Inversion of Magnetotelluric Data Constrained by Borehole Logs and Reflection Seismic Sections

PING YAN

颜 萍
Abstract

This thesis presents two new algorithms for doing constrained Magnetotelluric (MT) inversion based on an existing Occam 2D inversion program. The first algorithm includes borehole resistivity logs as prior information to constrain resistivity directly in the vicinity of boreholes. The second algorithm uses reflection seismic data as prior constraints to transfer structural information from seismic images to 2D resistivity models. These two algorithms are efficient (proved through tests of synthetic examples) and widely applicable. In this thesis, they have been successfully applied to the COSC (Collisional Orogeny in the Scandinavian Caledonides) MT data.

The COSC project aims to study the mountain belt dynamics in central Sweden by drilling two 2.5 km deep boreholes. MT data were collected to locate the main décollement that separates the overlying Caledonian allochthons and the underlying Precambrian basement, as the main décollement is associated with very conductive Alum shale. The previous interpretation based on part of the COSC seismic profile (CSP) was that the main décollement was located along a reflection with depth of 4.5 km underneath Åre and ~3 km underneath Mörsil, in central Jämtland.

The MT resistivity model reveals a very conductive layer in the central and western parts of the profile, the top of which coincides with the first seismic reflection. This means that the first conductive alum shale layer occurs at less than 1 km depth, supporting a new interpretation of the main décollement at shallower depth. In a re-interpretation of the CSP data based on the MT model, the main décollement occurs a few hundred metres below the top of the conductor and is coincident with a laterally continuous seismic reflection. Further, the overlying seismic reflections resemble imbricated alum shale of the Lower Allochthon. MT inversion using seismic constraints from CSP gives further support to the new interpretation.

Moreover, MT investigations were conducted in the Alnö alkaline and carbonatite ring-intrusion complex in Sweden. 2D and 3D resistivity models inverted from MT data together with resistivity and porosity laboratory measurements delineate a fossil magma chamber as a resistive anomaly surrounded by electrically conductive up-doming and ring-shaped faults and fractures.

Keywords: Magnetotellurics, constrained inversion, borehole logs, seismic, COSC, Alnö alkaline and carbonatite, airborne VLF

Ping Yan, Department of Earth Sciences, Geophysics, Villavägen 16, Uppsala University, SE-75236 Uppsala, Sweden.

© Ping Yan 2016

ISSN 1651-6214
ISBN 978-91-554-9694-4
urn:nbn:se:uu:diva-303498 (http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-303498)
Dedicated to my family
Supervisors & Committee

**Supervisor**
Dr. Thomas Kalscheuer  
Department of Earth Sciences – Geophysics, Uppsala University, Uppsala, Sweden

**Assistant Supervisor**
Senior Professor Laust Pedersen  
Department of Earth Sciences – Geophysics, Uppsala University, Uppsala, Sweden

Dr. María García Juanatey  
Department of Earth Sciences – Geophysics, Uppsala University, Uppsala, Sweden

**Faculty Opponent**
Professor Andreas Junge  
Institute of Geosciences, Johann Wolfgang Goethe University, Frankfurt am Main, Germany

**Examination Committee**
Dr. Ute Weckmann  
Helmholtz-Zentrum Potsdam, Deutsches GeoForschungsZentrum GFZ, Potsdam, Germany

Associate Professor Maxim Smirnov  
Department of Civil, Environmental and Natural resources engineering, Luleå University of Technology, Luleå, Sweden

Professor Ólafur Gudmundsson  
Department of Earth Sciences – Geophysics, Uppsala University, Uppsala, Sweden

Dr. Gerhard Schwarz (reserv)  
SGU, Uppsala, Sweden
List of Papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.


II. Yan, P., Kalscheuer, T., Hedin, P., García Juanatey, M.A. (2016) 2D magnetotelluric inversion using seismic information as constraints, application in the COSC project. Manuscript to be submitted.


Reprints were made with permission from the respective publishers.

An additional journal article, which was published during my Ph.D. studies, but is not included in the thesis, is:

## Contents

1. Introduction ........................................................................................................... 13

2. Magnetotelluric method ..................................................................................... 16  
   2.1 Maxwell's equations ..................................................................................... 16  
   2.2 Magnetotelluric transfer functions ............................................................ 17  
   2.3 Time-series processing .............................................................................. 17  
   2.4 Distortion by shallow inhomogeneities .................................................... 19  
      2.4.1 Zhang et al.'s distortion model ......................................................... 19  
      2.4.2 Bahr's equal phases ......................................................................... 20  
      2.4.3 Phase tensor .................................................................................... 21

3. Fundamentals of other related geophysical methods ........................................ 23  
   3.1 Borehole logging methods ......................................................................... 23  
      3.1.1 Normal logging .............................................................................. 23  
      3.1.2 Laterolog ......................................................................................... 24  
   3.2 Reflection seismic method ......................................................................... 25  
   3.3 Airborne VLF method .............................................................................. 26

4. Magnetotelluric inverse theory ........................................................................... 27  
   4.1 Occam inversion ......................................................................................... 27  
   4.2 Damped Occam inversion ......................................................................... 28  
   4.3 Inversion constrained by borehole logs ..................................................... 28  
   4.4 Inversion constrained by reflection seismic data ...................................... 29  
      4.4.1 Review of existing constrained inversion schemes ....................... 29  
      4.4.2 Newly proposed constrained inversion scheme ........................... 30

5. Geological background and regional setting .................................................... 34  
   5.1 The Åre district .......................................................................................... 34  
      5.1.1 Geological background .................................................................. 34  
      5.1.2 Geophysical background .............................................................. 38  
      5.1.3 MT survey ...................................................................................... 41  
   5.2 The Alnö district .......................................................................................... 43  
      5.2.1 Geological background .................................................................. 43  
      5.2.2 Geophysical background .............................................................. 44  
      5.2.3 MT survey ...................................................................................... 45
6. Summary of papers ................................................................. 46
   6.1 Paper I: A magnetotelluric investigation of the Scandinavian Caledonides in western Jämtland, Sweden, using the COSC borehole logs as prior information ................................................ 46
      6.1.1 Distortion analysis .................................................. 46
      6.1.2 2D inversion models .............................................. 48
      6.1.3 Interpretation and conclusions ............................... 49
   6.2 Paper II: 2D magnetotelluric inversion using seismic information as constraints, application in the COSC project ............................................. 52
      6.2.1 Two synthetic models ............................................ 52
      6.2.2 Application to the COSC MT data ....................... 53
      6.2.3 Interpretation and Conclusions .............................. 55
   6.3 Paper III: 3D magnetotelluric modelling of the Alnö alkaline and carbonatite ring complex, central Sweden ........................................... 57
      6.3.1 Distortion analysis ............................................... 57
      6.3.2 2D and 3D inversion models ................................. 57
      6.3.3 Laboratory resistivity measurements ..................... 59
      6.3.4 Interpretation and conclusions .............................. 60

7. Conclusions and outlook ..................................................... 63
   7.1 Conclusions ............................................................... 63
   7.2 Outlook ................................................................. 64

8. Summary in Swedish ........................................................ 66

Acknowledgements ............................................................... 68

References .............................................................................. 70
### Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two-Dimensional</td>
</tr>
<tr>
<td>3D</td>
<td>Three-Dimensional</td>
</tr>
<tr>
<td>3D/2D</td>
<td>Three-Dimensional local structure and Two-Dimensional regional structure</td>
</tr>
<tr>
<td>CCT</td>
<td>Central Caledonian Transect</td>
</tr>
<tr>
<td>CDP</td>
<td>Common Depth Point</td>
</tr>
<tr>
<td>CF</td>
<td>Caledonian Front</td>
</tr>
<tr>
<td>CMP</td>
<td>Common-Mid Point</td>
</tr>
<tr>
<td>COSC</td>
<td>Collisional Orogeny in the Scandinavian Caledonides</td>
</tr>
<tr>
<td>CSP</td>
<td>COSC Seismic Profile</td>
</tr>
<tr>
<td>DLL</td>
<td>Dual Laterolog</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>GPR</td>
<td>Ground Penetrating Radar</td>
</tr>
<tr>
<td>ICDP</td>
<td>International Scientific Drilling</td>
</tr>
<tr>
<td>LLD</td>
<td>Deep Laterolog</td>
</tr>
<tr>
<td>LLS</td>
<td>Shallow Laterolog</td>
</tr>
<tr>
<td>MGS</td>
<td>Minimum Gradient Support</td>
</tr>
<tr>
<td>MT</td>
<td>Magnetotellurics</td>
</tr>
<tr>
<td>OSG</td>
<td>Operational Support Group</td>
</tr>
<tr>
<td>REE</td>
<td>Rare Earth Elements</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>SGU</td>
<td>Geological Survey of Sweden (Sveriges geologiska undersökning)</td>
</tr>
<tr>
<td>SI</td>
<td>International System of units</td>
</tr>
<tr>
<td>SNC</td>
<td>Seve Nappe Complex</td>
</tr>
<tr>
<td>SP</td>
<td>Self-Potential</td>
</tr>
<tr>
<td>TE</td>
<td>Transverse Electric</td>
</tr>
<tr>
<td>TM</td>
<td>Transverse Magnetic</td>
</tr>
<tr>
<td>VLF</td>
<td>Very Low Frequency</td>
</tr>
</tbody>
</table>
1. Introduction

Magnetotellurics (MT) is a passive electromagnetic (EM) geophysical technique that aims to constrain the resistivity structure of the Earth at depths between tens of meters to hundred kilometers, by measuring natural electric and magnetic fields. These natural fields are caused at high frequencies (>10 Hz) by global thunderstorms and at low frequencies (<10 Hz) by the interaction between the solar wind and the Earth’s ionosphere and magnetosphere. Since the MT method is efficient and less expensive than seismic or other active EM methods, it has been widely used in both scientific and commercial projects. In this thesis, MT has been applied in two districts: Åre and Alnö. For the MT investigation of the Åre district, two new additions to an existing algorithm for 2D inversion of MT data were developed aiming at a) inclusion of borehole resistivity logs as prior information to constrain resistivity directly in the vicinity of boreholes and b) using reflection seismic images as prior constraints to transfer structural information from seismic images to 2D resistivity models.

In the Åre district, MT data were collected in connection with the Collisional Orogeny in the Scandinavian Caledonides (COSC) scientific continental drilling project (Gee et al., 2010; Lorenz et al., 2015a, 2015b, 2011), which aims to study both the Caledonian nappes and the underlying Precambrian basement in central Sweden by drilling two 2.5 km deep boreholes. The first borehole, COSC-1 drilled near the town Åre in 2014, penetrated a thick portion of the Lower Seve Nappe. The future second borehole COSC-2 is expected to be drilled through the main décollement to investigate both the Lower Allochthon and the basement. Performing an MT survey in this area aimed at locating the main décollement between the allochthons and the autochthonous basement, as the main décollement is associated with Alum shale that is very conductive (Gee, 1972). **Paper I** and **paper II** are based on the research on the COSC MT dataset.

In **paper I**, normal MT data processing, dimensionality and strike analyses, and 1D and 2D inversions were performed. Moreover, to improve the resistivity model, a constrained MT inversion algorithm using borehole resistivity logs as prior information was proposed. In the subsequent inversions of the COSC MT data, borehole resistivity logs from COSC-1 in the western part of the profile lead to local improvement of the resistivity model in the metamorphic units penetrated by COSC-1. In the central and eastern parts of
the profile, the MT models exhibit an abrupt transition to a highly conductive layer of $<1\,\Omega\cdot\text{m}$ resistivity at a depth of 1000-300 m. This conductive layer was interpreted to be associated with alum shales. Except for the easternmost part of the profile the thickness of this layer was poorly constrained. However, based on the MT inversion model, a laterally continuous seismic reflection 300-500 m below the top of the conductive layer and still inside the layer was interpreted as the main décollement.

