we studied the morphology of the Stokes profiles in individual spectral lines. We also computed mean Stokes profiles using the least-squares deconvolution technique (LSD, see Donati et al., 1997) with our multiprofile LSD code and the methodology described by Kochukhov et al. (2010). Specifically, we studied LSD Stokes profiles for Fe-peak and rare earth element lines. Besides Stokes parameters we also computed “null” polarization spectra, which contain no polarization signal coming from the star. Null spectra serve as a quality control allowing us to assess sources of spurious instrumental polarization signals and crosstalks between the Stokes parameters. Our analysis of the null LSD profiles showed that the crosstalk between Stokes $V$ and $QU$ parameters does not hamper the interpretation of the observational data. The crosstalk level from circular to linear polarization does not exceed 0.5% (measured in Stokes $V$ amplitude).

Using the LSD Stokes profiles, we calculated magnetic observables, such as the net linear polarization and mean longitudinal magnetic field. These observables were used to improve the rotational period of the star. We compared these measurements with previous broad-band linear polarization observations of HD 24712 (Leroy, 1995) data and determined the parameters of the dipolar magnetic field topology using the so-called “canonical” model introduced by Landolfi et al. (1993). Our analysis showed that the magnetic observables can be reasonably reproduced by a dipolar model, albeit with few discrepancies at some rotational phases. We attributed these discrepancies to the effects of chemical abundance spots.

The high signal-to-noise ratio of the data allowed us to reliably measure the mean line-of-sight magnetic field $\langle B_z \rangle$ from the cores of H$\alpha$ and H$\beta$ lines,
which probe high atmospheric layers. We compared these measurements with the ones for Fe-peak and rare earth elements and showed that any radial gradient of the magnetic field is less than the observational and systematic errors. This finding refutes previous claims of a fast decrease of the magnetic field strength in the radial direction in the atmospheres of Ap stars.

Finally, our analysis of the rotational modulation of the Hα core showed that rotational modulation of the core-wing anomaly of Balmer lines is synchronized with the variability of the magnetic field and metal line strengths. The core-wing anomaly for Ap stars is manifested as an unusually sharp transition between the Stark-broadened wings and Doppler dominated core when compared to normal stars of similar spectral types or to spectrum synthesis calculations with standard stellar model atmospheres. This anomaly indicates a non-standard atmospheric structure of Ap stars.

Our analysis showed that this anomaly is largest at phase ≃ 0, which coincides with the maximum of the magnetic field and maximum abundance of rare earth elements. This strengthens the suggestion by Shulyak et al. (2009) that the core-wing anomaly is related to stratification of rare earth elements.

3.2 Paper II

The second paper presents a magnetic Doppler imaging study of HD 24712. Our analysis was based on the same data studied in paper I. This is the very first magnetic Doppler imaging analysis of a cool Ap star based on four Stokes parameter observations.

We recovered magnetic field geometry and surface abundance maps from 16 spectral lines of three chemical elements: Fe, Nd, and Na. The spectral lines were chosen based on the presence of strong polarization signals in the Stokes $QUV$ parameters. Magnetic inversions showed that the surface magnetic field of HD 24712 has a dominant dipolar component with marginal contribution from higher-order harmonics.

The magnetic field map derived from the analysis of Fe, Nd, and Na lines was used to recover the abundance distribution map of Ca. Our analysis shows that the Fe and Ca abundance maps show enhancements near the magnetic equator with an under abundance patch centered at the positive (visible) magnetic pole. The recovered abundance maps of Nd and Na show a strong anti-correlation. Nd is highly abundant around the visible magnetic pole, while Na is overabundant around the negative magnetic pole.

We compared the magnetic field map of HD 24712 with results of four Stokes parameter magnetic Doppler imaging studies of 53 Cam (Kochukhov et al., 2004a) and α2 CVn (Kochukhov & Wade, 2010; Silvester et al., 2014a). We found that HD 24712, which is the least massive and probably the oldest star in this three object sample, has the simplest magnetic field. This gives some support to the hypothesis, proposed by Braithwaite & Nordlund (2006),
that Ap stars with dipole-like fields are older than stars with more complex magnetic field topologies.

Finally, we found that our abundance maps are not consistent with the theoretical models of atomic diffusion in the presence of magnetic fields. It appears that a major improvement of theoretical models is needed to explain abundance distributions on the surfaces of Ap stars.

3.3 Paper III

In our third paper we developed the first three-dimensional Doppler imaging method that can recover the full three-dimensional abundance distribution of a chemical element in the atmospheres of Ap stars. We then applied this inversion technique to the spectroscopic observations of HD 24712 that were studied in paper I, and reconstructed the first three-dimensional abundance distribution of Fe and Ca in the atmosphere of an Ap star.