In paper II, a new MT inversion algorithm was developed to transfer structural information from reflection seismic images to 2D resistivity models using weights on the horizontal and vertical smoothness constraints that were modified according to the seismic amplitude envelope, and the efficiency of the new method was demonstrated through two synthetic models. The choice of a newly introduced stabilization parameter for the constrained inversion and its impact on model smoothness and convergence of the new inversion scheme were discussed. Moreover, experiments with wrong or shifted locations of the input constraints helped to evaluate the stability of the new method. Finally, we obtained an improved resistivity model of the COSC study area applying the proposed scheme to the inversion of the COSC MT data and using seismic constraints from the envelope of the COSC seismic profile (CSP; Juhlin et al., 2016). The seismically constrained inversion model exhibited a much thinner conductive alum shale layer than the inversion model computed using regular smoothness constraints from paper I.

In the Alnö district, an MT survey was conducted to study the resistivity distribution of the Alnö alkaline and carbonatite ring-complex in central Sweden. Dimensionality and strike analyses showed 3D structure in this area. 2D and 3D MT inversions were performed and compared. Moreover, to help interpret the obtained resistivity models, resistivity laboratory measurements were performed on surface rock samples which were collected over various parts of the complex. These studies helped to delineate a solidified fossil magma chamber as a resistive anomaly surrounded by electrically conductive ring-type fault systems and up-doming faulted and fractured systems. The results were presented in paper III.

This thesis consists of a comprehensive summary and three accompanying paper. The comprehensive summary begins with a brief overview about the research methodologies used in this thesis. The theoretical concepts of the MT method are given in section 2, including the principle of MT, how MT time-series are processed to get transfer functions and three important approaches for dimensionality, distortion and strike analyses. The fundamentals of other geophysical techniques used in the thesis are discussed in section 3. Section 4 starts with the normal Occam and damped Occam inversion algorithms, and then presents in detail the two newly proposed constrained inversion schemes. Section 5 gives more comprehensive information about
the geological and geophysical background in the two districts studied using MT surveys. Section 6 provides a summary of the three papers, which is followed by conclusions and outlook in section 7. The thesis ends with a summary in Swedish in section 8.
2. Magnetotelluric method

2.1 Maxwell's equations

The principle of MT can be described by Maxwell's equations. Maxwell's equations are four partial differential equations that describe the relationships between the electric and magnetic fields, the electric charges and currents. With the International System of units (SI), they have the following forms:

\[ \nabla \cdot \mathbf{D} = \rho_e, \quad \text{Gauss's law} \quad (2.1) \\
\nabla \cdot \mathbf{B} = 0, \quad \text{Gauss's law for magnetism} \quad (2.2) \\
\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \text{Faraday's law} \quad (2.3) \\
\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \quad \text{Ampère's law} \quad (2.4) \\
\]

where \( \mathbf{D} \) (C/m\(^2\)) is the electric displacement, \( \mathbf{B} \) (Wb/m\(^2\)) is the magnetic induction, \( \mathbf{E} \) (V/m) is the electric field, \( \mathbf{H} \) (A/m) is the magnetic field, \( \rho_e \) (C/m\(^3\)) is the electric charge density and \( \mathbf{J} \) (A/m\(^2\)) is the current density.

Their constitutive relations in an isotropic medium are:

\[ \mathbf{D} = \varepsilon \mathbf{E}, \quad (2.5) \]
\[ \mathbf{B} = \mu \mathbf{H}, \quad (2.6) \]
\[ \mathbf{J} = \sigma \mathbf{E}, \quad (2.7) \]

where \( \varepsilon, \mu \) and \( \sigma \) are scalar quantities over time and frequency. \( \varepsilon \) (F/m) is the dielectric permittivity, \( \mu \) (H/m) is the magnetic permeability and \( \sigma \) (S/m) is the electrical conductivity.

As MT measures the natural electric and magnetic fields with frequencies \( f \) range from \( 10^3 \) Hz to \( 10^4 \) s, two assumptions are applicable (Vozoff, 1972): (1) the primary electromagnetic field is a plane wave propagating down to the earth surface; (2) for not extremely low conductivity structures, \( \sigma \gg \omega \varepsilon \) (\( \omega = 2\pi f \)), so that the displacement currents \( \partial \mathbf{D}/\partial t \) can be neglected relative to the current density \( \mathbf{J} \) (quasi-stationary approximation).
2.2 Magnetotelluric transfer functions

The horizontal electric field components are related to the horizontal magnetic field components by the MT impedance tensor (a transfer function) with the form

\[
\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \mathbf{Z} \mathbf{H} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}.
\]  

(2.8)

In a 2D Earth, the impedance tensor can be written as

\[
\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} = \begin{pmatrix} 0 & Z_{TE} \\ Z_{TM} & 0 \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix},
\]  

(2.9)

with the x axis parallel to the strike direction. The transverse electric (TE) mode consists of \(E_x, H_y\) and \(H_z\), and is therefore sensitive to along-strike conductors. The transverse magnetic (TM) mode consists of \(H_x, E_y\) and \(E_z\), and is effective at locating interfaces between regions of differing resistivity.

From the impedance tensor \(\mathbf{Z}\), apparent resistivity \(\rho_a\) and phase \(\varphi\) can be calculated by

\[
\rho_a = \frac{1}{\omega \mu} |Z_{i,j}|^2, \quad \varphi = \tan^{-1} \left( \frac{\text{Im} Z_{i,j}}{\text{Re} Z_{i,j}} \right), \quad i, j = x, y.
\]  

(2.10)

An important invariant of the MT impedance tensor \(\mathbf{Z}\) is called the determinant impedance (also referred to as the effective impedance) and defined by \(Z_{\text{DET}} = \sqrt{Z_{xx}Z_{yy} - Z_{xy}Z_{yx}}\). The determinant invariant \(Z_{\text{DET}}\) is important, because it is rotationally invariant and less prone to introducing artifacts from 3D effects into 2D models. Thus, inverting directly \(Z_{\text{DET}}\) can get robust 2D model when 2D condition is not fulfilled (Pedersen and Engels, 2005).

The vertical magnetic field component is related to the horizontal magnetic components by the vertical magnetic transfer function (also referred to as tipper)

\[
H_z = \mathbf{T} \mathbf{H} = \begin{pmatrix} T_x & T_y \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}.
\]  

(2.11)

From the vertical magnetic transfer function, induction arrows can be mapped and point away from internal current concentrations using the Wiese convention (Wiese, 1962).

2.3 Time-series processing

The purpose of MT time-series processing is to compute frequency-domain estimates of transfer functions from the collected time series of electric and magnetic fields, using fast Fourier transforms and robust statistical techniques (Figure 2.1; Tissen, 2015). The application of the later technique can
dramatically improve the quality of estimates of the impedance tensor (Egbert, 1997; Jones et al., 1989).

Another important technique used in time-series processing to get higher quality data is the remote reference technique (Gamble et al., 1979). In this technique, noise at the measurement location can be removed through using the magnetic data recorded simultaneously at a remote (reference) location, with the assumptions that the noise contributions at the two locations are uncorrelated while the source signals recorded at the local and remote sites are correlated.

*Figure 2.1. Work flow of MT time-series processing (modified after Tissen, 2015).*
2.4 Distortion by shallow inhomogeneities

Galvanic (non-inductive) distortion of the regional fields by ‘small scale’ local structures (shallow inhomogeneities) can result in poor interpretations in application of MT method. This problem has been recognized for over thirty years. Many approaches, both mathematically based and physically based, have been proposed to analyze and remove the distortions, among which Zhang et al.’s distortion analysis method (Zhang et al., 1987), Bahr’s distortion approach (Bahr, 1991) and phase tensor (Caldwell et al., 2004) are three of the most important tools and applied in the strike and dimensionality analyses in both the Åre and Alnö districts.

The theory of the distortion has been well described by Groom and Bailey (1991, 1989), Groom and Bahr (1992) and Chave and Smith (1994). Under the quasi-static pre-Maxwellian approximation (neglecting displacement currents), in source-free and non-magnetic media, assuming that the regional field is homogeneous across the inhomogeneities, the observed local horizontal electric \( \mathbf{E} \) and magnetic field \( \mathbf{B} \) can be related to the regional ones \( \mathbf{E}_R \) and \( \mathbf{B}_R \) with two second-rank tensors \( \mathbf{C} \) and \( \mathbf{D} \)

\[
\begin{align*}
\mathbf{E} &= \mathbf{C} \cdot \mathbf{E}_R, \\
\mathbf{B} &= \mathbf{B}_R + \mathbf{D} \cdot \mathbf{E}_R.
\end{align*}
\]  

(2.12)  

(2.13)

In the low-frequency limit, the magnetic effects of the charges \( \mathbf{D} \cdot \mathbf{E}_R \) can be ignored, so that the observed impedance \( \mathbf{Z} \) is

\[
\mathbf{Z} = \mathbf{C} \cdot \mathbf{Z}_R.
\]  

(2.14)

In a general coordinate system,

\[
\mathbf{Z}(\theta) = \mathbf{R}(\theta) \cdot \mathbf{C} \cdot \mathbf{Z}_R \cdot \mathbf{R}^T(\theta),
\]

(2.15)

where \( \theta \) denotes the angle between the observation reference frame and the strike direction of the 2D structures, and \( \mathbf{R} \) is the rotation matrix with the form

\[
\mathbf{R}(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}.
\]  

(2.16)

2.4.1 Zhang et al.'s distortion model

Zhang et al. (1987) described the impedance in a different but similar way as

\[
\mathbf{Z} = (\mathbf{I} + \mathbf{P}) \cdot \mathbf{Z}_R,
\]

(2.17)

where \( \mathbf{P} \) is the distortion tensor. Assume the principal model, in which both the local and regional structures are considered to be 2D, with the local strike \( \theta_l \) and the regional strike \( \theta_r \). In an arbitrary direction,

\[
\mathbf{P}(\theta - \theta_l) = \mathbf{R}(\theta - \theta_l) \cdot \begin{bmatrix}
P_{xx} & 0 \\
0 & P_{yy}
\end{bmatrix} \cdot \mathbf{R}^T(\theta - \theta_l),
\]

(2.18)
\[
Z_R(\theta - \theta_r) = R(\theta - \theta_r) \cdot \begin{bmatrix}
0 & Z_{R,xy} \\
Z_{R,yx} & 0
\end{bmatrix} \cdot R^T(\theta - \theta_r).
\]  

(2.19)

Thus, the impedance tensor in the direction of the regional strike is given by
\[
Z(0) = \begin{bmatrix}
-P_2 \sin 2\Delta \theta Z_{R,yy} & (1 + P_1 + P_2 \cos 2\Delta \theta)Z_{R,xy} \\
(1 + P_1 - P_2 \cos 2\Delta \theta)Z_{R,yx} & -P_2 \sin 2\Delta \theta Z_{R,xy}
\end{bmatrix},
\]  

(2.20)

where \( \Delta \theta = \theta_r - \theta \), \( P_1 = (P_{xy} + P_{yy})/2 \) and \( P_2 = (P_{xy} - P_{yy})/2 \). Therefore, the following relations can be seen (also true for models where the local structure is 3D):
\[
Z_{xx}(0) = \beta Z_{yx}(0),
\]

(2.21)
\[
Z_{yy}(0) = \gamma Z_{xy}(0),
\]

(2.22)

where
\[
\beta = \frac{-P_2 \sin 2\Delta \theta}{(1 + P_1 - P_2 \cos 2\Delta \theta)},
\]

(2.23)
\[
\gamma = \frac{-P_2 \sin 2\Delta \theta}{(1 + P_1 + P_2 \cos 2\Delta \theta)}.
\]

(2.24)

The regional strike direction \( \theta_l \) can be found by minimizing the following function:
\[
Q = |Z_{xx} - \beta Z_{yx}|^2 + |Z_{yy} - \gamma Z_{xy}|^2,
\]

(2.25)

where \( \beta = \langle z_{xx}, z_{yx} \rangle / \langle z_{yx}, z_{yx} \rangle \) and \( \gamma = \langle z_{yy}, z_{xy} \rangle / \langle z_{xy}, z_{xy} \rangle \), and the angle brackets denote the dot product. Here vector notations of the impedances tensor elements are used, because \( \beta \) and \( \gamma \) can be estimated over several adjacent periods and stations.

2.4.2 Bahr's equal phases

The impedance tensor \( Z \) rotated to the strike direction is given by
\[
Z(0) = R(0) \cdot C \cdot Z_R \cdot R^T(0) = C \cdot Z_R =
\begin{bmatrix}
c_{11} & c_{12} \\
c_{21} & c_{22}
\end{bmatrix}
\begin{bmatrix}
0 & Z_{xy} \\
Z_{yx} & 0
\end{bmatrix}
\begin{bmatrix}
c_{12} Z_{yx} & c_{11} Z_{xy} \\
c_{22} Z_{yx} & c_{21} Z_{xy}
\end{bmatrix}.
\]

(2.26)

As \( c_{ij} \) are real numbers, in the first column, the phases of the two elements only depend on \( Z_{yx} \). Similarly, in the second column, the phases of the two elements only depend on \( Z_{xy} \). Bahr discovered the important fact, that for each column of the impedance tensor in the direction of the regional strike \( Z(0) \), the phases of the two elements are equal. Thus, his approach of finding the strike direction was to rotate the impedance tensor until the condition was met as closely as possible (Bahr, 1991).

Bahr also proposed a dimensionality tool called the phase-sensitive skew (Bahr, 1991)
\[ \eta = \frac{[[D_1 S_2] - [S_1 D_2]]^{1/2}}{|D_2|}, \]  
(2.27)

where

\[ S_1 = Z_{xx} + Z_{yy}, \quad S_2 = Z_{xy} + Z_{yx}, \]  
(2.28)

\[ D_1 = Z_{xx} - Z_{yy}, \quad D_2 = Z_{xy} - Z_{yx}, \]  
(2.29)

\[ [D_1, S_2] = \text{Re} D_1 \text{Im} S_2 - \text{Re} S_2 \text{Im} D_1, \]  
(2.30)

and \( \eta > 0.3 \) is usually considered to indicate 3D data.

### 2.4.3 Phase tensor

Taking account the real and imaginary parts, \( Z = X + iY \) and \( Z_R = X_R + iY_R \), the phase tensor \( \Phi \) is given by

\[ \Phi = X^{-1} Y = (C \cdot X_R)^{-1} C \cdot Y_R = X_R^{-1} C^{-1} CY_R = X_R^{-1} Y_R = \Phi_R. \]  
(2.31)

Thus the phase tensor is not distorted by the galvanic effects in spite of the dimensionality of structure. Four parameters, the maximum value \( \Phi_{\text{max}} \), minimum value \( \Phi_{\text{min}} \), the skew angle \( \beta \) and the non-invariant angle \( \alpha \), were introduced to present the second-rank phase tensor (Caldwell et al., 2004):

\[ \Phi_{\text{max}} = \left( \Phi_1^2 + \Phi_3^2 \right)^{1/2} + \left( \Phi_1^2 + \Phi_3^2 - \det(\Phi) \right)^{1/2}, \]  
(2.32)

\[ \Phi_{\text{min}} = \left( \Phi_1^2 + \Phi_3^2 \right)^{1/2} - \left( \Phi_1^2 + \Phi_3^2 - \det(\Phi) \right)^{1/2}, \]  
(2.33)

\[ \beta = \frac{1}{2} \tan^{-1} \left( \frac{\Phi_3}{\Phi_1} \right) = \frac{1}{2} \tan^{-1} \left( \frac{\Phi_{xy} - \Phi_{yx}}{\Phi_{xx} + \Phi_{yy}} \right), \]  
(2.34)

\[ \alpha = \frac{1}{2} \tan^{-1} \left( \frac{\Phi_{xy} + \Phi_{yx}}{\Phi_{xx} - \Phi_{yy}} \right), \]  
(2.35)

where

\[ \Phi_1 = \left( \Phi_{xx} + \Phi_{yy} \right)/2, \]  
(2.36)

\[ \det(\Phi) = \Phi_{xx} \Phi_{yy} - \Phi_{xy} \Phi_{yx}, \]  
(2.37)

\[ \Phi_3 = \left( \Phi_{yx} - \Phi_{xy} \right)/2. \]  
(2.38)

Then, the phase tensor can be represented by an ellipse, the axes of which represent the principal axes and values of the tensor, with a rotation of \( \alpha - \beta \) from the observer’s coordinate system (Figure 2.2). In the cases of 1D or 2D (and 3D/2D) earth, \( \beta \) is zero. The 2D regional strike direction is \( \alpha \) in 2D (and 3D/2D).
Figure 2.2. Graphical illustration of the phase tensor (modified after Caldwell et al., 2004). The lengths of the major and minor axes are proportional to the maximum value $\Phi_{\text{max}}$ and minimum value $\Phi_{\text{min}}$. The direction of the major axis is rotated by $\alpha - \beta$ from the observer’s coordinate system.
3. Fundamentals of other related geophysical methods

3.1 Borehole logging methods

Two types of borehole resistivity logs are available in COSC-1, one is the normal logging conducted by Lund University, consisting of short and long normal logs with self-potential (SP), and the other is the dual laterolog (DLL) resistivity conducted by the Operational Support Group (OSG) of the International Scientific Drilling (ICDP), consisting of shallow (LLS) and deep (LLD) laterologs.

Borehole resistivity logs were presented to help the geological interpretation of MT inversion models. Additionally, both the long normal resistivity log and the deep laterolog were averaged and set as borehole constraints in MT inversion in paper I.

3.1.1 Normal logging

For normal logging device (Figure 3.1, Robinson and Çoruh, 1988), the current sink electrode B is far away from the source electrode A, and a potential electrode M, which is close to A, is far away from a second potential electrode N (usually set on the surface). The difference in potential $v_M$ between M and N is measured. Although $v_M$ is affected by all the material in a spherical shell of radius AN, the influence from the material close to M is much larger than those far away from M.

The apparent resistivity of the middle point between AM is given by

$$\rho(\xi) = \frac{v_M(\xi)}{l} = \frac{4\pi AM}{l},$$

(3.1)

where $l$ is the constant current supplied by the source electrode A, and AM (spacing) is the distance between A and M. Typical AM spacing is 16 inches for a short normal resistivity log and 64 inches for a long normal resistivity log.
As the current in the normal logging system is vertical, following Pedersen et al. (1992), the average resistivity over a certain interval \((z - \Delta z, z + \Delta z)\) is defined as

\[
\bar{\rho}_{\text{ver}}(z) = \frac{1}{2\Delta z} \int_{z-\Delta z}^{z+\Delta z} \rho(\xi) d\xi. \tag{3.2}
\]

![Diagram of normal logging device](image)

*Figure 3.1.* Schematic diagram of the normal logging device (modified after Robinson and Çoruh, 1988). The source electrode A is far away from the sink electrode B. Two potential electrodes M and N are far away from each other. Electrodes A and M are close to each other and the distance between is called spacing. The potential difference between M and N is measured.

### 3.1.2 Laterolog

For laterolog device, the current is forced to flow nearly horizontally into the formation by a set of guard electrodes. Therefore, the effect of the mud and invaded zone on the log response is reduced. The dual-laterolog (DLL) device used by OSG is a combination of two groups of laterolog-3 (LL3; Figure 3.2). The average resistivity over a certain interval \((z - \Delta z, z + \Delta z)\) is defined as (Pedersen et al., 1992)

\[
\frac{1}{\bar{\rho}_{\text{hor}}(z)} = \frac{1}{2\Delta z} \int_{z-\Delta z}^{z+\Delta z} \frac{1}{\rho(\xi)} d\xi. \tag{3.3}
\]

Compared with the normal logging method, the laterolog is much more sensitive to thin beds.
3.2 Reflection seismic method

Reflection seismic is an exploration geophysical method that uses the principles of seismology and a known source (i.e. time and location of the source) to image geological structures of the Earth's subsurface from reflected seismic waves. The method is sensitive to the seismic impedance (seismic wave velocity × density) contrast of the rocks that the seismic wave propagates through. Since the reflected pulses are weak, multiple shots and receivers are used. Energy from different shots that originate from reflectors in the same part of the subsurface are stacked in order to enhance the signal. The seismic traces which have a common reflection point is called common-mid point (CMP) or common depth point (CDP) (Yilmaz, 2001). After seismic data processing, the reflections are migrated to their true geological positions in the subsurface (Yilmaz, 2001).

From seismic data, many components can be obtained by measurement or computations. These components of the seismic data are called seismic attributes (Taner et al., 1979; Taner, 2001). Among others, seismic trace envelope (SE, Figure 3.3) is an important seismic attribute, which is defined by

\[ SE(t) = \sqrt{T^2(t) + H^2(t)}, \]  

(3.4)
where $T(t)$ denotes the seismic trace, $H(t)$ is Hilbert’s transform of $T(t)$. The seismic envelope represents mainly the seismic impedance (the product of seismic velocity and density) contrasts and its amplitude is proportional to reflectivity (Figure 3.3). Hence, the seismic envelope is a useful attribute to highlight discontinuities.

![Figure 3.3. Illustration of seismic envelope $SE(t)$ and its relationship with trace $T(t)$, impedance and reflectivity. The seismic envelope only has positive amplitude, which is proportional to reflectivity.](image)

In the Åre district, a 54-km-long COSC reflection seismic profile (CSP) existed and was used to help MT interpretation in paper I. In paper II, the CSP seismic envelope data were used as input constraints to assign the weights to the regularization operator in the MT inversion. For the Alnö district, three reflection seismic sections were available and used to support the geological interpretation of the MT models in paper III.

### 3.3 Airborne VLF method

The Geological Survey of Sweden (SGU) has conducted airborne very low frequency (VLF) measurements in Sweden for many years, using two transmitters in different directions with different frequencies (14-25 kHz) to collect three component magnetic field data. Apparent resistivity maps can be calculated from the magnetic transfer functions defined in equation 2.11 using a simple transformation introduced by Becken and Pedersen (2003). The airborne VLF data can be used to interpret the upper few tens to few hundred meters of the crust in a qualitative way.

The apparent resistivity maps from the airborne VLF data are available in both the Åre and Alnö districts and presented in paper I and III, respectively.
4. Magnetotelluric inverse theory

4.1 Occam inversion

Occam’s inverse algorithm is to generate a smooth resistivity model, through adding the model roughness as the regularization operator to the objective function to stabilize the inversion (Constable et al., 1987; deGroot-Hedlin and Constable, 1990).

The objective function $U$ which is going to be minimized during inversion is given by

$$U_{occam}[\mathbf{m}, \lambda] = Q_d[\mathbf{m}] + \lambda R_m[\mathbf{m}], \quad (4.1)$$

where $\mathbf{m} = (m_1, \cdots, m_M)^T$ is the model to be solved for, $\lambda$ is the Lagrange multiplier, $Q_d$ is the data misfit function and $R_m$ is the model roughness. $R_m$ is added as a regularization, because the MT inverse problem is ill-posed (Tikhonov and Arsenin, 1977). $Q_d$ and $R_m$ are of the following forms

$$Q_d[\mathbf{m}] = (\mathbf{d} - \mathbf{F}[\mathbf{m}])^T \mathbf{W}_d^T \mathbf{W}_d (\mathbf{d} - \mathbf{F}[\mathbf{m}]) - Q_d^*, \quad (4.2)$$

$$R_m[\mathbf{m}] = \mathbf{m}^T \mathbf{W}_m^T \mathbf{W}_m \mathbf{m}, \quad (4.3)$$

where $\mathbf{d}$ is the data set of $N$ field measurements, $\mathbf{F}[\mathbf{m}]$ represents the $N$ data predicted for a model $\mathbf{m}$, the superscript $T$ indicates transposition, $\mathbf{W}_d$ represents the weighting matrix of data (diagonal and based on the reciprocal data uncertainties), $Q_d^*$ denotes the target data misfit, and $\mathbf{W}_m$ denotes the weighting matrix of the model (smoothness constraints). For a 2D model with $x$ axis in parallel with the strike direction, the roughness can be expressed by

$$R_m = \alpha_y \| \partial_y \mathbf{m} \|^2 + \alpha_z \| \partial_z \mathbf{m} \|^2, \quad (4.4)$$

where $\alpha_y$ and $\alpha_z$ are factors for horizontal and vertical smoothness, and $\| \cdot \|^2$ denotes the Euclidean L2 norm. In this equation, $\partial_y$ and $\partial_z$ are the roughening matrices in the $y$ and $z$ direction with the following forms

$$\partial_y = \begin{bmatrix}
\partial_{y_1} \\
\partial_{y_2} \\
\vdots \\
\partial_{y_M}
\end{bmatrix}, (\mathbf{M}_z \cdot \mathbf{M}_y \times \mathbf{M}_z \cdot \mathbf{M}_y), \quad (4.5)$$
\[
\tilde{\mathbf{z}} = \begin{bmatrix}
\tilde{z}_1 \\
\tilde{z}_2 \\
\vdots \\
\tilde{z}_{M_y}
\end{bmatrix}, \quad (M_y \times M_y \times M_z \times M_y), \quad (4.6)
\]

where \(M_y\) and \(M_z\) are the number of cells of the model in the \(y\) and \(z\) direction, \(\tilde{\mathbf{y}}_i\) and \(\tilde{\mathbf{z}}_i\) \((i = 1, 2, \ldots, M_y)\) are block matrices and given by

\[
\tilde{\mathbf{y}}_i = \begin{bmatrix}
-1 \\
-1 \\
\vdots \\
-1 \\
1
\end{bmatrix}, \quad (M_z \times 2M_z), \quad (4.7)
\]

\[
\tilde{\mathbf{z}}_i = \begin{bmatrix}
-1 & 1 \\
-1 & 1 \\
\vdots & \vdots \\
0 & -1 \\
0 & 1
\end{bmatrix}, \quad (M_z \times M_z). \quad (4.8)
\]