The three-dimensional inversions confirmed the step-like variations of chemical elements, suggested by previous one-dimensional abundance studies (Shulyak et al., 2009). We discovered a strong correlation between the surface abundance derived with two-dimensional magnetic Doppler imaging and vertical stratification of Fe: higher surface abundance corresponds to a transition region that occurs higher in the atmosphere and vice versa. A similar correlation for Ca could not be established.

Theoretical modeling of atomic diffusion in the atmospheres of chemically peculiar, magnetic stars (Alecian & Stift, 2010; Alecian, 2015) has progressed tremendously in recent years, providing equilibrium stratification profiles as a function of the local magnetic field strength and inclination for a range of effective temperatures. We compared our results to these computations and found general disagreement between the theory and observations. The theoretical computations predict the presence of belt-like structures around the magnetic equator or abundance spots around areas of horizontal field line. Our inversion results do not show such correlations between the three-dimensional abundance distributions of Fe and Ca and the magnetic field of HD 24712. Furthermore, theoretical diffusion models predict the formation of overabundance inhomogeneities in the high atmospheric layers, however, we could not find evidence for this in our three-dimensional distributions of Fe and Ca.

The lack of a satisfactory agreement between observations and theory calls for a more intense collaboration between observational and theoretical studies, aimed at testing the limits of both approaches. On the theoretical side, additional physical effects, particularly time-dependent diffusion, need to be taken into account. On the other hand, it is also necessary to apply 3D-MDI to a statistically significant stellar sample of Ap stars. Future Doppler imaging studies should aim to investigate three-dimensional distribution of chemical elements for a wide range of field geometries and stellar parameters.
3.4 Paper IV

The fourth paper presents a magnetic Doppler imaging study of the Ap star HD 125248. This object is one of the well-known chemically peculiar stars, for which a wealth of historical observations exists but no recent analyses based on modern spectropolarimetric data have been undertaken.

We performed the analysis of HD 125248 based on the phase-resolved, four Stokes parameter observations obtained with the HARPSpol instrument. The full set of spectropolarimetric observations is comprised of 36 individual Stokes parameter observations, which have signal-to-noise ratio of 200–300 and cover the entire rotational period of the star.

We used these data and revised the atmospheric parameters of the star, \( T_{\text{eff}} = 9850 \pm 250 \) K and \( \log g = 4.05 \pm 0.10 \). The detailed abundance analysis confirmed that HD 125248 has a mean chemical composition typical of other Ap stars. We discovered significant vertical stratification effects for the \( \text{Fe}^{2+} \) and \( \text{Cr}^{2+} \) ions. At the same time, we did not find large differences in the abundances of different ions of the rare earth elements reported for cooler Ap stars (e.g., Ryabchikova et al., 2004).

We employed the least-squares deconvolution technique (LSD, see Donati et al., 1997) with our multiprofile LSD code (Kochukhov et al., 2010) and computed the LSD Stokes profiles for Fe-peak and rare earth element lines. From these LSD Stokes profiles we calculated magnetic observables, such as the net linear polarization and mean longitudinal magnetic field. These measurements were then used to refine the rotational period of the star.

The magnetic Doppler imaging analysis of HD 125248 revealed a largely poloidal and quasi-dipolar magnetic field with two spots of different polarity and field strength. In addition, small-scale magnetic features were found to be superimposed on this global field topology. The resulting surface abundance maps of Fe, Cr, Ce, Nd, Gd, and Ti show abundance contrasts of 0.9–3.5 dex. Among these elements, Cr is overabundant around the negative magnetic pole and has an abundance range of 3.5 dex, Fe shows the least deviations for a homogeneous surface abundance. The rare earth elements and Ti are over-abundant near the positive magnetic pole.

We compared the magnetic field topology of HD 125248 with the results obtained for other Ap stars using four Stokes parameter magnetic inversions. This comparison indicates that a correlation between the field complexity and stellar mass or age is probably present, as already suggested by Rusomarov et al. (2015) and Silvester et al. (2015). The abundance distribution maps derived in the magnetic Doppler imaging analysis, do not correlate with the horizontal field strength in a way predicted by theoretical studies of atomic diffusion in the presence of magnetic fields (Alecian & Stift, 2010).
4. Future developments

Magnetic fields and abundance inhomogeneities of bright Ap/Bp stars have been studied for over half a century. During that time however it has not been possible to confidently place the observed nearby field stars in an evolutionary sequence from the zero-age to the terminal-age main sequence. With the appearance of new generation spectropolarimeters, for example HARPSpol at the ESO 3.6m telescope and ESPaDONs at CFHT, it has become possible to study fainter magnetic stars in open clusters (e.g., Landstreet et al., 2007).

Mean abundance studies of magnetic stars in open clusters have revealed that the average magnetic field strength of Bp stars with masses 2–5 $M_{\odot}$ reduces quite strongly with the main sequence age. The chemical peculiarities of Bp stars with masses $\approx 3.5 M_{\odot}$ tend to converge toward solar abundances by the end of the main sequence life (Bailey et al., 2014).