### 4.2 Damped Occam inversion

For the damped Occam inverse algorithm (Kalscheuer et al., 2010; Lines and Treitel, 1984), a Marquardt-Levenberg damping term, \(\epsilon (\mathbf{m}_{k+1} - \mathbf{m}_k)^T (\mathbf{m}_{k+1} - \mathbf{m}_k)\), is added to the objective function \(U\) to further stabilize the inversion process. Here, \(k\) represents the iteration number and \(\epsilon\) denotes the damping factor. So the objective function \(U\) will have the following form

\[
U_{\text{damocc}}[\mathbf{m}_{k+1}, \lambda, \epsilon] = Q_d[\mathbf{m}_{k+1}] + \lambda R_m[\mathbf{m}_{k+1}] + \epsilon (\mathbf{m}_{k+1} - \mathbf{m}_k)^T (\mathbf{m}_{k+1} - \mathbf{m}_k). \quad (4.9)
\]

In practice, we would first do Occam inversion to the MT data with a homogeneous half space as the initial model. Then we put the preferred obtained Occam inversion model from the first round inversion as the initial model and perform a second round of Damped Occam inversion to the MT data. Thus, we can get a model which is close to the preferred Occam inversion model but with better data fit (lower RMS).

### 4.3 Inversion constrained by borehole logs

Similarly, to apply borehole resistivity logs as constraints to the Occam and damped Occam MT inversion, we propose adding another borehole term \(Q_{BH}\) to the objective function \(U_{\text{occam}}\) (equation 4.1) and \(U_{\text{damocc}}\) (equation 4.9), where
\[ \alpha Q_{BH}[m_{k+1}] = \alpha (m_{k+1} - m_{bh})^T W_{bh}^T W_{bh} (m_{k+1} - m_{bh}). \] (4.10)

In this equation, \( \alpha \) is the weighting for the borehole term and given as a fixed value, \( m_{bh} \) is the average borehole resistivity log over each MT model cell calculated from equations 3.2 and 3.3 and \( W_{bh} \) is the weighting matrix for borehole data (the standard deviations of the borehole data within each model cell) and is diagonal with non-zero diagonals only where model cells coincide with the borehole.

The proposed borehole constrained inversion scheme was easily implemented in the 2D MT inversion package Emilia (Kalscheuer et al., 2010). The deep laterolog from COSC-1 were applied as constraints in MT inversion in paper I. Although effect of this technique has limited lateral extent and the resulting resistivity models were not strongly affected by the borehole constraints from COSC-1, synthetic examples demonstrated that constraints from future borehole COSC-2 would improve the resistivity model significantly.

4.4 Inversion constrained by reflection seismic data

4.4.1 Review of existing constrained inversion schemes

Normal Occam and damped Occam inversion can give us a smooth model. However, the smooth model would not always be what we want. As it is very common that seismic data is available in the same research area, many geophysicists have been working on how to translate a priori information from seismic data and use it in the EM inversion to obtain an improved resistivity model. One approach is to link the electrical conductivity to seismic velocity and do both EM and seismic inversions simultaneously (Chen et al., 2007; Gao et al., 2012; Hoversten et al., 2006). The method has been proved to be efficient but the problem is that the resistivity-velocity interrelationships could not be easily determined. Another approach is the cross-gradient regularization method, which has been applied successfully in many areas (e.g., Gallardo and Meju, 2007, 2004, 2003).

In recent years, geophysicists have been doing research on constrained inversions, in which different weights are assigned to the entries of the regularization operator. The simplest method, called tear-zone inversion, was to assign zero to the weights on the boundaries which are defined by seismic or ground penetrating radar (GPR) data (e.g. Doetsch et al., 2012; Favetto et al., 2007; García Juanatey et al., 2013; Muñoz et al., 2010). However, the input
constraints must be very accurate; otherwise, significant artifacts could be introduced in the EM inversion models.

To overcome this problem, improved methods have been proposed, in which, instead of zero, small weights are assigned to the defined boundaries so that the smoothness will be relaxed instead of being totally ignored.

Brown et al. (2012) suggested using a seismically weighted regularization operator based on the gradient in seismic velocity:

\[
R_m = \left\| W_{sm, gs} \partial m \right\|^2 = \left\| \frac{\beta \partial m}{(\partial v^2 + \beta^2)^{1/2}} \right\|^2,
\]

(4.11)

where \( v \) is the seismic velocity and \( \beta \) is a stabilization parameter. The algorithm is essentially an extension of the minimum gradient support (MGS) inversion (Portniaguine and Zhdanov, 1999), in which the gradient of the resistivity was used in the denominator instead of the gradient of velocity.

To get the seismic velocity model, inversion of seismic waveforms is required. Since full seismic waveform inversion is still impractical for 2D and 3D models, Brown et al.’s (2012) algorithm and examples are for 1D models.

Zhou et al. (2014) introduced an image-guided inversion, in which a certain small weight (0.5) was assigned to the regularization at the determined edges and large weight (20) at the coherent cells. The edges and coherent cells were determined from a guiding (seismic) image, through some image processing.

Wiik et al. (2015) suggested adding a spatially varying matrix function \( \Phi \) to the regularization operator. The function \( \Phi \) is based on the local structural consistency in the seismic image:

\[
R_m = \left\| \Phi \partial m \right\|^2 = \left\| K e^{-5c^2} \partial m \right\|^2,
\]

(4.12)

where \( c \) denotes the coherency in the seismic cube and \( K \) represents the factor which controls the level.

The validities of these methods have been demonstrated by the authors using both synthetic examples and field datasets.

### 4.4.2 Newly proposed constrained inversion scheme

For our proposed constrained inversion scheme, the basic idea is the same as above constrained inversions: assigning small weights to the regularization operator where boundaries are assumed. The weighted regularization operator has a similar form as in Brown et al. (2012). However, instead of using the gradient of seismic velocity, our weighted regularization operator is
based on the distribution of the reflection seismic envelope and its directional gradients. As we have stated above, the seismic envelope is proportional to reflectivity and capable of highlighting discontinuities. Directional gradients are included to get directional information. Hence, our approach is believed to be more directly accessible and conceptually appealing than Zhou et al.’s (2014) and Wiik et al.’s (2015) approaches.

Our regularization operator has the following form:

\[
R_m = \|W_{sae,y} \partial_y m\|^2 + \|W_{sae,z} \partial_z m\|^2
\]

\[
= \sum_{j=1}^{M_y} \sum_{i=2}^{M_x} W_{sae,y} j, i-1 (m_{j,i} - m_{j,i-1})^2
+ \sum_{i=1}^{M_y} \sum_{j=2}^{M_x} W_{sae,z} j-1,i (m_{j,i} - m_{j-1,i})^2. \tag{4.13}
\]

In the equation, \(W_{sae,y}\) and \(W_{sae,z}\) are the weights to the roughness in the horizontal and vertical directions, respectively, and have the following forms partly similar to Brown et al.’s (2012) regularization:

\[
W_{sae,y,j,i} = \frac{1}{(s_{ae,y,j,i}^2 + \beta^2)^{1/2}} = \frac{1}{\left(\frac{(s_{ae,y,j,i})^2}{\beta} + 1\right)^{1/2}},
\]

\[
j = (1, M_z), \; i = (1, M_y - 1), \tag{4.14}
\]

\[
W_{sae,z,j,i} = \frac{1}{(s_{ae,z,j,i}^2 + \beta^2)^{1/2}} = \frac{1}{\left(\frac{(s_{ae,z,j,i})^2}{\beta} + 1\right)^{1/2}},
\]

\[
j = (1, M_z - 1), \; i = (1, M_y), \tag{4.15}
\]

where \(\beta\) is a stabilization factor which controls the effects of the input seismic constraints, and \(S_{ae,y}\) and \(S_{ae,z}\) depend on the seismic envelope and its fractional directional gradients.

Figure 4.1 was drawn to help demonstrate how to calculate \(S_{ae,y}\) and \(S_{ae,z}\) from the seismic envelope. The black solid lines represent the grid lines of the model cells (\(M_y + 1\) vertical lines and \(M_z + 1\) horizontal lines in total including the outer edges of the model grid) with their centers indicated by the black dashed lines.

First, let us look at the blue rectangle in Figure 4.1 and consider the horizontal smoothing across the black vertical cell edge \((jy, iy)\) (where \(jy = 1, 2, \cdots, M_z\) and \(iy = 1, 2, \cdots, M_y - 1\)). The horizontal value \(S_{ae,y}(jy, iy)\) for this vertical edge is:

\[
S_{ae,y}(jy, iy) = \frac{(SE_1 + SE_3) \times \frac{1}{2} \Delta y_{iy} + (SE_2 + SE_4) \times \frac{1}{2} \Delta y_{iy+1}}{\frac{1}{2} \Delta y_{iy} + \frac{1}{2} \Delta y_{iy+1}} \cdot \sqrt{\left(\frac{\partial SE}{\partial y}\right)^2 + \left(\frac{\partial SE}{\partial z}\right)^2 + y^2}, \tag{4.16}
\]
\[
\begin{align*}
\frac{\partial SE}{\partial y} &= \frac{(SE_2+SE_4)-(SE_1+SE_3)}{\frac{1}{2} \Delta y_{ij} + \frac{1}{2} \Delta y_{ij+1}}, \\
\frac{\partial SE}{\partial z} &= \frac{(SE_3+SE_4)-(SE_1+SE_2)}{\frac{1}{2} \Delta z_{ij}},
\end{align*}
\] (4.17) (4.18)

where \(SE_1, SE_2, SE_3\) and \(SE_4\) represent the average seismic envelopes within each quarter-cell inside the blue block in Figure 4.1, and \(\Delta y\) and \(\Delta z\) denote the distances between the vertical and horizontal grid lines, respectively. The fractional directional gradient factor \(\sqrt{\left(\frac{\partial SE}{\partial y}\right)^2 + \left(\frac{\partial SE}{\partial z}\right)^2 + \gamma^2}\), which measures the ratio of horizontal to overall change in seismic reflectivity and has a range between 0 and 1, is added to prevent small horizontal weights from entering into the regularization where the seismic reflectivity is strong but varies little in the horizontal direction. To preclude a division-by-zero error, a very small number \(\gamma\) is added.

Similarly, for the vertical smoothing across the horizontal edge \((jz, iz)\) (where \(jz = 1, 2, \ldots, M_z - 1\) and \(iz = 1, 2, \ldots, M_y\); the red block in Figure 4.1), the vertical value \(S_{ae,z(jz,iz)}\) is:

\[
S_{ae,z(jz,iz)} = \frac{(SE_2+SE_4) \times \frac{1}{2} \Delta z_{jz} + (SE_3+SE_4) \times \frac{1}{2} \Delta z_{jz+1}}{\frac{1}{2} \Delta z_{jz} + \frac{1}{2} \Delta z_{jz+1}} \cdot \sqrt{\left(\frac{\partial SE}{\partial y}\right)^2 + \left(\frac{\partial SE}{\partial z}\right)^2 + \gamma^2},
\] (4.19)

\[
\frac{\partial SE}{\partial y} = \frac{(SE_2+SE_4)-(SE_1+SE_3)}{\frac{1}{2} \Delta y_{iz}},
\] (4.20)

\[
\frac{\partial SE}{\partial z} = \frac{(SE_3+SE_4)-(SE_1+SE_2)}{\frac{1}{2} \Delta z_{jz} + \frac{1}{2} \Delta z_{jz+1}}.
\] (4.21)

The proposed seismically constrained inversion scheme was easily implemented in the 2D MT inversion package Emilia (Kalscheuer et al., 2010).
Figure 4.1. Illustration of the algorithm to calculate the roughness weights in both horizontal (the blue rectangle) and vertical (the red rectangle) directions from the seismic amplitude envelope. The black solid lines represent the grid lines of the model cells with their centers indicated by the black dashed lines.
5. Geological background and regional setting

5.1 The Åre district

The Scandinavian Caledonides were formed during the Scandan collisional orogeny when continent Laurentia was being underthrust by Baltica in the Silurian and Early Devonian period after the closure of the Iapetus Ocean located between the continents Laurentia and Baltica during the Ordovician (Figure 5.1; Gee et al., 2008; Lorenz et al., 2015a). As a result of the thrusting, nowadays the Scandinavian Caledonides consists of many different types of thrust sheets (allochthons) which spread several hundreds of kilometres on the top of the Precambrian basement (Figure 5.2; Corfu et al., 2014). Given that the transportation of these Caledonian allochthons is similar to that recognized in the Himalayas, the study of the Scandinavian Caledonides plays an important role in understanding the Himalayan-type orogeny (Dewey, 1969; Gee et al., 2010; Labrousse et al., 2010). The transect from Trondheim to Östersund through the Central Scandes (Figure 5.2) is a typical area to study the Scandinavian Caledonides and has been investigated by many geologists and geophysicists in the past hundred years (Asklund, 1938; Dyrelius, 1980; Gee, 1975; Hurich, 1996; Hurich et al., 1989; Juhojuntti et al., 2001; Palm et al., 1991; Törnebohm, 1888). Based on their research, the COSC drilling project (Gee et al., 2010; Lorenz et al., 2015a, 2015b, 2011) was launched to study both the Caledonian nappes and the underlying Precambrian basement in western Jämtland (yellow square in Figure 5.2) with two fully cored boreholes supported by seismic, MT and other geological and geophysical surveys.

5.1.1 Geological background

The Scandinavian mountain belt consists of the Precambrian autochthon of the Fennoscandian Shield overlain by the Caledonian allochthons. The main décollement, which is associated with autochthonous Cambrian carbon-bearing black shales, separates the autochthon and allochthons. The former consists of granites, porphyrites, gneisses, migmatites and volcanic rocks. The latter is divided into four major groups based on their origin: the Lower, Middle, Upper and Uppermost Allochthons. The rocks in these four groups of allochthons are derived from the Baltoscandian platform, the Baltoscandi-
an rifted margin, the terranes of the Iapetus Ocean and the Laurentian margin, respectively (Figure 5.1; Corfu et al., 2014).

In our study area of western Jämtland (Figure 5.3; Gee et al., 1985), only the Lower and Middle Allochthons were developed and they are separated by a several meters thick zone of mylonites and phyllonites. The Lower Allochthon includes, from the bottom to the top, sedimentary rocks of low metamorphic grade, Cambrian black Alum shale, Ordovician limestones and Ordovician to Silurian greywackes (turbidites). The Middle Allochthon comprises sedimentary successions of higher metamorphic grade. The lowermost unit is the Offerdal Nappe with feldspathic psammites as its main component. Overlying the Offerdal Nappe is the Särv Nappe, which is dominated by metasandstones intersected by abundant dolerite dyke-swarms. Thrust over the Särv Nappe is the Seve Nappe Complex (SNC), which is characterized by amphibolites, gneisses, psammitic schists and migmatites. The Köli Nappe of the Upper Allochthon, comprising lower grade rocks of oceanic affinity, is finally thrust on top of the SNC to the west of our study area.

Figure 5.1. Position of the Scandinavian Caledonides in the Silurian and Early Devonian period (modified from Lorenz et al., 2015a).

The SNC is further divided into three units: the Lower, Middle and Upper Seve Nappes. The Lower Seve Nappe consists of similar rock types as in the underlying Särv Nappe, but with higher metamorphic grade the rocks are deformed in amphibolite facies (Gee et al., 2010). A mylonite zone indicates the transition from the underlying Särv Nappe to the Lower Seve Nappe (Arnbom, 1980; Lorenz et al., 2015a). The Middle Seve Nappe is represented by the Åreskutan Mountain, which is dominated by granulite facies migmatites and gneisses (Arnbom, 1980). These rocks show evidence for ultra-high pressure metamorphism (Klonowska et al., 2015; Majka et al., 2014). A
A ductile shear zone of 50 m thickness has been mapped as the boundary between the Lower Seve Nappe and the Middle Seve Nappe (Majka et al., 2012). The Upper Seve Nappe, which is only found farther north in the Scandinavia, consists of garnet micaschists and amphibolites (Gee et al., 2008).

Due to late orogenic deformation, all the nappes in the Scandinavian Caledonides are folded by several antiforms and synforms with mainly N-S-trending and tens of kilometres wavelengths (Figure 5.2). In western Jämtland, the Olden-Oviksfjäll Antiform is flanked by the Åre Synform in the west and the Offerdal Synform in the east (Figure 5.3).

Figure 5.2. Tectonostratigraphic map of Scandinavian Orogen and the cross-section through the Central Scandes from Trondheim to Östersund (modified after Gee et al. (2010). The yellow rectangle represents our measurement area. The first borehole COSC-1 is located near the town Åre.
Figure 5.3. Bedrock geological map of the study area (yellow rectangle in Figure 5.2) with the locations of the COSC-1 borehole (yellow star in the northwest, Lorenz et al., 2015a), the new broadband MT stations (blue dots) and the recent reflection seismic profile (CSP; continuous yellow lines; Hedin et al., 2012; Juhlin et al., 2016). In addition, the previous MT stations (green diamonds, Korja et al., 2008) and reflection seismic profile (yellow dashed line, Juhojuntti et al., 2001) along the CCT profile are also shown. The map is modified after Strömberg et al. (1984) and @ Geological Survey of Sweden [I2014/00601].
5.1.2 Geophysical background

Many geophysical investigations have been done in western Jämtland. In the 1980s, potential field data collection (Dyrelius, 1986, 1980), petrophysical sampling (Elming, 1980), seismic refraction survey (Palm, 1984) and MT (Agustsson, 1986) provided fundamental information of the crustal structures.

In the 1990s, to study the entire crust down to the Moho, a 250-km-long reflection seismic profile was conducted along the Central Caledonian Transect (CCT; yellow dashed line in Figure 5.3; Hurich et al., 1989; Juhojuntti et al., 2001; Palm et al., 1991) in the Tröndelag-Jämtland region in Norway and Sweden, followed by a 60 broadband MT station survey along the CCT profile in Sweden (green diamonds in Figure 5.3; Korja et al., 2008). The CCT seismic profile revealed that the crust is ~40-50 km thick with strong reflections in the uppermost 20 km and a gently westward-dipping reflection was interpreted as a sole thrust reaching a depth of ~7 km by the Swedish-Norwegian border (Palm et al., 1991). The resistivity model inverted from the CCT MT data showed a shallow resistive layer with thickness of ~2 km overlying a highly conductive layer to the west of the Caledonian front (CF). The conductive layer was interpreted as a combination of graphite-bearing alum shales (0.1–0.01 Ωm; Gee, 1972) and resistive rocks from the basement, and perhaps, also from of the Caledonian allochthons.

Moreover, airborne very low frequency (VLF) data was collected in western Jämtland by SGU, to study the near-surface structures (Figure 5.4). The apparent resistivity from the VLF data revealed that the most resistive rocks in this area are the SNC, followed by Silurian turbidites and then Ordovician turbidites.

In connection with the COSC drilling project, a high-resolution 2D reflection seismic profile was acquired in the Åre-Hallen area from 2010 to 2014 (CSP; yellow solid line in Figure 5.3; Hedin et al., 2012; Juhlin et al., 2016). Figure 5.5 shows the CSP image overlain by the geologic interpretations from Juhlin et al. (2016) with two possible locations of the main décollement. The first interpretation was based on part of the CSP and suggested by Hedin et al. (2012), in which the main décollement was interpreted to be located along the reflection with a depth of 4.5 km in the west. The second interpretation was based on both the MT data and the re-interpretation of the full CSP with a much shallow depth of the main décollement. The first borehole COSC-1, targeted the SNC. From the study by Hedin et al. (2012), the SNC was revealed to have a maximum depth extent of 2.5 km, which was also supported by the 3D inverse modelling of ground gravity and aeromagnetic data (Hedin et al., 2014). To provide more precise information around the COSC-1 borehole, 3D seismic data (Hedin et al., 2015) were acquired in 2014 after drilling was completed.
Figure 5.4. Apparent resistivity map of the airborne VLF data for the measurement area (courtesy of SGU) with the locations of our MT stations (small and big blue dots) and the boundaries between different geological units (black solid lines). Note that, the linear features in the VLF map (marked by black dashed lines) are caused by power lines. In addition, the VLF data shown in the figure was compiled from three datasets which were collected in different years, and thus the map is discontinuous across the lines W-E = 440 km and S-N = 6996.5 km (marked by arrows).
Figure 5.5. The COSC reflection seismic profile (CSP) overlain with geological interpretations (modified after Juhlin et al. 2016). The interpretation in a) is from Hedin et al. (2012) which is based on part of the CSP section; while the interpretation in b) is from Juhlin et al. (2016) which considers the resistivity models inverted from our MT data.
COSC-1, located near the town of Åre in Jämtland (yellow star in Figure 5.3), sampled a 2.5 km vertical section through ~2350 m of the Lower Seve Nappe and ~150 m of underlying lithologies (Figure 5.6; Lorenz et al., 2015a). The Lower Seve Nappe is dominated by gneisses and amphibolites. Below 1710 m depth, mylonite begins to occur with increasing thickness and frequency with depth. Below 2350 m depth, the core is dominated by lower-grade metasedimentary rocks. Aside from the core, a large number of geophysical, geochemical, petrophysical, hydrogeological and geothermal data were collected, including two types of borehole resistivity logs (normal resistivity logging and DLL; Figure 5.6). Generally, the DLL logs are one order of magnitude higher than the normal resistivities but the variations of the curves look similar. This phenomenon is quite normal because the current systems are different. The SP curve varies oppositely to resistivity logging curves.

Comparing the lithology and the resistivity logs, some conclusions can be drawn:

a. within the same rock type, resistivity curves (in particular the laterologs) vary strongly, indicating inhomogeneity of the rocks;

b. the first strong variations in the SP curve (600-800 m) seem to be related to the meta-gabbro units;

c. between 1710 and 2350 m depth, where mylonite bands occurs frequently, the SP curve oscillates significantly while the resistivities are very high, indicating that mylonite has higher resistivity than gneisses and amphibolites;

d. below 2350 m depth, the SP curve becomes smooth and the resistivity curves decrease, indicating that the rock unit becomes more homogeneous and the dominated lower-grade metasedimentary rocks have lower resistivity than mylonite.

COSC-2, still in the planning phase, is expected to penetrate the main décollement to study both the Lower Allochthon and the Precambrian basement. Based on an integrated geophysical-geological interpretation, COSC-2 has been proposed to be located at around 21 km along the CSP profile (Juhlin et al., 2016).