Magnetic Doppler imaging of Ap and Bp stars in open clusters can provide information about the magnetic field topologies and surface abundance distributions as a function of age. A systematic magnetic Doppler imaging study of the properties of Ap and Bp stars will therefore provide a new insight into the physical processes that shape their atmospheres and magnetic fields. In particular, it will become possible to understand the secular evolution of fossil magnetic fields and an interplay of these fields with the angular momentum in magnetic stars.

Future three-dimensional Doppler imaging studies have the potential to provide an entirely new observational picture of atomic diffusion in the atmospheres of Ap stars. Investigations of the three-dimensional distribution of chemical elements for a wide range of field geometries and stellar parameters, are required to uncover how exactly atomic diffusion depends on those parameters and to guide future theoretical research of chemical transport processes. Of particular interest is three-dimensional Doppler imaging of rare earth elements, which accumulate in the upper atmospheric layers of Ap stars. We did not perform such study in this thesis due to the complexities arising from the simultaneous treatment of non-LTE line formation, horizontal chemical spots and vertical stratification.

Vår kunskap om magnetfält hos medelmassiva magnetiska stjärnor bygger nästan helt på observationer av cirkulärpolariserat ljus, vilket historiskt sett har begränsat vår förståelse av stjärnornas magnetism. Med tillkomsten av nya generationens spektropolarimetrar, som HARPSpol på 3.6 m European Southern Observatory (ESO) teleskopet och ESPaDOnS på Canada-France-Hawaii teleskopet, har det blivit möjligt att erhålla fasupplösta och högupplösta spektropolarimetriska observationer i alla fyra Stokes parametrar.

För att analysera de spektropolarimetriska observationerna använder vi oss av magnetisk Doppleravbildning (MDI) för att rekonstruera magnetfältet och producera kartor som visar hur koncentrationen av olika grundämnen förändras över stjärnytan. Genom tillämpningen av MDI på spektropolarimetriska observationer av magnetiska stjärnor har en ny, tidigare okänd, grad av komplexitet i stjärnornas magnetfält avslöjats. Den nya informationen är av intresse för att förstå utvecklingen och strukturen av magnetfält i medelmassiva och massiva stjärnor.

Vi observerade de magnetiska, kemiskt säregna, Ap stjärnorna HD 24712 (DO Eri, HR 1217) och HD 125248 (CS Vir, HR 5355) i fyra Stokes parametrar med spektropolarimetern HARPSpol på ESO 3.6 m teleskopet. De resulterande spektrumen har ett högt signal-brusförhållande och superb upplösning som långt överträffar kvaliteten av befintliga Stokes observationer av någon annan stjärna förutom solen.

Vi studerade variationer i stjärnspektrumet och av magnetiska observabler hos HD 24712 som en funktion av rotationsfasen (artikel I). I MDI studien av HD 24712, tolkade vi fasupplösta spektropolarimetriska observationer i alla fyra Stokes parametrar (artikel II). Denna analys visade att magnetfältet hos HD 24712 har en stark dipolkomponent med försumbara småskaliga bidrag. Samtidigt som den magnetiska kartläggningen rekonstruerade vi också yt fördelningen av grundämnen Fe, Nd, Na, och Ca.
I artikel III, utvecklade vi för första gången en Dopplertomografiteknik som gör det möjligt att rekonstruera fördelningen av grundämnen i tre rumsriktningar. Vi tillämpade denna nya teknik på observationer av stjärnan HD 24712, och rekonstruerade ämneshaltsfördelningen av Fe och Ca i dess atmosfär. Vi jämförde resultaten från vår tredimensionella Doppleravbildning av HD 24712 med resultaten från teoretiska förutsägelser om diffusion av grundämnen i magnetiserade stjärnatmosfärer, och fann skillnader mellan de två. Enligt teoretiska modeller ska höga koncentrationer av dessa ämnen finnas i moln högt upp i atmosfären, medan våra resultat av den tre-dimensionella fördelningen av Fe och Ca visar att deras koncentration istället minskar med höjden. Våra resultat kommer att ligga till grund för framtida teoretiska studier av detta slag.

I artikel IV använde vi MDI på spektropolarimetriska observationer av stjärnan HD 125248. De rekonstruerade, detaljerade, kartorna av ämneshaltsfördelningen och magnetfältet hos HD 125248 visar på kraftiga inhomogeniteter i fördelningen av Cr och andra sällsynta jordartsmetaller, och att magnetfältet har en topologi som är markant olik ett klassiskt dipolfält.

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A doctoral dissertation from the Faculty of Science and Technology, Uppsala University, is usually a summary of a number of papers. A few copies of the complete dissertation are kept at major Swedish research libraries, while the summary alone is distributed internationally through the series Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology. (Prior to January, 2005, the series was published under the title “Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology”.)