5.1.3 MT survey

The main décollement is dominated by Alum shale, which has very low resistivity (Gee, 1972). Hence, conducting an MT survey to map the resistivity structure down to several kilometres can help locate the main décollement. In 2013, MT data were collected at 83 stations along the CSP in western Jämtland (blue dots in Figure 5.3).
Figure 5.6. Logs of borehole COSC-1 with rock units identified from the retrieved core (Lorenz et al., 2015a, 2015b). Two types of borehole resistivity logs are available: one is the normal logging conducted by Lund University, consisting of long normal (purple curve) and short (light blue curve) logs with self-potential (SP, black curve); the other is the dual laterolog (DLL) resistivity conducted by ICDP, consisting of deep (LLD, dark blue curve) and shallow (LLS, red curve) laterologs. The Lower Seve Nappe is interpreted to end at 2350 m depth. Between 1710 m and 2350 m, rocks were increasingly mylonitised. At the bottom, the core is dominated by lower-grade metasedimentary rocks.
5.2 The Alnö district

As one of the largest (radius ~2.5 km) and earliest detected alkaline and carbonatite intrusions in the world (Figure 5.7; Kresten, 1990, 1980; Von Eckermann, 1948), the Alnö ring-complex in central Sweden offers a great opportunity to study carbonatites. Carbonatites are igneous rocks with a composition of more than 50% carbonate minerals (Le Maitre et al., 1989). They are of great relevance for the understanding of crustal and mantle processes, because the formation of the carbonatite melts have been reported to be related to the melt of the mantle, liquid immiscibility and crystal fractionation (Bell et al., 1998). Moreover, carbonatites have potential economic benefits, as rare earth elements (REE) are often concentrated in carbonatites and alkaline silicate rocks (Berger et al., 2009; Kanazawa and Kamitani, 2006).

5.2.1. Geological background

The Alnö complex was intruded into Paleoproterozoic migmatites and gneisses at the ages of 584±7 Ma (Meert et al., 2007). The main alkaline and carbonatite rocks in the complex are (in the order of the suggested emplacement): pyroxenites, ijolite series (melteigeite-ijolite-urtite), nepheline syenite and carbonatites (Kresten, 1990; Morogan and Woolley, 1988; Vuorinen, 2005). Based on the composition of the trace elements, the carbonatites in the Alnö area are divided into two types: sövite (white color and coarse grained) and alvikite (dark color and fine grained) (Le Bas, 1999). Inside the alkaline and carbonatite intrusion, carbonatite semicircular ring dykes exist with relatively large volumes of sövite. Surrounding the intrusion, the migmatitic wall rocks have been metasomatised to fenite (Morogan and Woolley, 1988).
5.2.2. Geophysical background

Kresten (1980) suggested the presence of a dome-shaped magma chamber, which has supplied the dipping radial dykes and cone-sheets, >1.5 km below the surface.
Three high-resolution reflection seismic profiles (black lines in Figure 5.7; Andersson et al., 2013) were conducted across the intrusion to unravel the structure of the Alnö complex at depth. Seismic data suggested the depth of the magma chamber to be ~3 km and the thickness to be less than 1 km. Moreover, up-doming-shaped reflectors on the seismic sections were interpreted as major weak zones which surround a caldera-related ring-type fault system.

Gravity and magnetic data (Figure 5.8) indicate that the main intrusion has higher density and higher magnetic susceptibility compared to the surroundings. Gamma radiation maps (Almgren et al., 2008) shows concentrations of uranium and thorium in the Alnö complex center. Airborne VLF data suggest that the shallow rocks have resistivities of between 570 Ωm to 8000 Ωm.

![Figure 5.8. Maps of potential data in the Alnö area: a) Bouguer gravity and b) magnetic anomaly (Andersson et al., 2013).](image)

5.2.3. MT survey

To get the resistivity distribution of the subsurface of the Alnö ring-complex, MT data were collected at 34 sites in 2013 (Figure 5.7). The stations were distributed so that 2D and 3D inversions could be conducted. To help the interpretation of the obtained resistivity models, resistivity laboratory measurements were performed on surface rock samples which were collected over various parts of the complex.

The processing and inversion of the MT data in the Alnö area is shown in paper III, together with the resistivity laboratory measurements.
6. Summary of papers

6.1 Paper I: A magnetotelluric investigation of the Scandinavian Caledonides in western Jämtland, Sweden, using the COSC borehole logs as prior information

In this paper, aside from the normal processing and inversion of the COSC MT dataset, the borehole resistivity logs from COSC-1 are set as prior information in the MT inversion, using the proposed borehole constrained inversion algorithm. Although the inverted resistivity model using borehole constraints from COSC-1 only exhibits small improvement in the vicinity of COSC-1, future constraints from COSC-2 are expected to lead to significant improvement of the model below the alum shale cover. Integrating the CSP section, airborne VLF data, information from COSC-1 and the surface geological map, the final resistivity model is interpreted. The most important finding from MT data study is that the boundary, which separates the overlying shallow resistive layer and the underlying conductive layer in the central and eastern parts of the profile, coincides with the first seismic reflections, indicating that the shallowest alum shales occur at less than 1 km depth. However, to determine the location of the main décollement, further research needs to be done.

6.1.1 Distortion analysis

After data editing, 59 sites are left. Bahr’s skew (Bahr, 1991), phase tensor skew (Caldwell et al., 2004) and the misfit values $\sqrt{Q}$ using Zhang et al.’s distortion model (e.g. Bastani et al., 2011; Smirnov and Pedersen, 2009) with fixed strike angles (no matter what the angle is), show big values for the western 13 sites and small values for the rest of the sites in the central and eastern parts of the profile. Moreover, for most of the sites with periods shorter than ~1 s in the central and eastern parts of the profile, the apparent resistivities and phases of $Z_{xy}$, $Z_{yx}$ and $Z_{DET}$ are almost the same and their polar diagrams are nearly circular. These demonstrate that the structure in the western part of the profile is 3D and in the rest of the profile it can be assumed to be approximately 1D at shallow depths and approximately 2D at greater depths.
Figure 6.1 shows the strike analysis using Zhang et al.’s distortion model (Zhang et al., 1987), Bahr’s equal phase approach (Bahr, 1991) and the phase tensor method (Caldwell et al., 2004). All three methods show almost the same strike angles, with N30°E as the preferred strike angle for the central and eastern parts of the profile at periods of more than ~1 s.

**Figure 6.1.** Distortion analysis using a) Zhang et al.’s distortion model (Zhang et al., 1987), b) Bahr’s equal phase algorithm (Bahr, 1991) and c) the phase tensor approach (Caldwell et al., 2004). In the left panels, the blue rose plots show all the strike results from the respective right panels; while the rose plots in black show only those strike angles marked in grey for periods of more than ~1 s with N30°E as the preferred strike angle. They are plotted separately, because dimensionality analysis shows 3D structure for the western 13 sites, whereas for the remaining sites it shows 1D structure for periods shorter than 0.1 s and 2D for periods longer than 0.1 s. d) shows the Q-misfit functions assuming N30°E as the strike direction, proving the result from dimensionality analysis.
6.1.2 2D inversion models

The determinant impedances are preferred in 2D inversions, because inverting directly from them can give robust 2D model even when 2D conditions are not exactly fulfilled (Pedersen and Engels, 2005). First, we perform normal inversion using the Occam inversion scheme with a homogeneous half space as the initial model. Second, the preferred Occam inversion model from this first inversion round is set as the initial model in a second inversion round, using a damped Occam inversion scheme. The resistivity model from the second round of inversion is shown in Figure 6.2.

Next, the deep laterolog from COSC-1 are set as borehole constraints in MT inversions. Similarly, two rounds of inversion are performed. The resistivity model after the second inversion round, using borehole logs as prior constraints, is compared with the normal inversion model in Figure 6.3. The difference between the two models only occurs around the COSC-1 borehole and in other places the constrained inversion result is the same as the normal inversion model. Thus, the interpretation is the same no matter which model is used as the final model.

Figure 6.2. The 2D resistivity model inverted from the determinant impedances using a damped Occam scheme. The red line in the west denotes the location of COSC-1 borehole.
6.1.3 Interpretation and conclusions

The constrained inversion model with deep laterolog as prior information is chosen as our preferred model and it is shown in Figure 6.4, overlain with the CSP section.

In the westernmost 10 km, where distortion analysis shows predominantly 3D structures, the model shows a resistor R1 in the top ~3 km depth. A very resistive feature in the airborne VLF data correlates with the SNC, as mapped in the bedrock geology, suggesting that the SNC is very resistive (the most resistive rock unit in the area). In the COSC-1 core, the lithologies of the SNC extend down to 2350 m, coinciding with a change in the reflective pattern observed in both the SNC and the 3D seismic data acquired post-drilling. The COSC-1 borehole does not fully penetrate the shear zone at the base of the Lower Seve Nappe and the 3D reflection seismics (Hedin et al., 2015) suggest that it may extend down to ~2.7 km depth beneath the COSC-1 drill site. The borehole resistivity curves from the COSC-1 borehole indicate the underlying rocks are more conductive than the SNC, consistent with the MT model. R1 is thus explained perfectly by the SNC.

In the central and eastern parts of the profile, the resistivity model shows a resistive layer L2 overlying a conductive layer L3 and the boundary between them correlates remarkably well with the shallowest reflection in the CSP section. Considering the surface geological map, Ordovician and Silurian greywackes are the most likely candidates for the resistive layer L2, consistent with the seismic interpretation by Hedin et al. (2012). Furthermore,
Silurian rocks are more resistive than Ordovician greywackes, as the resistivity in SL is higher than the rest of the profile, in agreement with the conclusion from the airborne VLF data. The high conductivity observed in the underlying layer L3 can be well explained by graphite-bearing alum shale (probably imbricated and mixed with rocks from the Caledonian allochthons and basement). However, the thickness of the alum shale is not resolved from the normal inversion MT model. The cause of the deep conductor DC is left to be discovered in the future.

The main décollement may be located either inside layer L3 (along a laterally continuous seismic reflector marked by a blue dashed line in Figure 6.4; based on the MT investigation and a re-interpretation of the CSP image by Juhlin et al., 2016) or along the deep seismic reflection at 4.5 km depth underneath the western end of the profile (red dashed line in Figure 6.4; from the seismic interpretation by Hedin et al., 2012). The MT model gives more support for the first interpretation, because the alum shales start to occur at less than 1 km depth. Since the upper part of the conductive layer L3 may be associated with the alum shales of the Lower Allochthon, it is natural to expect the main decollement to occur at somewhat greater depth inside L3 as it is associated with autochthonous Precambrian alum shale. In the MT model, the thickness of the conductive layer L3 is not resolved and the resistivity model loses resolution at more than 2 km depth in most of the areas. Hence, the location of the main décollement is not resolved based on the MT resistivity model alone. However, the lateral continuity of the seismic reflector marked by a blue dashed line in Figure 6.4 and the pattern of seismic reflectivity of the units immediately overlying this reflector, which resembles imbricated sedimentary units, offers strong support for a shallow main decollement.
Figure 6.4. The final preferred 2D resistivity model overlain with CSP section. The final model is computed from the determinant impedances with deep laterolog from COSC-1 as prior information. Sensitivity analysis result using Schwalenberg et al. (2002) method is represented by transparency (the partly transparent area has scaled total sensitivities between $10^{-4.5}$ and $10^{-5}$). R1 – SNC; L2 – units of the Lower Allochthon; SL – Silurian turbidites; L3 – alum shales (probably mixed with rocks from the Lower Allochthons and the basement); DC – undefined conductive structure in the basement. The location of the main décollement is not resolved, and it may be located along the blue dashed line or along the red dashed line.
6.2 Paper II: 2D magnetotelluric inversion using seismic information as constraints, application in the COSC project

As stated in paper I, the location of the main décollement was not resolved by the MT data alone. To further improve the resistivity model, in this paper we try to do constrained inversion of the COSC MT data with the CSP as prior information. First, we propose a new scheme of doing MT inversion with seismic constraints, which has been presented in section 4.4. Then, we demonstrate the efficiency of the scheme through two synthetic models. The choice of the newly introduced parameter for the constrained inversion is discussed. Moreover, experiments with wrong or shifted locations of the input constraints help us to evaluate the stability of the new method. Finally, applying the proposed scheme to the inversion of the COSC MT data with seismic constraints from the envelope of the CSP, an improved resistivity model is obtained.

6.2.1. Two synthetic models

Two synthetic models are tested: the first one is a four-block model with two resistors and two conductors (Figure 6.5); the second one is a synthetic COSC model which is based on the normal inversion model from the COSC MT data and other existing geophysical and geological information in western Jämtland. Both model tests show that the proposed constrained scheme is very efficient; almost all the structures (both shallow/deep resistors/conductors) can be resolved with correct locations of the boundaries. The effects of the constraints can be controlled by setting the value of the parameters $\frac{s_{ae,y}}{\beta}$ and $\frac{s_{ae,z}}{\beta}$ in equations 4.14 and 4.15, as described in section 4.4.2. If the input constraints are trustworthy, using big values (bigger than 1) of $\frac{s_{ae,y}}{\beta}$ and $\frac{s_{ae,z}}{\beta}$ can give a much improved resistivity model than normal inversions. However, too big values will result in the inversion to be unable to converge. On the other hand, with untrustworthy input constraints, small values of $\frac{s_{ae,y}}{\beta}$ and $\frac{s_{ae,z}}{\beta}$ should be given, otherwise, the constrained inversion process will force the resistivity contrasts to occur at the inaccurate boundaries and lead to worse data fit.
6.2.2. Application to the COSC MT data

Before being used as constraints in the COSC MT data inversion, the CSP envelope is edited to remove the noise and the coordinates are projected to match the corresponding MT stations (Figure 6.6a and 6.6b). Following equations 4.16-4.21, $S_{ae,y}$ and $S_{ae,z}$ are calculated (Figure 6.6c and 6.6d) with a stabilizing factor $\gamma = 0.002$.

To better match the CSP data, a new mesh with a finer spacing in the vertical direction than the mesh in paper I is used in this paper. Using regular smoothness constraints, the final model from a two-step inversion using an initial Occam inversion and a subsequent damped Occam inversion for this new mesh is shown in Figure 6.7a. In the seismically constrained inversion, different values of $\beta$ between 0.1 (the inversion does not converge with $\beta$ below 0.1) and 3 ($\beta$ should be smaller than 3 to ensure that the most important reflections affect the inversion) are tested. In the end, $\beta$ is set to be 0.5 in the final preferred model (Figure 6.7b), because the effects of the seismic constraints are sufficiently pronounced and the data fit is comparable with the normal inversion model.

The main difference between the two models in Figure 6.7 is that between 10 km to 50 km distance along the profile, the thickness of the conductive layer L3 is less than 1 km in the seismically constrained inversion model (Figure 6.7b), while in the regular inversion model (Figure 6.7a), the whole structure underneath the resistive layer L2 is totally conductive.
Figure 6.6. The envelope of the CSP section before a) and after b) data cleaning and coordinate shifting to match locations of MT stations. The calculated matrices $S_{ae,y}$ and $S_{ae,z}$ for weighting of smoothness constraints are plotted in c) and d).

Figure 6.7. a) damped Occam inversion model of the COSC MT data computed using regular smoothness constraints and b) damped Occam inversion model computed using the CSP envelope as input constraints.
6.2.3. Interpretation and Conclusions

Figure 6.8 shows the constrained inversion model in Figure 6.7b overlain by the CSP section with the same symbols used in the final model in paper I in Figure 6.4.

In the constrained inversion model, the thickness of the conductive layer is restricted to less than 1 km. Forward modeling tests show that setting the deeper structure more conductive brings worse data fits. Therefore, the constrained resistivity model supports the first interpretation in paper I, with a shallow main décollement (the blue dashed line in Figure 6.8), and the second interpretation (the red dashed line in Figure 6.8) would be ignored, if the resistivity-reflectivity relationship is good in the research area. Moreover, the proposal to put COSC-2 at around 21 km along the CSP profile is supported from the constrained MT resistivity model as well. COSC-2 would penetrate through Silurian greywackes (SL), Ordovician rocks (L2), the main décollement in L3 and reach the Proterozoic basement.

The proposed constrained algorithm improves the MT resistivity model efficiently by adding constraints from seismic envelope data. The choice of the newly introduced stabilization parameter $\beta$ is simple. The scheme can easily be implemented in existing 2D or even 3D inversion programs. Furthermore, the application is very broad, as it is common that reflection seismic (or GPR) data is available in the same area where MT (or other EM) data is collected.
Figure 6.8. The seismically constrained 2D resistivity model overlain with the CSP section. The symbols are the same as in Figure 6.4. The constrained resistivity model gives more support to the first interpretation of the location of the main décollement in paper I (the blue dashed line) and the current proposal of placing the COSC-2 drill site around 21 km along the profile.
6.3 Paper III: 3D magnetotelluric modelling of the Alnö alkaline and carbonatite ring complex, central Sweden

In this paper, the MT data of the Alnö ring-complex are processed. The edited impedance tensors have periods between 0.003 s and 10 s. Strike and dimensionality analyses from the impedance tensors and induction arrows from the tipper suggest a 3D structure in this area. 3D inversion of the MT data is conducted, together with 2D inversions of determinant impedance data collected along three profiles coinciding with the seismic lines. To help interpret the inversion models, resistivities and porosities of the rock samples are measured in the laboratory.

6.3.1 Distortion analysis

Maps of the surface geology, gravity and magnetic anomalies (Figures 5.7 and 5.8) suggest a 3D structure for the Alnö intrusion. For the MT data, the three distortion analysis methods presented in section 2.4 are used, together with induction arrows. Both Bahr’s 3D/2D skew (Bahr, 1991) and the phase tensor skew (Caldwell et al., 2004) show big values, indicating that the structure cannot be approximated in 2D. The calculated strike angles vary from 30° west of north to 50° east of north and no regional strike direction could be determined.

6.3.2 2D and 3D inversion models

As 2D inversion of the determinant impedance can get robust features of the resistivity distribution (Pedersen and Engels, 2005) when the structure is 3D, MT stations next to the three seismic lines (Alnö1-3) are selected to perform 2D inversions using the determinant impedances. The 2D inversion resistivity models are shown in Figures 6.9a-c, with three big conductors BC1-3 dominating the main intrusion at <3000 m depth.

Full impedance tensors are used in the 3D inversion using the software WSINV3DMT (Siripunvaraporn et al., 2005). The depth slices of the 3D inversion resistivity model at six depths (80m, 200m, 800m, 1400m, 2500m and 4500m) are shown in Figure 6.10. In the 3D resistivity model, major conductors (red area in Figure 6.10) occupy the center at depths between ~800 m and ~3000 m and they are surrounded by an arch-shaped resistor (R2) in the northeast.

To compare with the 2D inversion models, corresponding cross-section are extracted from the 3D inversion model (Figures 6.9d-f). Comparing between Figures 6.9a-c and Figures 6.9d-f, the major conductors (BC1-3) are
located at similar positions in the 2D and 3D models. However, the 3D inversion model contains more features at shallow depths.

Figure 6.9. Resistivity models from 2D inversion of the determinant impedances (a, b and c) along the seismic sections and corresponding cross-sections from the 3D inversion model (d, e and f).
6.3.3 Laboratory resistivity measurements

Thirty rock samples of typical rock types (red diamonds in Figure 5.7) were collected. Subsequently, they were cut into cubes (~50 mm length) and fully saturated by water before their resistivities were measured. To get statistical data, each rock was measured twice along three directions with nine different frequencies. The effects caused by anisotropy and the applied frequencies could be neglected, comparing with the difference between each rock types. The measurement results for all the rock types are shown as box and whisker diagrams in Figure 6.11. Alvikite and nepheline syenite are the most resistive rock types; while sövite, pyroxenite and fenite are the most conductive rocks. The country rocks (migmatite and granite) have intermediate resistivities.

Besides the compositions of rocks, the resistivity of rocks is determined by porosity, pore content and degree of saturation, according to Archies’s law (Archie, 1942; Glover, 2016; Palacky, 1988). To identify which parame-
ter controls the resistivity of each rock type, relative porosities are measured together with density. The sequence of rock type with porosity from low to high is: alvikite, pyroxenite/ijolite/nepheline syenite, granite/migmatite, fenite and sövite.

Generally, the resistivity decreases with increasing porosity. But pyroxenite is an exception. Pyroxenite rocks consists of mainly pyroxene, which is dominated by $Fe^{+2}$ and $Fe^{+3}$. Therefore, pyroxenite is very conductive, even though the porosity is very low. The most dense rock types are pyroxenite, ijolite and alvikite, consistent with the result in Andersson (2015).

![Graph showing resistivity of different types of rocks](image)

*Figure 6.11.* Laboratory resistivity measurements of the typical rock types. For each box, from top to bottom, the values of the maximum, the upper quartiles, the median, the lower quartiles and the minimum are denoted.

### 6.3.4 Interpretation and conclusions

Figure 6.12 shows the cross-sections from the 3D inversion model in Figures 6.9d-f, overlain by corresponding reflection seismic sections from Andersson et al. (2013).

For the major conductors, BC1 correlates with the up-doming reflectors and BC2 and BC3 are associated with the edge of the caldera center. As porosity is the key factor of the resistivities of the rocks in the Alnö area and the most conductive rock type is the coarse carbonatite (sövite), the major conductors (BC1-3) are interpreted to be related to the fracture and fault systems (possibly in combination with the coarse-grained carbonatite ring-dyke systems). The arch-shaped resistor (R2) in the northeastern part might be caused by a large volume of nepheline syenite rocks, because the other outcropping rock types in this area (coarse-grained carbonatite, fenite and pyroxenite) are very conductive. Considering the surface geological map, the shallow resistors SR1 and SR2 can be explained by nepheline syenite and
ijolite rocks; while the shallow conductors SC1 and SC2 might be caused by fenite possibly in combination with sövite.

The presence of a solidified magma chamber at a depth of about 3-4 km, which is suggested from the seismic data, is supported by the MT data. In the MT inversion resistivity models, the area of the magma chamber is dominated by a moderate resistor surrounded by ring-shaped conductors. This moderate resistor can be explained well by a solidified magma chamber which has very low porosity. However, the MT data suggest a smaller lateral extent of the magma chamber than what Andersson et al. (2013) suggested based on the seismic data.

Integrating previous geophysical and geological data and resistivity measurements in the laboratory, the resistivity models inverted from the MT data provide complementary information for the Alnö ring-complex. The MT models depict the magma chamber in the center as moderate resistor and the surrounding and overlying caldera-related ring-type fault system as conductors.
Figure 6.12. Cross-sections of the 3D resistivity model overlain with the seismic images from Andersson et al. (2013).
7. Conclusions and outlook

7.1 Conclusions

This thesis presents two newly proposed constrained MT inversion algorithms and studies of the applications of MT in Åre and Alnö districts. The overall conclusions drawn are listed in the following with respect to the methodologies and the results of the MT investigations in the two districts.

The algorithm for using borehole resistivity logs as constrains in MT inversion works very well. Although borehole constraints from COSC-1 only produces small improvement in the vicinity of COSC-1 in the inverted resistivity model, synthetic tests show that future constraints from COSC-2 are expected to lead to much more significant improvement.

The algorithm for using seismic data (envelope) as constraints in MT inversion improves significantly the inversion models of both synthetic examples and the COSC MT data. The basic idea behind this method is similar to other’s approaches (Brown et al., 2012, Zhou et al., 2014 and Wiik et al., 2015), but the newly algorithm is more widely applicable. The choice of the introduced stabilization parameter is simple and the scheme can be easily adopted in other inversion programs or problems (e.g. constraining resistivity models computed from ERT data using GPR images)..

The three important tools for strike and dimensionality analyses used in this thesis (Zhang et al., 1987; Bahr, 1991; and Caldwell et al., 2004) produce similar result in the two MT investigations, because theoretically they treat the distortion in similar ways.

In the Åre district, the most important finding from MT investigation is that the first conductive alum shales layer occurs at less than 1 km depth in the central and western parts of the profile. Moreover, the top of the conductive layer coincides well with the first seismic reflections. The MT resistivity model supports the new interpretation by Juhlin et al. (2016) that the main décollement may be located several hundred meters below the top of the conductive layer along a laterally continuous seismic reflection. However, the depth of the conductor was not resolved in normal 2D MT inversion, leaving the old interpretation by Hedin et al. (2012) as an alternative option, in which the main décollement is located much deeper. The resistivity model improved by using constraints from the reflection seismic data (envelope)
gives further support to the new interpretation of the main décollement. From the MT and seismic investigations in this area, placing the second borehole COSC-2 around 21 km along the CSP profile would allow for the possibility to study Silurian greywackes, Ordovician rocks, the main décollement and the Proterozoic basement.

In the Alnö area, laboratory measurements showed that the resistivity decreases with increasing porosity for most rock types except for pyroxenite. Sövite (coarse-grained, white color) and alvikite (fine-grained, dark color) carbonatites are the most conductive and the most resistive rock types, respectively. The MT models depicted the magma chamber in the center as moderate resistor and the surrounding and overlying caldera-related ring-type fault system as conductors.

7.2 Outlook

The proposed constrained MT inversion algorithms can be applied widely in other research areas, especially the scheme of using seismic data as constraints. Resistivity models might get improved and thus help the interpretation in those areas. Since we implemented reflection seismic and borehole logging constraints in the object-oriented inversion code Emilia (Kalscheuer et al., 2010), both approaches can be directly employed also in the 2D inversion of ERT data and the 2D joint inversion of RMT and ERT data. Obviously, also 2D inversion of ERT data using structural constraints from GPR images is directly possible. It can be expected that such approaches will be employed to study geo-engineering, geo-hazard, hydrological (including saltwater intrusion and contaminant transport) and tectonic problems.

It might be worth to compare the three tools for dimensionality, distortion and strike analyses presented in this thesis with another widely used tool (McNeice and Jones, 2001) which is based on Groom and Bailey’s (1989) approach.

In the Åre district, more research needs to be performed. Our next step is to perform 3D MT inversions to determine the 3D resistivity distribution in western Jämtland, especially around borehole COSC-1. Improved geological understanding in this area is expected to be obtained. Moreover, before the start of drilling, collecting more MT data around the proposed location of the second borehole COSC-2 (around 21 km along the CSP profile) would further help to ensure that COSC-2 penetrates the main décollement.

In the Scandinavian Caledonides, many MT stations have been deployed (Figure 7.1a; Cherevatova et al., 2015a, 2015b, 2014). Comparing the resistivity distributions of all these cross-sections would provide more information to understand the transportation of the Scandinavian Caledonides.
The airborne VLF data which is available for most of the area in Scandinavia could also play an important role in the study.

Moreover, as was stated in section 5, the movement of the Caledonian allochthons is similar to that recognized in the Himalayas (Dewey, 1969; Gee et al., 2010; Labrousse et al., 2010). Many MT investigations have been performed in the Himalayas (Figure 7.1; Unsworth, 2010; Zhang, 2016). Thus, it is worthwhile to collect all the research (not only MT, but also all the other geophysical and geological research) in these two areas and compare the resistivity distributions of the crustal and upper mantle structures in these two big mountain chains. However, this work would require extensive international collaboration.

*Figure 7.1. Coverage of a) Scandes and b) Himalayas with MT stations (modified after Cherevatova et al., 2015a, 2015b; Unsworth, 2010; Zhang, 2016). In b) the yellow and red circles denote the MT stations conducted within the SinoProbe Project between 2010 and 2013 (1099 stations in total); the rest circles represent the MT stations conducted before 2010.*
Denna doktorsavhandling behandlar huvudsakligen tillämpningar av magnetotellurik (MT) vid två platser i Sverige: Åre och Alnö. Utöver MT-mätningar så har ytterligare ett antal geologiska och geofysiska dataset insamlade i samma områden använts; till exempel seismik, tyngdkraft, magnetdata, VLF samt litologiska data och resistivitet från borrhål. Det viktigast bidraget är dock att två inversionsalgoritmer för MT, med resistivitetelogning från borrhål och seismisk data som a priori information, presenteras. Förbättrad tolkning har erhållits genom studier av MT-data från de två platserna.

Mätningarna av MT i Åre-området genomfördes i samband med det vetenskapliga djupborrprojektet ”Collisional Orogeny in the Scandinavian Caledonides” (COSC) (Gee et al., 2010; Lorenz et al., 2011, 2015a, 2015b). Syftet med COSC är att studera de Kaledoniska skällorna och det underliggande prekambriska urberget i centrala Sverige. Syftet med att mäta MT i detta området var att hjälpa till att hitta den primära överskjutningszonen mellan skällorna och urberget, eftersom det är känt att detta är associerat med alunskiffer som har hög elektrisk konduktivitet (Gee, 1972). Artikel I och II är baserade på analys utförd på MT-data COSC.

Normal processering av MT-data med dimensionalitet- och strykningsanalyser samt 1D- och 2D-inversion genomfördes och presenteras i artikel I. Eftersom data från resistivitetelogning av borrhålet COSC-1 fanns tillgängligt så föreslås i artikeln en MT-inversionsalgoritm som använder borrhålsresistivitet som a priori information. Resistivitetsmodellen från inversionen visade bara förbättringar nära borrhålet COSC-1, men inkludering av borrhålsdata från borrhålet COSC-2 förväntas ge signifikanta förbättringar av modellen under alunskifferlagren. Det viktigaste resultatet från analysen av MT-data i artikel I är att gränsen mellan det överliggande ytliga resistiva lagret och det underliggande elektriskt ledande lagret i de centrala och östliga delarna av profilen sammanfaller med den första seismiska reflektorn. Detta indikerar att den ytligaste alunskiffeln ligger på mindre än 1 km djup. Resistivitetsmodellen från MT styrker tolkningen att den primära överskjutningszonen kan vara lokalisert inom det elektriskt ledande lagret, flera hundra meter under denna övre gräns. En annan tolkning är möjlig, enligt vilken den primära överskjutningszonen återfinns längs en djupare seismisk reflektor på 4.5 km djup under den västliga delen av profilen (detta är base-
rat på den seismiska tolkningen av Hedin et al., 2012). Den senare tolkningen ska inte förkastas då djupet till det elektriskt ledande lagret inte lösts och alunskiffer kan också existera i den Undre Alloktonen. Vidare analyser av COSC MT-data måste utföras för att utröna placeringen av den primära överskjutningszonen.


MT-mätningar genomfördes i Alnö-området för att skapa en modell av hur resistiva kroppar är distribuerade i alkalin- och karbonatitkomplexet på norra Alnön utanför Sundsvall. Dimensionalitets- och strykningsanalyser påvisar 3D strukturer i komplexet. 2D och 3D MT-inversioner utfördes och resultaten jämfördes. Som stöd för tolkningen av resistivitetsmodellerna så genomfördes också resistivitetsmätningar i laboratorium på bergartsprover insamlade från hållar i området. Relativ porositet och densitet mättes också på proverna. Mätningarna i laboratorium visade att resistiviteten ökar vid ökad porositet för alla testade bergarter utom pyroxenit. Grovkornig vit karbonatit (sövit) och finkornig mörk karbonatit (alvikit) är de bergarter med högst respektive lägst elektrisk ledningsförmåga. MT-modellerna ger en bild av den stelnade magmakammaren i centrum som en moderat resistor och det omgivande och överliggande kaldera-relaterade ringformade förkastningssystemet som en elektrisk ledare. Denna studie presenteras i artikel III.

More than four years ago, when I was writing my application to the China Scholarship Council (CSC), I had no clue how the PhD life in Uppsala University would be. Now I am going to submit my thesis and defend my PhD. Time flies so fast. Looking back at the four years, I can say that applying for a PhD in Uppsala University in Sweden is the most right decision I have made in my life. It is my good fortune having the chance to learn from intelligent supervisors and to work with wonderful colleagues. Over the four years, I have received various supports and help from people both inside and outside the department. It is my honor to express my gratitude to all of them.

I would like to thank my main supervisor Thomas Kalscheuer. You are really an excellent supervisor with strong knowledge. You have guided me throughout in the last two years so that I could have headed in the right direction. I cannot remember how many times when we sat together and you took a paper and explained patiently to me all those hard stuff. You are so intelligent with lots of ideas. Whenever I encountered a problem in my research, you could always come up with an idea to solve it. I am also grateful for your help with the revisions of the manuscripts.

I am very thankful to my second supervisor Laust Pedersen, who accepted my application and provided me an interesting research direction. Thank you for your guidance and support, especially in the first two years. I am always impressed by how wise you are. Your expertise and enthusiasm has always been an inspiration to me. The EM group people are at ease and feel supported having you by side.

I am indebted to María García Juanatey, who became my third supervisor. I had very good memories working with you in the field. Thank you for investing a lot of time in discussions and sharing your experience and scripts in MT data processing. You have always been a warm person with very good skills for writing and communication, which I have always admired.

Christopher Juhlin and Alireza Malehmir are very much thanked for providing interesting projects and for giving useful comments for the manuscripts. I am grateful to Ari Tryggvason for providing a nice office so that I could have enjoyed the quite time in writing the thesis and manuscripts. Maxim Smirnov is thanked for sharing his data processing and plotting programs and for his patience in replying all the questions. Special thanks to Chunling Shan, Peter Hedin, Magnus Andersson and Bjarne Almqvist for your excellent co-work and for proof reading the thesis. Magnus is also very
much thanked for letting me keep his sew machine for years. Peter is also thanked for reading and recording Swedish for me. Gratitude also goes to Jochen Kamm, Suman Mehta, Théo Berthet, David Gee and Henning Lorenz for many useful discussions.

I would also like to thank Harbe, Karin, Angeliki, Ershad and Milad for sharing the office. Thank you for the nice memories with lots of talks and laughs. Emil Lundberg, Tomas Nord and Simon Östling are thanked for helping me with solving computer problems. Lutz Kübler from SGU is especially thanked for his technical and instructional assistance with the equipment when I was measuring the resistivities. It was really a pleasure to work with Kristina Juhlin and Josefin Andersson in the field. You girls showed me how to be a female in Sweden: live with passion, train to be strong and work hard and independently.

Thanks to all the Chinese friends in the department: Fengjiao Zhang, Fei Huang, Shunguo Wang, Lichuan Wu, Le Gao, Fan Zheng, Lanyun Miao, Hongling Deng, Jianliang Wang, Lei Liu, Liang Tian, Zhibing Yang, Zhiliang Zhang, Peng He and Peng Yi. To me, you are my family in Sweden. I will always miss those lunches, dinners and trips we had. I cannot thank you enough for your friendship and various help with both my PhD study and personal life.

I would like to thank dear Pianpian, Na, Lily, Marsha, Jiejie and Ge for your precious friendships. Without your company, it could have been hard to be alone here for four years. Alice, Jie, Li, Beichen, Xiaoqing, and Tianqing, it is really my honor to be your roommate. Living with you for more than one year gave me the chance to see how great personalities you have. Thank you for all the precious time. Thank you Shihuai for giving me food when I was too lazy or busy to cook, for so many times! Special thanks go to Chenyu and Mingyu for helping me move. Of course, Yanshuang, thank you for helping draw the cover of the thesis and for providing very helpful medical knowledge. You are so humor and funny and will be the best doctor wherever you will be.

I am extremely thankful to Bianca, Hugo, Johan, Rickard, Marcus, Marian and Irina for practicing Swedish with me, for teaching me how to cook typical Swedish food and for telling me interesting stuff. You guys have always been so patient even though my speaking is poor. I wouldn’t have fallen in love with Swedish without you.

CSC is acknowledged for financially supporting my PhD study. I would also like to extend my deepest gratitude to Prof. Yangang Wu, who was my supervisor when I was doing my master program in Jilin University. Thank you for your support and love.

Last but not least, I would like to thank my parents and brother for your unconditional love and support during the last four years. Although we are 7600 km away, I know you are always there and supporting me. Thank you for believing in me and giving me freedom to pursue my dreams whatever they are.
References


A doctoral dissertation from the Faculty of Science and Technology, Uppsala University, is usually a summary of a number of papers. A few copies of the complete dissertation are kept at major Swedish research libraries, while the summary alone is distributed internationally through the series Digital Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology. (Prior to January, 2005, the series was published under the title “Comprehensive Summaries of Uppsala Dissertations from the Faculty of Science and Technology”.)